Transverse momentum-like resolution variables and power suppressed contributions

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Outline

- Slicing method
- q_T - subtraction
- Exploring different jet resolution variables: $Y_{N,N+1}$, k_T^{ness}, k_T^{FSR}.
- Power corrections
- Application to $e^+e^- \to 2j$ @ NLO
- Conclusions and outlook

Introduction

- Phase space integrals for higher-order corrections to cross-sections have **infrared divergences** that have to be regularized.

- There are many different subtraction schemes for handling IR divergences. We will focus on **slicing/non-local subtraction.**

- The idea of the slicing method @ N^kLO is to define a **resolution variable** X such that:

  1. In the region $X > 0$ we have **1 resolved emission**, there are only $N^{k-1}LO$-type singularities.

  2. The N^kLO unresolved limits occur **only at** $X = 0$.

Slicing method

- For a process with n jets at the Born level we can **split** the N^kLO correction to cross section **using the resolution variable**:

$$\int d\sigma_{N^kLO} = \int d\sigma_{N^kLO}\,\theta(r_{cut} - X) + \int d\sigma^{R}_{N^{k-1}LO}\,\theta(X - r_{cut})$$

- We can **approximate the integral** in the unresolved region by taking the **soft and collinear limits**:

$$\int d\sigma_{N^kLO}\,\theta(r_{cut} - X) = \int d\sigma^{sing}_{N^kLO}\theta(r_{cut} - X) + \mathcal{O}(r^{\ell}_{cut}) = \int \mathscr{H} \otimes d\sigma_{LO} - \int d\sigma^{CT}_{N^kLO}\,\theta(X - r_{cut}) + \mathcal{O}(r^{\ell}_{cut})$$

- The N^kLO cross-section is then:

$$\int d\sigma_{N^kLO} = \int \mathscr{H} \otimes\, d\sigma_{LO} + \int \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}\right]_{X>r_{cut}} + \mathcal{O}(r^{\ell}_{cut})$$

- The computation is performed using a **small but finite** value of r_{cut}. This means that the size of power correction affects the performance of our method.

q_T - subtraction (0-jet case)

- q_T can be used as resolution variable for processes that **do not involve jets** at the Born level. [Catani, Grazzini (2007)]

- It can distinguish the **transition $0 \to 1$ jet**.

- q_T - subtraction has been applied for the production of **color-singlet and heavy-quark @ $NNLO$.** MATRIX: [Grazzini, Kallweit, Wiesemann (2017)][Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019, 2020)]

- It has been applied @ N^3LO **for Drell-Yan** [Chen, Gehrmann, Glover, Huss, Yang, Zhu (2021)][Camarda, Cieri, Ferrera (2021)][Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli (2022)][Neumann, Campbell (2022)] and **Higgs** [Billis, Dehnadi, Ebert, Michel, Tackmann (2021)] production.

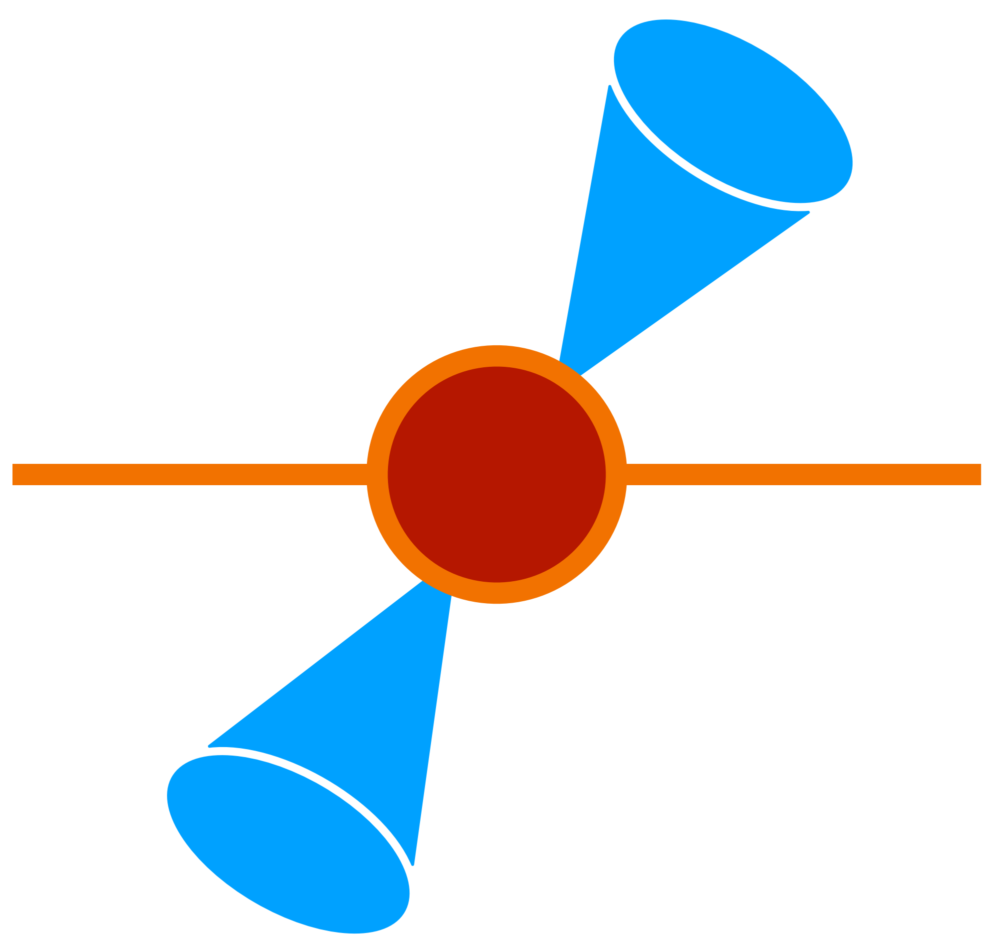
- **Drawback:** it cannot regularize final-state **collinear** singularities, i.e. it cannot distinguish the transition $N \to N+1$jets for $N \neq 0$.

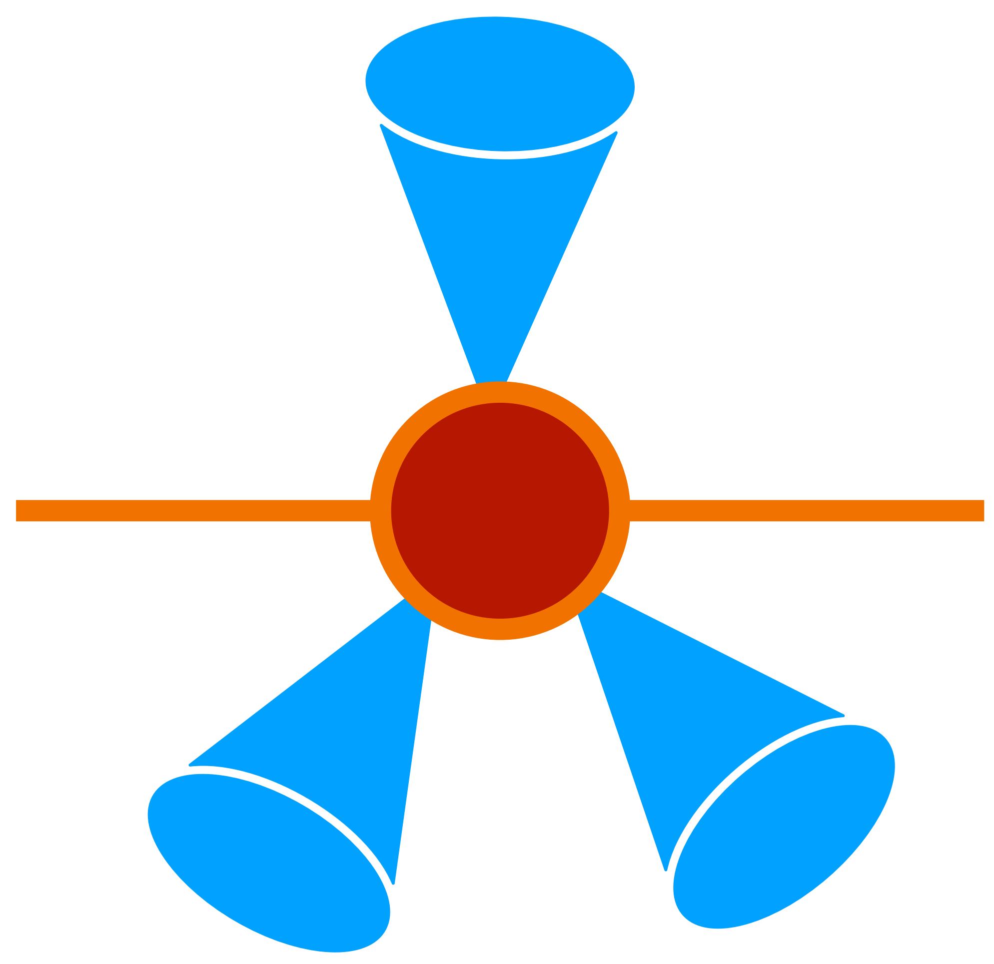
q_T - subtraction (0-jet case): power corrections

- Various analytical calculations confirmed **quadratic power corrections** ($r_{cut}^2 \log r_{cut}$) of q_T subtraction for colourless final-state.
- However, power corrections **may get worse** if we apply fiducial cuts or if we consider an emission from massive final-state quarks.
- **2-body fiducial cuts**: linear power corrections in r_{cut}. Examples: symmetric cuts on Drell-Yan, Higgs 2-body decay.
- **Photon isolation**: linear and logarithmic enhanced power corrections: proportional to r_{cut}, $r_{cut} \log r_{cut}$. Examples: vector boson pair production involving photons.
- **Massive final-state emitters**: linear power corrections in r_{cut}. Example: heavy quarks, NLO EW, mixed QCD-EW to Drell-Yan.

N-Jet resolution variable

- An N-jet resolution variable has to smoothly capture the transition **from N to $N+1$ jet configuration**.





- The first proposal for such a variable is **N-Jettiness (τ_N).** [Stewart, Tackmann, Waalewijn (2010)]
- It has been successfully applied as resolution variable in hadronic collisions for processes with **1 jet up to NNLO** [Boughezal, Focke, Giele, Liu, Petriello (2015)] [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (2016)].
- N-Jettiness exhibits **linear logarithmic enhanced power corrections** already @ NLO.

Exploring jet resolution variables

- We want to explore other jet resolution variables.
- We would like to have **linear** or even **quadratic power corrections**.
- We want to investigate **why** resolution variables **have different power correction scaling**.
- In this talk we will discuss and compare **three jet resolution variables** for e^+e^- collisions: Y_{23}, k_T^{ness}, k_T^{FSR}.

$Y_{N,N+1}$ resolution variable

Definition of the variable

- From now on we will focus on e^+e^- collisions.
- Consider the (dimensionless) distance between final-state partons and among final-state partons and the beam normalized to $Q^2 = (p_a + p_b)^2$:

$$d_{ij} = 2\, min(E_i^2, E_j^2)\frac{(1 - \cos\theta_{ij})}{Q^2}$$

$$d_{iB} = \frac{p_{i,T}^2}{Q^2}$$

- Run the k_T jet-clustering algorithm until $N+1$ proto-jets are left.
- When $N+1$ proto-jets are left, $Y_{N,N+1}$ is the square root of the minimum among all the d_{ij} and the d_{iB}:

$$Y_{N,N+1} = \sqrt{\min_{i,j}\{d_{ij}, d_{iB}\}}$$

- In the collinear limit this variable coincides with the transverse momentum w.r.t. the collinear axis.

$Y_{2,3}$ - subtraction

Counterterm and finite piece, $e^+e^- \to q\bar{q}@NLO$

- The counterterm can be obtained considering the divergent part of the integral over the radiation variables of the real matrix element below the cut in the collinear limit:

$$8\pi\alpha_s\mu^{2\epsilon}|\mathscr{M}_B|^2\int d\phi_{rad}\frac{1}{s_{ij}}P_{qq}(z,\epsilon)\theta(r_{cut}-Y_{2,3}) = -\,d\sigma^{CT} + \text{finite term} + \epsilon\text{-poles} + \mathcal{O}(r_{cut})$$

$$d\sigma^{CT} = d\sigma_{LO}\frac{\alpha_S}{\pi}C_F\left(2\log^2 r_{cut} + 3\log r_{cut}\right)$$

- To obtain the correct finite piece we have to add the **soft wide-angle contribution**:

$$8\pi\alpha_S C_F|\mathscr{M}_B|^2\int d\phi_{rad}\left[(-\mathrm{T}_1\cdot\mathrm{T}_2)\,\omega_{12} - \mathrm{T}_1^2\omega_1 - \mathrm{T}_2^2\omega_2\right]\theta(r_{cut}-Y_{23}) = |\mathscr{M}_B|\,C_F\frac{\alpha_s}{2\pi}\frac{\pi^2}{6}$$

$$\omega_1 = \frac{p_1\cdot p_2}{(p_1\cdot k)(p_1+p_2)\cdot k} \qquad \omega_2 = \frac{p_1\cdot p_2}{(p_2\cdot k)(p_1+p_2)\cdot k} \qquad \omega_{12} = \frac{p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \qquad \omega_{12} = \omega_1+\omega_2$$

- Since the soft wide angle contribution does not vanish, we expect a linear scaling.

$Y_{2,3}$ - subtraction

Power corrections

- Slicing using Y_{23} variable has **linear power correction** in r_{cut}. This can be seen by analytically integrating the real matrix element (not approximated) above the cut:

$$d\sigma_{LO} C_F \frac{\alpha_S}{2\pi} \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \theta(Y_{23} - r_{cut}) \qquad x_i = \frac{2E_i}{Q}$$

- We computed this integral obtaining the complete dependence on r_{cut} of the power corrections.

- The final result is a large expression involving function of r_{cut} that contains logarithms up to weight 2. Power corrections up to $\mathcal{O}(r_{cut})$ are:

$$d\sigma_{LO} C_F \frac{\alpha_S}{\pi} \left(\frac{5}{4} - \frac{\pi^2}{12} + 3\log 2 + 3\log r_{cut} + 2\log^2 r_{cut} + (2\, arcsinh(1) - 4\sqrt{2}) r_{cut} + \mathcal{O}(r_{cut}^2) \right)$$

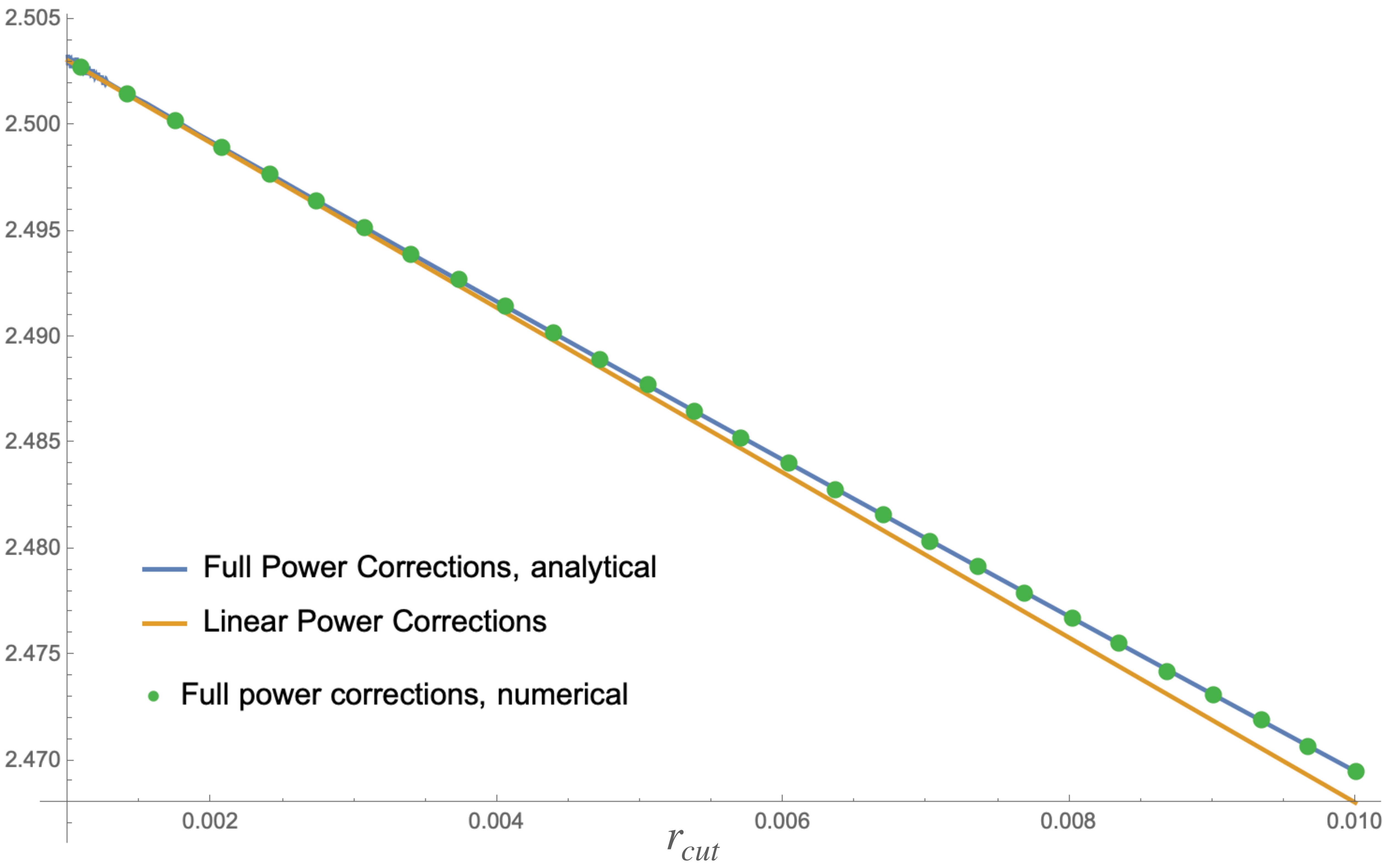
Cancelled by the counterterm

Linear Power Correction

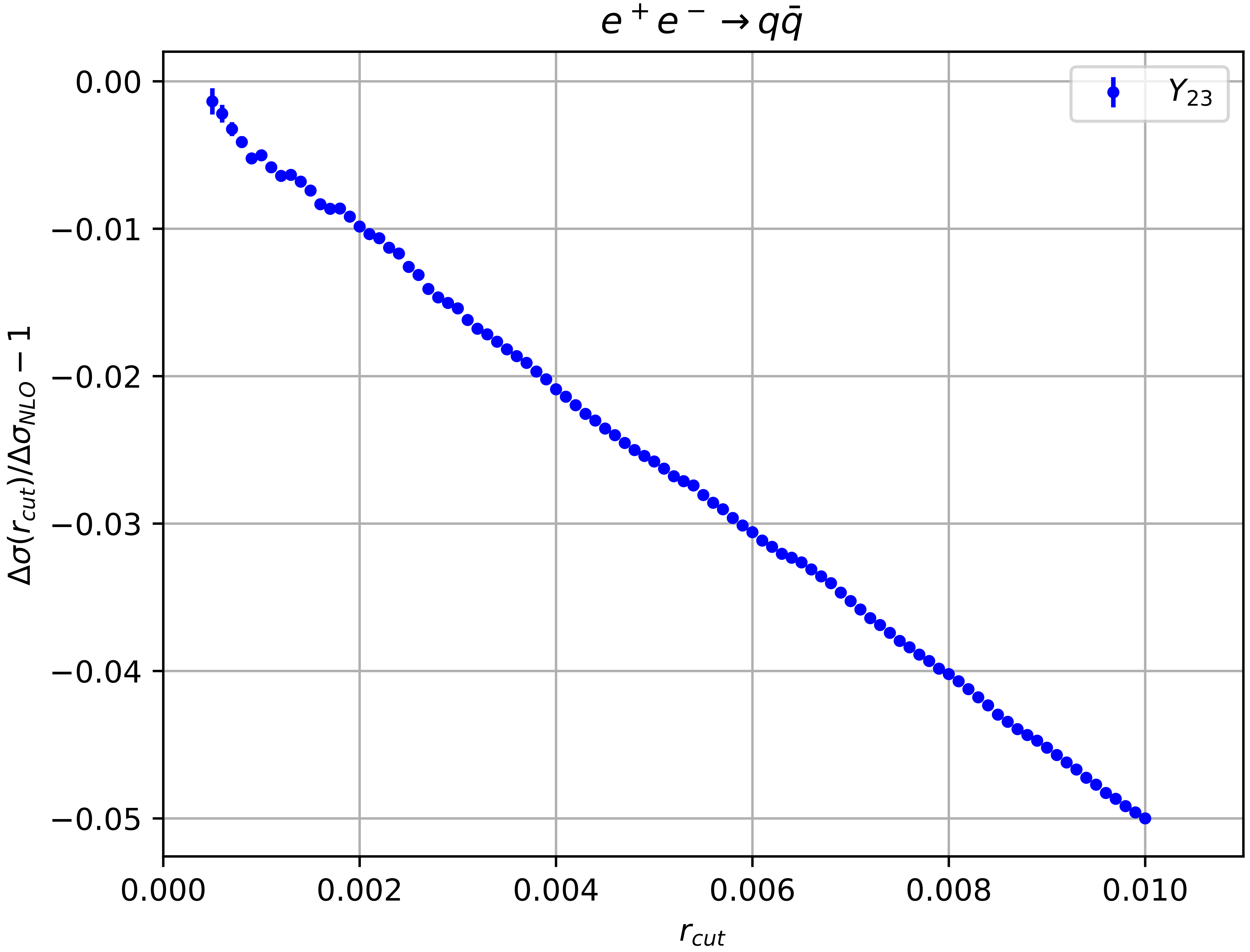
$Y_{2,3}$ - subtraction

Power corrections

$$\left(\int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \theta(Y_{23} - r_{cut}) \right) - 3 \log r_{cut} - 2 \log^2 r_{cut}$$



Numerical application



k_T^{ness} resolution variable

- The definition of k_T^{ness} is similar to the one of $Y_{2,3}$ but we use the distance among partons:

$$d_{ij} = \min(k_{i,T}^2, k_{j,T}^2)\frac{\Delta R_{ij}^2}{Q^2} \qquad \Delta R_{ij}^2 = \Delta\eta_{ij}^2 + \Delta\phi_{ij}^2$$

- Since $Y_{2,3}$ **and** k_T^{ness} **coincide in the collinear limit**, the counterterm is the same as the one for $Y_{2,3}$.

- We have a non-vanishing soft wide-angle contribution:

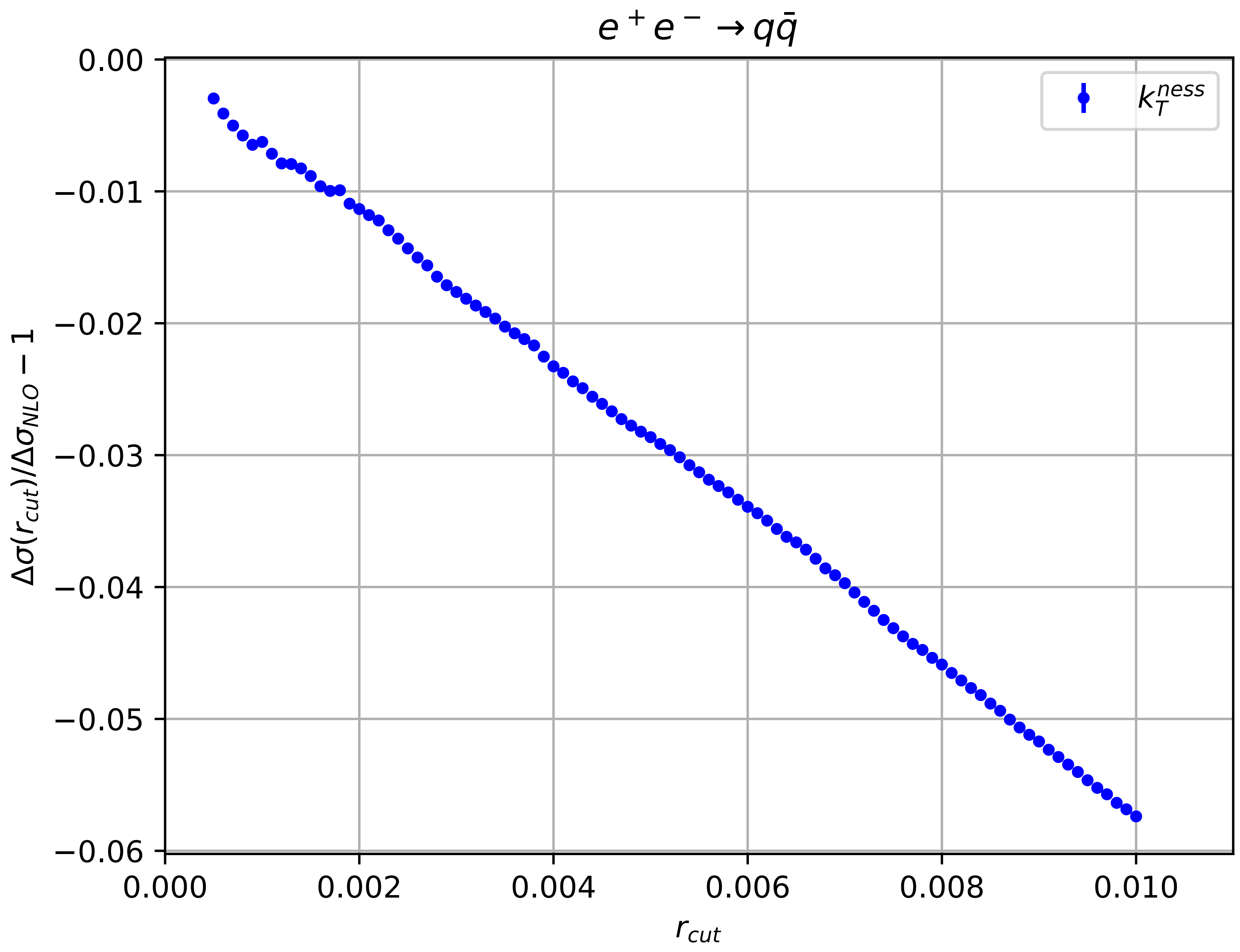
$$\int d\phi_{rad}\left\{\omega_{12}[\theta(\Delta R_{1k}^2 - \Delta R_{2k}^2)\,\theta(r_{cut}^2 - d_{2k}) + \theta(\Delta R_{2k}^2 - \Delta R_{1k}^2)\,\theta(r_{cut}^2 - d_{1k})] - \omega_1(r_{cut}^2 - d_{1k}^{\|}) - \omega_2(r_{cut}^2 - d_{2k}^{\|})]\right\} =$$

$$= 2\int d\phi_{rad}\,\left\{\omega_1\theta(\Delta R_{1k}^2 - \Delta R_{2k}^2)[\theta(r_{cut}^2 - d_{2k}) - \theta(r_{cut}^2 - d_{1k})] + \omega_1[\theta(r_{cut}^2 - d_{1k}) - \theta(r_{cut}^2 - d_{1k}^{\|})]\right\} \longrightarrow \boxed{\text{Finite in } d = 4 \text{ dimensions}}$$

- The soft contribution has been computed numerically as a two-folded integral.

k_T^{ness}- subtraction

Numerical application



- k_T^{ness}-subtraction has been successfully applied to hadronic collisions @NLO [Buonocore, Grazzini, Haag, Rottoli, Savoini (2022)] for dijet and trijet processes and has been implemented in the MATRIX framework.

- Power corrections are **linear** in r_{cut} for hadronic processes too.

k_T^{FSR} resolution variable

- How to reach a quadratic scaling?

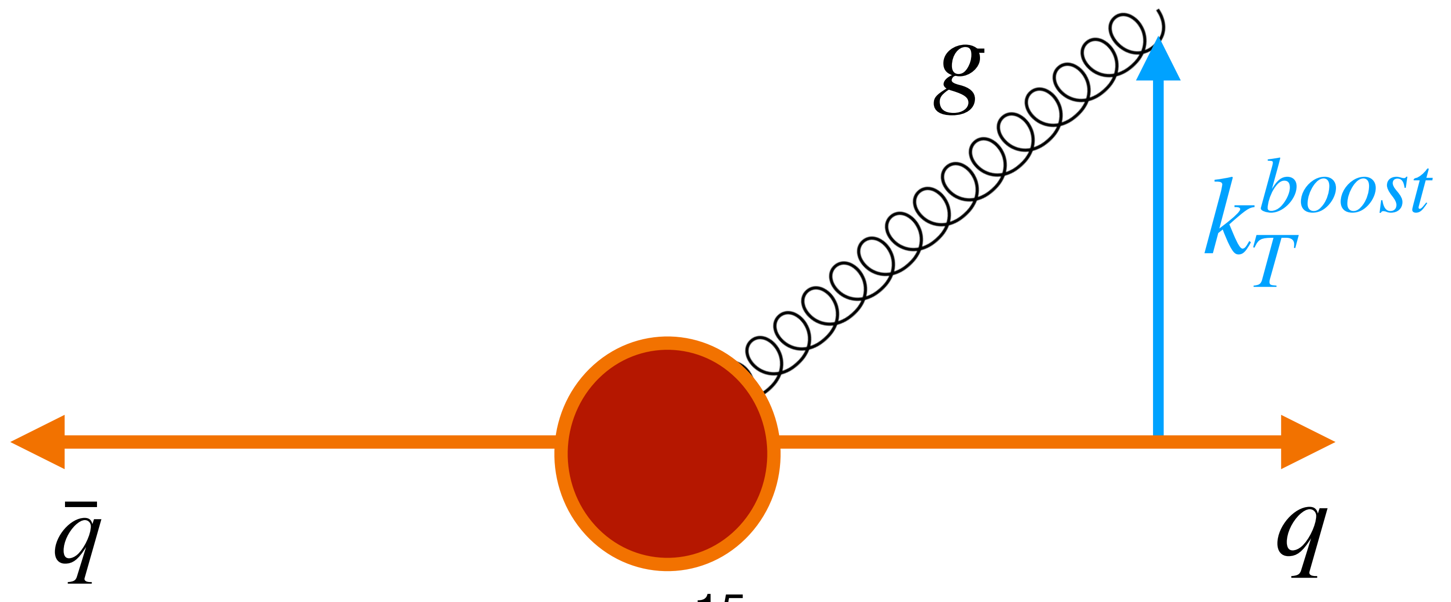
- The idea is to define a variable that has the **same properties of q_T** for colour-singlet production (that scales quadratically).

- It has to coincide with the **transverse momentum** in the singular limit.

- The soft wide-angle contribution **vanishes**.

- @ NLO, we consider the reference frame in which q and $\bar{q}$ are back-to-back. k_T^{FSR} is defined as the transverse momentum of the emitted gluon in that frame with respect to the $q\bar{q}$ axis.

- This variable is **specific for $2 \to 3$ jet transition**.

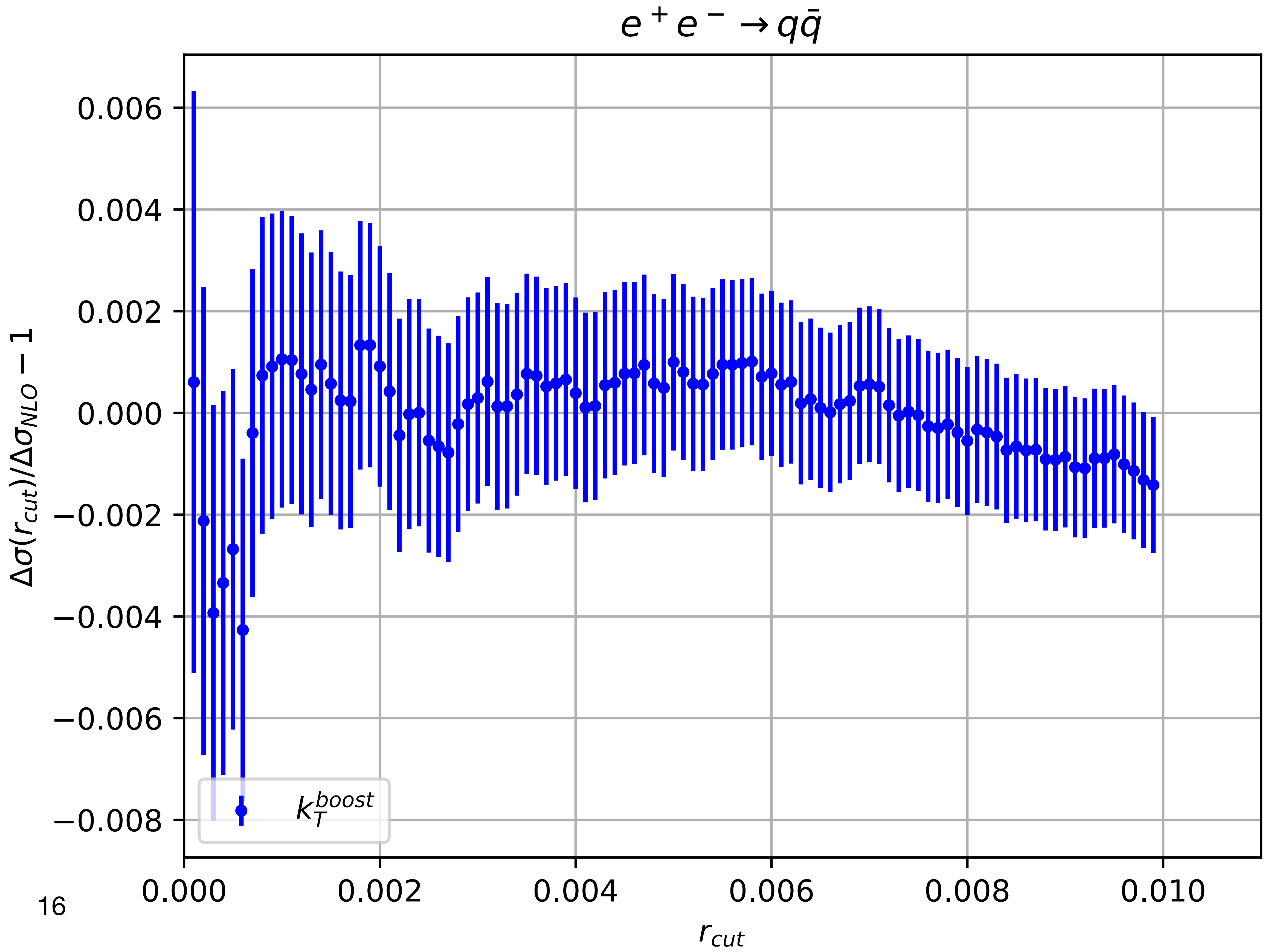


k_T^{FSR} - subtraction

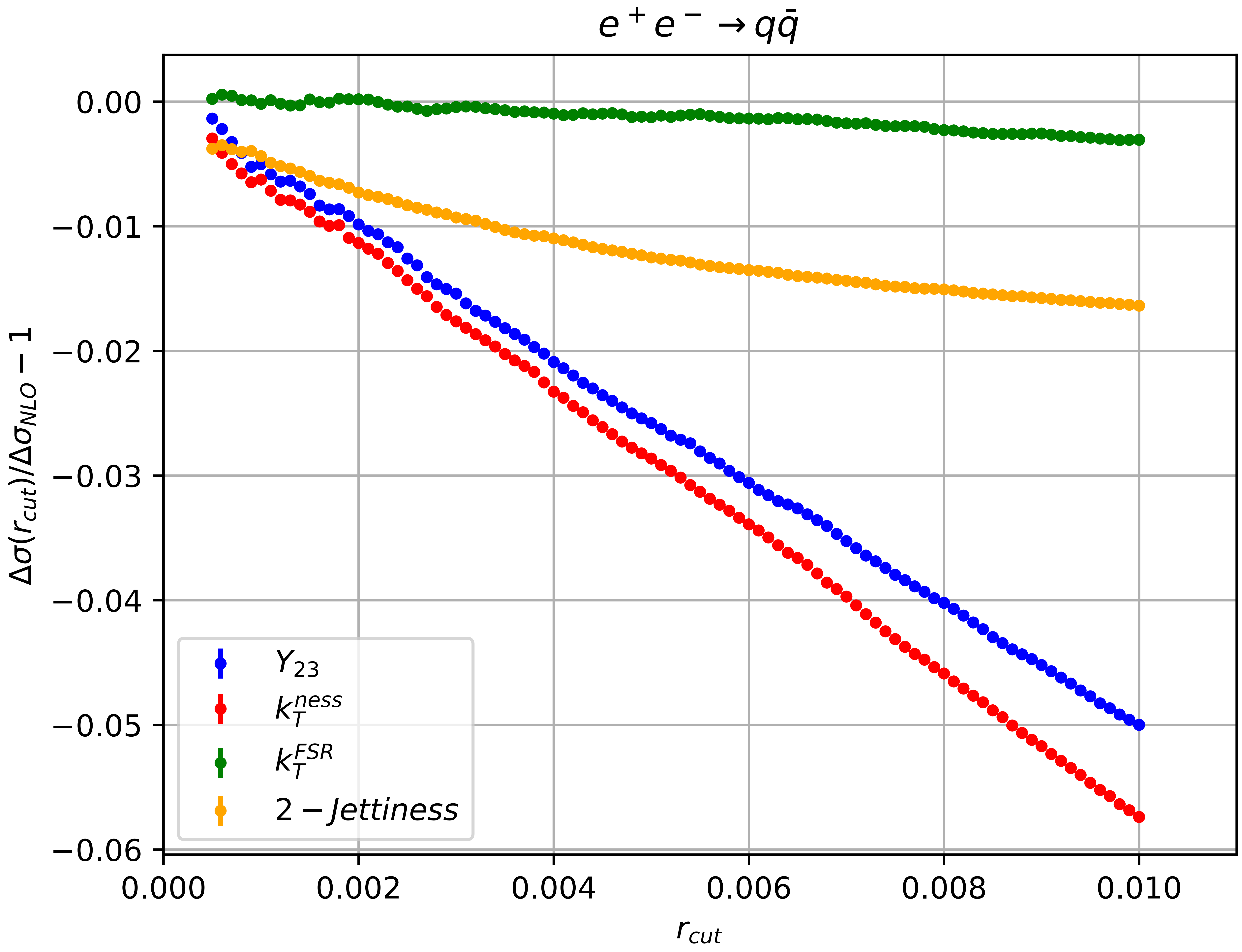
- The **counterterm** for k_T^{boost}- subtraction is the same as the one for the other two variables.
- Soft wide-angle contribution vanishes:

$$2C_F \int d\phi_{rad}\, \omega_1 \left[\theta(r_{cut}^2 - k_T^{FSR}) - \theta(r_{cut}^2 - k_T^{FSR,\parallel})\right] = 0, \text{ since } k_T^{FSR} = k_T^{FSR,\parallel}$$

We observe **quadratic power corrections**!



Comparison among the variables



Conclusions and outlook

We considered resolution variables that in the collinear limit are: $k_T^a\, e^{-b|\eta|}$

Resolution Variable	Dependence on rapidity	Dependence on kT	Soft wide-angle contribution	Scaling
N-Jettiness	✓	✓	✓	Logarithmic en.
$Y_{2,3}$	✗	✓	✓	Linear
k_T^{ness}	✗	✓	✓	Linear
k_T^{FSR}	✗	✓	✗	Quadratic

Future Goals:
- Deeper investigation of scaling properties.
- Extensions to NNLO.