High Precision for Hard Processes (HP2) Newcastle upon Tyne 22nd Sept. 2022

g-2 puzzles: a status update

UNIVERSITY OF LIVERPOOL

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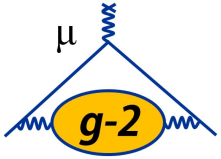
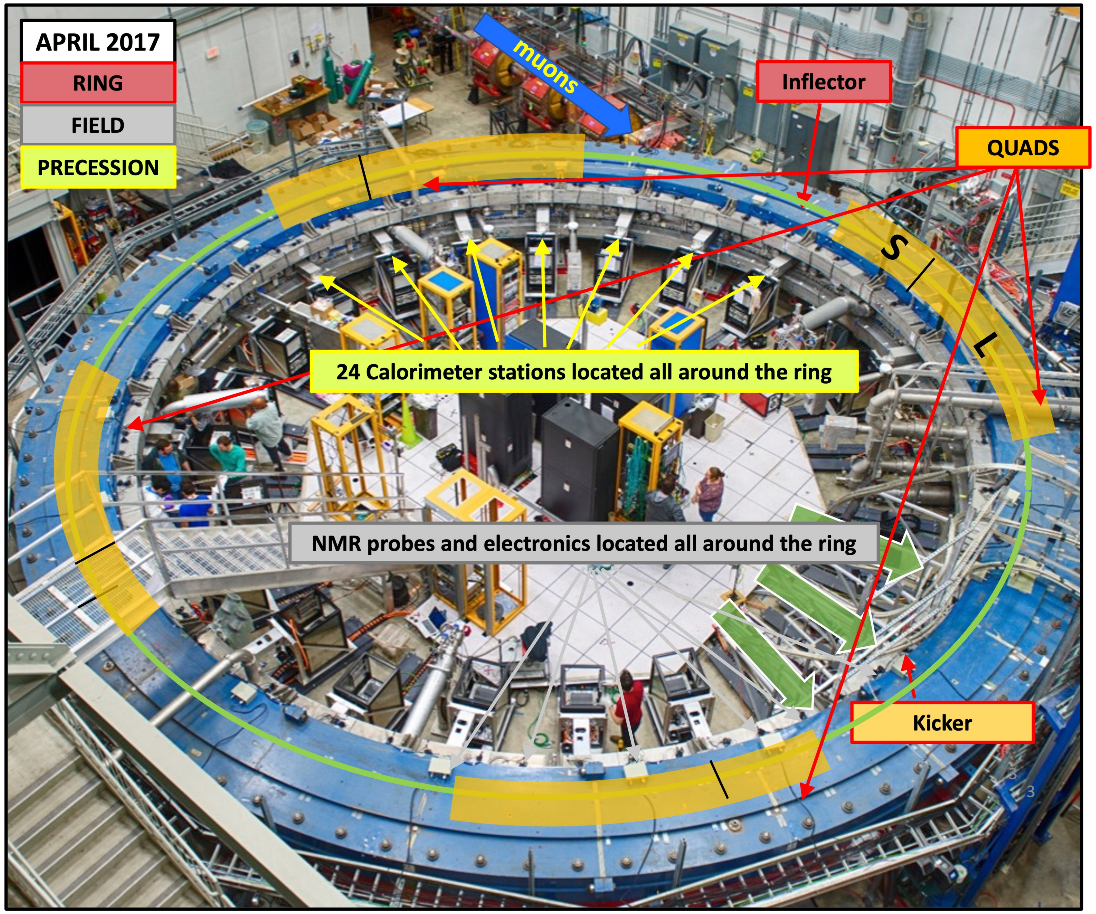
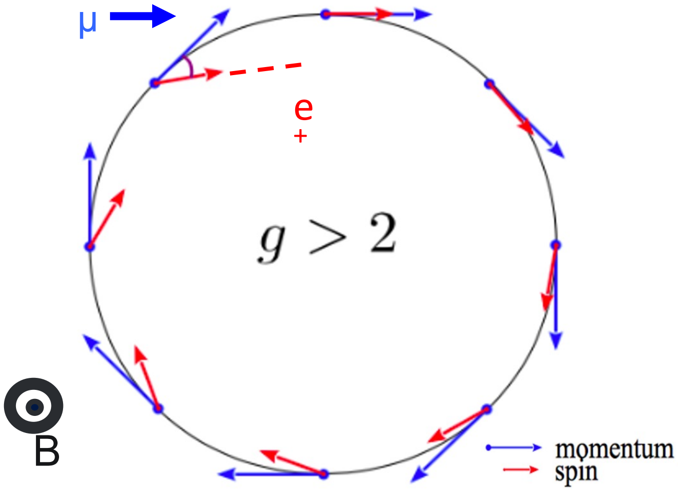


- Introduction: status of g-2
- Main ingredients for the SM prediction of $a_\mu^{\mathrm{HVP,\ LO}}$
- Recent progress from the Lattice: Window Fever
- Outlook & pathways to solving the puzzles

Measurement of the Muon g-2 by E989 at Fermilab

- Beam of polarized muons injected in storage ring
- Both the muon spin and momentum precess
- Because g is slightly greater than 2 the spin precesses faster than the momentum





$$a_\mu = \omega_a \frac{eB}{mc}$$

3ppb

0.0003ppb

$$a_\mu = \boxed{\frac{\omega_a}{\tilde{\omega}_p}} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

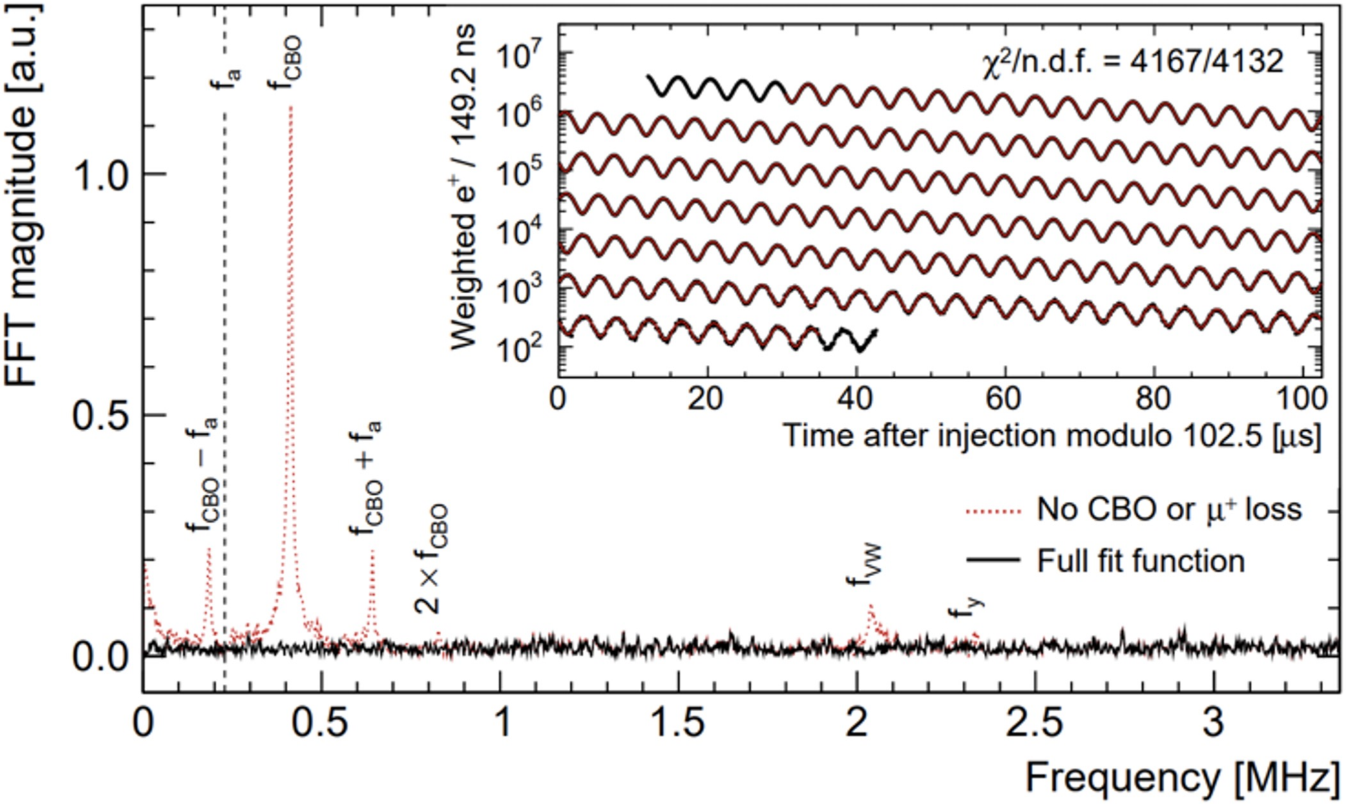
What we measure

22ppb

Material thanks to Becky Chislett's talk at the recent TI Edinburgh workshop 5-9 Sept.

Measurement of the Muon g-2 by E989 at Fermilab

Fit of ω_a from `wiggle plot':



Need to account for acceptance changes due to beam motions and slow effects on the exponential due to muon losses

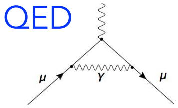
Simple 5-parameter fit
χ^2 / ndf = 8191 / 4149

Fit with extra terms
χ^2 / ndf = 4005 / 4134

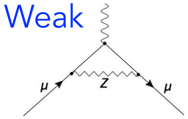
Material thanks to Becky Chislett's talk at the recent TI Edinburgh workshop 5-9 Sept.

SM prediction from Theory Initiative vs. Experiment

White **P**aper [T. Aoyama et al, *Phys. Rept.* 887 (2020) 1-166]
(132 authors, 82 institutions, 21 countries)

QED


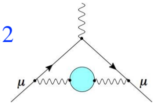
+… $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak


+… $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

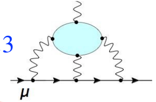
Hadronic…

…Vacuum Polarization (HVP)

α^2


+… $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

…Light-by-Light (HLbL)

α^3


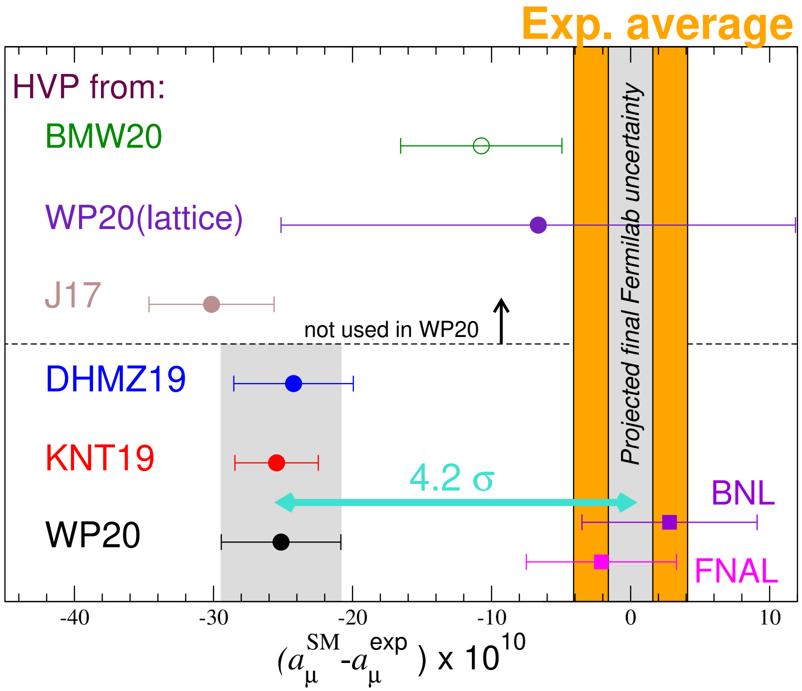
+… $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

(0.37 ppm)

➥

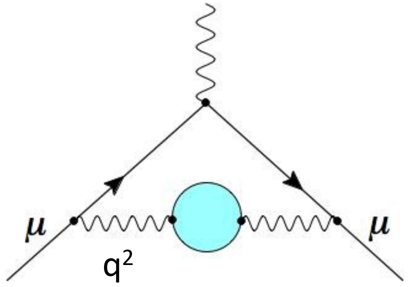
Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm
[*Phys. Rev. Lett.* 126 (2021) 14, 141801]

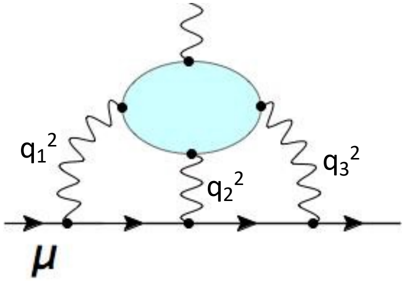
(0.35 ppm)



- ➤ SM uncertainty dominated by hadronic contributions, now with **δ HVP > δ HLbL**

a_μ^{hadronic} : non-perturbative, the limiting factor of the SM prediction





- **Q:** What’s in the hadronic (**Vacuum Polarisation** & **Light-by-Light scattering**) blobs?

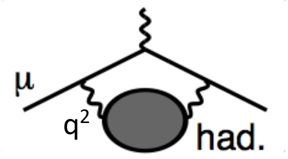
  A: Anything `**hadronic**’ the virtual photons couple to, i.e. **quarks + gluons** + photons

  But: low q^2 photons dominate loop integral(s) ➠ cannot calculate blobs with perturbation theory

- **Two very different** (model independent) **strategies:**

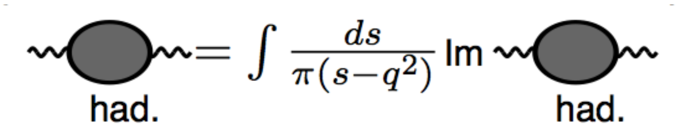
  1. use wealth of hadronic data, `**data-driven dispersive methods**’:
     - data combination from many experiments, **radiative corrections** required
  2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `**lattice QCD**’:
     - finite size, finite lattice spacing, artifacts from lattice actions, **QCD + QED** needed
     - numerical Monte Carlo methods require large computer resources

$a_\mu^{\rm HVP}$: Basic principles of **dispersive** data-driven method



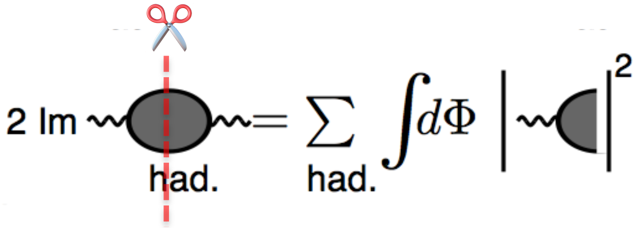
One-loop diagram with hadronic blob =

integral over q^2 of virtual photon, 1 HVP insertion



Causality ➠ analyticity ➠ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➠ Optical Theorem:

imaginary part (`cut diagram’) =

sum over |cut diagram|^2, i.e.

∝ sum over all total hadronic cross sections

$$\boxed{a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)}$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

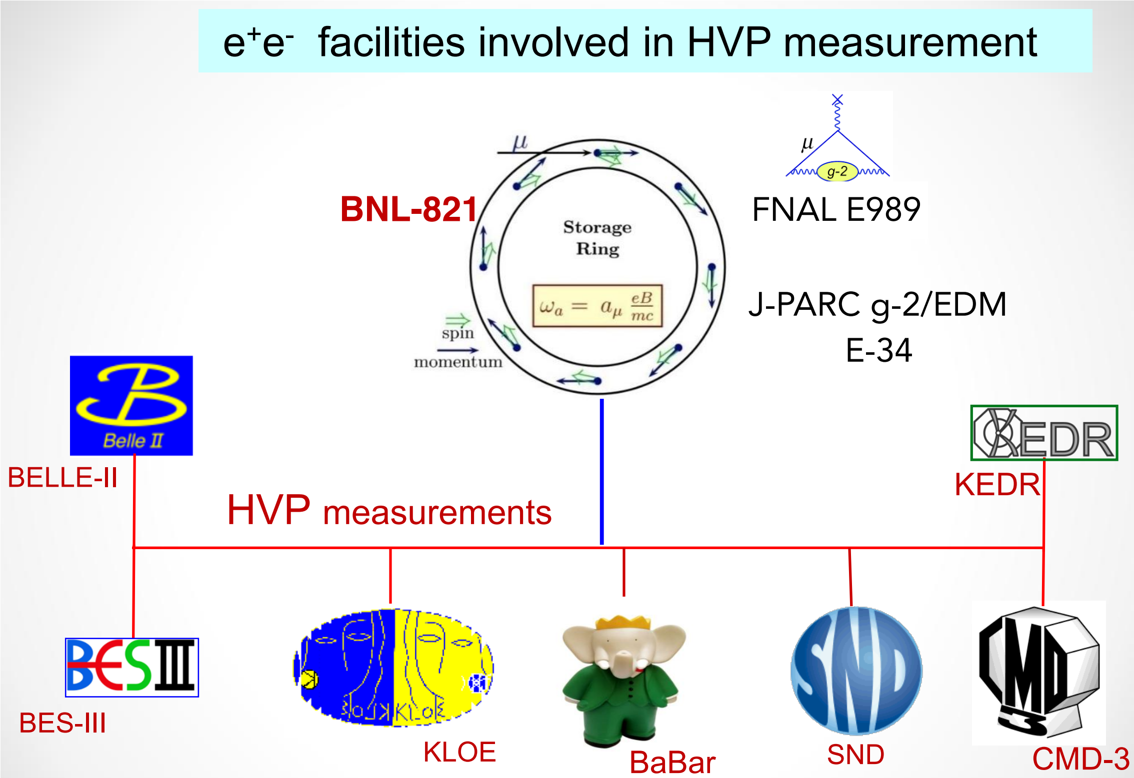
$\Longrightarrow$ Lower energies more important

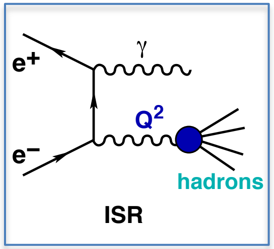
$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

- Total hadronic cross section $\boldsymbol{\sigma_{\rm had}}$ from > 100 data sets for **e^+e^- ➞ hadrons** in > 35 final states
- Uncertainty of $a_\mu^{\rm HVP}$ prediction from statistical & systematic uncertainties of input data
- KNT use pQCD only at large s, **no modelling** of $\sigma_{\rm had}(s)$, direct data integration

a_μ^{HVP} : Recent (of 25+ years) experiments providing input $\sigma_{had}(s)$ data

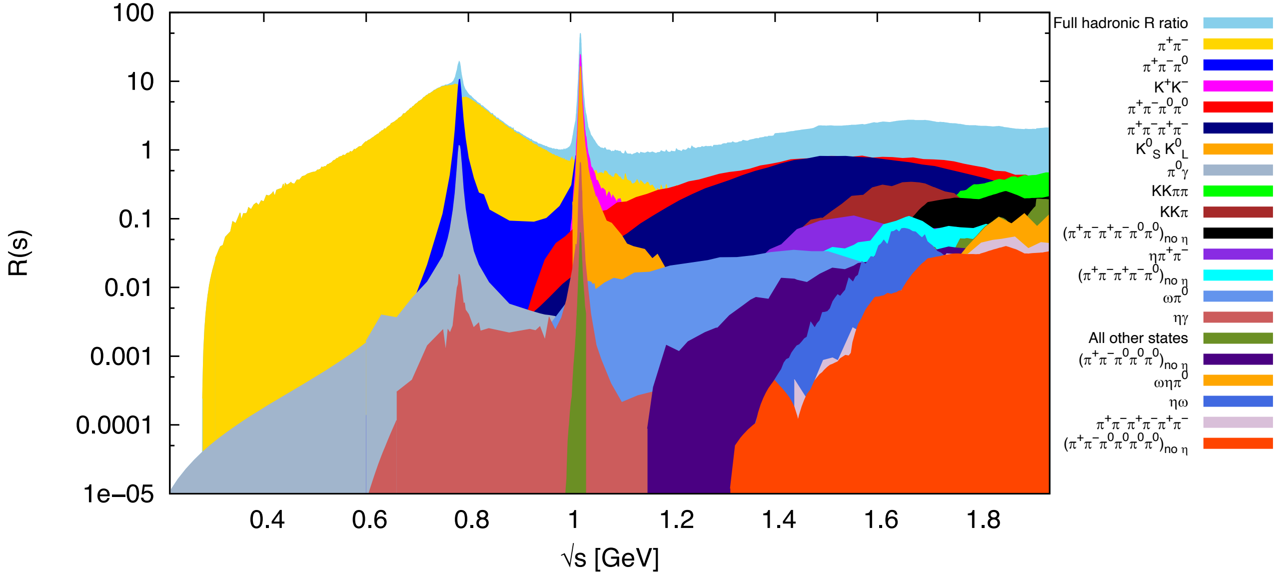
S. Serednyakov (for SND) @ HVP KEK workshop





- Different methods: \`**Direct Scan**' (tunable e^+e^- beams) &
  \`**Radiative Return**' (Initial State Radiation scan at fixed cm energy) ↗
- Over last decades detailed studies of **radiative corrections** & **Monte Carlo Generators** for $\sigma_{had}(s)$
  - ➤ **RadioMonteCarLow** Working Group report: *Eur. Phys. J.* C66 (2010) 585-686
  - ➤ full NLO radiative corrections in ISR MC *Phokhara*: Campanario et al, PRD 100(2019)7,076004

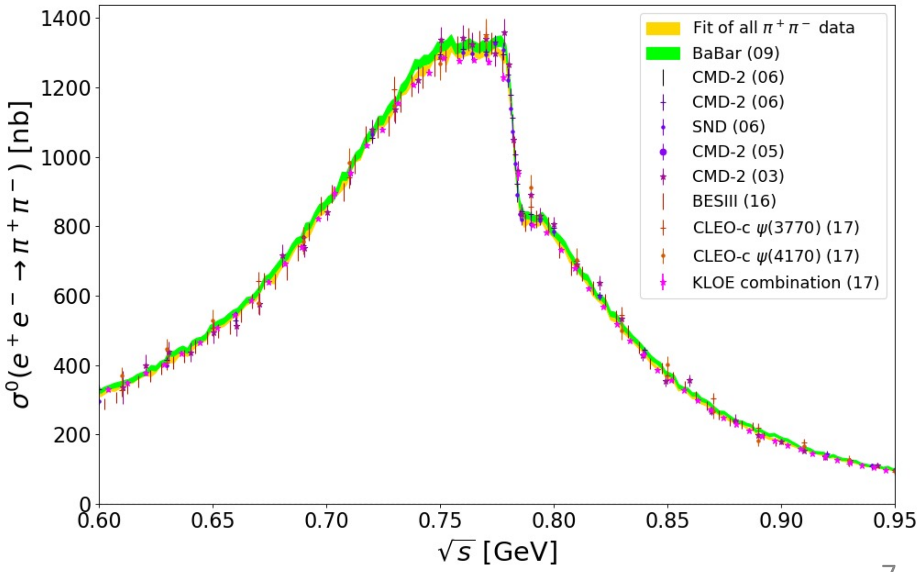
a_μ^{HVP} : **Landscape** of $\sigma_{had}(s)$ data & most important $\boldsymbol{\pi^+\pi^-}$ channel



[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of $2\boldsymbol{\pi}$

[KNT19, PRD101, 014029]

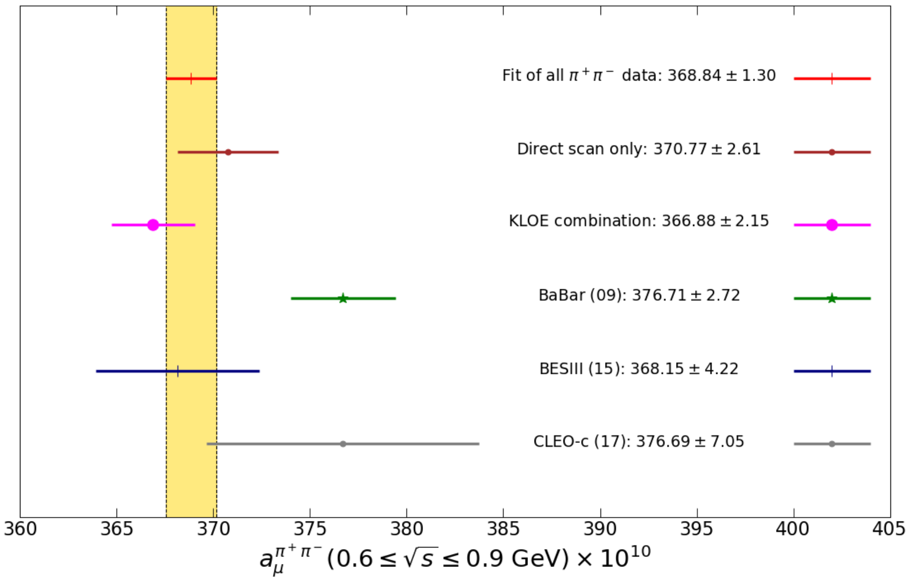


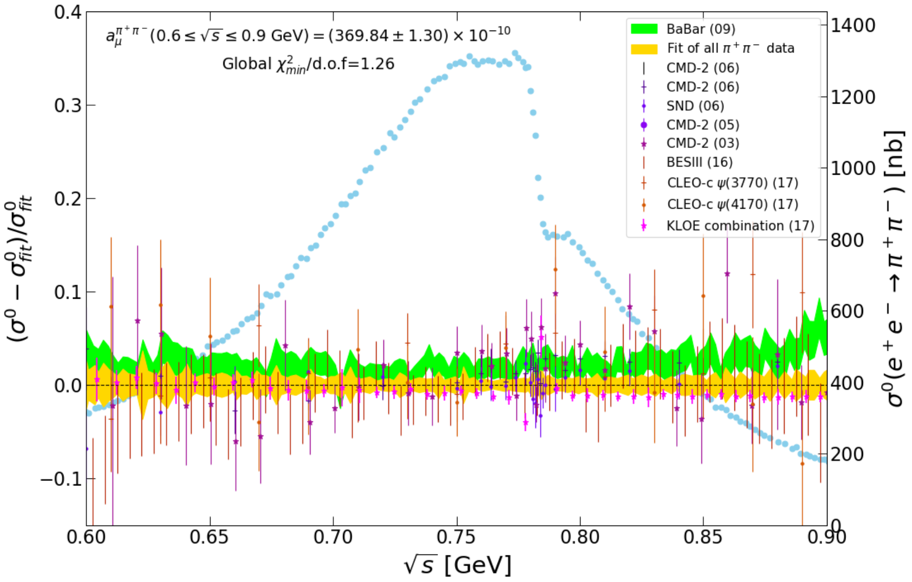
$\boldsymbol{\pi^+\pi^-}$:

- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from 6 **independent experiments** with different systematics and different radiative corrections
- Data sets from Radiative Return dominate

a_μ^{HVP} : $\pi^+\pi^-$ channel KLOE vs. Babar a puzzle, enlarges WP error

[Plots from KNT19]

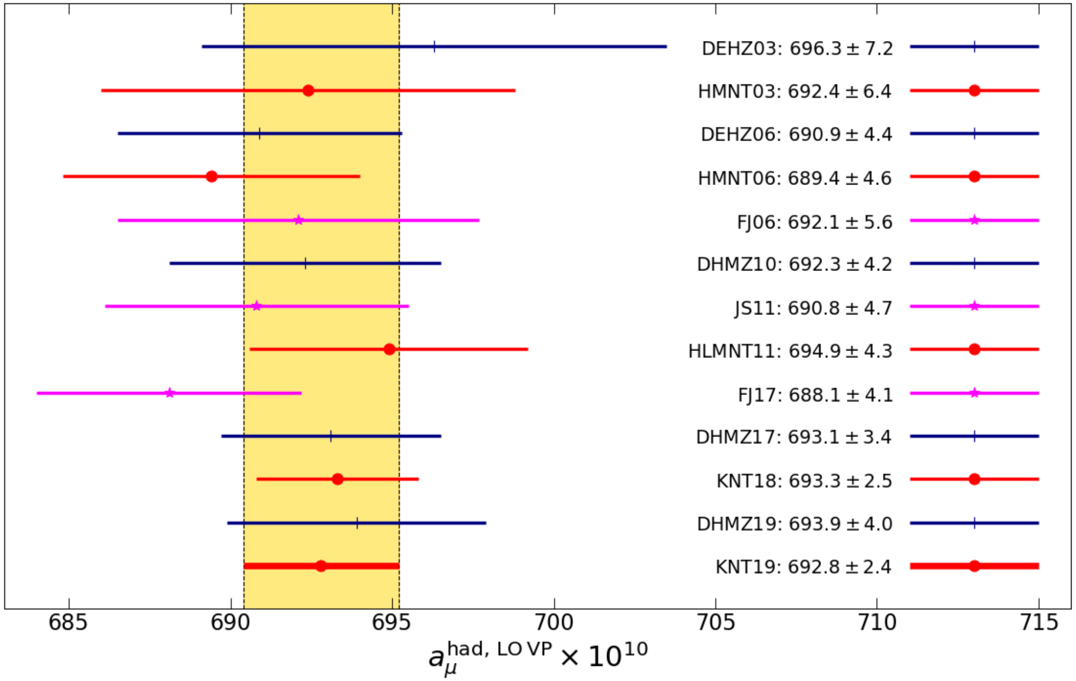




- **Tension** between different sets, especially between the most precise 4 sets from **BaBar** and **KLOE**
- Inflation of error with **local χ^2_{min}** accounts for tensions, leading to a **~14% error inflation**
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods (KNT vs. DHMZ)

- Differences in data and methods accounted for in **WP merging procedure, leading to enlarged, conservative error** for a_μ^{HVP}.

a_μ^{HVP} : History plot (data based predictions). Pies



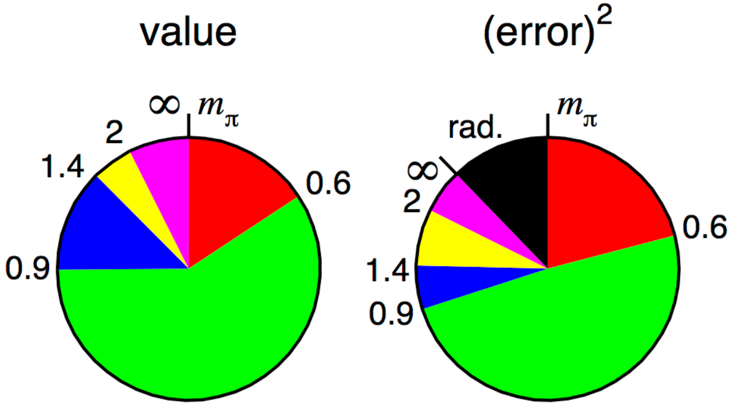
- **Stability** and **consolidation** over two decades thanks to more and better data input and improved compilation procedures

- Compare with **merged** DHMZ & KNT WP20 value:

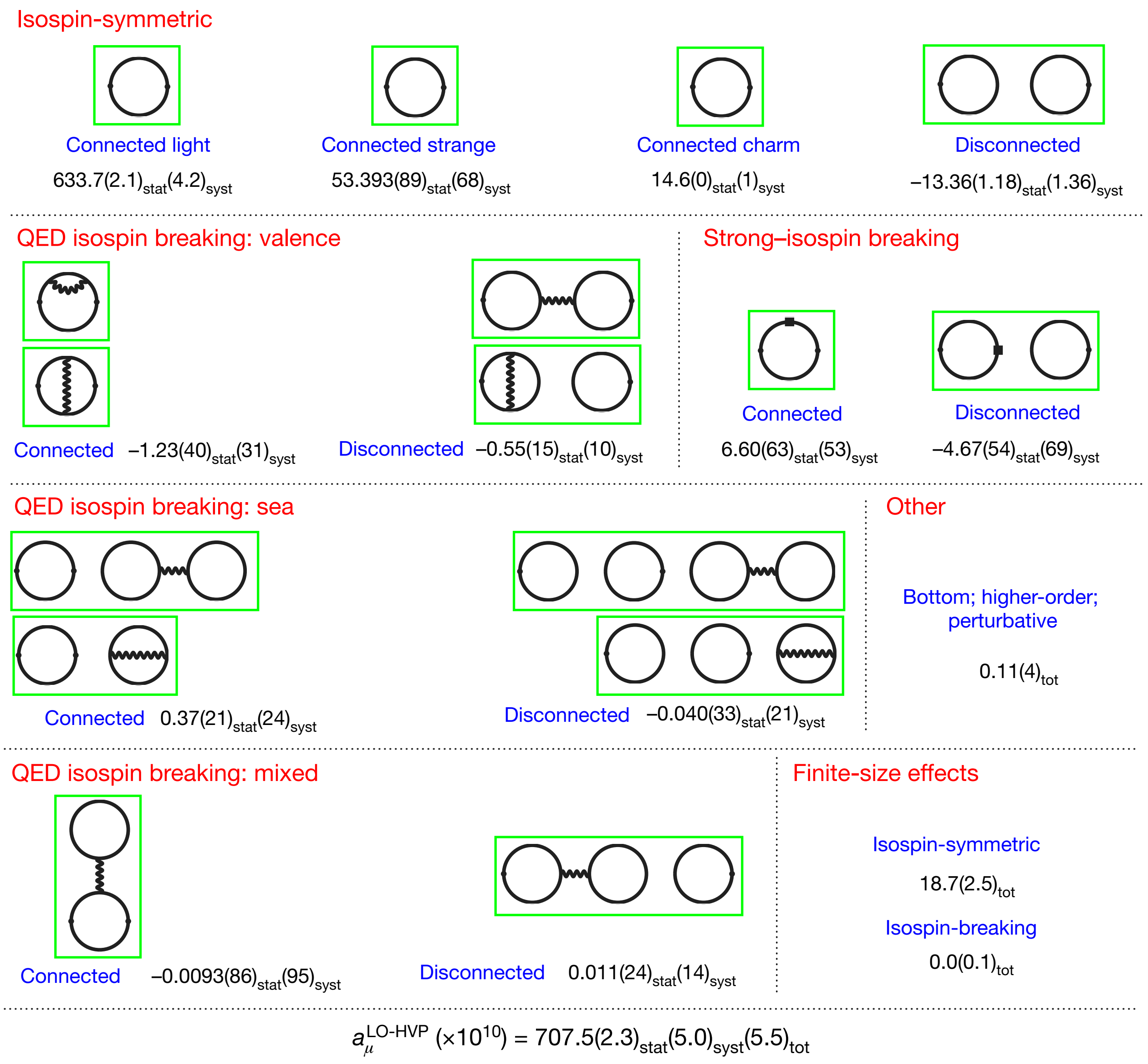
$a_\mu^{\text{had, LO VP}}$(WP20) = 693.1(4.0)×$10^{-10}$

Pie diagrams for KNT compilation:

- error still dominated by two pion channel
- significant contribution to error from additional, **conservative uncertainty from radiative corrections**



a_μ^{HVP} : Lattice result from BMW [Borsanyi et al., Nature 2021]



Isospin-symmetric

Connected light $633.7(2.1)_{stat}(4.2)_{syst}$

Connected strange $53.393(89)_{stat}(68)_{syst}$

Connected charm $14.6(0)_{stat}(1)_{syst}$

Disconnected $-13.36(1.18)_{stat}(1.36)_{syst}$

QED isospin breaking: valence

Connected $-1.23(40)_{stat}(31)_{syst}$

Disconnected $-0.55(15)_{stat}(10)_{syst}$

Strong–isospin breaking

Connected $6.60(63)_{stat}(53)_{syst}$

Disconnected $-4.67(54)_{stat}(69)_{syst}$

QED isospin breaking: sea

Connected $0.37(21)_{stat}(24)_{syst}$

Disconnected $-0.040(33)_{stat}(21)_{syst}$

Other

Bottom; higher-order; perturbative

$0.11(4)_{tot}$

QED isospin breaking: mixed

Connected $-0.0093(86)_{stat}(95)_{syst}$

Disconnected $0.011(24)_{stat}(14)_{syst}$

Finite-size effects

Isospin-symmetric

$18.7(2.5)_{tot}$

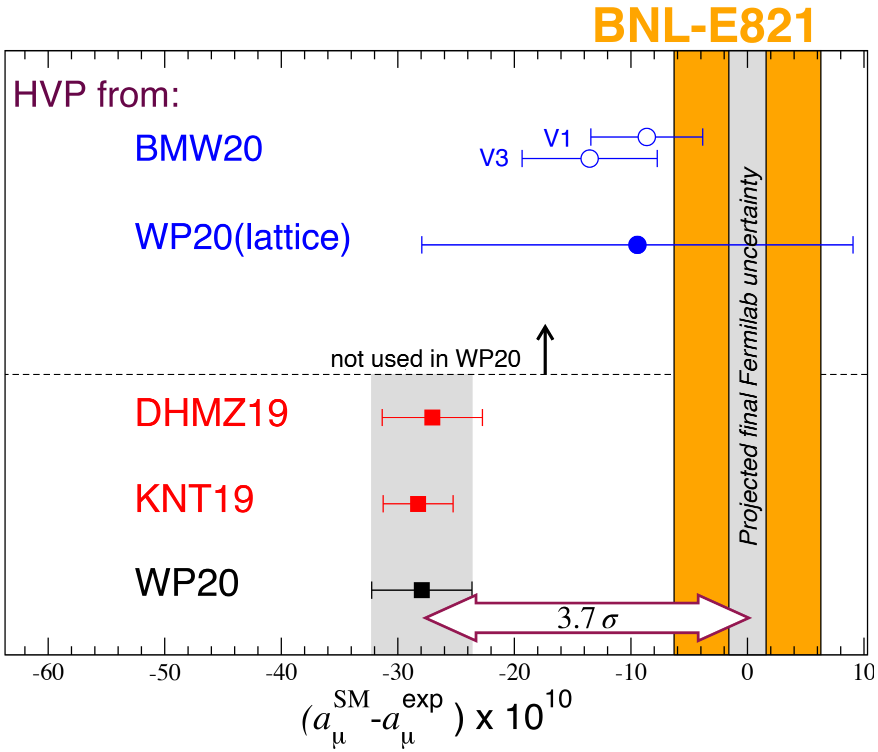
Isospin-breaking

$0.0(0.1)_{tot}$

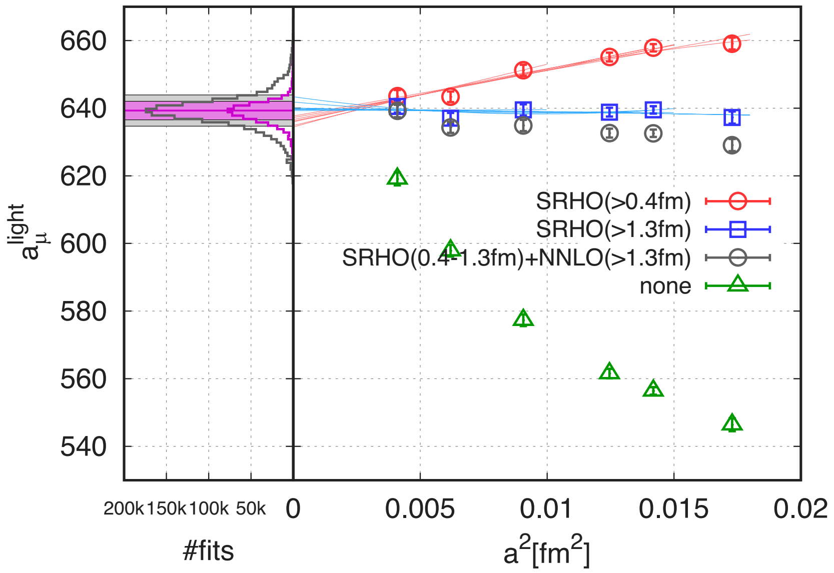
$a_\mu^{\text{LO-HVP}}$ $(\times 10^{10}) = 707.5(2.3)_{stat}(5.0)_{syst}(5.5)_{tot}$

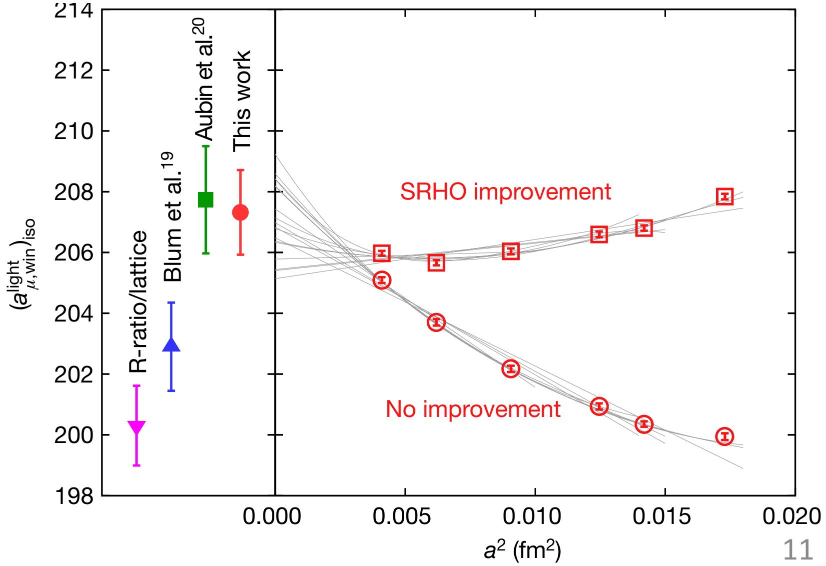
- First lattice prediction with errors matching the data-driven approach
- Current-current corr., summed over all distances and integrated over time (TMR)
- Using a L~6fm lattice (11fm for finite size corrections)
- Physical quark masses
- Strong and QED isospin breaking corrs.

a_μ^{HVP} : Tension between data-driven & BMW. Systematics



BMW20 [Borsanyi et al, arXiv:2002.12347, 2021 Nature]





BMW20: large systematics from **continuum limit,**
large taste-breaking corrections (`SRHO')

- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central `**window**' compared to other lattice and data-driven results (**3.7σ** tension)

a_μ^{HVP}: **Window method** for more detailed comparison

$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)}$$

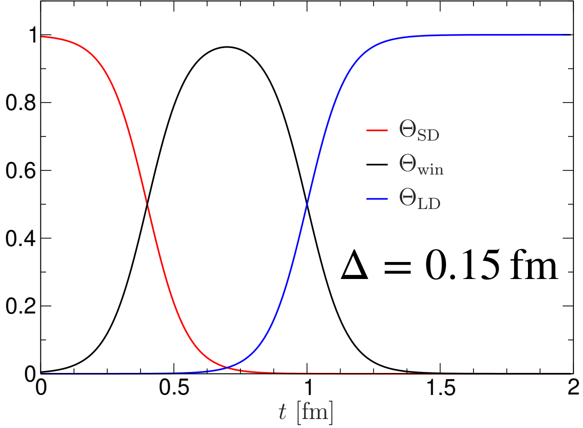
- Use windows in Euclidean time to consider the different time regions separately.

Short Distance (SD) $t : 0 \to t_0$

Intermediate (W) $t : t_0 \to t_1$

Long Distance (LD) $t : t_1 \to \infty$

$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$

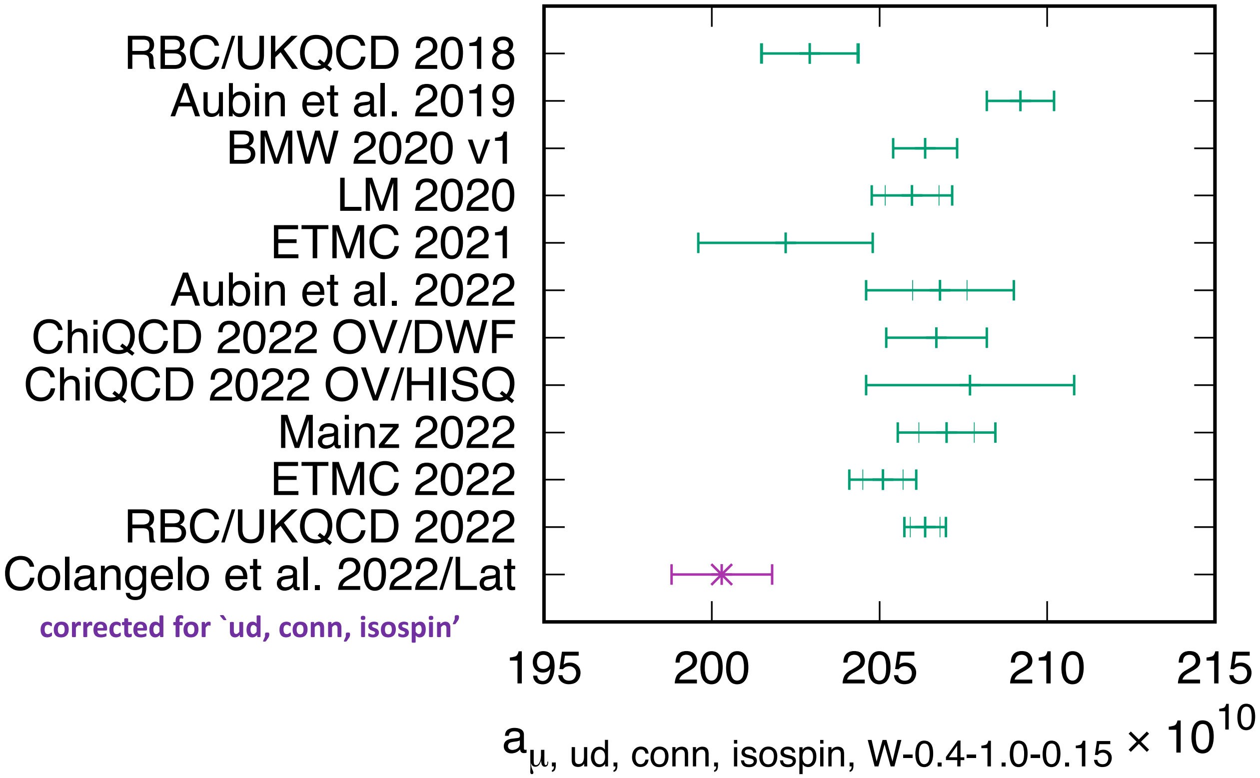


- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

a_μ^{HVP} : Window Fever

Plot from C Lehner's talk at the recent TI Edinburgh workshop 5-9 Sept.



corrected for `ud, conn, isospin'

Another ~4σ puzzle:

- Lattice QCD `easiest' in the middle window
- Comparison not direct, but heavier quark and iso-spin breaking contributions unlikely to change much
- So why is there such a large disagreement w. the data?

- **3.9σ tension** betw. RBC/UKQCD 2022 and data-driven

  [Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner (22)]
- + FNAL/HPQCD/MILC result unblinded 206.1(1.2). (LatticeNET workshop Benasque)
- **Agreement** of different lattice results, check of **universality** betw. lattice methods

a_μ^{HVP} : **Window Fever,** where to go from here

- Shorter term: further window studies, with short- and long-distance windows needed to better understand the emerged discrepancy

- Longer term: full **a_μ** with high precision from other lattice collaborations

- For now there is a puzzle

- **Could σ_{had} be _so_ wrong?** → independent check via **MUonE**, see following talks

  - If cross sections would shift up at energies above ~1-2 GeV, this would change **$\Delta\alpha(M_Z^2)$** and the SM **EW precision fits** would be in trouble

    [Crivellin, Hoferichter, Manzari, Montull ('20) / Keshavarzi, Marciano, Passera, Sirlin ('20) / Malaescu, Schott ('20)]

  - Most important $\pi^+\pi^-$ channel constrained by analyticity and unitarity. First detailed comparisons of lattice with data-driven window evaluations show that to reconcile data-driven with lattice ~40% of the shift must come from above 1 GeV for any reasonable cross section shifts (so not only $\pi^+\pi$- would need change)

    [Colangelo at recent LatticeNET workshop in Benasque 11-17 Sept. 2022]

Pathways to solving the puzzles

- No easy way out!
- BSM at high scales? Many explanations for `4.2**σ**’ puzzle, few seem natural
- BSM `faking’ low $\boldsymbol{\sigma}_{\mathbf{had}}$? Possible but not probable

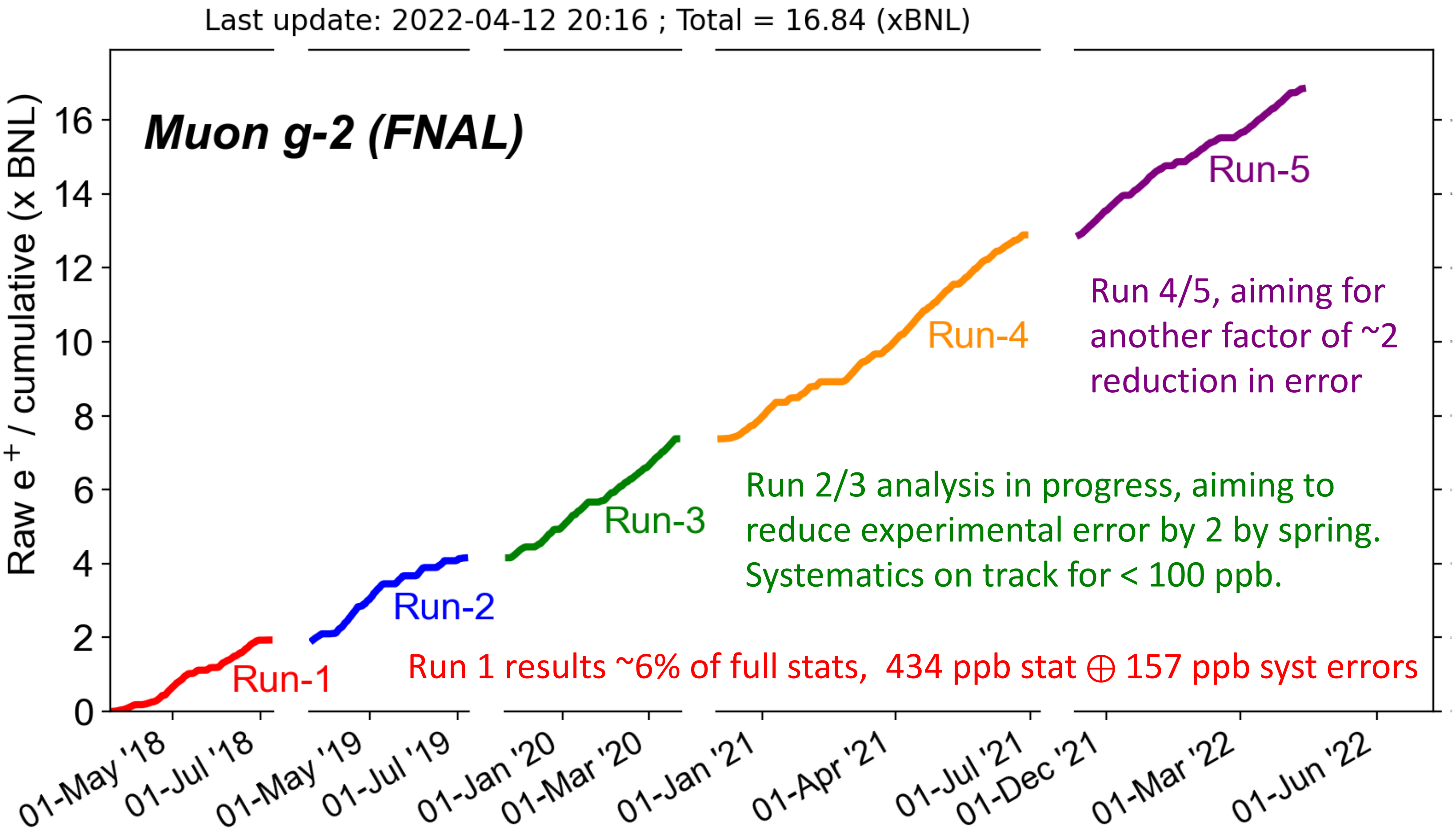
[DiLuzio, Masiero, Paradisi, Passera *Phys.Lett.B* 829 (2022) 137037]

…or even new hadronic states (like sexa-quarks [Farrar, 2206.13460]) ?

- To avoid any possible bias, **blinded analyses** are now standard, both for experiments (g-2 and $\boldsymbol{\sigma}_{\mathbf{had}}$) and lattice
- KNT just started blinding their combination machinery too, in preparation for the next major inclusion of new data
- More than 20 new sets already available (since KNT19),
  but no big changes expected, and the current WP prediction still stands

Outlook: g-2 experiment E989 at FNAL

FNAL E989: Run-2, 3, 4, 5 already on tape (but still blinded)



Run 4/5, aiming for another factor of ~2 reduction in error

Run 2/3 analysis in progress, aiming to reduce experimental error by 2 by spring. Systematics on track for < 100 ppb.

Run 1 results ~6% of full stats, 434 ppb stat ⊕ 157 ppb syst errors

Material thanks to Becky Chislett’s talk at the recent TI Edinburgh workshop 5-9 Sept.

Outlook: g-2 experiments

FNAL: Run-6

Run-6 will start in October this year, the final data taking of the experiment

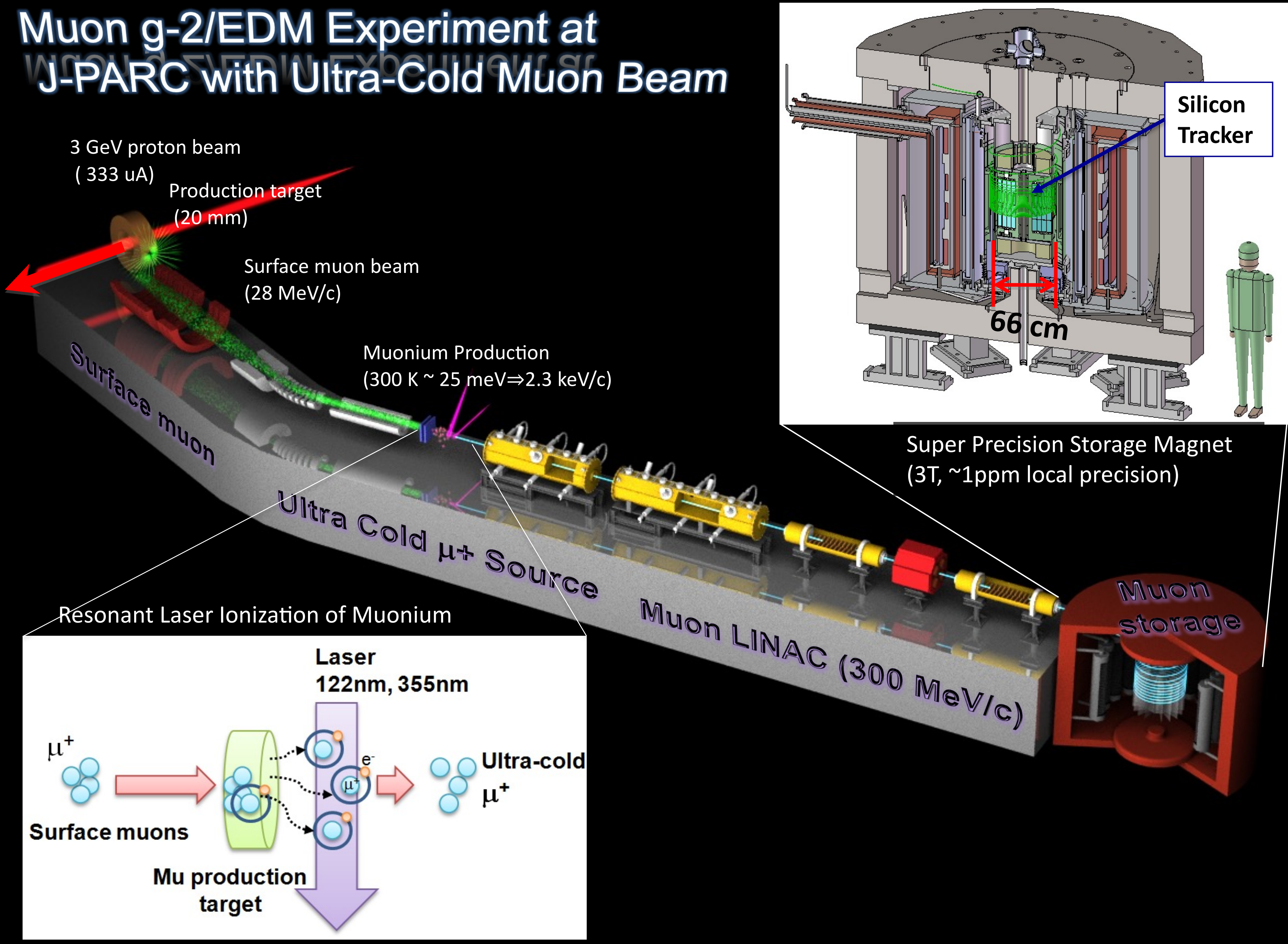
- This was planned to take mu- data but this is no longer possible as the FNAL directorate have said there isn’t enough resources
- Running parasitically sharing the beam with Mu2e
- We will take data with mu+ which will both
    - Increase our statistics
    - Allow for further systematic studies
- Some further reductions in systematics due to improvements, extra studies and increased statistics

J-PARC:

- New experiment with completely different approach under construction at J-PARC
- Will have no electric field and run at low muon momentum ➡
- Data taking planned to start in FY 2027 with first results two years later

Material thanks to Becky Chislett’s talk at the recent TI Edinburgh workshop 5-9 Sept.

Muon g-2/EDM Experiment at J-PARC with Ultra-Cold Muon Beam



3 GeV proton beam
(333 uA)

Production target
(20 mm)

Surface muon beam
(28 MeV/c)

Muonium Production
(300 K ~ 25 meV⇒2.3 keV/c)

Super Precision Storage Magnet
(3T, ~1ppm local precision)

Resonant Laser Ionization of Muonium

Outlook / Conclusions

- The still **unresolved muon g-2 discrepancy** has triggered a lot of experimental & theory activities, including experiments, the Muon g-2 Theory Initiative & **lattice**

- **Much progress** has been made for **HLbL** which previously was seen as the bottleneck

- For **HVP dispersive**, the **TI published a conservative & robust consensus**

  - no significant changes since WP20, but

  - soon (game-changing?) **new data for 2π** and other channels from

    BaBar, CMD-3, BES III and Belle II

  ➤ if they will `agree', the error $a_\mu^{\text{HVP, LO (dispersive)}}$ will go down significantly

  -- but further theory input (NNLO rad. corrs. & MCs) will become crucial, too

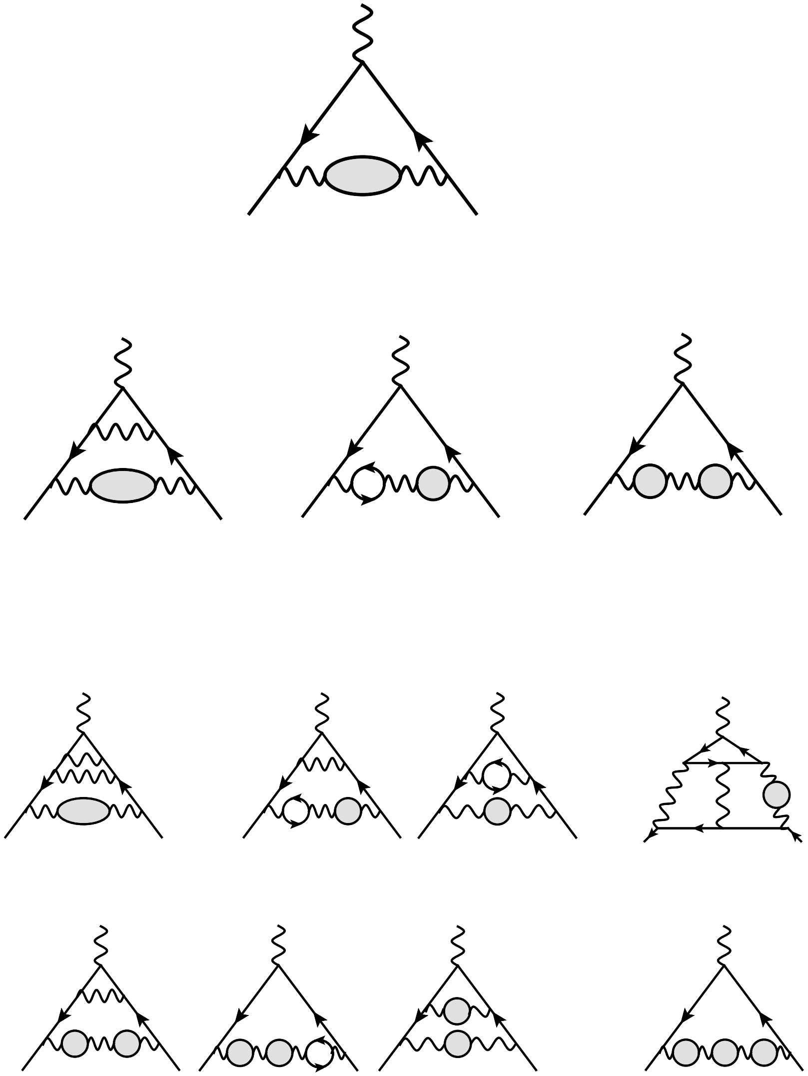
  ➤ the resolution of the puzzle in the crucial 2π channel requires new data

  ➤ this may (not) help to solve the **new puzzle with lattice HVP predictions**

...after many years there is still a lot to learn from g-2. BSM?

Extras

a_μ^{HVP} : Higher orders & power counting; WP20 values in 10^{-11}



➤ All hadronic blobs also contain photons,
i.e. real + virtual corrections in $\sigma_{had}(s)$

- LO: 6931(40)

- NLO: - 98.3(7)

  from three classes of graphs:
  - 207.7(7) + 105.9(4) + 3.4(1) [KNT19]
  (photonic, extra e-loop, 2 had-loops)

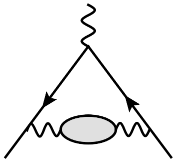
- NNLO: 12.4(1) [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]

  from five classes of graphs:
  8.0 - 4.1 + 9.1 - 0.6 + 0.005

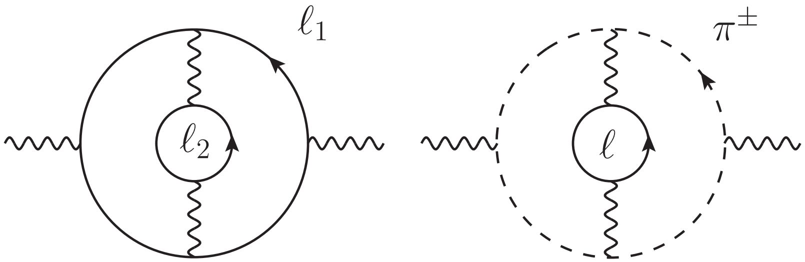
➥ good convergence,
iterations of hadronic blobs _very_ small

➠ `double-bubbles' very small:

a_μ^{HVP} : short detour: Higher orders Double Bubbles

- What if the blob in  is a `double-bubble' ?

- Purely leptonic graphs (left diagram below) are part of four-loop QED corrections



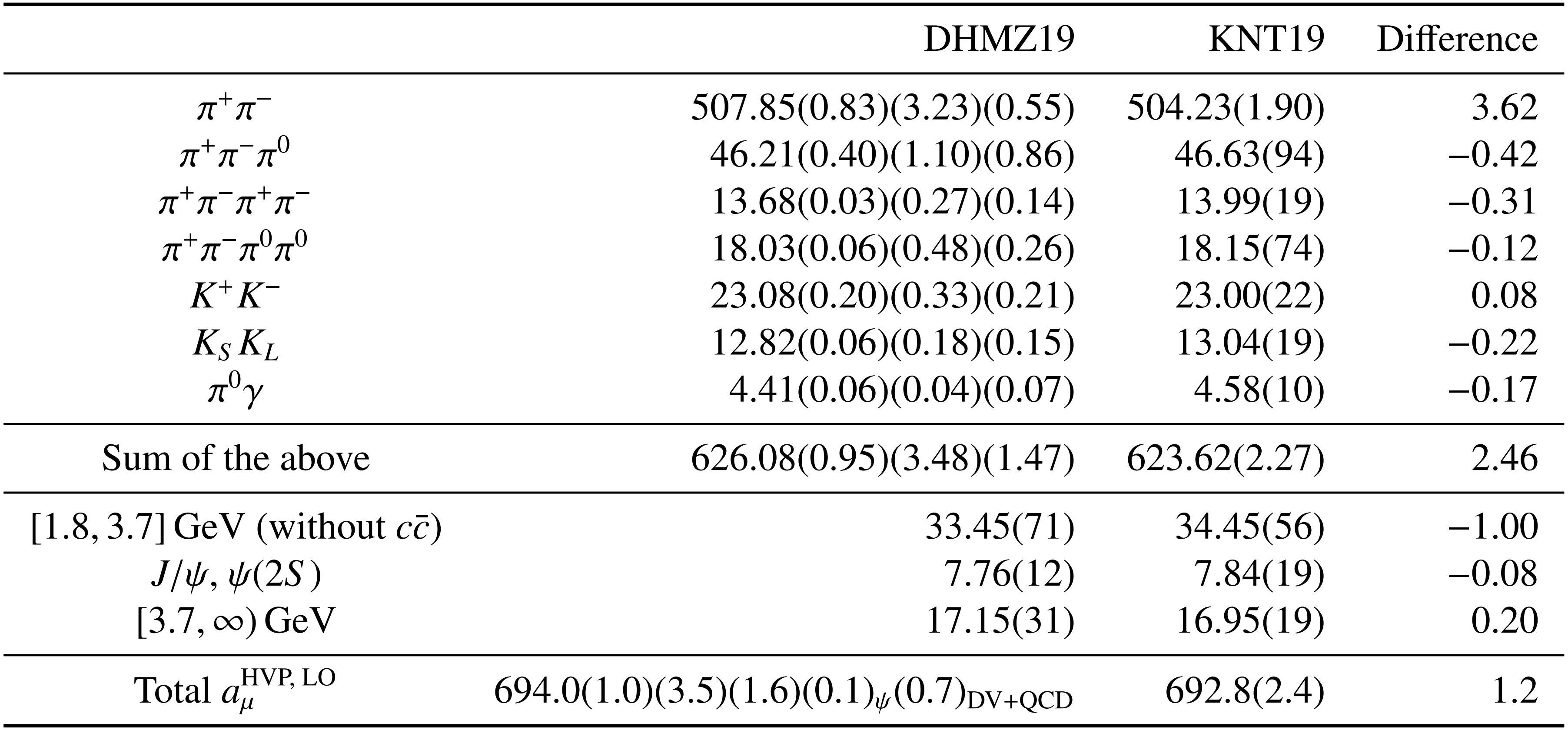
- But possibly enhanced contributions from mixed hadronic-leptonic double bubble graphs (right diagram above) are not included in the hadronic **NNLO** HVP corrections quoted above

- Our recent work has estimated these remaining NNLO contributions to $\mathbf{a_\mu}$ to be **below 1 × $\mathbf{10^{-11}}$** and hence not critical at the level of the experimental accuracy

M Hoferichter + TT, *Phys. Rev. Lett.* 128 (2022) 11, 112002

HVP: White Paper comparison

Detailed comparisons by-channel and energy range between direct integration results:



	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, HHKS 2019]

HVP: White Paper merging procedure

Conservative merging procedure developed during 2019 Seattle TI workshop:

- Accounts for the different results obtained by different groups based on the same or similar experimental input

- Includes correlations and their different treatment as much as possible

- Allows to give one recommended (merged) result, which is conservative w.r.t. the underlying (and possibly underestimated) systematic uncertainties

- Note: Merging leads to a bigger error estimate compared to individual evaluations; error `corridor' defined by embracing choices goes far beyond χ^2_{min} inflation

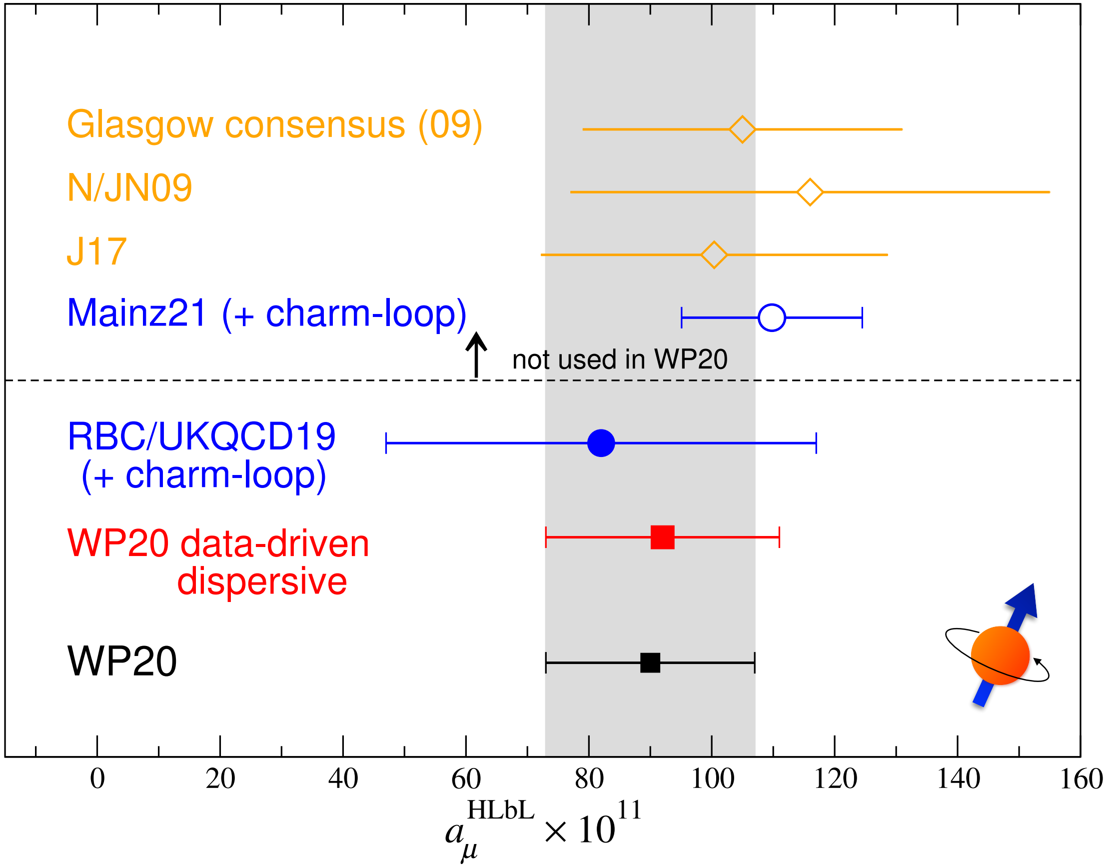
➠ $a_\mu^{\text{HVP, LO}} = 693.1\ (4.0) \times 10^{-10}$ is the result used in the WP `SM2020' value

- This result does not include lattice, but is compatible with published full lattice results apart from the BMW prediction:

  $a_\mu^{\text{HVP, LO}}$ (BMW) = 707.5 (5.5) × 10^{-10} [Nature 2021] ⤳ **1.5/2.1 σ** tension w. exp/WP20

Many efforts are ongoing to understand this new puzzle, see e.g. talks at the TI workshop Edinburgh

a_μ^{HLbL} : WP Status/Summary of Hadronic Light-by-Light contributions



hadronic models + pQCD

lattice QCD + QED (after WP)

lattice QCD + QED

data-driven

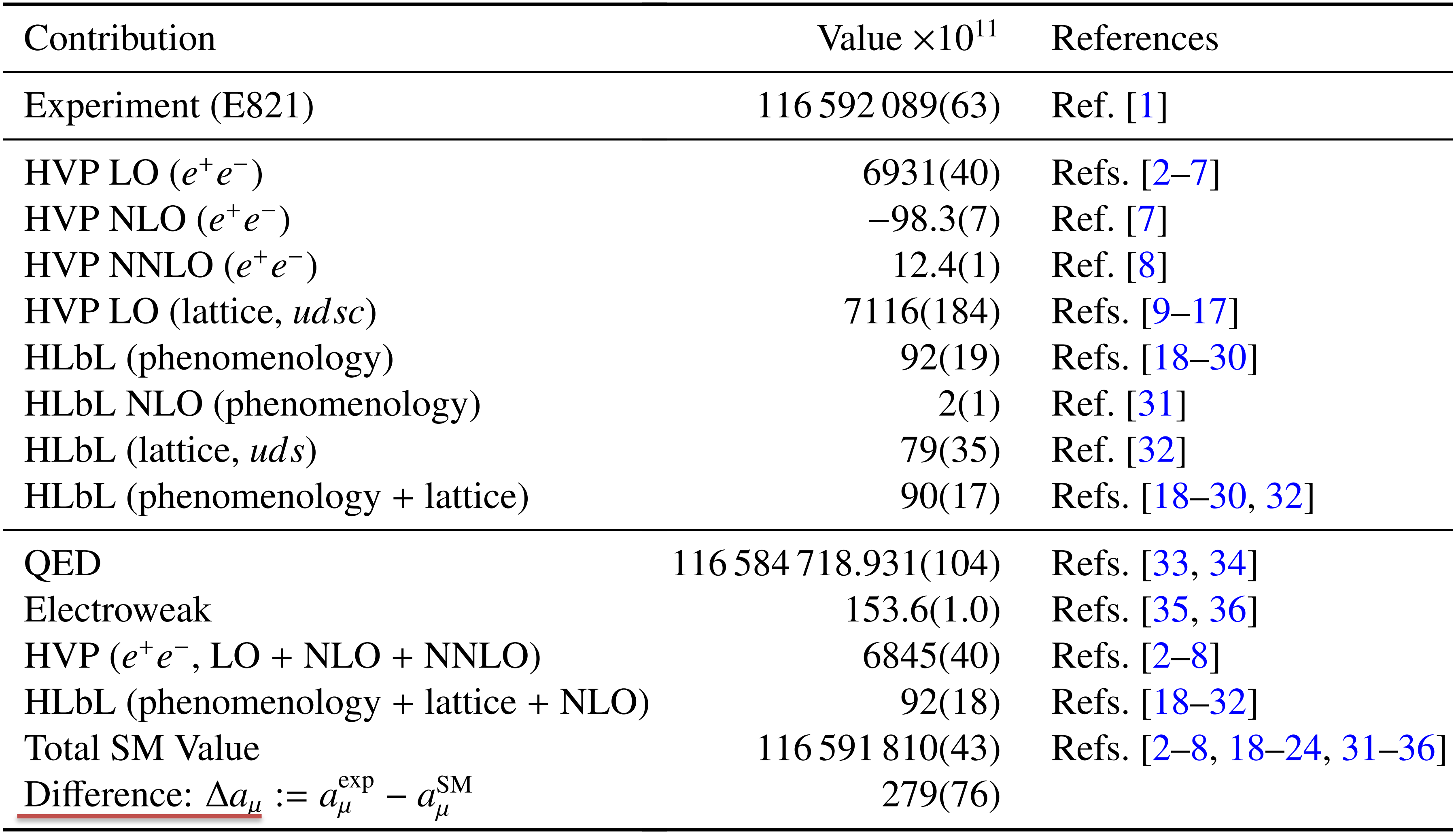
TI White Paper 2020 value:

$a_\mu^{HLbL} = 92\ (18) \times 10^{-11}$ ✓

- data-driven dispersive & lattice results have confirmed the earlier model-based predictions
- **uncertainty much better under control** and at 0.15ppm already **sub-leading compared to HVP**
- lattice predictions now competitive, good prospects for combination and error reduction to ≤ 10%

a_μ (SM): White Paper https://doi.org/10.1016/j.physrep.2020.07.006

White Paper [T. Aoyama et al, arXiv:2006.04822], 132 authors, 82 institutions, 21 countries



Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\exp} - a_\mu^{\mathrm{SM}}$	279(76)	

w.r.t. BNL only