High precision at low energy: the case of MUonE

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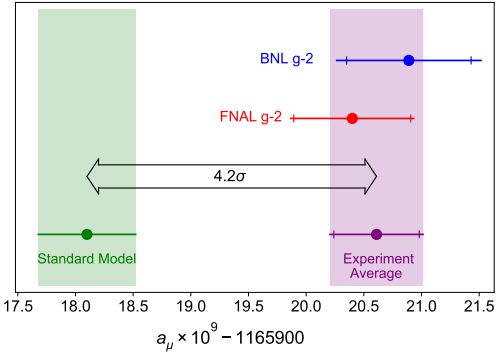
HP2 2022
High Precision for Hard Processes

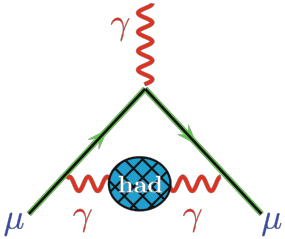
Discovery Museum, Newcastle upon Tyne, 20-22 September 2022

Starting point: muon $g-2$



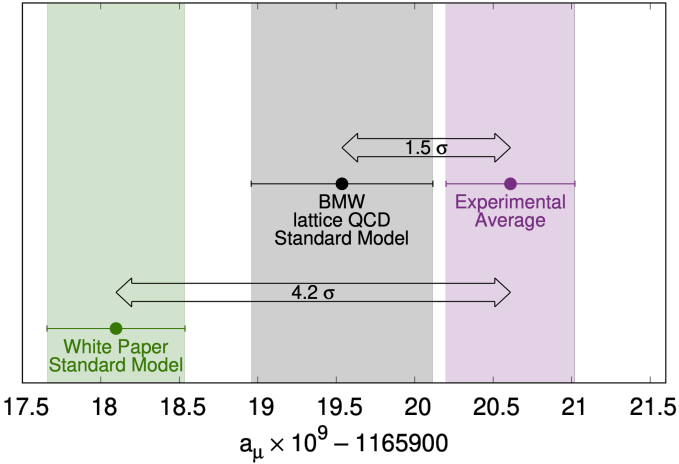
B. Abi et al., Phys. Rev. Lett. 126 (2021) 14, 141801 [arXiv:2104.03281[hep-ex]]

largest Th. unc. source: HVP



see talk by T. Teubner

An additional puzzle in theoretical predictions

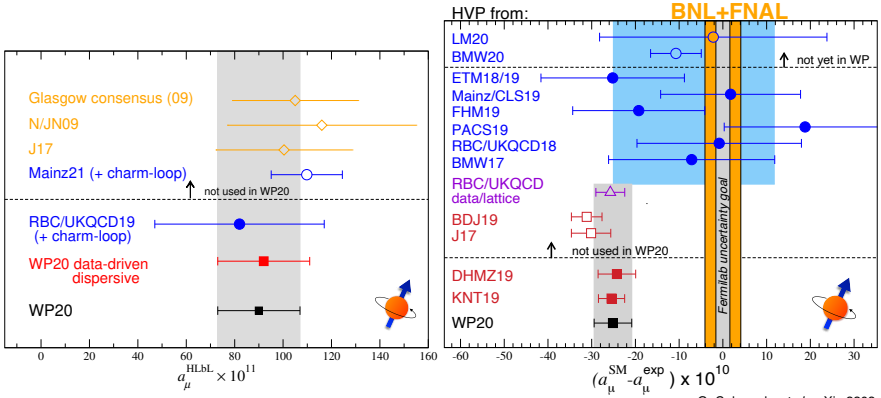


T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166

B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801.

Borsanyi, S. *et al.* *Nature* **593**, 51–55 (2021).

A recent summary



G. Colangelo *et al.* arXiv:2203.1581 (Snowmass 2021)

recent new developments e.g.

- Lattice 2022 (8-13 August 2022)
- Fifth Plenary Workshop of the Muon g-2 Theory Initiative (5-9 September 2022)

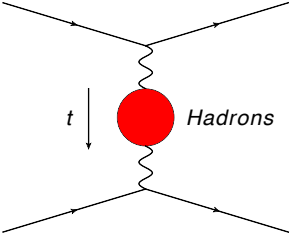
A third independent determination more than welcome

- G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  Measuring the leading hadronic contribution to the muon g-2 via μe scattering
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

- Alternatively (exchanging s and x integrations in $a_\mu^{\rm HLO}$)

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$
$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$



e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193

⇝ The hadronic VP correction to the running of α enters

⇝ Essentially the same formula used in lattice QCD calculation of $a_\mu^{\rm HLO}$

★ $\Delta\alpha_{\rm had}(t)$ (and $a_\mu^{\rm HLO}$) can be directly measured in a (single) experiment involving a space-like scattering process

Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

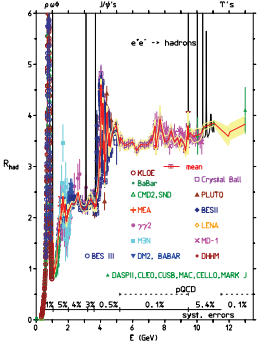
★ **Still a data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

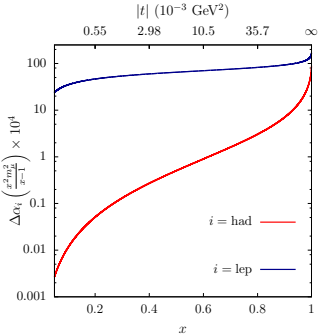
- By modifying the kernel function $\frac{\alpha}{\pi}(1-x)$, also $a_\mu^{\rm HNLO}$ and $a_\mu^{\rm HNNLO}$ can be provided

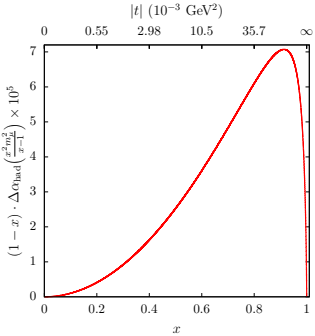
Balzani, Laporta, Passera, **arXiv:2112.05704 [hep-ph]**

From time-like to space-like evaluation of a_μ^{HLO}

Time-like $\mapsto$ **Space-like**







Smooth function

- $\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required
- $\mapsto$ **Space-like:** in principle, one single experiment, *it's a one-loop effect, very high accuracy needed*

MUonE: $\mu e \to \mu e$ @CERN

Abbiendi et al., EPJC 77 (2017) 3, 139

Abbiendi et al., *Letter of Intent: the MUonE project, CERN-SPSC-2019-026, SPSC-I-252 (2019)*

- Scattering μ's on e's in a low Z target looks like an ideal process (fixed target experiment)
- **It is a pure t-channel process at tree level**
- **The M2 muon beam ($E_\mu \simeq 160$ GeV) is available at CERN**
- $\sqrt{s} \simeq 0.4$ GeV and $-0.143 < t < 0$ GeV^2
- We can cover 87% of the a_μ^{HLO} space-like integral (and extrapolate to $x \to 1$)
- With ~ 3 years of data taking, a statistical accuracy of 0.3% on a_μ^{HLO} can be achieved

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\text{had}}$$

$\Delta\alpha_{\text{had}}$ is a 0.1% effect in this region $\to$ to measure it at 1%, σ must be controlled at the 10^{-5} level

statistics and (main) systematic uncertainties

- **statistics**: CERN muon beam M2 ($E = 150$ GeV), $1.3 \cdot 10^7$ μ/s with a target (Be/C) with total thickness of 60 cm $\Longrightarrow L \sim 1.5 \cdot 10^7$nb^{-1} $\Longrightarrow$ statistical sensitivity ~ 0.3% on $a_\mu^{HLO} (\sim 20 \cdot 10^{-11})$ in about 3 yrs of data taking

Sistematics

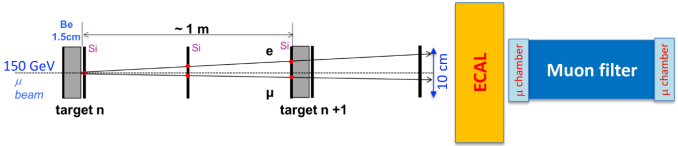
- (main) experimental sources
  - multiple scattering: E_e in normalization region much lower than in signal region
    Effect $\sim 1/E \Longrightarrow$ it affects signal and normalization in different way
  - absolute μ beam energy scale, 5 MeV $\Longrightarrow 10^{-5}$ effect
  - angular intrinsec resolution $(\sim 1\%)$
  - longitudinal alignment $(\sim 10 \mu m)$

- theoretical: higher order radiative corrections modify the shapes
  - order of magnitude estimate, barring infrared logs and setting $c_{i,j} \sim 10$
  - $c_{1,1} \left(\frac{\alpha}{\pi}\right) L \sim 0.2$ $\quad c_{1,0} \left(\frac{\alpha}{\pi}\right) \sim 2.5 \cdot 10^{-2}$
  - $c_{2,2} \left(\frac{\alpha}{\pi}\right)^2 L^2 \sim 5 \cdot 10^{-3}$ $\quad c_{2,1} \left(\frac{\alpha}{\pi}\right)^2 L \sim 5 \cdot 10^{-4}$ $\quad c_{2,0} \left(\frac{\alpha}{\pi}\right)^2 \sim 5 \cdot 10^{-5}$
  - $c_{3,3} \left(\frac{\alpha}{\pi}\right)^3 L^3 \sim 1.5 \cdot 10^{-4}$ $\quad c_{3,1} \left(\frac{\alpha}{\pi}\right)^3 L^2 \sim 1.5 \cdot 10^{-5}$ $\quad c_{3,0} \left(\frac{\alpha}{\pi}\right)^3 L \sim 1.5 \cdot 10^{-6}$
  - the most advanced technologies for NNLO calculations and higher order resummation and matching are needed

On the experimental side

- a modular apparatus has been proposed (40 independent tracking stations)

G. Abbiendi et al., LoI CERN-SPSC-2019-026, SPSC-I-252, CERN



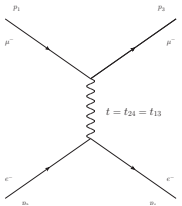
- whole acceptance covered with a 10×10 cm^2 silicon sensor
- thin targets equivalent to 60 cm
- ECal and Muon filter after last station, for PID and background rejection
- two Beam Tests already done at CERN (2017 and 2018)
  1. Multiple Scattering measurements
  2. selection of a clean sample of elastic events

G. Abbiendi et al., arXiv:1905.11677

G. Abbiendi et al., arXiv:2021.11111

- Further Beam Test in October 2022
- 3 weeks Test Run in 2023 (proof of concept of the experimental proposal)
- 10 stations before LHC LS3 (2026) with first measurements of a_μ^{HVP} with $\sim 1\%$ accuracy

First step towards precision: QED NLO



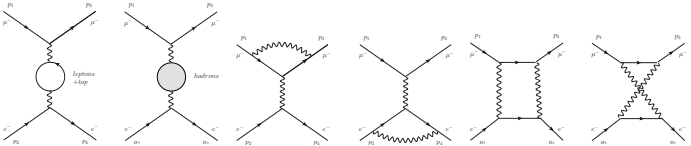
- analytical expression for tree level

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

(Van Nieuwenhuizen 1971, D'Ambrosio 1983, Kukhto et al. 1987, Bardin, Kalinovskaya 1997)

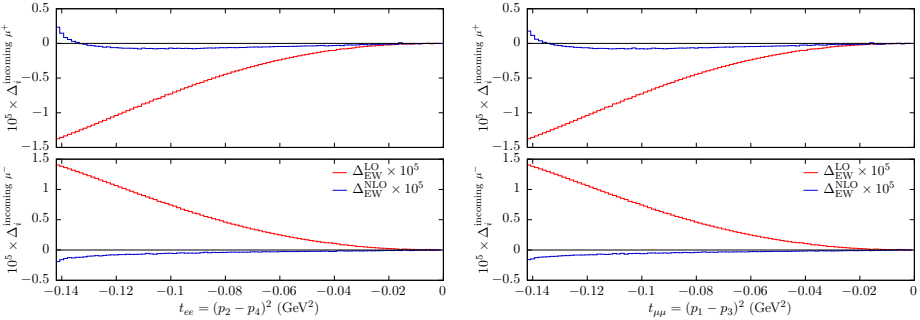
- NLO virtual diagrams



- and corresponding real emission diagrams
- **NLO matrix elements** calculated with finite m_μ and m_e mass effects and a **Monte Carlo** program, MESMER, has been taylored to the fixed target kinematics

Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini, arXiv:1811.06743; JHEP 02 (2019) 155

Weak interaction effects (LO and NLO)



Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini, arXiv:1811.06743

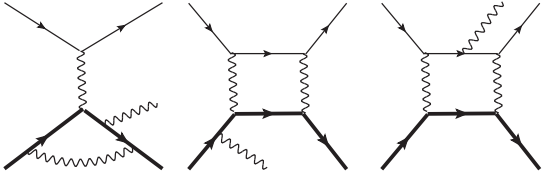
- **tree-level Z-exchange important** at the 10^{-5} level
- purely weak RCs (in QED NLO units) at a few 10^{-6} level

Second step, towards *photonic* radiative corrections at **NNLO**

Carloni Calame et al., JHEP 11 (2020) 028

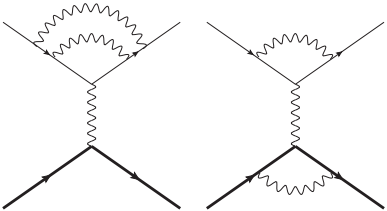
calculated exactly

- $|$ NLO virtual diagrams $|^2$
- interference of LO $\mu e \to \mu e \gamma$ amplitude with



$+$ many others

- interference of LO $\mu e \to \mu e$ amplitude with



$+$ many others

2-loop QED vertex form factors borrowed from **Mastrolia and Remiddi, NPB 664 (2003) 341**

Second step, towards *photonic* radiative corrections at **NNLO**

JHEP 11 (2020) 028

- interference of LO $\mu e \to \mu e$ amplitude with

approximated à la YFS

$+$ many others

⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the infra-red divergent structure

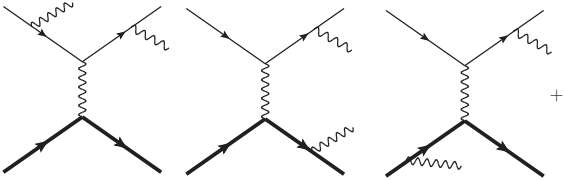
$$\widetilde{\mathcal{A}}^{\alpha^2} = \underbrace{\mathcal{A}_e^{\alpha^2} + \mathcal{A}_\mu^{\alpha^2} + \mathcal{A}_{e\mu,\,\mathsf{1L\times 1L}}^{\alpha^2}}_{\textbf{exact}} + \underbrace{\frac{1}{2} Y_{e\mu}^2 \mathcal{T} + Y_{e\mu}\left(Y_e + Y_\mu\right)\mathcal{T} + \left(Y_e + Y_\mu\right)\mathcal{A}_{e\mu}^{\alpha^1,\mathsf{R}} + Y_{e\mu}\mathcal{A}^{\alpha^1,\mathsf{R}}}_{\textbf{YFS approximated}}$$

Second step, towards *photonic* radiative corrections at **NNLO**

JHEP 11 (2020) 028

- squared absolute value of

calculated exactly



$+$ many others

⇝ **we estimate the subset of amplitudes in YFS approximation to miss terms of order**

$$\left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(m_\mu^2/m_e^2\right) \simeq 5 \times 10^{-4}$$

- going beyond this requires the full two-loop virtual amplitudes

R. Bonciani *et al.*, PRL 128 (2022) 2; **see talks by J. Ronca and Y. Ulrich**

⇝ detailed comparisons ongoing with the independent Monte Carlo code McMule (PSI)

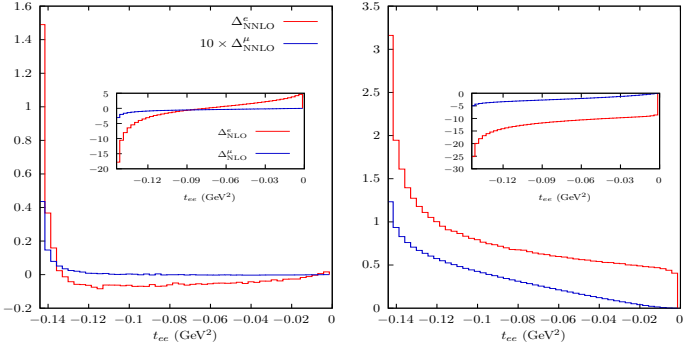
NNLO results

JHEP 11 (2020) 028

- Showing

$$\Delta^i_{\text{NNLO}} \equiv 100 \times \frac{d\sigma^i_{\text{NNLO}} - d\sigma^i_{\text{NLO}}}{d\sigma_{\text{LO}}}$$

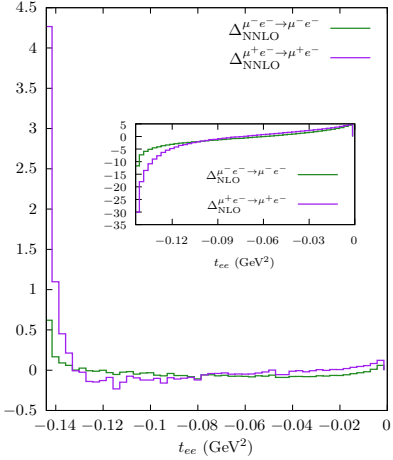
⇝ exact NNLO radiation from electron or muon leg, with or without acoplanarity cut

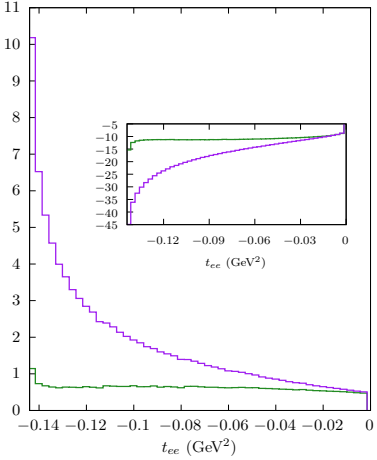


(approximated) photonic NNLO results

JHEP 11 (2020) 028

⇝ approximated NNLO[1] radiation for incoming μ^+ or μ^-, with or without acoplanarity cut



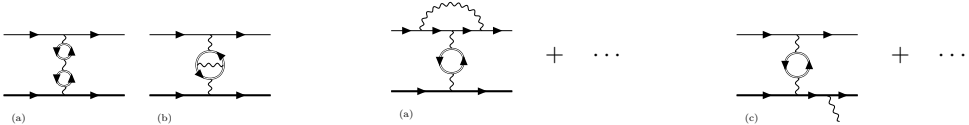


[1] of course with "double boxes" in YFS approximation

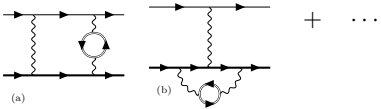
NNLO Virtual leptonic pairs (vacuum polarization insertions)

E. Budassi et al., JHEP 11 (2021) 098

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes



- interfered with $\mu e \to \mu e$ amplitude



Here the 2-loop integral is evaluated with dispersion relation techniques

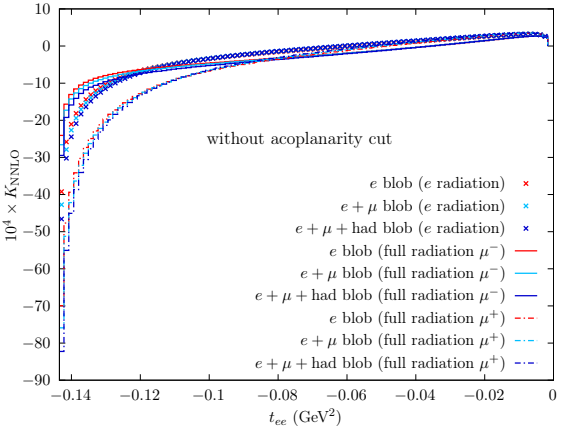
used in the past for Bhabha: Actis et al., Phys. Rev. Lett. 100 (2008) 131602; Carloni Calame et al., JHEP 07 (2011) 126

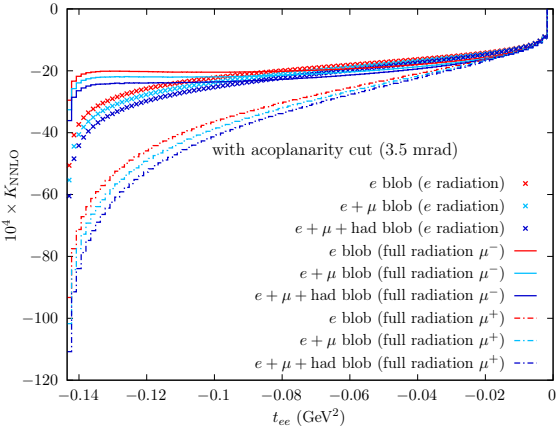
and for hadr. corr. in MUonE: Fael & Passera, PRL 122 (2019) 19

$$\frac{g_{\mu\nu}}{q^2 + i\epsilon} \to g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{R_\ell(z)}{q^2 - z + i\epsilon} = g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_\ell^2}{2z}\right) \sqrt{1 - \frac{4m_\ell^2}{z}}$$

Virtual pair effects $K_{\mathrm{NNLO}} = d\sigma^{\alpha^2}_{N_f}/d\sigma_{\mathrm{NLO}}$

JHEP 11 (2021) 098

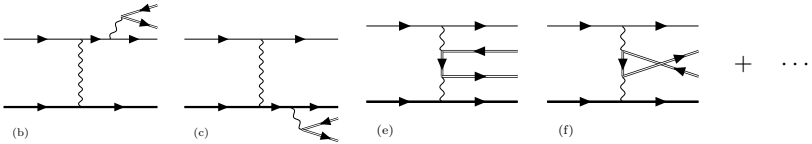




Real pair emissions

E. Budassi et al., JHEP 11 (2021) 098

- they also contribute at NNLO
- squared absolute vaule of



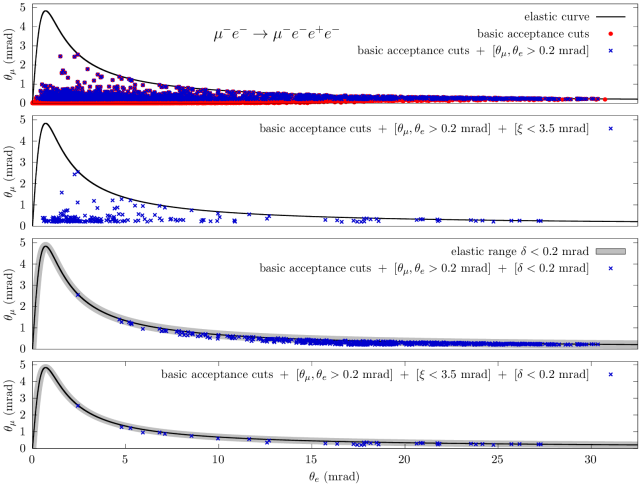
- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large (reducible) background, because of the presence of "peripheral" diagrams

⇝ A set of experimental cuts is needed to get rid of it.
In addition to basic cuts (exactly one muon-like and one electron-like, with $E \geq 1$ GeV, particle in the detector), we consider

1. $\theta_{\mu\text{-like}}, \theta_{e\text{-like}} \geq \theta_c = 0.2$ mrad
2. acoplanarity ≤ 3.5 mrad
3. geometric distance from the elastic curve in the $[\theta_\mu, \theta_e]$ plane < 0.2 mrad

Real e^+e^- pairs

E. Budassi et al., JHEP 11 (2021) 098

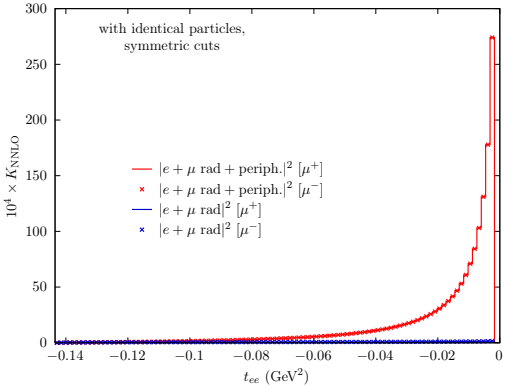


⇝ only 0.007% of $\mu e \to \mu e\, e^+ e^-$ events survives the combination of the three cuts

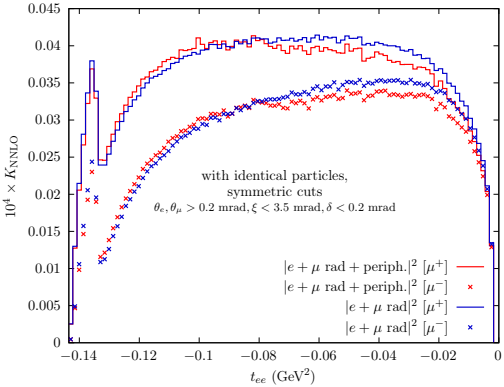
Real e^+e^- pairs

E. Budassi et al., JHEP 11 (2021) 098

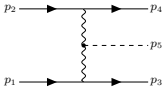
only basic cuts



with extra cuts



- **π^0 production**



  - The process $\mu e \to \mu e \pi^0$ with $\pi^0 \to \gamma\gamma$ as possible background, using a phenomenological model for the $\gamma^\star\gamma^\star\pi^0$ effective vertex

  ⇝ not an issue in the signal region

E. Budassi et al., PLB 829 (2022) 137138

  ⇝ perhaps to be considered for NP searches in phase space region outside the signal one

- **robustness of the measurement against possible New Physics "contamination" has been studied**

A. Masiero, P. Paradisi and M. Passera, arXiv:2002.05418

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, arXiv:2002.04822

- **interesting proposals for New Physics searches at MUonE (new light mediators)**
  - invisibly decaying light Z' in $\mu e \to \mu e Z'$

Asai et al., arXiv:2109.10093

  - long-lived mediators with displaced vertex signatures

Galon et al., arXiv:2202.08843

  - through scattering off the target nuclei $\mu N \to \mu N X$

Grilli di Cortona and E. Nardi, arXiv:2204.04227

Summary

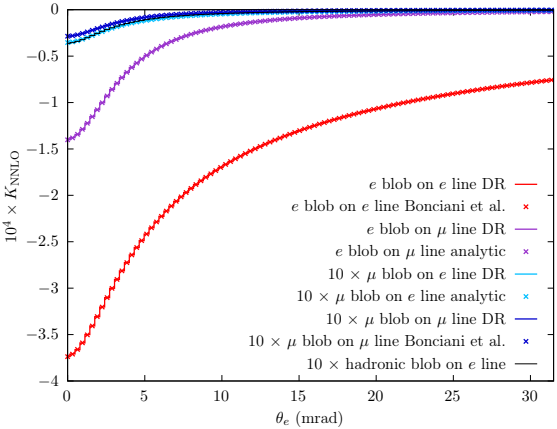
- Carloni Calame et al., PLB 746 (2015), 325
- Abbiendi et al., Eur. Phys. J. C77 (2017), 139
- Mastrolia et al., JHEP 11 (2017) 198
- Di Vita et al., JHEP 09 (2018) 016
- Alacevich et al., JHEP 02 (2019) 155
- Fael and Passera, PRL 122 (2019) 19, 192001
- Fael, JHEP 02 (2019) 027
- Carloni Calame et al., JHEP 11 (2020) 028
- Banerjee et al., SciPost Phys. 9 (2020), 027
- Banerjee et al., EPJC 80 (2020) 6, 591
- Budassi et al., JHEP 11 (2021) 098
- Balzani et al., arXiv:2112.05704 [hep-ph]
- Bonciani et al., PRL 128 (2022) 2, 022002
- Budassi et al., PLB 829 (2022) 137138

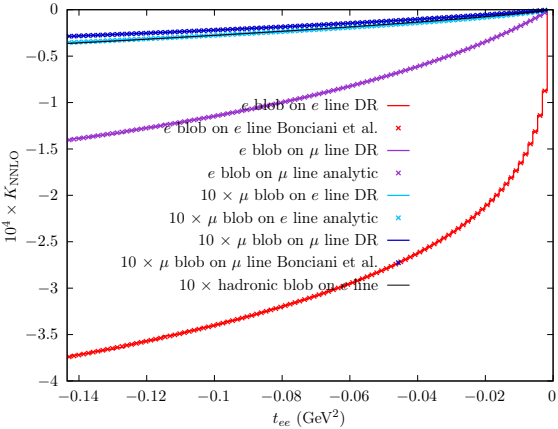
- A lively theory community is active to provide state-of-the-art calculations to match the required accuracy for meaningful data analysis
- Independent numerical codes (Monte Carlo generators and/or integrators) are developed and cross-checked to validate high-precision calculations. Chiefly
  - ✓ **Mesmer** in Pavia
    `github.com/cm-cc/mesmer`
  - ✓ **McMule** at PSI/IPPP
    `gitlab.com/mule-tools/mcmule`
- An international MUonE collaboration is growing

THANK YOU

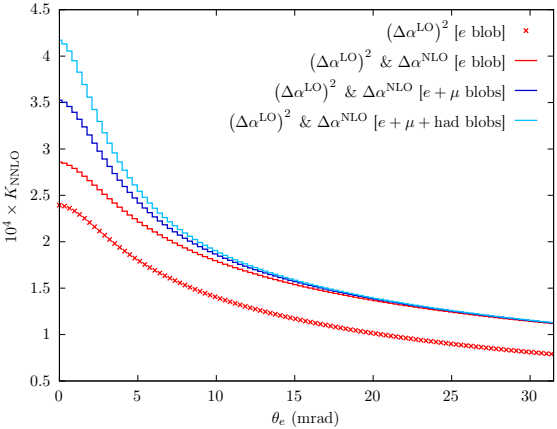
SPARES

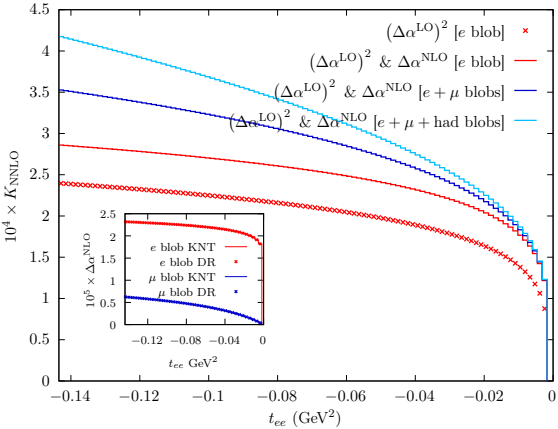
Virtual leptonic (and hadronic NNLO) vertex corrections





Virtual leptonic (and hadronic) NNLO VP corrections





Virtual pair effects

E. Budassi et al., JHEP 11 (2021) 098