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# NNLO calculations for low-energy experiments

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non-exhaustive list of experiments as implemented in McMULE

process	experiment	physics motivation
$e\mu \rightarrow e\mu$	MUonE	HVP to $(g - 2)_\mu$
$\ell p \rightarrow \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge normalisation
$e^-e^- \rightarrow e^-e^-$	Prad 2	$\sin \theta_W^2$ at low $Q^2$
$e^+e^- \rightarrow e^+e^-$	MOLLER, ...	luminosity measurement
$ee \rightarrow ll$	any $e^+e^-$ collider	$R$ -ratio
$ee \rightarrow \gamma\gamma$	VEPP, BES, Daphne, ... Belle	$\tau$ properties dark searches
$ee \rightarrow \gamma\gamma$	Daphne	luminosity measurement
$ee \rightarrow \gamma\gamma$	any $e^+e^-$ collider	
$\mu \rightarrow \nu\bar{\nu}e$	MEG DUNE	ALP searches beam-line profiling

QCD @ LHC	$\Leftrightarrow$	QED @ low & medium energy	
non-abelian	$\gtrsim$	abelian	matrix elements somewhat easier
non-abelian	$\gg$	abelian	IR structure <b>much easier</b> ①
massless fermions	$\ll$	massive fermions	loop amplitudes <b>much harder</b> ②
jets	$<$	exclusive w.r.t. collinear radiation	numerics <b>harder</b> $\supset \log(m^2/Q^2) \equiv L$ <b>much harder</b> for small masses ③

### stealing from QCD

- master integrals (reduction and computation), automated tools, EFT methods
- use dimensional regularisation for IR singularities, not photon mass
- use subtraction method for phase-space integration, not slicing method
- for the future: match fixed-order result to parton shower

soft singularities exponentiate [Yennie, Frautschi, Suura 61]

- universal soft limit  $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure  $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} = \text{finite}$

use this to construct an all-order subtraction scheme FKS<sup>ℓ</sup>

- nothing complicated needed higher than  $\mathcal{O}(\epsilon^0)$
- only one universal CT:  $\hat{\mathcal{E}}$

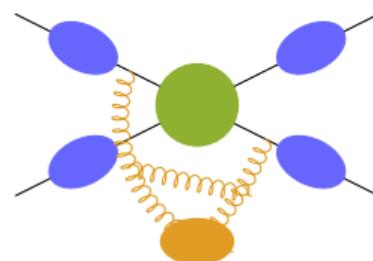
$$\underbrace{\int d\Phi_\gamma \text{ (diagram with grey blob)}}_{\text{divergent and complicated}} = \underbrace{\int d\Phi_\gamma \left( \text{ (diagram with grey blob)} - \text{ (diagram with green blob)} \right)}_{\text{complicated but finite}} + \underbrace{\int d\Phi_\gamma \text{ (diagram with green blob)}}_{\text{divergent but easy}}$$

masses are physical in QED  $\Rightarrow$  keep masses

- drop polynomially suppressed terms at two-loop  $\rightarrow$  error  $\sim \left(\frac{\alpha}{\pi}\right)^2 \log \frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions  
[Penin 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g.  $ee \rightarrow ee$  at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$

- soft: process-dependent  $S = 1 + \text{fermion loops}$   
 $\rightarrow$  compute separately anyway to combine with hadron loops
- collinear: universal  $Z$ , converts  $1/\epsilon \rightarrow \log(m^2/Q^2)$
- hard: massless calculation



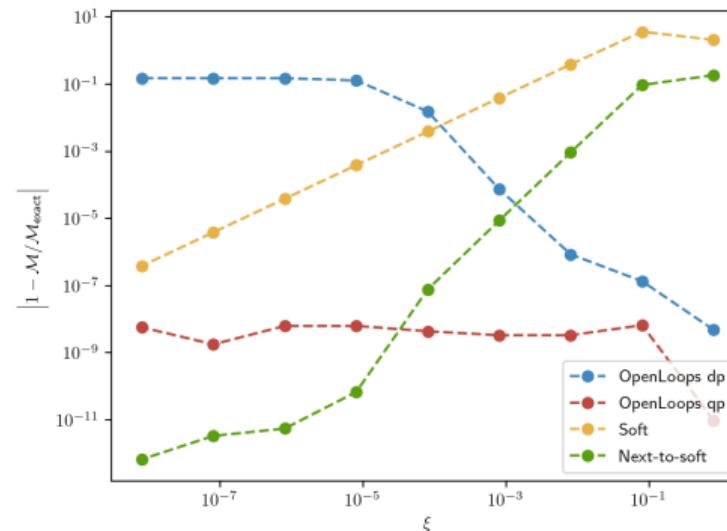
real-virtual corrections trivial in principle, extremely delicate numerically



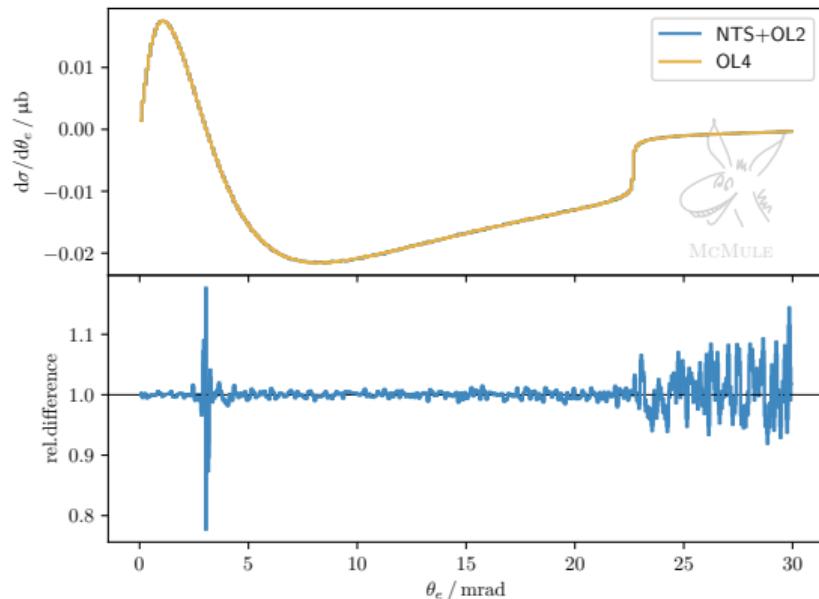
example  $ee \rightarrow ee\gamma$

[Banerjee, Engel, Schalch, Signer, YU 21]

- soft limit (of collinear emission)  
 $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs dp, qp, eikonal, NTS
- stability problem solved & speed-up



test next-to-soft stabilisation vs OL4 (OpenLoops quad) for  $\mu e \rightarrow \mu e$  real-virtual

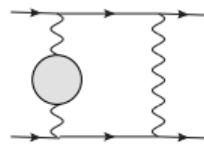


- same statistics, same result
  - 70 days vs 4 days
  - integrated results for different cuts
- ⇒ this is **not** an approximation but a numerical tool

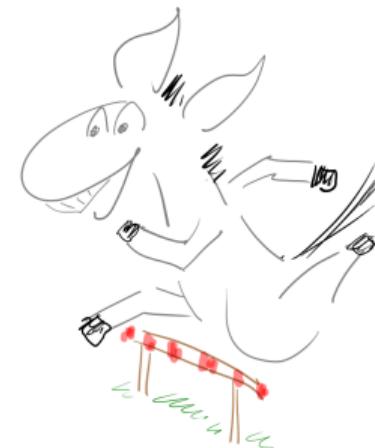
NTS	OL4
-0.29268(4)	-0.29267(4)
-0.44789(6)	-0.44778(6)
-0.64662(9)	-0.64649(9)

## a few more hurdles

- VP diagrams for  $e/\mu/\tau/\text{had}/\dots$  numerically with full mass dependence

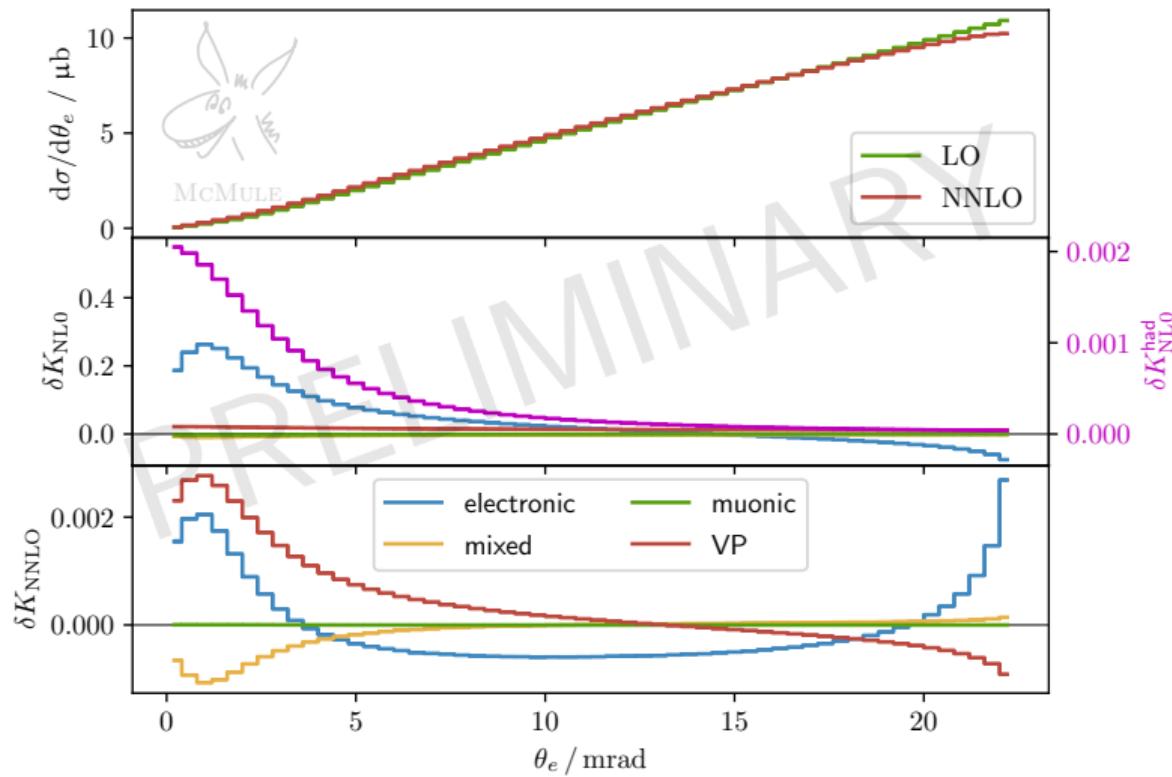


- collinear pseudo-singularities  $\lim_{\gamma \rightarrow 0} \triangle(p_\gamma, p_i) \Rightarrow L$
  - phase-space tuning s.t.  $\cos \triangle \sim x_i$
- $\Rightarrow$  at most one small angle  $\rightarrow$  FKS partitioning



[Signer 22]

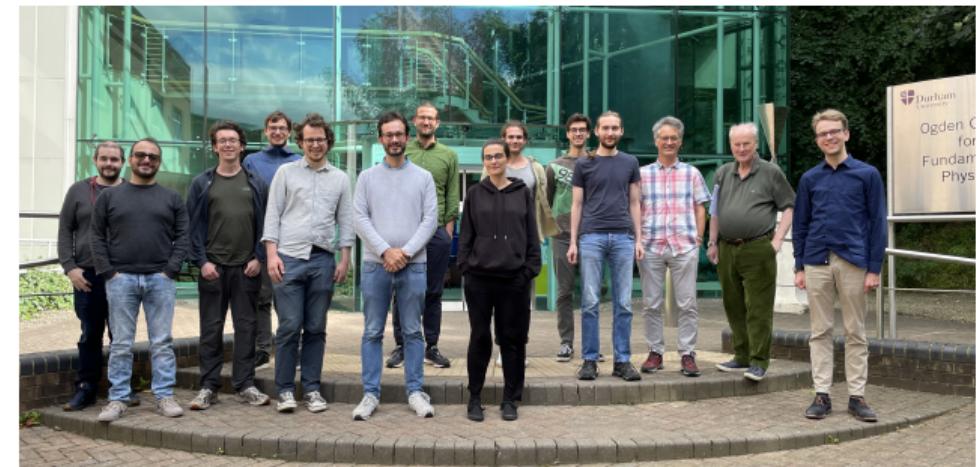
$E_{\text{beam}}^\mu = 150 \text{ GeV}$ ,  $E_e > 1 \text{ GeV}$ ,  $\theta_\mu > 0.3 \text{ mrad}$  [Broggio, Engel, Ferroglio, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]



the NNLO era is here, not only for QCD, also for QED

### future steps

- NNLO QED $\oplus$  EW
  - NNLO QED $\oplus$  PS
  - higher energies
  - massification for real corrections
  - collinear stabilisation
- N<sup>3</sup> LO for  $\gamma^* \rightarrow ll$   
 $\Rightarrow$  Workstop in Durham





f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (PSI), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (→ Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Durham), A.Gurgone (Pavia)



McMULE  
[mule-tools.gitlab.io](https://mule-tools.gitlab.io)