# Zero-jettiness resummation for top-quark pair production at the LHC 

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Based on arXiv:2111.03632, S. Alioli, A. Broggio, MAL


## Motivation

- Top-quark properties are highly interesting (vacuum stability, large coupling to Higgs sector)
- Pair production known at NNLO - why do we need more precision for $t \bar{t}$ ?

NNLO study of $t \bar{t}$ with decay in 1901.05407 shows why! Extrapolation from fiducial to inclusive phase space is done using NLO event generators desirable to have NNLO+PS calculations.

1901.05407 A. Behring, M. Czakon, A. Mitov, A. Papanastasiou,

## Matching NNLO to parton showers in an event generator

- Several methods available to match NNLO to PS, mostly formulated for colour-singlet processes.
- Recently, NNLO+PS for $t \bar{t}$ available via MINNLOPS formalism.
- Higher-order resummation can improve description of observables, included in NNLO+PS formulation via Gen Eva.




## NNLO+PS with the Geneva method

What is needed to do NNLO+PS with Geneva?
See also talks by G. Marinelli, D. Napoletano

- NLO calculations for $t \bar{t}, t \bar{t}+j e t$ matched to PS
- Resummed calculation at NNLL' in a resolution variable.

The resummed calculation can come from anywhere! Options?

- $q_{T}$ resummation, either via SCET (NNLL in 1307.2464) or direct QCD (NNLL in 1408.4564, 1806.01601, NNLL' ingredients in 1809.01459, 1901.04005)
- Full resummation ingredients not publicly available.
- $N$-jettiness resummation - used for colour-singlet in Gen eva, must be adapted for $t \bar{t}$.

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## $N$-jettiness

- $N$-jettiness is a global physical observable with definitions for hadron colliders in terms of beam $q_{a, b}$ and jet-directions $q_{j}$

$$
\mathcal{T}_{N}=\frac{2}{Q} \sum_{k} \min \left\{q_{a} \cdot p_{k}, q_{b} \cdot p_{k}, q_{1} \cdot p_{k}, \ldots, q_{N} \cdot p_{k}\right\}
$$



- $\mathcal{T}_{N} \rightarrow 0$ for $N$ pencil-like jets, $\mathcal{T}_{N} \gg 0$ spherical limit.
- Definition must be adapted with top-quarks in final state when calculating $\mathcal{T}_{0}$, choose to treat them like EW particles and exclude them from the sum.


## Resummation from EFT

- Effective field theories such as SCET separate energy scales
- This allows factorisation theorems to be derived, which feature only single scale objects
- Each single scale object can be evaluated at fixed order, at a scale where no large logs are present
- RGE running is used to evolve all pieces to a common scale and resum large logs.



## Factorisation for N -jettiness

For colour-singlet production, the factorisation theorem reads

$$
\begin{aligned}
\frac{\mathrm{d} \sigma^{\mathrm{NNLL}}{ }^{\prime}}{\mathrm{d} \Phi_{0} \mathrm{~d} \mathcal{T}_{0}}=\sum_{i j} \int & \mathrm{~d} t_{a} \mathrm{~d} t_{b} B_{i}\left(t_{a}, x_{a}, \mu\right) B_{j}\left(t_{b}, x_{b}, \mu\right) \\
& \times H_{i j}\left(\Phi_{0}, \mu\right) S\left(\mathcal{T}_{0}-\frac{t_{a}+t_{b}}{Q}, \mu\right) .
\end{aligned}
$$

For $t \bar{t}$, initial- and final-state lines can 'talk' to each other through exchange of soft gluons. Hard and soft functions are matrices in colour space!

We have derived the $t \bar{t}$ case using soft-collinear effective theory and heavy-quark effective theory - it reads

$$
\begin{aligned}
\frac{\mathrm{d} \sigma^{\mathrm{NNLL}}}{\mathrm{~d} \Phi_{0} \mathrm{~d} \mathcal{T}_{0}}=\sum_{i j} \int & \mathrm{~d} t_{a} \mathrm{~d} t_{b} B_{i}\left(t_{a}, x_{a}, \mu\right) B_{j}\left(t_{b}, x_{b}, \mu\right) \\
& \times \operatorname{Tr}\left\{\mathrm{H}_{i j}\left(\Phi_{0}, \mu\right) \mathrm{S}\left(\mathcal{T}_{0}-\frac{t_{a}+t_{b}}{Q}, \Phi_{0}, \mu\right)\right\}
\end{aligned}
$$

## Factorisation for N -jettiness

In Laplace space, convolutions between functions become products, solving evolution equations is easier. Factorisation formula reads:

$$
\begin{aligned}
\mathcal{L}\left[\frac{\mathrm{d} \sigma^{\text {NNLL }^{\prime}}}{\mathrm{d} \Phi_{0} \mathrm{~d} \mathcal{T}_{0}}\right]=\sum_{i j} \tilde{B}_{i} & \left(\ln \left(\frac{M \kappa}{\mu^{2}}\right)\right) \tilde{B}_{j}\left(\ln \left(\frac{M \kappa}{\mu^{2}}\right)\right) \\
& \times \operatorname{Tr}\left\{\mathrm{H}_{i j} \tilde{S}\left(\ln \frac{\kappa^{2}}{\mu^{2}}\right)\right\}
\end{aligned}
$$

Soft function is a polynomial in $L=\ln \kappa^{2} / \mu^{2}$ with function-valued coefficients.

## The hard function

The hard function arises from the matching of QCD onto SCET - can be extracted from colour-decomposed loop amplitudes. At one-loop, was first computed in 1003.5827.

It obeys the RG equation:

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \mathrm{H}\left(M, \beta_{\mathrm{t}}, \theta, \mu\right)=\Gamma_{H}\left(M, \beta_{\mathrm{t}}, \theta, \mu\right) \mathrm{H}\left(M, \beta_{\mathrm{t}}, \theta, \mu\right)+\text { h.c. }
$$

with solution

$$
\mathbf{H}\left(M, \beta_{\mathrm{t}}, \theta, \mu\right)=\mathbf{U}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}, \mu\right) \mathbf{H}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}\right) \mathbf{U}^{\dagger}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}, \mu\right)
$$

where

$$
\mathbf{U}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right)=\exp \left[2 S\left(\mu_{h}, \mu\right)-a_{\Gamma}\left(\mu_{h}, \mu\right)\left(\ln \frac{M^{2}}{\mu_{h}^{2}}-i \pi\right)\right] \mathbf{u}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}, \mu\right)
$$

## The hard function

$\mathbf{U}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right)=\exp \left[2 S\left(\mu_{h}, \mu\right)-a_{\Gamma}\left(\mu_{h}, \mu\right)\left(\ln \frac{M^{2}}{\mu_{h}^{2}}-i \pi\right)\right] \mathbf{u}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right)$
We have split the anomalous dimension

$$
\Gamma_{H}\left(M, \beta_{t}, \theta, \mu\right)=\Gamma_{\text {cusp }}\left(\alpha_{S}\right)\left(\ln \frac{M^{2}}{\mu^{2}}-i \pi\right)+\gamma^{h}\left(M, \beta_{t}, \theta, \alpha_{S}\right)
$$

into cusp and non-cusp parts.
Double and single logarithmic resummation is provided by the functions

$$
\begin{aligned}
S\left(\mu_{a}, \mu_{b}\right) & =-\int_{\alpha_{s}\left(\mu_{a}\right)}^{\alpha_{S}\left(\mu_{b}\right)} \mathrm{d} \alpha \frac{\Gamma_{\text {cusp }}(\alpha)}{\beta(\alpha)} \int_{\alpha_{s}\left(\mu_{a}\right)}^{\alpha} \frac{\mathrm{d} \alpha^{\prime}}{\beta\left(\alpha^{\prime}\right)}, \\
a_{\Gamma}\left(\mu_{a}, \mu_{b}\right) & =-\int_{\alpha_{s}\left(\mu_{a}\right)}^{\alpha_{5}\left(\mu_{b}\right)} \mathrm{d} \alpha \frac{\Gamma_{\text {cusp }}(\alpha)}{\beta(\alpha)} .
\end{aligned}
$$

## The hard function

$$
\mathrm{U}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}, \mu\right)=\exp \left[2 S\left(\mu_{h}, \mu\right)-a_{\Gamma}\left(\mu_{h}, \mu\right)\left(\ln \frac{M^{2}}{\mu_{h}^{2}}-i \pi\right)\right] \mathbf{u}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}, \mu\right)
$$

The off-diagonal, non-cusp evolution is instead provided by the colour matrix

$$
\mathrm{u}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}, \mu\right)=\mathcal{P} \exp \int_{\alpha_{s}\left(\mu_{h}\right)}^{\alpha_{\mathrm{s}}(\mu)} \frac{\mathrm{d} \alpha}{\beta(\alpha)} \gamma^{h}\left(M, \beta_{\mathrm{t}}, \theta, \alpha\right),
$$

where $\mathcal{P}$ specifies the path-ordering operator. We evaluate the matrix exponential $\mathbf{u}$ as a series expansion in $\alpha_{s}$.

## Evaluating the non-cusp evolution matrices

Diagonalise by finding the matrix $\wedge$ s.t.

$$
\gamma_{D}^{h(0)}=\boldsymbol{\Lambda}^{-1} \gamma^{h(0)} \boldsymbol{\Lambda}
$$

Expanding in $\alpha_{s}$ then gives

$$
\begin{aligned}
\mathrm{u}^{\mathrm{NNLL}}\left(\beta_{t}, \theta, \mu_{h}, \mu\right)= & {\left[\Lambda\left(1+\frac{\alpha_{s}(\mu)}{4 \pi} K\right)\left(\left[\frac{\alpha_{S}\left(\mu_{h}\right)}{\alpha_{S}(\mu)}\right]^{\frac{\overline{\frac{\gamma}{h}}^{2(0)}}{2 \beta_{0}}}\right)_{D}\right.} \\
& \left.\left(1-\frac{\alpha_{s}\left(\mu_{h}\right)}{4 \pi} K\right) \Lambda^{-1}\right]_{\mathcal{O}\left(\alpha_{s}\right)}
\end{aligned}
$$

Matrix $K$ has entries given by

$$
K_{i j}=\delta_{i j} \vec{\gamma}_{i}^{h(0)} \frac{\beta_{1}}{2 \beta_{0}^{2}}-\frac{\left[\boldsymbol{\Lambda}^{-1} \gamma^{h(1)} \boldsymbol{\Lambda}\right]_{i j}}{2 \beta_{0}+\vec{\gamma}_{i}^{h(0)}-\vec{\gamma}_{j}^{h(0)}} .
$$

## The soft function

Our factorisation formula defines a new soft function which must be computed to at least one-loop.


## The soft function

Soft function RG equation in Laplace space given by

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \tilde{\mathrm{~S}}_{B}\left(L, \beta_{\mathrm{t}}, \theta, \mu\right)=\left[\Gamma_{\mathrm{cusp}} L-\gamma^{\mathrm{s}^{\dagger}}\right] \tilde{\mathrm{S}}_{B}\left(L, \beta_{\mathrm{t}}, \theta, \mu\right)+\text { h.c. }
$$

Given the one-loop soft function, we can solve this at fixed order to obtain the logarithmic terms of the two-loop function. The boundary term remains undetermined and must be computed separately.

All-order solution in momentum space given by

$$
\begin{aligned}
\mathrm{S}_{B}\left(l^{+}, \beta_{t}, \theta, \mu\right)= & \exp \left[4 \mathrm{~S}\left(\mu_{\mathrm{s}}, \mu\right)+2 a_{\gamma^{B}}\left(\mu_{\mathrm{s}}, \mu\right)\right] \\
& \times \mathrm{u}^{\dagger}\left(\beta_{t}, \theta, \mu, \mu_{\mathrm{S}}\right) \tilde{\mathrm{S}}_{B}\left(\partial_{\eta_{s}}, \beta_{\mathrm{t}}, \theta, \mu_{\mathrm{S}}\right) \mathrm{u}\left(\beta_{t}, \theta, \mu, \mu_{\mathrm{S}}\right) \\
& \times \frac{1}{l^{+}}\left(\frac{l^{+}}{\mu_{\mathrm{S}}}\right)^{2 \eta_{\mathrm{s}}} \frac{e^{-2 \gamma_{\varepsilon} \eta_{s}}}{\Gamma\left(2 \eta_{\mathrm{s}}\right)}
\end{aligned}
$$

where $\eta_{s} \equiv-2 a_{\Gamma}\left(\mu_{\mathrm{s}}, \mu\right)$.

## The beam function

Beam functions are given by convolutions of perturbative kernels with the normal PDFs $f_{i}(x, \mu)$ :

$$
B_{i}(t, z, \mu)=\sum_{j} \mathcal{I}_{i j}(t, z, \mu) \otimes f_{j}(z, \mu)
$$

The $\mathcal{I}_{i j}$ are available up to $\mathrm{N}^{3} \mathrm{LO}$ and are process independent.
RG equation in Laplace space given by

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \tilde{B}_{i}\left(L_{c}, z, \mu\right)=\left[-2 \Gamma_{\text {cusp }}\left(\alpha_{s}\right) L_{c}+\gamma_{i}^{B}\left(\alpha_{s}\right)\right] \tilde{B}_{i}\left(L_{c}, z, \mu\right),
$$

with solution in momentum space
$B(t, z, \mu)=\exp \left[-4 S\left(\mu_{B}, \mu\right)-a_{\gamma^{B}}\left(\mu_{B}, \mu\right)\right] \tilde{B}\left(\partial_{\eta_{B}}, z, \mu_{B}\right) \frac{1}{t}\left(\frac{t}{\mu_{B}^{2}}\right)^{\eta_{B}} \frac{e^{-\gamma_{E} \eta_{B}}}{\Gamma\left(\eta_{B}\right)}$
where $\eta_{B} \equiv 2 a_{\Gamma}\left(\mu_{B}, \mu\right)$ and the collinear log is given by
$L_{c}=\ln \left(M \kappa / \mu^{2}\right)$.
1401.5478,1405.1044 J. Gaunt, M. Stahlhofen, F. Tackmann

## Resummed $\mathcal{T}_{0}$ distribution

We can combine our solutions for the hard, soft and beam functions to obtain:

$$
\begin{aligned}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Phi_{0} \mathrm{~d} \mathcal{T}_{0}}= & U\left(\mu_{h}, \mu_{B}, \mu_{\mathrm{S}}, L_{h}, L_{s}\right) \\
\times & \operatorname{Tr}\left\{\mathbf{u}\left(\beta_{t}, \theta, \mu_{h}, \mu_{\mathrm{S}}\right) \mathrm{H}\left(M, \beta_{\mathrm{t}}, \theta, \mu_{h}\right) \mathbf{u}^{\dagger}\left(\beta_{t}, \theta, \mu_{h}, \mu_{\mathrm{S}}\right)\right. \\
& \left.\tilde{\mathrm{S}}_{B}\left(\partial_{\eta_{\mathrm{s}}}+L_{\mathrm{S}}, \beta_{t}, \theta, \mu_{\mathrm{S}}\right)\right\} \\
\times & \tilde{B}_{a}\left(\partial_{\eta_{B}}+L_{B}, z_{a}, \mu_{B}\right) \tilde{B}_{b}\left(\partial_{\eta_{B}^{\prime}}+L_{B}, z_{b}, \mu_{B}\right) \frac{1}{\mathcal{T}_{0}^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_{E} \eta_{\mathrm{tot}}}}{\Gamma\left(\eta_{\text {tot }}\right)}
\end{aligned}
$$

where $L_{s}=\ln \left(M^{2} / \mu_{\mathrm{s}}^{2}\right), L_{h}=\ln \left(M^{2} / \mu_{h}^{2}\right), L_{B}=\ln \left(M^{2} / \mu_{B}^{2}\right)$,
$\eta_{\text {tot }}=2 \eta_{\mathrm{S}}+\eta_{\mathrm{B}}+\eta_{\mathrm{B}}^{\prime}$. Valid at arbitrary logarithmic order.

## Resummed $\mathcal{T}_{0}$ distribution

We have:

- The hard function at 1-loop (some 2-loop ingredients in principle known but not included)
- The soft function at 1-loop, with logarithmic 2-loop terms
- The beam function at 2-loops.

This is enough to resum large logarithms at NNLL. Including the known 2-loop terms of the soft function, we miss only terms in the hard and soft at 2 -loops $\propto \delta\left(\mathcal{T}_{0}\right)$ - we call this NNLL' ${ }^{\prime}$.

## Constructing approximate NNLO distributions

By evaluating our master formula with fixed scales, we can construct an approximate ( N )NLO formula which should reproduce the FO behaviour for $\mathcal{T}_{0}>0$ in the small $\mathcal{T}_{0}$ limit.



## Numerical results for resummed $\mathcal{T}_{0}$




## Matching to fixed order

We use an additive matching to the fixed order calculation:

$$
\frac{\mathrm{d} \sigma^{\text {match }}}{\mathrm{d} \mathcal{T}_{0}}=\frac{\mathrm{d} \sigma^{\text {resum }}}{\mathrm{d} \mathcal{T}_{0}}+\frac{\mathrm{d} \sigma^{\mathrm{FO}}}{\mathrm{~d} \mathcal{T}_{0}}-\left[\frac{\mathrm{d} \sigma^{\text {resum }}}{\mathrm{d} \mathcal{T}_{0}}\right]_{\mathrm{FO}}
$$

Profile scales are used to smoothly turn off the resummation.



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$$

Profile scales are used to smoothly turn off the resummation.




## Conclusions

- We have derived a factorisation formula for the 0-jettiness observable in $t \bar{t}$ production using SCET+HQET.
- We have calculated the relevant soft function at 1-loop order with partial 2-loop information.
- Using this and known hard and beam functions, we are able to resum large logs up to approximate NNLL' accuracy.
- Our matched calculation is the most accurate available for a jet resolution variable in tet production.
- Future knowledge of the 2-loop hard and soft functions will allow a full NNLL' resummation.
- Applications to NNLO+PS event generation in Geneva as well as NNLO slicing computations in MCFM.

Backup slides

## Numerical results for resummed $\mathcal{T}_{0}$

We evaluate $u$ by expansion - should we also expand U?



## Profile scales

- Resummation is switched off via profile scales - when hard, beam and soft scales become equal, RGE evolution stops.
- Scales are continuous functions of the resolution variable.
- Transition points determined by examination of size of singular vs nonsingular contribution as a function of $\tau$.



[^0]:    1307.2464 Li H.T., Li C.S., Shao D.Y., Yang L.L., Zhu H.X.
    1408.4564 S. Catani, M. Grazzini, A. Torre, 1806.01601 S. Catani, M. Grazzini, H. Sargsyan, 1901.04005 S. Catani, S. Devoto, M. Grazzini, S. Kallweit,
    J. Mazzitelli, H. Sargsyan
    1809.01459 R. Angeles-Martinez, M. Czakon, S. Sapeta

