# Two-loop Beam Functions for Jet-veto resummation

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with G. Bell, K. Brune and M. Wald

based on arXiv:2207.05578

High Precision for Hard Processess
22 September, 2022

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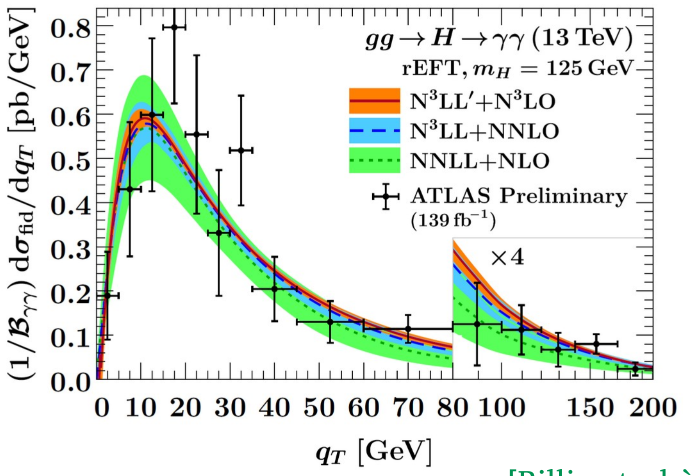
# Plan

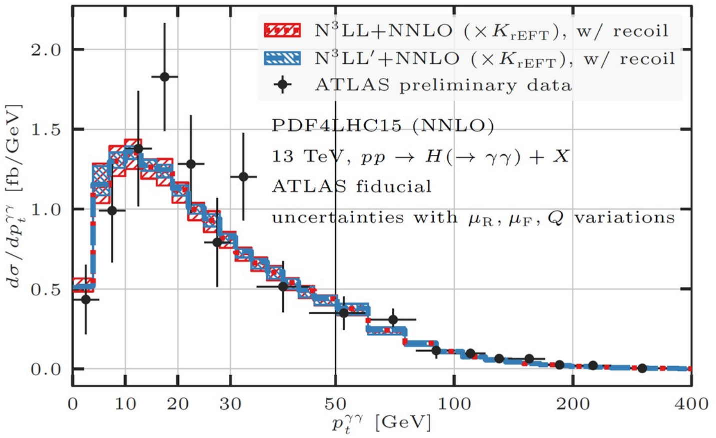
- Overview
- Calculational details
- Results
- Outlook

# Motivation

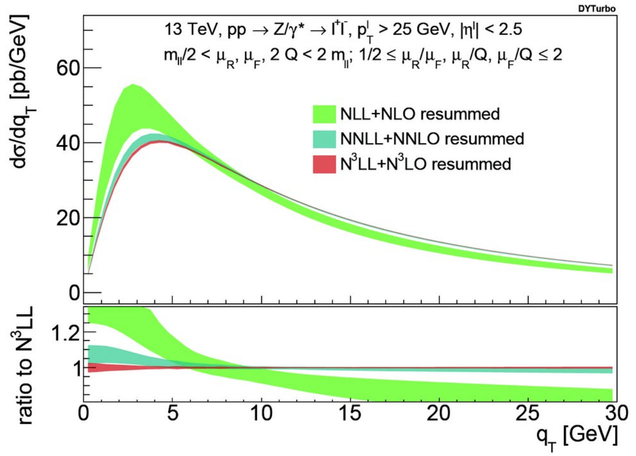
- Resummation is useful to correctly describe observables at colliders




[Billis et al `21]




[Re et al `21]




[Camarda et al `21]

- Often experimental analyses involve cut on extra radiations to enhance signal-background ratio – jet vetoes

See talk by K. Ellis

  - Reject events based on transverse momenta $p_T^{\rm jet} < p_T^{\rm veto}$

[ Becher, Neubert, +Rothen `12,`13
Banfi, Monni, Salam, Zanderighi `12
Stewart, Tackmann, Walsh, Zuberi `13]

  - Veto is also possible on Rapidity

[Gangal et al `14,`20,
Hornig et al `17,
Michel et. al, 18]

# Motivation

- Jet-vetoes introduce large logarithms which needs to be resummed

$$\ln\left(p_T^{\text{veto}}/Q\right)$$

- Measurement at NNLO

$$\omega_2(\{k,l\}) = \theta(\Delta - R)\max\{|\vec{k}_T|,|\vec{l}_T|\} + \theta(R-\Delta)|\vec{k}_T + \vec{l}_T|$$

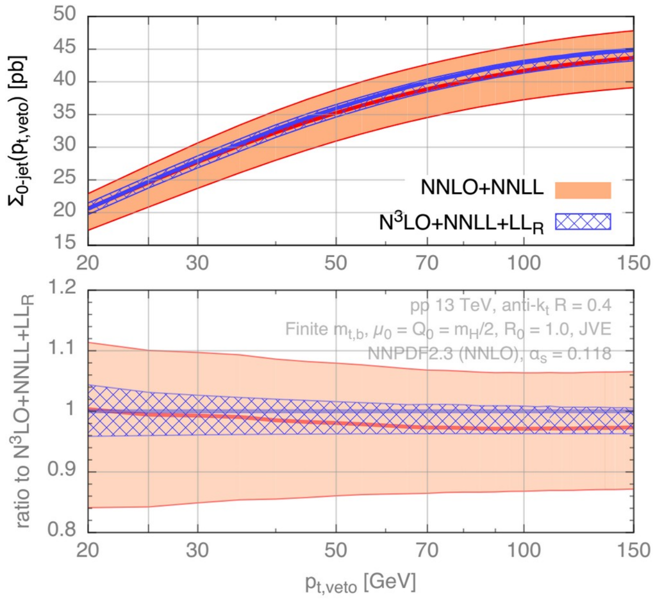
- State-of-the-art N3LO+N2LL+LLR [Banfi et.al. \`16]

  Small jet-radius resummation at LL [DasGupta et.al. \`14]

- Recent explicit computation at NNLO

  with transverse momentum as a reference

[Abreu et.al. \`22]



# Factorization

- Factorization in **SCET** for color-singlet processes

$$\mathrm{d}\sigma \simeq \boxed{H(\mu_f)} \prod_i \boxed{B_i(\mu_f)} \otimes \boxed{S(\mu_f)}$$

**Hard interaction**

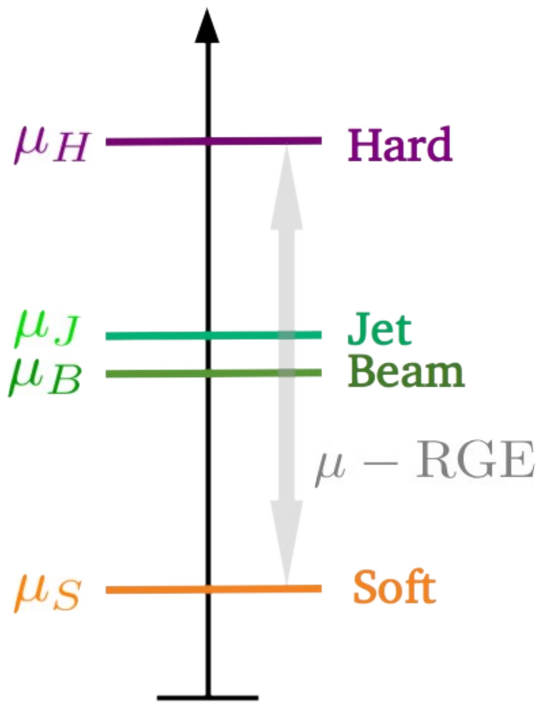
**ISR**

**Soft radiation**

- Each function can be calculated perturbatively
- Resummation is performed by calculating them at their characteristic scales and evolving them to a common scale.

# Resummation

- Resummation through RGE.




- Hard function RGE :

$$\frac{\mathrm{d}\ln H(Q,\mu)}{\mathrm{d}\ln\mu} = \gamma_{\mathrm{H}}(Q,\mu)$$

  - Hard anomalous dimension

$$\boxed{\gamma_{\mathrm{H}}(Q,\mu) = \Gamma_{\mathrm{cusp}}(\alpha_S)\ln\frac{Q}{\mu} + \gamma^H(\alpha_S)}$$

$$H(Q,\mu) = H(Q,\mu_H)U(\mu_H,\mu)$$

Boundary term
(free from large logs)
$\mu_H \simeq Q$

Evolution kernel
(resums large logs)
$\ln^n(Q/\mu)$

$$U(\mu_H,\mu) = \exp\left[\int_{\mu_H}^{\mu} d\ln\mu'\,\gamma_{\mathrm{H}}(Q,\mu')\right]$$

# Ingredients for Resummation

- Need all the anomalous dimensions & matching coefficients

$$\underbrace{\Gamma_{\rm cusp},\quad \gamma^H,\quad c_H,}_{\text{Observable-independent}}\quad \underbrace{\gamma^B,\quad c_B,\quad \gamma^S,\quad c_S}_{\text{Observable-dependent}}$$

  - Hard anomalous dims are known to 3-loops (some cases 4-loops)

[Becher, Neubert, `10
**GD**, Moch, Vogt `19,`20,
Manteuffel, Panzer, Schabinger
+Huber, +Yang `20, `22
Lee, Manteuffel, Schabinger, Smirnov, Smirnov,
Steinhauser,+Huber,+Chakraborty `22]

- Beam, Soft quantities are computed on a case-by-case basis
- Need two-loop matching coefficients to achieve **NNLL′** accuracy

| | $\Gamma_{\rm cusp}, \beta$ | $\gamma^{H,B,J,S}$ | Boundary term ($c_{H,B,J,S}$) |
|---|---|---|---|
| NLL | 2-loop | 1-loop | 1 |
| NLL′ | 2-loop | 1-loop | $\alpha_S$ |
| NNLL | 3-loop | 2-loop | $\alpha_S$ |
| NNLL′ | 3-loop | 2-loop | $\alpha_S^2$ |

This talk!

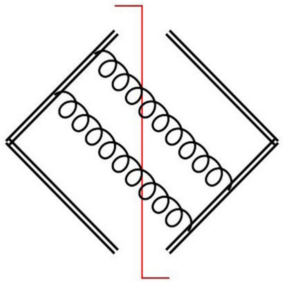
# Automation

- Set up a general framework to automatically calculate general class of observables

$$d\sigma \simeq H(\mu_f) \prod_i B_i(\mu_f) \otimes S(\mu_f)$$

- **Hard function** → underlying form factor LiteRed, Fire, Reduze, Air, Kira ...

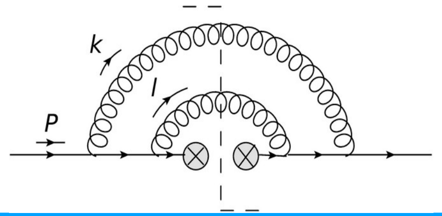
[Lee, `13,
Smirnov, Chukharev, `19,
von Manteuffel, Studerus, `12,
Anastasiou, Lazopoulos, `10
Maierhöfer, Usovitsch, Uwer, `17]



- **Soft functions** → 2-particle final state
  - NNLO di-jet soft functions: **SoftSERVE** [Bell, Rahn, Talbert `18, `20]

  Thrust, C-parameter, Angularities, Hemisphere masses, Threshold, $P_T$, Jet-veto resummation ...

- **Beam functions** → 2-particle final state
  - Non-trivial matching onto pdfs! [Bell, Brune, GD, Wald `21, (in progress) ]



# Beam function

- Proton matrix element of renormalized operator composed of partonic fields (with additional dependence on measurement)
  - Contains non-perturbative contribution – match onto PDF!

Matching also holds wrt partonic states!

Quark-Quark kernel

$$\mathcal{B}_{qq}(x,\tau,\mu) = \sum_{i\in X_c} \delta\Big((\bar{n}\cdot P)(1-x) - \bar{n}\cdot k_{X_c}\Big) \langle P|\bar{\chi}|X_c\rangle \frac{\bar{n}}{2} \langle X_c|\chi|P\rangle \mathcal{M}(\tau,\{k_i\})$$

- In the limit $\tau^{-1} \gg \Lambda_{QCD}$

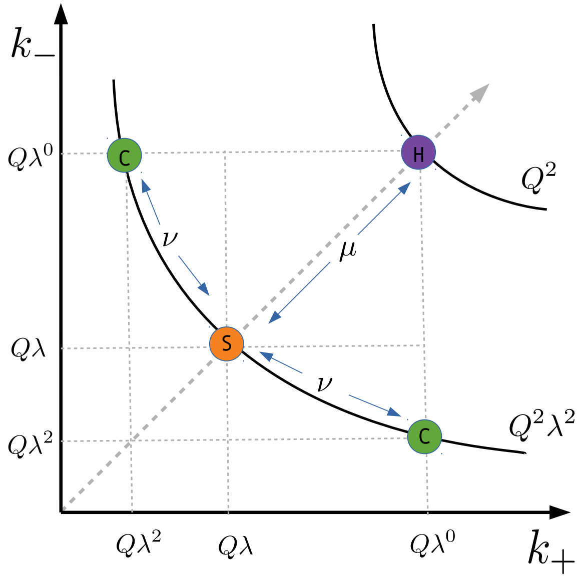
$$\mathcal{B}_{qq}(x,\tau,\mu) = \sum_i \int \frac{\mathrm{d}z}{z} \mathcal{I}_{qi}\Big(\frac{x}{z},\tau,\mu\Big) f_{iq}(z,\mu)$$

*IR-divergent*

*IR-finite*
Matching coefficients

*IR-divergent*
Partonic PDF

# Rapidity divergence




- **SCET-II observables**
  - Suffers from additional rapidity divergences
  - Rapidity logs can be resummed to all orders
  - Follow the collinear-anomaly approach. [Becher, Neubert, `10]
  - Introduce analytic symmetric regulators at the phase space level [Becher, Bell, `11]

**Modified PSP**

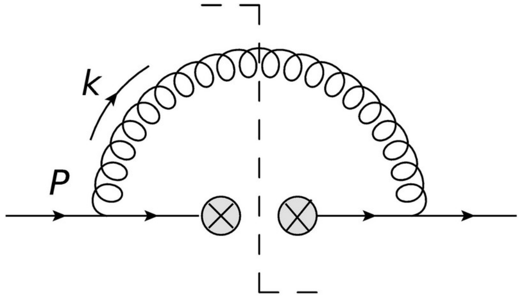
$$\int \frac{d^d k}{(2\pi)^d}(2\pi)\delta_+(k^2)\left(\frac{\nu}{k_- + k_+}\right)^{\alpha}$$

**Collinear = Anti-collinear**

# NLO

- An automated framework exists at NLO

[K. Brune's master thesis `18]




- Matrix element - LO splitting kernel

$$P^{(0)}_{q\to gq^*}(x_1) = C_F \frac{1}{x_1}\left[1+\bar{x}_1^2 - \epsilon x_1^2\right]$$

[Altarelli & Parisi `77]

- Phase space

$$x_1 = \frac{k_-}{P_-}, \quad k_T = \sqrt{k_+ k_-}, \quad t_k = \frac{1-\cos(\theta_k)}{2}$$

$$\left\{k_-, k_T, \theta_k\right\} \to \left\{x_1, k_T, t_k\right\}$$

# Measurement-$P_{veto}$

- Cumulant Beam function

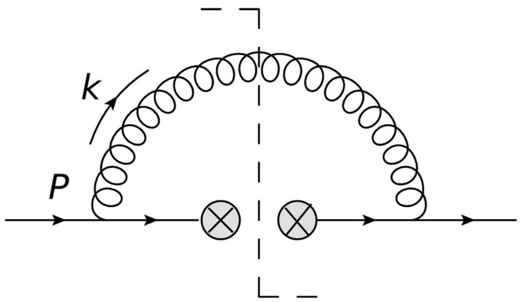
$$\mathcal{M}(p_T^{\text{veto}};\{k_i\}) = \theta(p_T^{\text{veto}} - \omega(\{k_i\}))$$

$$\omega_1(\{k\}) = |\vec{k}_T|$$
$$\omega_2(\{k,l\}) = \theta(\Delta - R)\max\{|\vec{k}_T|,|\vec{l}_T|\} + \theta(R-\Delta)|\vec{k}_T + \vec{l}_T|$$

- Measurement function for single emission (in Laplace Space)

$$\mathcal{M}_1(x_1,\tau;k) = \exp\left[-\tau k_T\left(\frac{k_T}{x_1 P_-}\right)^n f(t_k)\right]$$

Non-zero in singular limits of ME




- $P_T$ – Veto

$$n = 0,\ f(t_k) = 1$$

$k_T$ – dependence is trivial! → Perform the integration analytically!

- Master Formula

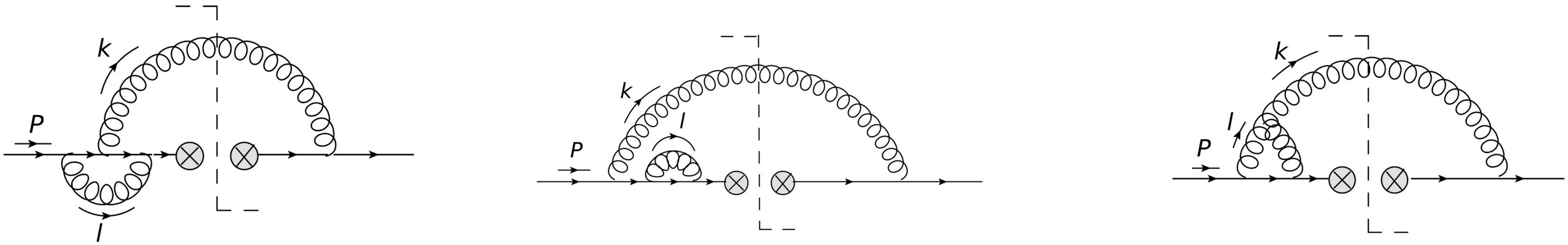
$$\mathcal{B}^{(1)}_{qq}(x_1,\tau) \sim \Gamma\left(\frac{-2\epsilon}{1+n}\right) x_1^{-1-\frac{2n\epsilon}{1+n}-\alpha}\left[x_1 P^{(0)}_{q\to gq^*}(x_1)\right]\int_0^1 dt_k (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon} f(t_k)^{\frac{2\epsilon}{1+n}}$$

All singularities are factorised !

- Matrix Element

related to NLO collinear splitting kernel - $P^{(1)}_{q\to gq^*}(x)$

[Bern, Chalmers, +Del Duca, +Kilgore, +Schmidt `95, `99, Kosower, Uwer, `99 Sborlini, de Florian, Rodrigo `13]




- Phase space & measurement function follow NLO type
- Master formula

$$\mathcal{B}^{(2),RV}_{qq}(x_1,\tau,\mu) \sim V(\epsilon)\,\Gamma\left(\frac{-4\epsilon}{1+n}\right) x_1^{-1-\frac{4n\epsilon}{1+n}-\alpha}\,\mathcal{W}(x_1)\int_0^1 dt_k (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon}\; f(t_k)^{\frac{4\epsilon}{1+n}}$$

$\sim \epsilon^{-2} + \mathcal{O}(\epsilon^{-1})$

All Phase space singularities are factorised !

# Real-Real

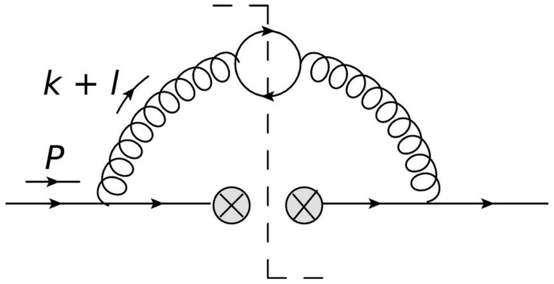
- Matrix element - LO triple collinear splitting kernels

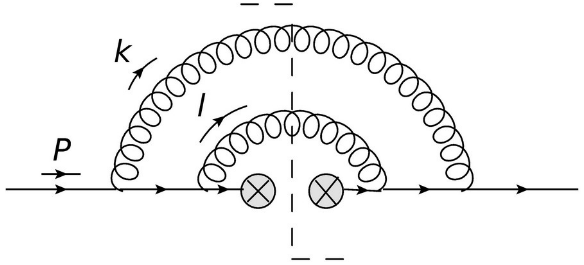
$$P^{(0),C_F T_F}_{q\to q'\bar{q}'q^*}, \quad P^{(0),id}_{q\to q\bar{q}q^*}, \quad P^{(0),C_F^2}_{q\to ggq^*}, \quad P^{(0),C_A C_F}_{q\to ggq^*}$$

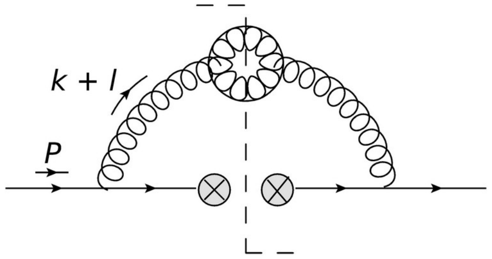
[Catani, Grazzini `98, `99
Campbell, Glover `98]

- Complicated divergence structures from ME

$$\frac{1}{s_{12}^2 s_{123}^2 (x_1+x_2)^2}, \quad \frac{1}{s_{13}s_{23}x_1x_2}, \quad \frac{1}{s_{13}s_{123}x_1x_2}, \quad \frac{1}{s_{13}s_{123}x_2\bar{x}_3}, \cdots$$



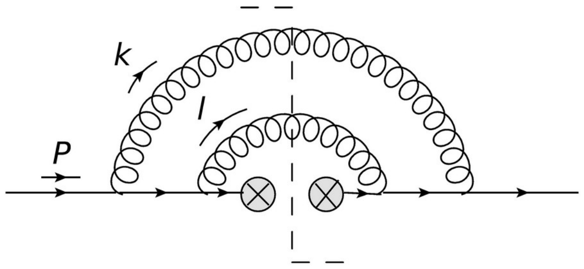




...

# Real-Real

- 2-particle phase space




$$a = \frac{k_- l_T}{l_- k_T}, \quad b = \frac{k_T}{l_T}, \quad x_{12} = \frac{k_- + l_-}{P_-}, \quad q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$

$$\Big\{k_-, k_T, l_-, l_T, \theta_k, \theta_l, \theta_{kl}\Big\} \to \Big\{a, b, x_{12}, q_T, t_k, t_l, t_{kl}\Big\}$$

- Measurement function – two real emissions

$$\mathcal{M}_2(\tau, \{k, l\}) = \exp\left[-\tau q_T \left(\frac{q_T}{x_{12} P_-}\right)^n \mathcal{F}(x_{12}, a, b, t_{kl}, t_k, t_l)\right]$$

  - Exact form depends on parametrization & ME divergences

$$\mathcal{F}(x_{12}, a, b, t_{kl}, t_k, t_l) \quad \Rightarrow$$

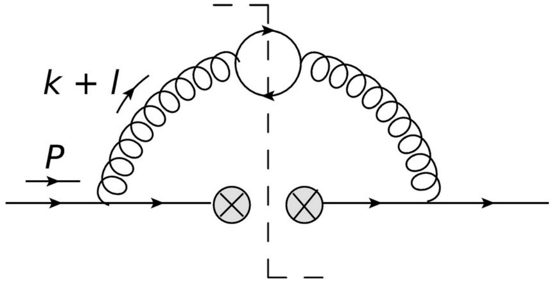
ensure it does **NOT** vanish in the singular limits of ME !

# Real-Real : $C_F T_F n_f$

$$\left\{k_-, k_T, l_-, l_T, \theta_k, \theta_l, \theta_{kl}\right\} \to \left\{a, b, x_{12}, q_T, t_k, t_l, t_{kl}\right\}$$

- $q_T$-dependence is trivial → Perform the integration analytically
- Remap **{a,b}** to unit hypercube → **4 sectors**

Exploit k – l symmetry: **2 sectors**




- Avoid distributions in $x_{12}$

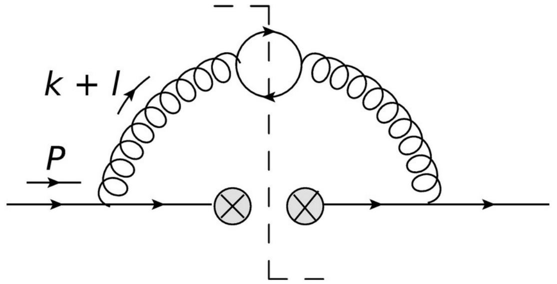
work in **Laplace-Mellin space**!

Also possible to keep the distributions in $x_{12}$

- Collinear divergence `k||l` still overlaps:

$$\bar{a} \Rightarrow u\bar{v}$$
$$t_{kl} \Rightarrow u^2 v/(1-u\bar{v})$$

$$(\bar{a}^2 + 4at_{kl})^{-1} \xRightarrow{\text{NLT}} u^{-1}$$




- Master formula:

$$\mathcal{B}_{qq}^{(2),nf}(N_1,\tau) \sim \mathcal{C}(n,\epsilon)\left(\frac{\nu}{Q}\right)^{2\alpha}\int_0^1 dx_{12}\; db\; du\; dv\; dt_l\; dt_5\; x_{12}^{-1-2\alpha-\frac{4n\epsilon}{1+n}}\; u^{-1-2\epsilon}\; \bar{x}_{12}^{N_1-1+2\alpha+\frac{4n\epsilon}{1+n}} \times$$
$$\mathcal{G}(x_{12},b,u,v,t_l,t_5)\mathcal{F}(x_{12},b,u,v,t_l,t_5)^{\frac{4\epsilon}{1+n}}$$

finite function

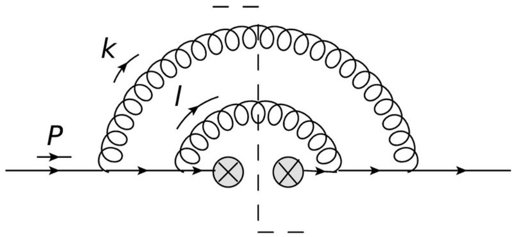
non-zero in the singular limit of ME

**All singularities factorize !**

# Real-Real : $C_F^{\ 2}$

- Divergence structures

$$\frac{1}{s_{12}s_{13}\bar{x}_1\bar{x}_3},\quad \frac{1}{s_{12}s_{123}\bar{x}_1\bar{x}_3},\cdots \frac{1}{s_{13}s_{23}x_1x_2},\quad \frac{1}{s_{13}s_{123}x_1x_2},\cdots$$




- Complications due to many overlapping divergences in ME

- Additional complications from measurements

$$\omega_{p_T-veto} = \sqrt{\frac{a}{(a+b)(1+ab)}}\left[\Theta(\Delta - R)\max(1,b) + \Theta(R-\Delta)\sqrt{(1-b)^2 + 4b(1-t_{kl})}\right]$$

Must stay non-zero in the physical limits $a \to 0, b \to 0$

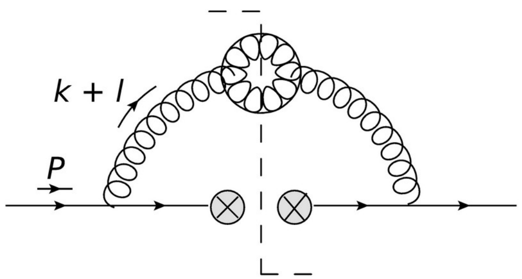
- General strategy:
  - Use of **non-linear transformations**
  - **Sector decomposition** [Heinrich `08]

**~28 sectors**

# Real-Real : $C_A C_F$

- Divergence structures (in addition to all color structures)

$$\frac{1}{s_{12}s_{13}x_2\bar{x}_3}, \quad \frac{1}{s_{12}s_{123}x_1x_2}, \ldots$$




- Calculation follows similar to $C_F^2$ structures. ~ **new 14 sectors**
- Implementation:
  - Automated Mathematica code
  - Laurent expansion & integration in **pySecDec** **(Cuba)** [Heinrich et.al. `17 Hahn, `04, `14]

# Cumulant Measurement

- Revisit measurement $\mathcal{M}(p_T^{\text{veto}};\{k_i\}) = \theta(p_T^{\text{veto}} - \omega(\{k_i\}))$

$$\omega_1(\{k\}) = |\vec{k}_T| \qquad \omega_2(\{k,l\}) = \theta(\Delta - R)\max\{|\vec{k}_T|,|\vec{l}_T|\} + \theta(R-\Delta)|\vec{k}_T + \vec{l}_T|$$

- Our framework calculates in Laplace space

$$\int_0^\infty dp_T^{\text{veto}} e^{-\tau p_T^{\text{veto}}}\theta(p_T^{\text{veto}} - \omega(\{k_i\})) = \frac{1}{\tau}\exp(-\tau\omega(\{k_i\}))$$

- Revert back to Momentum space

$$\mathcal{S}_R^{\text{bare}} \Rightarrow \frac{e^{\gamma_E(2\epsilon+\alpha)}}{\Gamma(1-2\epsilon-\alpha)}\mathcal{S}_R^{\text{bare}}\,, \qquad \mathcal{S}_{RV}^{\text{bare}} \Rightarrow \frac{e^{\gamma_E(4\epsilon+\alpha)}}{\Gamma(1-4\epsilon-\alpha)}\mathcal{S}_{RV}^{\text{bare}}\,, \qquad \mathcal{S}_{RR}^{\text{bare}} \Rightarrow \frac{e^{\gamma_E(4\epsilon+2\alpha)}}{\Gamma(1-4\epsilon-2\alpha)}\mathcal{S}_{RR}^{\text{bare}}$$

$$\mathcal{B}_R^{\text{bare}} \Rightarrow \frac{e^{\gamma_E(2\epsilon)}}{\Gamma(1-2\epsilon)}\mathcal{B}_R^{\text{bare}}\,, \qquad \mathcal{B}_{RV}^{\text{bare}} \Rightarrow \frac{e^{\gamma_E(4\epsilon)}}{\Gamma(1-4\epsilon)}\mathcal{B}_{RV}^{\text{bare}}\,, \qquad \mathcal{B}_{RR}^{\text{bare}} \Rightarrow \frac{e^{\gamma_E(4\epsilon)}}{\Gamma(1-4\epsilon)}\mathcal{B}_{RR}^{\text{bare}}$$

# SCET-II renormalization

Soft function from **SoftSERVE**

- Collinear anomaly approach

$$\left[\mathcal{S}(\tau,\mu,\nu)\mathcal{I}_{qq}(N_1,\tau,\mu,\nu)\mathcal{I}_{\bar{q}\bar{q}}(N_2,\tau,\mu,\nu)\right]_{q^2} \overset{\alpha=0}{=} \left(\bar{\tau}^2 q^2\right)^{-F_{q\bar{q}}(\tau,\mu)} \widehat{I}_{qq}(N_1,\tau,\mu)\widehat{I}_{\bar{q}\bar{q}}(N_2,\tau,\mu)$$

[Becher, Neubert, `10]

- RGE : Anomaly coefficients

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}F_{q\bar{q}}(\tau,\mu) = 2\Gamma_{\text{cusp}}(\alpha_S)$$

Finite non-logarithmic coefficient → $d_2$

- RGE: Matching coefficients

$L = \ln(\bar{\tau}\mu)$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\widehat{I}_{qq}(N_1,\tau,\mu) = \left[2\Gamma_{\text{cusp}}(\alpha_S)L + 2\gamma^B(\alpha_S)\right]\widehat{I}_{qq}(N_1,\tau,\mu) - 2\sum_i \widehat{I}_{qi}(N_1,\tau,\mu)\widehat{P}_{iq}(N_1,\mu)$$

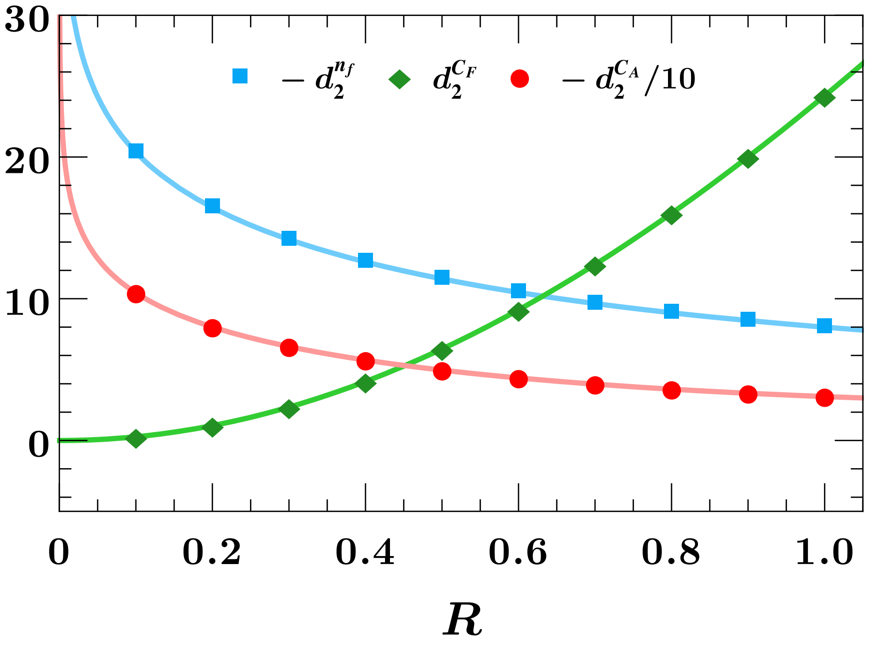
- Solution → renormalized matching coefficients

Finite remainder coefficient $\widehat{\mathrm{I}}_{qq}(N_1)$ — Non-logarithmic piece

# Results : Anomalies

- Anomaly coefficients

  - depends on the jet radius at 2-loop




- Non-cusp anomalous dimensions

| $\gamma_1^B$ | Analytical[1] | This Work |
|---|---|---|
| $\gamma_1^{n_f}$ | $-11.395$ | $-11.395(2)$ |
| $\gamma_1^{C_F}$ | 10.610 | 10.611(19) |
| $\gamma_1^{C_A}$ | 4.637 | 4.640(17) |

[1. Becher, Neubert, +Rothen `12,`13]

# Results : Matching Kernels

- Matching coefficients at two loops also depend on the Jet radius R
  - Seven independent kernels for the quark case

$$
\begin{aligned}
\widehat{I}^{(2)}_{q\leftarrow q}(N) &= C_F^2\,\widehat{I}^{(2,C_F)}_{q\leftarrow q}(N) + C_F C_A\,\widehat{I}^{(2,C_A)}_{q\leftarrow q}(N) + C_F T_F n_f\,\widehat{I}^{(2,n_f)}_{q\leftarrow q}(N) + C_F T_F\,\widehat{I}^{(2,T_F)}_{q\leftarrow q}(N)\,,\\
\widehat{I}^{(2)}_{q\leftarrow g}(N) &= C_F T_F\,\widehat{I}^{(2,C_F)}_{q\leftarrow g}(N) + C_A T_F\,\widehat{I}^{(2,C_A)}_{q\leftarrow g}(N)\,,\\
\widehat{I}^{(2)}_{q\leftarrow \bar{q}}(N) &= C_F(C_A-2C_F)\,\widehat{I}^{(2,C_{AF})}_{q\leftarrow \bar{q}}(N) + C_F T_F\,\widehat{I}^{(2,T_F)}_{q\leftarrow q}(N)\,,\\
\widehat{I}^{(2)}_{q\leftarrow q'}(N) &= \widehat{I}^{(2)}_{q\leftarrow \bar{q}'}(N) = C_F T_F\,\widehat{I}^{(2,T_F)}_{q\leftarrow q}(N)\,.
\end{aligned}
$$

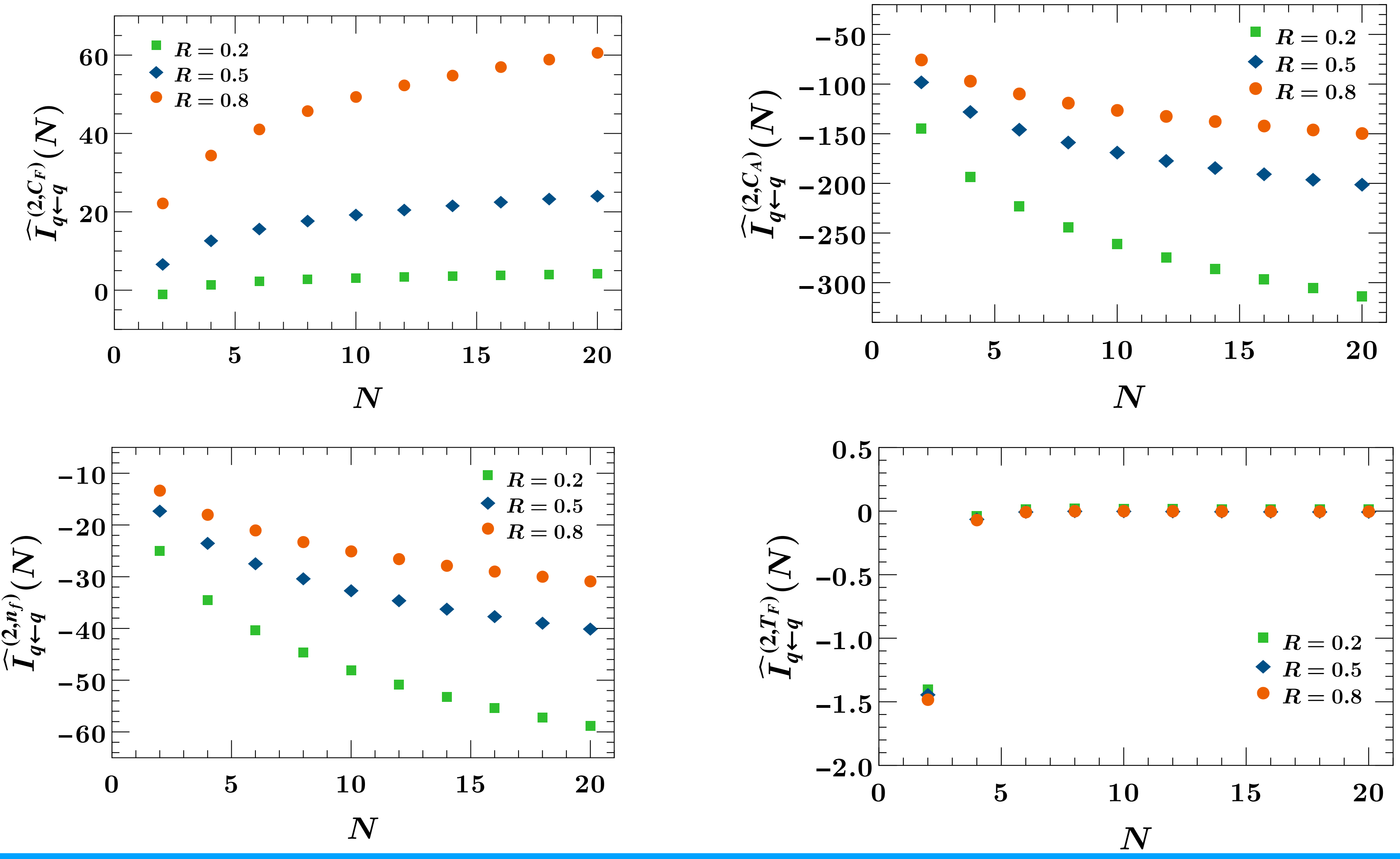
[G. Bell, **GD**, K. Brune, M. Wald `22]

  - Six kernels for the gluon case

$$
\begin{aligned}
\widehat{I}^{(2)}_{g\leftarrow g}(N) &= C_A^2\,\widehat{I}^{(2,C_A^2)}_{g\leftarrow g}(N) + C_A T_F n_f\,\widehat{I}^{(2,C_A T_F n_f)}_{g\leftarrow g}(N) + C_F T_F n_f\,\widehat{I}^{(2,C_F T_F n_f)}_{g\leftarrow g}(N)\,,\\
\widehat{I}^{(2)}_{g\leftarrow q}(N) &= C_F^2\,\widehat{I}^{(2,C_F^2)}_{g\leftarrow q}(N) + C_F C_A\,\widehat{I}^{(2,C_F C_A)}_{g\leftarrow q}(N) + C_F T_F n_f\,\widehat{I}^{(2,C_F T_F n_f)}_{g\leftarrow q}(N)\,.
\end{aligned}
$$

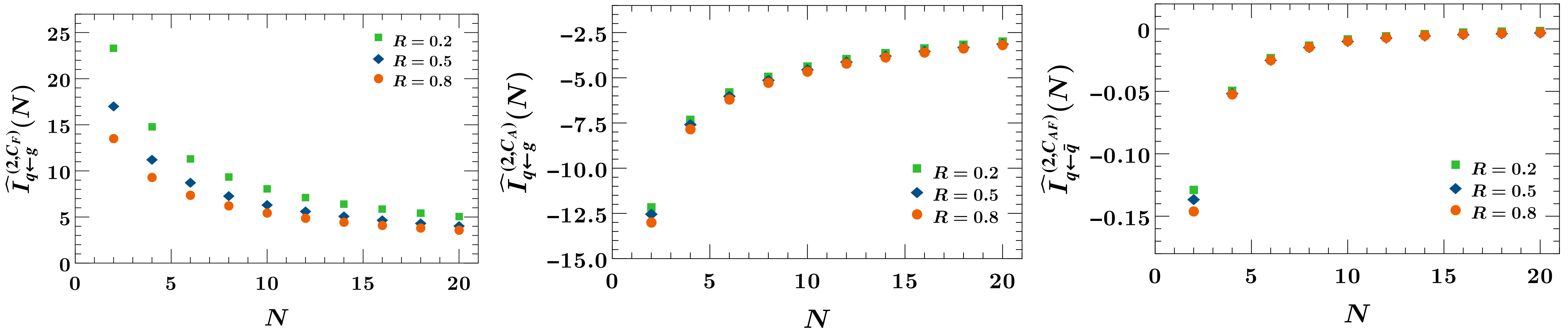
[G. Bell, **GD**, K. Brune, D-Y Shao, M. Wald (to appear soon!)]

- Quark matching kernels



# Results : Matching Kernels

- Quark matching kernels



# Outlook

- An automated framework to calculate **Beam** functions for a wide class of obervables at NNLO.
- New results for **$P_T$-veto**
- Future plans:
  - Implementation in a C++ code - BeamSERVE to be integrated with resummed framework *e.g.* **SCETlib** - **GENEVA**
  - Application to phenomenology!

**Thank you for your attention!**