

Projected transverse momentum resummation in the top-antitop pair production at LHC

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based on the paper “Projected transverse momentum resummation in the top-antitop pair production at LHC”, to appear

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Fixed-order calculations

1. NLO QCD: [Nucl. Phys. B 303 (1988)/Phys. Rev. D 40 (1989)/Nucl. Phys. B 351 (1991)/Nucl. Phys. B 373 (1992)]
2. NNLO QCD: [Czakon:2013goa, Czakon:2015owf, Czakon:2016ckf, Czakon:2017dip]
[Czakon:2017wor, Catani:2019iny, Catani:2020tko]
3. NNLO QCD+ t/\bar{t} decay:
[Gao:2012ja, Brucherseifer:2013iv, Catani:2019hip, Behring:2019iiv, Czakon:2020qbd]
4. electroweak corrections
[Bernreuther:2010ny, Kuhn:2006vh, Bernreuther:2006vg, Kuhn:2013zoa, Hollik:2011ps]
[Pagani:2016caq, Gutschow:2018tuk, Denner:2016jyo, Czakon:2017wor]
5.

Resummation in the various kinematics

1. the mechanic threshold $M_{t\bar{t}} \rightarrow \hat{s}$
[Kidonakis:2014pj, Kidonakis:2010dk, Kidonakis:2009ev, Kidonakis:2014isa, Ahrens:2010zv, Ferroglio:2012ku, Ferroglio:2013awa, Pecjak:2016nee, Czakon:2018nun, Hinderer:2014qta]
2. the production threshold $M_{t\bar{t}} \rightarrow 2m_t$
[Beneke:2009ye, Beneke:2010da, Beneke:2011mq, Pilcup:2018ndt, Ju:2020otc, Ju:2019mqc]
3. TM → This Work
[Zhu:2012ts, Li:2013mia, Catani:2014qha, Catani:2017tuc, Catani:2018mei]
4. the jettiness limit $\mathcal{T}_0 \rightarrow 0$ [Alioli:2021ggd]
5. MINNLO_{PS} [Mazzitelli:2020jio, Mazzitelli:2021mmm]
6.

Experimental measurements on the processes $pp \rightarrow t\bar{t} + X$

Measurements on the process $\sigma(pp \rightarrow t\bar{t} + X)$ at LHC

1) Inclusive cross section $\sigma_{t\bar{t}}$:

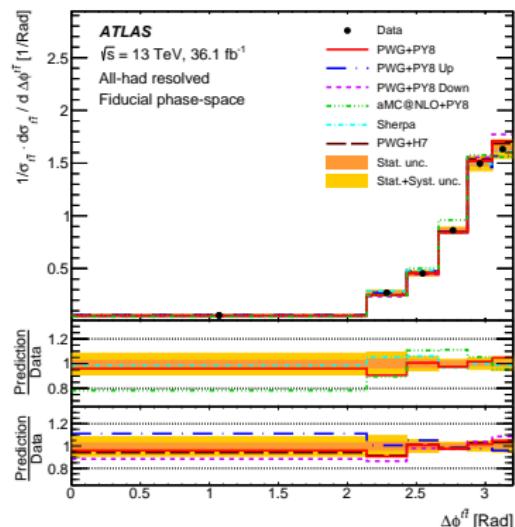
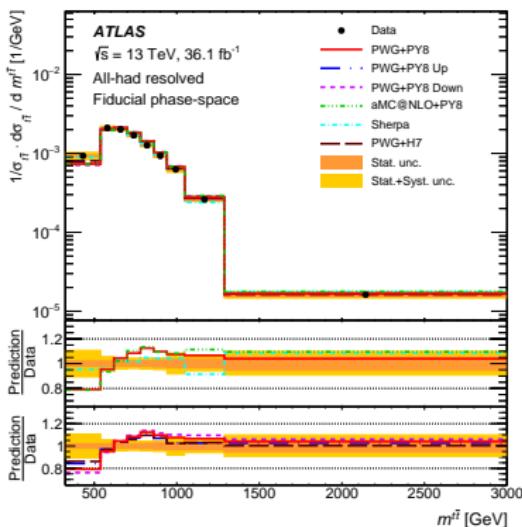
A) ATLAS: [ATLAS:2021xhc,1910.08819,2006.13076...]

B) CMS: [1711.03143,2112.09114,1911.13204]

2) Differential cross sections $d\sigma_{t\bar{t}}(pp \rightarrow t\bar{t} + X)/d\mathcal{O}$:

A) ATLAS: [2006.09274,2202.12134]

B) CMS: [1904.05237,2108.02803,2008.07860,CMS:2022uea]

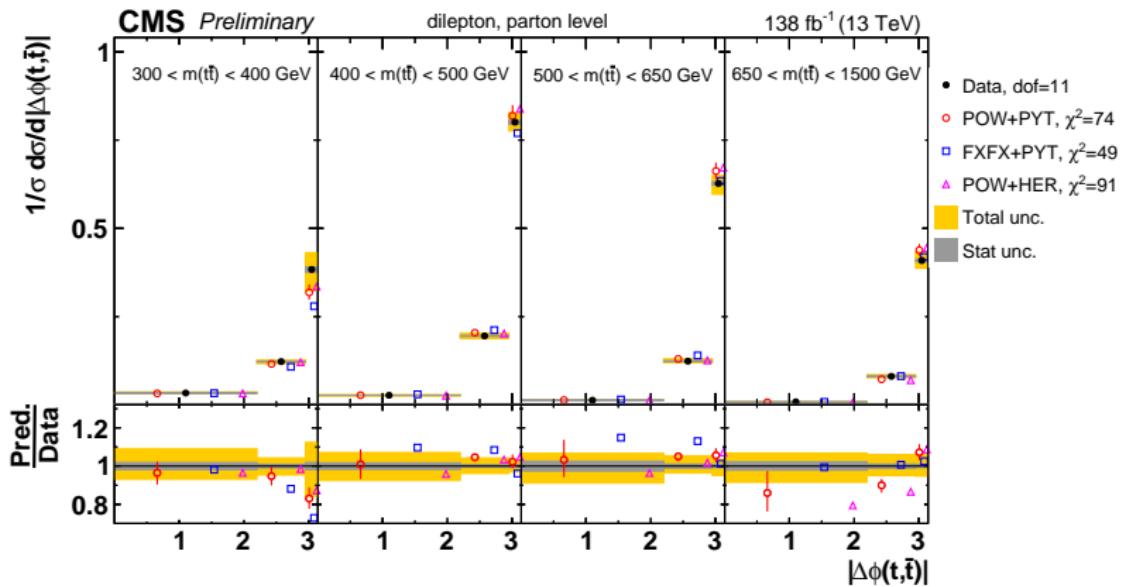


[2006.09274]

[2006.09274]

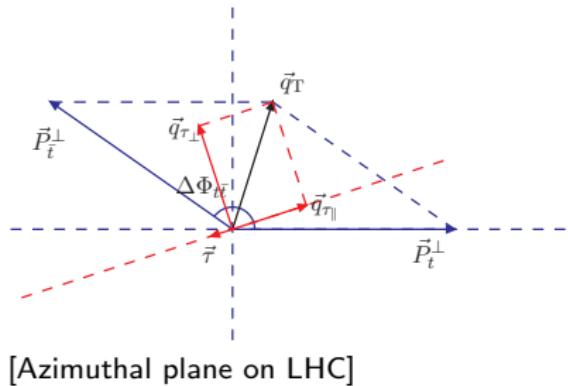
Experimental measurements

- Measurements on the differential cross sections $d\sigma_{t\bar{t}}(pp \rightarrow t\bar{t} + X)/d\mathcal{O}$
 - ATLAS: [2006.09274, 2202.12134]
 - CMS: [1904.05237, 2108.02803, 2008.07860, CMS:2022uae]



[CMS:2022uae]

Azimuthal separation between top and antitop quarks



- A $\Delta\Phi_{t\bar{t}}$ is the azimuthal separation of the top and antitop quark.

$$\Delta\Phi_{t\bar{t}} \equiv \pi - \Delta\phi_{t\bar{t}} \equiv \arccos \left[\frac{\vec{p}_{t,\text{T}} \cdot \vec{p}_{\bar{t},\text{T}}}{|\vec{p}_{t,\text{T}}| |\vec{p}_{\bar{t},\text{T}}|} \right]$$

- B Akin to the observable $\vec{q}_T = \vec{P}_t^\perp + \vec{P}_{\bar{t}}^\perp$, the $\Delta\phi^{t\bar{t}}$ spectrum is also sensitive to the soft&collinear emissions within the region $\Delta\phi_{t\bar{t}} \rightarrow 0$

$$\frac{d\sigma_{t\bar{t}}}{d\Delta\phi^{t\bar{t}}} \sim \sum_{m,n} d_{m,n} \frac{\alpha_s^n \ln^m \Delta\phi^{t\bar{t}}}{\Delta\phi^{t\bar{t}}} + \dots$$

- C Beneficial from the resummation of those singular behaviours

Projected transverse momentum distributions $d\sigma_{t\bar{t}}/dq_\tau$

The spectrum of $\Delta\phi^{t\bar{t}}$ is a special case of $d\sigma_{t\bar{t}}/dq_\tau$ with $\vec{\tau} \perp \vec{p}_t^\perp$

- A In the limit $\Delta\phi^{t\bar{t}} \rightarrow 0$, the expansion can be applied,

$$\Delta\phi^{t\bar{t}} = \pi - \arccos \left[\frac{\vec{p}_{t,T} \cdot \vec{p}_{\bar{t},T}}{|\vec{p}_{t,T}| |\vec{p}_{\bar{t},T}|} \right] = \frac{q_{t\bar{t},T}^y}{|\vec{p}_{t,T}|} + \dots$$

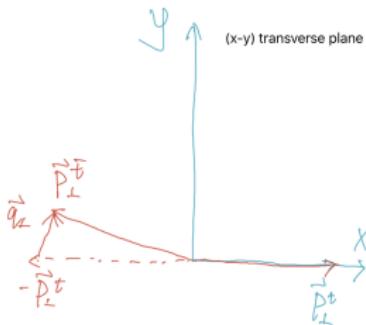
- B An approximate relationship up to leading power in $\Delta\phi^{t\bar{t}}$:

$$\frac{d\sigma_{t\bar{t}}}{d\Delta\phi^{t\bar{t}}} \rightarrow \frac{d\sigma_{t\bar{t}}}{d|q_{t\bar{t},T}^y|}$$

- C Generic circumstance:

$\vec{\tau}$ -2D unit reference vector: $q_\tau = |\vec{q}_{t\bar{t},T} \cdot \vec{\tau}|$

$$\frac{d\sigma_{t\bar{t}}}{dq_\tau}$$



Projected transverse momentum distributions $d\sigma_{t\bar{t}}/dq_\tau$

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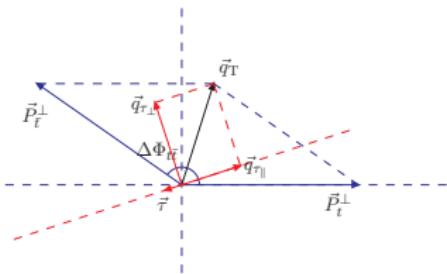
$$\frac{d\sigma_{t\bar{t}}}{dq_\tau}$$

B Numeric implementations:

$$q_\tau = q_{T,\text{out}}, \quad \text{if } \vec{\tau} = \pm \vec{n} \times \frac{\vec{P}_t^\perp}{|\vec{P}_t^\perp|};$$

$$q_\tau = q_{T,\text{in}}, \quad \text{if } \vec{\tau} = \pm \frac{\vec{P}_t^\perp}{|\vec{P}_t^\perp|},$$

$$\Delta\Phi_{t\bar{t}} \equiv \cos^{-1} \left[\frac{\vec{P}_t^\perp \cdot \vec{P}_{\bar{t}}^\perp}{|\vec{P}_t^\perp| |\vec{P}_{\bar{t}}^\perp|} \right] \sim \pi - \frac{q_{T,\text{out}}}{|\vec{P}_t^\perp|} + \mathcal{O}(\lambda_\tau^2).$$



Factorization

QCD factorisation formula

[Collins:1989gx]

$$\frac{d\sigma_{t\bar{t}}}{dM_{t\bar{t}}^2 d^2 \vec{P}_t^\perp dY_{t\bar{t}} dq_\tau} = \sum_{\text{sign}[P_t^z]} \frac{1}{16s(2\pi)^6} \int d^2 \vec{q}_T \Theta_{\text{kin}} \delta(q_\tau - |\vec{q}_T \cdot \vec{\tau}|) \frac{\Sigma_{t\bar{t}}}{E_{t\bar{t}} |P_t^z|}$$

where

$$\begin{aligned} \Theta_{\text{kin}} &= \Theta\left[\sqrt{s} - M_T^{t\bar{t}} - |\vec{q}_T|\right] \Theta\left[M_T^{t\bar{t}} - m_T^t - m_T^{\bar{t}}\right] \\ &\times \Theta\left[\sinh^{-1}\left(\sqrt{\frac{(M_{t\bar{t}}^2 + s)^2}{4s M_T^{t\bar{t}}^2} - 1}\right) - |Y_{t\bar{t}}|\right], \end{aligned}$$

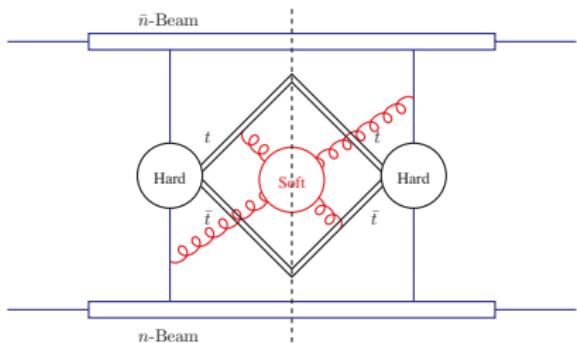
and,

$$\begin{aligned} \Sigma_{t\bar{t}} &= \sum_{i,j} \int_0^1 \frac{dx_n}{x_n} \frac{dx_{\bar{n}}}{x_{\bar{n}}} f_{i/N}(x_n) f_{j/\bar{N}}(x_{\bar{n}}) \sum_r \int \prod_m d\Phi_{k_m} \\ &\overline{\sum_{\text{hel,col}}} |\mathcal{M}(i+j \rightarrow t+\bar{t}+X)|^2 (2\pi)^4 \delta^4 \left(p_i + p_j - p_t - p_{\bar{t}} - \sum_m k_m \right) \end{aligned}$$

Factorization

Asymptotic expansion and the leading regions

$$\frac{d\sigma_{t\bar{t}}}{dq_\tau} \sim \sigma_B^{t\bar{t}} \sum_{m,n} \left[\frac{\alpha_s(M_{t\bar{t}})}{4\pi} \right]^m \left[\underbrace{c_{m,n}^{(0)} \frac{\ln^n(\lambda_\tau)}{\lambda_\tau}}_{\text{LP}} + \underbrace{c_{m,n}^{(1)} \ln^n(\lambda_\tau)}_{\text{NLP}} + \underbrace{c_{m,n}^{(2)} \lambda_\tau \ln^n(\lambda_\tau)}_{\text{N}^2\text{LP}} + \dots \right],$$



Momentum Modes for the LP terms

Hard : $k_h \sim M_{t\bar{t}} [\mathcal{O}(1), \mathcal{O}(1), \mathcal{O}(1)]_n,$

Soft : $k_s \sim M_{t\bar{t}} [\mathcal{O}(\lambda_\tau), \mathcal{O}(\lambda_\tau), \mathcal{O}(\lambda_\tau)]_n,$

Beam- n : $k_c \sim M_{t\bar{t}} [\mathcal{O}(1), \mathcal{O}(\lambda_\tau^2), \mathcal{O}(\lambda_\tau)]_n,$

Beam- \bar{n} : $k_{\bar{c}} \sim M_{t\bar{t}} [\mathcal{O}(\lambda_\tau^2), \mathcal{O}(1), \mathcal{O}(\lambda_\tau)]_n,$

Factorization

Asymptotic expansion and the leading regions

Phase space:

$$\begin{aligned}\Theta_{\text{kin}} \rightarrow & \Theta(\sqrt{s} - M_{t\bar{t}}^2) \Theta\left(M_{t\bar{t}} - 2\sqrt{m_t^2 + (\vec{P}_t^\perp)^2}\right) \\ & \times \Theta\left[\sinh^{-1}\left(\frac{s - M_{t\bar{t}}^2}{2M_{t\bar{t}}\sqrt{s}}\right) - |Y_{t\bar{t}}|\right] + \mathcal{O}(\lambda_\tau)\end{aligned}$$

Squared amplitudes and PDFs SCET_{II}+HQET [Bauer:2002aj, Lange:2003pk, Beneke:2003pa, Neubert:1993mb]

$$\Sigma_{t\bar{t}} \rightarrow \frac{8\pi^2}{M_{t\bar{t}}^2} \sum_{\kappa} \int d^2 \vec{b}_T \exp\left(i \vec{b}_T \cdot \vec{q}_T\right) \tilde{\Sigma}_{t\bar{t}}^{[\kappa]}(\vec{b}_T, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}, \mu, \nu) \mathcal{W}_t \mathcal{W}_{\bar{t}} + \mathcal{O}(\lambda_\tau),$$

where $\kappa \in \{g_n g_{\bar{n}}, q_n^i \bar{q}_{\bar{n}}^j, q_{\bar{n}}^i \bar{q}_n^j\}$,

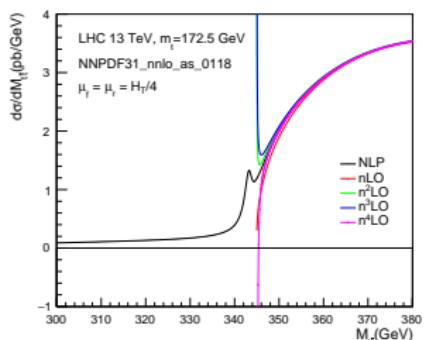
Factorization

Asymptotic expansion and the leading regions

Absence of the interaction in the heavy parton correlation if $M_{t\bar{t}} \gtrsim 2m_t$

$$\mathcal{W}_t = \frac{1}{4m_t N_c} \mathbf{Tr} \langle 0 | h_{v_t}(0) | t \rangle \langle t | \bar{h}_{v_t}(0) \frac{1 + \not{p}_t}{2} | 0 \rangle = 1,$$
$$\mathcal{W}_{\bar{t}} = -\frac{1}{4m_t N_c} \mathbf{Tr} \langle 0 | \bar{\chi}_{v_{\bar{t}}}(0) | \bar{t} \rangle \langle \bar{t} | \frac{1 - \not{p}_{\bar{t}}}{2} \chi_{v_{\bar{t}}}(0) | 0 \rangle = 1,$$

Coulomb enhancement if $M_{t\bar{t}} \rightarrow 2m_t$ [Beneke:2009ye, Beneke:2010da, Beneke:2011mq, Ju:2019lwp, Ju:2020otc]



[Ju:2020otc]

$$\mathcal{W}_t \otimes \mathcal{W}_{\bar{t}} \rightarrow J_c(\Delta E_{t\bar{t}}) \propto \sum_{k=1}^{+\infty} \frac{\alpha_s^k}{\beta_{t\bar{t}}^k}$$

$$\text{where } \beta_{t\bar{t}} \equiv \sqrt{1 - 4m_t^2/M_{t\bar{t}}^2}$$

Factorization

Asymptotic expansion and the leading regions

Absence of the interaction in the heavy parton correlation if $M_{t\bar{t}} \gtrsim 2m_t$

$$\mathcal{W}_t = \frac{1}{4m_t N_c} \mathbf{Tr} \langle 0 | h_{v_t}(0) | t \rangle \langle t | \bar{h}_{v_t}(0) \frac{1 + \gamma_t}{2} | 0 \rangle = 1,$$

$$\mathcal{W}_{\bar{t}} = -\frac{1}{4m_t N_c} \mathbf{Tr} \langle 0 | \bar{\chi}_{v_{\bar{t}}}(0) | \bar{t} \rangle \langle \bar{t} | \frac{1 - \gamma_{\bar{t}}}{2} \chi_{v_{\bar{t}}}(0) | 0 \rangle = 1,$$

Coulomb enhancement if $M_{t\bar{t}} \rightarrow 2m_t$ [Beneke:2009ye, Beneke:2010da, Beneke:2011mq, Ju:2019lwp, Ju:2020otc]

$$\mathcal{W}_t \otimes \mathcal{W}_{\bar{t}} \rightarrow J_c(\Delta E_{t\bar{t}})$$

Numeric Calculations

Imposing $M_{t\bar{t}} > 400$ GeV which guarantees $\beta_{t\bar{t}} \gtrsim 0.5$ being away from the Coulomb singularities

Factorization

Ingredients from the soft-collinear radiations

$$\begin{aligned} \widetilde{\Sigma}_{t\bar{t}}^{[q_n^i \bar{q}_{\bar{n}}^j]} &\rightarrow \mathcal{B}_n^{[q_i]}(\eta_n, b_T, \mu, \nu) \mathcal{B}_{\bar{n}}^{[\bar{q}_j]}(\eta_{\bar{n}}, b_T, \mu, \nu) \sum_{\alpha, \beta} \left\{ \mathcal{H}_{\alpha\beta}^{[q_n^i \bar{q}_{\bar{n}}^j]}(M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, \mu) \right. \\ &\quad \times \mathcal{S}_{[q_n \bar{q}_{\bar{n}}]}^{\alpha\beta}(\vec{b}_T, \mu, \nu) \Big\}, \\ \widetilde{\Sigma}_{t\bar{t}}^{[g_n g_{\bar{n}}]} &\rightarrow \sum_{\alpha, \beta, h_n, h'_{\bar{n}}, h_{\bar{n}}, h'_n} \left\{ \mathcal{B}_{h'_{\bar{n}} h_{\bar{n}}}^{[g_{\bar{n}}]}(\eta_{\bar{n}}, \vec{b}_T, \mu, \nu) \mathcal{B}_{h'_n h_n}^{[g_n]}(\eta_n, \vec{b}_T, \mu, \nu) \right. \\ &\quad \times \mathcal{H}_{\alpha\beta; h'_{\bar{n}} h_{\bar{n}}; h'_n h_n}^{[g_n g_{\bar{n}}]}(M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, \mu) \mathcal{S}_{[g_n g_{\bar{n}}]}^{\alpha\beta}(\vec{b}_T, \mu, \nu) \Big\}, \end{aligned}$$

1) Hard function:

Color-helicity dependent amplitudes, Recola@NLO [Actis:2012qn, Actis:2016mpe]. RGE@N²LO.

Grid-based results [Chen:2017jvi]; Partial analytic results [DiVita:2019ipl, Badger:2021owl, Mandal:2022vju]

2) Quark-beam function:

N³LO [Luo:2020epw, Luo:2019szz]

3) Gluon-beam function:

N³LO@helicity-conserving FF [Luo:2020epw, Luo:2019szz];

N²LO@helicity-flipping FF [Luo:2019bmw, Gutierrez-Reyes:2019rug, Catani:2022sg];

4) Soft function:

Azimuthally averaged N⁽²⁾LO [Zhu:2012ts, Li:2013mia, Angeles-Martinez:2018mqh];

Azimuthally resolved NLO in CSS [Catani:2014qha, Catani:2021cb];

Azimuthally resolved NLO in exp rapidity regulator → This work;

Factorization

Azimuthally resolved soft function with the exponential rapidity regulator

Exponential regulator [Li:2016axz] to curb the rap div

$$\mathcal{S}_{[\kappa]}^{\alpha\beta}(\vec{b}_T, \mu, \nu) \equiv \sum_{m=0} \left[\frac{\alpha_s(\mu)}{4\pi} \right]^m \mathcal{S}_{[\kappa]}^{\alpha\beta,(m)}(\vec{b}_T, \mu, \nu), \quad (1)$$

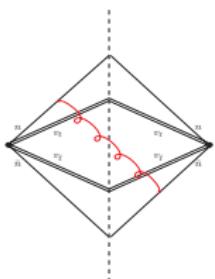
where

$$\mathcal{S}_{[\kappa]}^{\alpha\beta,(0)}(\vec{b}_T, \mu, \nu) = \delta_{\alpha\beta},$$

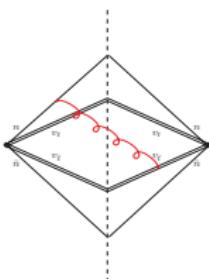
$$\mathcal{S}_{[\kappa]}^{\alpha\beta,(1)}(\vec{b}_T, \mu, \nu) = \sum_{a,b} \langle c_\kappa^\alpha | \mathbf{T}_a \cdot \mathbf{T}_b | c_\kappa^\beta \rangle \mathcal{I}_{ab}(\vec{b}_T, \mu, \nu)$$

Mellin-Barnes (MB)

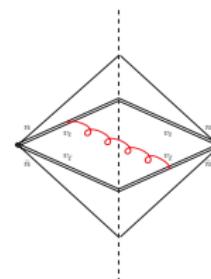
transformation [Smirnov:1999gc, Tausk:1999vh] & MBtools [Czakon:2005rk, Ochman:2015fho, Czakon:Hepforge]



(a) Light-light correlation



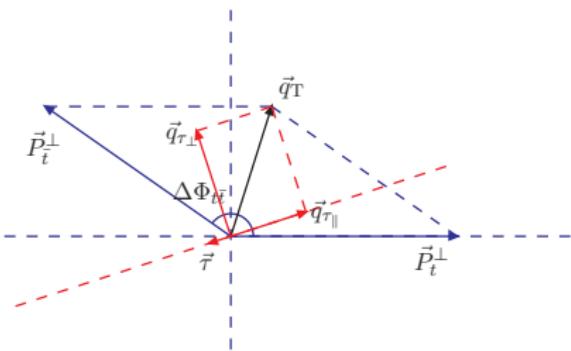
(b) Light-heavy correlation



Factorization

Combining the ingredients aforementioned, performing the multipole/asymptotic expansion in $\lambda_\tau = q_\tau/M_{t\bar{t}}$, and integrating out the rejection components $q_{\tau\perp}$ and $b_{\tau\perp}$, we arrive at the leading power factorization formula

$$\frac{d\sigma_{t\bar{t}}}{dM_{t\bar{t}}^2 d^2 \vec{P}_t^\perp dY_{t\bar{t}} dq_\tau} \rightarrow \sum_\kappa \int_{-\infty}^\infty db_{\tau\parallel} \cos(b_{\tau\parallel} q_\tau) \tilde{\Sigma}_{t\bar{t}}^{[\kappa]}(b_{\tau\parallel} \vec{\tau}, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}, \mu, \nu)$$



A Definitions:

$$\vec{b}_T = \vec{b}_{\tau\perp} + \vec{b}_{\tau\parallel} = b_{\tau\perp} \vec{\tau} \times \vec{n} + b_{\tau\parallel} \vec{\tau},$$

$$\vec{q}_T = \vec{q}_{\tau\perp} + \vec{q}_{\tau\parallel} = q_{\tau\perp} \vec{\tau} \times \vec{n} + q_{\tau\parallel} \vec{\tau}.$$

Resummation

$d\sigma_{t\bar{t}}/dq_\tau$ is free of azimuthally asymmetric divergence !!

Proof:

$$\begin{aligned} \frac{d\sigma_{t\bar{t}}}{dM_{t\bar{t}}^2 d^2 \vec{P}_t^\perp dY_{t\bar{t}} dq_\tau} &\sim \int_{-\infty}^{\infty} db_{\tau_\parallel} \cos(b_{\tau_\parallel} q_\tau) L_M^n \left\{ s_{m,n}(\beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) \right. \\ &\quad \left. + a_{m,n}(\text{sign}[b_{\tau_\parallel}], \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) \right\} \\ &= \left\{ 2 s_{m,n}(\beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) + a_{m,n}(+1, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) \right. \\ &\quad \left. + a_{m,n}(-1, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) \right\} \mathcal{F}_\tau^{(n)}(q_\tau, M_{t\bar{t}}), \end{aligned}$$

where the function $\mathcal{F}_\tau^{(n)}$ is defined as,

$$\mathcal{F}_\tau^{(n)}(q_\tau, M_{t\bar{t}}) = \int_0^\infty db_{\tau_\parallel} \cos(b_{\tau_\parallel} q_\tau) L_M^n.$$

and

$$\mathcal{F}_\tau^{(0)}(q_\tau, M_{t\bar{t}}) = 0, \quad \mathcal{F}_\tau^{(1)}(q_\tau, M_{t\bar{t}}) = -\frac{\pi}{q_\tau}, \quad \mathcal{F}_\tau^{(2)}(q_\tau, M_{t\bar{t}}) = -\frac{2\pi}{q_\tau} \ln \left[\frac{M_{t\bar{t}}^2}{4q_\tau^2} \right], \dots$$

Azimuthally asymmetric divergences in the generic circumstances

[Nadolsky:2007ba, Catani:2010pd, Catani:2014qha, Catani:2017tuc]

Logarithmic exponentiations with RGE and RaGE

[Chiu:2011qc, Chiu:2012ir, Li:2016axz, Li:2016ctv]

- A Renormalisation group equations (RGE) →
exponentiate the visibility-associated logs
- B Renormalisation rapidity group equations (RaGE) →
exponentiate the rapidity-associated logs

Master formula for resummed spectra

$$\frac{d\sigma_{t\bar{t}}^{\text{res}}}{dM_{t\bar{t}}^2 d^2 \vec{P}_t^\perp dY_{t\bar{t}} dq_\tau} \rightarrow \sum_\kappa \int_{-\infty}^{\infty} db_{\tau_{||}} \cos(b_{\tau_{||}} q_\tau) \tilde{\Sigma}_{t\bar{t}}^{\text{res}, [\kappa]}(b_{\tau_{||}} \vec{\tau}, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}),$$

Resummation

Master formula for resummed spectra

$$\frac{d\sigma_{t\bar{t}}^{\text{res}}}{dM_{t\bar{t}}^2 d^2 \vec{P}_t^\perp dY_{t\bar{t}} dq_\tau} \rightarrow \sum_{\kappa} \int_{-\infty}^{\infty} db_{\tau_{||}} \cos(b_{\tau_{||}} q_\tau) \tilde{\Sigma}_{t\bar{t}}^{\text{res}, [\kappa]}(b_{\tau_{||}} \vec{\tau}, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}),$$

where

$$\begin{aligned} \tilde{\Sigma}_{t\bar{t}}^{\text{res}, [q_n^i \bar{q}_{\bar{n}}^j]}(\vec{b}_T, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) &= \left(\frac{1}{2N_c}\right)^2 \mathcal{D}_{[q_n^i \bar{q}_{\bar{n}}^j]}^{\text{res}}(b_T, M_{t\bar{t}}, \mu_h, \mu_b, \mu_s, \nu_b, \nu_s) \mathcal{B}_n^{[q_i]}(\eta_n, b_T, \mu_b, \nu_b) \\ &\times \mathcal{B}_{\bar{n}}^{[\bar{q}_j]}(\eta_{\bar{n}}, b_T, \mu_b, \nu_b) \sum_{\{\alpha, \beta\}} \left\{ \mathcal{S}_{[q_n \bar{q}_{\bar{n}}]}^{\alpha_1 \beta_1}(\vec{b}_T, \mu_s, \nu_s) \left[\mathcal{V}_{\alpha_1 \alpha_2}^{[q_n \bar{q}_{\bar{n}}]}(\beta_{t\bar{t}}, x_t, \mu_s, \mu_h) \right]^* \right. \\ &\times \left. \mathcal{V}_{\beta_1 \beta_2}^{[q_n \bar{q}_{\bar{n}}]}(\beta_{t\bar{t}}, x_t, \mu_s, \mu_h) \mathcal{H}_{\alpha_2 \beta_2}^{[q_n^i \bar{q}_{\bar{n}}^j]}(M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, \mu_h) \right\}, \\ \tilde{\Sigma}_{t\bar{t}}^{\text{res}, [g_n g_{\bar{n}}]}(\vec{b}_T, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) &= \left(\frac{1}{N_c^2 - 1}\right)^2 \mathcal{D}_{[g_n g_{\bar{n}}]}^{\text{res}}(b_T, M_{t\bar{t}}, \mu_h, \mu_b, \mu_s, \nu_b, \nu_s) \\ &\times \sum_{\{\alpha, \beta, h\}} \left\{ \mathcal{B}_{h'_{\bar{n}} h_{\bar{n}}}^{[g_{\bar{n}}]}(\eta_{\bar{n}}, \vec{b}_T, \mu_b, \nu_b) \mathcal{B}_{h'_{\bar{n}} h_n}^{[g_n]}(\eta_n, \vec{b}_T, \mu_b, \nu_b) \mathcal{S}_{[g_n g_{\bar{n}}]}^{\alpha_1 \beta_1}(\vec{b}_T, \mu_s, \nu_s) \right. \\ &\times \left. \left[\mathcal{V}_{\alpha_1 \alpha_2}^{[g_n g_{\bar{n}}]}(\beta_{t\bar{t}}, x_t, \mu_s, \mu_h) \right]^* \mathcal{V}_{\beta_1 \beta_2}^{[g_n g_{\bar{n}}]}(\beta_{t\bar{t}}, x_t, \mu_s, \mu_h) \right. \\ &\times \left. \mathcal{H}_{\alpha_2 \beta_2; h'_{\bar{n}} h_{\bar{n}}; h'_n h_n}^{[g_n g_{\bar{n}}]}(M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, \mu_h) \right\}. \end{aligned}$$

Resummation

Matching resummed spectra on the fixed-order outputs within the multiplicative scheme [Banfi:2012jm, Banfi:2012yh, Bizon:2018foh]

$$\begin{aligned}\frac{d\sigma_{t\bar{t}}^{\text{mat}}}{d\mathcal{Q}} &\equiv \left\{ \left[\frac{d\sigma_{t\bar{t}}^{\text{res}}}{d\mathcal{Q}} - \frac{d\sigma_{t\bar{t}}^s(\mu_{\text{mat}})}{d\mathcal{Q}} \right] f_{\text{tran}}(\mathcal{Q}, c_m, r_m) + \frac{d\sigma_{t\bar{t}}^s(\mu_{\text{mat}})}{d\mathcal{Q}} \right\} \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \\ &= f_{\text{tran}}(\mathcal{Q}, c_m, r_m) \left(\frac{d\sigma_{t\bar{t}}^{\text{res}}}{d\mathcal{Q}} \right) \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \Big|_{\text{exp}} \\ &\quad + \left\{ 1 - f_{\text{tran}}(\mathcal{Q}, c_m, r_m) \right\} \frac{d\sigma_{t\bar{t}}^{\text{FO}}(\mu_{\text{mat}})}{d\mathcal{Q}} + \dots,\end{aligned}$$

where

A $\mathcal{Q} \in \{q_{T,\text{out}}, q_{T,\text{in}}, \Delta\phi_{t\bar{t}}\}$ and the definitions read,

$$\begin{aligned}q_\tau &= q_{T,\text{out}}, \quad \text{if } \vec{\tau} = \pm \vec{n} \times \frac{\vec{P}_t^\perp}{|\vec{P}_t^\perp|}; \\ q_\tau &= q_{T,\text{in}}, \quad \text{if } \vec{\tau} = \pm \frac{\vec{P}_t^\perp}{|\vec{P}_t^\perp|}, \\ \Delta\Phi_{t\bar{t}} &\equiv \cos^{-1} \left[\frac{\vec{P}_t^\perp \cdot \vec{P}_{\bar{t}}^\perp}{|\vec{P}_t^\perp||\vec{P}_{\bar{t}}^\perp|} \right] \sim \pi - \frac{q_{T,\text{out}}}{|\vec{P}_t^\perp|} + \mathcal{O}(\lambda_\tau^2).\end{aligned}$$

Resummation

Matching resummed spectra on the fixed-order outputs within the multiplicative scheme [Banfi:2012jm, Banfi:2012yh, Bizon:2018foh]

$$\begin{aligned} \frac{d\sigma_{t\bar{t}}^{\text{mat}}}{d\mathcal{Q}} &\equiv \left\{ \left[\frac{d\sigma_{t\bar{t}}^{\text{res}}}{d\mathcal{Q}} - \frac{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})}{d\mathcal{Q}} \right] f_{\text{tran}}(\mathcal{Q}, c_m, r_m) + \frac{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})}{d\mathcal{Q}} \right\} \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \\ &= f_{\text{tran}}(\mathcal{Q}, c_m, r_m) \left(\frac{d\sigma_{t\bar{t}}^{\text{res}}}{d\mathcal{Q}} \right) \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \Big|_{\text{exp}} \\ &\quad + \left\{ 1 - f_{\text{tran}}(\mathcal{Q}, c_m, r_m) \right\} \frac{d\sigma_{t\bar{t}}^{\text{FO}}(\mu_{\text{mat}})}{d\mathcal{Q}} + \dots , \end{aligned}$$

where

- A $\mathcal{Q} \in \{q_{T,\text{out}}, q_{T,\text{in}}, \Delta\phi_{t\bar{t}}\}$
- B f_{tran} represents the transition function.

$$f_{\text{tran}}(\mathcal{Q}, c_m, r_m) = \begin{cases} 1, & \mathcal{Q} \leq c_m - r_m ; \\ 1 - \frac{(\mathcal{Q} - c_m + r_m)^2}{2r_m^2}, & c_m - r_m < \mathcal{Q} \leq c_m ; \\ \frac{(\mathcal{Q} - c_m - r_m)^2}{2r_m^2}, & c_m < \mathcal{Q} \leq c_m + r_m ; \\ 0, & c_m + r_m \leq \mathcal{Q} , \end{cases}$$

with

$$\begin{aligned} c_m^{\text{def}} &= 30 \text{ GeV} , r_m^{\text{def}} = 20 \text{ GeV} , & \text{if } \mathcal{Q} \in \{q_{T,\text{out}}, q_{T,\text{in}}\} ; \\ c_m^{\text{def}} &= 0.3 , r_m^{\text{def}} = 0.2 , & \text{if } \mathcal{Q} = \Delta\phi_{t\bar{t}} . \end{aligned}$$

Resummation

Matching resummed spectra on the fixed-order outputs within the multiplicative scheme [Banfi:2012jm, Banfi:2012yh, Bizon:2018foh]

$$\begin{aligned}\frac{d\sigma_{t\bar{t}}^{\text{mat}}}{d\mathcal{Q}} &\equiv \left\{ \left[\frac{d\sigma_{t\bar{t}}^{\text{res}}}{d\mathcal{Q}} - \frac{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})}{d\mathcal{Q}} \right] f_{\text{tran}}(\mathcal{Q}, c_m, r_m) + \frac{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})}{d\mathcal{Q}} \right\} \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \\ &= f_{\text{tran}}(\mathcal{Q}, c_m, r_m) \left(\frac{d\sigma_{t\bar{t}}^{\text{res}}}{d\mathcal{Q}} \right) \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \Big|_{\text{exp}} \\ &\quad + \left\{ 1 - f_{\text{tran}}(\mathcal{Q}, c_m, r_m) \right\} \frac{d\sigma_{t\bar{t}}^{\text{FO}}(\mu_{\text{mat}})}{d\mathcal{Q}} + \dots,\end{aligned}$$

where

- A $\mathcal{Q} \in \{q_{T,\text{out}}, q_{T,\text{in}}, \Delta\phi_{t\bar{t}}\}$
- B f_{tran} represents the transition function.
- C $\mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \equiv \frac{d\sigma_{t\bar{t}}^{\text{FO}}(\mu_{\text{mat}})/d\mathcal{Q}}{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})/d\mathcal{Q}}.$

Resummation

Matching resummed spectra on the fixed-order outputs within the multiplicative scheme [Banfi:2012jm, Banfi:2012yh, Bizon:2018foh]

$$\begin{aligned}\frac{d\sigma_{t\bar{t}}^{\text{mat}}}{dQ} &\equiv \left\{ \left[\frac{d\sigma_{t\bar{t}}^{\text{res}}}{dQ} - \frac{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})}{dQ} \right] f_{\text{tran}}(Q, c_m, r_m) + \frac{d\sigma_{t\bar{t}}^{\text{s}}(\mu_{\text{mat}})}{dQ} \right\} \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \\ &= f_{\text{tran}}(Q, c_m, r_m) \left. \left(\frac{d\sigma_{t\bar{t}}^{\text{res}}}{dQ} \right) \mathcal{R}_{\text{fs}}(\mu_{\text{mat}}) \right|_{\text{exp}} \\ &\quad + \left\{ 1 - f_{\text{tran}}(Q, c_m, r_m) \right\} \frac{d\sigma_{t\bar{t}}^{\text{FO}}(\mu_{\text{mat}})}{dQ} + \dots,\end{aligned}$$

Definition of the logarithmic accuracies

Logarithmic accuracy	$\mathcal{H}, \mathcal{S}, \mathcal{B}$	Γ_{cusp}	$\gamma_{t,h,s,b}$
NLL	$\mathcal{O}(\alpha_s^0)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$
N^2LL	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$
N^2LL'	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$
aN ² LL'	$\mathcal{O}(\alpha_s^2)^{\text{log}}$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$

Table: Precision prerequisites on the anomalous dimensions and the fixed-order functions for a given logarithmic accuracy.

Numerical Results

- $\mathcal{Q} \in \{q_{T,\text{out}}, q_{T,\text{in}}, \Delta\phi_{t\bar{t}}\}$ and the definitions read,

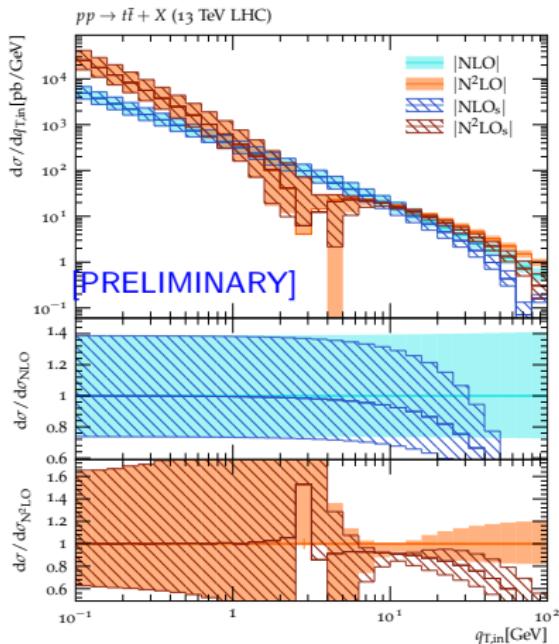
$$q_T = q_{T,\text{out}}, \quad \text{if } \vec{\tau} = \pm \vec{n} \times \frac{\vec{P}_t^\perp}{|\vec{P}_t^\perp|};$$

$$q_T = q_{T,\text{in}}, \quad \text{if } \vec{\tau} = \pm \frac{\vec{P}_t^\perp}{|\vec{P}_t^\perp|},$$

$$\Delta\Phi_{t\bar{t}} \equiv \cos^{-1} \left[\frac{\vec{P}_t^\perp \cdot \vec{P}_{\bar{t}}^\perp}{|\vec{P}_t^\perp||\vec{P}_{\bar{t}}^\perp|} \right] \sim \pi - \frac{q_{T,\text{out}}}{|\vec{P}_t^\perp|} + \mathcal{O}(\lambda_\tau^2).$$

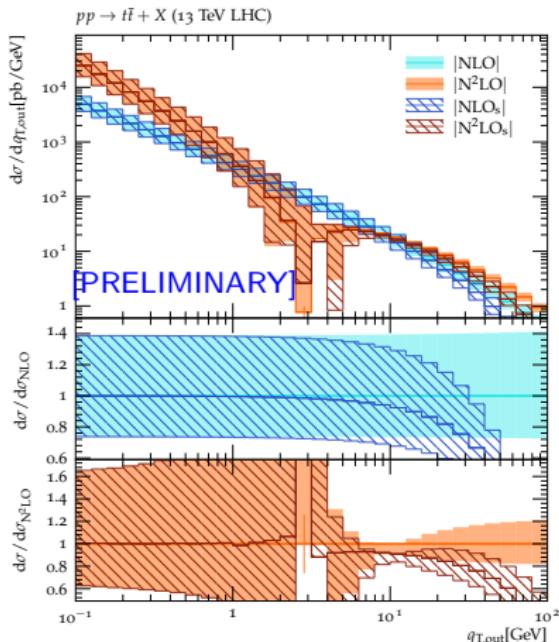
- $\mu_h = \nu_b = M_{t\bar{t}}$, $\nu_s = \mu_b = \mu_s = b_0/b_T$,
- Throughout this paper, we take all the input parameters (including electroweak coupling as well as the involved masses and widths) [PDG].
- The PDFs utilized in this work is NNPDF3.1 from [Ball:2017nwa].
- Imposing $M_{t\bar{t}} > 400$ GeV which amounts to $\beta_{t\bar{t}} \gtrsim 0.5$ in a bid to circumvent the Coulomb singularities.

Validation



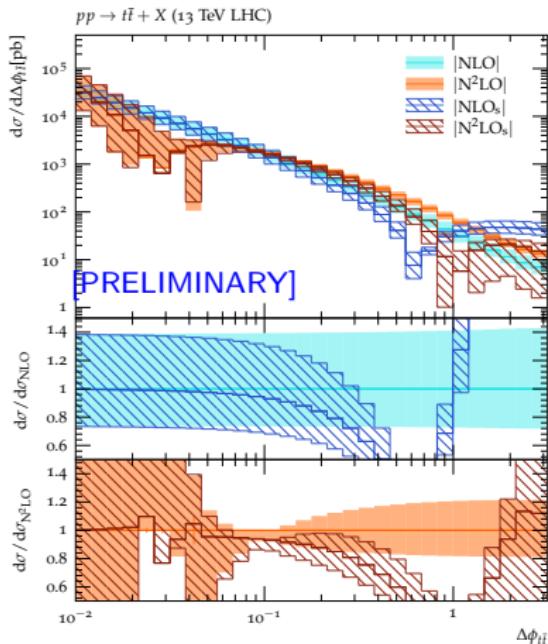
- $q_{T,\text{in}} \equiv |\vec{q}_\perp^{t\bar{t}} \cdot \vec{\tau}|$ where $\vec{\tau} \parallel \vec{P}_\perp^t$
 - Uncertainties: matching scale
[$2M_{t\bar{t}}, M_{t\bar{t}}/2$]
 - Obvious agreements between approximate and exact results in $q_T \rightarrow 0$

Validation



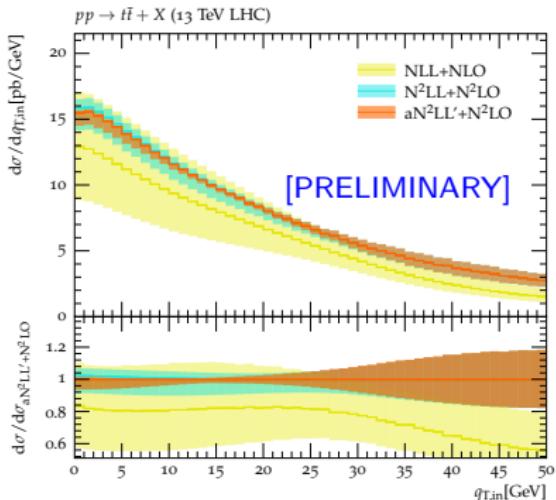
- $q_{T,\text{out}} \equiv |\vec{q}_{\perp}^{t\bar{t}} \cdot \vec{\tau}|$ where $\vec{\tau} \perp \vec{P}_{\perp}^t$
- Uncertainties: matching scale $[2M_{t\bar{t}}, M_{t\bar{t}}/2]$
- Obvious agreements between approximate and exact results in $q_T \rightarrow 0$

Validation



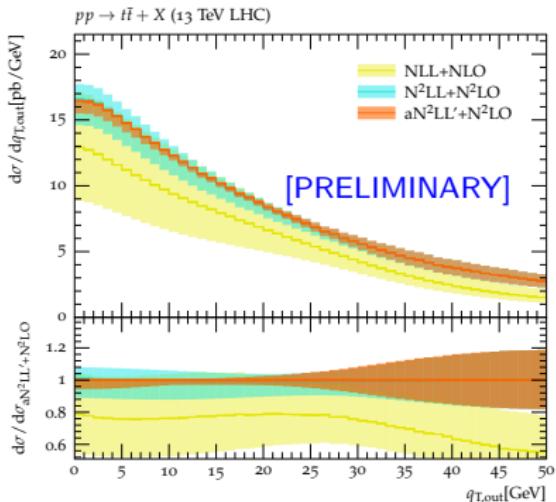
- $\Delta\phi_{t\bar{t}} \equiv \pi - \cos^{-1} \left[\frac{\vec{P}_t^\perp \cdot \vec{P}_{\bar{t}}^\perp}{|\vec{P}_t^\perp| |\vec{P}_{\bar{t}}^\perp|} \right] \sim \frac{q_{T,\text{out}}}{|\vec{P}_t^\perp|} + \mathcal{O}(\lambda_\tau^2)$
- Uncertainties: matching scale $[2M_{t\bar{t}}, M_{t\bar{t}}/2]$
- Obvious agreements between approximate and exact results in $q_T \rightarrow 0$

Resummation



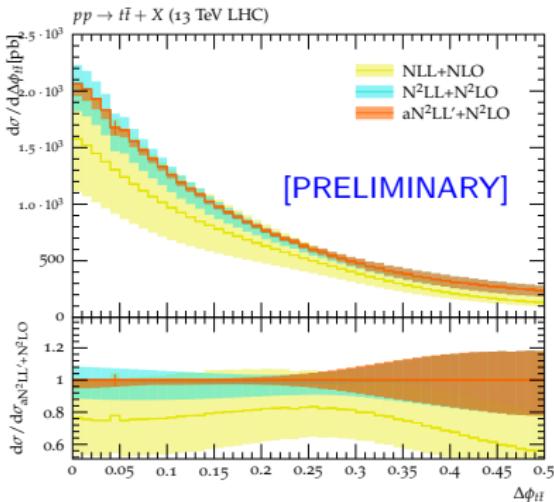
- Uncertainties: intrinsic scales $[2\mu, \mu/2]$ & parameters $\{c_m, r_m\}$ of f_{tran}
- Asymptotic regime
 - The central values are close to each other.
 - With the accuracy growing, the uncertainties decrease considerably.
 - The error bands of higher accuracy are contained by those with lower accuracy.

Resummation



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Conclusion

Upshot

- In the context of SCET_{II}+HQET, we derive the leading power factorization formula of the q_τ spectra on the $t\bar{t}$ production
- Akin to $|\vec{q}_\perp^{t\bar{t}}|$, q_τ is another observable insensitive to azimuthal asymmetric divergences.
- Based on the framework of RGE and RaGE, we carry out the resummation for three particular observables $\mathcal{Q} \in \{q_{T,\text{out}}, q_{T,\text{in}}, \Delta\phi_{t\bar{t}}\}$ at the accuracies NLL, NNLL, and the approximate NNLL'.
- With the increase in the logarithmic accuracy, the manifest reductions in the theoretical uncertainties are observed.