A NLL accurate Parton Shower algorithm in Sherpa

Florian Herren

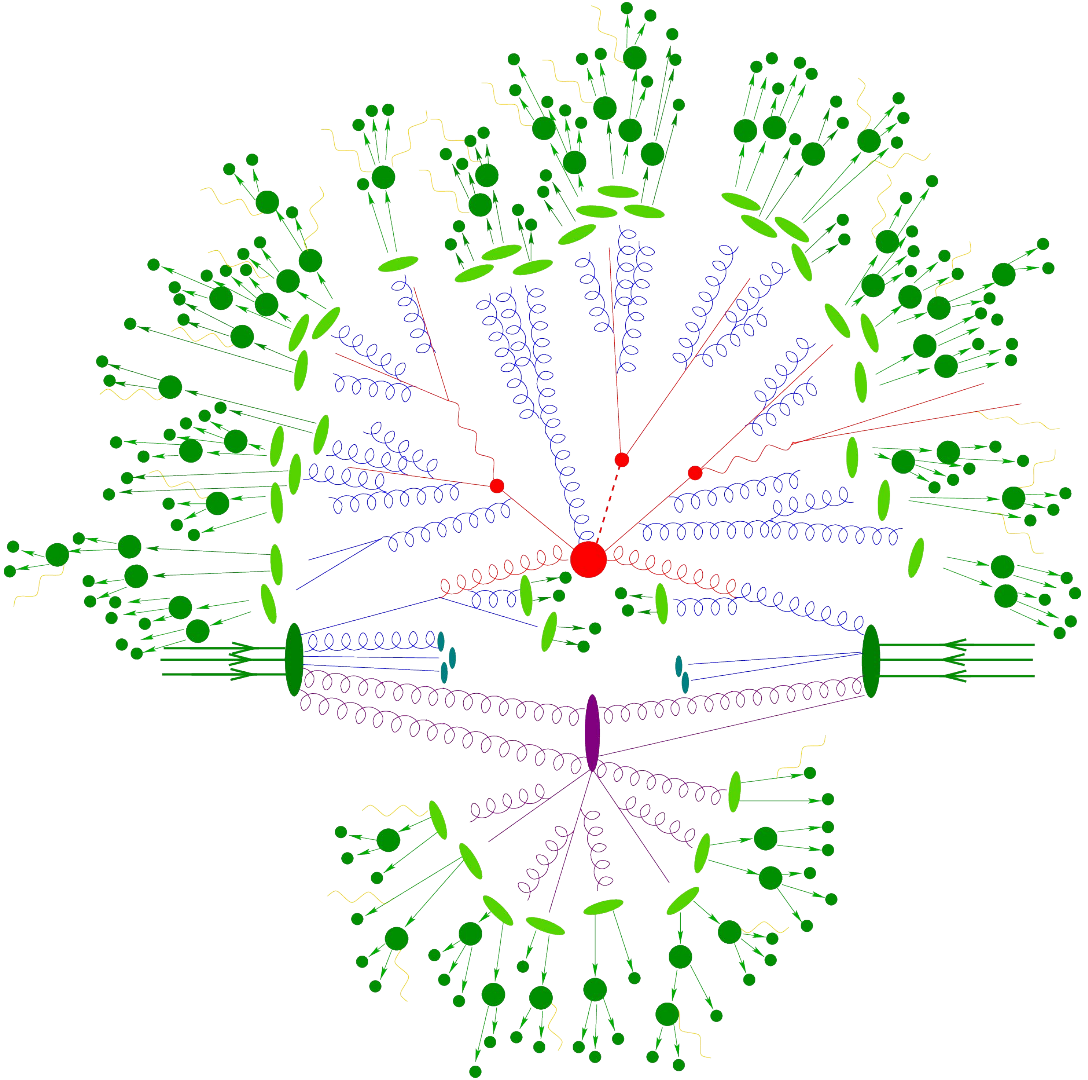
2208.06057, in collaboration with Stefan Höche, Frank Krauss, Daniel Reichelt & Marek Schönherr

Event Generators

Crucial for precision Collider Physics

Combine different physics at different scales:

- Hard Process
- Parton Shower
- Underlying Interaction
- Hadronization
- QED FSR
- Hadron Decays

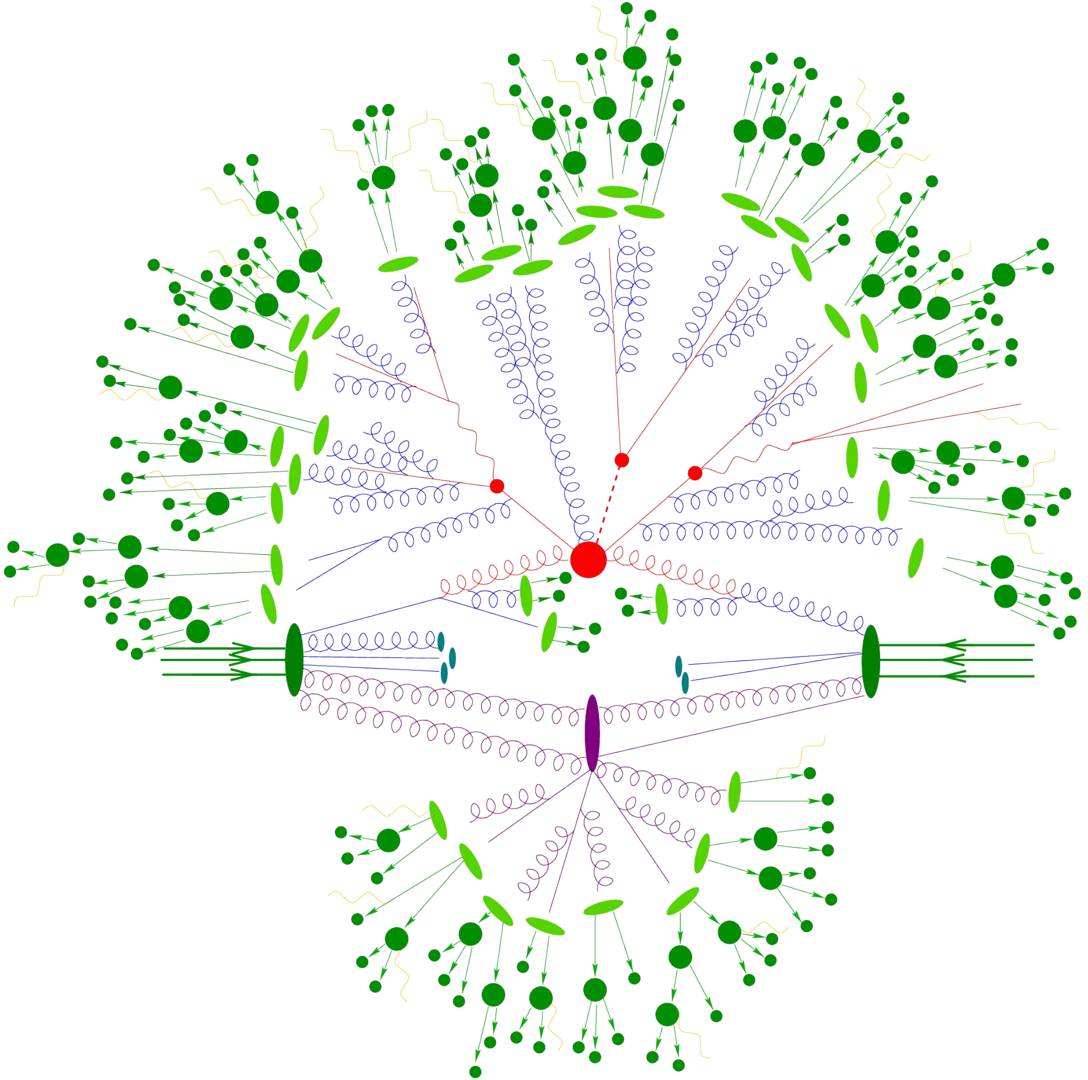


NLL Showers

Criteria for NLL accuracy:

- Generate correct square tree-level ME when one kinematic variable (angles, kt) for two emissions differ significantly and another one is similar
- Reproduce NLL results for rIRC safe observables → Subsequent Emissions don't change previous ones significantly

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] 2002.11114



Soft Radiation

Factorisation in the soft limit:

$$ {}_{n}\langle 1,\ldots,n|1,\ldots,n\rangle_{n} = -8\pi\alpha_s \sum_{i,k\neq j} {}_{n-1}\langle 1,\ldots,\cancel{j},\ldots,n\big|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}\big|1,\ldots,\cancel{j},\ldots,n\rangle_{n-1} $$

Eikonal factor:

$$ w_{ik,j} = \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2}\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} $$

Implementing the Eikonal in the collinear limit leads to double-counting of soft singularities

[Marchesini, Webber] *Nucl.Phys.B* 310 (1988) 461-526

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$\boxed{W_{ik,j}}$

Additive matching of singularities:

$$ W_{ik,j} = \tilde{W}^{i}_{ik,j} + \tilde{W}^{k}_{ki,j} $$

$$ \tilde{W}^{i}_{ik,j} = \frac{1}{2}\left(W_{ik,j} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right) $$

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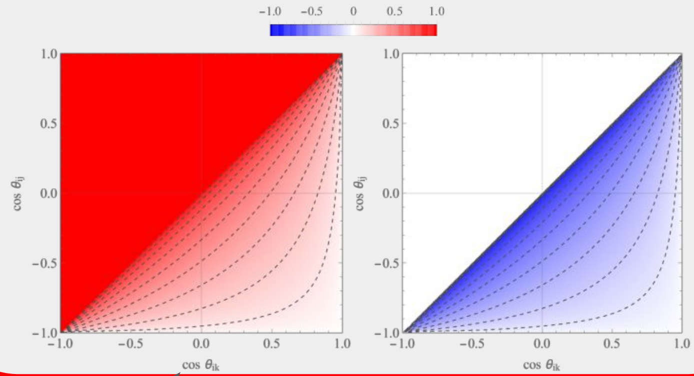
Option 2: Implement radiator differentially



Option 1:
Angular Ordering $\rightarrow$ Spoils NGLs

Azimuthal average:

$$
\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\tilde{W}^i_{ik,j} = \frac{\theta(\theta_{ik}-\theta_{ij})}{1-\cos\theta_{ij}}
$$

Soft Radiation

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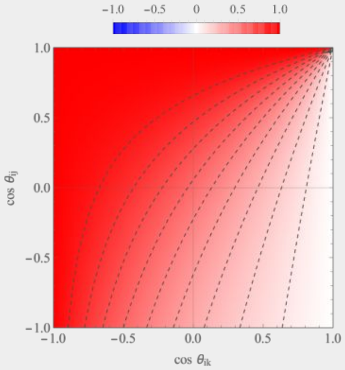
Multiplicative matching of singularities:

$$ W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j} $$

$$ \bar{W}^i_{ik,j} = W_{ik,j}\frac{1-\cos\theta_{jk}}{2-\cos\theta_{ij}-\cos\theta_{jk}} $$

[Catani, Seymour] hep-ph/9605323

Implement radiator differentially



Azimuthal average:

$$ \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\bar{W}^i_{ik,j} = \frac{1}{\sqrt{(A^i_{ik,j})^2-(B^i_{ik,j})^2}} $$

$$ A^i_{ij,k} = \frac{2-\cos\theta_{ij}(1+\cos\theta_{ik})}{1-\cos\theta_{ik}} $$

$$ B^i_{ij,k} = \frac{\sqrt{(1-\cos^2\theta_{ij})(1-\cos^2\theta_{ik})}}{1-\cos\theta_{ik}} $$

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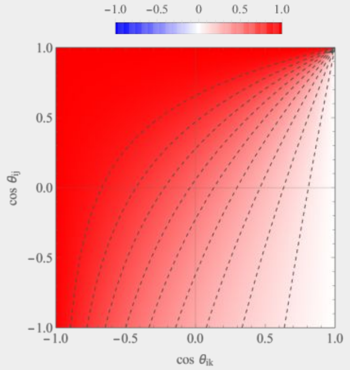


$$ \frac{1}{2p_ip_j}P_{(ij)i}(z) \to \frac{1}{2p_ip_j}P_{(ij)i}(z)+\delta_{(ij)i}\left[\frac{\bar{W}^{i}_{ik,j}}{E_j^2} - w^{(\mathrm{coll})}_{ik,j}(z)\right] $$

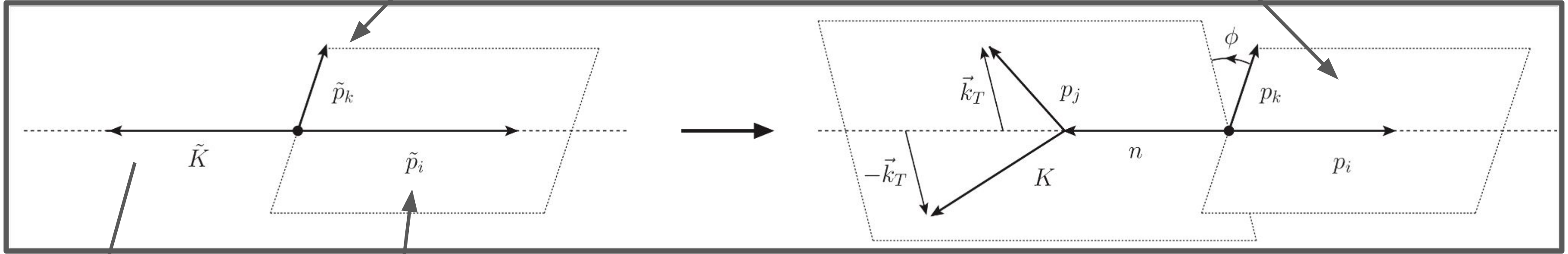
Momentum Mapping

Main Idea:
maintain directions of hard particles exactly

$$p_i = z\tilde{p}_i \qquad z = \frac{p_i n}{(p_i + p_j)n}$$

$$p_k = \tilde{p}_k$$

Color Spectator



Color neutral System

Emitter

Need to find K and p_j such that:

$$K^2 = \tilde{K}^2 \quad p_j \to (1-z)\tilde{p}_i$$

Shift:
$$n = \tilde{K} + (1-z)\tilde{p}_i$$

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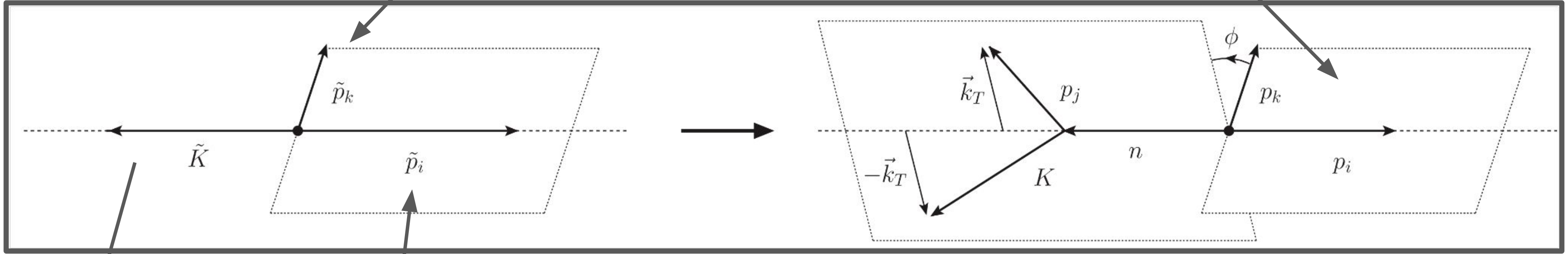


Color neutral System

Emitter

$$v = \frac{p_i p_j}{p_i \tilde{K}} \qquad \kappa = \frac{\tilde{K}^2}{2\tilde{p}_i \tilde{K}}$$

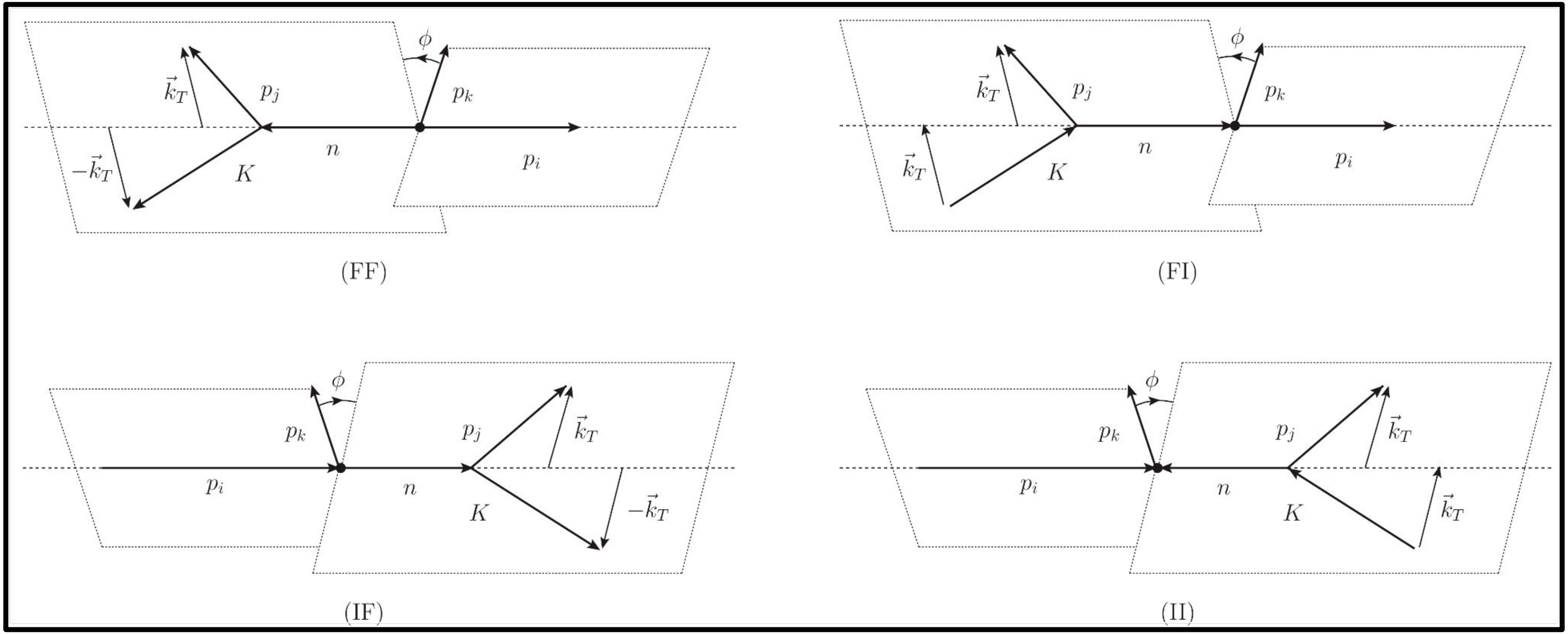
$$p_j = (1-z)\tilde{p}_i + v\left(\tilde{K} - (1-z+2\kappa)\tilde{p}_i\right) + k_\perp$$
$$K = \tilde{K} - v\left(\tilde{K} - (1-z+2\kappa)\tilde{p}_i\right) - k_\perp$$

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \rightarrow \Lambda^\mu_\nu(K,\tilde{K})p_l^\nu$$

Recoil

Momentum mapping works for initial and final state emitters/spectator
→ e+ e-, pp, DIS, … all treated on same footing



Recoil

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

Define

$$X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$
$$= v\big(\tilde{K}^\mu - (1 - z + 2\kappa)\,\tilde{p}_i^\mu\big) + k_\perp^\mu$$

At most $\mathcal{O}(k_\perp)$ in logarithmically enhanced region

Recoil

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$$\Lambda^\mu{}_\nu(K, \tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right] \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}$$

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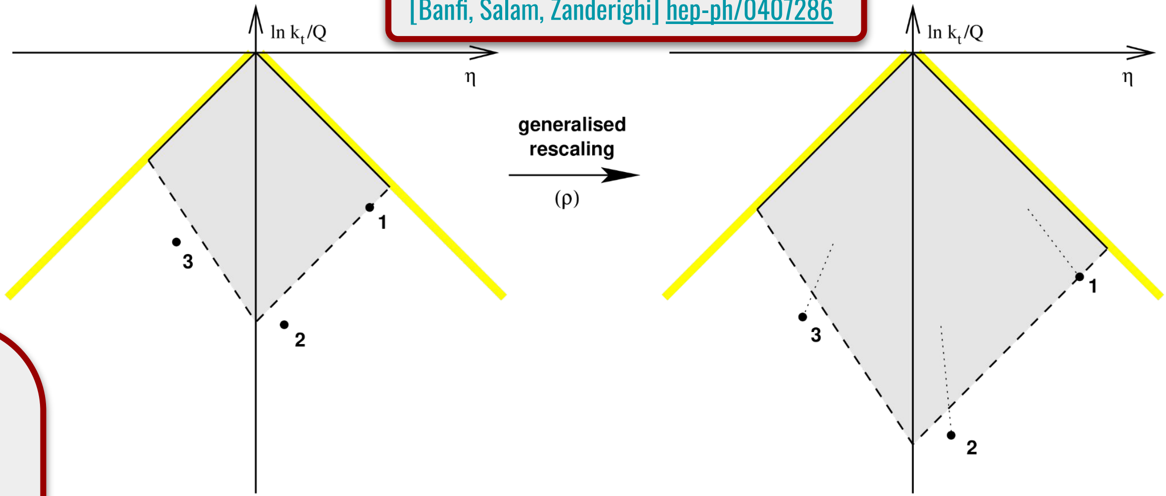
Suppressed by

$\mathcal{O}(k_\perp/K)$

$$\Lambda^\mu_\nu \approx g^\mu_\nu + \frac{K_\rho X_\sigma}{K^2}T^{\mu\rho\sigma}_\nu + \mathcal{O}(k_\perp^2)$$

Recoil

[Banfi, Salam, Zanderighi] hep-ph/0407286



For one emission kinematic variables in the Lund plane scale like:

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln \rho}{a+b}$$

$$\xi_l = \frac{\eta_l}{\eta_{l,\max}}$$

where a = 1 and b = 0 for Alaric

Working in the rest frame of the color dipole, the other momenta scale like:

$$\tilde{p}_l^0 \sim \rho^{1-\xi_l}$$

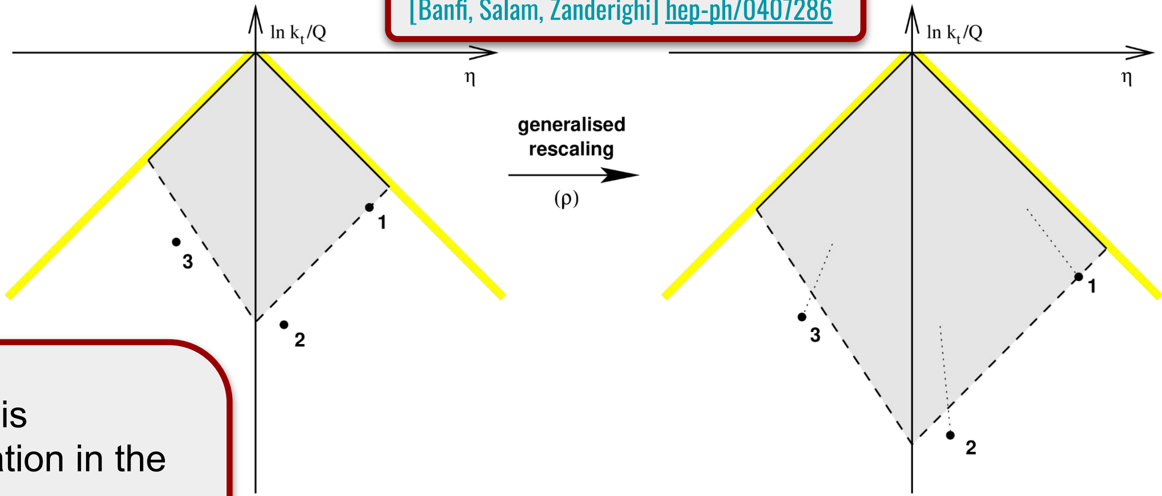
$$\tilde{p}_l^{1,2} \sim \rho$$

$$\tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

for $\rho \to 0$

Recoil

[Banfi, Salam, Zanderighi] hep-ph/0407286



Scaling under an additional emission is determined by the Lorentz transformation in the limit $\rho \to 0$:

$$\Delta p_l^\mu = 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,\tilde{K}^\mu - \frac{\tilde{p}_l X}{\tilde{K}^2}\,\tilde{K}^\mu + \frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,X^\mu$$

Scaling becomes:

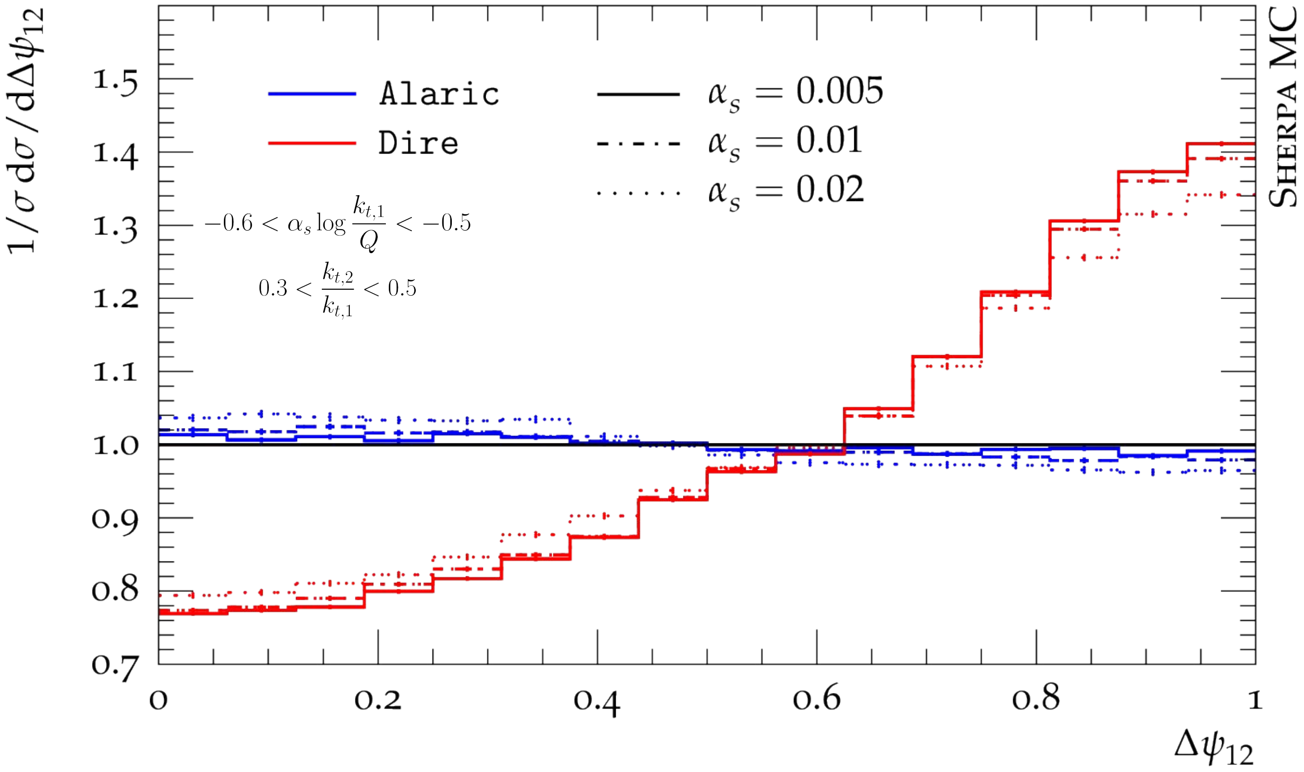
$$\Delta p_l^0 \sim \rho^{1-\xi_l}X^0 + \rho^{2-\xi_l-\max(\xi_i,\xi_j)}\tilde{K}^0 + \rho^{1-\xi_l}X^0 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^0 \sim \rho^{1-\xi_l}$$

$$\Delta p_l^{1,2} \sim \rho^{1-\xi_l}X^{1,2} \sim \rho^{2-\xi_l} \qquad \tilde{p}_l^{1,2} \sim \rho$$

$$\Delta p_l^3 \sim \rho^{1-\xi_l}X^3 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

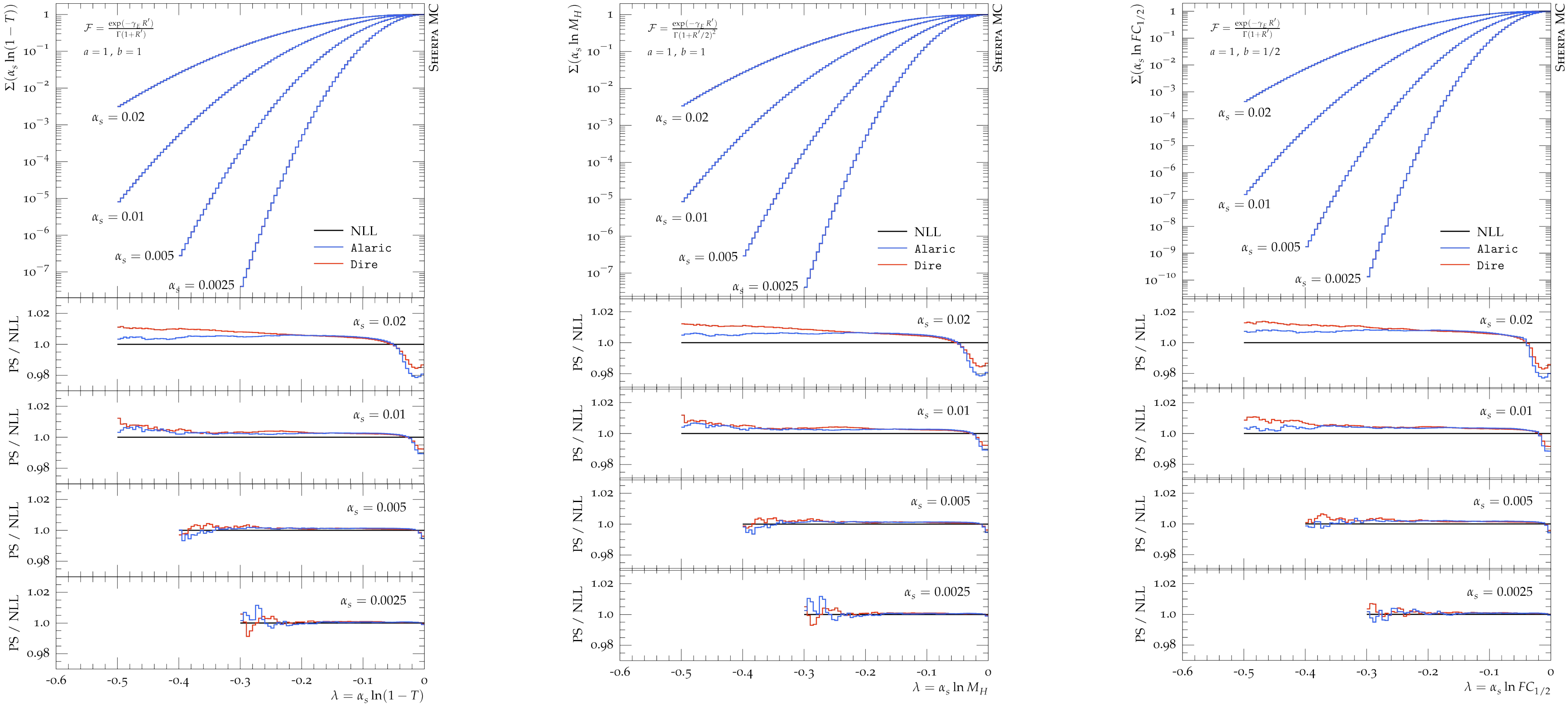
Numerical Tests

Azimuthal angle between two Lund plane declusterings
Tests soft and rapidity separated emissions



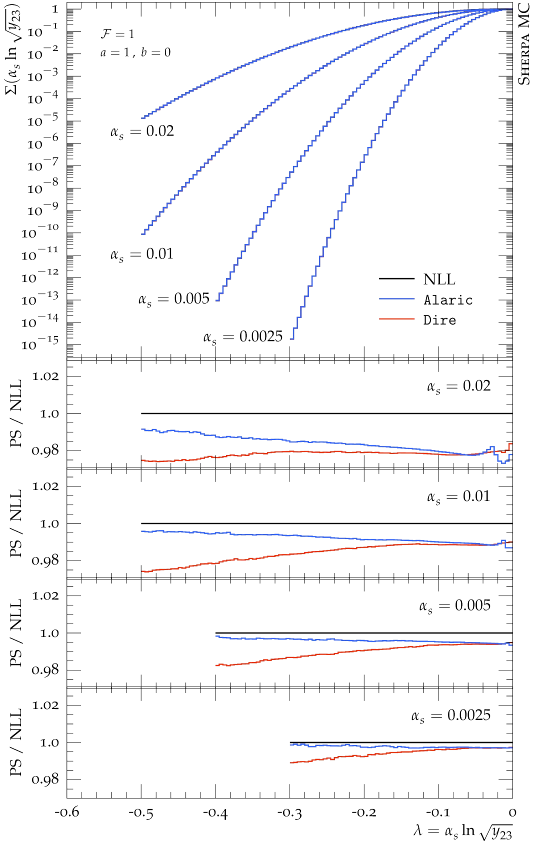
Numerical Tests

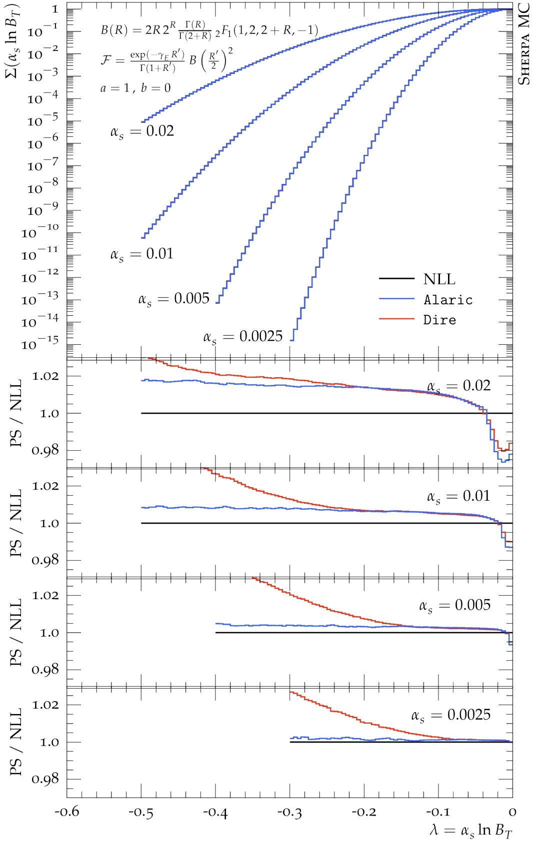
For Thrust, Heavy Jet mass and Fractional Energy Correlators with b = 1, both Dire and Alaric are NLL

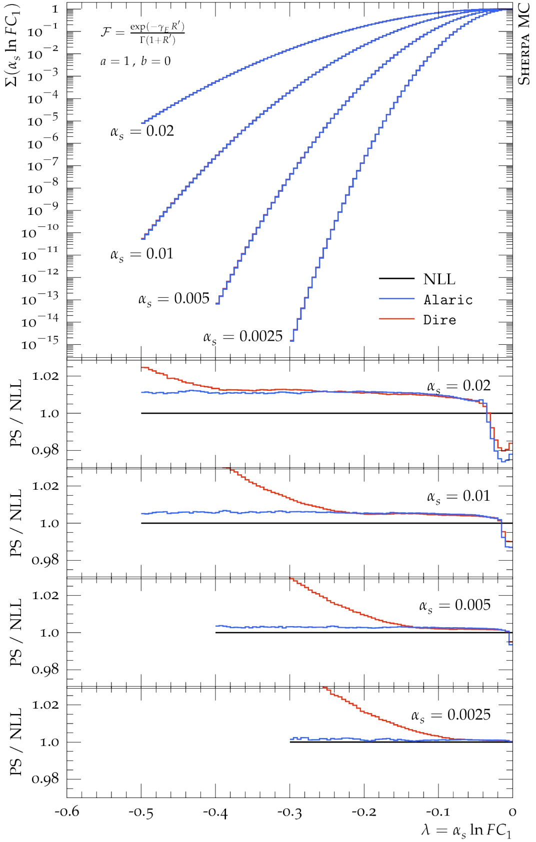


Numerical Tests

For the Two-Jet rate, total Broadening and FC with b = 1 Alaric and Dire differ



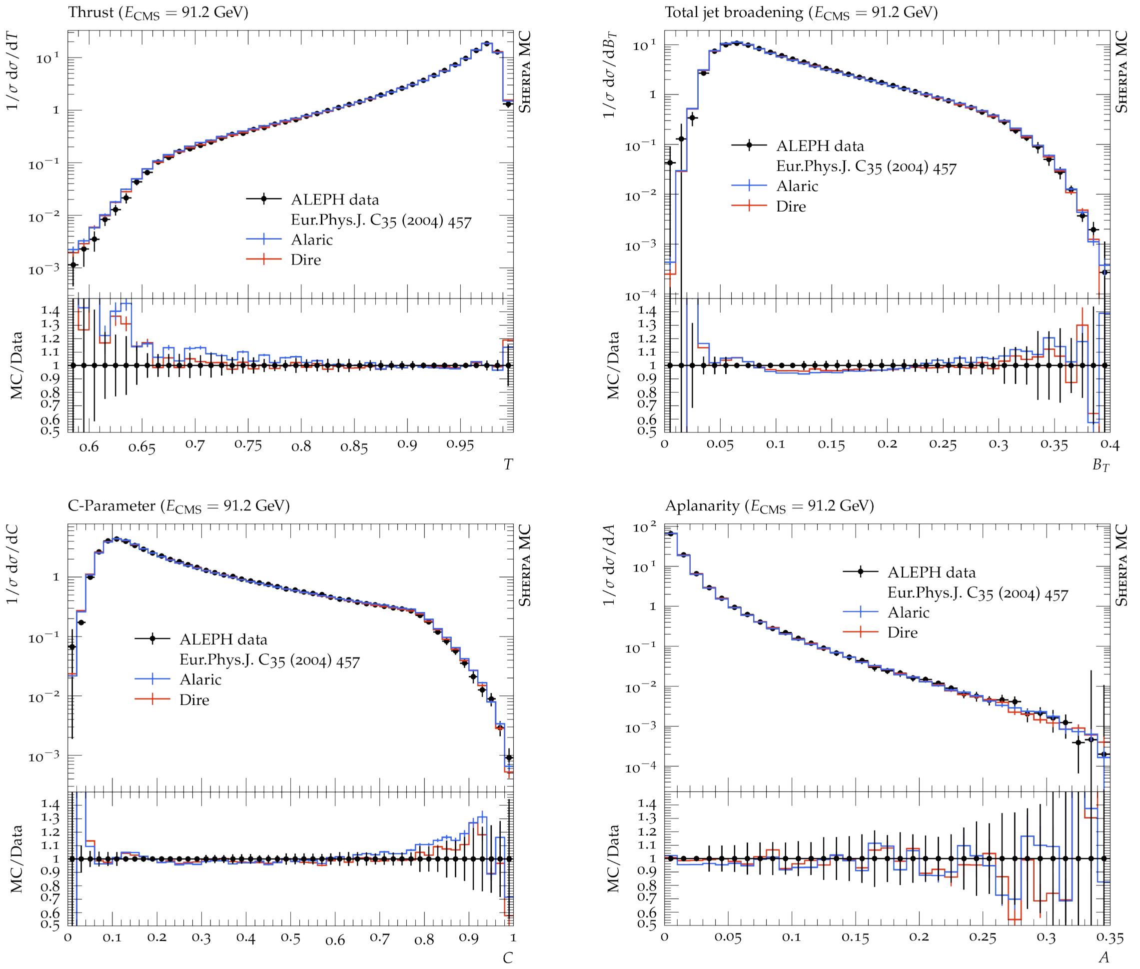




Let’s look at Data

Details:
- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:
- Perturbative region to the right, except for thrust
- Some deviations for Broadening and Aplanarity
- MECs and massive quarks will improve situation



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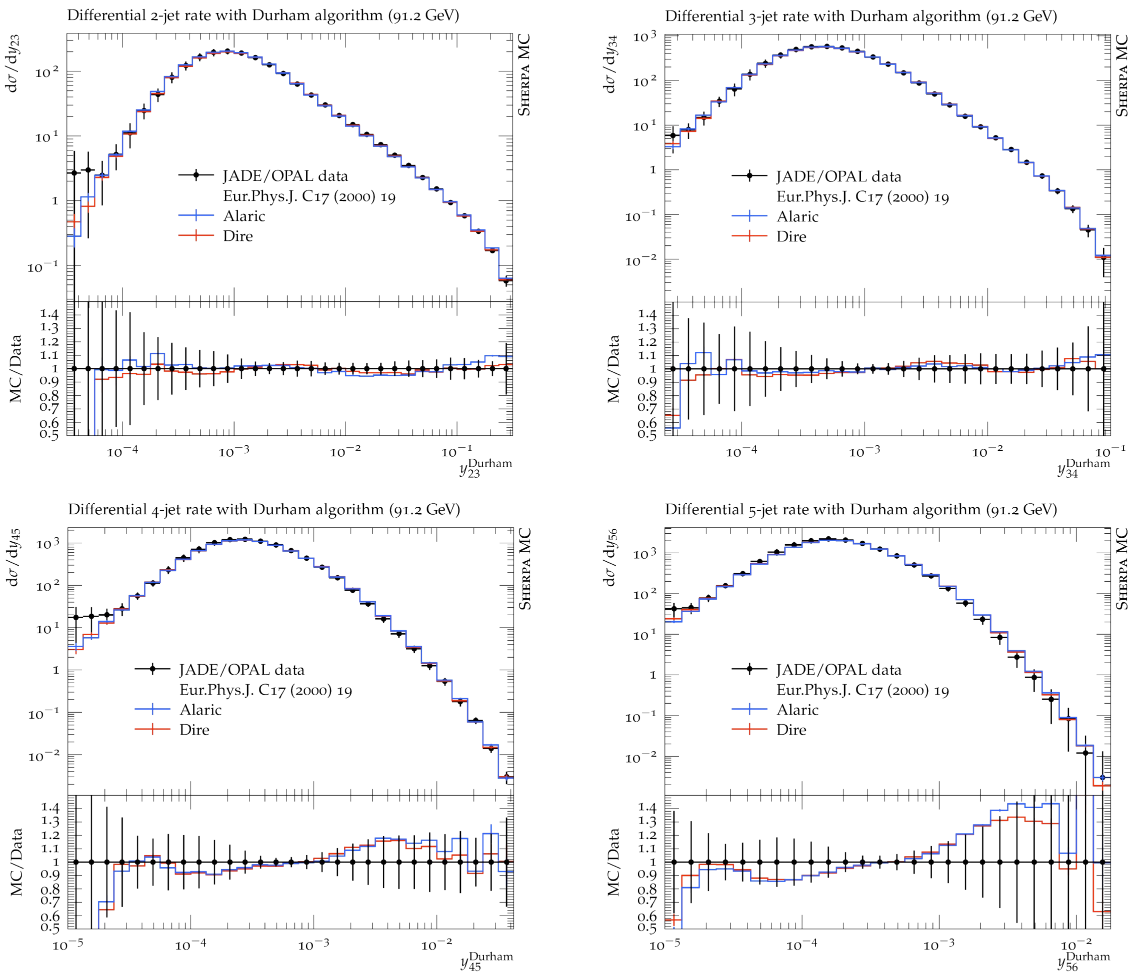
Comments:
- Perturbative region to the right
- b-quark mass corresponds to $y \approx 2.8 \times 10^{-3}$



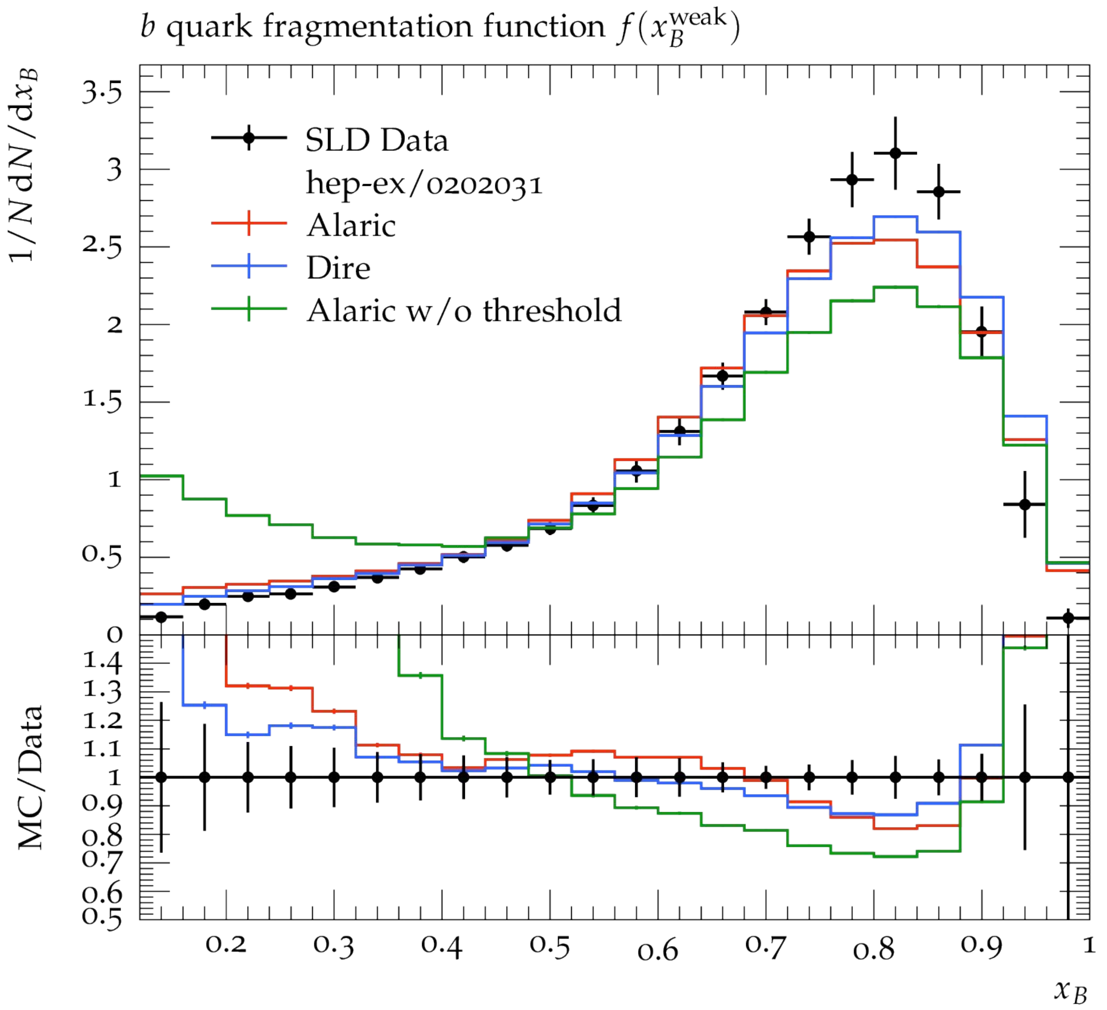
Let’s look at Data

Details:

- CMW scheme
- Massless b- and c-quarks
- Flavour thresholds
- Hadronization through Lund string fragmentation

Comments:

- Low values of x dominated by g → bb
- Large values of x dominated by b → bg and hadronization



Conclusion

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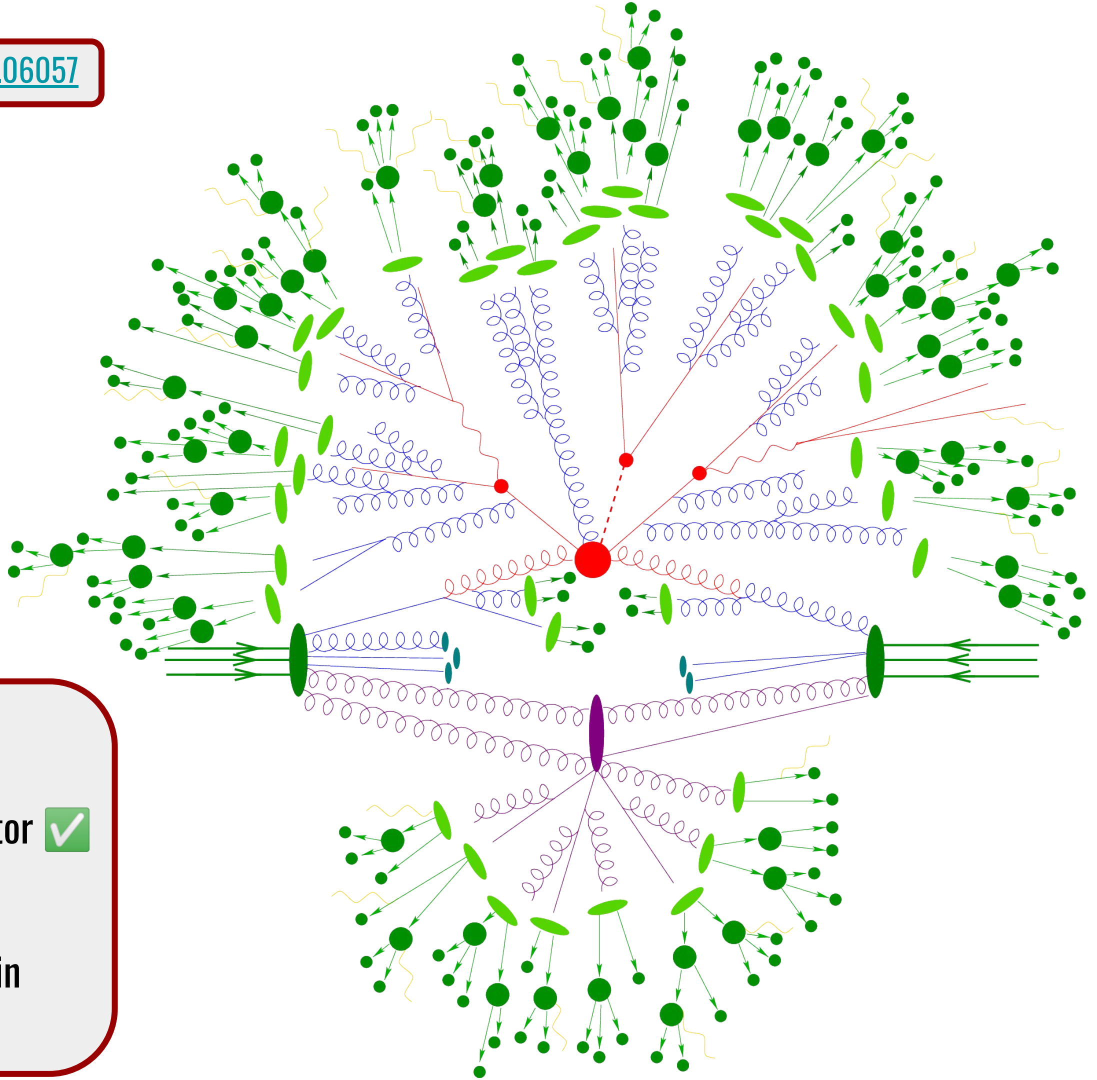
- We presented a new NLL accurate Parton Shower Algorithm: Alaric
- First dipole-like algorithm to disentangle colour and kinematics
- Strict positivity of evolution kernels
- Momentum mapping preserves directions of hard partons

Additional developments:

- Initial state emitter and spectator ✅
- Initial state emitter and final state spectator ✅
- NLO matching ✅
- Implementation in Sherpa ✅

Add massive quarks, higher order corrections, spin correlations, subleading colour,...



Backup

NLO Matching

Alaric shares many similarities with
Catani-Seymour identified particle subtraction
$\rightarrow$ MC@NLO matching straightforward
Follow [Höche, Liebschner, Siegert] 1807.04348

Combined integrated subtraction term for identified parton production with a partonic fragmentation function:

$$\int_{m+1} \mathrm{d}\sigma^S + \int_m \mathrm{d}\sigma^C = \frac{1}{2} \sum_{i=g,q,\bar{q}} \sum_{\tilde{\imath}=1}^{m} \int_0^1 \frac{\mathrm{d}z}{z^{2-2\epsilon}} \int_m \mathrm{d}\sigma^B\left(p_1, \ldots, \frac{p_i}{z}, \ldots, p_m\right) \otimes \hat{\mathbf{I}}_{\tilde{\imath}i}^{(\mathrm{FS})}$$

Insertion operator:

$$\hat{\mathbf{I}}_{\tilde{\imath}i}^{(\mathrm{FS})} = \delta(1-z)\mathbf{I}_{\tilde{\imath}i} + \mathbf{P}_{\tilde{\imath}i} + \mathbf{H}_{\tilde{\imath}i}$$

$$\mathbf{I}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; \epsilon) = -\frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}} \mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left(\frac{4\pi\mu^2}{2p_i p_k}\right)^{\epsilon} \mathcal{V}_{\tilde{\imath}i}(\epsilon)$$

$$\mathbf{P}_{\tilde{\imath}i}\left(p_1, \ldots, \frac{p_i}{z}, \ldots, p_m; z; \mu_F\right) = \frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}} \mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \ln \frac{z\mu_F^2}{2p_i p_k} \delta_{\tilde{\imath}i} P_{\tilde{\imath}i}(z)$$

$$\mathbf{H}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; n; z) = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}} \mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left[\tilde{K}^{\tilde{\imath}i}(z) + \bar{K}^{\tilde{\imath}i}(z) + 2P_{\tilde{\imath}i}(z) \ln z + \mathcal{L}^{\tilde{\imath}i}(z; p_i, p_k, n)\right]$$

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Non-trivial integral:

$$\int_0^1 \mathrm{d}z\, \mathbf{H}_{\tilde{\imath}i}(p_1, \ldots, p_i, \ldots, p_m; n; z) = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left\{ \mathcal{K}^{\tilde{\imath}i} + \delta_{\tilde{\imath}i} \operatorname{Li}_2\left(1 - \frac{2\tilde{p}_i\tilde{p}_k\, \tilde{K}^2}{(\tilde{p}_i\tilde{K})(\tilde{p}_k\tilde{K})}\right) - \int_0^1 \mathrm{d}z\, P_{\mathrm{reg}}^{qq}(z) \ln \frac{n^2 \tilde{p}_i\tilde{p}_k}{2z(\tilde{p}_i n)^2} \right\}$$