Two-loop helicity amplitudes for $W\gamma + j$ production

(based on hep-ph/2201.04075)

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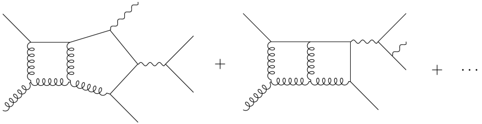
Typical workflow of loop amplitude computations

1. Draw all relevant Feynman diagrams:



2. Write down the integrand:

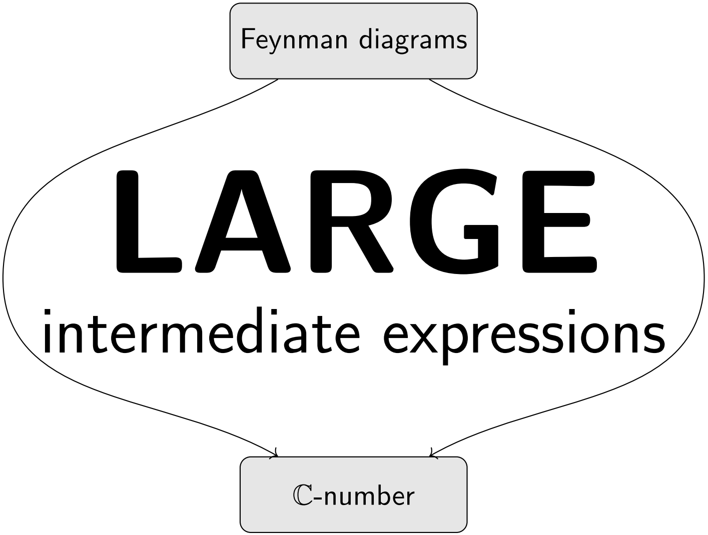
$$A = \sum_{T \in topologies} \int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\sum_i c_i(\{p\}) m_i(\{k,p\})}{\prod_{j \in T} D_j(\{k,p\})}$$

3. Reduce the amplitude onto a set of master integrals:

$$A = \sum_j d_j(\epsilon, p) \times MI_j(\epsilon, p)$$

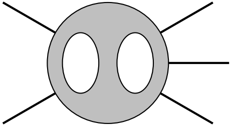
4. Evaluate the result at a chosen phase-space point

Complexity


Feynman diagrams
LARGE
intermediate expressions
$\mathbb{C}$-number

Complexity

- Complexity increases with **loop order** and **multiplicity**.
- Current QCD frontier: $2 \to 3$ scattering at NNLO.

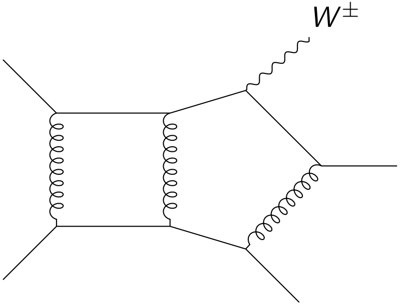


- Massless case: results for all relevant Feynman integrals available.
- One external mass: results for all planar + some non-planar integrals now available.

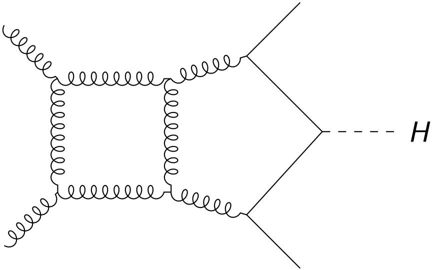
one-mass, planar:	Nov '15	[Papadopoulos, Tommasini, Wever]	(one penta-box, MPLs)
	May '20	[Abreu, Ita, Moriello, Page, Tschernow, Zeng]	(DEs+numerical sols)
	Sep '20	[Canko, Papadopoulos, Syrrakos]	(MPLs)
	Dec '20	[Syrrakos]	(1L pentagon, MPLs)
	Oct '21	[Chicherin, Sotnikov, Zoia]	(2L pentagon functions)
one-mass, non-planar	Oct '19	[Papadopoulos, Wever]	(one hexa-box, MPLs)
	July '21	[Abreu, Page, Ita, Tschernow]	(hexa-box, DEs+numerical sols)

Recent work

- $pp \to W/H + b\bar{b}$ at 2L (leading colour, massless b quarks)



[Badger, Hartanto, Zoia, Feb '21]

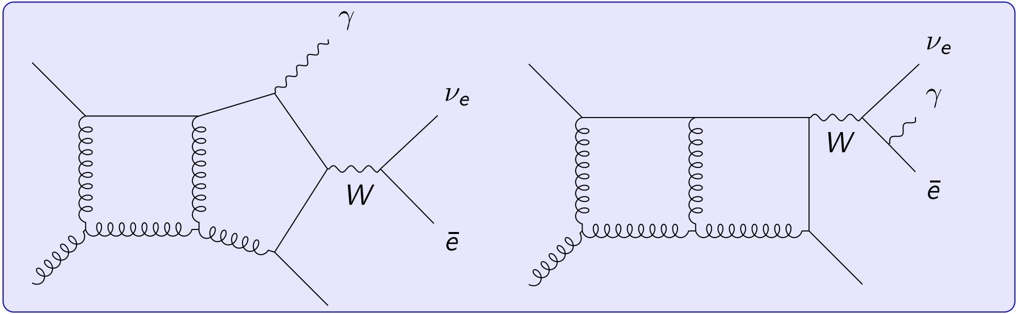


[Badger, Hartanto, **Kryś**, Zoia, July '21]

- $W(\to \ell\bar{\ell}') + 4$-partons at 2L (leading colour, massless quarks)
  [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, Oct '21]

Recent work

- $pp \to W(\to \bar{e}\nu_e)\gamma + j$ at 2L (leading colour, massless quarks)
- Detach the leptonic $W-$boson decay and only compute the W production amplitudes



[Badger, Hartanto, **Kryś**, Zoia, Jan '22]

- Important for precision SM tests and constraining BSM physics

Finite fields

- To avoid analytic complexity in intermediate steps, use numerical evaluations over **finite fields**
- We work with **rational numbers** modulo a large prime number:

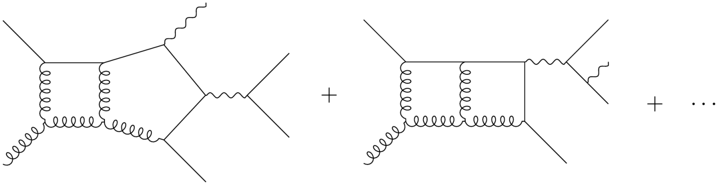
$$q = \frac{a}{b} \longrightarrow q \bmod p \equiv \left(a \times \left(b^{-1} \bmod p\right)\right) \bmod p$$

$$\frac{3}{7} \equiv 2 \bmod 11$$

- One can reconstruct the analytic result from its many numerical evaluations
- `FiniteFlow` [Peraro, '19]

$W\gamma + j$

- Generate the amplitudes starting from Feynman diagrams:



- Compute two kinds of $W-$production amplitudes:
  - $A_{6,u/d}(p_1, p_2, p_3, p_4, p_5, p_6) = A^{\mu}_{5,u/d}(p_1, p_2, p_3, p_4, p_W)\ L_{A,\mu}(p_5, p_6)$
  - $A_{6,e/W}(p_1, p_2, p_3, p_4, p_5, p_6) = A^{\mu}_{4}(p_2, p_3, p_4, \tilde{p}_W)\ L^{e/W}_{B,\mu}(p_1, p_5, p_6)$
- Decompose the amplitudes in the basis of external momenta and define contracted amplitudes:

$$A^{\mu}_5 = p^{\mu}_1 a_1 + p^{\mu}_2 a_2 + p^{\mu}_3 a_3 + p^{\mu}_4 a_4$$
$$\tilde{A}_{5,i} = p_i \cdot A_5$$

$W\gamma + j$

- Perform the $4D$ tensor decomposition on $\tilde{A}_{5,i}$ using **physical projectors** [Peraro, Tancredi, '19], [Peraro, Tancredi, '21]
- The contracted amplitudes can be written as:

$$\tilde{A}_{5,i} = \sum_{T \in topologies} \int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\sum_i c_i(\{p\}) m_i(\{k,p\})}{\prod_{j \in T} D_j(\{k,p\})}$$

- Coefficients c_i are functions of external kinematics only and are expressed through the five-point Mandelstam invariants:

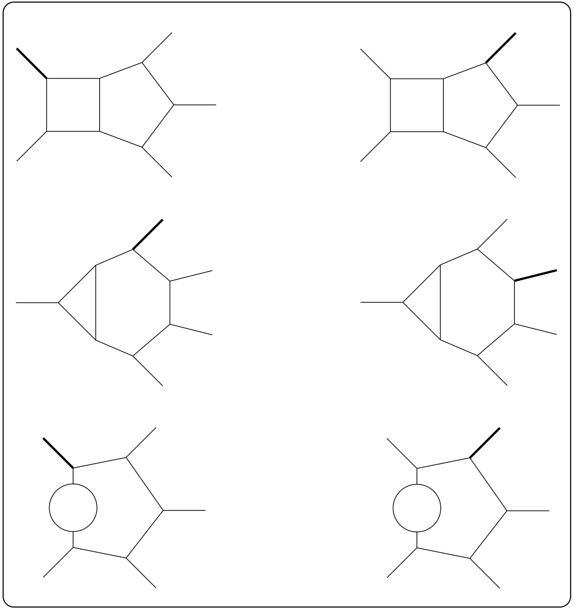
$$\vec{s}_5 = \{s_{12}, s_{23}, s_{34}, s_{123}, s_{234}, s_{56}\},$$

as well as the pseudo-scalar invariant $\mathrm{tr}_5 = 4i\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$.

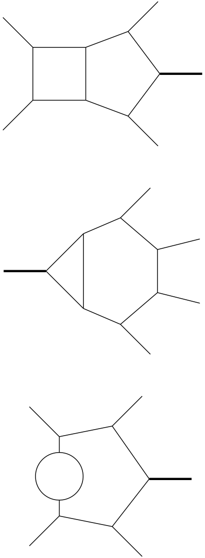
(Begin finite field sampling)

- The amplitude is mapped onto scalar integrals within 15 **maximal topologies**

$W\gamma + j$



$\times 2$



$W\gamma + j$

- Scalar integrals are IBP-reduced onto a **master integral basis** [Laporta, '01], [Lee, '13]

$$\tilde{A}_{5,i} = \sum_j d_j\,(\epsilon, p) \times MI_j(\epsilon, p)$$

- We work with MIs that satisfy canonical DEs [Henn, '13]:

$$\mathrm{d}\vec{MI} = \epsilon \left(\sum_{i=1}^{58} a_i \times \mathrm{d}\log w_i \right) \vec{MI}\,,$$

where the 'letters' w_i are algebraic functions of external kinematics [Abreu, Ita, Moriello, Page, Tschernow, Zeng, '20]

$W\gamma + j$

- Laurent expand the coefficients and map the MIs onto square roots and a basis of special functions $\{f\}$ related to the letters
- Subtract the poles to get the **finite remainder**:

$$\tilde{F}_{5,i} = \sum_j u_{i,j}(p) \times mon_j \left(\mathrm{tr}_5, \sqrt{\Delta_3}, \{f\}\right) \; ,$$

- Reconstruct the coefficients, now free of ϵ

(End finite field sampling)

Special functions

- The reconstructed finite remainders need to be evaluated at many phase-space points *and their permutations*
- Translate the basis of iterated integrals $\{f_i\}$ into **pentagon functions** $\{g_i\}$:

$$f_i = \sum_j \beta_{ij} \, \mathrm{mon}_j \left(\{g_k\}\right), \qquad \beta_{ij} \in \mathbb{Q}$$

- Pentagon function basis is closed under permutations:

$$(\sigma \circ g_i)\left(\vec{s}_5, \mathrm{tr}_5\right) = \sum_j \lambda_{ij}^{(\sigma)} \mathrm{mon}_j \left[\{g_k\left(\vec{s}_5, \mathrm{tr}_5\right)\}\right], \qquad \lambda_{ij}^{(\sigma)} \in \mathbb{Q}$$

- Efficient evaluation in `PentagonFunctions++` [Chicherin, Sotnikov, Zoia, Oct '21]

Simplifying rational coefficients

- Switch from $\vec{s}_5$ and tr_5 to **momentum twistors** $\vec{z}$:

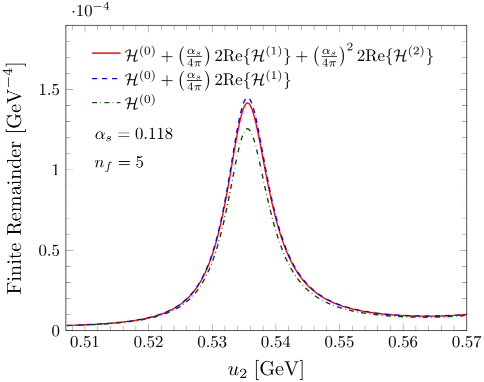
$$z_1 = s_{12}\,, \qquad z_2 = -\frac{\mathrm{tr}_+(1234)}{s_{12}s_{34}}\,,$$

$$z_3 = \frac{\mathrm{tr}_+(1341(5+6)2)}{s_{13}\,\mathrm{tr}_+\,(14(5+6)2)}\,, \qquad z_4 = \frac{s_{23}}{s_{12}}\,,$$

$$z_5 = -\frac{\mathrm{tr}_-(1(2+3)(1+5+6)(5+6)23)}{s_{23}\,\mathrm{tr}_-(1(5+6)23)}\,, \qquad z_6 = \frac{s_{456}}{s_{12}}\,.$$

- For each helicity amplitude, look for the most optimal re-parametrisation based on permuting external momenta
- Choose the one that leads to the lowest polynomial degrees
- Achieved compression of $\mathcal{O}(10^2)$ for the most complicated coefficients

Results



The finite remainders of the $u\bar{d}$ channel interfered with tree-level amplitudes, evaluated at a univariate phase-space slice.

$$p_1^\mu = u_1 \frac{\sqrt{s}}{2}(1,1,0,0) \qquad p_3^\mu = u_2 \frac{\sqrt{s}}{2}(1, \cos\theta, -\sin\phi\sin\theta, -\cos\phi\sin\theta)$$

$$p_2^\mu = \frac{\sqrt{s}}{2}(-1,0,0,-1) \qquad p_4^\mu = \frac{\sqrt{s}}{2}(-1,0,0,1) \qquad (p_5+p_6)^2 = M_{ll}^2$$

$$p_5^\mu = u_3 \frac{\sqrt{s}}{2}(1, \cos\theta_{ll}, -\sin\phi_{ll}\sin\theta_{ll}, -\cos\phi_{ll}\sin\theta_{ll})$$

Conclusion

- Calculated two-loop QCD amplitudes for $pp \to W\gamma + j$
- Implemented several tools to reduce the complexity of the reconstructed finite remainders
- The results are suitable for phenomenology
- Integrals for non-planar topologies needed to go beyond leading colour

Thank you!

Details of reconstruction

$$\tilde{F}_{5,i} = \sum_j u_{i,j}(p) \times mon_j \left(\mathrm{tr}_5, \sqrt{\Delta_3}, f\right)$$

1. Linear relations between rational coefficients:
   - Coefficients $u_{i,j}$ are not independent
   - Find relations between them and choose the independent ones based on the lowest polynomial degree

Details of reconstruction

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1. Linear relations between rational coefficients:
   - Coefficients $u_{i,j}$ are not independent
   - Find relations between them and choose the independent ones based on the lowest polynomial degree
2. Factor matching:
   - Aid the reconstruction by providing an ansatz of factors related to the letters
   - All denominator factors guessed + some of the numerator

Details of reconstruction

3 Simplifying the reconstructed coefficients:

- For each helicity configuration, search for an optimal re-parametrisation of momentum twistors to lower the polynomial degree of the coefficients
- Then, apply univariate partial fractioning, followed by `MultivariateApart` [Heller, von Manteuffel, '21] and `Singular` [Decker, Greuel, Pfister, Schönemann]

	$s_{12}=1$	linear relations	factor matching
$\tilde{F}^{(2),1\; h_1 h_2 h_3 h_4}_{5,i}$	$48/47$	$42/42$	$42/0$
$\tilde{F}^{(2),n_f\; h_1 h_2 h_3 h_4}_{5,i}$	$39/38$	$26/24$	$26/0$

Maximal numerator/denominator polynomial degrees of the finite remainder coefficients.

$$\tilde{F}_{5,i} = \sum_j u_{i,j}(p) mon_j \times \left(\mathrm{tr}_5, \sqrt{\Delta_3}, f\right)$$

$4D$ Tensor decomposition

Perform the $4D$ tensor decomposition on $\tilde{A}_{5,i}$:

- Parametrise the amplitude as a combination of form factors and tensor structures compatible with the symmetries of the process:

$$\tilde{A}_{5,i} = \sum_j \alpha_{i,j} T_j$$

- Form factors can be extracted by applying suitable projectors
- To reduce their number, work with helicity amplitudes and $4D$ projectors [Peraro, Tancredi, '19], [Peraro, Tancredi, '21]
- The number of **physical projectors** corresponds to the number of independent helicity amplitudes

Our workflow