New Developments in Parton Showers

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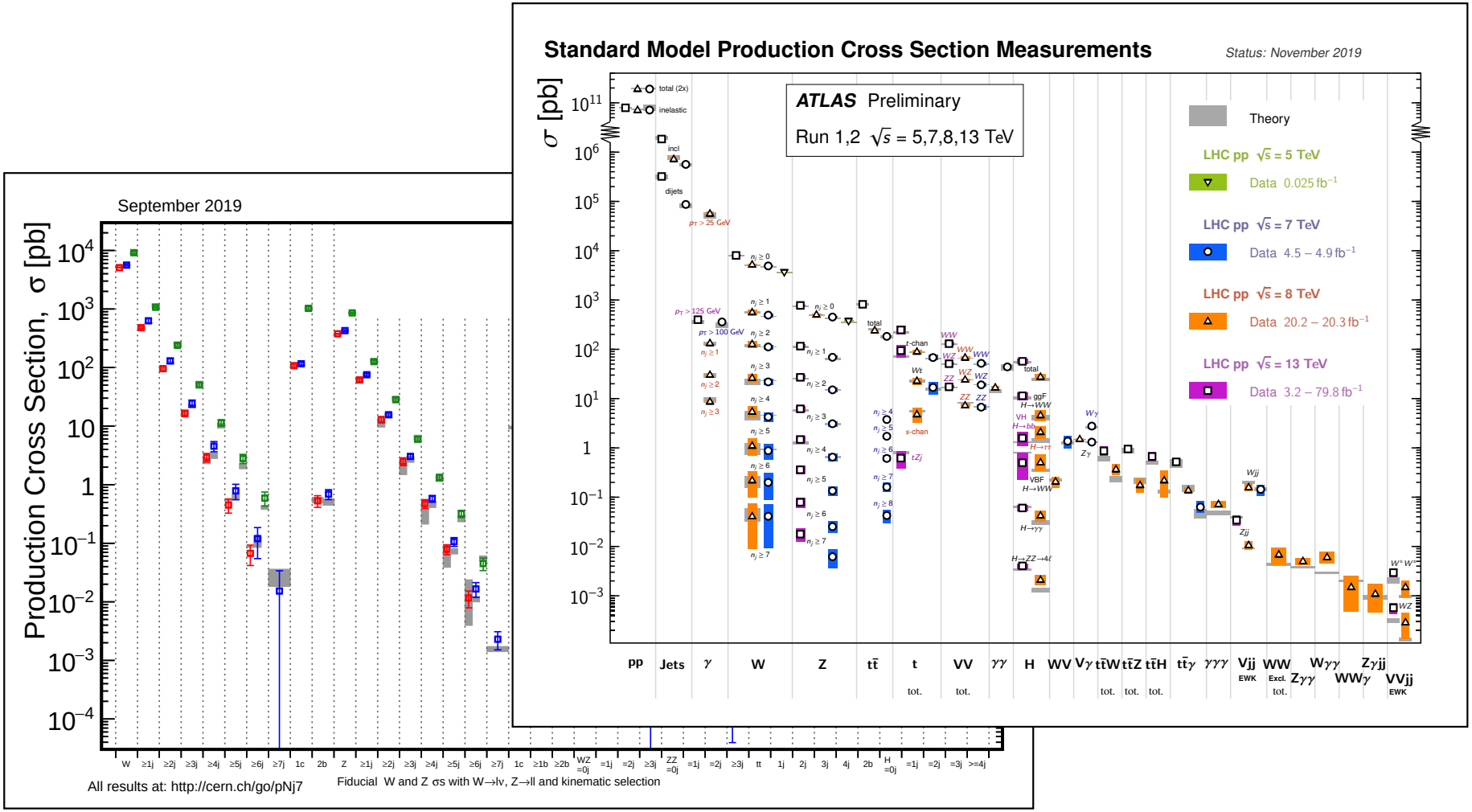
Fermi National Accelerator Laboratory

High Precision for Hard Processes

Newcastle upon Tyne, 22/09/2022

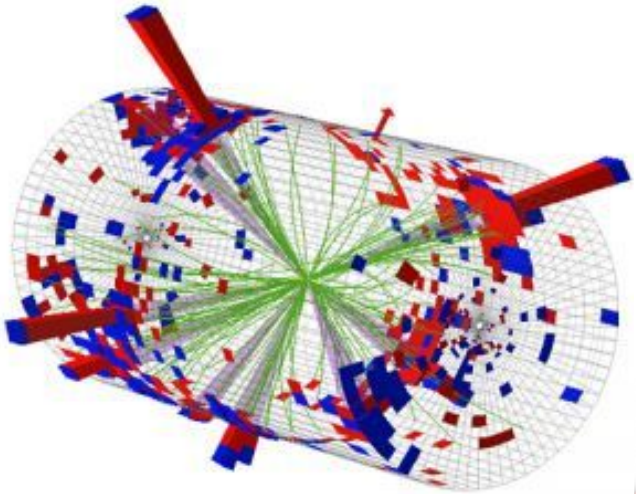
The Standard Model as we know it

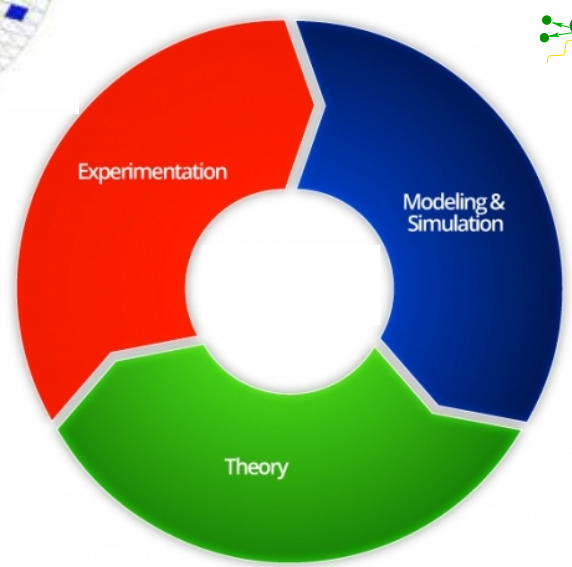


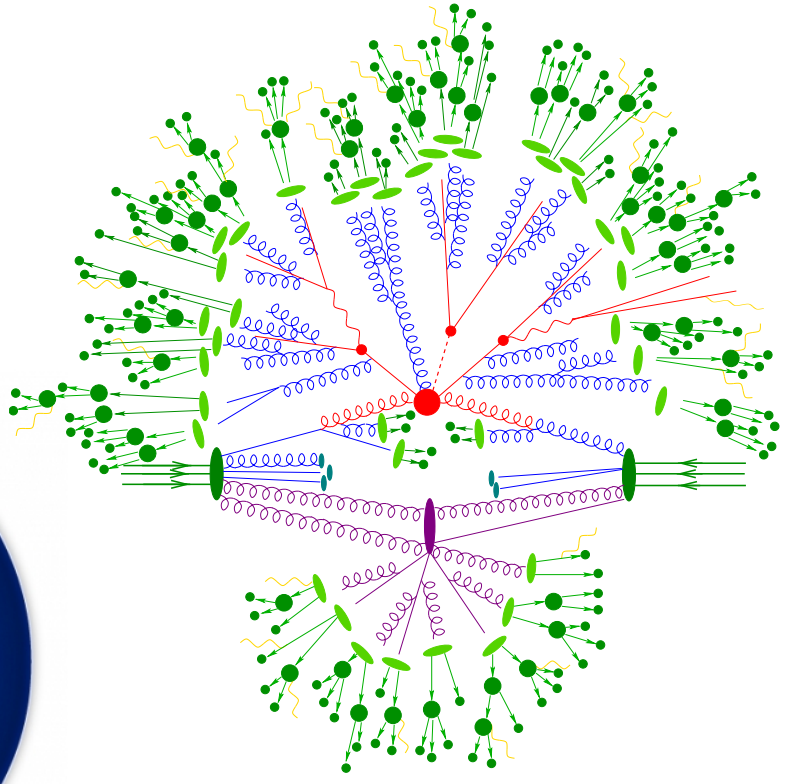
[ATLAS] https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults

[CMS] https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined

The toolkit





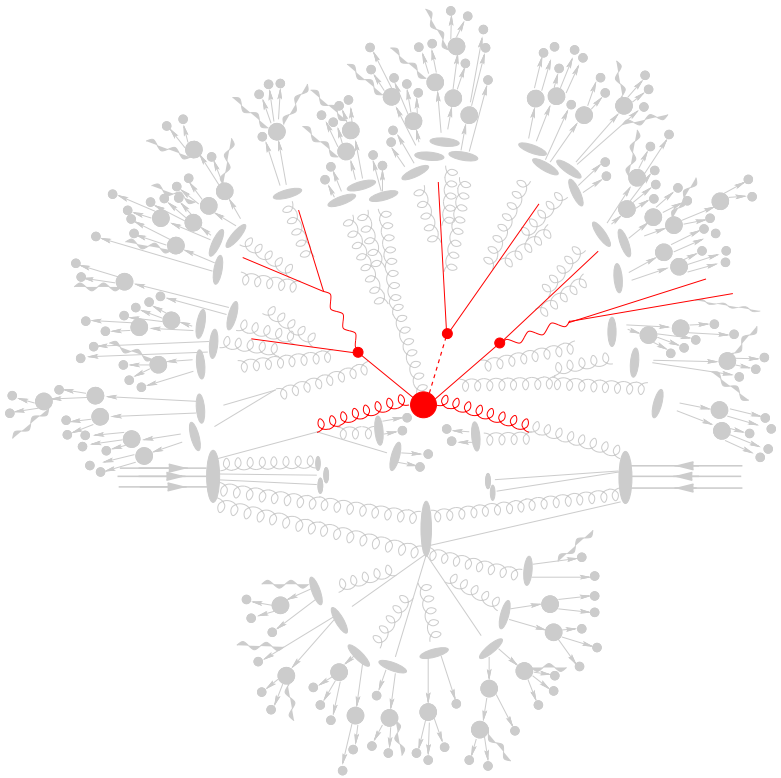


$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ i \bar{\psi} \not{D} \psi + h.c.$$

LHC event generators

[Buckley et al.] arXiv:1101.2599



- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \to X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

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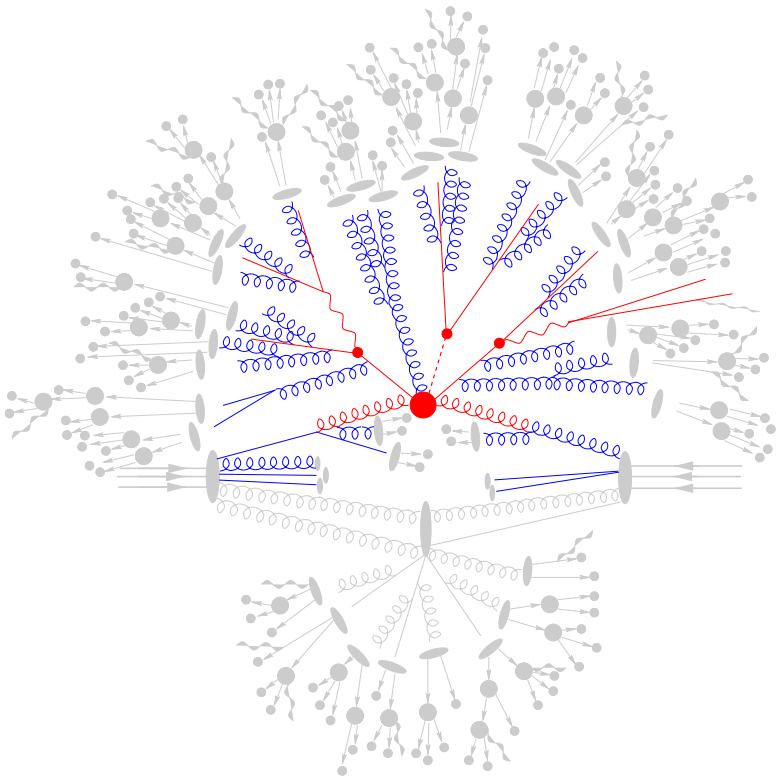

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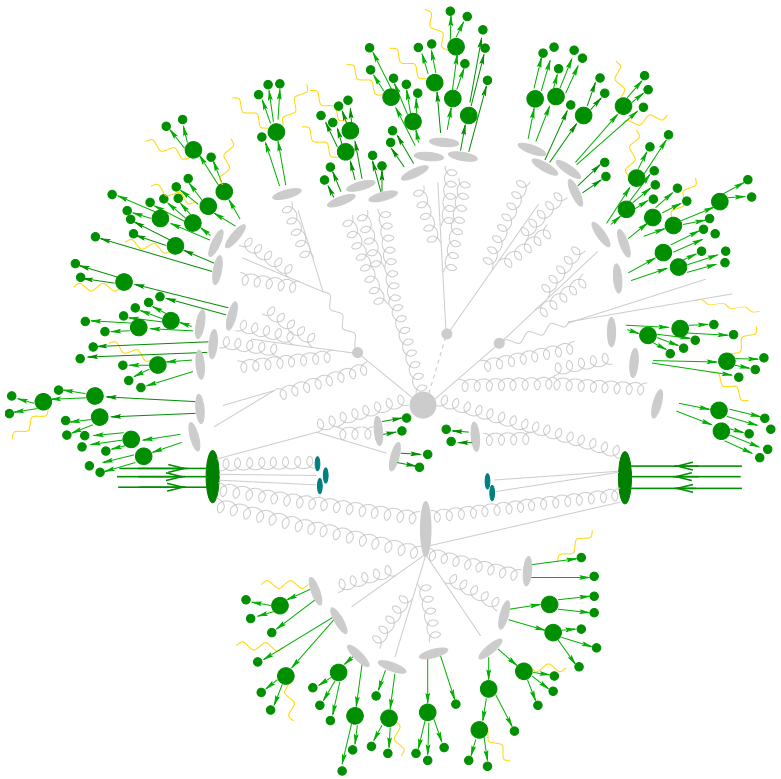

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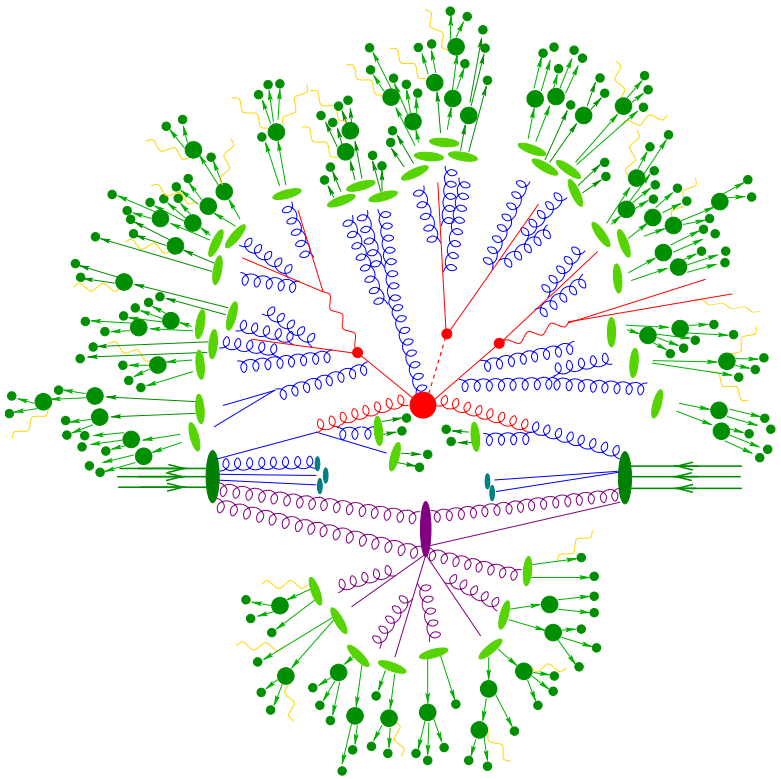

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Connection to QCD theory

- $\hat{\sigma}_{ij\to n}(\mu_F^2) \to$ Collinearly factorized fixed-order result at N^{x}LO

  Implemented in fully differential form to be maximally useful

  Tree level: $\mathrm{d}\Phi_n\, B_n$
  - Automated ME generators + phase-space integrators

  1-Loop level: $\mathrm{d}\Phi_n \left(B_n + V_n + \sum C + \sum I_n\right) + \mathrm{d}\Phi_{n+1}\left(R_n - \sum S_n\right)$
  - Automated loop ME generators + integral libraries + IR subtraction

  2-Loop level: It depends ...
  - Individual solutions based on SCET, q_T subtraction, P2B

- $f_i(x, \mu_F^2) \to$ Collinearly factorized PDF at N^{y}LO

  Evaluated at $O(1\mathrm{GeV}^2)$ and expanded into a series above $1\mathrm{GeV}^2$

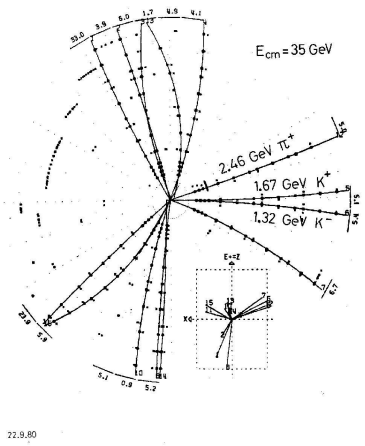
  DGLAP: $\dfrac{\mathrm{d}x\, x f_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g} \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \dfrac{\alpha_s}{2\pi} \left[z P_{ab}(z)\right]_+ \tau f_b(\tau, t)\, \delta(x - \tau z)$
  - Parton showers, dipole showers, antenna showers, ...

  Matching: $\mathrm{d}\Phi_n\, \dfrac{S_n}{B_n} \leftrightarrow \dfrac{\mathrm{d}t}{t}\mathrm{d}z\, \dfrac{\alpha_s}{2\pi}\, P_{ab}(z)$
  - MC@NLO, POWHEG, Geneva, MINNLO$_{\mathrm{PS}}$, ...

Co-design of simulations over the years

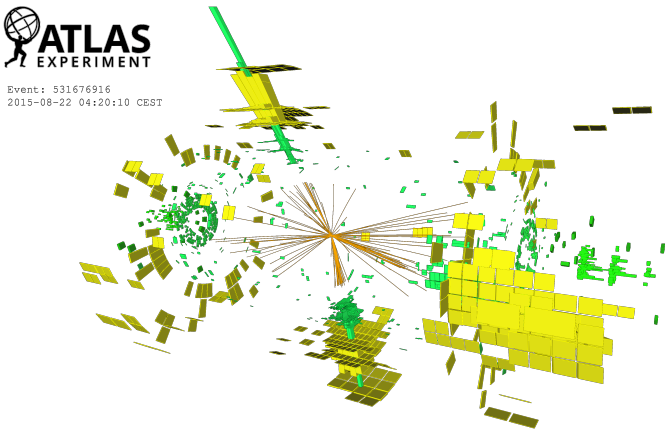
Accuracy → Precision

- 1985: Color coherent evolution; Backward evolution
- 1990: ME corrections; Color dipole formalism; Spin correlations
- 1995: Color dipole showers
- 2000: LO merging; NLO matching
- 2005: Dipole-like showers; Antenna showers; NLO merging
- 2010: Color ME corrections; NNLO matching
- 2015: Threshold effects; NLO splitting kernels; Higher-order PS framework
- 2018: Accuracy quantification; Amplitude evolution
- 2022: NLL & NGLs in one framework; Efficient $1/N_c$ corrections



$\sqrt{s} \times 500$ →

e^+e^- vs. pp



Directions of development

Much effort focused on parton-shower component

- Phenomenologically interesting: Drives jet production, b-tagging, ...
- Experimentally relevant: Often source of largest uncertainty

Fixed-order aspects

- Matching to NLO calculations
  - Negative weight fraction
  - Unweighting efficiency
- Matching to NNLO calculations
  - Semi-inclusive (Geneva, $\text{MINNLO}_{\text{PS}}$)
  - Fully differential (Vincia)

All-order aspects

- NLL precision
- Spin correlations in collinear & soft limit
- Sub-leading color effects
- Threshold effects
- Amplitude evolution

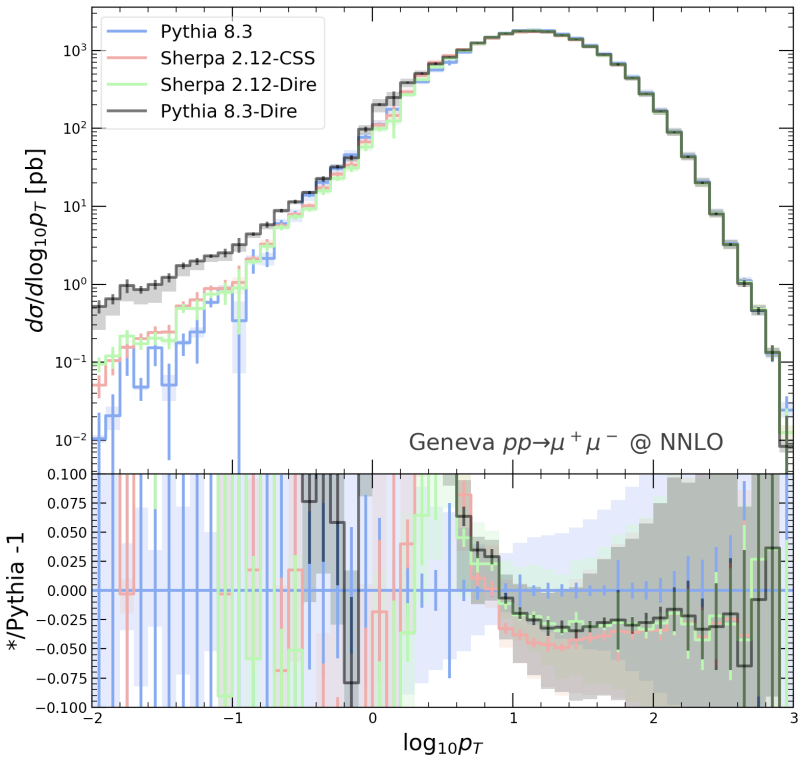
Fixed-order matching: Geneva

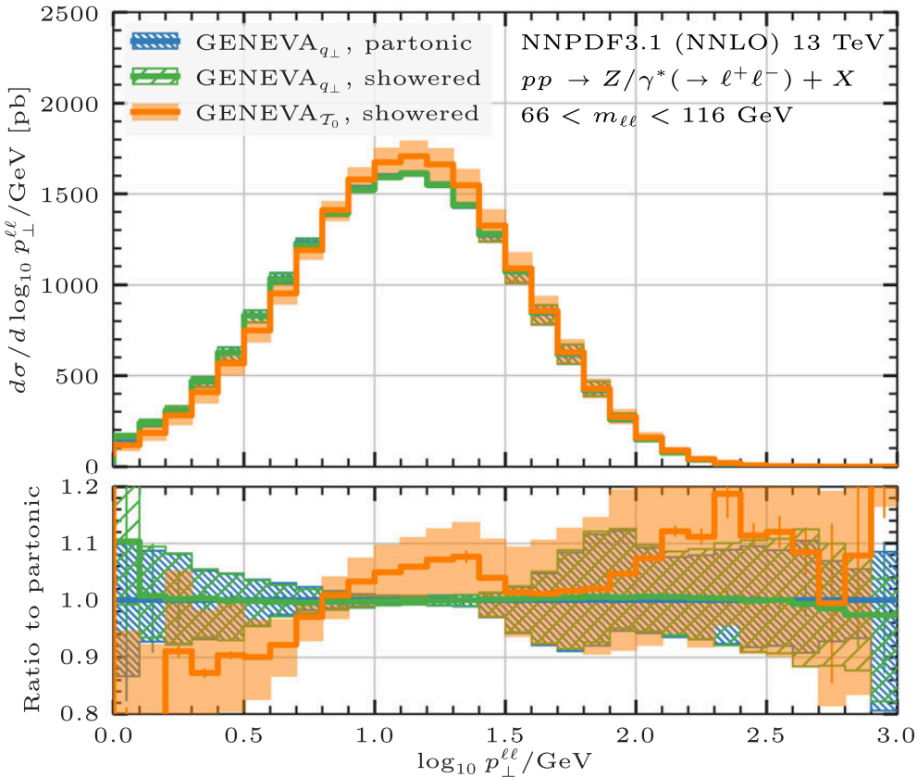
[D. Napoletano's talk]

- Use known resummation in jettiness / q_T & match to NNLO

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi\,\mathrm{d}r} = \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi\,\mathrm{d}r} - \frac{\mathrm{d}\sigma^{\mathrm{res.exp.}}}{\mathrm{d}\Phi\,\mathrm{d}r} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\Phi\,\mathrm{d}r}$$

- Match to shower by vetoing events with $r_N(\Phi_{N+M}) > r_N$



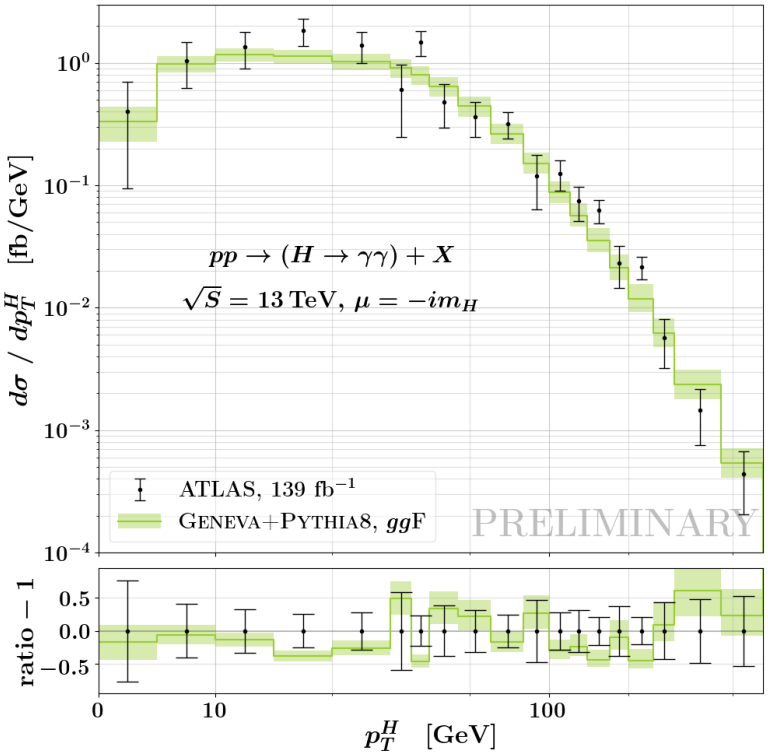


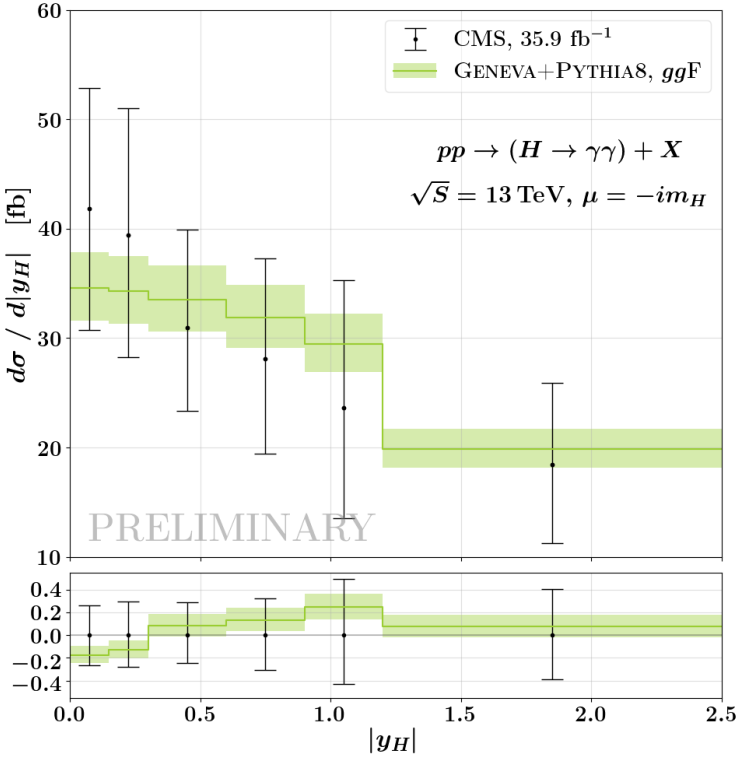
- Parton shower scheme uncertainty
- Choice of resolution variable

Fixed-order matching: Geneva

[G. Marinelli's talk]

- Comparison against experimental data





- $p_{T,H}$ and ATLAS data
- y_H and CMS data

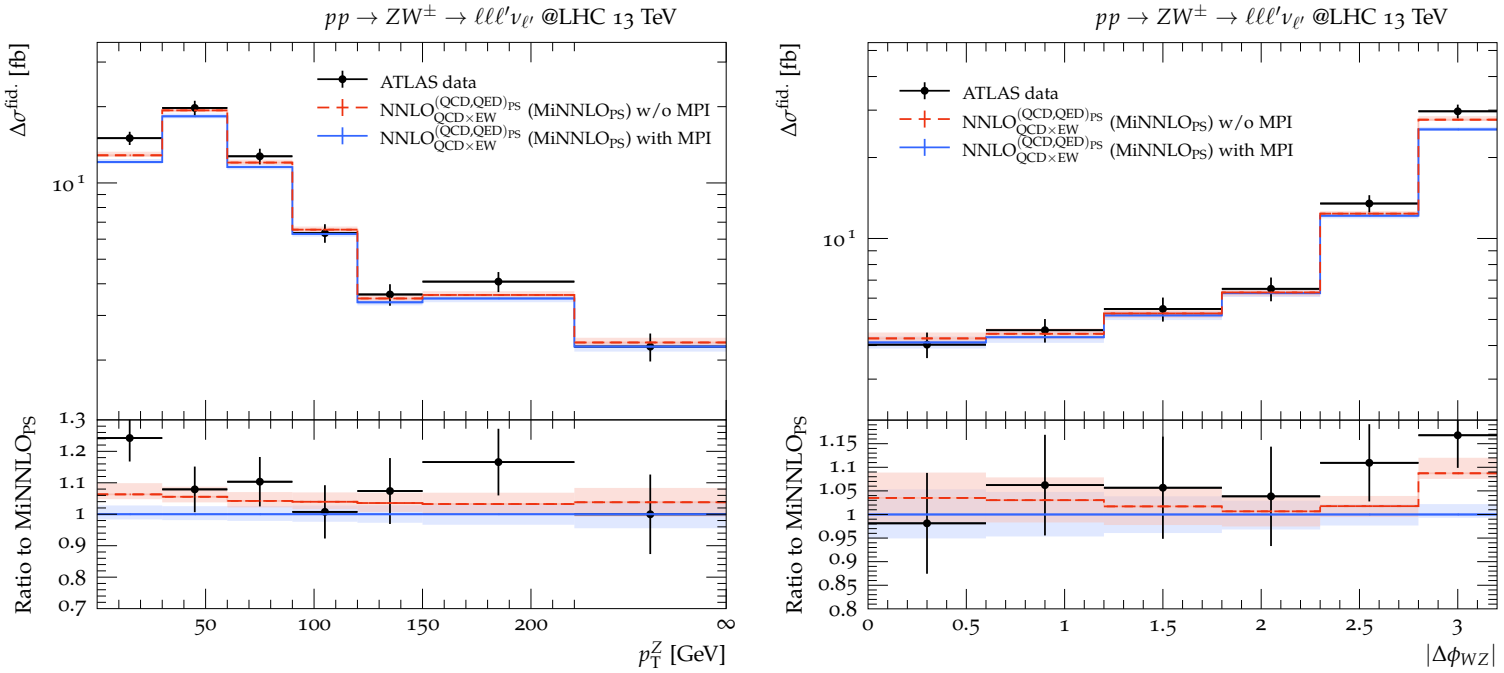
Fixed-order matching: MINNLO$_{PS}$

[S. Zanoli's talk]

- WZ production at NNLO QCD $\times$ NLO EW
- Various schemes to combine QCD & EW corrections
  $\rightarrow$ associated uncertainty estimates

[2208.12660]

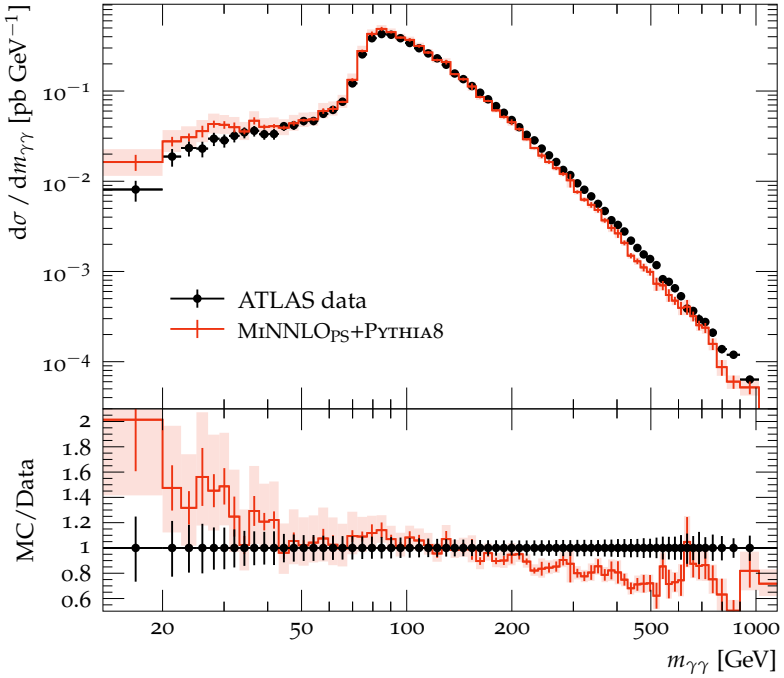
ATLAS data from Eur. Phys. J. C 79 (2019)

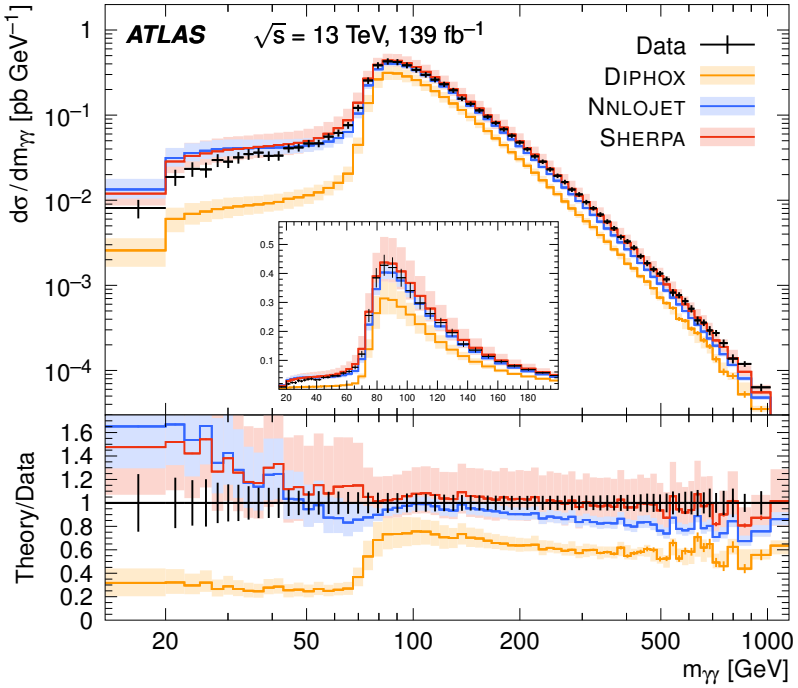


Fixed-order matching: MINNLO$_{PS}$

[A. Gavardi's talk]

- Di-photon production at the LHC
- QED singular contributions in real-emission corrections treated as fixed order → split off by damping function



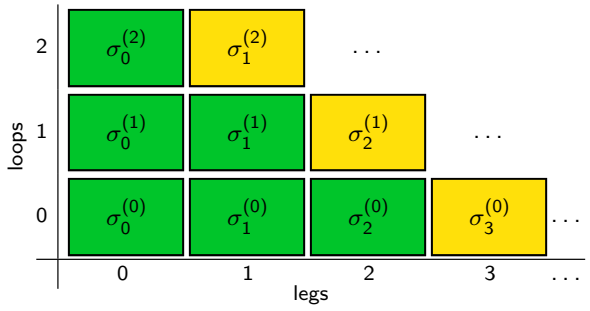


- Comparison between ATLAS data and MINNLO$_{PS}$
- Previous experimental analysis

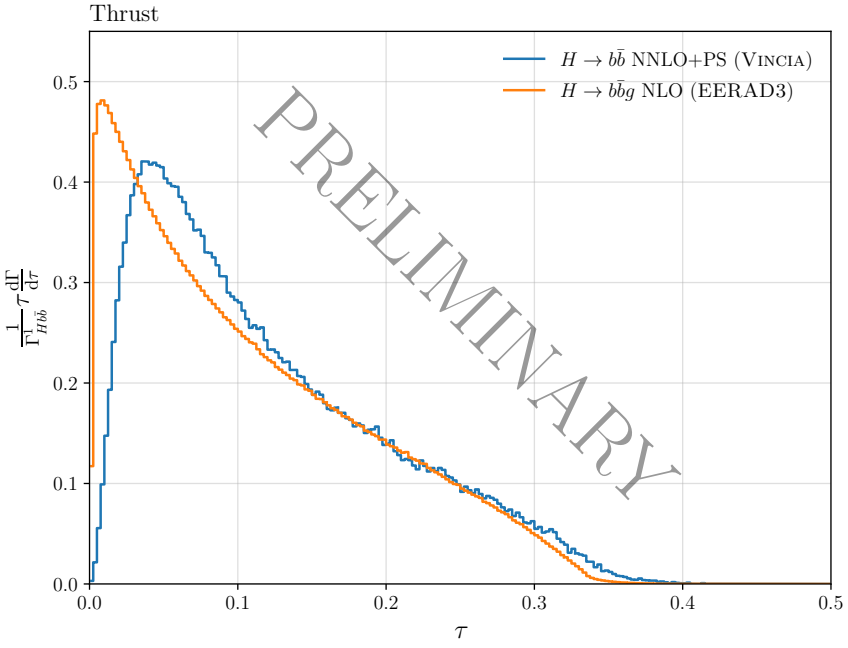
  [ATLAS] arXiv:2107.09330

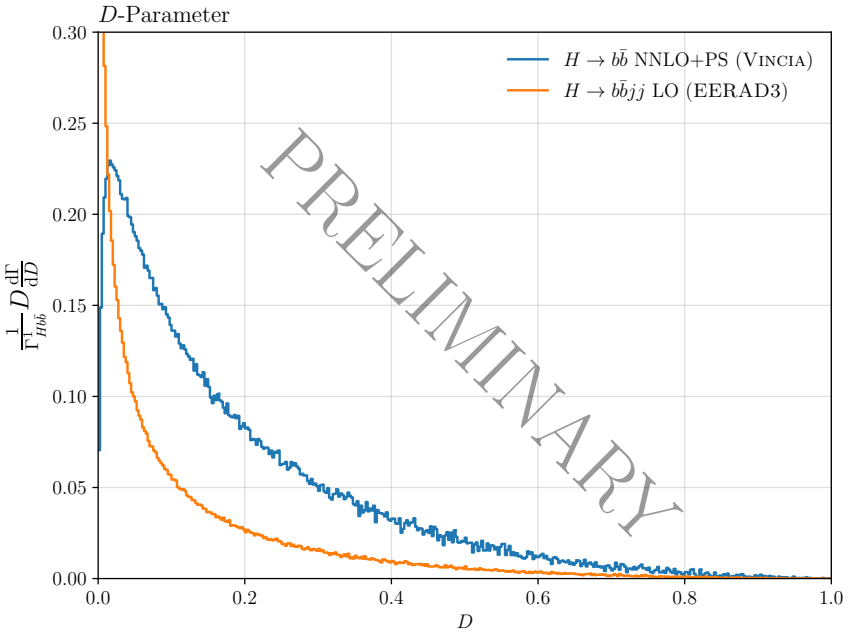
Fixed-order matching: Vincia

[C. Preuss' talk]



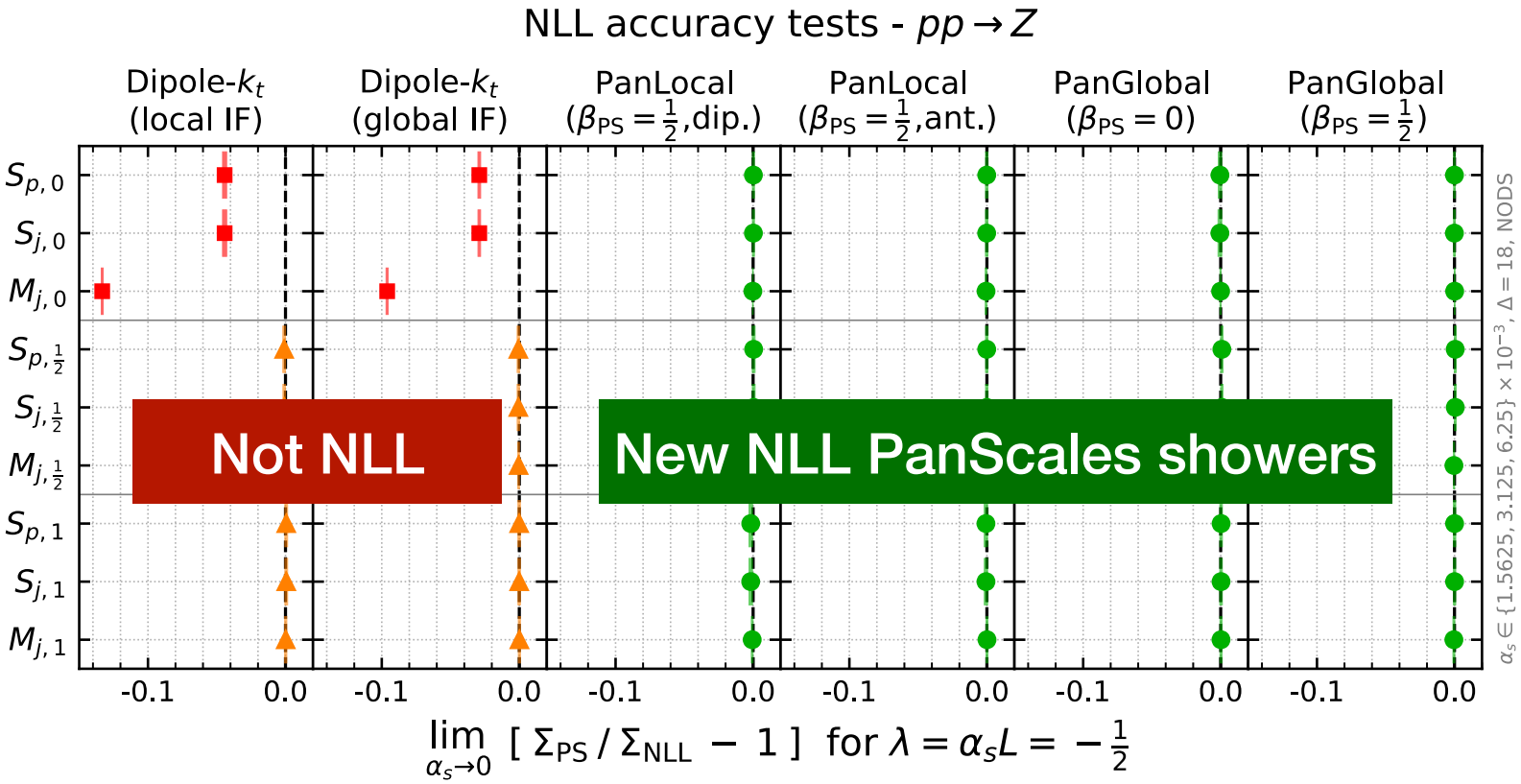
NNLO accuracy in $H \to 2j$ implies **NLO correction in first emission** and **LO correction in second emission.**





All-order results: PanScales

[R. Verhyen's talk]



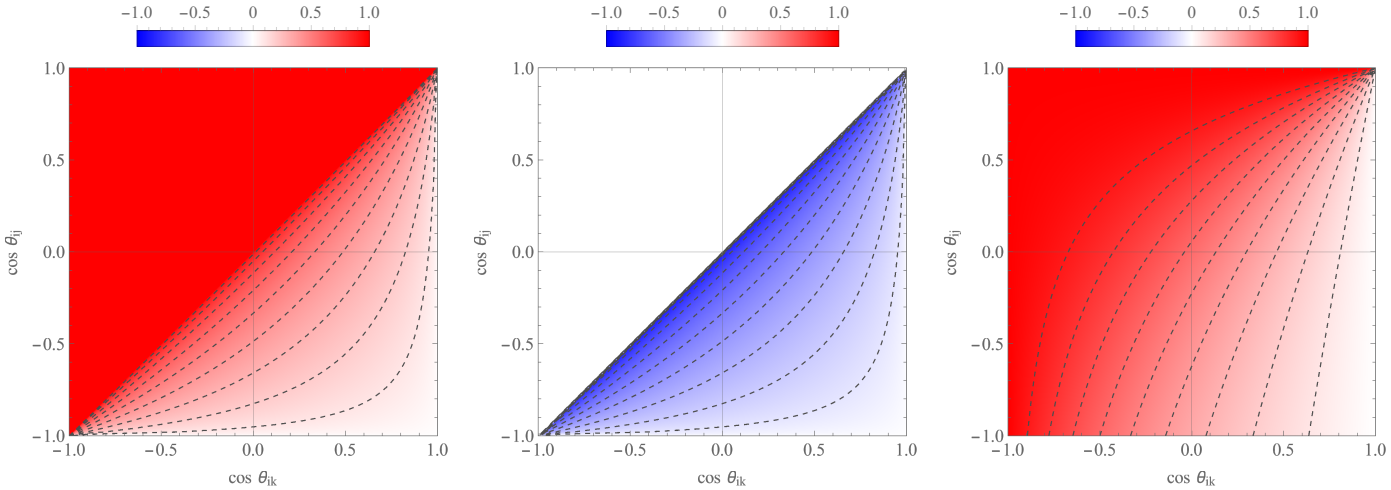
$$S_{p,\beta_{\rm obs}} = \sum_{i \in \rm particles} \frac{k_{t,i}}{Q} e^{-\beta_{\rm obs}|\eta_i|} \qquad S_{j,\beta_{\rm obs}} = \sum_{i \in \rm jets} \frac{k_{t,i}}{Q} e^{-\beta_{\rm obs}|\eta_i|} \qquad M_{j,\beta_{\rm obs}} = \max_{i \in \rm jets} \frac{k_{t,i}}{Q} e^{-\beta_{\rm obs}|\eta_i|}$$

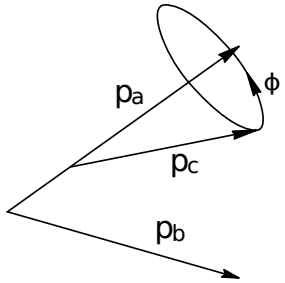
All-order results: Alaric

[F. Herren's talk]

Soft-collinear matching

- Angular ordering emerges after integration over azimuth
  Additive matching leading to AO is problematic for NGLs
  but multiplicative matching is not (SF remains probabilistic)





Momentum mapping

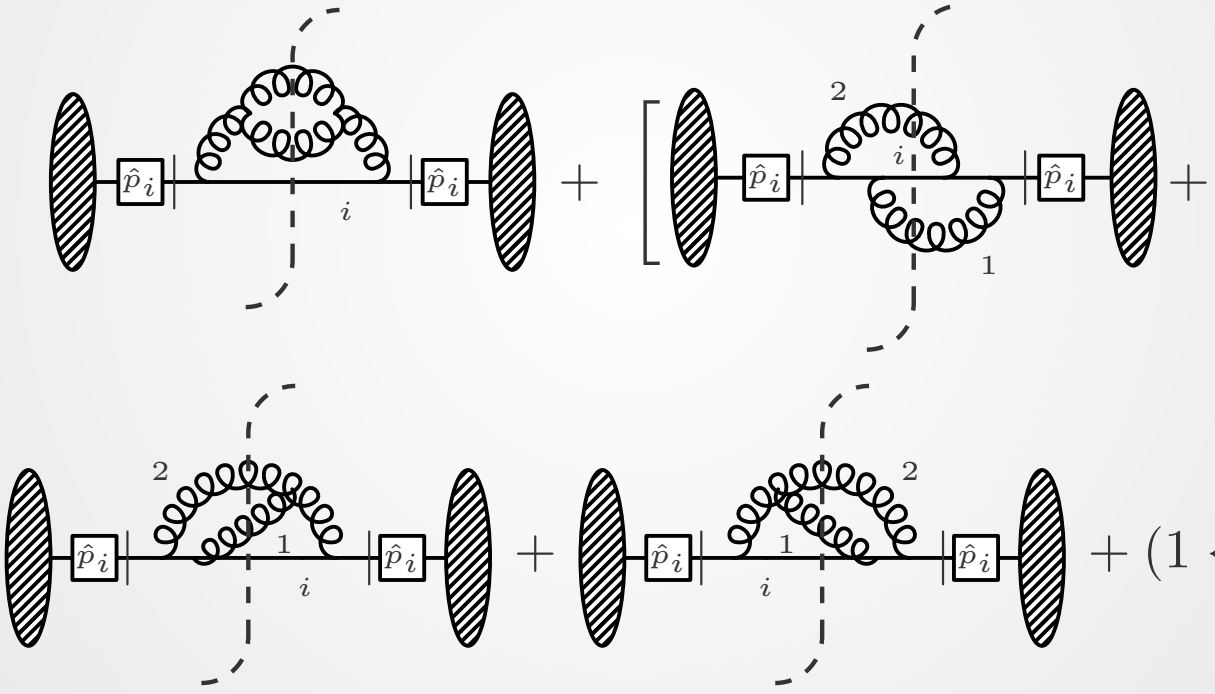
- Kinematic reconstruction preserves angular correlations between splitting parton and all other momenta
- Recoil absorbed by multipole $\rightarrow$ effect scales as $\sqrt{k_T^2/Q^2}$
- Analytic proof of NLL accuracy for global observables

All-order results: Multi-Emission Kernels

[M. Löschner's talk]

- Program to define higher-order splitting functions for parton showers
- Sudakov-like momentum decomposition → power counting
- Reproduces known soft & double-/triple-collinear splitting functions

$$\frac{\mu^{2\varepsilon}}{\hat{\alpha}^2 S_{i12}^2}\Bigg\{$$



$$+ (1 \leftrightarrow 2)\Bigg]\Bigg\}_{C_A C_F}$$

$$= \left(\frac{8\pi\alpha_S}{\hat{\alpha} S_{i12}}\mu^{\varepsilon}\right)^2 C_A C_F \langle \hat{P}_{ggq}^{\text{(non-Ab)}} \rangle \hat{\not{p}}_i + \mathcal{O}\left(\beta_{il}^{-3/2}\right).$$

Topics not discussed (much) in parallel talks

Highly successful interplay of parton showers & resummation

- Resummation enables computation of important results at NNLO
- Analytic resummation enables improvement of parton showers

Fixed-order

- N^3LO matching
- Truncated showers

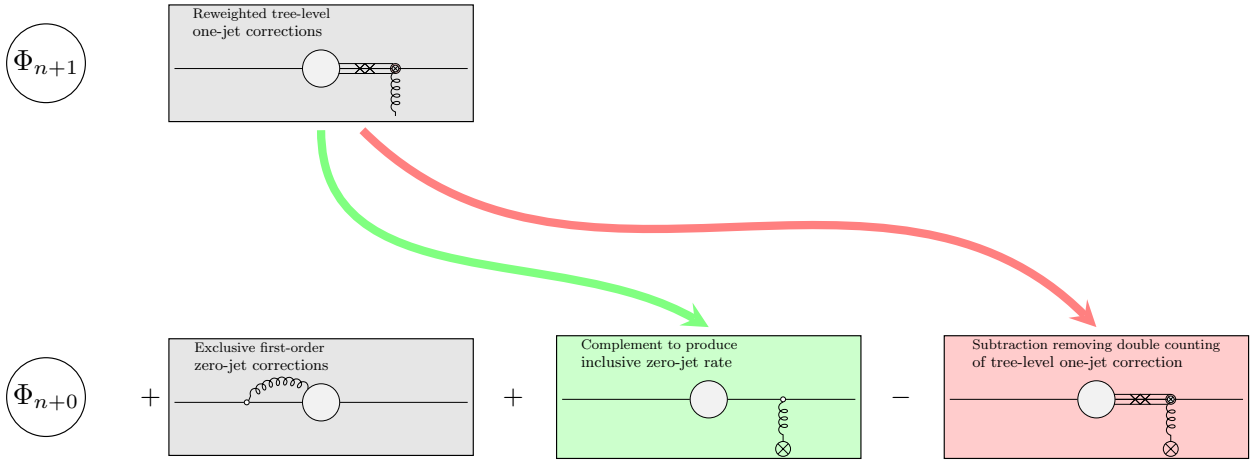
All-orders

- NLO splitting functions
- Kinematic edge effects

N^{3}LO matching

[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467

U(N)LOPS

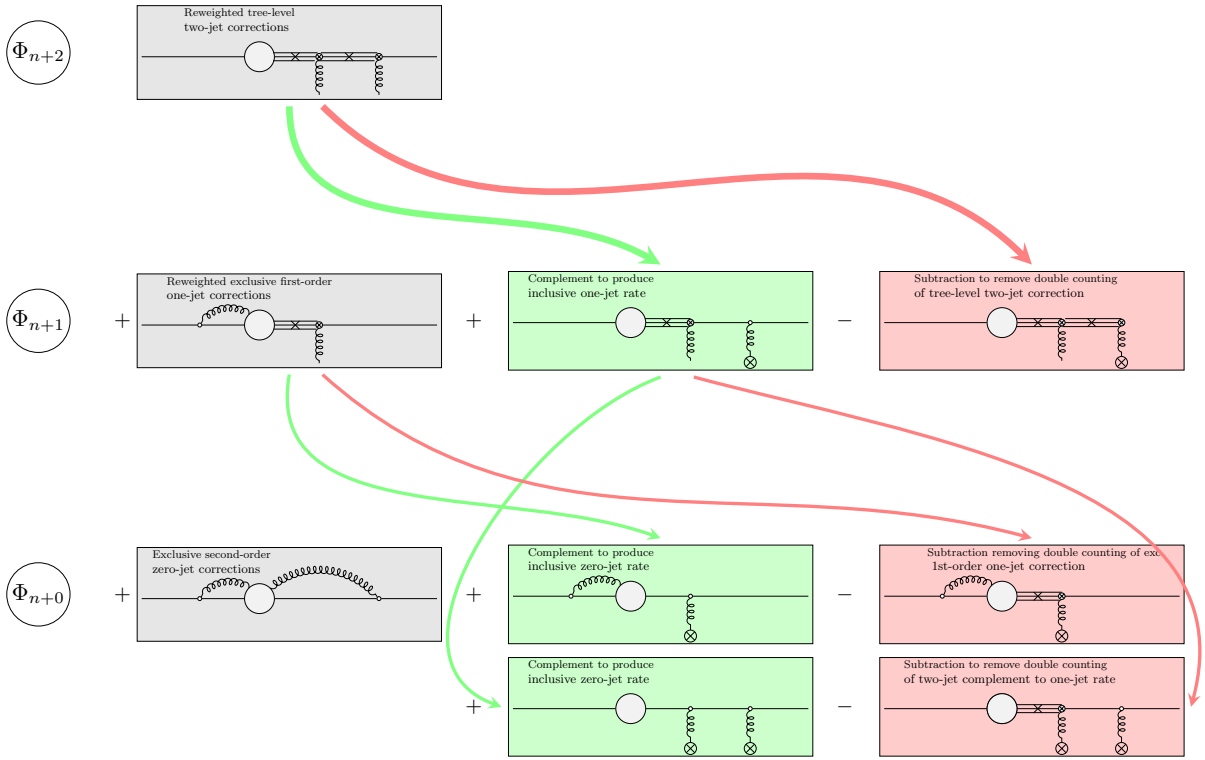


- Compute vetoed cross section & complete with real-emission
- Add Sudakov vetoed real-emission cross section & projection
- Can be implemented based on only two inputs (gray boxes)

N^{3}LO matching

[Lönnblad,Prestel] arXiv:1211.4827, [Li,Prestel,SH] arXiv:1405.3607

UN^{2}LOPS

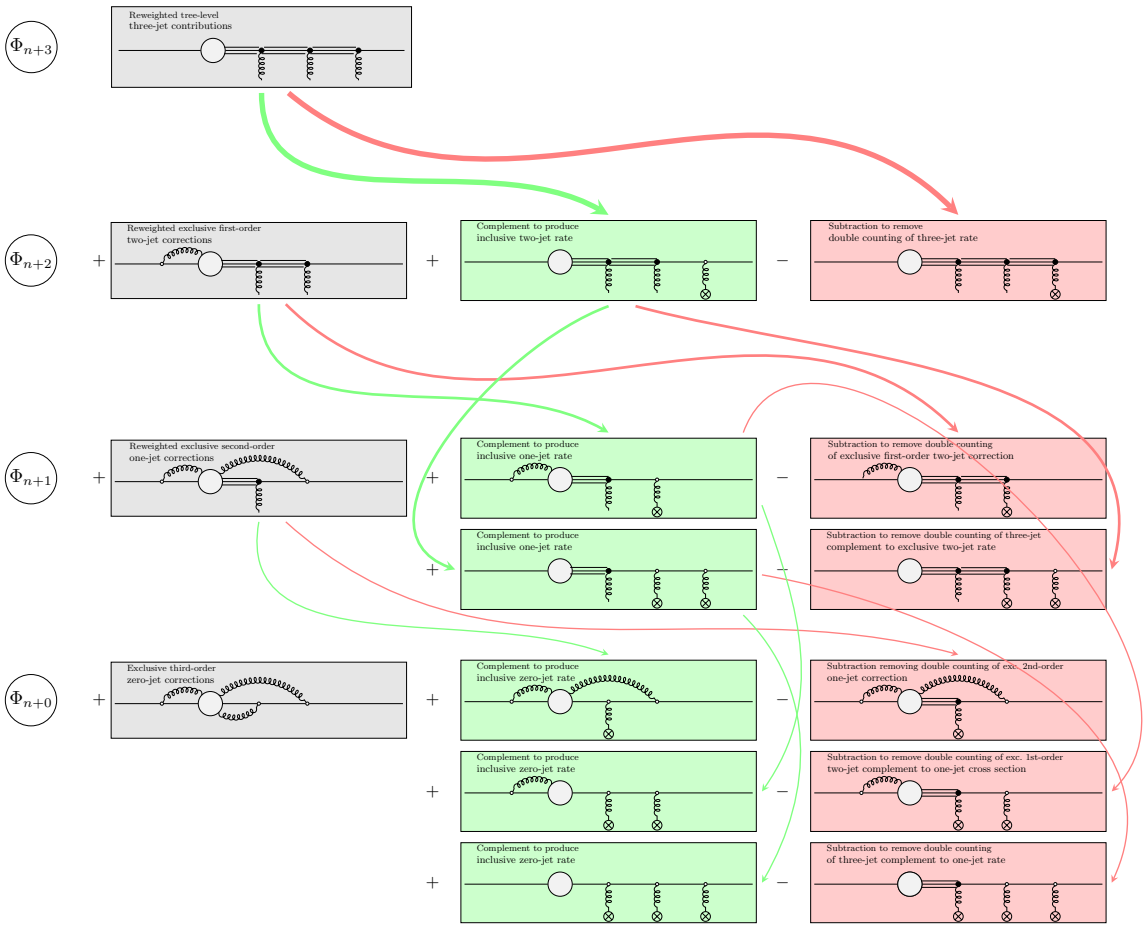


- Same idea as in ULOPS, but now also adding 2-loop contribution

N^{3}LO matching

[Prestel] arXiv:2106.03206, [Bertone,Prestel] arXiv:2202.01082

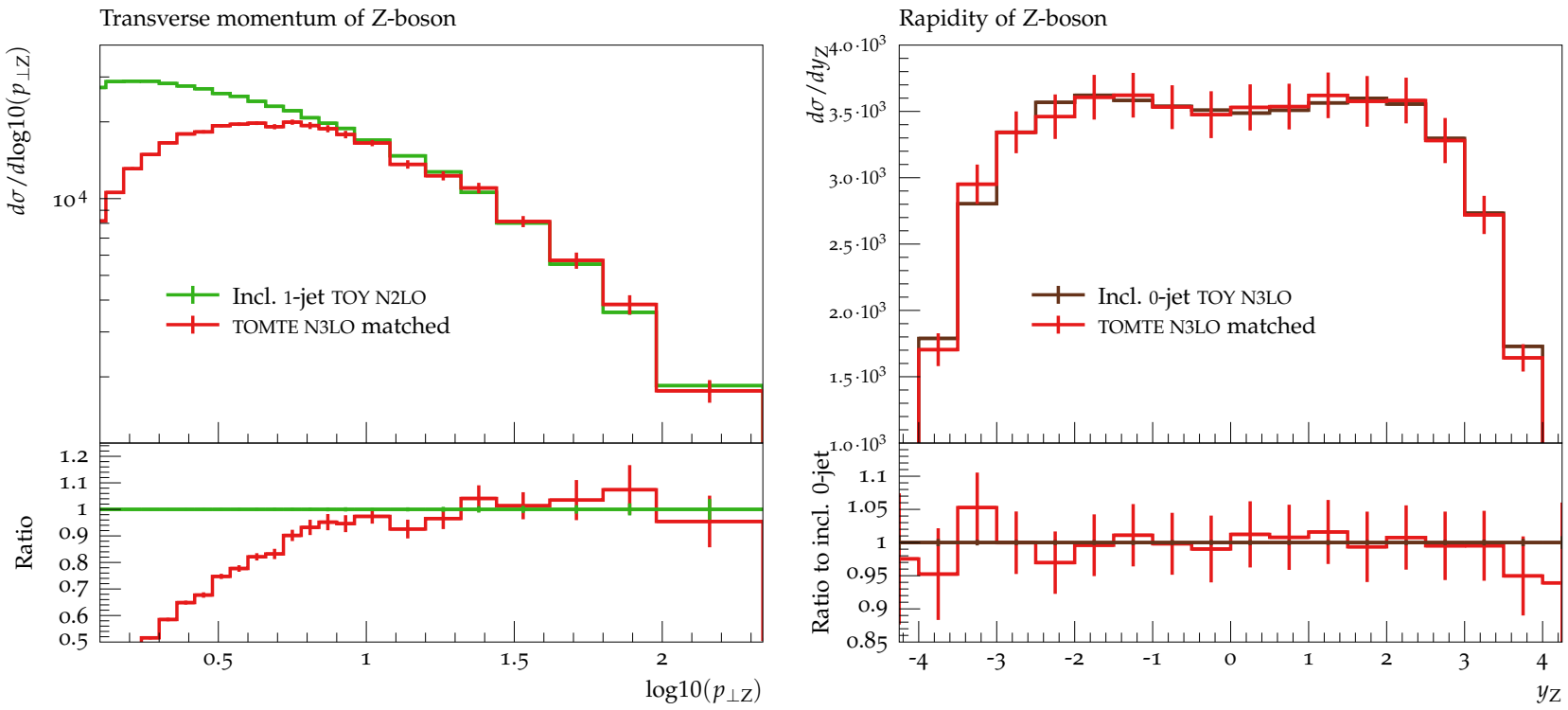
TOMTE



- Same idea as in UN^{2}LOPS, but now also adding 3-loop contribution
- Must pay careful attention to projections (relevant for all UN^{X}LOPS)

N^{3}LO matching in Drell-Yan

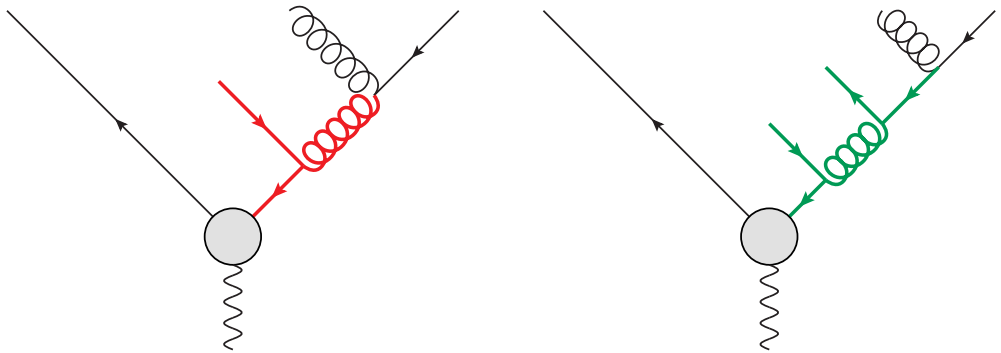
[Bertone,Prestel] arXiv:2202.01082



- Transverse momentum and rapidity in $pp \to Z$
- Stand-in fixed-order calculation for closure tests

Truncated unvetoed parton showers

- Matching parton showers with given ordering variable to resummed calculation with different observable requires implementation of the “difference” [Nason] hep-ph/0409146
- Momentum and flavor conserving implementation highly non-trivial
  Example: Two emissions may be allowed, while one may be not



- General algorithmic solution given by CKKW-L [Lönnblad] hep-ph/0112284

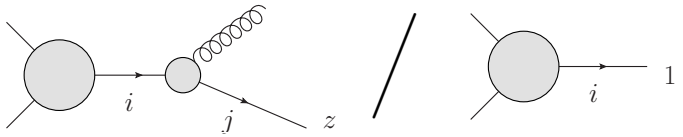
  see D. Napoletano’s talk for application in Geneva
  and R. Verheyen’s talk on PanScales

Collinear parton evolution at NLO

- Higher-order DGLAP evolution kernels obtained from factorization

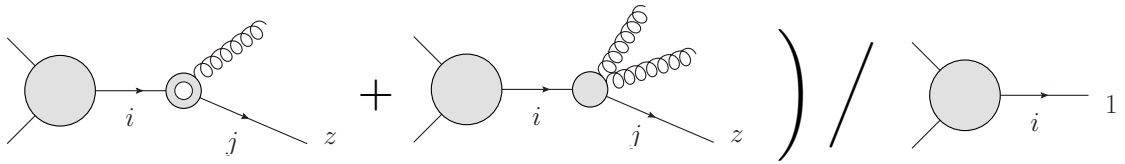
$$D_{ji}^{(0)}(z,\mu) = \delta_{ij}\delta(1-z) \qquad \leftrightarrow$$

$$D_{ji}^{(1)}(z,\mu) = -\frac{1}{\varepsilon}P_{ji}^{(0)}(z) \qquad \leftrightarrow$$



$$D_{ji}^{(2)}(z,\mu) = -\frac{1}{2\varepsilon}P_{ji}^{(1)}(z) + \frac{\beta_0}{4\varepsilon^2}P_{ji}^{(0)}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x}P_{jk}^{(0)}(x)P_{ki}^{(0)}(z/x)$$

$\leftrightarrow$



- $P_{ji}^{(n)}$ not probabilities, but sum rules hold ($\leftrightarrow$ unitarity constraint)
  In particular: Momentum sum rule identical between LO & NLO
- Can perform the NLO computation of $P_{ji}^{(1)}$ fully differentially using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Collinear parton evolution at NLO

[Prestel,SH] arXiv:1705.00742

- Example: Flavor-changing NLO splitting functions

$$P^{(1)}_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1}\left[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\right]$$

- Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$
  Difference finite in 4 dimensions $\rightarrow$ amenable to MC simulation
- Integrated subtraction term and factorization counterterm given by

$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

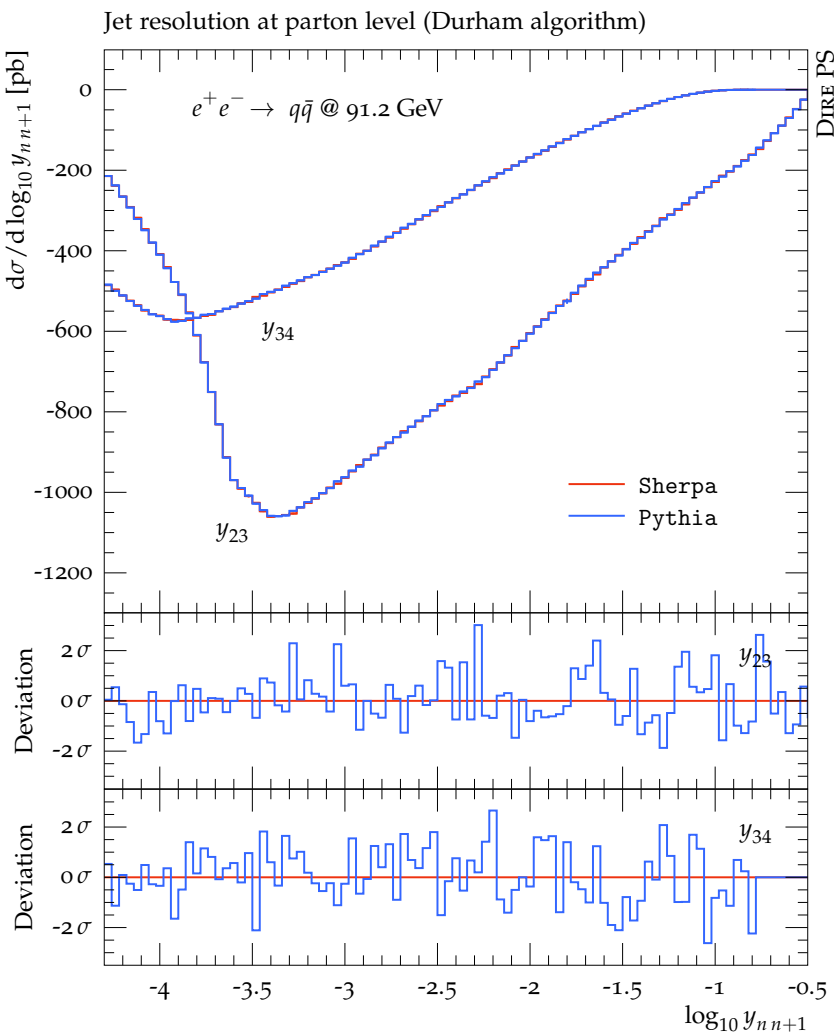
$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x}\left(P^{(0)}_{qg}(x) + \varepsilon \mathcal{J}^{(1)}_{qg}(x)\right)\frac{1}{\varepsilon}P^{(0)}_{gq}(z/x)$$

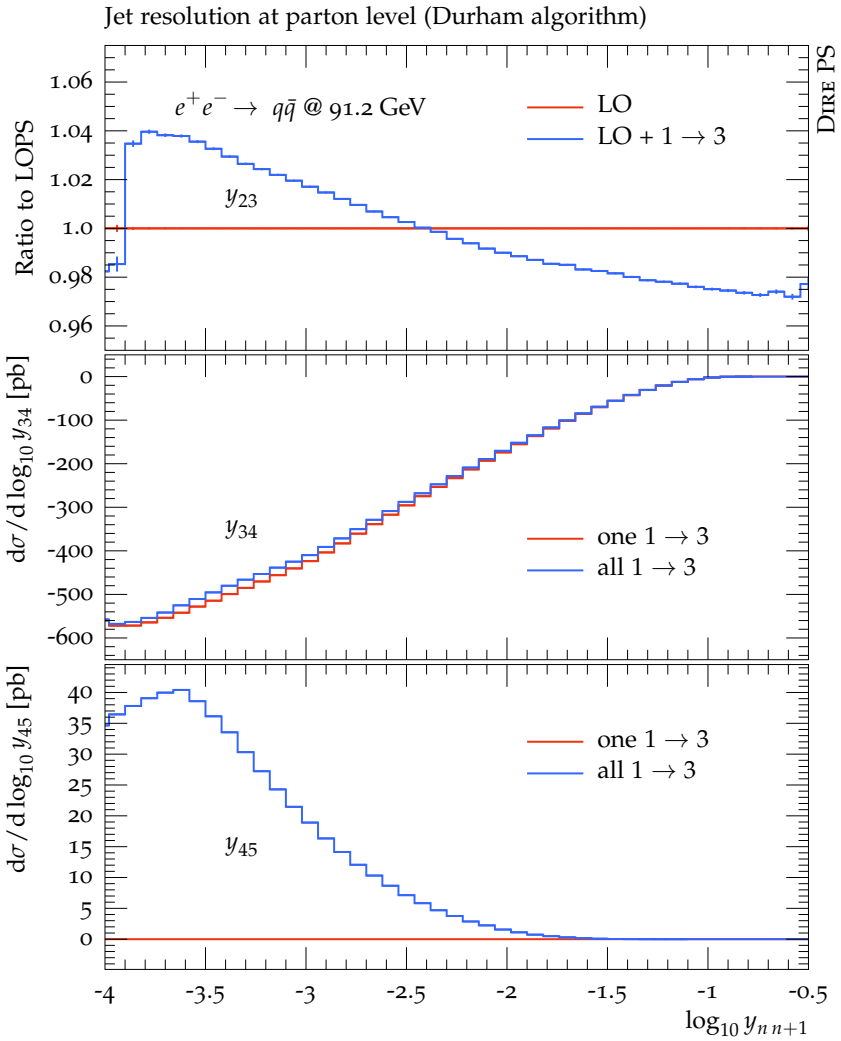
$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F\left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x\right)$$

- Analytical computation of I not needed, as $\mathrm{I} + \mathcal{P}/\varepsilon$ finite
  generate as endpoint at $s_{ai} = 0$, starting from integrand at $\mathcal{O}(\varepsilon)$
- All components of $P^{(1)}_{qq'}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

Effects on jet rates in $e^+e^- \to$ hadrons at LEP

[Gellersen,Prestel,SH] arXiv:2110.05964





Looking beyond logarithmic accuracy

- Provably NLL accurate parton showers solve long-standing problem
  NNLL seems on the horizon, but is it the obvious target?
- Revisit well-established result: Thrust or $FC_{1-\beta}$ in $e^+e^- \to$hadrons

- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\rm PS}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\rm min}}^{z_{\rm max}} dz\, \frac{\alpha_s\left(k_T^2\right)}{2\pi}\, C_F \left[\frac{2}{1-z} - (1+z)\right] \Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\rm NLL}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz\, \frac{\alpha_s\left(k_T^2\right)}{2\pi} \frac{2\,C_F}{1-z}\Theta(\eta) - \frac{\alpha_s(\xi)}{\pi}\, C_F B_q\right]$$

Origin of the $\alpha_s \to 0$ limit

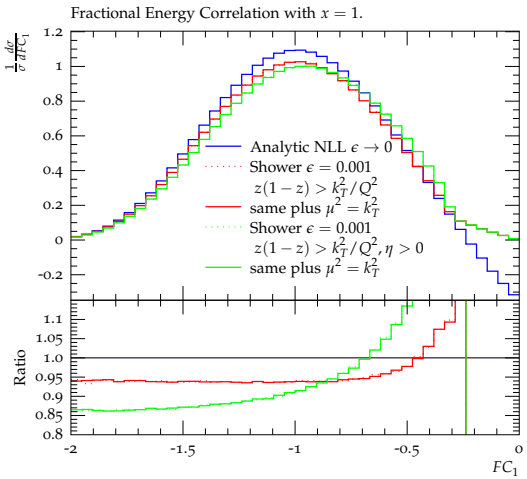
- Cumulative cross section $\Sigma(\tau) = e^{-R(\tau)}\mathcal{F}(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int \mathrm{d}^3 k_1 |M(k_1)|^2 \, e^{-R' \ln \frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=2}^{m+1} \int_{\varepsilon v_1}^{v_1} \mathrm{d}^3 k_i |M(k_i)|^2 \right) \times \Theta\big(\tau - V(\{p\}, k_1, \ldots, k_n)\big)$$

  - $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
  - Observable is recursively infrared and collinear safe
  - Hold $\alpha_s(Q^2) \ln \tau$ fixed, while taking limit $\tau \to 0$
    $\to$ Can factorize integrals and neglect kinematic edge effects
- Breaks momentum conservation and unitarity for finite τ
  $\to$ Clean NLL result, but unknown kinematic corrections
- How large are effects in regions of a typical measurement?

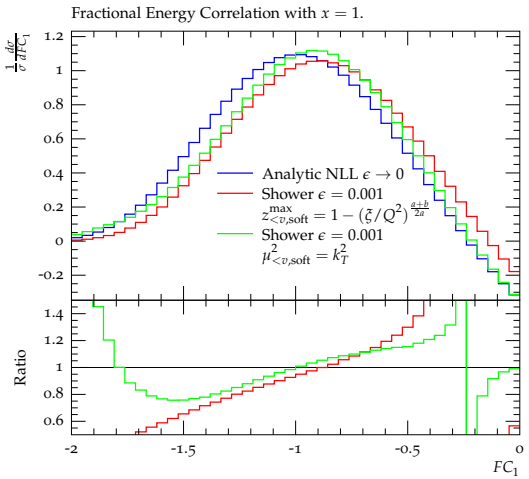
Numerical effects away from the limit

[Reichelt,Siegert,SH] arXiv:1711.03497



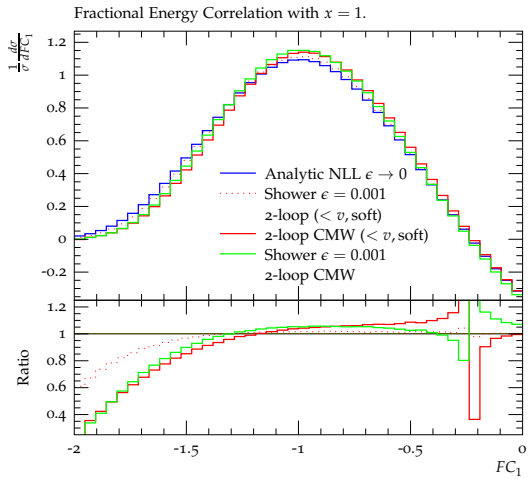
Single emission effects

- 4-mom conservation
- PS sectorization
- k_T scale in coll. terms



Multiple emission effects

- z bounds by unitarity
- k_T scale by unitarity



Effects of scale choice

- 2-loop CMW in all soft terms
- 2-loop CMW overall

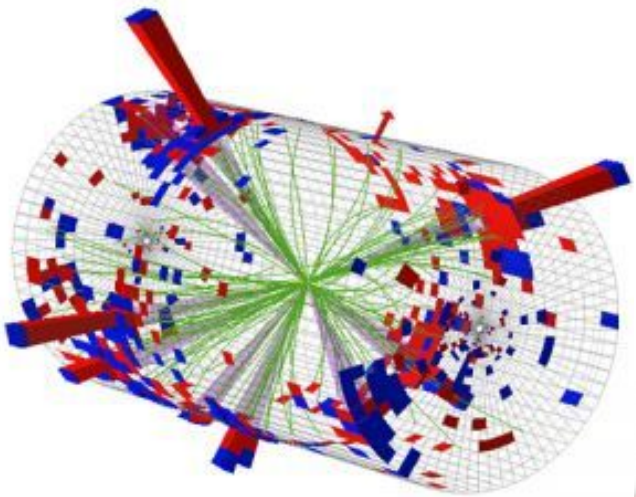
- Simplest process and simplest type of observable, still sizable differences away from $\tau \to 0$ limit
- How do we proceed to quantify precision in the intermediate region ("between" NLL and NLO) ?

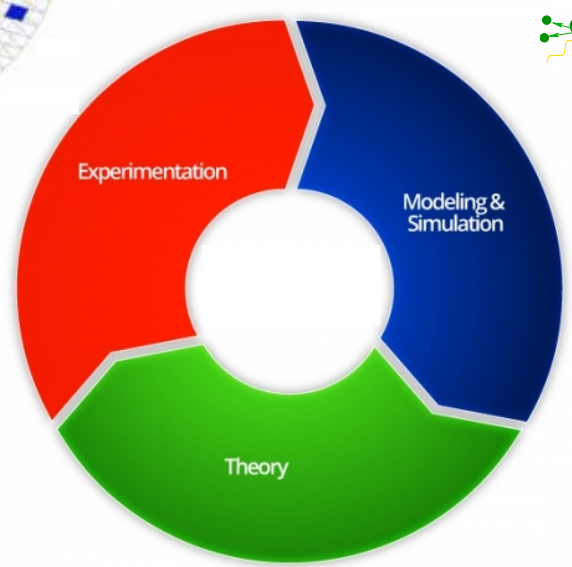
Summary

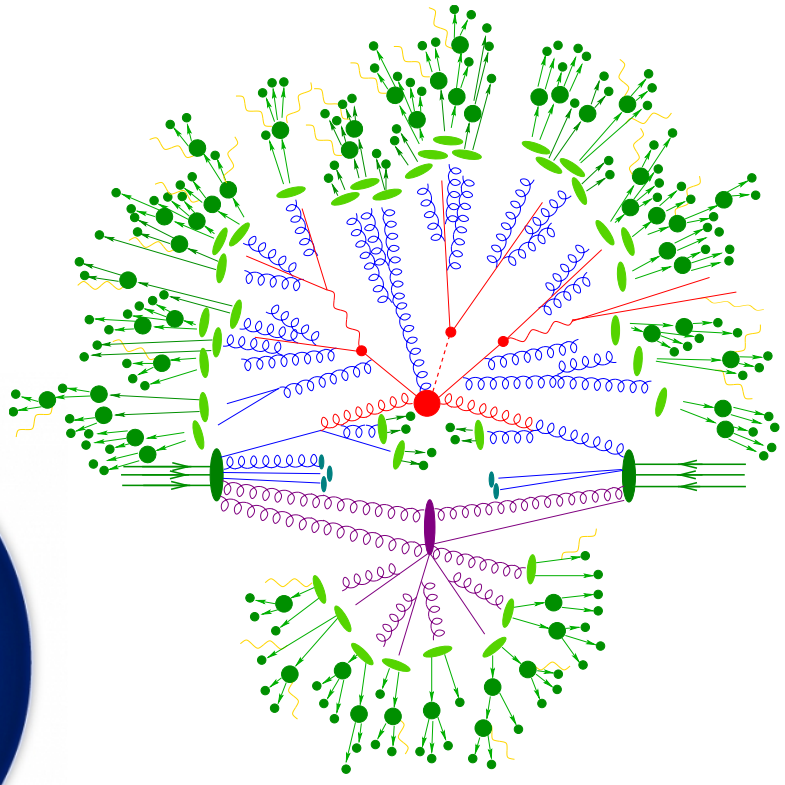
- Lots of activity in parton shower development ...
  - Logarithmic precision [PanScales,Herwig,Sherpa,...]
  - Higher-order kernels [Vincia,Sherpa,Herwig,...]
  - Interplay w/ NNLL [PanScales,...]
- ... and matching to fixed-order calculations
  - Improvements at NLO [Herwig,Pythia,Sherpa,...]
  - Resummation based [Geneva,MINNLO$_{\mathrm{PS}}$]
  - Fully differential [Vincia,UN^{X}LOPS,TOMTE]

- Still, many questions remain
  - Systematic treatment of kinematic edge effects
  - Massive quark production & evolution
  - Interplay with hadronization

Exciting times ahead!







$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ i \bar{\psi} \not{D} \psi + h.c.$$