Next-to-soft-virtual resummed prediction for pseudoscalar Higgs boson production at NNLO+$\overline{\text{NNLL}}$

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Durham University

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Introduction

Inclusive cross-section for pseudoscalar Higgs boson production :

$$\sigma^A\left(\tau, m_A^2\right) = \sigma^{A,(0)}\left(\mu_R^2\right) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\ \Phi_{ab}\left(y, \mu_F^2\right) \Delta_{ab}^A\left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right),$$

$$\text{where } \Phi_{ab}\left(y, \mu_F^2\right) = \int_y^1 \frac{dx}{x} f_a\left(x, \mu_F^2\right) f_b\left(\frac{y}{x}, \mu_F^2\right).$$

Definitions : $\sigma^{A,(0)}\left(\mu_R^2\right)$: Born cross-section, $\Phi_{ab}\left(y, \mu_F^2\right)$: Parton flux,
$\Delta_{ab}^A\left(\tau/y, m_A^2, \mu_R^2, \mu_F^2\right)$: Finite Partonic Coefficient Function,
a and b : Initial state partons, f_a and f_b : Parton distribution functions (PDFs).

- Partonic Coefficient Function near threshold, $z = \frac{\tau}{y} \to 1$:

$$\Delta_{ab} \sim \underbrace{a_i\left[\frac{\ln^i(1-z)}{(1-z)}\right]_+ + b\delta(1-z)}_{\text{Leading power (LP)/ Soft-Virtual (SV) corrections}} + \underbrace{c_i \ln^i(1-z)}_{\text{Next-to-Leading power (NLP)/ Next-to-soft virtual (NSV) corrections}} + \mathbf{d}.$$

Motivation

- FO QCD predictions experience various irregular logarithms.
  - Logs of UV & Collinear origin $\xrightarrow{\text{absorbed}}$ Renormalization & PDFs,
  - Soft regions $\xrightarrow{\text{witness}}$ Soft gluon emissions $\xRightarrow{\text{cancel with}}$ those of virtual gluons.
- Still, the soft-gluon-effects can be significant in kinematic configurations where high imbalance persists between real and virtual contributions $\Rightarrow$ **Threshold Region**.
- NSV logarithmic corrections are numerically sizeable ; often comparable or beyond SV ones.

- NSV logs contribute $\approx 25\%$ of the born in **gg→H** at a_s^3 while SV terms contribute -2.25%.

  - **Anastasiou, Duhr, Dulat et al. (2014)**

- NSV logs contribute 1.49% of the born in **DY** at a_s^3 while SV terms contribute only 0.02%.

  - **A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)**

m_A (GeV)	NNLL/NNLO (%)	$\overline{\text{NNLL}}$/NNLO (%)
125	11.8189	17.0234
700	12.8902	15.8511
1000	13.2377	16.2727
1500	14.8419	18.4658
2000	16.5992	21.0971

$\text{NNLL} = \text{FO}_{\text{NNLO}} + \text{SV}_{\text{resum}}$, and

$\overline{\text{NNLL}} = \text{FO}_{\text{NNLO}} + (\text{SV} + \text{NSV})_{\text{resum}}$.

Solution : Systematically sum these logs up to all orders $\Rightarrow$ **Resummation**.

Developments of Work

- Anastasiou, Duhr, Dulat et al. (2015)$\Rightarrow$ completed N^{3}LO prediction for scalar Higgs boson production *via* gluon fusion in the large top mass limit.
  The corrections to the cross-section were found to be $\approx 1\%$ at NNLO, and $\approx 2\%$ at N^{3}LO.
- FO cross-section for pseudoscalar Higgs boson production to NNLO accuracy :
  - R. V. Harlander and W. B. Kilgore (2002), & C. Anastasiou and K. Melnikov (2003) - similar but independent works.
  - V. Ravindran, J. Smith and W. van Neerven (2003) - alternative method.
- Development of the resummation formalism :
  - G. F. Sterman(1987),
  - S. Catani and L. Trentadue (1989),
  - V. Ravindran (2005, 2006),
  - **A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020).**
- **T. Ahmed, M. Bonvini, M. C. Kumar, P. Mathews, N. Rana, V. Ravindran, and L. Rottoli (2016)** $\Rightarrow$ FO computation at NNLO & approx. N^{3}LO + all-order threshold resummation.
- **T. Ahmed, M.C. Kumar, P. Mathews, N. Rana and V. Ravindran (2015)** $\Rightarrow$ N^{3}LO SV corrections to pseudoscalar Higgs boson production through gluon fusion.

Success of EFT

Calculations become simpler in the infinite quark mass limit ($m_X \ll 2m_t$) with increasing complexities at higher orders in the perturbation theory.

- In the case of scalar Higgs boson production, the difference between the exact and EFT results at NNLO were found to be within 1% - **A Success !**

  - R. V. Harlander, K. J. Ozeren (2009),
  A. Pak, M. Rogal, M. Steinhauser (2009),
  M. Czakon, R. V. Harlander *et al.* (2021).

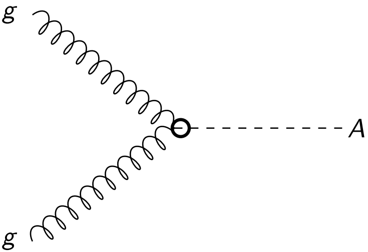
- Eventual observation $\Rightarrow$ the EFT approach, when rescaled with the exact LO results, provides a reasonably good approximation even at masses outside the region of formal validity.

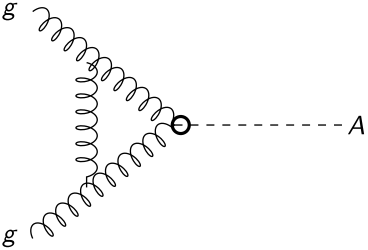
  - M. Spira, A. Djouadi, et al. (1995),
  R. Bonciani, G. Degrassi, A. Vicini (2007),
  C. Anastasiou, C. Duhr, F. Dulat, et al. (2016).

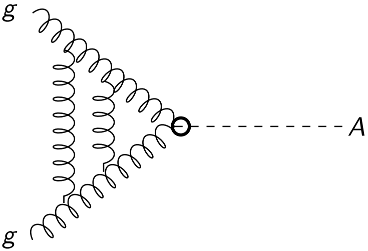
- The difference between the exact and EFT results at NLO reaches $\approx 10\%$ for $m_A = 500$ GeV, but does not increase much as m_A gets larger.

  - R.V. Harlander, S. Liebler, H. Mantler (2013),
  R.V. Harlander, S. Liebler, H. Mantler (2016).

Sample Feynman Diagrams







SV+NSV partonic CF near threshold - A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)

$$\Delta_{ab}^{X}\left(z,q^2,\mu_R^2,\mu_F^2\right) = \underbrace{\Delta_{ab}^{X,SV+NSV}(z,q^2,\mu_i^2)}_{\boxed{\delta(1-z),\ \mathcal{D}_i\ \log^i(1-z)}} + \underbrace{\Delta_{ab}^{X,hard}(z,q^2,\mu_i^2)}_{\boxed{\text{Regular terms in } z \text{ like } (1-z)^i}}$$

Mass factorised SV+NSV coefficient function for diagonal channels (since we will consider terms till NSV) :

$$\Delta_{c}^{X,SV+NSV}\left(z,q^2,\mu_R^2,\mu_F^2\right) = \mathcal{C}\exp\left\{\Psi_c^X\left(z,q^2,\mu_R^2,\mu_F^2,\varepsilon\right)\right\}\Big|_{\varepsilon=0}$$

The finite distribution for $c=g$ channel :

$$\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\varepsilon\right) = \left(\ln\left[Z_g^A\left(\hat{a}_s,\mu_R^2,\mu^2,\varepsilon\right)\right]^2 + \ln\left|\mathcal{F}_g^A\left(\hat{a}_s,Q^2,\mu^2,\varepsilon\right)\right|\right)\delta(1-z)$$
$$+\ 2\Phi_g^A\left(\hat{a}_s,q^2,\mu^2,z,\varepsilon\right) - 2\mathcal{C}\ln\Gamma_{gg}\left(\hat{a}_s,\mu_F^2,\mu^2,z,\varepsilon\right).$$

$Z_g^A \to$ overall operator UV renormalization constant, $\mathcal{F}_g^A \to$ form factors, $\Phi_g \to$ soft collinear distribution, $\Gamma_{gg} \to$ mass factorization kernels.

Constituent elements

- $\Phi_{\mathbf{g}}$: Has pole structure in ε similar to the residual divergences
  - Functional form : $\boxed{\Phi_{\mathbf{g}} = \Phi_{\mathbf{g}}^{\mathbf{SV}} + \Phi_{\mathbf{g}}^{\mathbf{NSV}}}$ where

$$\Phi_g^{SV}\left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\varepsilon}{1-z}\right) \hat{\phi}_g^{SV,(i)}(\varepsilon), \text{ and}$$

$$\Phi_g^{NSV}\left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \varphi_g^{NSV,(i)}(z, \varepsilon).$$

☞ $\hat{\phi}_{\mathbf{g}}^{\mathbf{SV},(\mathbf{i})}(\varepsilon) \Rightarrow$ **cusp** $(A_{g,i})$ **and soft** (f_g) **anomalous dimensions,**
z-independent constants, $\overline{\mathcal{C}}_{g,i}^{A}$**, and** $\overline{\mathcal{G}}_{g,i}^{A,k}$.

☞ $\boxed{\varphi_g^{NSV,(i)}(z,\varepsilon) = \varphi_{s,g}^{NSV,(i)}(z,\varepsilon) + \varphi_{f,g}^{NSV,(i)}(z,\varepsilon)}$

Constituent elements

$\varphi^{NSV,(i)}_{s,g}(z,\varepsilon) \rightarrow$ these singular coefficients should acquire a definite structure. For $g+g\rightarrow A$, we evaluated them to be

$$\varphi^{NSV,(1)}_{s,g}(z,\varepsilon) = -\frac{8C_A}{\varepsilon},$$

$$\varphi^{NSV,(2)}_{s,g}(z,\varepsilon) = \frac{8\beta_0 C_A}{\varepsilon^2} + \frac{1}{\varepsilon}\left\{ C_A^2\left(8\zeta_2 - \frac{268}{9}\right) + \frac{40C_A n_f}{9} + 16C_A^2\log(1-z)\right\}.$$

$\varphi^{NSV,(i)}_{f,g}(z,\varepsilon) \xRightarrow[\text{in terms of}]{\text{can be expressed}}$ certain finite coefficients $\mathcal{G}^{g}_{L,i}(z,\varepsilon)$

$$\varphi^{NSV,(1)}_{f,g}(z,\varepsilon) = \frac{1}{\varepsilon}\mathcal{G}^{g}_{L,1}(z,\varepsilon),$$

$$\varphi^{NSV,(2)}_{f,g}(z,\varepsilon) = \frac{1}{\varepsilon^2}\left\{-\beta_0\mathcal{G}^{g}_{L,1}(z,\varepsilon)\right\} + \frac{1}{2\varepsilon}\mathcal{G}^{g}_{L,2}(z,\varepsilon),$$

where $\mathcal{G}^{g,(j)}_{L,i}(z) \xrightarrow{\text{parameterized}} \mathcal{G}^{g,(j,k)}_{L,i}$ and $\log^k(1-z)\,, k=0,1,\cdots$.

Expansion coefficients

The parameterized finite coefficients, $\mathcal{G}_{L,i}^{g,(j,k)}$, are related to certain expansion coefficients, $\varphi_{f,g}^{NSV,(i)}$, as below :

$$\begin{aligned}\varphi_{g,1}^{(k)} &= \mathcal{G}_{L,1}^{g,(1,k)}, \qquad k = 0,1\\ \varphi_{g,2}^{(k)} &= \frac{1}{2}\mathcal{G}_{L,2}^{g,(1,k)} + \beta_0 \mathcal{G}_{L,1}^{g,(2,k)}, \qquad k = 0,1,2.\end{aligned}$$

Our Observation : The $\varphi_{g,i}^{(k)}$'s, for the scalar and the pseudoscalar Higgs boson productions in gluon fusion, are identical to each other till the two-loop level.

Earlier Observations : - **A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)**

- Same was noticed for the DY process and scalar Higgs production *via* bottom quark annihilation up to two-loop level.
- This failed for the quark annihilation process at third order for $k = 0,1$.

Hence, this behaviour at third order for the pseudoscalar Higgs boson production can be checked only when the corresponding explicit N^{3}LO results are available.

Resummation in Mellin space

$\Delta_{gg}^{A,SV+NSV}\left(z, q^2, \mu_R^2, \mu_F^2\right) \xrightarrow{\text{convenience}}$ Mellin (N-moment) space

- Convolutions $\Rightarrow$ Simple products.
  - $z \to 1$ translates to $N \to \infty$ near threshold.
  - Keep $\mathcal{O}(1/N)$ corrections.

$$\Delta_{g,N}(q^2, \mu_R^2, \mu_F^2) = C_0(q^2, \mu_R^2, \mu_F^2) \exp\left(\Psi_N^g(q^2, \mu_F^2)\right)$$

$$\Psi_N^g = \Psi_{\rm SV,N}^g + \Psi_{\rm NSV,N}^g$$

$$\Downarrow$$

$$\Psi_{\rm SV,N}^g = \log(g_0^g(a_s(\mu_R^2))) + g_1^g(\omega)\log(N) + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g_{i+2}^g(\omega);$$

$$\Psi_{\rm NSV,N}^g = \frac{1}{N}\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\left(\bar{g}_{i+1}^g(\omega) + h_i^g(\omega, N)\right), \text{ with } h_i^g(\omega, N) = \sum_{k=0}^{i} h_{ik}^g(\omega)\ \log^k(N).$$

$\boxed{\text{Coefficients } g_i^g, \bar{g}_i^g \text{ and } h_i^g \text{ are available}; C_0 \to \text{process-dependent coefficients.}}$

Determining the expansion coefficients

By exploiting the similarity between pseudoscalar and scalar Higgs !

- T. Ahmed, M. Bonvini, M. C. Kumar, P. Mathews, N. Rana, V. Ravindran, L. Rottoli (2016)

Conclusion $\Rightarrow$ The pseudoscalar result can be approximated from the available scalar Higgs results

$$\Downarrow$$

$$\Delta^{A}_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)=\frac{g_0\left(a_s\right)}{g_0^H\left(a_s\right)}\left[\Delta^{H}_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)+\delta\Delta^{A}_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)\right].$$

- $\delta\Delta^{A,NSV}_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)\rightarrow$ correction to the scalar Higgs coefficient functions,
- $g_0\left(a_s\right)$ and $g_0^H\left(a_s\right)\rightarrow$ constant functions of resummation for pseudoscalar and scalar Higgs, respectively.
- **Ratio :**

$$\frac{g_0\left(a_s\right)}{g_0^H\left(a_s\right)}=1+a_s\left(8C_A\right)+a_s^2\left[\frac{1}{3}\left\{-215\,C_A^2\ldots\right\}\right]+a_s^3\left[\frac{1}{81}\left\{68309\,C_A^3+\ldots\right\}\right].$$

Borrowing this Idea

- **T. Ahmed, M. Bonvini, et. al.** *arXiv :1606.00837 [hep-ph]*

A Conjecture to all higher orders.

- $\delta\Delta^{A}_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right)$ corrections vanish at the one-loop level.
- At two-loop level, these $\delta\Delta^{A}_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right)$ corrections contain only the next-to-next-to-soft terms.
- These observations $\xrightarrow[\text{conclusion}]{\text{lead to the}}$ $\delta\Delta^{A}_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right)$ corrections do not contain any NSV terms at $\mathcal{O}(a_s^3)$.

Consequence :

- $\delta\Delta^{A}_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right) = 0 \xrightarrow{\text{leads to}}$ the approximate $\mathrm{N^3LO}$ cross-sections denoted by $\mathrm{N^3LO}_A$.

Implications on our Analysis

Hence, we simply rescale the Higgs SV+NSV CF to obtain the corresponding one for the pseudoscalar using

$$\Delta_{gg}^{A,NSV}\left(z,q^2,\mu_R^2,\mu_F^2\right)=\frac{g_0\left(a_s\right)}{g_0^H\left(a_s\right)}\left[\Delta_{gg}^{H,NSV}\left(z,q^2,\mu_R^2,\mu_F^2\right)\right].$$

The ratio and the CF's are known up to NNLO $\xrightarrow{\text{leading to}}$ successful computation of $\Delta_{gg}^{A}\left(z,q^2,\mu_R^2,\mu_F^2\right)$ up to two-loop level.

To evaluate the SV+NSV CFs for pseudoscalar higgs boson production from gluon fusion, we follow the following procedure :

1. Using the analytical formalism. - A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)
2. Using the ratio, $g_0\left(a_s\right)/g_0^H\left(a_s\right)$, and combining it with the available scalar Higgs SV+NSV CFs. - T. Ahmed, M. Bonvini, M. C. Kumar, et. al. (2016)

$$\Downarrow$$

1 yields the corresponding pseudoscalar Higgs SV+NSV CFs in terms of the $\varphi_{g,i}^{(k)}$'s which are evaluated by comparison with the result from 2 .

Predicting higher logs

Significance of this method of computation :

- Obtained the SV+NSV CFs for pseudoscalar production up to $\mathcal{O}(a_s^3)$.
- Predicting coefficients of three highest logarithms of $\Delta_{gg}^{A,SV+NSV}$, from $\mathcal{O}(a_s^4)$ to $\mathcal{O}(a_s^7)$.

a_s^4	$\log^7(1-z)$	$\log^6(1-z)$	$\log^5(1-z)$
	$-4096/3\ C_A^4$	$98560/9\ C_A^4 - 7168/9\ n_f C_a^3$	$-335104/9\ C_a^4 + 174208/27\ n_f C_a^3 - 4096/27\ n_f^2 C_a^2 + 23552\ \zeta_2 C_a^4$
a_s^5	$\log^9(1-z)$	$\log^8(1-z)$	$\log^7(1-z)$
	$-8192/3\ C_a^5$	$96256/3\ C_a^5 - 8192/3\ n_f C_a^4$	$-131685640/81\ C_a^5 + 569216/81\ n_f C_a^4 - 81920/81\ n_f^2 C_a^3 + 262144/3\ \zeta_2 C_a^5$
a_s^6	$\log^{11}(1-z)$	$\log^{10}(1-z)$	$\log^9(1-z)$
	$-65536/15\ C_a^6$	$9490432/135\ C_a^6 - 180224/27\ C_a^5 n_f$	$-4458496/9\ C_a^6 + 8493056/81\ C_a^5 n_f - 327680/81\ C_a^4 n_f^2 + 671744/3\ \zeta_2 C_a^6$
a_s^7	$\log^{13}(1-z)$	$\log^{12}(1-z)$	$\log^{11}(1-z)$
	$-262144/45\ C_a^7$	$3309568/27\ C_a^7 - 1703936/135\ C_a^6 n_f$	$-92717056/81\ C_a^7 + 115835488/45\ C_a^6 n_f - 917504/81\ C_a^5 n_f^2 + 1310720/3\ \zeta_2 C_a^7$

Still we are left with certain other logarithms that cannot be predicted from previous order informations.

Relevance of Pseudoscalar studies

- Will prove beneficial if/when the pseudoscalar Higgs is discovered.
- Contribute towards establishing the CP properties of the discovered Higgs boson.

Speculations : The observed Higgs boson at the LHC can be an admixture of scalar-pseudoscalar states.

Exploring such possibilities had already started some time back :

- Y. Gao, A. V. Gritsan *et al.* (2010),
- P. Artoisenet *et al.* (2013),
- F. Maltoni, K. Mawatari, and M. Zaro (2014),
- M. Jaquier and R. Röntsch (2019).

Solution for the problems in the SM $\xrightarrow{\text{may lead to}}$ Possible new physics.

Requirement from theoretical physicists :

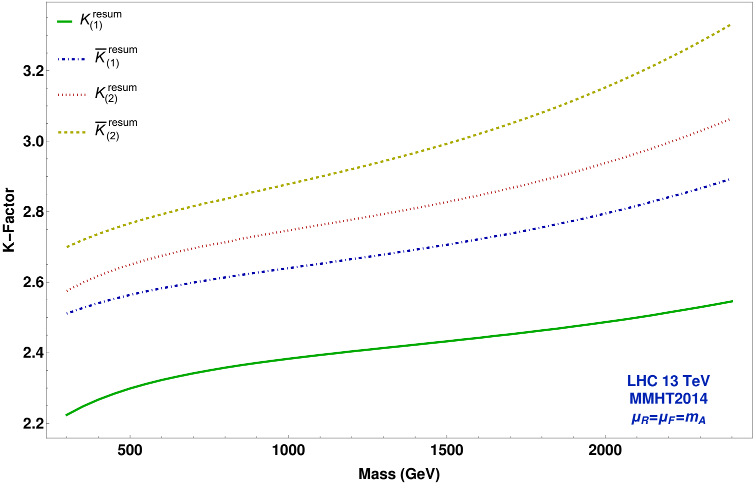
Precision calculations of the relevant observable corresponding to both scalar and pseudoscalar production processes to the same order of precision.

Numerical results

Assumptions :

- Based on EFT.
- 13 TeV C.O.M. energy at the LHC.
- $\cot\beta = 1$ (other values can be obtained by rescaling).
- $C_J^{(2)} = 0$ because of non-availability.
- PDF's : Corresponding MMHT 2014 up to NNLO ;
  MMHT 2014 NNLO at N^{3}LO (for non-availability).
- For NSV resummation $\Rightarrow$ Resum threshold logs only for gluon fusion channel.
- Theoretical uncertainties computed at $m_A = 125$ GeV, 700 GeV for seven point scale uncertainties, and by varying one scale & keeping the other fixed.
  - $\{\mu_R/m_A, \mu_F/m_A\} =$ $(0.5, 0.5), (0.5, 1.0), (1.0, 0.5), (1.0, 1.0), (1.0, 2.0), (2.0, 1.0)$ and $(2.0, 2.0)$.
  - $\{\mu_R/m_A$ or $\mu_F/m_A\} = \{0.5, 1.0, 2.0\}$ and the other scale fixed at m_A.

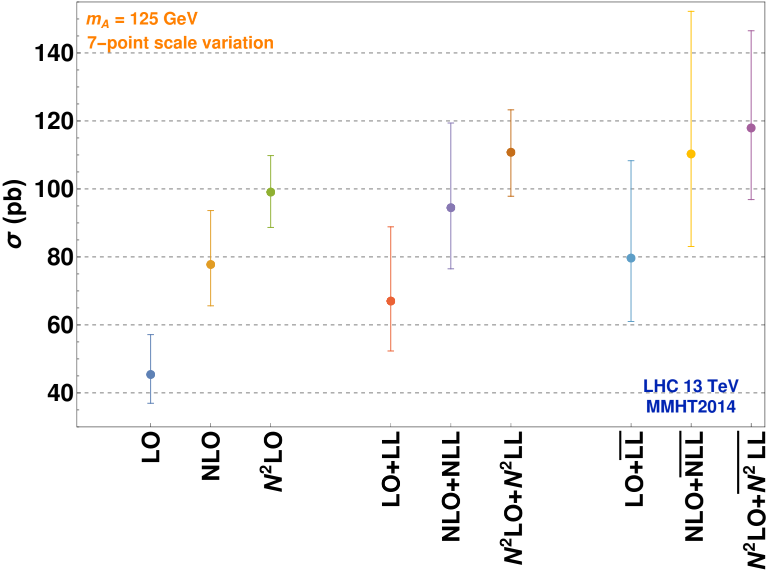
Resummed K-factor plot at NLO (K_1) and NNLO (K_2)



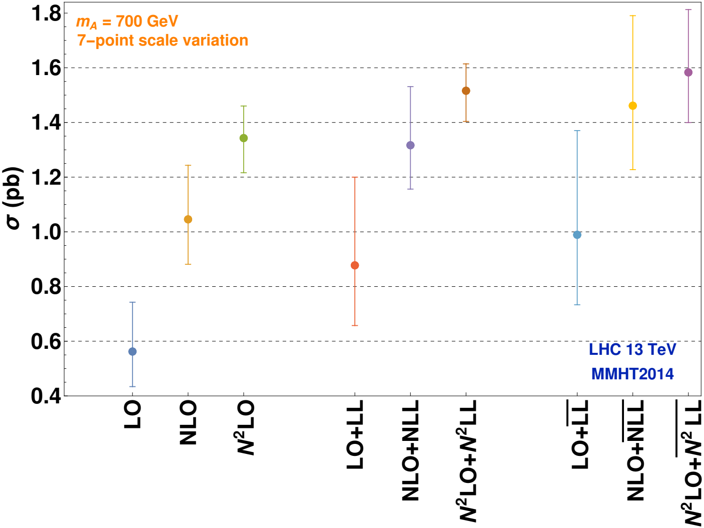
$$K^{\text{res}}_{(1)} = \frac{\sigma_{\text{NLO+NLL}}}{\sigma_{\text{LO}}},$$
$$K^{\text{res}}_{(2)} = \frac{\sigma_{\text{NNLO+NNLL}}}{\sigma_{\text{LO}}},$$
$$\overline{K}^{\text{res}}_{(1)} = \frac{\sigma_{\text{NLO+}\overline{\text{NLL}}}}{\sigma_{\text{LO}}},$$
$$\overline{K}^{\text{res}}_{(2)} = \frac{\sigma_{\text{NNLO+}\overline{\text{NNLL}}}}{\sigma_{\text{LO}}}.$$

- The $\overline{\text{NLL}}$ results increase the NLL results by about 30% (40%) in the low (high) mass region.
- The $\overline{\text{NNLL}}$ results, in a similar behavior, enhances the NNLL results by about 10% (30%) in the low (high) mass region.

7-point scale uncertainty plot for $m_A = 125$ GeV



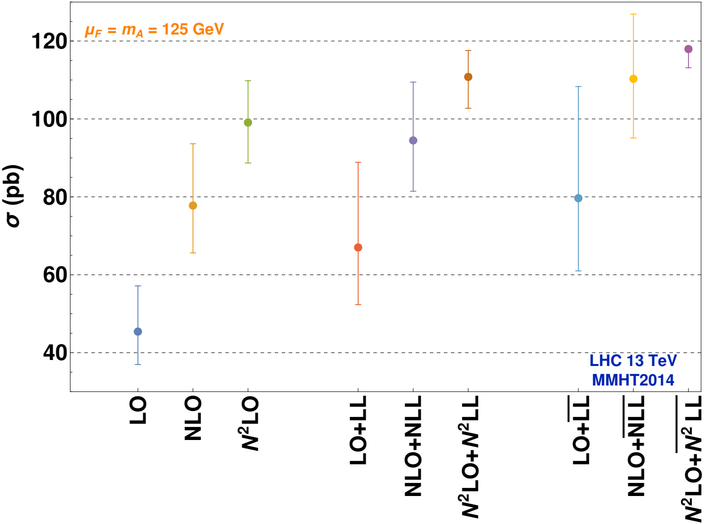
7-point scale uncertainty plot for $m_A = 700$ GeV



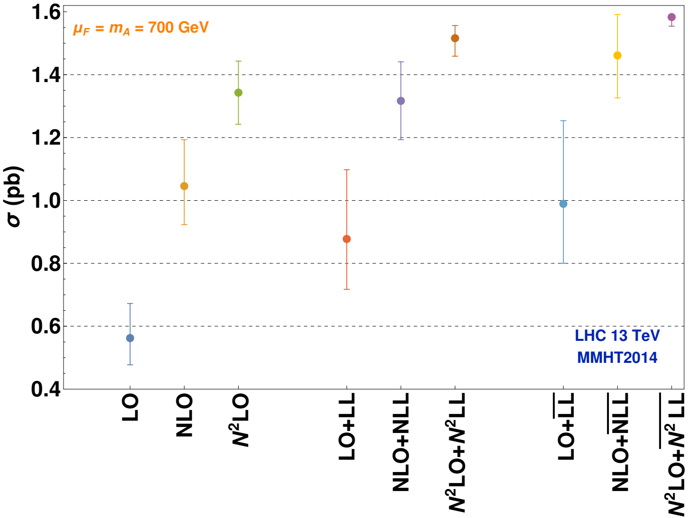
Observation $\Rightarrow$ The uncertainties reduce from NLO to NNLO, NLO+NLL to NNLO+NNLL, and NLO+$\overline{\text{NLL}}$ to NNLO+$\overline{\text{NNLL}}$ $\xRightarrow{\textbf{Problem}}$ *NSV resummation exhibits higher uncertainties than SV.*

Uncertainty plot for μ_F scale fixed at $m_A = 125$ GeV

To comprehend this unexpected behaviour $\xrightarrow{\text{we study}}$ scale variations due to μ_R and μ_F separately by varying one and keeping the other fixed at m_A.

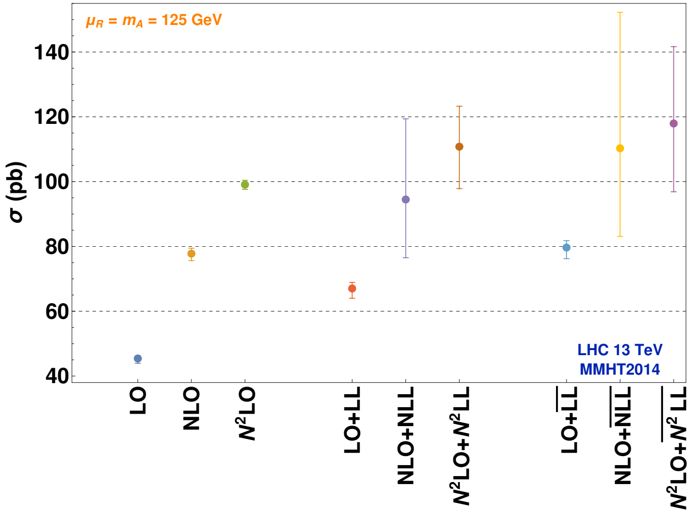


Uncertainty plot for μ_F scale fixed at $m_A = 700$ GeV



Observation : At the 2nd order, uncertainties reduce from FO results to resummed ones with the NSV results being more stable than the SV ones.

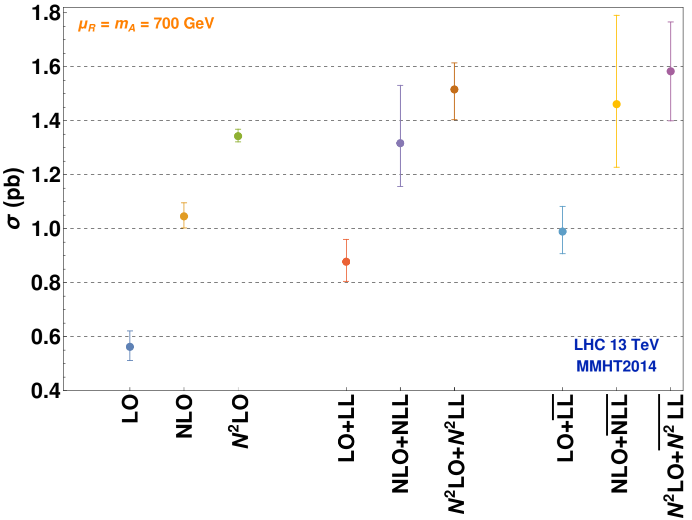
Uncertainty plot for μ_R scale fixed at $m_A = 125$ GeV



Observations : Results are in contrast to the μ_R variation ones, *i.e.*

- **NLO+$\overline{\text{NLL}}$ > NLO+NLL > NLO,**
- **NNLO+$\overline{\text{NNLL}}$ > NNLO+NNLL > NNLO.**

Uncertainty plot for μ_R scale fixed at $m_A = 700$ GeV



Conclusion : Contributions from other partonic channels for resummation because different partonic channels are expected to mix when the μ_F scale varies.

Possibility of scalar-pseudoscalar Higgs boson mixed state

Parameter : Mixing angle α.

- M. Jaquier, R. Röntsch (2019)

Consider a Higgs boson production, while neglecting its decay,

$\xrightarrow[\text{the results up to NNLO}]{\text{for any arbitrary value of } \alpha,}$ may be obtained by the simple rescaling formula below.

$$\sigma = \cos^2 \alpha \cdot \sigma_{\mathbf{H}} + \sin^2 \alpha \cdot \sigma_{\mathbf{A}}$$

K-Factor	$\alpha = 0$ **(pure scalar)**	$\alpha = \pi/2$ **(pure pseudoscalar)**	$\alpha = \pi/4$ **(mixed state)**	$\alpha = \pi/6$ **(mixed state)**
$K_{(1)}$	1.6990	1.7124	1.7083	1.7048
$K_{(2)}$	2.1571	2.1814	2.1741	2.1677
$K_{(1)}^{resum}$	2.0033	2.0803	2.0570	2.0368
$K_{(2)}^{resum}$	2.2785	2.4392	2.3907	2.3485
$\overline{K}_{(1)}^{resum}$	2.3425	2.4284	2.4025	2.3799
$\overline{K}_{(2)}^{resum}$	2.4737	2.5966	2.5595	2.5272

Our Observation

Changing the mixing angle α modifies the corresponding QCD corrections only by a few percent.

Consequence : Availibility of the pseudoscalar Higgs boson production cross-section to a precision comparable to that of the scalar Higgs

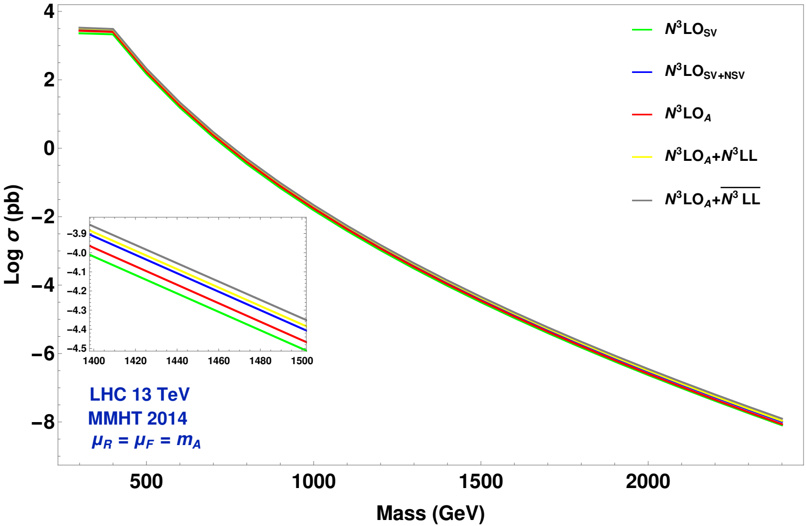
$\Downarrow$

In extracting the mixing angle, α, to a better accuracy.

☞ While studying Higgs decay processes, the simple reweighting formula above fails. Hence, the corresponding K-factors, similar to those given in the above table, get modified slightly $\xRightarrow{\textbf{Why ?}}$ Number of angular observables get involved.

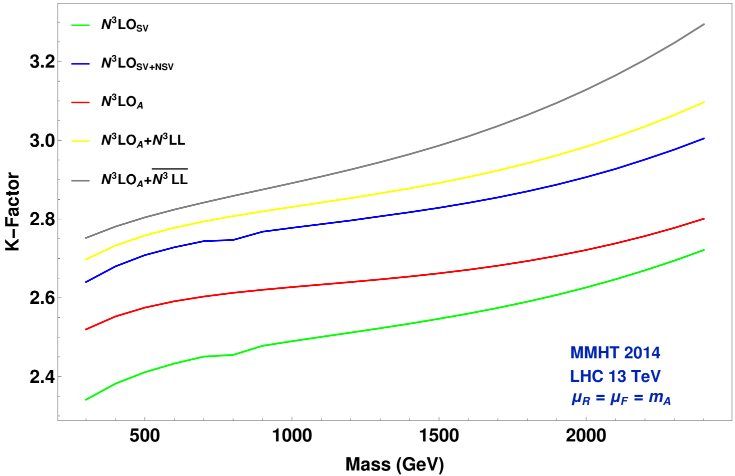
- M. Jaquier, R. Röntsch (2019)

N^3LO results : Cross-sections



Observation : Inclusion of NSV corrections at the FO or their resummation substantially increase the cross-sections.

N^3LO results : K-factors



Observations :

$$\begin{cases} N^3LO_{SV} \xrightarrow{\text{are closer to}} N^3LO_A \text{ in the high mass region,} \\ N^3LO_{SV+NSV} \xrightarrow{\text{are closer to}} N^3LO_A \text{ in the small mass region.} \end{cases}$$

Summary

Aim : NSV resummation for pseudoscalar Higgs boson production *via* gluon fusion to $\overline{\text{NNLL}}$ accuracy.

1. Compute the NSV corrections up to second order, and compare them with the corresponding FO corrections.
   - **Conclude** These corrections significantly impact the pseudoscalar production cross-section compared to the conventional SV logarithms.
2. Estimate theory uncertainties.
   - The 7-point scale uncertainties do not improve much after NSV resummation.
   - The μ_F scale variations increase the uncertainties.
   - For μ_R scale variations, the uncertainties reduce significantly.

   $$\Downarrow$$

   Conclude $\rightarrow$ The need of NSV contributions from other parton channels, & beyond NSV contributions in the gluon fusion channel.
3. Evaluate the production cross-sections for mixed scalar-pseudoscalar states.
   - Study their behavior for different values of the mixing angle, α.
   - **Conclude** $\Rightarrow$ QCD corrections change with α by a few percent.

Thank you...

]Threshold Limit

Threshold region corresponds to the limit

$\mathbf{z \to 1}$ **:** $z \equiv \frac{q^2}{\hat{s}} = \frac{\tau}{y}$ and $\tau = \frac{q^2}{S}$.

Emission of **soft and collinear gluons**

$\Rightarrow$ large logarithmic contributions.

- q^2 : invariant mass,
- S : Hadronic COM energy,
- $\hat{s}$: partonic COM energy,
- $y \equiv x_1, x_2$: partonic scaling variables.

Large gluon flux at the threshold significantly contributes to the hadronic cross-section.

$\Downarrow$

Spoils reliability of the FO predictions in these kinematic regions by ruining the perturbative nature of the series.

Solution : Resumming these large logarithms to all orders in the perturbation theory.

Physics in the threshold limit

Partonic Coefficient Function near threshold :

$$\Delta_{ab} \sim a_i \mathcal{D}_i + b\ \delta\left(1-z\right) + c_i \ln^i\left(1-z\right) + d$$

where $\mathcal{D}_i = \left[\frac{\ln^i\left(1-z\right)}{\left(1-z\right)}\right]_+$.

Constituent elements

- $\mathbf{Z_g^A}$: Removes UV divergences
  - Functional form : $\boxed{\mu_R^2 \frac{\mathbf{d}}{\mathbf{d}\mu_R^2} \ln \mathbf{Z_g^A} \left(\hat{\mathbf{a}}_s, \mu_R^2, \mu^2, \varepsilon\right) = \sum_{i=1}^{\infty} \mathbf{a}_s^i \gamma_{g,i}^A}$
- $\mathcal{F}_\mathbf{g}^\mathbf{A}$: deals with virtual corrections
  - Functional form : $\boxed{\ln \mathcal{F}_\mathbf{g}^\mathbf{A} \left(\hat{\mathbf{a}}_s, \mathbf{Q}^2, \mu^2, \varepsilon\right) = \sum_{i=1}^{\infty} \hat{\mathbf{a}}_s^i \left(\frac{\mathbf{Q}^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} \mathbf{S}_\varepsilon^i \hat{\mathcal{L}}_{g,i}^A (\varepsilon)}$

Dependents :

- $\gamma_{g,i} \rightarrow$ UV anomalous dimensions,
- $A_{g,i} \rightarrow$ cusp anomalous dimensions,
- $G_{g,i}^A(\varepsilon) \rightarrow$ resummation functions which decompose into
  1. process dependent $g_{g,j}^{A,i}$'s, and
  2. collinear (B_g), soft (f_g) and UV (γ_g) anomalous dimensions.

Constituent elements

- $\Gamma_{\mathbf{gg}}$: Removes soft and collinear (IR) divergences
  - Functional form : $\boxed{\Gamma_{\mathbf{gg}}\left(\mathbf{z},\mu_{\mathbf{F}}^{2},\varepsilon\right)=\delta\left(\mathbf{1}-\mathbf{z}\right)+\sum_{\mathbf{i=1}}^{\infty}\hat{\mathbf{a}}_{\mathbf{s}}^{\mathbf{i}}\left(\frac{\mu_{\mathbf{F}}^{2}}{\mu^{2}}\right)\mathbf{S}_{\varepsilon}^{\mathbf{i}}\Gamma_{\mathbf{gg}}^{(\mathbf{i})}\left(\mathbf{z},\varepsilon\right)}$,

    where

    the mass factorization kernels, $\Gamma_{gg}^{(i)}\left(z,\varepsilon\right)$'s, are expanded in negative powers of ε and the AP splitting kernels, $P_{gg}^{(i)}$'s.
- $\Phi_{\mathbf{g}}$: Has pole structure in ε similar to the residual divergences
  - Functional form : $\boxed{\Phi_{\mathbf{g}}=\Phi_{\mathbf{g}}^{\mathbf{SV}}+\Phi_{\mathbf{g}}^{\mathbf{NSV}}}$ where

$$\Phi_{g}^{SV}\left(\hat{a}_{s},q^{2},\mu^{2},z,\varepsilon\right)=\sum_{i=1}^{\infty}\hat{a}_{s}^{i}\left(\frac{q^{2}\left(1-z\right)^{2}}{\mu^{2}}\right)^{i\frac{\varepsilon}{2}}S_{\varepsilon}^{i}\left(\frac{i\varepsilon}{1-z}\right)\hat{\phi}_{g}^{SV,(i)}\left(\varepsilon\right), \text{ and}$$

$$\Phi_{g}^{NSV}\left(\hat{a}_{s},q^{2},\mu^{2},z,\varepsilon\right)=\sum_{i=1}^{\infty}\hat{a}_{s}^{i}\left(\frac{q^{2}\left(1-z\right)^{2}}{\mu^{2}}\right)^{i\frac{\varepsilon}{2}}S_{\varepsilon}^{i}\varphi_{g}^{NSV,(i)}\left(z,\varepsilon\right).$$

☞ $\hat{\phi}_{\mathbf{g}}^{\mathbf{SV},(\mathbf{i})}\left(\varepsilon\right)\Rightarrow$ **cusp** $(A_{g,i})$ **and soft** (f_g) **anomalous dimensions, z-independent constants,** $\overline{C}_{g,i}^{A}$**, and** $\overline{\mathcal{G}}_{g,i}^{A,k}$.