Effects of U(1) gauge bosons and small size instantons on axions at collider scales

based on work with Alexey Kivel and Felix Yu
(2207.08740, 2209.XXXXX)

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Recent Progress in Axion Theory and Experiment

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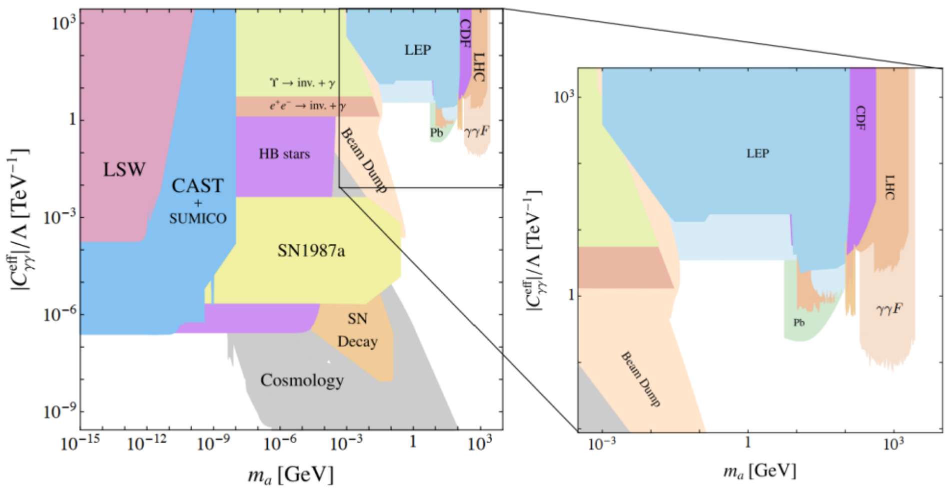
Motivation

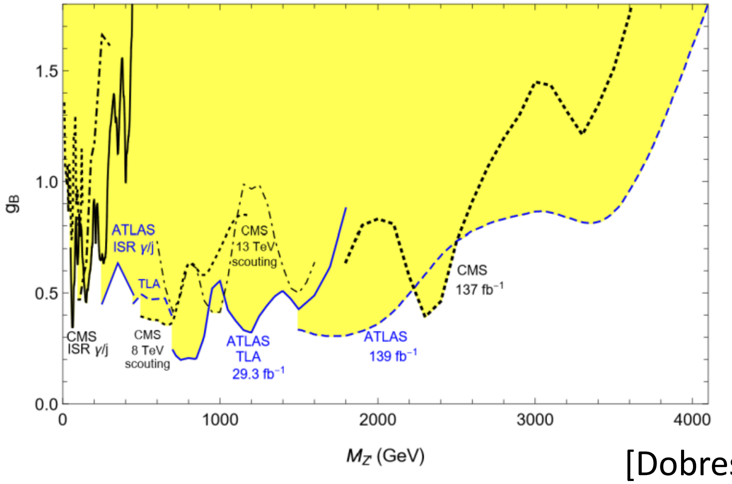
- ❑ Have well motivated candidates for new physics:
  - ❑ Axions and Axion-like particles (ALPs)
  - ❑ Additional U(1) gauge bosons (Z's)
- ❑ Collider probes for masses in the GeV-TeV range



[Bauer et al, 1808.10323]

Can we probe the QCD axion at collider scales?

Can we find interference effects between ALPs and Z's at collider scales?



[Dobrescu, Yu, 2112.04392]

Outline

1. **General Axion/ALP interactions**

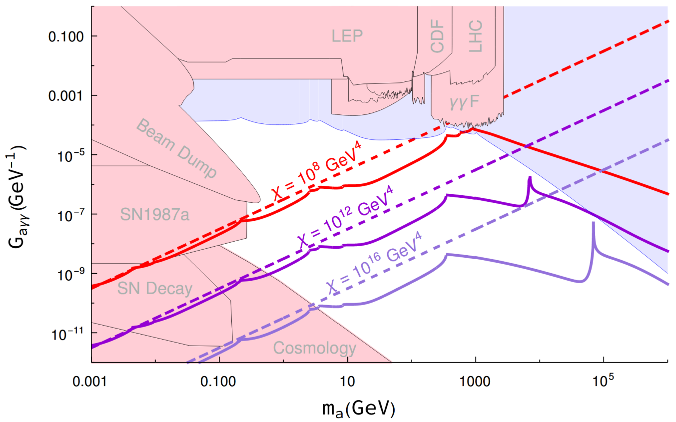
2. **DFSZ-like ALP with gauged baryon number**

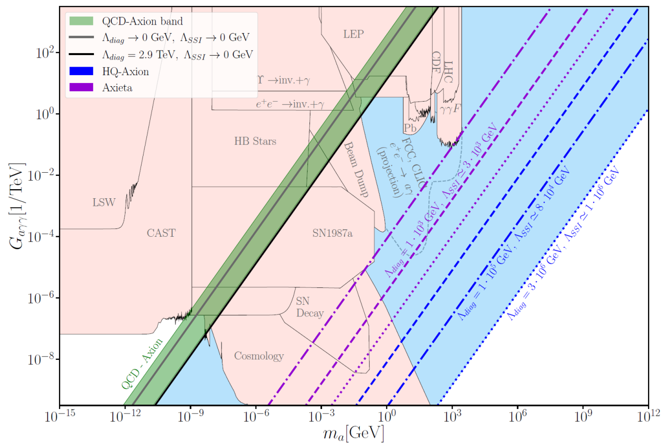
   ➢ Z' interactions

   ➢ Flavor-violating ALP interactions

3. **QCD Axion with small size instanton effects**

   ➢ Axion-Meson mixing with determinantal operator





General Axion/ALP interactions

- Motivated by Strong CP problem:
  - Have anomalous θ term $\mathcal{L} \supset -\bar{\theta}\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$ with two contributions $\bar{\theta} = \theta_{\text{QCD}} - \theta_F$
  - θ_{QCD} comes from instantons and θ_F from U(3) rotations of fermions into a real and diagonal mass basis
  - Measurements of neutron EDM constrain $\bar{\theta} < 10^{-10}$ [Pendlebury et al, 1509.04411]
    ⇒ suggests that θ_{QCD} and θ_F are aligned
- Can be solved by having a Peccei-Quinn (PQ) symmetry: [Peccei, Quinn, 1977]
  - Spontaneously broken global U(1) symmetry from additional complex scalar field
  - Axion is corresponding Goldstone boson, relaxes $\bar{\theta}$ to 0 via shift symmetry $a \to a + \bar{\theta}v_a$

General Axion/ALP interactions

- Three types of interactions at dim-5

$$\mathcal{L} \supset \frac{\partial_\mu a}{2f_a} \sum_f C_1^f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f - \sum_f m_f \bar{\psi}_f e^{iC_2^f a/f_a \gamma_5} \psi_f + \frac{a}{f_a} \sum_{I,J} C_3^{IJ} \frac{g_I g_J}{(4\pi)^2} F^a_{I\mu\nu} \tilde{F}^{a,\mu\nu}_J$$

- Two classes of models: In KSVZ models, ψ_f refers to additional heavy quarks
  In DFSZ models, ψ_f refers to SM fermions

- Can remove operator redundancy via fermion transformation

$$\psi_f \to e^{-iX_A^f \frac{\alpha}{2} \frac{a}{f_a} \gamma_5} \psi_f \,, \quad \bar{\psi}_f \to \bar{\psi}_f e^{-iX_A^f \frac{\alpha}{2} \frac{a}{f_a} \gamma_5}$$

$$C_1^f \to C_1^f + X_A^f \alpha \,, \quad C_2^f \to C_2^f - X_A^f \alpha \,, \quad C_3^{IJ} \to C_3^{IJ} - \alpha \sum_f T(R_{If}) X_A^f (Q_{IV}^f Q_{JV}^f + Q_{IA}^f Q_{JA}^f)$$

General Axion/ALP interactions

- Can rotate C_2^f term away to get general Axion/ALP Lagrangian with SM fields

$$\mathcal{L}_{\text{axion}}^{d\leq 5} \supset \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \frac{\partial_\mu a}{2f_a}J^\mu_{\text{PQ}} + C_{\gamma\gamma}\frac{e^2}{(4\pi)^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ C_{ZZ}\frac{e^2}{s_W^2 c_W^2}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{Z\gamma}\frac{e^2}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ C_{WW}\frac{g_L^2}{(4\pi)^2}\frac{a}{f_a}W_{\mu\nu}\tilde{W}^{\mu\nu} + C_{gg}\frac{g_s^2}{(4\pi)^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

- Get additional operators from flavor effects in DFSZ-like models

- Wilson coefficients also generated from integrating out heavy fermions

- Axion mass set by topological susceptibility $\chi = m_a^2 f_a^2, \quad \chi_{\text{QCD}} = \frac{m_u m_d}{(m_u+m_d)^2}m_\pi^2 f_\pi^2$

DFSZ-like ALP with gauged baryon number

[Kivel, JL, Yu, 2209.XXXXX]

DFSZ-like ALP with gauged baryon number

- First start with an ALP
- Can have mass at collider scale due to free topological susceptibility χ
- Depending on UV completion we can generate other particles at the same scale
- Take, for example, a UV completion with gauged baryon number, which adds a gauge boson Z’ and would be accessible at hadron colliders
- Get a minimal subset of parameters of the two extensions
- Want to find new collider signatures from interference effects

Z' interactions

- Additional U(1) gauge boson (Z'): vector boson with a mass and a kinetic mixing to the SM

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu} + \frac{\epsilon_{\text{mix}}}{2} B_{\mu\nu} B'^{\mu\nu} + \frac{m_{B'}^2}{2} B'_\mu B'^\mu + g' B'_\mu J^\mu_{Q'}$$

- Remove kinetic mixing term by diagonalization and canonical normalization of the gauge fields

- Leads to shift in masses and in currents

$$m_Z = m_{Z,\text{SM}} \left(1 + \frac{\epsilon_{\text{mix}}^2}{2} \frac{s_W^2 m_{Z,\text{SM}}^2}{m_{Z,\text{SM}}^2 - m_{B'}^2} + \mathcal{O}(\epsilon_{\text{mix}}^4) \right)$$

$$m_{Z'} = m_{B'} \left(1 + \frac{\epsilon_{\text{mix}}^2}{2} \frac{(m_{B'}^2 - c_W^2 m_{Z,\text{SM}}^2)}{m_{B'}^2 - m_{Z,\text{SM}}^2} + \mathcal{O}(\epsilon_{\text{mix}}^4) \right)$$

$$\begin{aligned} \mathcal{L} &\supset e A_\mu J^\mu_Q + \frac{e}{s_W c_W} Z^{\text{SM}}_\mu J^\mu_Z + g' B'_\mu J^\mu_{Q'} \\ &= e A_\mu J^\mu_Q + Z_\mu \left(\frac{e}{s_W c_W} J^\mu_Z - \epsilon_{\text{mix}} s_W g' \frac{m_Z^2}{m_Z^2 - m_{Z'}^2} J^\mu_{Q'} \right) \\ &\quad + Z'_\mu \left(g' J^\mu_{Q'} + \epsilon_{\text{mix}} e J^\mu_Q - \epsilon_{\text{mix}} \frac{e}{c_W} \frac{m_{Z'}^2}{m_{Z'}^2 - m_Z^2} J^\mu_Z \right) + \mathcal{O}(\epsilon_{\text{mix}}^2) \end{aligned}$$

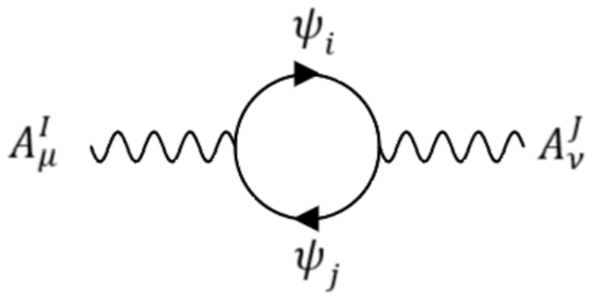
[Liu, Wang, Yu, 1704.00730]

Z' interactions

- For either U(1) being a subgroup of an SU(N) group, ϵ_{mix} appears at 1-loop and fermions fulfill a trace condition

$$\mathcal{A}_{5IJ} = \frac{1}{2}\sum_f N_f (Q^f_{IR}Q^f_{JR} + Q^f_{IL}Q^f_{JL}) = \sum_f N_f (Q^f_{IV}Q^f_{JV} + Q^f_{IA}Q^f_{JA}) = 0$$

- Can integrate out heavy fermions via



- Effective kinetic mixing parameter given by

$$\epsilon^{IJ}_{\text{eff}} = \frac{g_I g_J}{(4\pi)^2}\frac{4}{3}\sum_{m_f^2 \gg p^2} N_f (Q^f_{IV}Q^f_{JV} + Q^f_{IA}Q^f_{JA})\left(\frac{5}{3} + \ln\left(\frac{m_f^2}{p^2}\right) - \frac{1}{5}\frac{p^2}{m_f^2} + \mathcal{O}\left(\frac{p^4}{m_f^4}\right)\right)$$

DFSZ-like ALP with gauged baryon number

- Start with a two Higgs doublet model (2HDM) and SM fermions

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$		
Q_L^i	1/6	2	3		
u_R^i	2/3	1	3		
d_R^i	-1/3	1	3		
L_L^i	-1/2	2	1		
e_R^i	-1	1	1		
H_u	-1/2	2	1		
H_d	1/2	2	1		

DFSZ-like ALP with gauged baryon number

- Start with a two Higgs doublet model (2HDM) and SM fermions

- Add baryon number as gauge charge

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	$U(1)_B$
Q_L^i	1/6	2	3	1/3
u_R^i	2/3	1	3	1/3
d_R^i	-1/3	1	3	1/3
L_L^i	-1/2	2	1	0
e_R^i	-1	1	1	0
H_u	-1/2	2	1	0
H_d	1/2	2	1	0

DFSZ-like ALP with gauged baryon number

- Start with a two Higgs doublet model (2HDM) and SM fermions

- Add baryon number as gauge charge

- Add minimal set of new fermions (anomalons) which cancel gauge anomalies

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	$U(1)_B$
Q_L^i	1/6	2	3	1/3
u_R^i	2/3	1	3	1/3
d_R^i	-1/3	1	3	1/3
L_L^i	-1/2	2	1	0
e_R^i	-1	1	1	0
H_u	-1/2	2	1	0
H_d	1/2	2	1	0
L_L'	-1/2	2	1	-1
L_R'	-1/2	2	1	2
E_L'	-1	1	1	2
E_R'	-1	1	1	-1
N_L'	0	1	1	2
N_R'	0	1	1	-1

DFSZ-like ALP with gauged baryon number

- Start with a two Higgs doublet model (2HDM) and SM fermions
- Add baryon number as gauge charge
- Add minimal set of new fermions (anomalons) which cancel gauge anomalies
- Add two more scalar fields in order to break baryon number as well as PQ symmetry

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	$U(1)_B$
Q_L^i	1/6	2	3	1/3
u_R^i	2/3	1	3	1/3
d_R^i	-1/3	1	3	1/3
L_L^i	-1/2	2	1	0
e_R^i	-1	1	1	0
H_u	-1/2	2	1	0
H_d	1/2	2	1	0
L'_L	-1/2	2	1	-1
L'_R	-1/2	2	1	2
E'_L	-1	1	1	2
E'_R	-1	1	1	-1
N'_L	0	1	1	2
N'_R	0	1	1	-1
Φ_A	0	1	1	-3
Φ_B	0	1	1	3

DFSZ-like ALP with gauged baryon number

- Start with a two Higgs doublet model (2HDM) and SM fermions
- Add baryon number as gauge charge
- Add minimal set of new fermions (anomalons) which cancel gauge anomalies
- Add two more scalar fields in order to break baryon number as well as PQ symmetry
- Add a discrete symmetry which sets scalar and Yukawa interactions

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	$\mathbb{Z}_4$	$U(1)_B$
Q_L^i	1/6	2	3	+1	1/3
u_R^i	2/3	1	3	-i	1/3
d_R^i	-1/3	1	3	+1	1/3
L_L^i	-1/2	2	1	+1	0
e_R^i	-1	1	1	+1	0
H_u	-1/2	2	1	+i	0
H_d	1/2	2	1	+1	0
L'_L	-1/2	2	1	+1	-1
L'_R	-1/2	2	1	+i	2
E'_L	-1	1	1	+i	2
E'_R	-1	1	1	+1	-1
N'_L	0	1	1	+1	2
N'_R	0	1	1	-i	-1
Φ_A	0	1	1	-1	-3
Φ_B	0	1	1	+i	3

DFSZ-like ALP with gauged baryon number

- Start with a two Higgs doublet model (2HDM) and SM fermions
- Add baryon number as gauge charge
- Add minimal set of new fermions (anomalons) which cancel gauge anomalies
- Add two more scalar fields in order to break baryon number as well as PQ symmetry
- Add a discrete symmetry which sets scalar and Yukawa interactions
- PQ symmetry emerges as accidental global symmetry

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	$\mathbb{Z}_4$	$U(1)_B$	$U(1)_{PQ}$
Q_L^i	1/6	2	3	+1	1/3	X_Q
u_R^i	2/3	1	3	-i	1/3	X_Q-X_u
d_R^i	-1/3	1	3	+1	1/3	X_Q-X_d
L_L^i	-1/2	2	1	+1	0	X_L
e_R^i	-1	1	1	+1	0	X_L-X_d
H_u	-1/2	2	1	+i	0	X_u
H_d	1/2	2	1	+1	0	X_d
L_L'	-1/2	2	1	+1	-1	X'
L_R'	-1/2	2	1	+i	2	X'-X_B
E_L'	-1	1	1	+i	2	X'-X_d-X_B
E_R'	-1	1	1	+1	-1	X'-X_d
N_L'	0	1	1	+1	2	X'-X_u-X_B
N_R'	0	1	1	-i	-1	X'-X_u
Φ_A	0	1	1	-1	-3	$-X_A$
Φ_B	0	1	1	+i	3	$-X_B$

DFSZ-like ALP with gauged baryon number

- For the given choice of $\mathbb{Z}_4$ charges we get a scalar Lagrangian

$$\mathcal{L}_{\text{scalar}} \supset (D_\mu H_u)^\dagger (D^\mu H_u) + (D_\mu H_d)^\dagger (D^\mu H_d) + (D_\mu \Phi_A)^\dagger (D^\mu \Phi_A) + (D_\mu \Phi_B)^\dagger (D^\mu \Phi_B)$$
$$- V\left(|H_u|^2, |H_d|^2, |\Phi_A|^2, |\Phi_B|^2\right) - \lambda_{AB}\left(H_u^T H_d \Phi_A \Phi_B + \text{h.c.}\right) \ .$$

- Last term sets PQ charges $X \equiv X_u + X_d = X_A + X_B$
- Use non-linear representation $\Phi_A = \frac{v_A + h_A}{\sqrt{2}} e^{i a_A / v_A}$, …
- Physical vevs defined by $f_a = X v_a$ and

$$(v/2)^2 = \sum_{\{\Phi_i\}} Y_i^2 v_i^2 \ , \quad (3v')^2 = \sum_{\{\Phi_i\}} B_i^2 v_i^2 \ , \quad (X v_a)^2 = \sum_{\{\Phi_i\}} X_i^2 v_i^2$$

DFSZ-like ALP with gauged baryon number

- Find mixing between vevs, angular modes and radial modes (Higgs-basis)

$$v_u = v\sin\beta\ ,\quad v_d = v\cos\beta\ ,\quad v_A = v'\sin\beta'\ ,\quad v_B = v'\cos\beta'$$

$$\begin{pmatrix} a \\ A_0 \\ G_0 \\ G_B \end{pmatrix} = \begin{pmatrix} c_\beta s_\gamma & s_\beta s_\gamma & -c_{\beta'} c_\gamma & -s_{\beta'} c_\gamma \\ c_\beta c_\gamma & s_\beta c_\gamma & c_{\beta'} s_\gamma & s_{\beta'} s_\gamma \\ -s_\beta & c_\beta & 0 & 0 \\ 0 & 0 & -s_{\beta'} & c_{\beta'} \end{pmatrix} \begin{pmatrix} a_u \\ a_d \\ a_A \\ a_B \end{pmatrix} \qquad \begin{pmatrix} h \\ H_0 \\ h' \\ H'_0 \end{pmatrix} = \begin{pmatrix} s_\beta & c_\beta & 0 & 0 \\ c_\beta & -s_\beta & 0 & 0 \\ 0 & 0 & s_{\beta'} & c_{\beta'} \\ 0 & 0 & c_{\beta'} & -s_{\beta'} \end{pmatrix} \begin{pmatrix} h_u \\ h_d \\ h_A \\ h_B \end{pmatrix}$$

- Angle γ defined by

$$\sqrt{v_A^2\cos^4\beta' + v_B^2\sin^4\beta'} = v'\sin\beta'\cos\beta' = v_a\cos\gamma$$

$$\sqrt{v_u^2\cos^4\beta + v_d^2\sin^4\beta} = v\sin\beta\cos\beta = v_a\sin\gamma$$

DFSZ-like ALP with gauged baryon number

- The Yukawa Lagrangian contains

$$\mathcal{L}_{\text{Yukawa}} \supset - y_u^{ij} \bar{Q}_L^i H_u u_R^j - y_d^{ij} \bar{Q}_L^i H_d d_R^j - y_e^{ij} \bar{L}_L^i H_d e_R^j$$
$$- y_L \bar{L}'_R \Phi_B L'_L - y_E \bar{E}'_L \Phi_B E'_R - y_N \bar{N}'_L \Phi_B N'_R$$
$$- y_1 \bar{L}'_L H_d E'_R - y_2 \bar{L}'_R H_d E'_L - y_3 \bar{L}'_L H_u N'_R - y_4 \bar{L}'_R H_u N'_L + \text{h.c.}$$

- Have two CP violating phases in anomalon sector

$$\mathcal{L}_{\text{anom}} \supset - |y_L| \bar{L}'_R \Phi_B L'_L - |y_E| \bar{E}'_L \Phi_B E'_R - |y_1| e^{i\delta_{12}} \bar{L}'_L H_d E'_R - |y_2| e^{-i\delta_{12}} \bar{L}'_R H_d E'_L$$
$$- |y_N| \bar{N}'_L \Phi_B N'_R - |y_3| e^{i\delta_{34}} \bar{L}'_L H_u N'_R - |y_4| e^{-i\delta_{34}} \bar{L}'_R H_u N'_L + \text{h.c.},$$

DFSZ-like ALP with gauged baryon number

- Couplings to H_u and H_d induce mixing between anomalons

$$\mathcal{L}_{\text{anom,m}} \supset -\begin{pmatrix}\bar{e}'_L & \bar{E}'_L\end{pmatrix}\begin{pmatrix} m_L & m_{12}(1+\Delta_{12})(1+it_{12}) \\ m_{12}(1-\Delta_{12})(1-it_{12}) & m_E \end{pmatrix}\begin{pmatrix} e'_R \\ E'_R \end{pmatrix} - \begin{pmatrix}\bar{\nu}'_L & \bar{N}'_L\end{pmatrix}\begin{pmatrix} m_L & m_{34}(1+\Delta_{34})(1+it_{34}) \\ m_{34}(1-\Delta_{34})(1-it_{34}) & m_N \end{pmatrix}\begin{pmatrix} \nu'_R \\ N'_R \end{pmatrix} + \text{h.c.}$$

$$m_{ij} = \frac{m_i + m_j}{2}$$

$$\Delta_{ij} = \frac{m_i - m_j}{m_i + m_j}$$

$$t_{ij} = \tan\delta_{ij}$$

- Assuming $\Delta_{12} = 0 = \Delta_{34}$ and $t_{12} = 0 = t_{34}$ we can define

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha_E & -\sin\alpha_E \\ \sin\alpha_E & \cos\alpha_E \end{pmatrix}\begin{pmatrix} e' \\ E' \end{pmatrix}, \quad \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha_N & -\sin\alpha_N \\ \sin\alpha_N & \cos\alpha_N \end{pmatrix}\begin{pmatrix} \nu' \\ N' \end{pmatrix}$$

DFSZ-like ALP with gauged baryon number

- This induces flavor-violating ALP couplings to fermions

$$\mathcal{L}_{\text{anom},a}^{d\leq 4} \supset + iX_B \frac{a}{f_a}\cos(2\alpha_E)(m_{E_1}\bar{E}_1\gamma_5 E_1 - m_{E_2}\bar{E}_2\gamma_5 E_2) + iX_B\frac{a}{f_a}\sin(2\alpha_E)\frac{m_{E_1}+m_{E_2}}{2}(\bar{E}_1\gamma_5 E_2 + \bar{E}_2\gamma_5 E_1)$$
$$+ iX_B \frac{a}{f_a}\cos(2\alpha_N)(m_{N_1}\bar{N}_1\gamma_5 N_1 - m_{N_2}\bar{N}_2\gamma_5 N_2) + iX_B\frac{a}{f_a}\sin(2\alpha_N)\frac{m_{N_1}+m_{N_2}}{2}(\bar{N}_1\gamma_5 N_2 + \bar{N}_2\gamma_5 N_1)$$
$$+ iX_d \frac{a}{f_a}\sin(2\alpha_E)\frac{m_{E_1}-m_{E_2}}{2}(\bar{E}_1 E_2 - \bar{E}_2 E_1) + iX_u\frac{a}{f_a}\sin(2\alpha_N)\frac{m_{N_1}-m_{N_2}}{2}(\bar{N}_1 N_2 - \bar{N}_2 N_1)\ .$$

- Can be generalized to

$$\mathcal{L}_{\psi,m} \supset -\bar{\psi}\mathbf{M}\psi + i\frac{a}{f_a}\bar{\psi}\{\mathbf{M},\mathbf{X}_A\}\gamma_5\psi + i\frac{a}{f_a}\bar{\psi}[\mathbf{M},\mathbf{X}_V]\psi + \mathcal{O}\left(\frac{1}{f_a^2}\right)$$
$$= -\bar{\psi}\exp\left(i(\mathbf{X}_V - \mathbf{X}_A\gamma_5)\frac{a}{f_a}\right)\mathbf{M}\exp\left(-i(\mathbf{X}_V + \mathbf{X}_A\gamma_5)\frac{a}{f_a}\right)\psi + \mathcal{O}\left(\frac{1}{f_a^2}\right)$$

$$\mathbf{X}_{V,E} = \frac{X_d}{2}\begin{pmatrix}\cos(2\alpha_E) & \sin(2\alpha_E)\\ \sin(2\alpha_E) & -\cos(2\alpha_E)\end{pmatrix}$$

$$\mathbf{X}_{A,E} = \frac{X_B}{2}\begin{pmatrix}\cos(2\alpha_E) & \sin(2\alpha_E)\\ \sin(2\alpha_E) & -\cos(2\alpha_E)\end{pmatrix}$$

Flavor-violating ALP couplings

- Set up general interaction Lagrangian with gauge groups indexed by I and real scalar fields indexed by K, in Higgs basis, aligned to vev v_K

$$
\begin{aligned}
\mathcal{L}_\psi \supset & i\bar{\psi}\gamma^\mu\partial_\mu\psi + \sum_I g_I \bar{\psi} A_\mu^I \gamma^\mu (\mathbf{Q}_{IV} + \mathbf{Q}_{IA}\gamma_5)\psi \\
& - \bar{\psi}\exp\left(i(\mathbf{X}_V - \mathbf{X}_A\gamma_5)\frac{a}{f_a}\right)\mathbf{M}\exp\left(-i(\mathbf{X}_V + \mathbf{X}_A\gamma_5)\frac{a}{f_a}\right)\psi \\
& - \sum_K \phi_K \bar{\psi}\exp\left(i\mathbf{X}_V\frac{a}{f_a}\right)\mathbf{Y}_K\exp\left(-i\mathbf{X}_V\frac{a}{f_a}\right)\psi + \mathcal{O}\left(\frac{1}{f_a^2}\right)
\end{aligned}
$$

- Perform fermion transformation to find operator basis

$$
\psi \to \exp\left(i(\mathbf{X}_V + \mathbf{X}_A\gamma_5)\frac{a}{f_a}\right)\psi, \quad \bar{\psi} \to \bar{\psi}\exp\left(-i(\mathbf{X}_V - \mathbf{X}_A\gamma_5)\frac{a}{f_a}\right)
$$

Flavor-violating ALP couplings

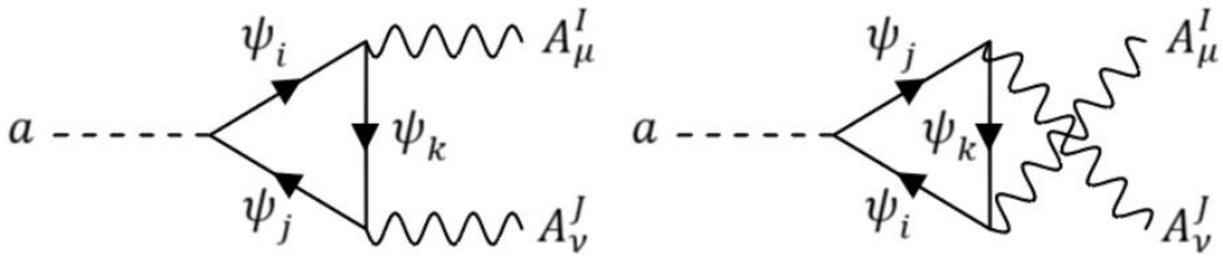
- Find generalized ALP interaction basis

$$\mathcal{L}_a^{d\leq 5} \supset -\frac{\partial_\mu a}{2f_a}\bar{\psi}\gamma^\mu(\mathbf{C}_{1V}+\mathbf{C}_{1A}\gamma_5)\psi + \frac{i}{2}\frac{a}{f_a}\bar{\psi}[\mathbf{M},\mathbf{C}_{2V}]\psi + \frac{i}{2}\frac{a}{f_a}\bar{\psi}\{\mathbf{M},\mathbf{C}_{2A}\}\gamma_5\psi$$
$$+\frac{i}{2}\frac{a}{f_a}\sum_K \phi_K\bar{\psi}[\mathbf{Y}_K,\mathbf{C}_{KV}]\psi + \frac{i}{2}\frac{a}{f_a}\sum_K \phi_K\bar{\psi}\{\mathbf{Y}_K,\mathbf{C}_{KA}\}\gamma_5\psi$$
$$+\underbrace{\frac{i}{2}\frac{a}{f_a}\sum_I g_I\bar{\psi}A_\mu^I\gamma^\mu[\mathbf{Q}_{IV}+\mathbf{Q}_{IA}\gamma_5,\mathbf{C}_{IV}+\mathbf{C}_{IA}\gamma_5]\psi}_{} + \frac{a}{f_a}\sum_{I,J} C_3^{IJ}\frac{g_I g_J}{(4\pi)^2}F^a_{I\mu\nu}\tilde{F}_J^{a,\mu\nu}$$

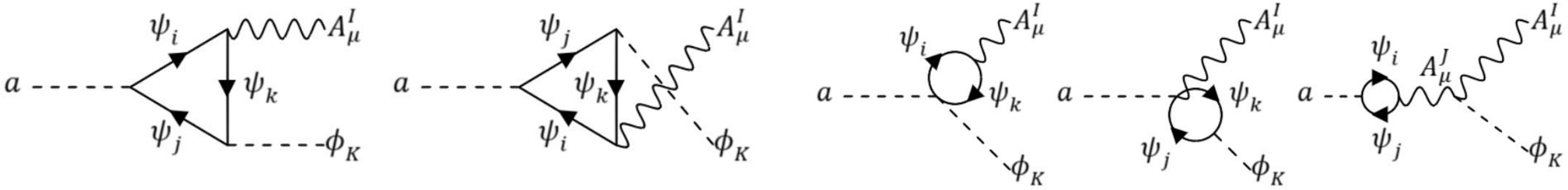
Only appears for flavor-violating couplings, but also for the W interaction in DFSZ, since $X_u \neq X_d$

Flavor-violating ALP couplings

- Can now evaluate Wilson coefficients for an ALP coupling to two gauge bosons



- and for an ALP coupling to one gauge boson and one real scalar field



DFSZ-like ALP with gauged baryon number

- The resulting ALP Lagrangian at dim-5 reads

$$\mathcal{L}_{\text{axion}}^{d\leq 5} \supset + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \frac{\partial_\mu a}{2f_a}J^\mu_{\text{PQ}} + C_{\gamma\gamma}\frac{e^2}{(4\pi)^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ C_{ZZ}\frac{e^2}{s_W^2 c_W^2}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{Z\gamma}\frac{e^2}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ C_{Z'Z'}\frac{g_B^2}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}'^{\mu\nu} + C_{Z'\gamma}\frac{g_B e}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{F}^{\mu\nu} + C_{Z'Z}\frac{g_B e}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}^{\mu\nu}$$
$$+ C_{WW}\frac{g_L^2}{(4\pi)^2}\frac{a}{f_a}W_{\mu\nu}\tilde{W}^{\mu\nu} + C_{gg}\frac{g_s^2}{(4\pi)^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - C_{Zh}hZ_\mu\partial^\mu a - C_{Z'h}hZ'_\mu\partial^\mu a$$
$$+ i\frac{a}{f_a}\frac{e}{\sqrt{2}s_W}(W_\mu^-(X_d J_W^{+\mu} - X_u J_{W,l}^{+\mu}) + \text{h.c.}) + \mathcal{O}(h', H_0, H_0', A_0)\ .$$

DFSZ-like ALP with gauged baryon number

- Have new operators involving the Z'

$$\mathcal{L}_{\text{axion}}^{d\leq 5} \supset + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \frac{\partial_\mu a}{2f_a} J^\mu_{\text{PQ}} + C_{\gamma\gamma}\frac{e^2}{(4\pi)^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$+ C_{ZZ}\frac{e^2}{s_W^2 c_W^2}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{Z\gamma}\frac{e^2}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$+ C_{Z'Z'}\frac{g_B^2}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}'^{\mu\nu} + C_{Z'\gamma}\frac{g_B e}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{F}^{\mu\nu} + C_{Z'Z}\frac{g_B e}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}^{\mu\nu}$$

$$+ C_{WW}\frac{g_L^2}{(4\pi)^2}\frac{a}{f_a}W_{\mu\nu}\tilde{W}^{\mu\nu} + C_{gg}\frac{g_s^2}{(4\pi)^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - C_{Zh}hZ_\mu\partial^\mu a - C_{Z'h}hZ'_\mu\partial^\mu a$$

$$+ i\frac{a}{f_a}\frac{e}{\sqrt{2}s_W}(W^-_\mu(X_d J^{+\mu}_W - X_u J^{+\mu}_{W,l}) + \text{h.c.}) + \mathcal{O}(h', H_0, H'_0, A_0)\ .$$

DFSZ-like ALP with gauged baryon number

- Have operators involving the Higgs boson

$$
\begin{aligned}
\mathcal{L}_{\text{axion}}^{d\leq 5} \supset & + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \frac{\partial_\mu a}{2 f_a} J^\mu_{\text{PQ}} + C_{\gamma\gamma}\frac{e^2}{(4\pi)^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} \\
& + C_{ZZ}\frac{e^2}{s_W^2 c_W^2}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{Z\gamma}\frac{e^2}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{F}^{\mu\nu} \\
& + C_{Z'Z'}\frac{g_B^2}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}'^{\mu\nu} + C_{Z'\gamma}\frac{g_B e}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{F}^{\mu\nu} + C_{Z'Z}\frac{g_B e}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}^{\mu\nu} \\
& + C_{WW}\frac{g_L^2}{(4\pi)^2}\frac{a}{f_a}W_{\mu\nu}\tilde{W}^{\mu\nu} + C_{gg}\frac{g_s^2}{(4\pi)^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - C_{Zh} h Z_\mu \partial^\mu a - C_{Z'h} h Z'_\mu \partial^\mu a \\
& + i\frac{a}{f_a}\frac{e}{\sqrt{2}s_W}\left(W^-_\mu\left(X_d J^{+\mu}_W - X_u J^{+\mu}_{W,\ell}\right) + \text{h.c.}\right) + \mathcal{O}(h', H_0, H'_0, A_0)\ .
\end{aligned}
$$

DFSZ-like ALP with gauged baryon number

- Have an operator from commutator interaction

$$\mathcal{L}^{d\leq 5}_{\text{axion}} \supset +\frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \frac{\partial_\mu a}{2f_a}J^\mu_{\text{PQ}} + C_{\gamma\gamma}\frac{e^2}{(4\pi)^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ C_{ZZ}\frac{e^2}{s_W^2 c_W^2}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{Z\gamma}\frac{e^2}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ C_{Z'Z'}\frac{g_B^2}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}'^{\mu\nu} + C_{Z'\gamma}\frac{g_B e}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{F}^{\mu\nu} + C_{Z'Z}\frac{g_B e}{s_W c_W}\frac{1}{(4\pi)^2}\frac{a}{f_a}Z'_{\mu\nu}\tilde{Z}^{\mu\nu}$$
$$+ C_{WW}\frac{g_L^2}{(4\pi)^2}\frac{a}{f_a}W_{\mu\nu}\tilde{W}^{\mu\nu} + C_{gg}\frac{g_s^2}{(4\pi)^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - C_{Zh}hZ_\mu\partial^\mu a - C_{Z'h}hZ'_\mu\partial^\mu a$$
$$+ i\frac{a}{f_a}\frac{e}{\sqrt{2}s_W}(W^-_\mu(X_d J^{+\mu}_W - X_u J^{+\mu}_{W,\not{l}}) + \text{h.c.}) + \mathcal{O}(h', H_0, H'_0, A_0)\ .$$

DFSZ-like ALP with gauged baryon number

- Wilson coefficients are not independent
- Set by properties of the anomalons

$$\Sigma_E = \frac{m_{E_1} + m_{E_2}}{f_a} \qquad \Delta_E = \frac{m_{E_1} - m_{E_2}}{f_a}$$

- ALP-diphoton coupling suppressed by mass difference of anomalons
- Most ALP-Z' couplings suppressed by kinetic mixing
- For ALP-Higgs couplings the coefficient for Z Higgs is suppressed

$$C_{\gamma\gamma} = -\frac{8}{3} X_B \frac{\Delta_E}{\Sigma_E^3} \cos(2\alpha_E) \frac{m_a^2}{f_a^2} + \mathcal{O}\left(\frac{1}{f_a^3}, \Delta_E^2\right),$$

$$C_{\gamma Z} = -\frac{X_B}{4} + \mathcal{O}\left(\frac{1}{f_a}, \Delta_E^2\right),$$

$$C_{\gamma Z'} = +\frac{X_B}{4} \frac{\epsilon_{\text{eff}} e}{g_B c_W} \frac{m_{Z'}^2}{m_{Z'}^2 - m_Z^2} + \mathcal{O}\left(\frac{1}{f_a}, \epsilon_{\text{eff}}^2, \Delta_E^2\right),$$

$$C_{ZZ} = -\frac{X_B}{4}(1 - 2s_W^2) + \mathcal{O}\left(\frac{1}{f_a}, \epsilon_{\text{eff}}^2, \Delta_E^2, \Delta_N^2\right),$$

$$C_{ZZ'} = +\frac{X_B}{4}(1 - 2s_W^2) \frac{\epsilon_{\text{eff}} e}{g_B c_W} \frac{m_{Z'}^2}{m_{Z'}^2 - m_Z^2} + \mathcal{O}\left(\frac{1}{f_a}, \epsilon_{\text{eff}}^2, \Delta_E^2, \Delta_N^2\right),$$

$$C_{Z'Z'} = -\frac{X_B}{4}(1 - 2s_W^2) \frac{\epsilon_{\text{eff}}^2 e^2}{g_B^2 c_W^2} \frac{m_{Z'}^4}{(m_{Z'}^2 - m_Z^2)^2} + \mathcal{O}\left(\frac{1}{f_a}, \epsilon_{\text{eff}}^3, \Delta_E^2, \Delta_N^2\right),$$

$$C_{WW} = -\frac{X_B}{2} + \mathcal{O}\left(\frac{1}{f_a}, \Delta_E^2, \Delta_N^2, \Delta_{EN}^2\right),$$

$$C_{hZ} = +\frac{X_B}{2} \frac{g_B \epsilon_{\text{eff}} s_W^2 c_W}{e} \frac{m_Z^2}{m_Z^2 - m_{Z'}^2} \frac{v}{f_a} \Sigma_M^2 \left(1 - 6\frac{m_Z^2(m_Z^2 - m_a^2 - m_h^2)}{\lambda(m_Z^2, m_a^2, m_h^2)}\right) + \mathcal{O}\left(\frac{1}{f_a^2}, \epsilon_{\text{eff}}^2, \Delta_E, \Delta_N, \Delta_M\right),$$

$$C_{hZ'} = -\frac{X_B}{2} \frac{v}{f_a} \Sigma_M^2 \left(1 - 6\frac{m_{Z'}^2(m_{Z'}^2 - m_a^2 - m_h^2)}{\lambda(m_{Z'}^2, m_a^2, m_h^2)}\right) + \mathcal{O}\left(\frac{1}{f_a^2}, \epsilon_{\text{eff}}^2, \Delta_E, \Delta_N, \Delta_M\right)$$

DFSZ-like ALP with gauged baryon number

- Get an f_a dependence for Z' mass and anomalon masses

$$\Sigma_E = \frac{m_{E_1} + m_{E_2}}{f_a}$$

$$\Delta_E = \frac{m_{E_1} - m_{E_2}}{f_a}$$

$$m_{Z'} = 3 g_B v' = \frac{3 c_\gamma g_B f_a}{X s_{\beta'} c_{\beta'}} \,, \qquad \frac{\Sigma_{E,N} \pm \Delta_{E,N}}{2} f_a \approx \frac{f_a}{X s_{\beta'} \sqrt{2}} \equiv m_{\text{anom}}$$

- Have a limit on f_a from L3 and ALEPH bounds on anomalon mass

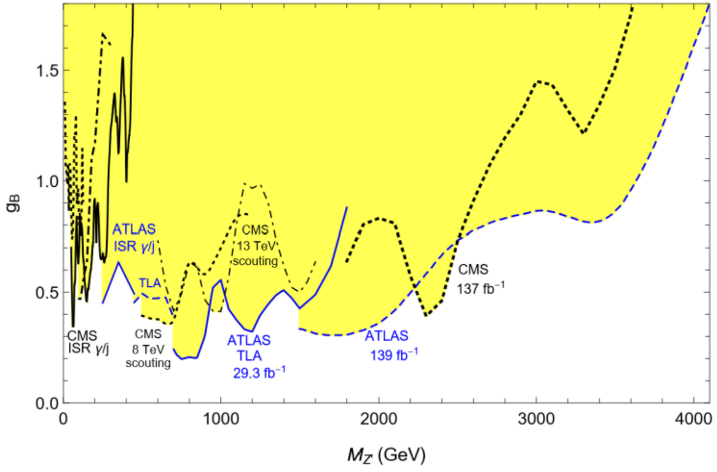
$$m_{E_1} = \frac{|y_L| + |y_E|}{2} \frac{c_{\beta'} v'}{\sqrt{2}} - \frac{|y_1| + |y_2|}{2} \frac{c_\beta v}{\sqrt{2}} \sqrt{1 + \cot(2\alpha_E)^2} > 90\text{GeV}$$

$$\Leftrightarrow \quad f_a^2 > X^2 v^2 s_\beta^2 c_\beta^2 + X^2 s_{\beta'}^2 \left(\frac{2\sqrt{2} \cdot 90\text{GeV}}{|y_L| + |y_E|} + \frac{|y_1| + |y_2|}{|y_L| + |y_E|} c_\beta v \sqrt{1 + \cot(2\alpha_E)^2} \right)^2$$

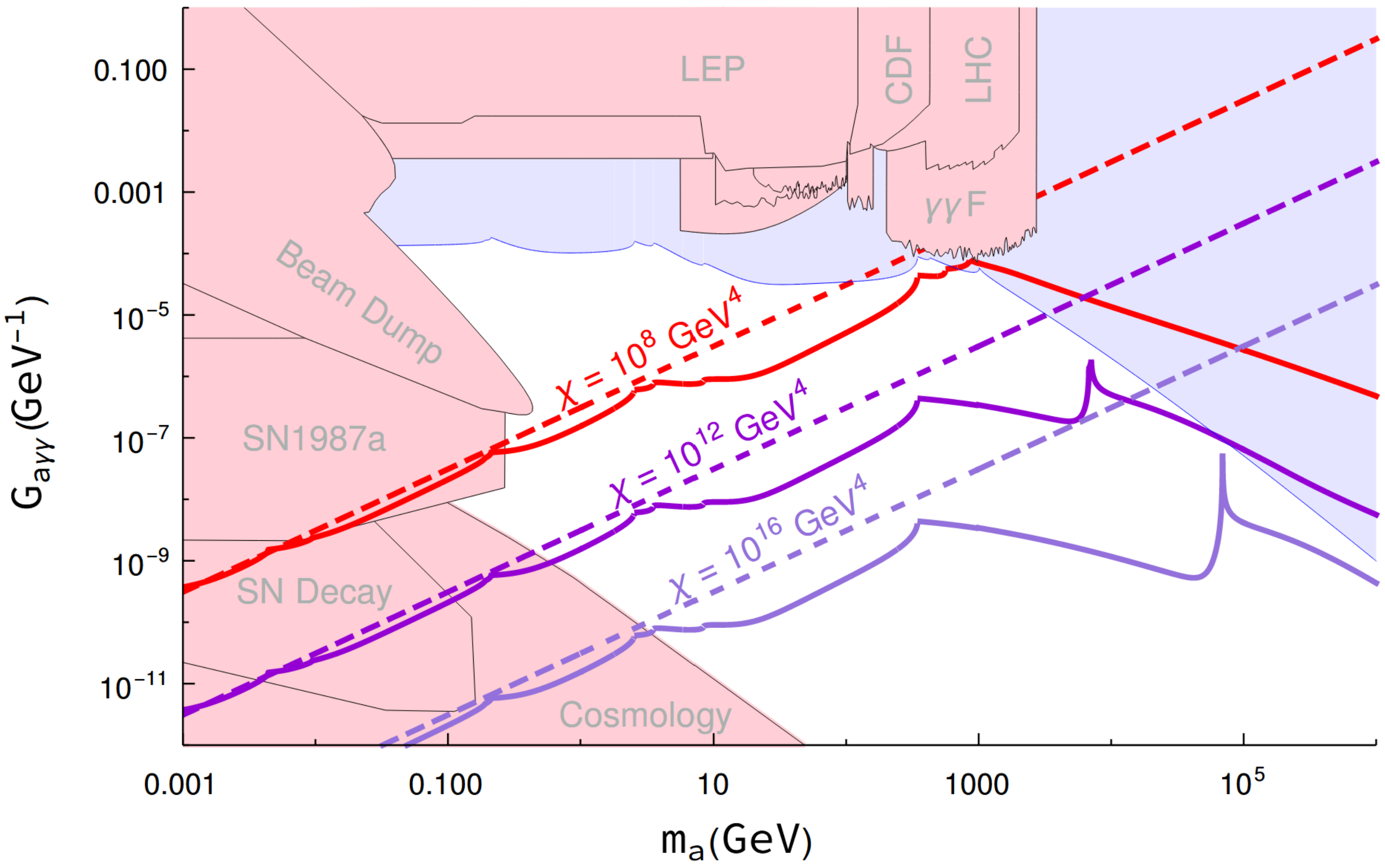
- Corresponds to $f_a \gtrsim$ 15 GeV

- Anomalons decouple for $m_{Z'} < m_{\text{anom}} \Leftrightarrow g_B < \dfrac{c_{\beta'}}{3\sqrt{2} c_\gamma} \approx \dfrac{1}{6}$

[Dobrescu, Yu, 2112.04392]

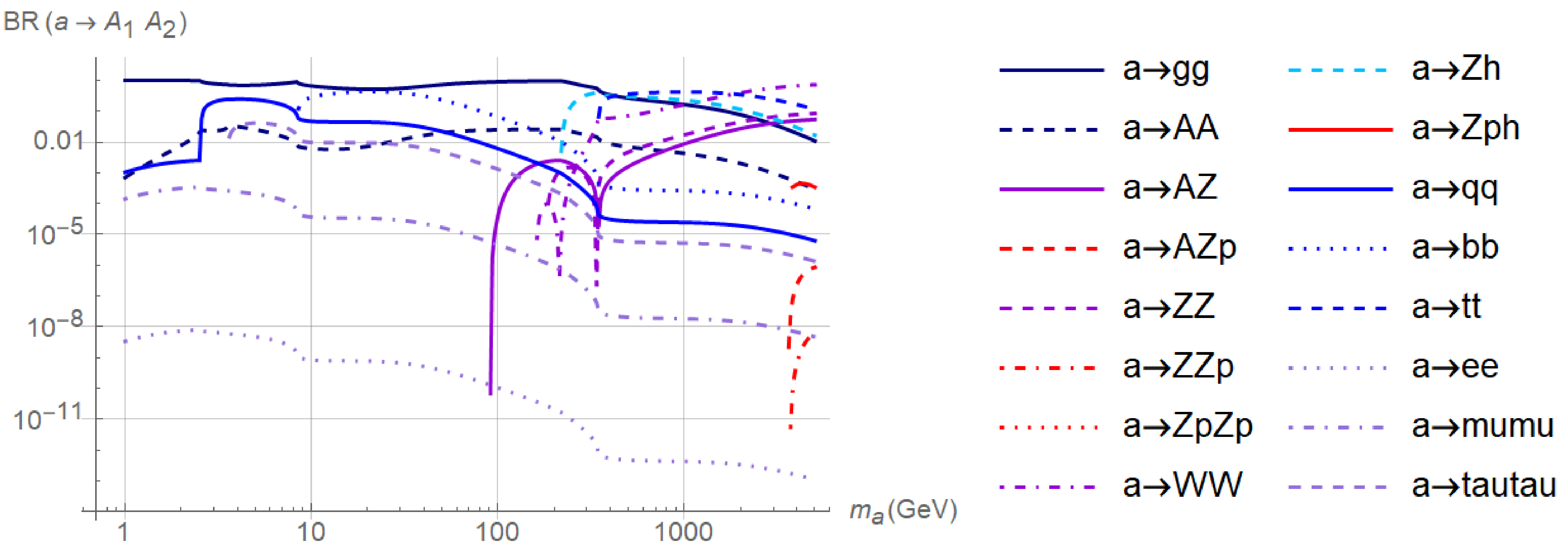


DFSZ-like ALP with gauged baryon number



DFSZ-like ALP with gauged baryon number

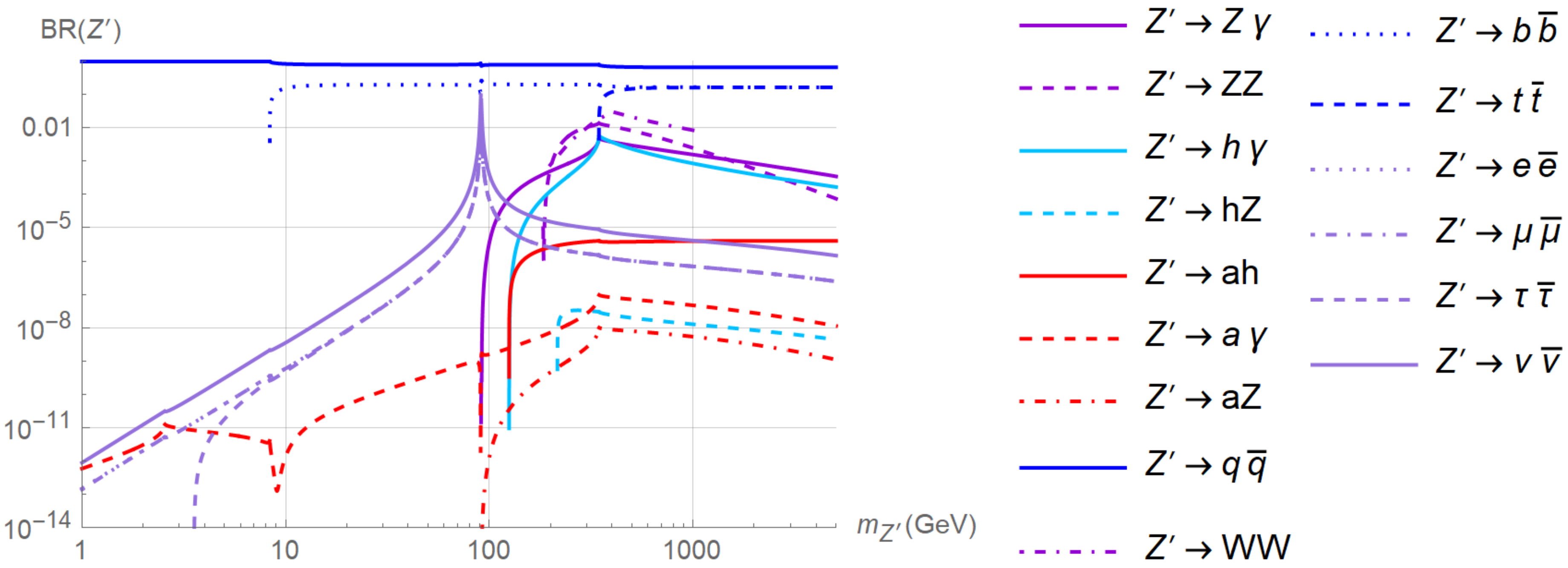
- ALP branching ratios for f_a=250 GeV, g_B=0.1, β=π/4, β'=π/4 ⇒ $m_{Z'}$=3.6 TeV



- In Progress: identify testable cross sections, apply to LHC measurements

DFSZ-like ALP with gauged baryon number

- Z' branching ratios for f_a=1 TeV, m_a=100 MeV, β=π/4, β'=π/4



- In Progress: identify testable cross sections, apply to LHC measurements

QCD Axion with small size instanton effects

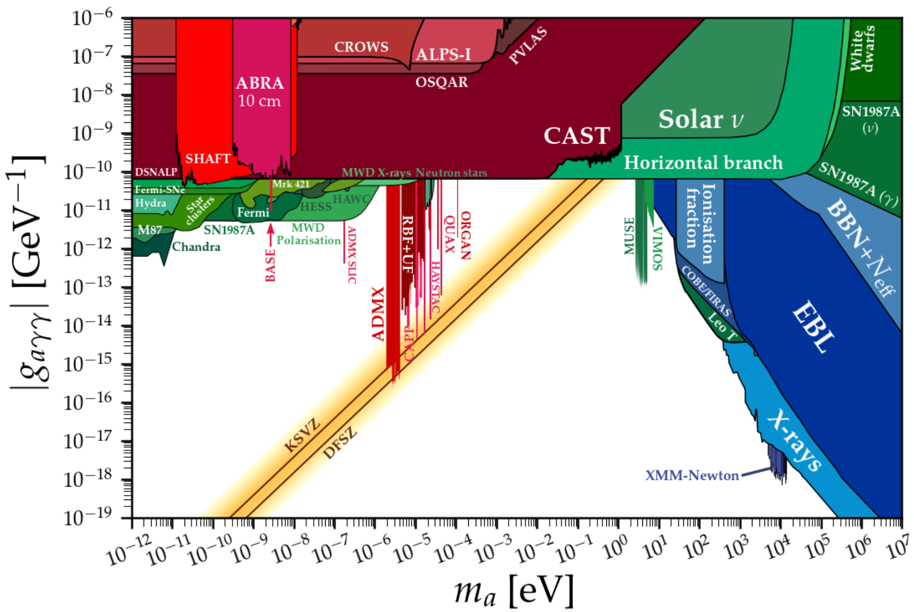
[Kivel, JL, Yu, 2207.08740]

QCD Axion with small size instanton effects

- In contrast to before, let's fix the topological susceptibility for having a QCD axion

[O'Hare, 2020]

$$\chi = m_a^2 f_a^2, \quad \chi_{\rm QCD} = \frac{m_u m_d}{(m_u + m_d)^2} m_\pi^2 f_\pi^2$$



- For standard KSVZ or DFSZ model excluded for m_a>1 eV
- Question: Can we change $\chi_{\rm QCD}$ to find the QCD axion at colliders?
- Several proposals to use small size instantons as in [Gaillard et al, 1805.06465]
- Involves a confining SU(N) group at a higher scale
- We construct a new framework to calculate the small size instanton effects

Axion-Meson mixing

- ❑ Want to find a mass matrix for mesons and axion
- ❑ Need to consider QCD confinement and chiral symmetry breaking
- ❑ QCD becomes strongly coupled at Λ_{QCD}
- ❑ Quarks form a condensate $\langle \bar{q} q \rangle \equiv v^3$ roughly at $v \sim \Lambda_{QCD}$
- ❑ For two flavors, condensate breaks chiral symmetry $SU(2)_L \times SU(2)_R \to SU(2)_{\text{Isospin}}$
- ❑ Get a chiral effective Lagrangian with pions as Goldstone bosons

$$\mathcal{L}_{\text{ChEFT}} = \frac{1}{4} F_\pi^2 \, \text{Tr}\left(D^\mu \Sigma D_\mu \Sigma^\dagger \right) + \frac{1}{2} F_\pi^2 \mu \, \text{Tr}\left(\Sigma M \right) + \text{h.c.} \qquad \Sigma = \exp(2 i \pi^a t^a / F_\pi)$$

Axion-Meson mixing

- Meson masses determined by quark condensate

$$\bar{u}_L u_R \approx |\langle \bar{u}_L u_R \rangle| \exp\big(i(\theta_{\pi^0} + \theta_{\eta'})\big) = \frac{v^3}{2} \exp\big(i(\theta_{\pi^0} + \theta_{\eta'})\big) \ ,$$

$$\bar{d}_L d_R \approx |\langle \bar{d}_L d_R \rangle| \exp\big(i(-\theta_{\pi^0} + \theta_{\eta'})\big) = \frac{v^3}{2} \exp\big(i(-\theta_{\pi^0} + \theta_{\eta'})\big) \ ,$$

$$\bar{s}_L s_R \approx |\langle \bar{s}_L s_R \rangle| \exp\big(i\theta_{\eta'}\big) = \frac{v^3}{2} \exp\big(i\theta_{\eta'}\big) \sim \frac{v^3}{2} \ ,$$

- Adding direct axion-fermion couplings the potential for the axions and mesons reads

$$\mathcal{L} \supset -m_u \bar{u}_L e^{ic_2^u \frac{a}{F_a}} u_R - m_d \bar{d}_L e^{ic_2^d \frac{a}{F_a}} d_R - m_s \bar{s}_L e^{ic_2^s \frac{a}{F_a}} s_R + \text{ h.c.}$$

$$\approx -m_u v^3 \cos\big(\theta_{\pi^0} + \theta_{\eta'} + c_2^u \theta_a\big) - m_d v^3 \cos\Big(-\theta_{\pi^0} + \theta_{\eta'} + c_2^d \theta_a\Big) - m_s v^3 \cos\big(\theta_{\eta'} + c_2^s \theta_a\big)$$

Axion-Meson mixing

- Instanton effects encoded by ‘t Hooft determinantal operator [‘t Hooft, Phys. Rept. 142, 357 (1986)]

- Schematically with complex mass matrix $\tilde{M}$

$$\mathrm{Tr}(\Sigma\tilde{M}) \Leftrightarrow \tilde{M}\bar{Q}Q \qquad\qquad \theta\langle G\tilde{G}\rangle = Ke^{-i\theta}\mathrm{Det}(\bar{Q}Q) \Leftrightarrow Ke^{-i\theta}\mathrm{Det}(\Sigma)$$

- K is instanton amplitude

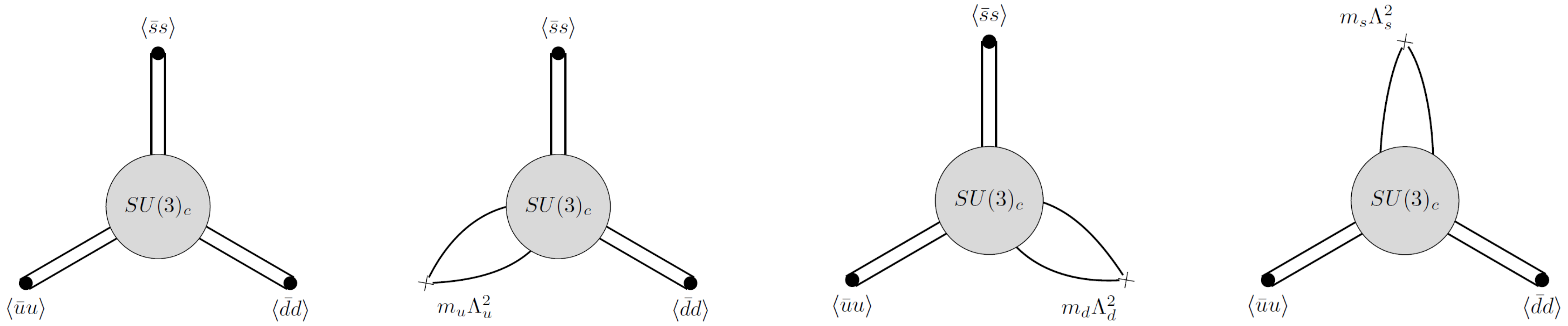
$$K^{4-N_f} \sim \int \frac{\mathrm{d}\rho}{\rho^{5-N_f}} \exp\left[\frac{-2\pi}{\alpha(\mu)}\right] \qquad\qquad \mu = 1/\rho$$

- More explicitly we have

$$-\frac{a}{F_a} c_3^G \frac{g_s^2}{32\pi^2} G\tilde{G} \qquad \Leftrightarrow \qquad \mathcal{L}_{\det} = (-1)^{N_f} K^{4-3N_f} \left(\prod_{i=1}^{N_f} \det(\bar{q}_L^i q_R^i)\right) e^{-ic_3^G \frac{a}{F_a}} + \mathrm{h.c.}$$

Axion-Meson mixing

- Find potential by using instanton flower diagrams



- The contributions to axion and meson potentials are

$$\mathcal{L} \supset -m_u v^3 \cos\left(\theta_{\pi^0} + \theta_{\eta'} + c_2^u \theta_a\right) - m_d v^3 \cos\left(-\theta_{\pi^0} + \theta_{\eta'} + c_2^d \theta_a\right) - \frac{v^9}{4K^5} \cos\left(2\theta_{\eta'} - c_3^G \theta_a\right)$$
$$- \frac{v^6}{2K^5} m_u \Lambda_u^2 \cos\left(\theta_{\pi^0} + \theta_{\eta'} - c_3^G \theta_a\right) - \frac{v^6}{2K^5} m_d \Lambda_d^2 \cos\left(-\theta_{\pi^0} + \theta_{\eta'} - c_3^G \theta_a\right) \; ,$$

Axion-Meson mixing

- Defines mass matrix $\mathcal{L} = \frac{1}{2}\left(a \quad \eta' \quad \pi^0\right) M^2 \left(a \quad \eta' \quad \pi^0\right)^T$
- Mass matrix elements given by

$$
\begin{aligned}
\mathcal{L} \supset\ & \frac{1}{2}\theta_a^2 \left(v^3 (m_u (c_2^u)^2 + m_d (c_2^d)^2) + (c_3^G)^2 \left(\frac{v^9}{4K^5} + \frac{v^6}{2K^5} m_u \Lambda_u^2 + \frac{v^6}{2K^5} m_d \Lambda_d^2 \right) \right) \\
& + \frac{1}{2}\theta_{\eta'}^2 \left(m_u v^3 + m_d v^3 + \frac{v^9}{K^5} + \frac{v^6}{2K^5} m_u \Lambda_u^2 + \frac{v^6}{2K^5} m_d \Lambda_d^2 \right) \\
& + \frac{1}{2}\theta_{\pi^0}^2 \left(m_u v^3 + m_d v^3 + \frac{v^6}{2K^5} m_u \Lambda_u^2 + \frac{v^6}{2K^5} m_d \Lambda_d^2 \right) \\
& + \theta_a \theta_{\eta'} \left(v^3 (m_u c_2^u + m_d c_2^d) - c_3^G \left(\frac{v^9}{2K^5} + \frac{v^6}{2K^5} m_u \Lambda_u^2 + \frac{v^6}{2K^5} m_d \Lambda_d^2 \right) \right) \\
& + \theta_a \theta_{\pi^0} \left(v^3 (m_u c_2^u - m_d c_2^d) + c_3^G \left(-\frac{v^6}{2K^5} m_u \Lambda_u^2 + \frac{v^6}{2K^5} m_d \Lambda_d^2 \right) \right) \\
& + \theta_{\eta'} \theta_{\pi^0} \left(m_u v^3 - m_d v^3 + \frac{v^6}{2K^5} m_u \Lambda_u^2 - \frac{v^6}{2K^5} m_d \Lambda_d^2 \right) \ .
\end{aligned}
$$

Axion-Meson mixing

- Pion mass and η‘ mass constrain

$$v = 336.3\ \text{MeV}\ , \quad \Lambda_{\eta'} = 239.3\ \text{MeV}\ , \quad \Lambda_{\text{inst}} = 261.7\ \text{MeV}$$

$$\Rightarrow K = 582.6\ \text{MeV}, \quad L = 1289.5\ \text{MeV}\ ,$$

- Cross check: Mass for the KSVZ axion

$$(m_a^2 F_a^2)^{\text{KSVZ}} = \Lambda_{\eta'}^4 + 2\mu\Lambda_{\text{inst}}^3 - \frac{(2\Lambda_{\eta'}^4 + 2\mu\Lambda_{\text{inst}}^3)^2(m_+ v^3 + 2\mu\Lambda_{\text{inst}}^3)}{F_{\pi^0}^2 m_{\pi^0}^2 F_{\eta'}^2 m_{\eta'}^2}\ ,$$

$$\Rightarrow m_a^{\text{KSVZ}} = 8.4\ \mu\text{eV}\frac{10^{12}\ \text{GeV}}{F_a}\ ,$$

- Cross check: Mass for the DFSZ axion

$$(m_a^2 F_a^2)^{\text{DFSZ}} = 2c_2^2(2\Lambda_{\eta'}^4 + \mu\Lambda_{\text{inst}}^3) - \frac{4c_2^2(2\Lambda_{\eta'}^4 + \mu\Lambda_{\text{inst}}^3)^2(m_+ v^3 + 2\mu\Lambda_{\text{inst}}^3)}{F_{\pi^0}^2 m_{\pi^0}^2 F_{\eta'}^2 m_{\eta'}^2}\ .$$

$$\Rightarrow m_a^{\text{DFSZ}} = 15\mu\text{eV}\frac{10^{12}\text{GeV}}{F_a}\ ,$$

QCD Axion with small size instanton effects

- Application to model in [Gaillard et al, 1805.06465] which contains additional SU(N) symmetry

- Add two new massless fermions and add successive symmetry breaking at two scales

	$SU(3)_{\rm diag}$	$SU(3)_c$	$SU(3')$	$SU(6)$
Q	$\bar{\square}$	$\square$	1	20
q	$\square$	1	$\square$	1

$$SU(6) \times SU(3') \xrightarrow{\Lambda_{\rm CUT}} SU(3)_c \times SU(3)_{\rm diag} \xrightarrow{v_{\rm diag}} SU(3)_c$$

- Lagrangian contains

$$\begin{aligned}
\mathcal{L} \supset\; & \bar{Q}_{I,i} \left(i\delta_{IJ}\; \delta_{ij}\; \partial\!\!\!/ - g_{\rm diag}\; T^A_{IJ} A\!\!\!/^A_{\rm diag} \delta_{ij} - g_s\; \delta_{IJ} T^a_{ij} A\!\!\!/^a \right) Q_{J,j} \\
& + \bar{q}_{I,i'} \left(i\delta_{IJ}\; \delta_{i'j'} \partial\!\!\!/ - g_{\rm diag}\; T^A_{IJ} A\!\!\!/^A_{\rm diag} \delta_{i'j'} - g'\; \delta_{IJ} T^b_{i'j'} A\!\!\!/^{\prime,b} \right) q_{J,j'} \\
& + \theta_{\rm diag} \frac{\alpha_{\rm diag}}{8\pi} G_{\rm diag} \tilde{G}_{\rm diag} + \bar{\theta} \frac{\alpha_s}{8\pi} G\tilde{G} + \theta' \frac{\alpha'}{8\pi} G'\tilde{G}' + \frac{(g')^2\, \Lambda^2_{\rm CUT}}{2} A'_\mu A'^\mu
\end{aligned}$$

QCD Axion with small size instanton effects

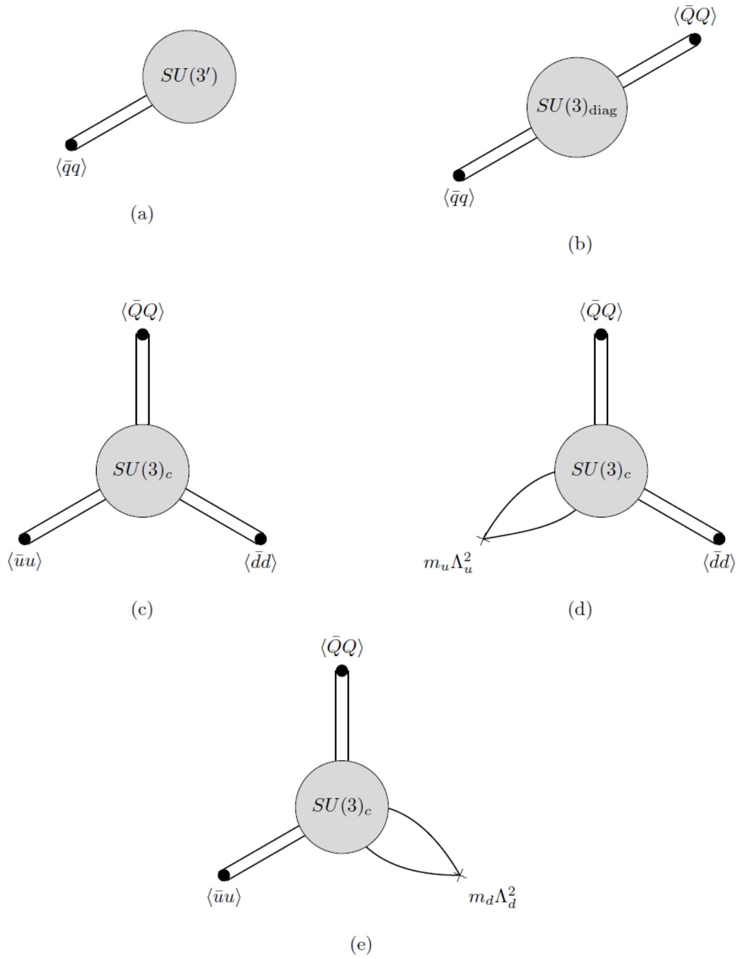
- Additional quarks form condensate

$$\bar{Q}_L Q_R \approx |\langle \bar{Q}_L Q_R \rangle| \exp\left(i\frac{\sqrt{6}a}{F_a}\right) = \frac{v_{\rm diag}^3}{2}\exp\left(i\frac{\sqrt{6}a}{F_a}\right)$$

$$\bar{q}_L q_R \approx |\langle \bar{q}_L q_R \rangle| \exp\left(i\frac{2\eta_d}{F_a}\right) = \frac{v_{\rm diag}^3}{2}\exp\left(i\frac{2\eta_d}{F_a}\right),$$

- Instanton contribution from instanton flower diagrams

$$\mathcal{L} \supset -K' v_{\rm diag}^3 \cos\left(\frac{2\eta_d}{F_a}\right) - \frac{v_{\rm diag}^6}{2K_{\rm diag}^2}\cos\left(\frac{2\eta_d}{F_a} + \frac{\sqrt{6}a}{F_a}\right)$$
$$- m_u v^3 \cos\left(\frac{\pi^0}{F_{\pi^0}} + \frac{\eta'}{F_{\eta'}}\right) - m_d v^3 \cos\left(-\frac{\pi^0}{F_{\pi^0}} + \frac{\eta'}{F_{\eta'}}\right) - \frac{v_{\rm diag}^3 v^9}{4K^8}\cos\left(\frac{\sqrt{6}a}{F_a} + 2\frac{\eta'}{F_{\eta'}}\right)$$
$$- \frac{v_{\rm diag}^3 v^6 m_u \Lambda_u^2}{2K^8}\cos\left(\frac{\sqrt{6}a}{F_a} + \frac{\eta'}{F_{\eta'}} + \frac{\pi^0}{F_{\pi^0}}\right) - \frac{v_{\rm diag}^3 v^6 m_d \Lambda_d^2}{2K^8}\cos\left(\frac{\sqrt{6}a}{F_a} + \frac{\eta'}{F_{\eta'}} - \frac{\pi^0}{F_{\pi^0}}\right)$$



QCD Axion with small size instanton effects

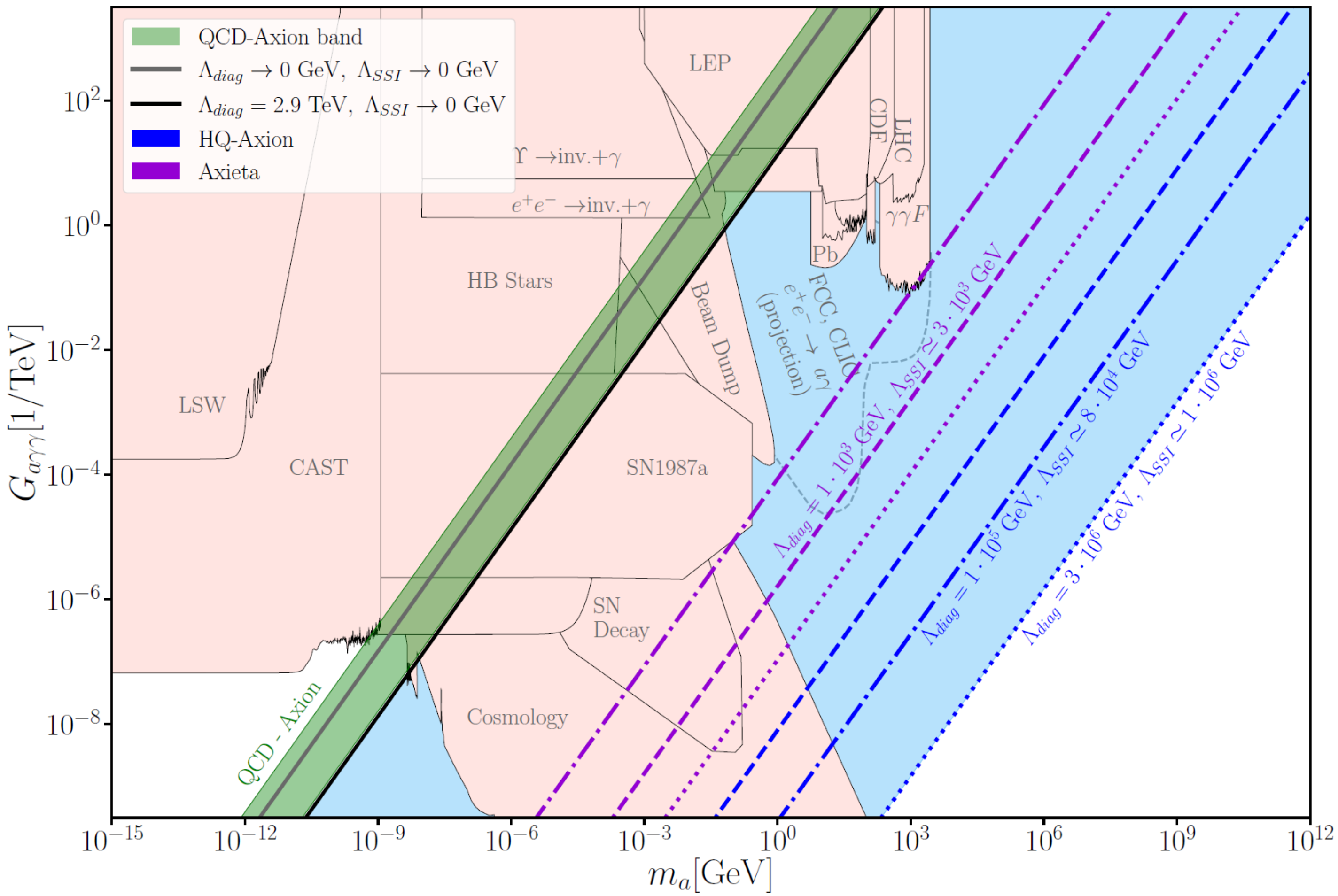
- Get coupling to photons via transformation

$$\begin{pmatrix} a_{1,m} \\ a_{2,m} \\ \eta'_m \\ \pi^0_m \end{pmatrix} = V^T \begin{pmatrix} a_1 \\ a_2 \\ \eta' \\ \pi^0 \end{pmatrix}$$

- For $E_2 = 0$ the interaction Lagrangian reads

$$\begin{aligned}\mathcal{L} \supset &-\frac{1}{4}\left(\left(\frac{\alpha_e}{2\pi F_a}E_1\right)v_{1,1} + \left(\frac{\alpha_e}{2\pi F_d}E_2\right)v_{1,2} + G_{\eta'\gamma\gamma}v_{1,3} + G_{\pi^0\gamma\gamma}v_{1,4}\right)a_{1,m}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ &-\frac{1}{4}\left(\left(\frac{\alpha_e}{2\pi F_a}E_1\right)v_{2,1} + \left(\frac{\alpha_e}{2\pi F_d}E_2\right)v_{2,2} + G_{\eta'\gamma\gamma}v_{2,3} + G_{\pi^0\gamma\gamma}v_{2,4}\right)a_{2,m}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ \simeq &-\frac{1}{4}\left(\frac{\alpha_e}{2\pi F_a}\right)(E_1 - \Delta_1)\,a_{1,m}F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}\left(\frac{\alpha_e}{2\pi F_d}\right)\left(\frac{F_d}{F_a}E_1 v_{2,1} - \Delta_2\right)a_{2,m}F_{\mu\nu}\tilde{F}^{\mu\nu}\end{aligned}$$

QCD Axion with small size instanton effects



QCD Axion with small size instanton effects

- Second Model in [Gaillard et al, 1805.06465] contains bifundamental Δ_2

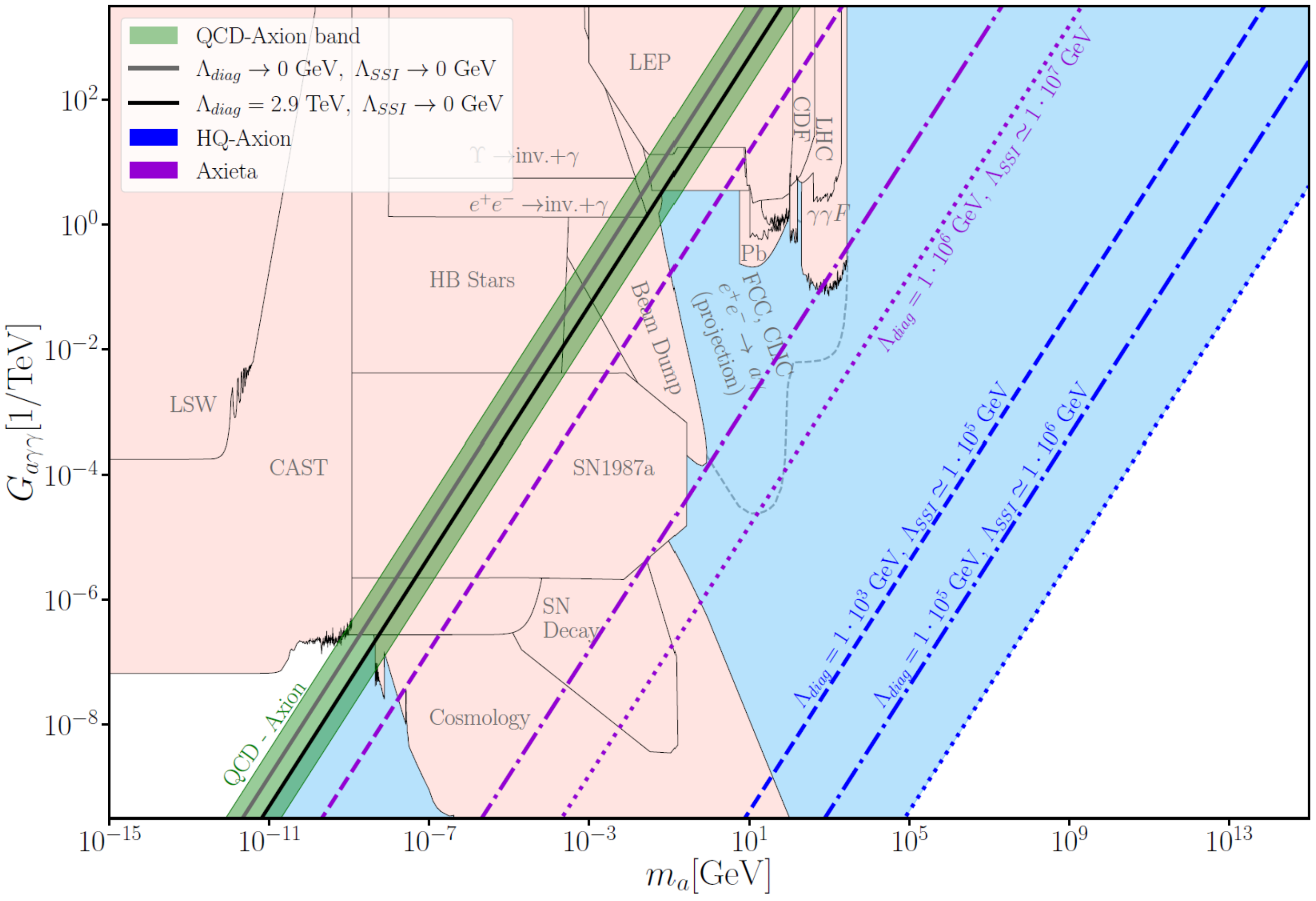
	$SU(3)_{\text{diag}}$	$SU(3)_c$	$SU(3')$	$SU(6)$
Q	$\bar{\square}$	$\square$	1	20
Δ_2	-	-	$\bar{\square}$	$\square$

- Here, we get two axions at different scales F_a and F_d

$$m_{a'}^2 F_a^2 = 144 \left(\Lambda_{\text{diag}}^4 + \Lambda_{\text{SSI}}^4 - \Lambda_{\text{diag}}^4 \left(1 + \frac{(m_a^2 F_a^2)^{\text{KSVZ}}}{\Lambda_{\text{diag}}^4} \right)^{-1} \right) ,$$

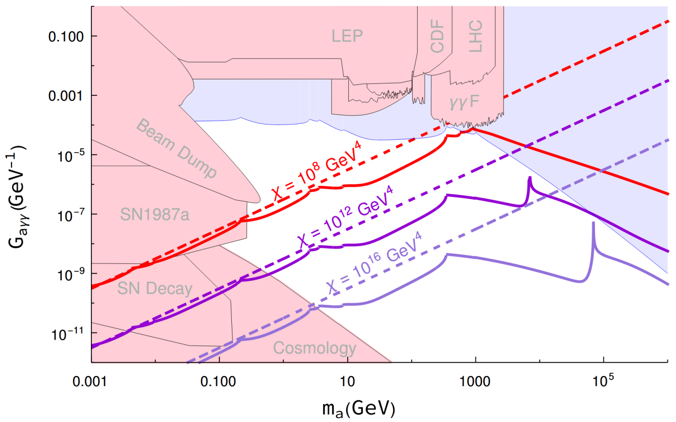
$$m_a^2 F_d^2 = 6\Lambda_{\text{diag}}^4 + 6(m_a^2 F_a^2)^{\text{KSVZ}} + 144 \frac{F_d^2}{F_a^2} \left(\Lambda_{\text{diag}}^4 \left(1 + \frac{(m_a^2 F_a^2)^{\text{KSVZ}}}{\Lambda_{\text{diag}}^4} \right)^{-1} \right)$$

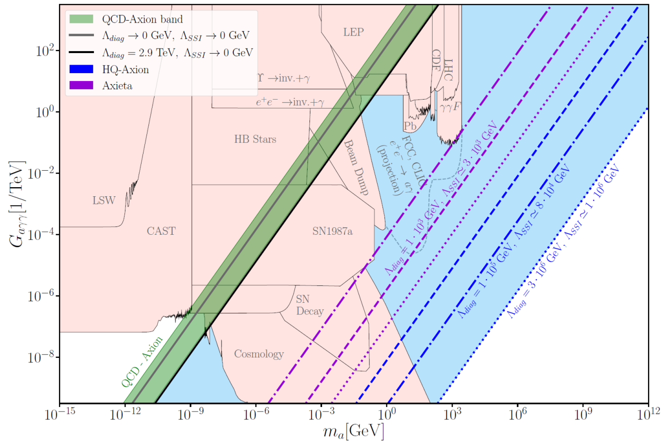
QCD Axion with small size instanton effects



Conclusion and Outlook

- ❑ Have discussed two possibilities to find exotic axion/ALP effects at colliders
- ❑ For DFSZ-like ALP with gauged baryon number we found new ALP Z' couplings and threshold effects from anomalons
- ❑ Used a generically flavor-violating ansatz for fermion couplings
- ❑ Small Size instantons shift the QCD axion mass to collider scales
- ❑ Used 't Hooft determinantal operator approach
- ❑ Ongoing: Application to LHC measurements
- ❑ Next: Test other U(1)' extensions [JL, Najjari, Yu, 22XX.XXXXX]





Backup

DFSZ-like ALP with gauged baryon number

- Matrix element for ALP coupling to two gauge bosons in heavy fermion limit

$$
\begin{aligned}
i\mathcal{M} =& -i\frac{g_I g_J}{(4\pi)^2}\frac{2}{f_a} p_{1\alpha} p_{2\beta} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{1\mu} \epsilon^*_{2\nu} C^{IJ}_{\text{eff}} \\
=& -i\frac{g_I g_J}{(4\pi)^2}\frac{2}{f_a} p_{I\alpha} p_{J\beta} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{I\mu} \epsilon^*_{J\nu} \Bigg(C_3^{IJ} - \sum_{i,j,k} N_k \frac{2C_0(0,0,0,m_i,m_j,m_k)}{\lambda(m_a^2, m_I^2, m_J^2)} \times \\
& \times \Bigg((m_i + m_j)(C^{ij}_{1A} + C^{ij}_{2A}) \Big((Q^{ik}_{IV} Q^{kj}_{JV} - Q^{ik}_{IA} Q^{kj}_{JA}) m_k m_a^2 (m_a^2 - m_I^2 - m_J^2) \\
& + (Q^{ik}_{IV} Q^{kj}_{JV} + Q^{ik}_{IA} Q^{kj}_{JA}) (m_i m_J^2 (m_a^2 + m_I^2 - m_J^2) + m_j m_I^2 (m_a^2 - m_I^2 + m_J^2)) \Big) \\
& + (m_i - m_j)(C^{ij}_{1V} + C^{ij}_{2V}) \Big((Q^{ik}_{IA} Q^{kj}_{JV} - Q^{ik}_{IV} Q^{kj}_{JA}) m_k m_a^2 (m_a^2 - m_I^2 - m_J^2) \\
& + (Q^{ik}_{IV} Q^{kj}_{JA} + Q^{ik}_{IA} Q^{kj}_{JV}) (m_i m_J^2 (m_a^2 + m_I^2 - m_J^2) - m_j m_I^2 (m_a^2 - m_I^2 + m_J^2)) \Big) \Bigg) \\
& + \sum_{i,j,k} N_k \Big(C^{ij}_{1A} (Q^{ik}_{IV} Q^{kj}_{JV} + Q^{ik}_{IA} Q^{kj}_{JA}) + C^{ij}_{1V} (Q^{ik}_{IV} Q^{kj}_{JA} + Q^{ik}_{IA} Q^{kj}_{JV}) \Big) \Bigg) + \mathcal{O}\left(\frac{m^2_{a,I,J}}{m^2_{i,k,j}} \right)
\end{aligned}
$$

DFSZ-like ALP with gauged baryon number

- Matrix element for ALP coupling to gauge boson and scalar in heavy fermion limit

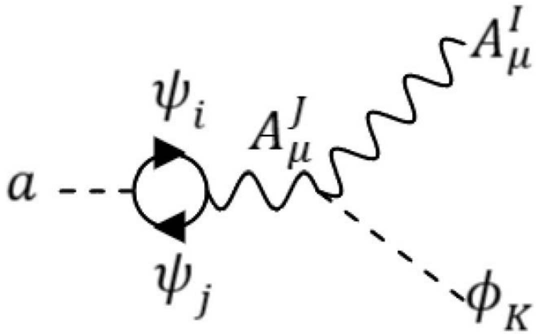
$$
\begin{aligned}
i\mathcal{M} = &-\frac{i}{(4\pi)^2}\frac{g_I}{f_a}\epsilon^*_{I\mu}\sum_{i,j,k} N_k \Bigg((m_i+m_j)((C^{ji}_{1A}+C^{ji}_{2A})Q^{ik}_{IA}Y^{kj}_K + (C^{ij}_{1A}+C^{ij}_{2A})Y^{jk}_K Q^{ki}_{IA}) \\
&\Bigg(B_0(0,m_k,m_i)(2p^\mu_a - p^\mu_I) - \frac{2C_0(0,0,0,m_i,m_j,m_k)}{\lambda(m_a^2,m_K^2,m_I^2)}\times \\
&\times \Bigg(\Big(m_I^2(m_I^2-m_a^2-m_K^2)(m_i-m_j)(m_j+m_k) - \lambda(m_a^2,m_K^2,m_I^2)(m_i m_k + m_j^2)\Big)p^\mu_a \\
&\quad + \Big(m_a^2(m_a^2-m_I^2-m_K^2)(m_i-m_j)(m_j+m_k) - \lambda(m_a^2,m_K^2,m_I^2)(m_i m_j - m_j^2)\Big)p^\mu_I\Bigg)\Bigg) \\
&+ (m_i+m_k)((C^{ji}_{1A}+C^{ji}_{KA})Q^{ik}_{IA}Y^{kj}_K + (C^{ij}_{1A}+C^{ij}_{KA})Y^{jk}_K Q^{ki}_{IA})B_0(0,m_k,m_i)p^\mu_I \\
&+ (m_j-m_k)((C^{ji}_{1A}+C^{ji}_{IA})Q^{ik}_{IA}Y^{kj}_K + (C^{ij}_{1A}+C^{ij}_{IA})Y^{jk}_K Q^{ki}_{IA})B_0(0,m_j,m_k)(p^\mu_I - p^\mu_a)\Bigg) \\
&+ \mathcal{O}\left(\frac{m^2_{a,I,K}}{m^2_{i,k,j}}\right).
\end{aligned}
$$

DFSZ-like ALP with gauged baryon number

- Last diagram in ALP scalar vector coupling

- Define scalar vector interaction as

$$\mathcal{L} \supset \frac{1}{2}(\phi_K + v_K)^2 \left(\sum_I g_I Q_I^K A_{I\mu}\right)\left(\sum_J g_J Q_J^K A_J^\mu\right)$$



- Matrix element reads

$$i\mathcal{M} = \frac{-i}{(4\pi)^2}\frac{g_I}{f_a}\sum_{J,L}\frac{g_J^2 Q_I^K Q_J^K v_K v_L}{m_J^2}$$

$$\sum_{i,j,k} N_k(m_i+m_j)((C_{1A}^{ji}+C_{2A}^{ji})Q_{JA}^{ik}Y_L^{kj}+(C_{1A}^{ij}+C_{2A}^{ij})Y_L^{jk}Q_{JA}^{ki})B_0(0,m_i,m_j)\epsilon_{I\mu}^* p_a^\mu$$

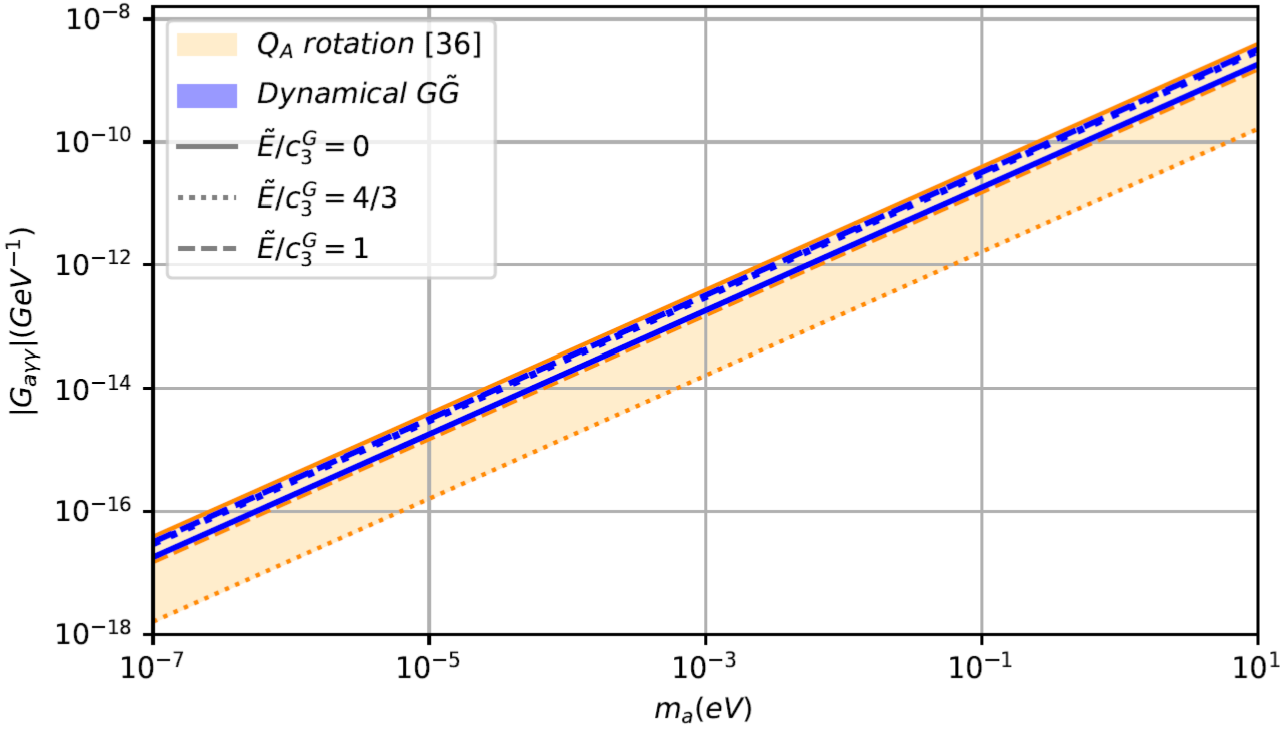
- Reduces divergent terms to

$$i\mathcal{M}_{\text{tot}} = \frac{i}{(4\pi)^2}\frac{g_I}{f_a}\frac{1}{\epsilon}\epsilon_{I\mu}^*\mathcal{T}_-\left(\mathbf{Y}_K, \mathbf{M}, \mathbf{Q}_{IA}, \mathbf{C}_{2A} - \mathbf{C}_{IA}\right)p_a^\mu + \mathcal{O}(\epsilon^0)$$

$$\frac{g_J Q_I^K v_L}{m_J} = \delta_{IJ}\delta_{KL}$$

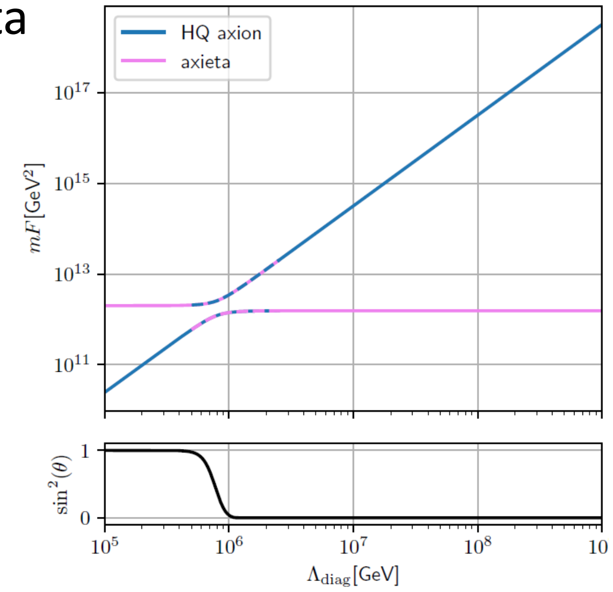
QCD Axion with small size instanton effects

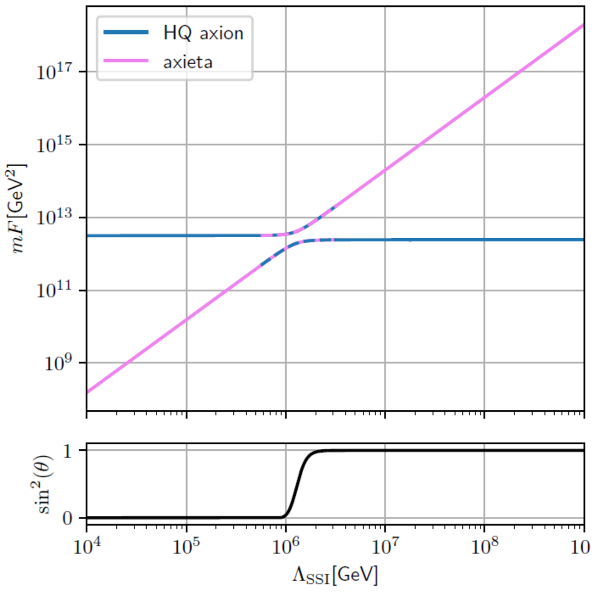
- Cross check QCD axion band



QCD Axion with small size instanton effects

- Avoided crossing between axion and axieta





$$m_a^2 F_a^2 = 4(\Lambda_{\mathrm{diag}}^4 + \Lambda_{\mathrm{SSI}}^4) - 24\Lambda_{\mathrm{diag}}^8 \times \left| 2\Lambda_{\mathrm{SSI}}^4 - \Lambda_{\mathrm{diag}}^4 - 3(m_a^2 F_a^2)^{\mathrm{KSVZ}} - \sqrt{\left(2\Lambda_{\mathrm{SSI}}^4 - \Lambda_{\mathrm{diag}}^4 - 3(m_a^2 F_a^2)^{\mathrm{KSVZ}}\right)^2 + 24\Lambda_{\mathrm{diag}}^8} \right|^{-1}$$

$$m_{\eta_d}^2 F_a^2 = 2\Lambda_{\mathrm{SSI}}^4 + 5\Lambda_{\mathrm{diag}}^4 + 3(m_a^2 F_a^2)^{\mathrm{KSVZ}} + \sqrt{\left(2\Lambda_{\mathrm{SSI}}^4 - \Lambda_{\mathrm{diag}}^4 - 3(m_a^2 F_a^2)^{\mathrm{KSVZ}}\right)^2 + 24\Lambda_{\mathrm{diag}}^8}\,,$$