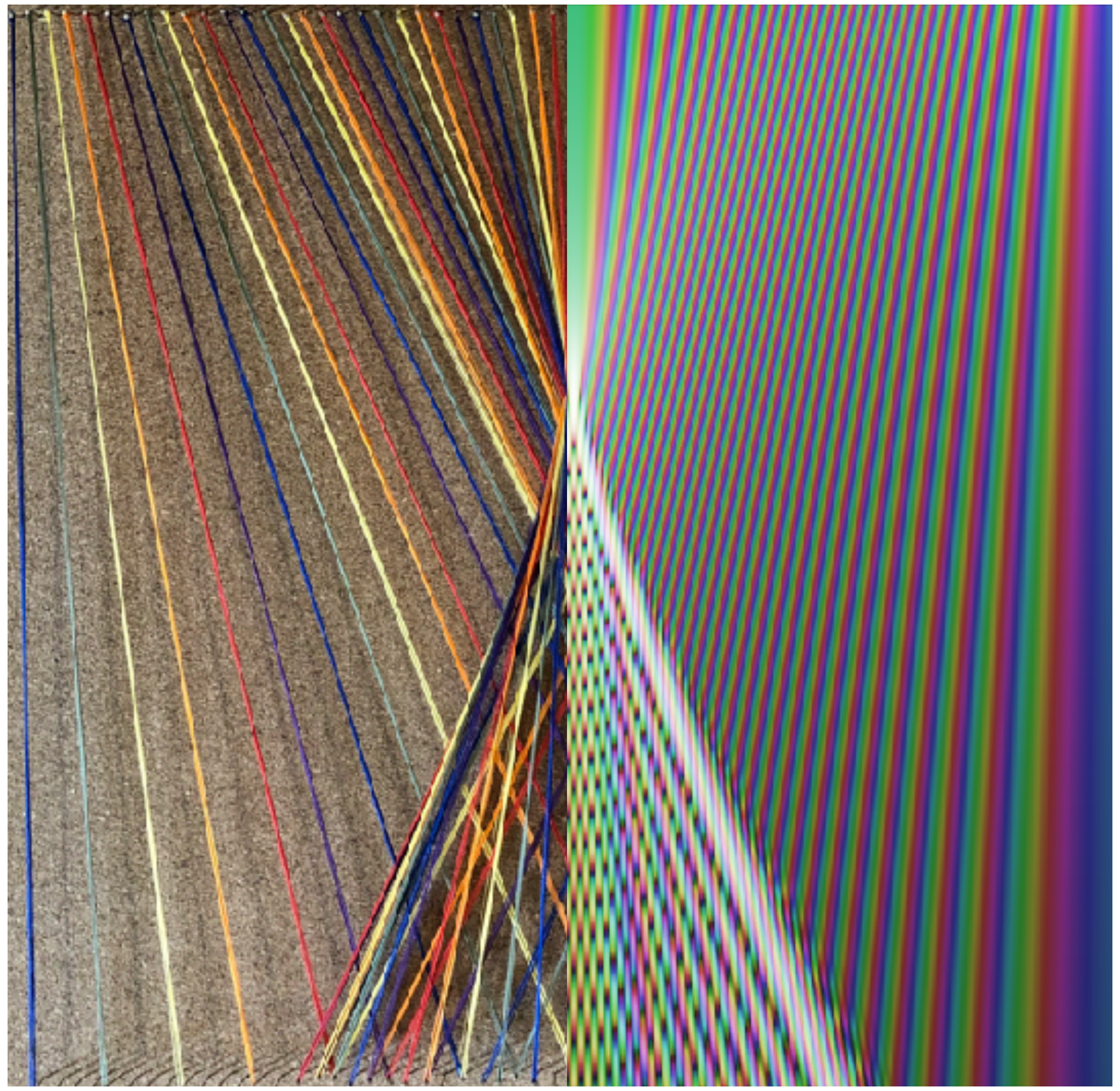
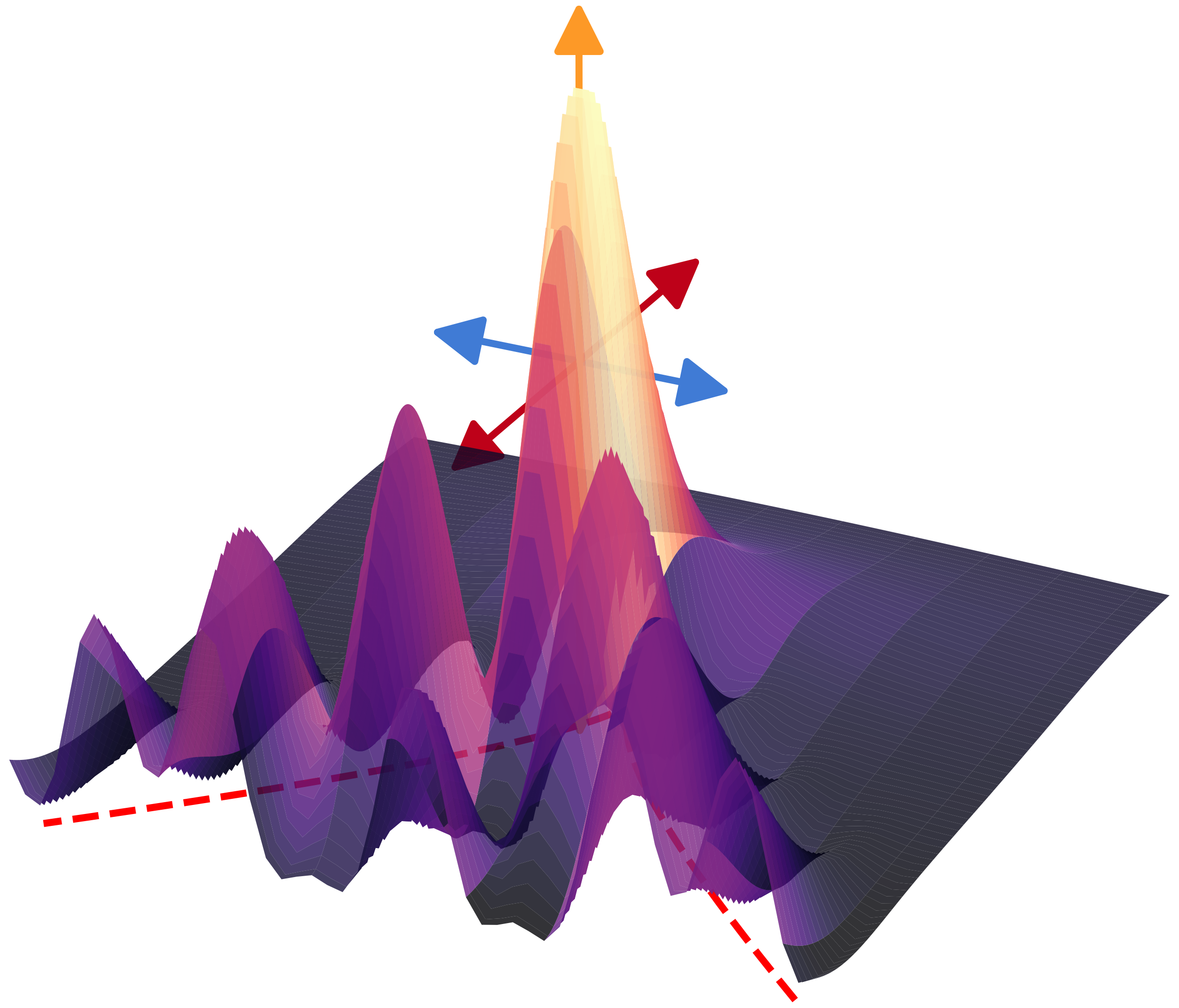
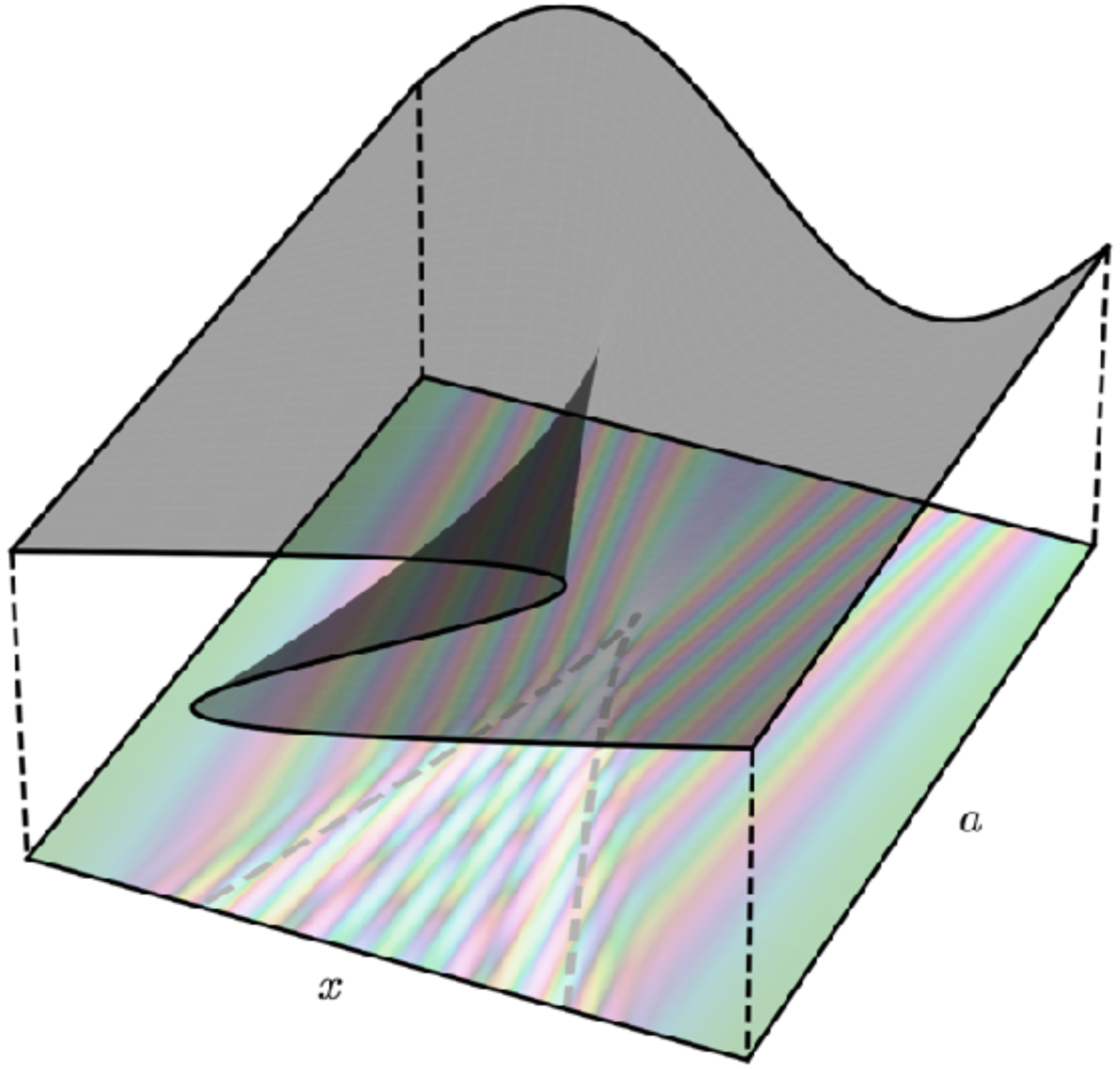
Making (dark matter) waves:

Untangling wave interference for multi-streaming dark matter

OJAp (5) 2022; (2206.11918)







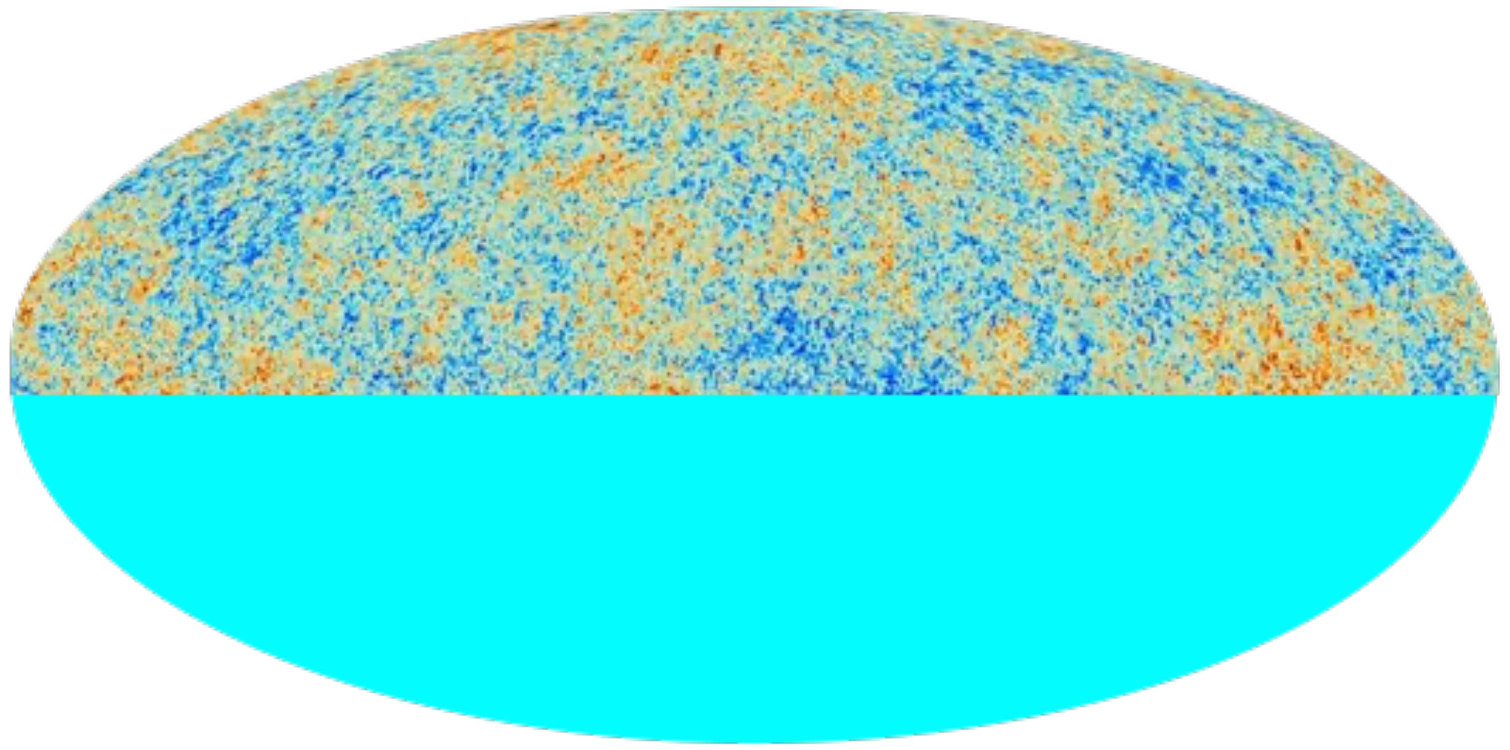
Recent Progress in Axion Theory and Experiment
Durham, *5-8 Sept 2022*

Alex Gough
(they/them)
with Cora Uhlemann

Newcastle University

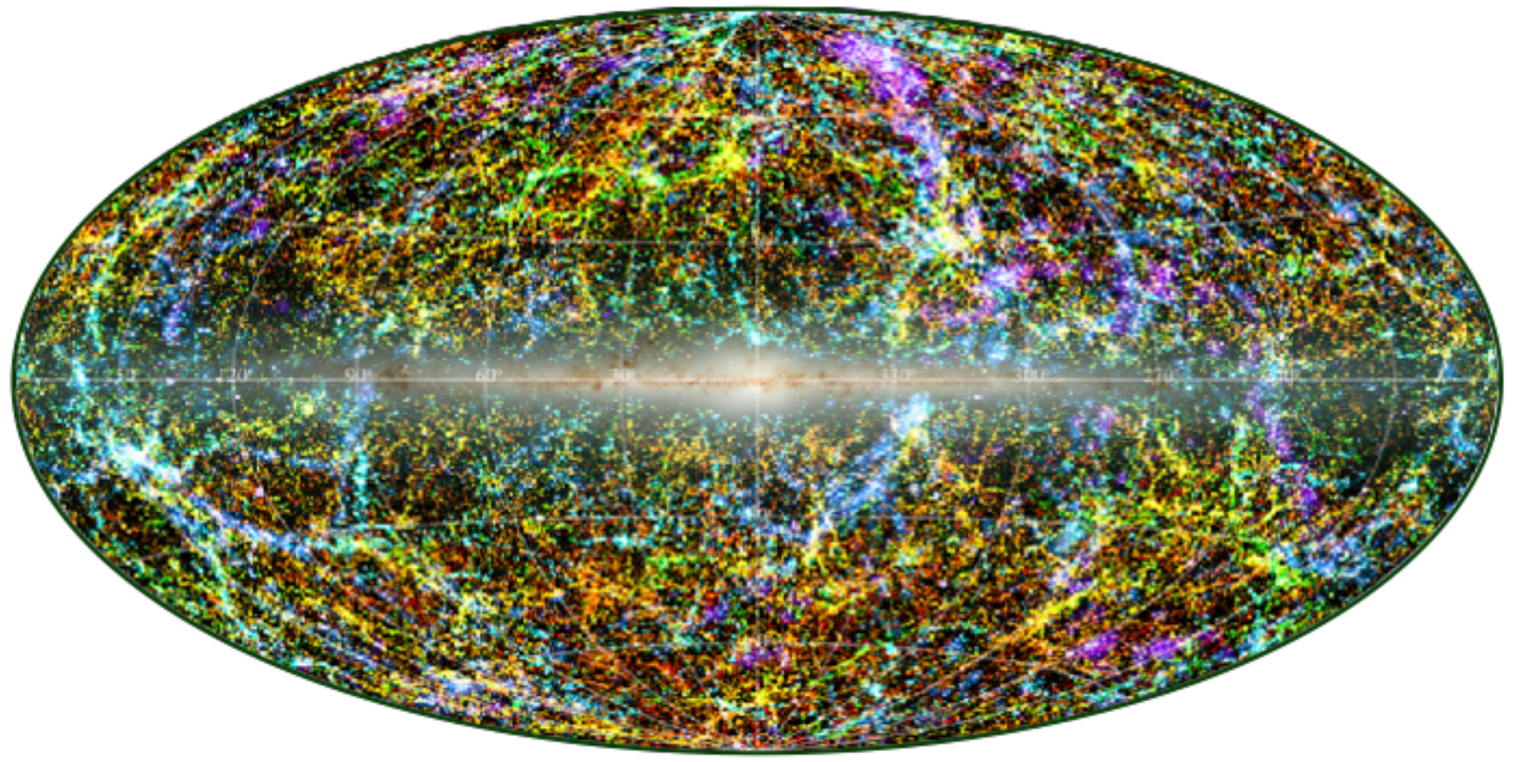
Big questions

Afterglow
of the early universe



nearly uniform

Cosmic web
of galaxies



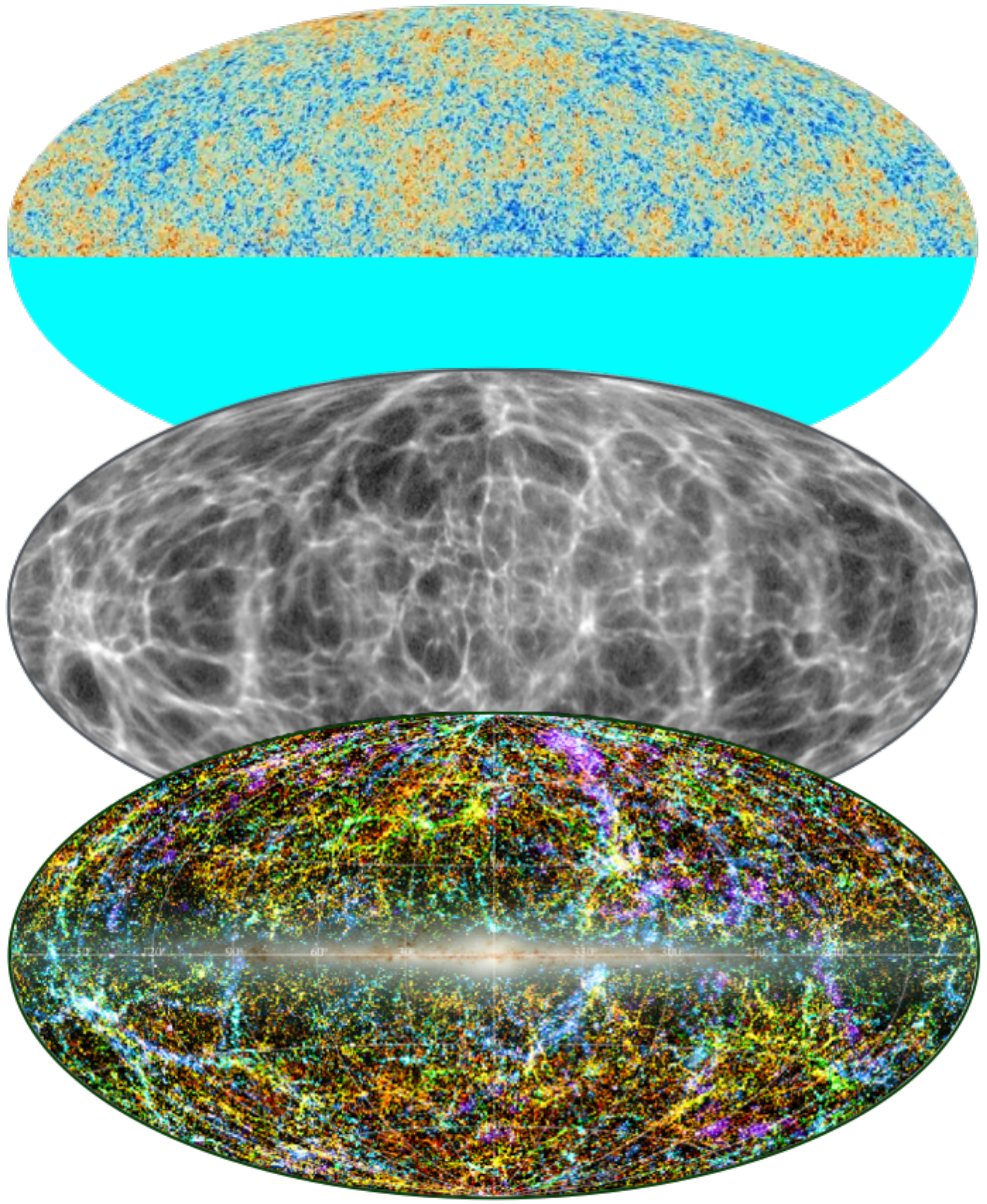
rich structure

Big questions

Afterglow
of the early universe

Skeleton
of dark matter

Cosmic web
of galaxies

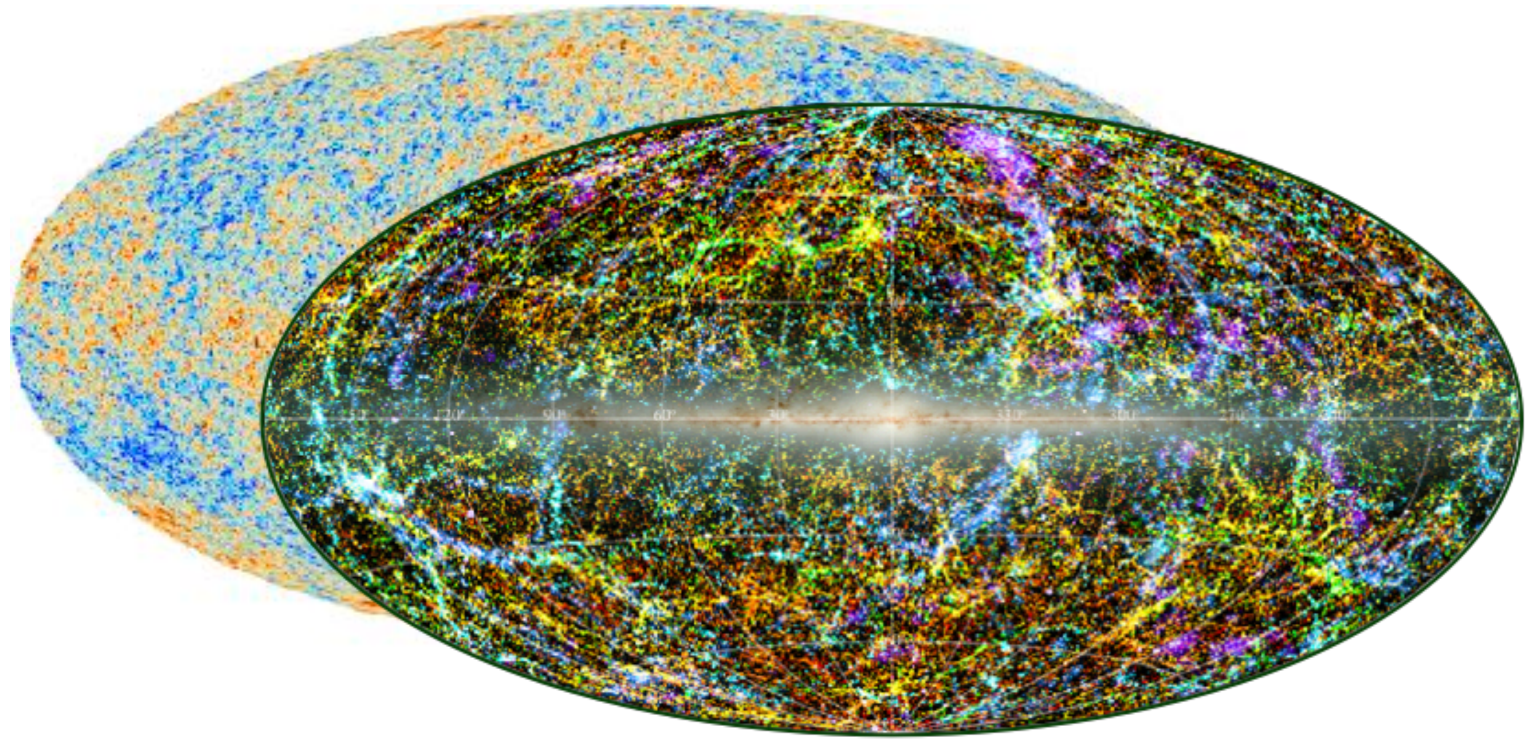


nearly uniform

rich structure

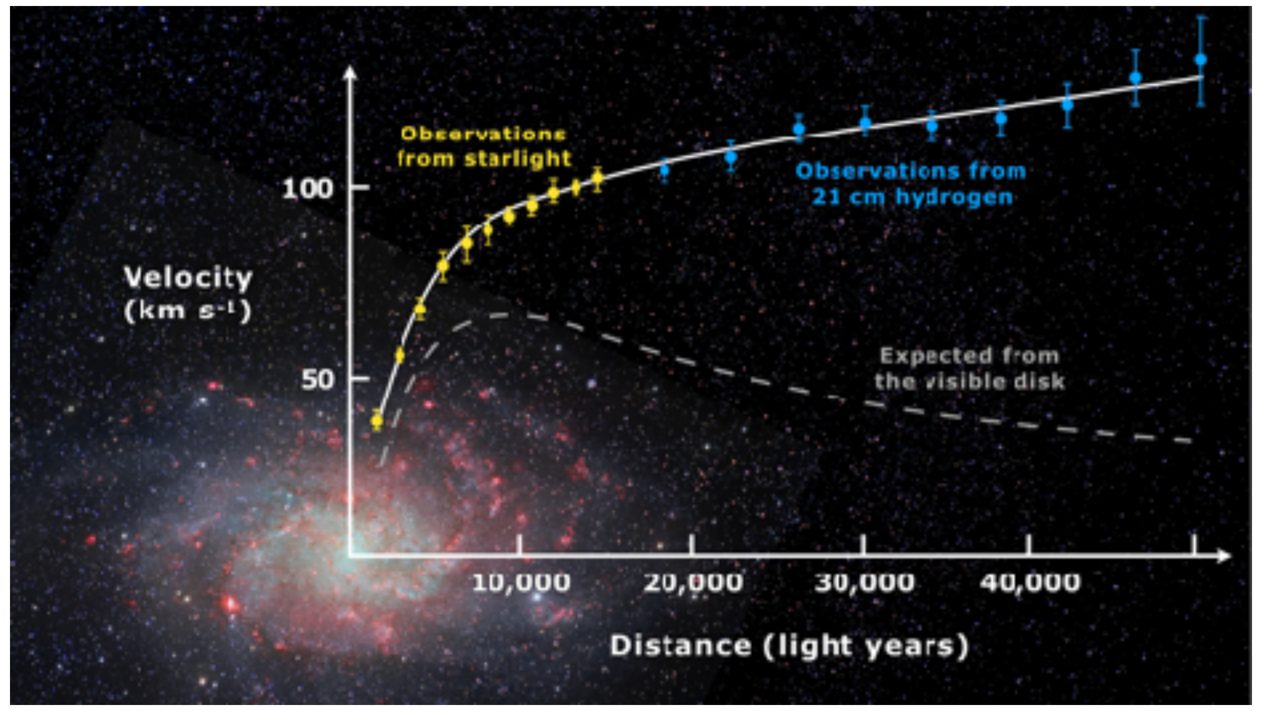
Evidence for dark matter

CMB/LSS



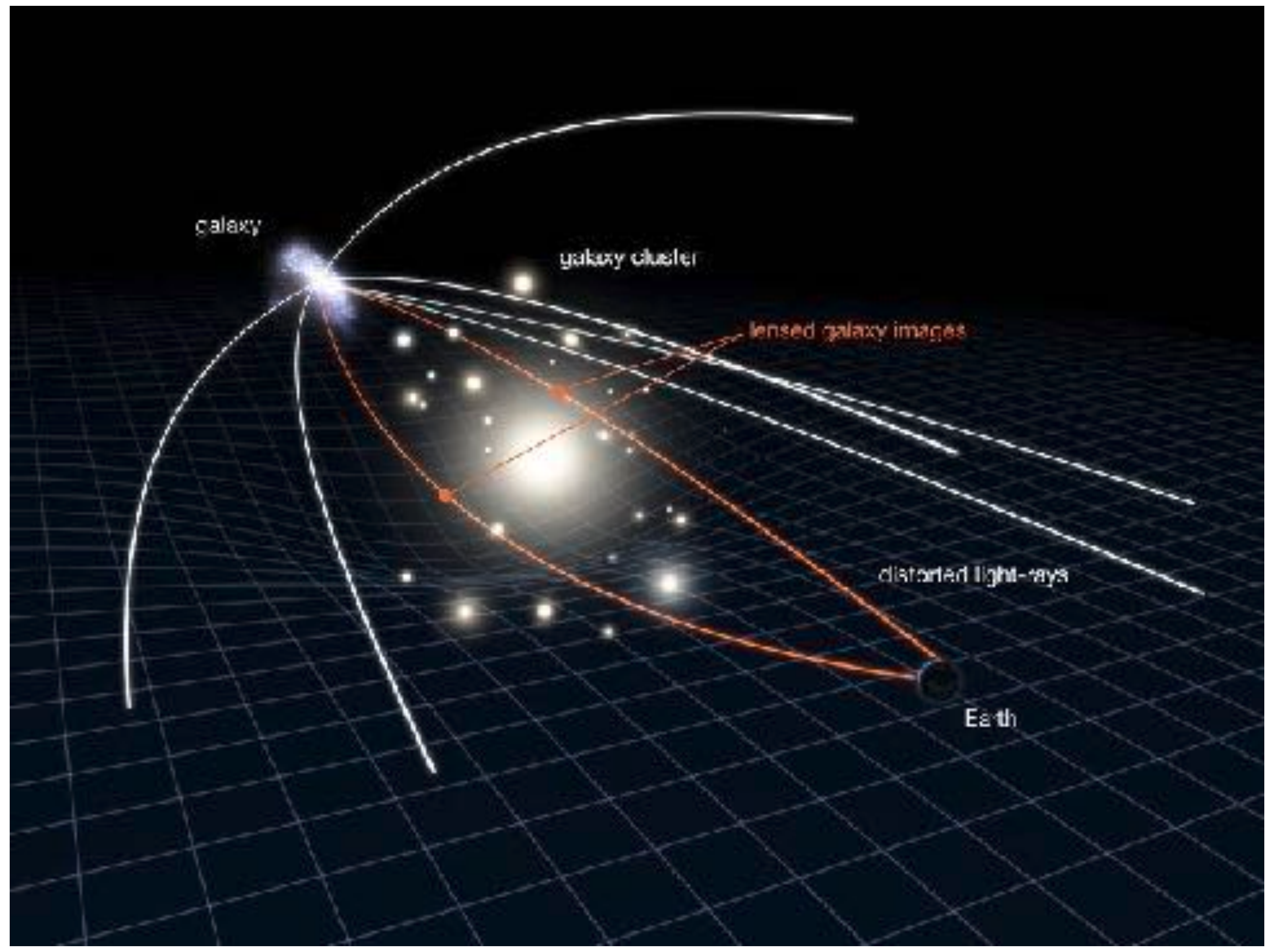
Thermal history, structure growth, anisotropies

Galaxies



Rotation curves, mass fraction, distribution

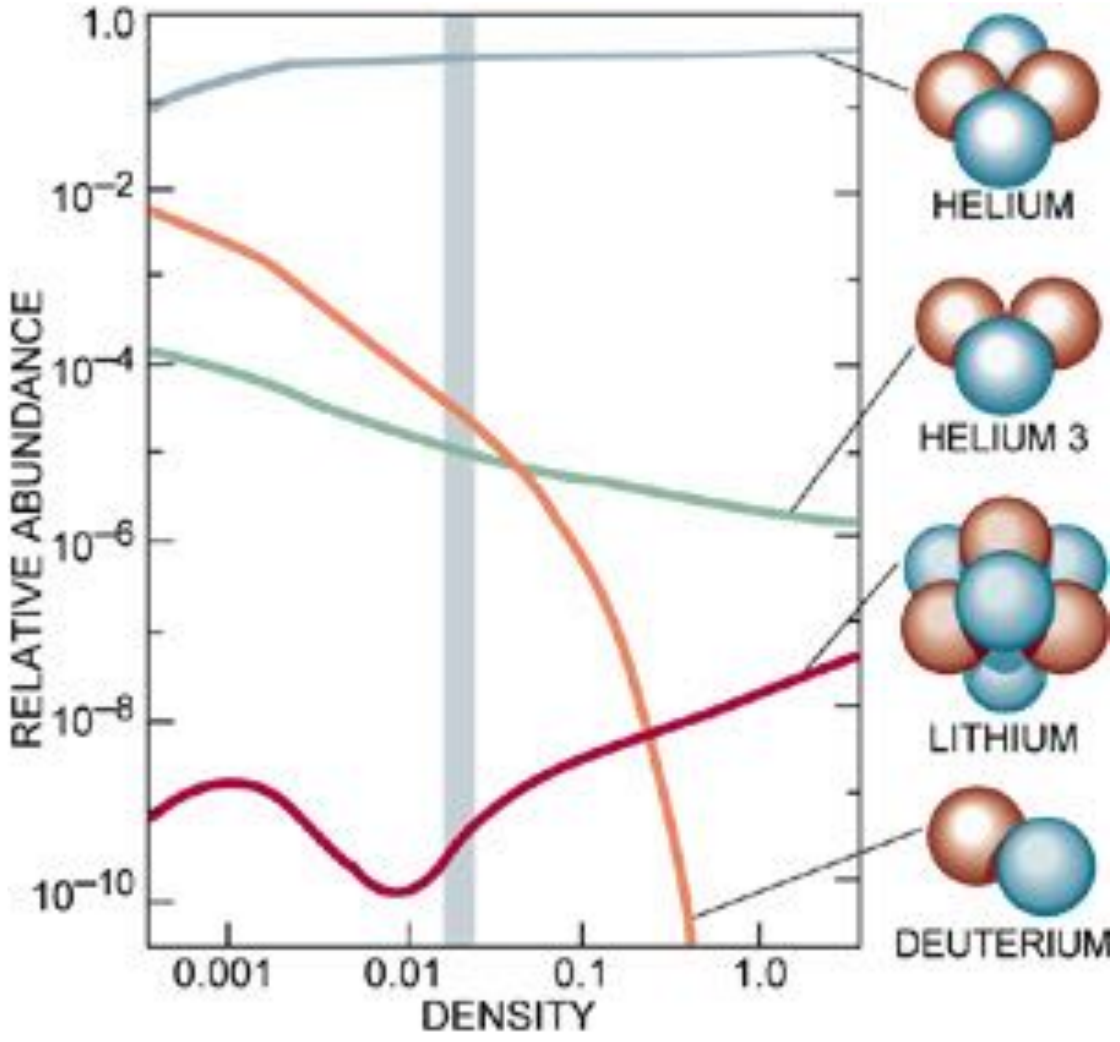
Gravitational lensing



Strong lensing, weak lensing, shape, structure

Clusters & collisions

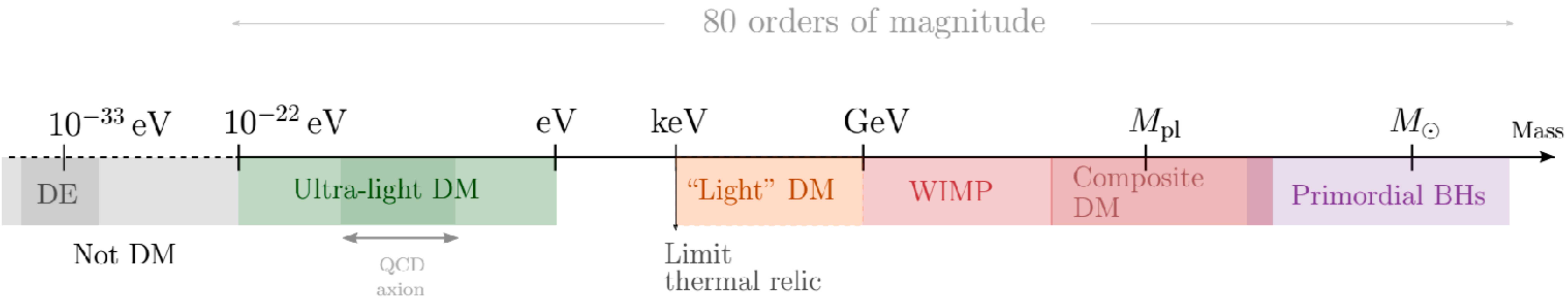
Distribution, separation from collisional matter, self-interaction



Big Bang Nucleosynthesis

Relative abundance of baryons

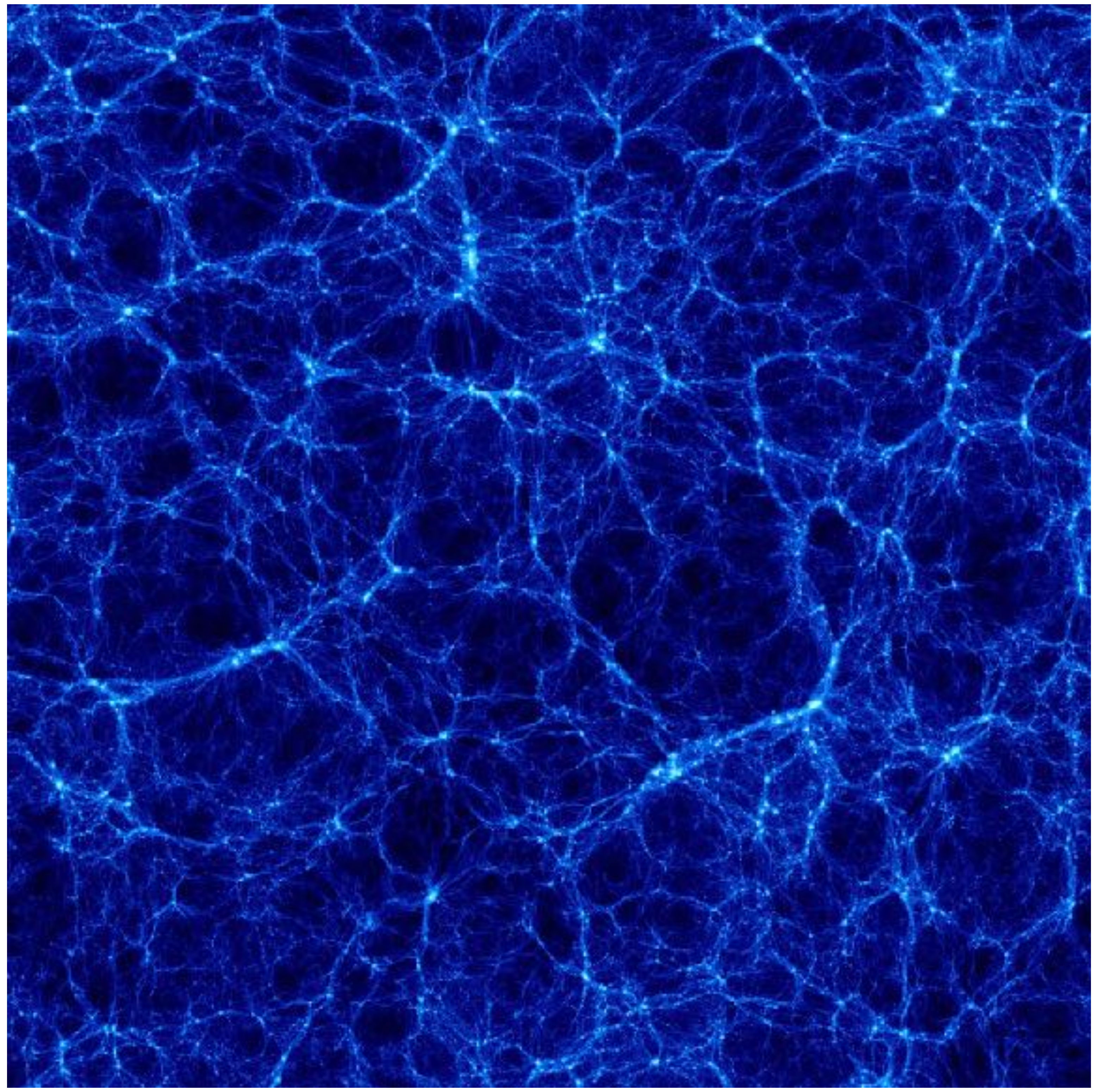
Dark matter mass



Ferreira A&A Review 2020

The standard picture

- Cold: moves slower than *c*
- Pressureless: clusters efficiently
- Dark: no/weak electromagnetic interactions
- Collisionless: no/weak self interactions or with baryons
- Abundant: $\approx$ 5x more DM than baryons.



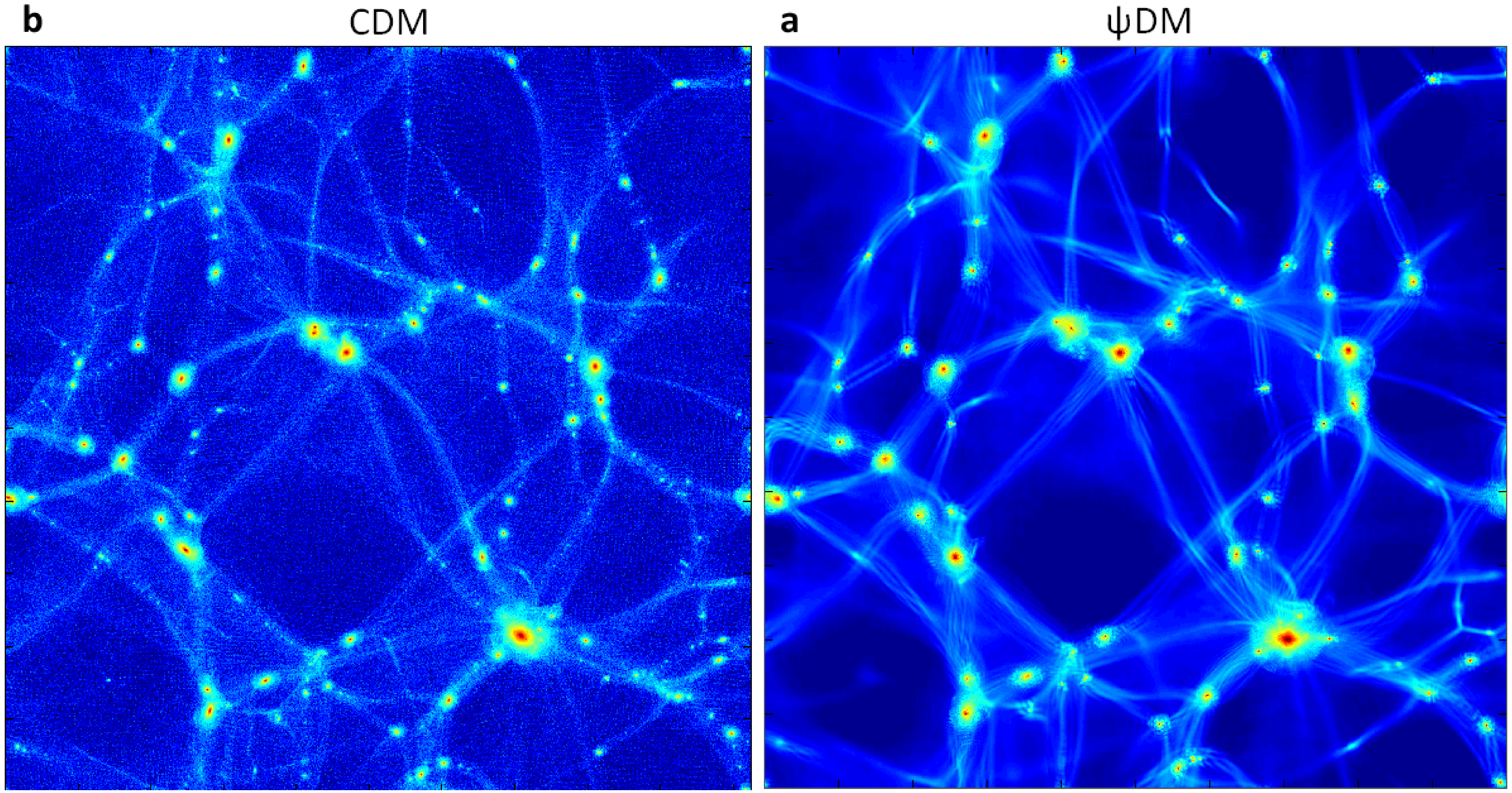
ΛCDM

Wave dark matter

Spot the difference

- Same large scale network as CDM

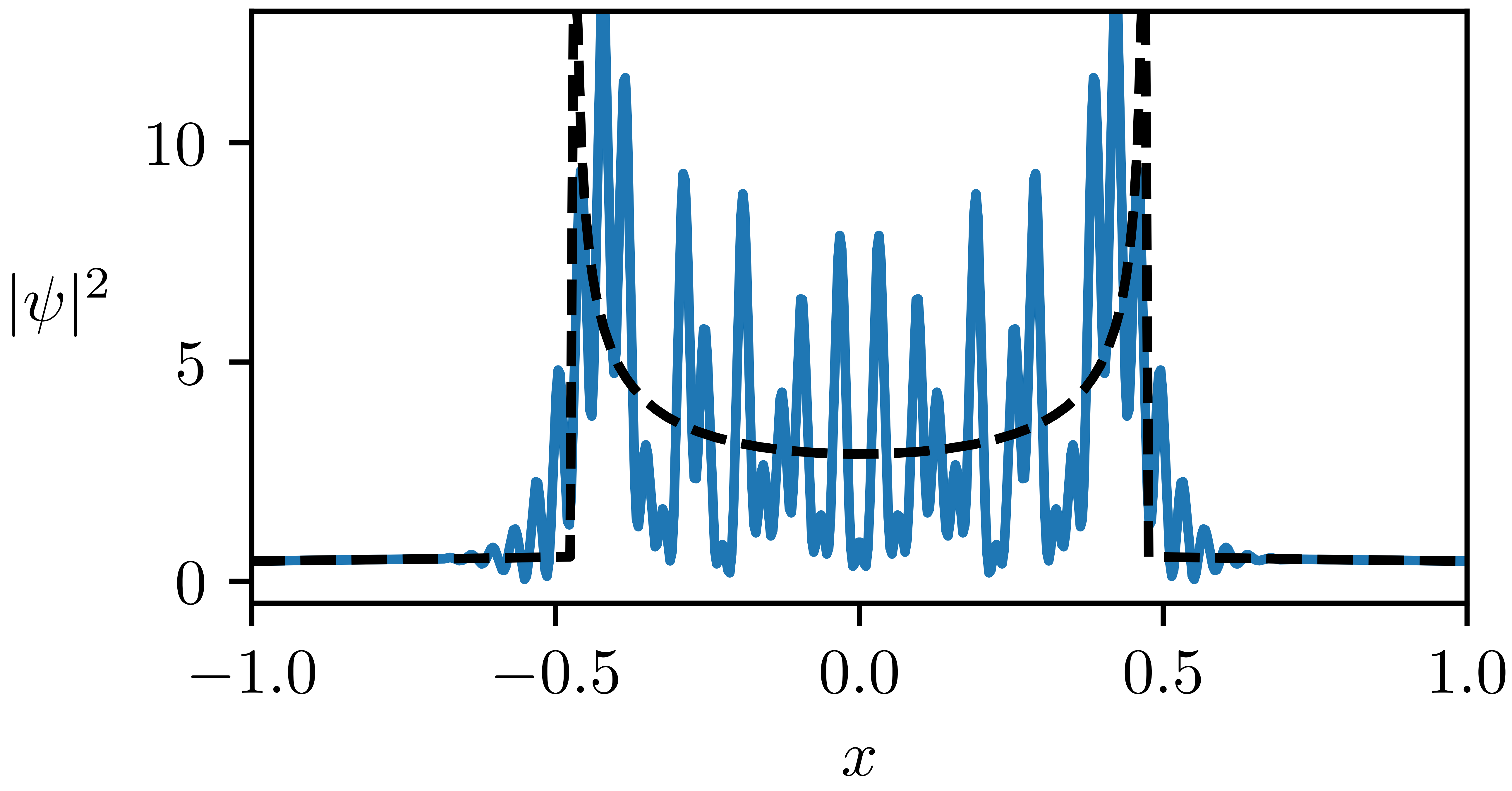
- Wave interference “decorates” the cosmic web

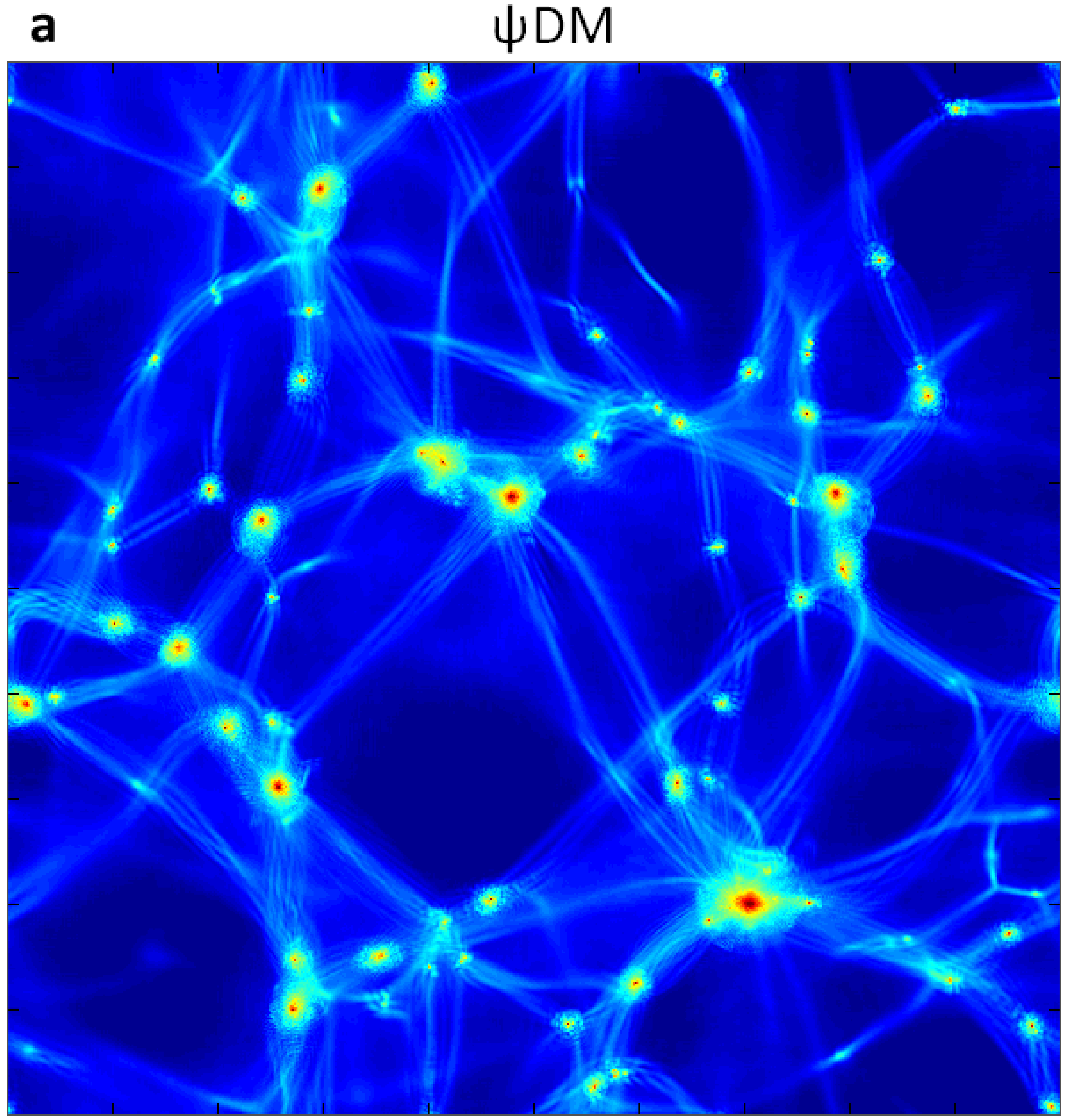


Schive ++ Nature Phys. Lett, `15

astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Wave dark matter





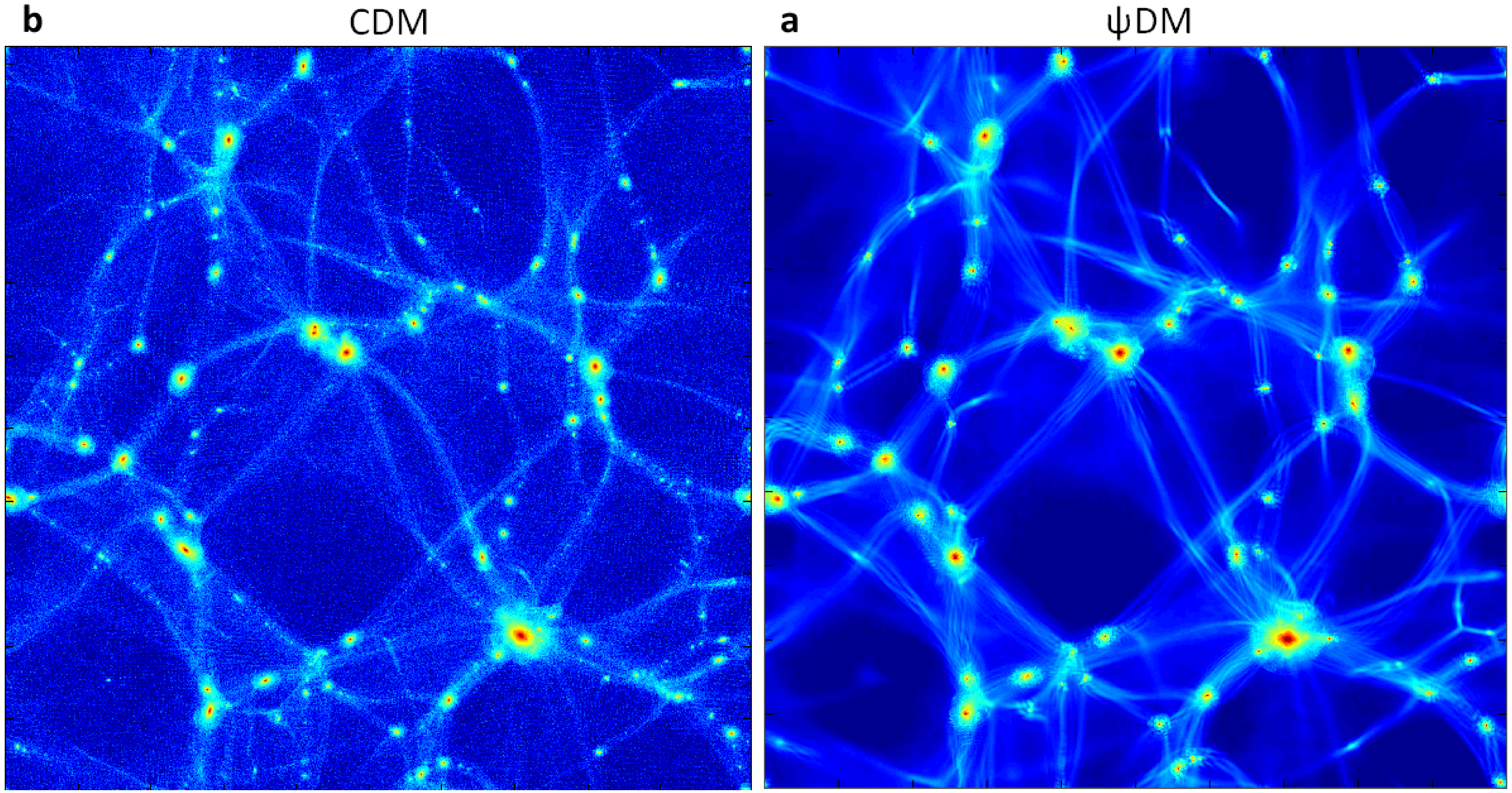
Schive ++ Nature Phys. Lett, `15

astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Wave dark matter

Why do we care?

- True wavelike dark matter (axions etc)
- Rich phenomenology
- Universal features (tool even for CDM)



Schive ++ Nature Phys. Lett, `15

astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Scalar field dynamics

- KG + FRW → homogeneous oscillations at frequency m

  ➡ change cosmological background, doesn’t drive collapse

- non-relativistic regime

$$\phi = \frac{1}{\sqrt{2m}}\left(\psi e^{-imt} + \psi^* e^{imt}\right)$$

complex classical field

- spatial fluctuations in ψ become structures

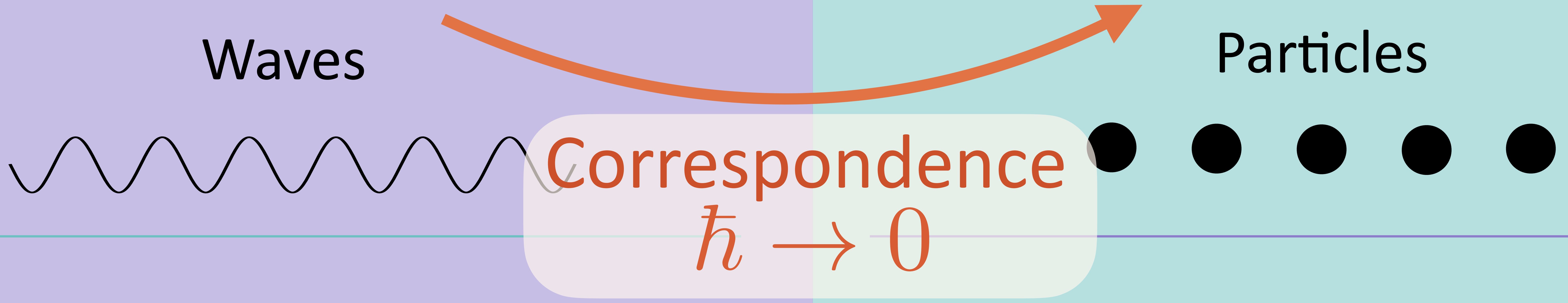
$$\psi(x,t) = \sqrt{\rho(x,t)}\,e^{i\phi_v(x,t)/\hbar}$$

Density

Velocity $v(x,t) = \nabla\phi_v(x,t)$

- full dynamics difficult/expensive (state of the art 1—10 Mpc box)…simple models can still be useful!

Simple models



Correspondence

$\hbar \to 0$

Wave Dark Matter

Waves

Evolution: Propagation

Simple model: free Schrödinger

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi$$

Cold Dark Matter

Particles

Evolution: Displacement

Simple model: Zel’dovich approximation

$$\boldsymbol{x} = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}$$

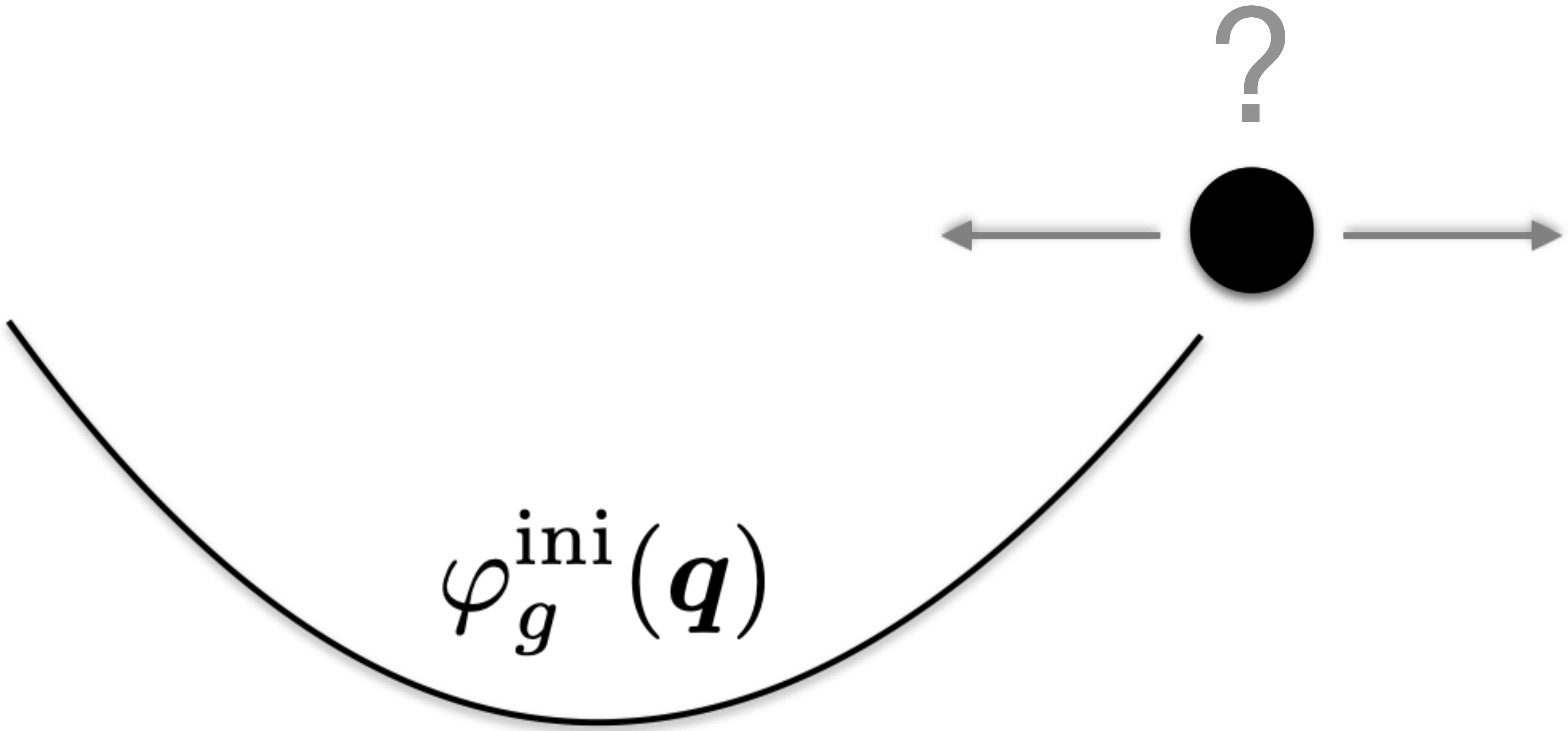
Widrow & Kaiser APJ `93
Coles `02, Uhelmann ++ `19

Classical phenomenology

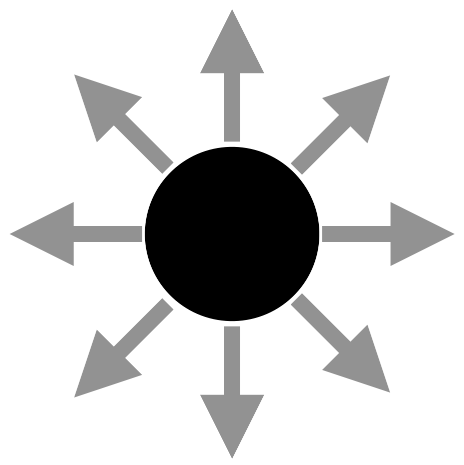
Approximate: shoot particles following initial potential

$$\boldsymbol{v}(\boldsymbol{q},a) = -\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}(\boldsymbol{q})$$

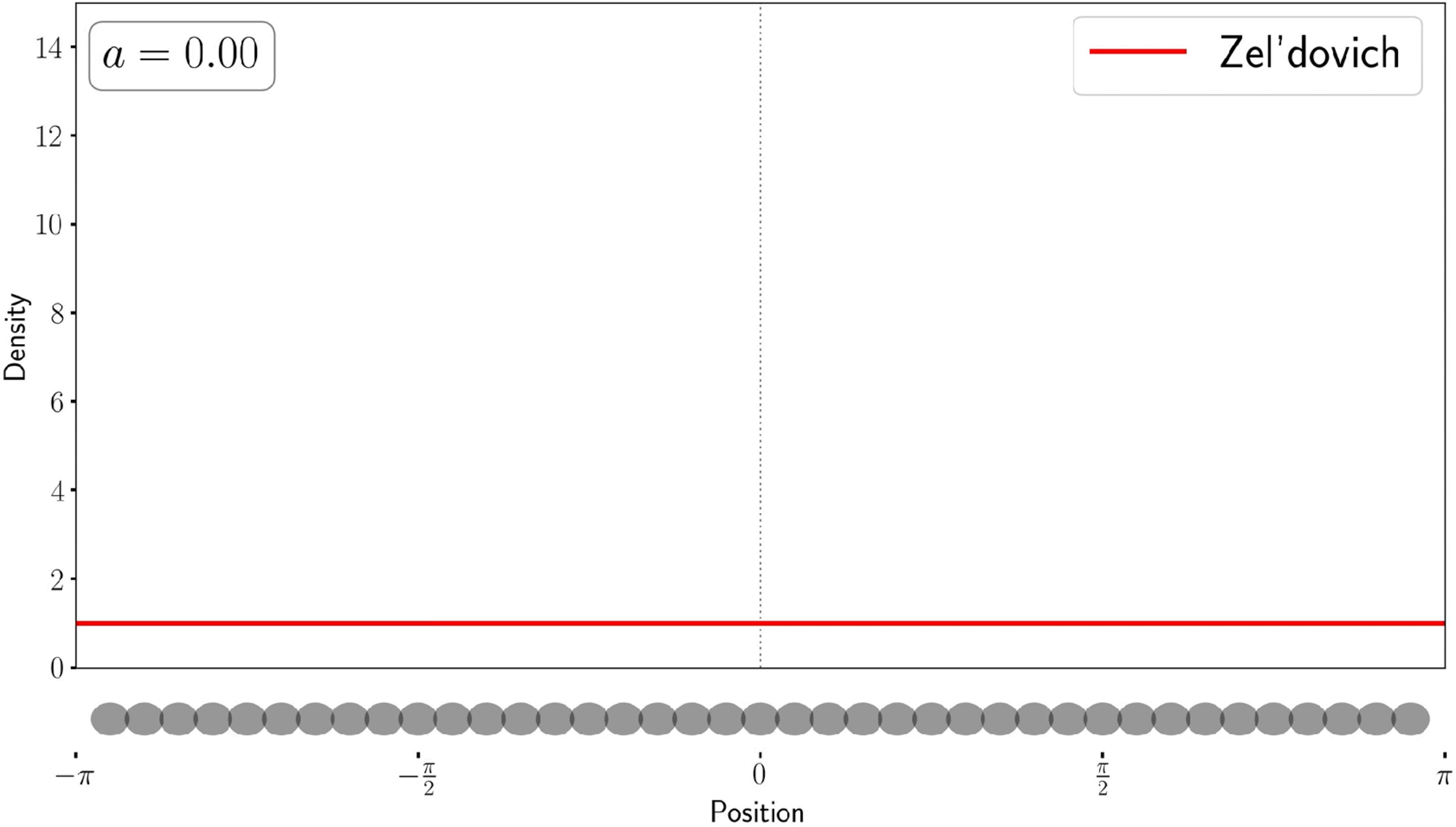
$$\boxed{\boldsymbol{x}(\boldsymbol{q},a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}(\boldsymbol{q}) \qquad \text{Zel'dovich approximation*}}$$

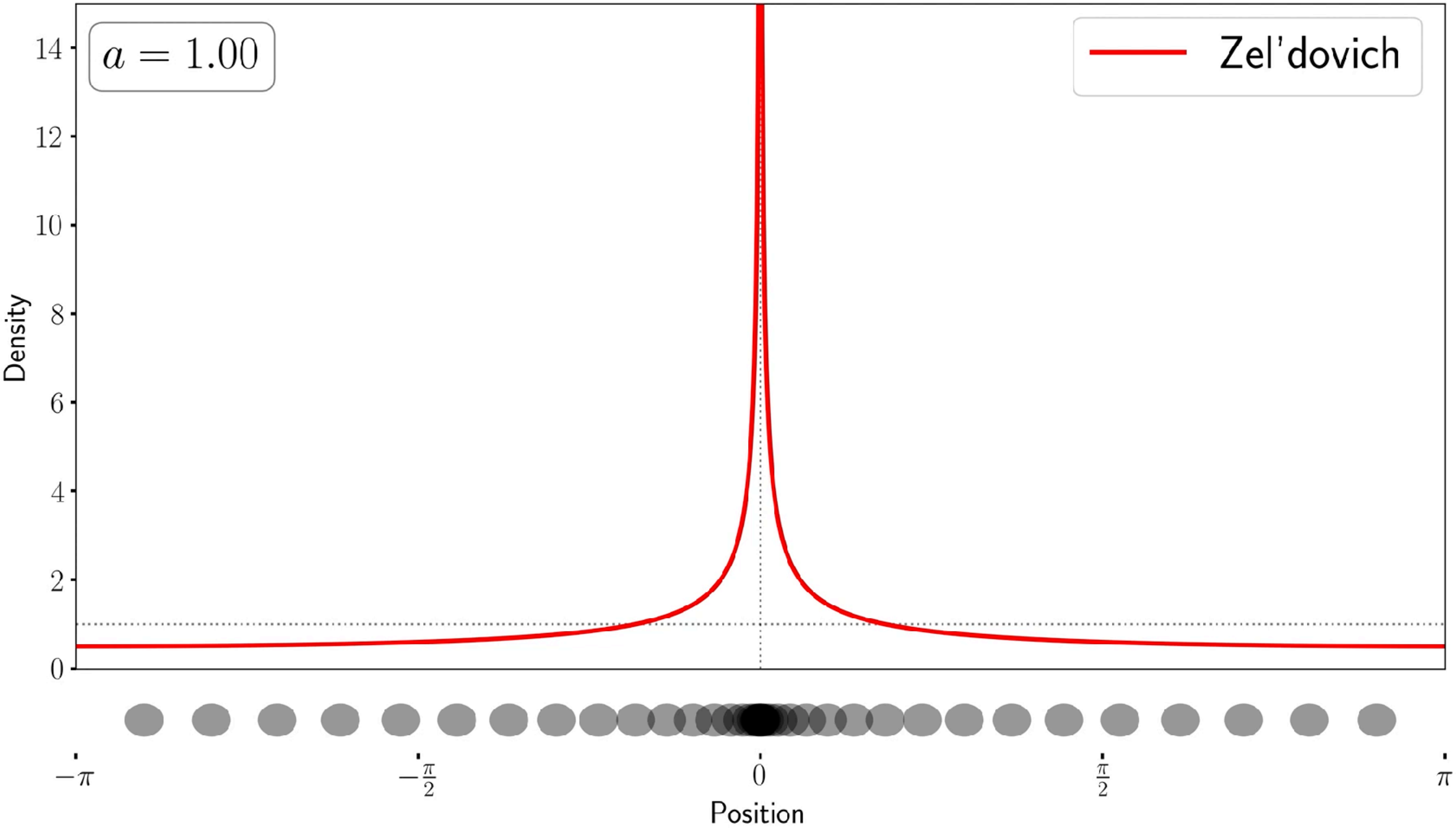


*(Lagrangian) perturbation theory:
ZA + tidal effects



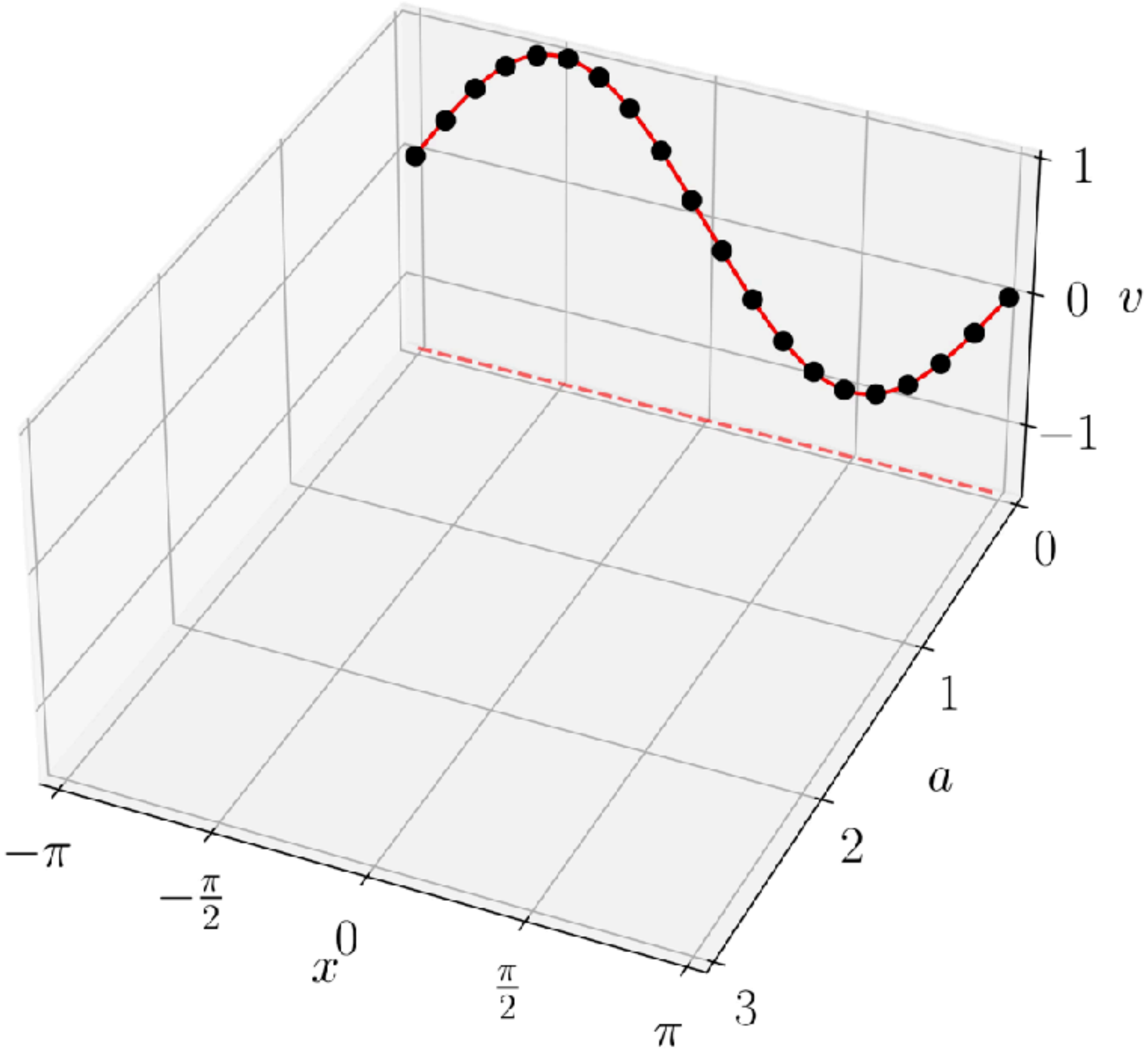
Zel'dovich A&A 1970


$a = 0.00$
Zel'dovich
Density
14
12
10
8
6
4
2
0
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position



a = 1.00
Zel'dovich
Density
0
2
4
6
8
10
12
14
Position
−π
−π/2
0
π/2
π

Multi-streaming



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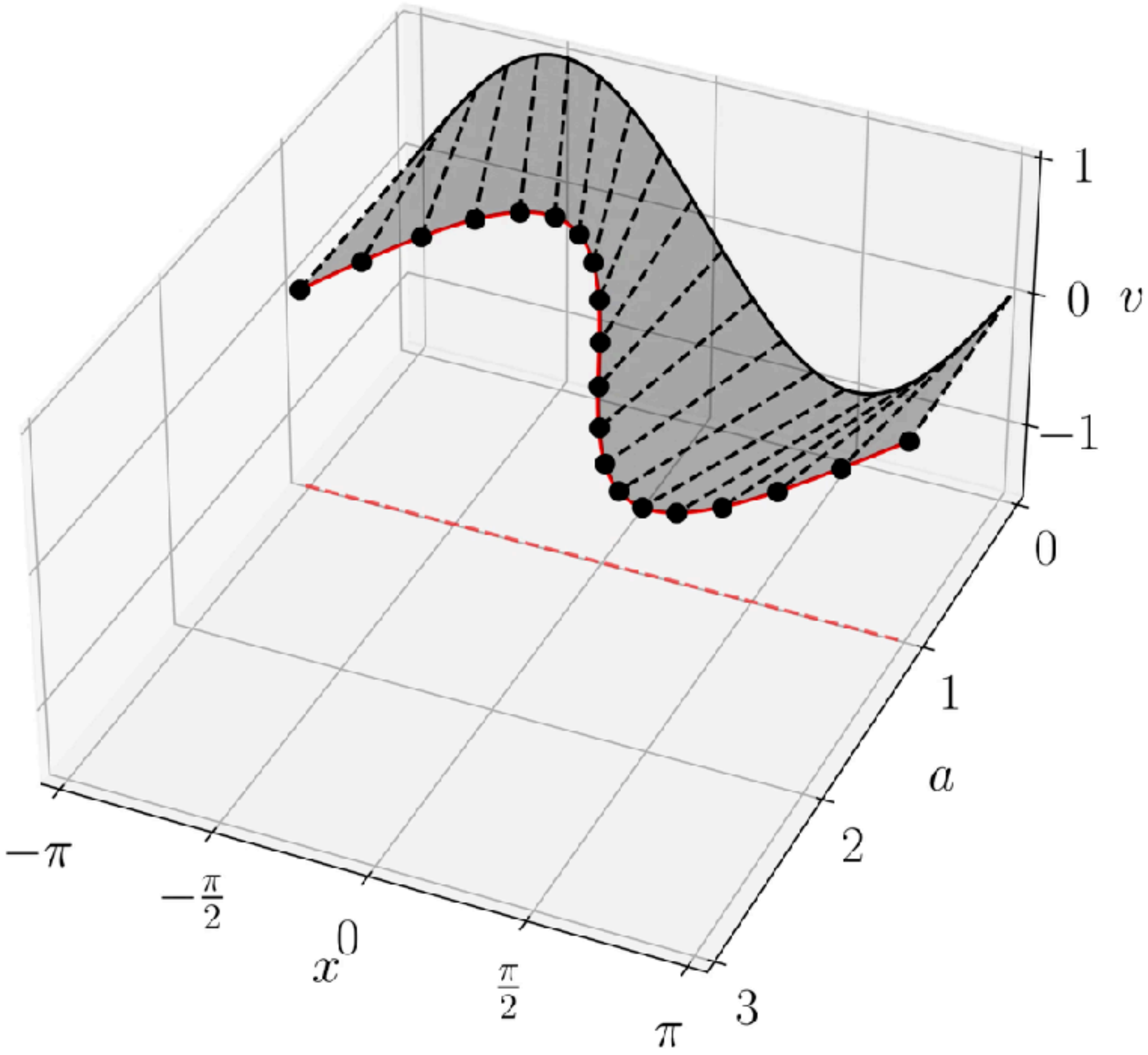
Multi-streaming



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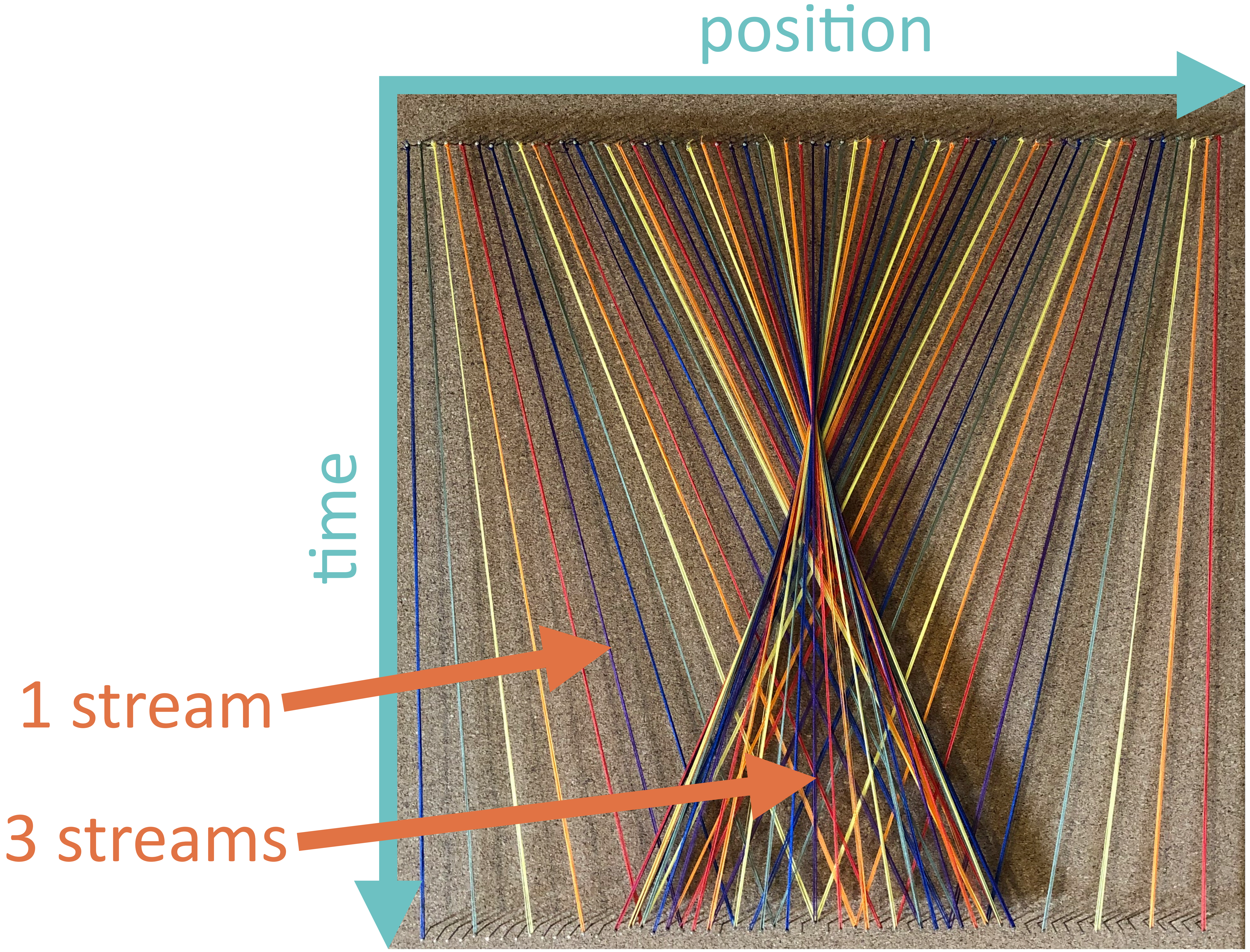
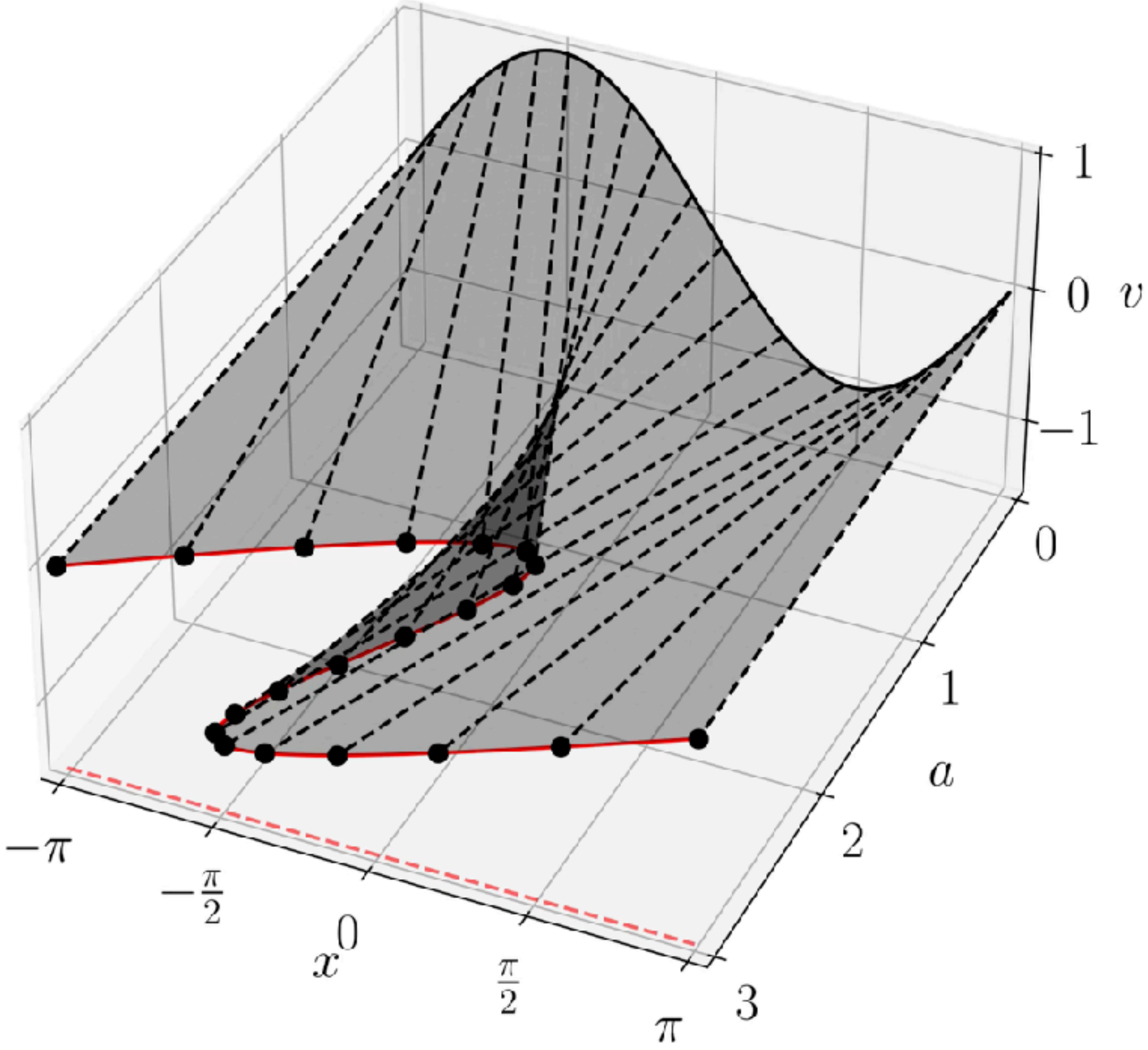
Multi-streaming





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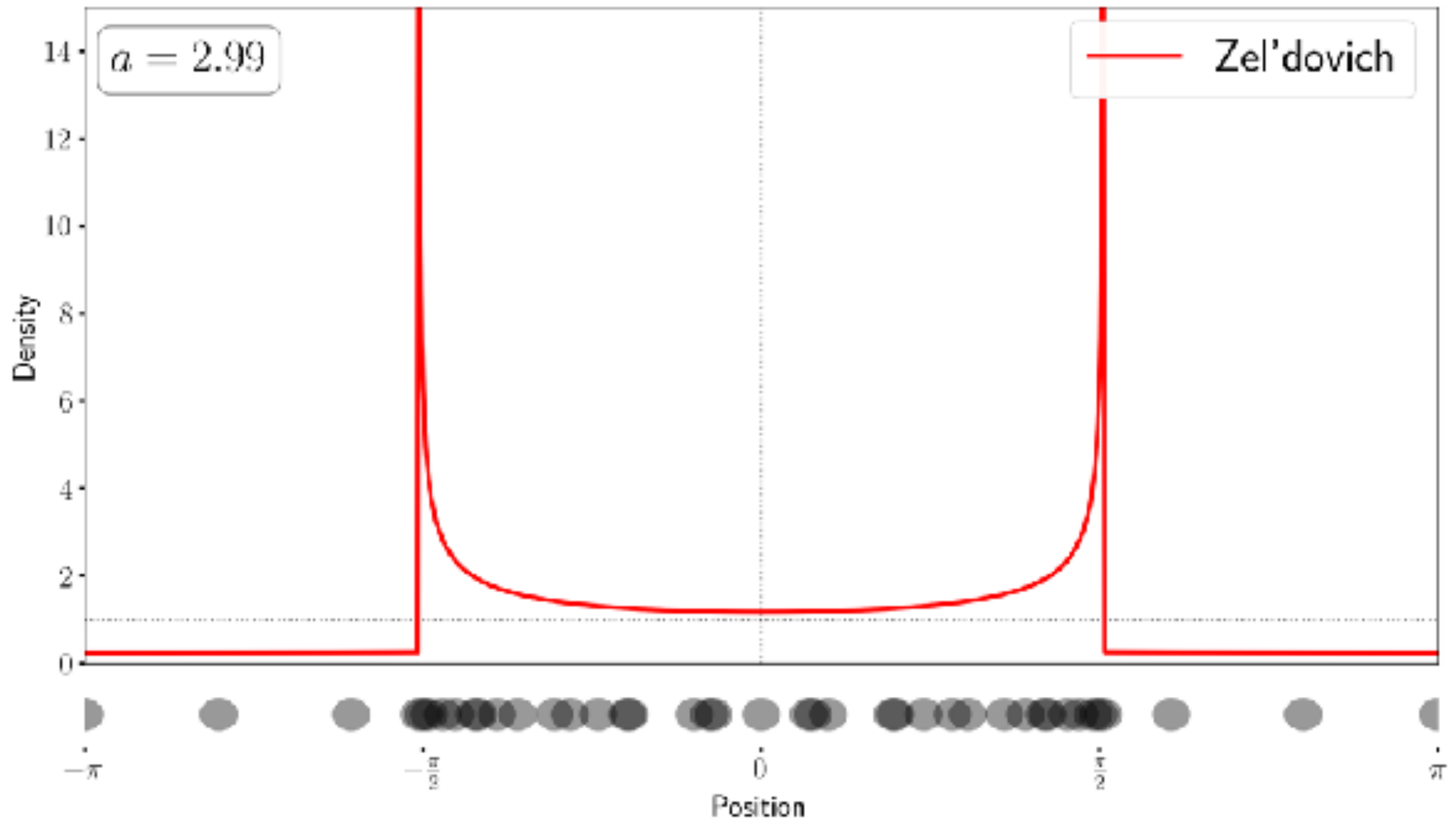
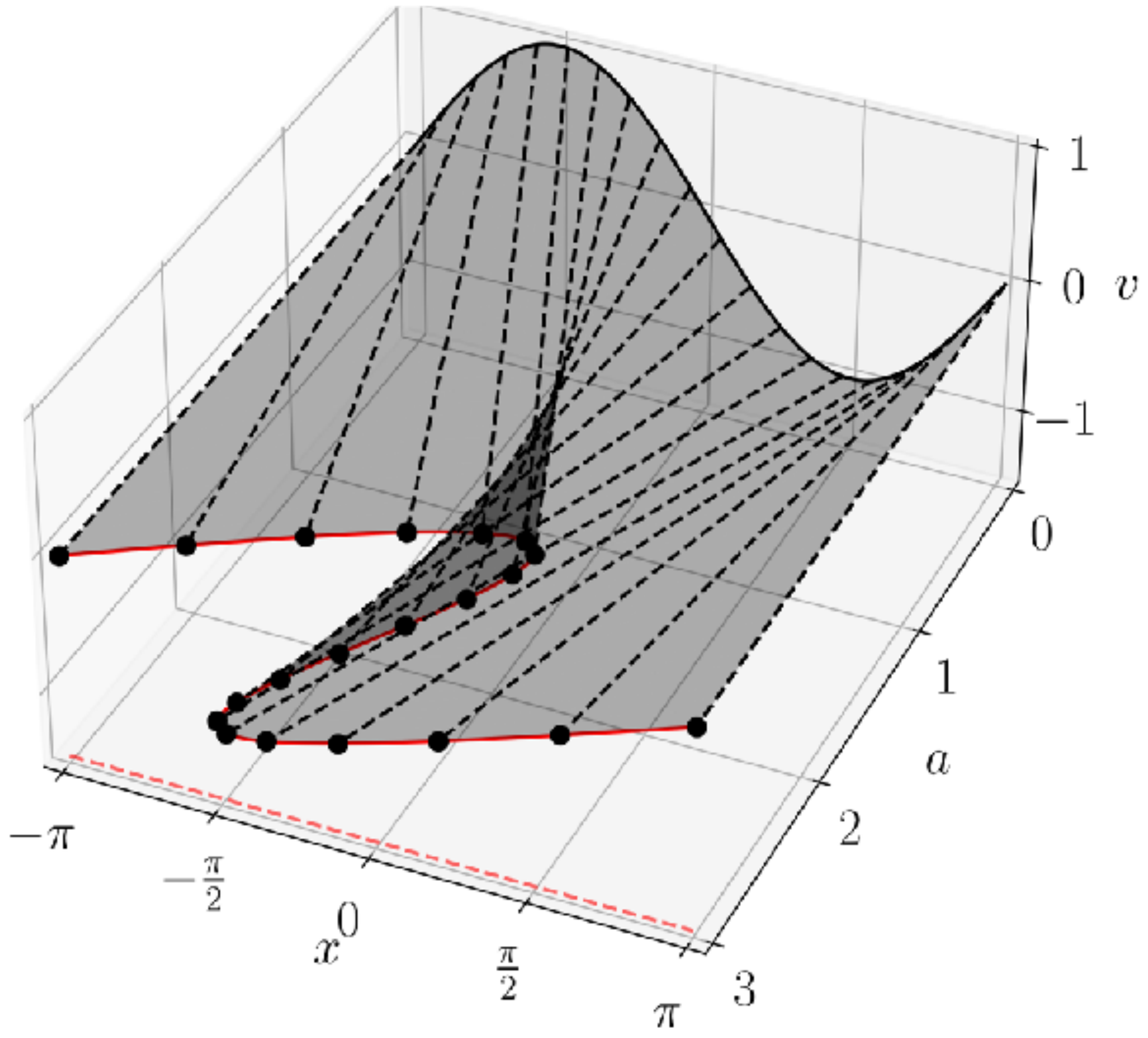
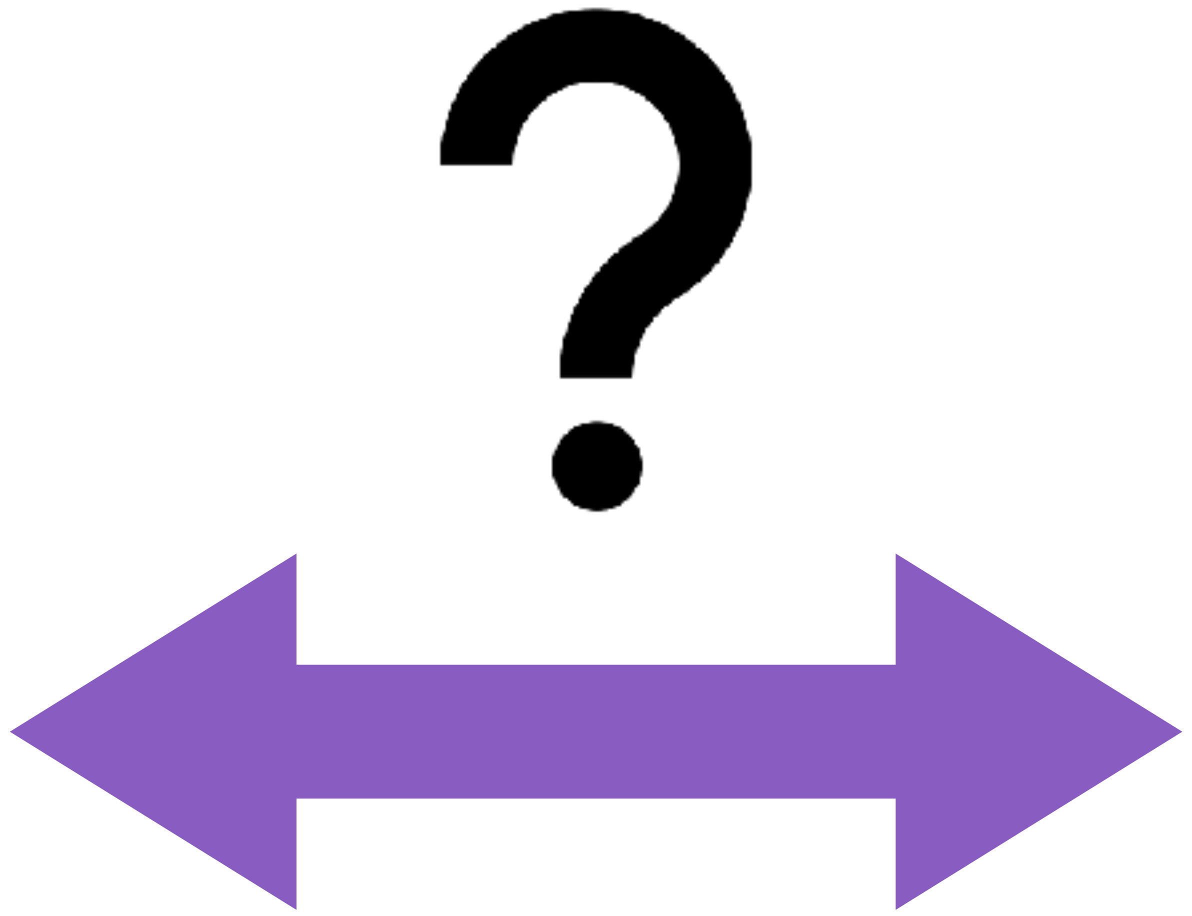
Particles to waves

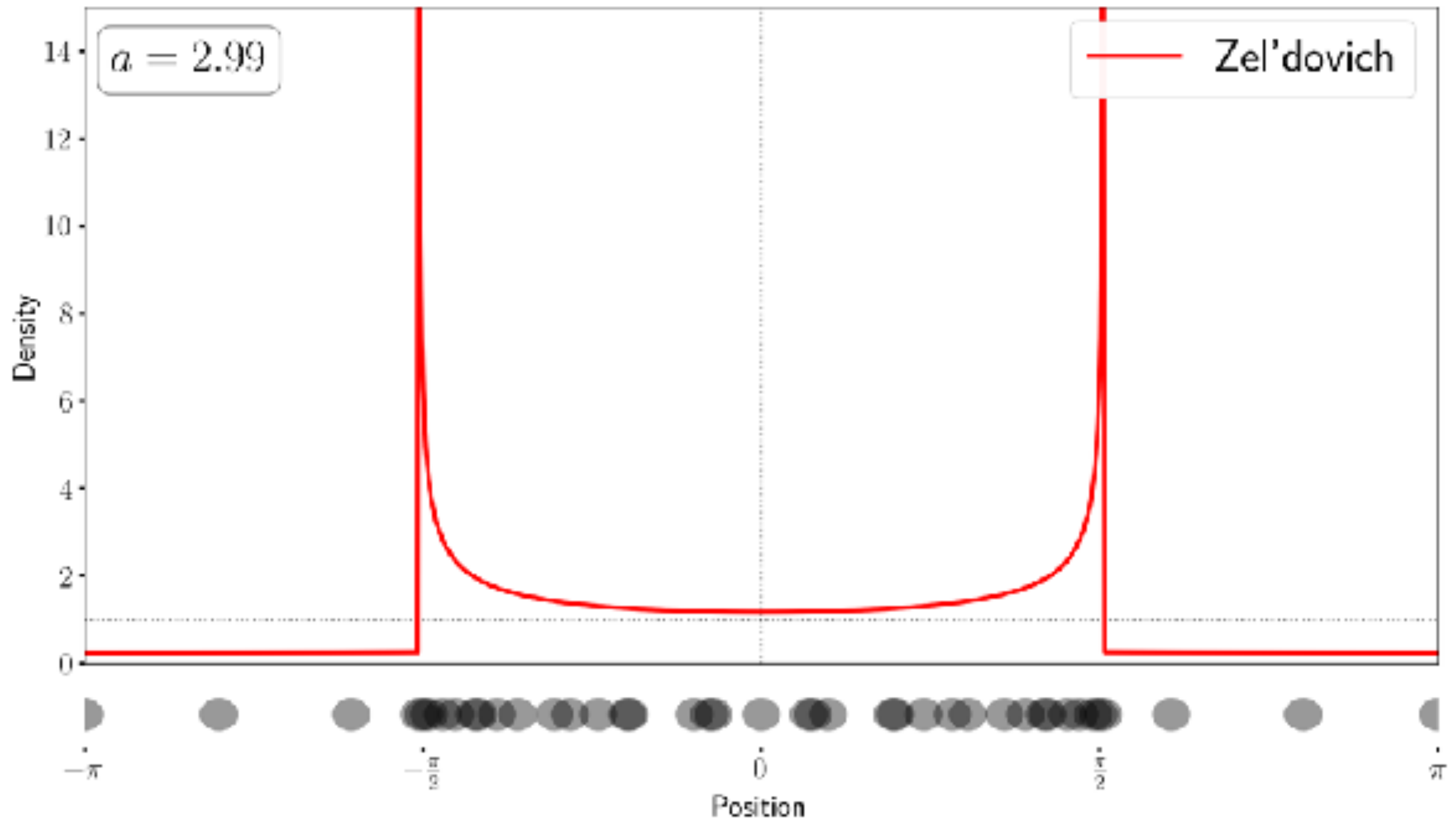


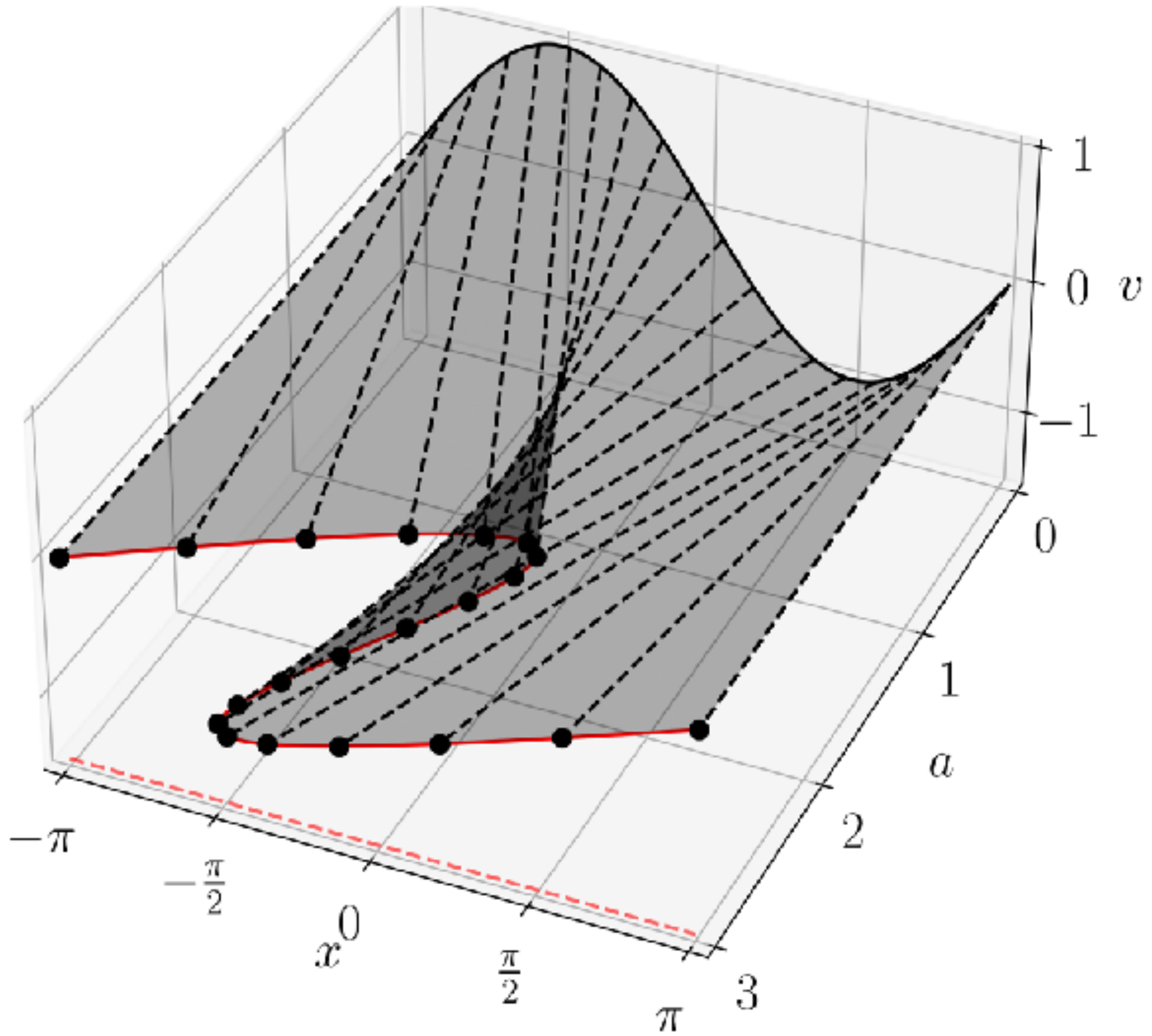




$$\psi(x, a)$$

Wave toy model





Initial conditions

$$\psi = \sqrt{\rho} e^{i\phi_v/\hbar}$$

Uniform density $\rho^{(\text{ini})}(q) = 1$

Sinusoid velocity $\phi_v^{(\text{ini})}(q) = \cos(q)$

Evolution

$$\boxed{\begin{gathered}\psi^{(\text{ini})}(q) = \exp\left(\frac{i}{\hbar}\cos(q)\right) \\ i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\partial_x^2\psi\end{gathered}}$$

Toy Model

Propagator formalism

Solving wavefunction

- Easy! Use your favourite method (e.g. FFT)
- Useful for us to write solution in particular form

$$\psi(x,a) \sim \int \mathrm{d}q \, \underbrace{K_0(q;x,a)\psi^{(\mathrm{ini})}(q)}_{\exp\left[\frac{i}{\hbar}\zeta(q;x,a)\right]}$$

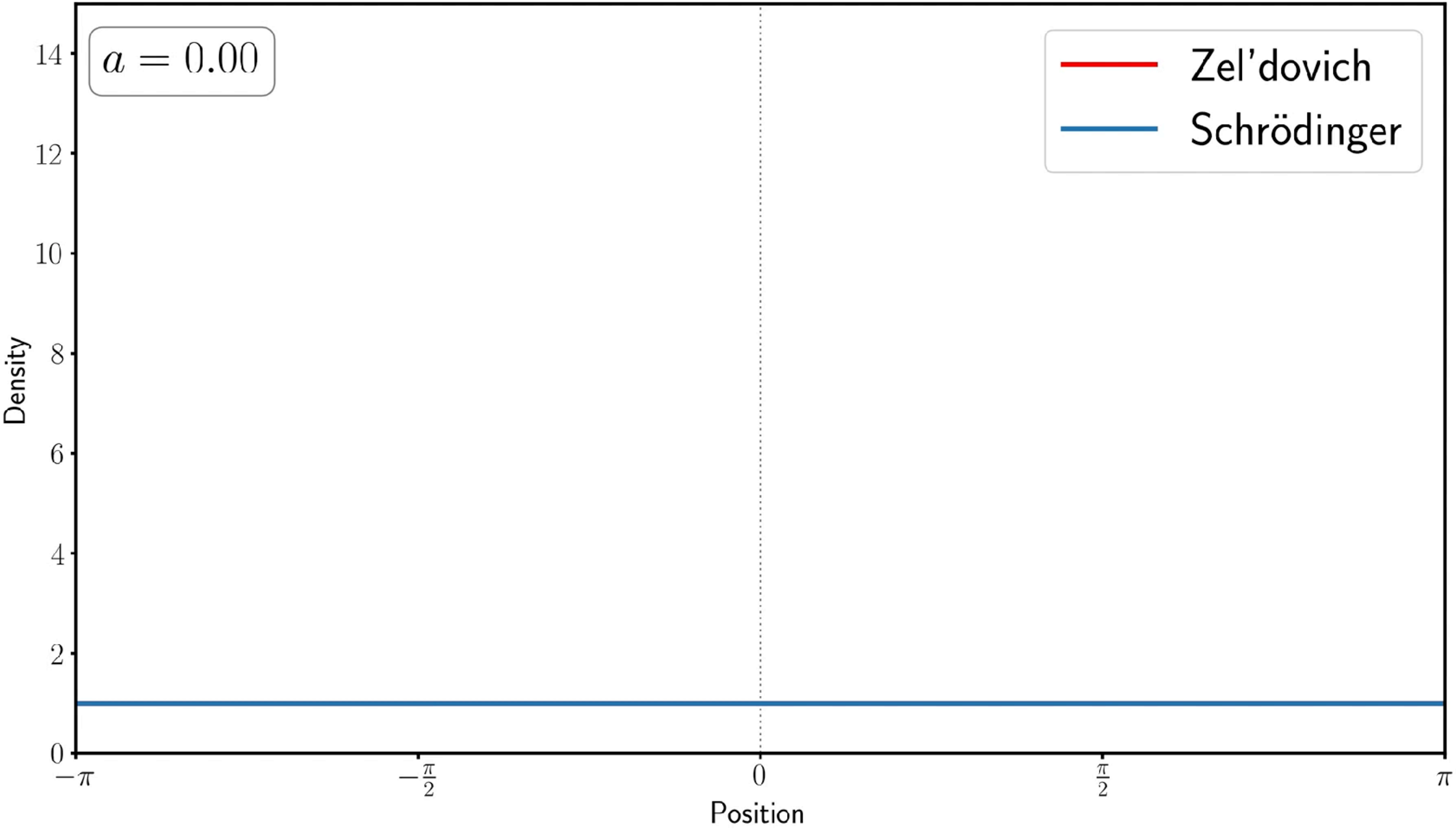
$$\psi^{(\mathrm{ini})}(q) = \exp\left(\frac{i}{\hbar}\cos(q)\right)$$

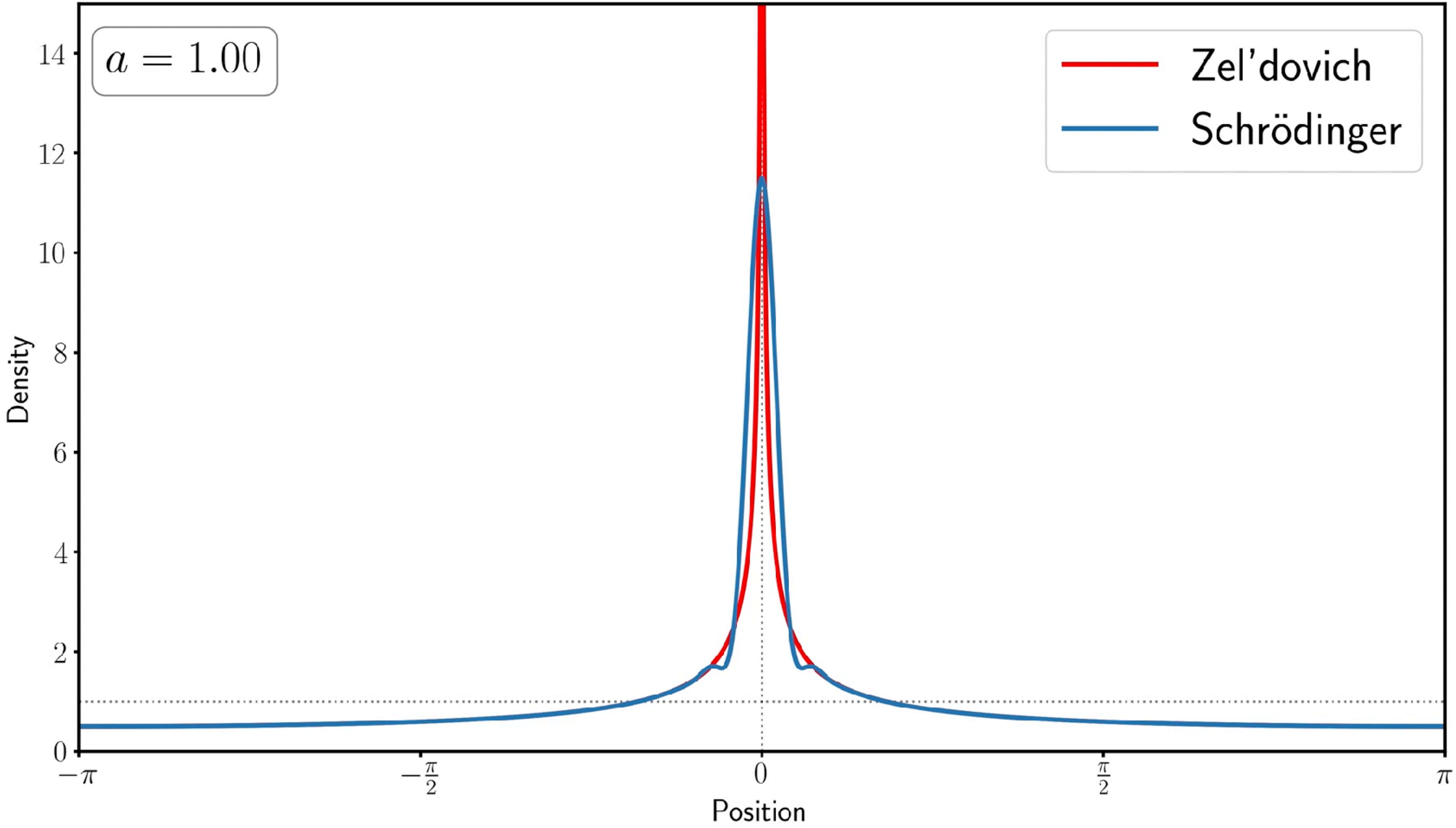
$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\partial_x^2\psi$$

Toy Model

- $\zeta(q;x,a)$ contains the *action* and the *initial conditions*

$$\psi(x,a) \propto \int \mathrm{d}q \; \exp\left(\frac{i}{\hbar}\left[\frac{(x-q)^2}{2a} + \cos(q)\right]\right)$$


$a = 0.00$
Zel'dovich
Schrödinger
Density
Position
0
2
4
6
8
10
12
14
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π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Zel'dovich
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Density
0
2
4
6
8
10
12
14
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position

Free wave evolution

Amplitude: brightness

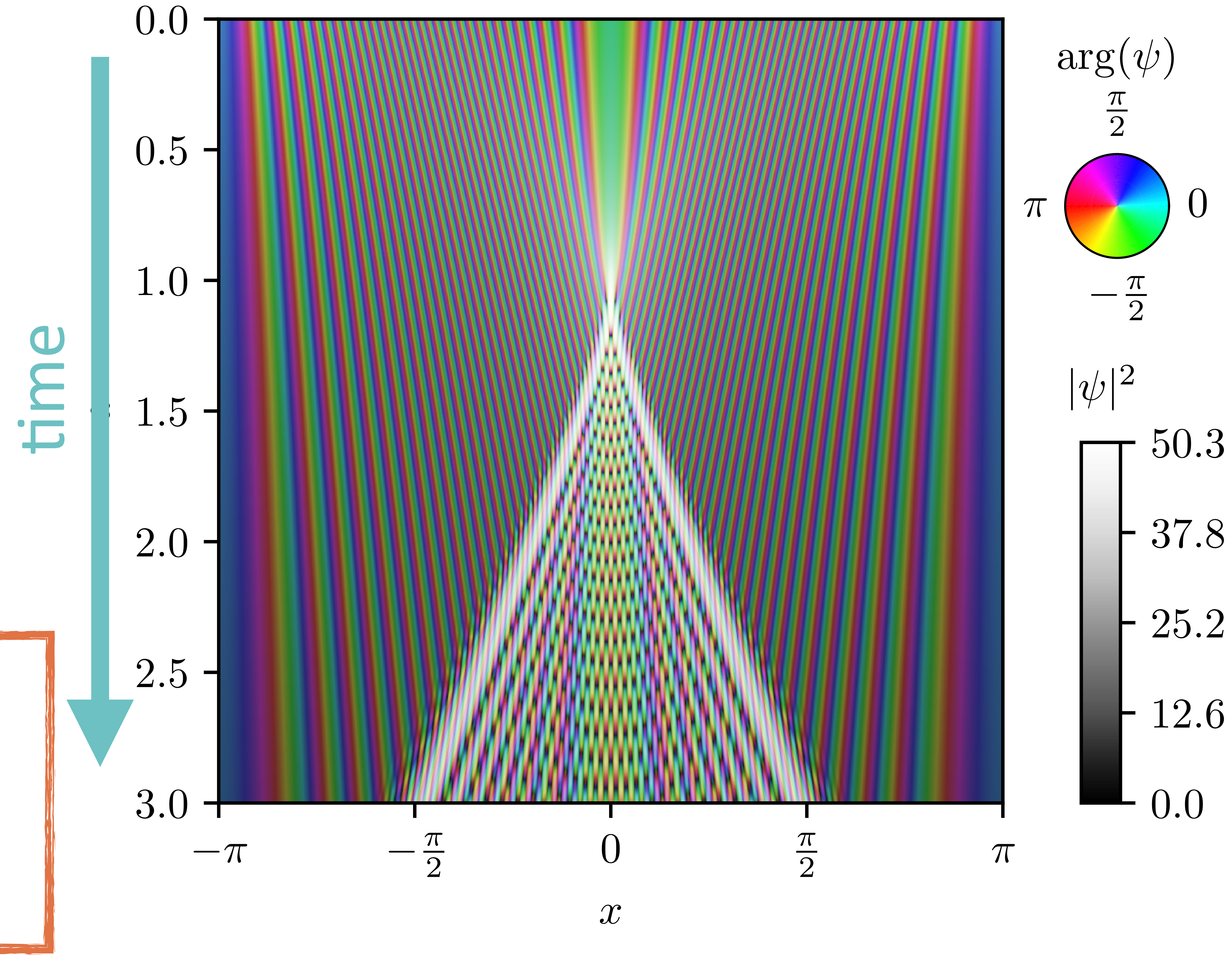
Phase: colour

Features

- Regularised caustic
- Interference

Understanding the caustic

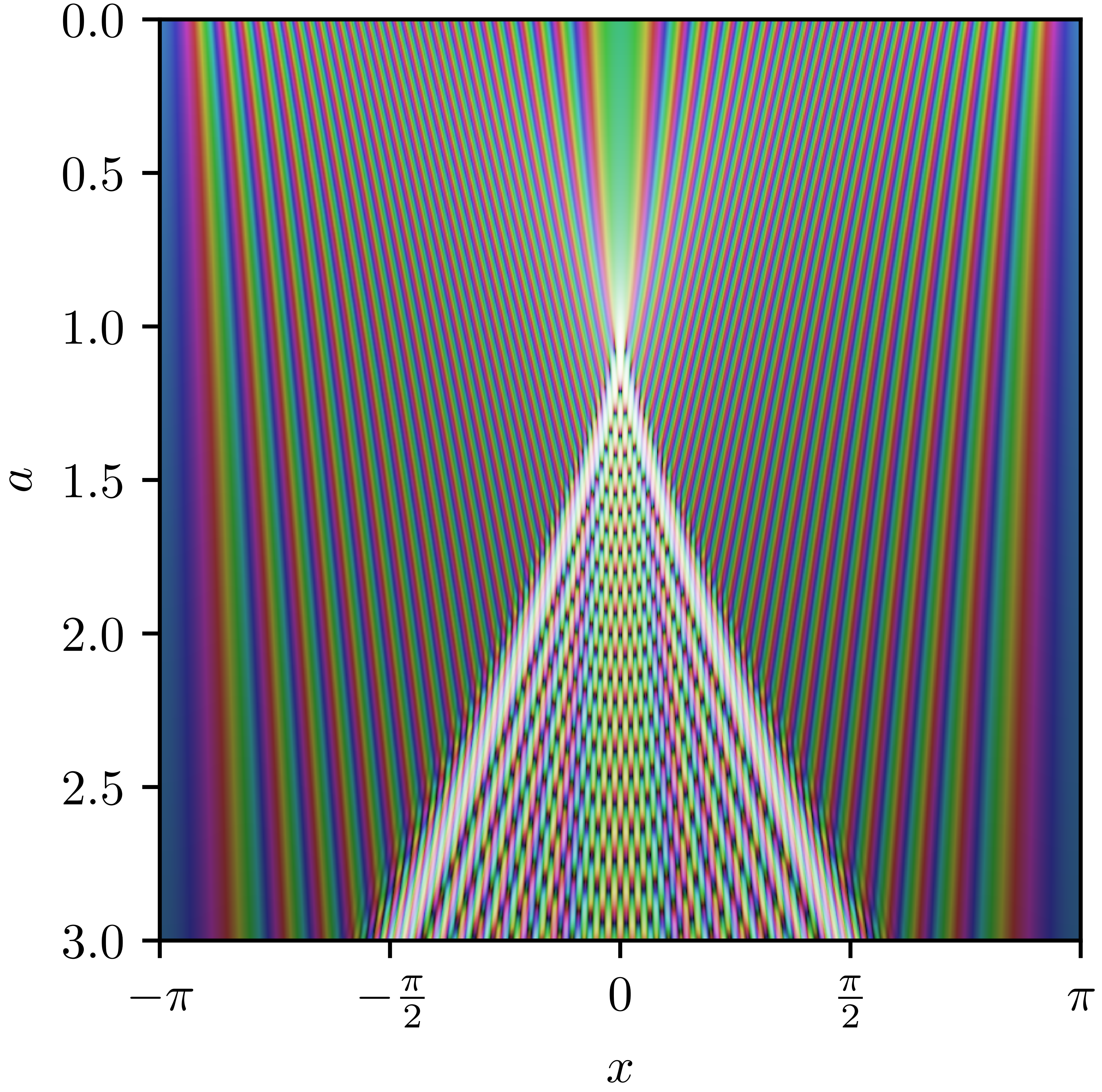
Interference features



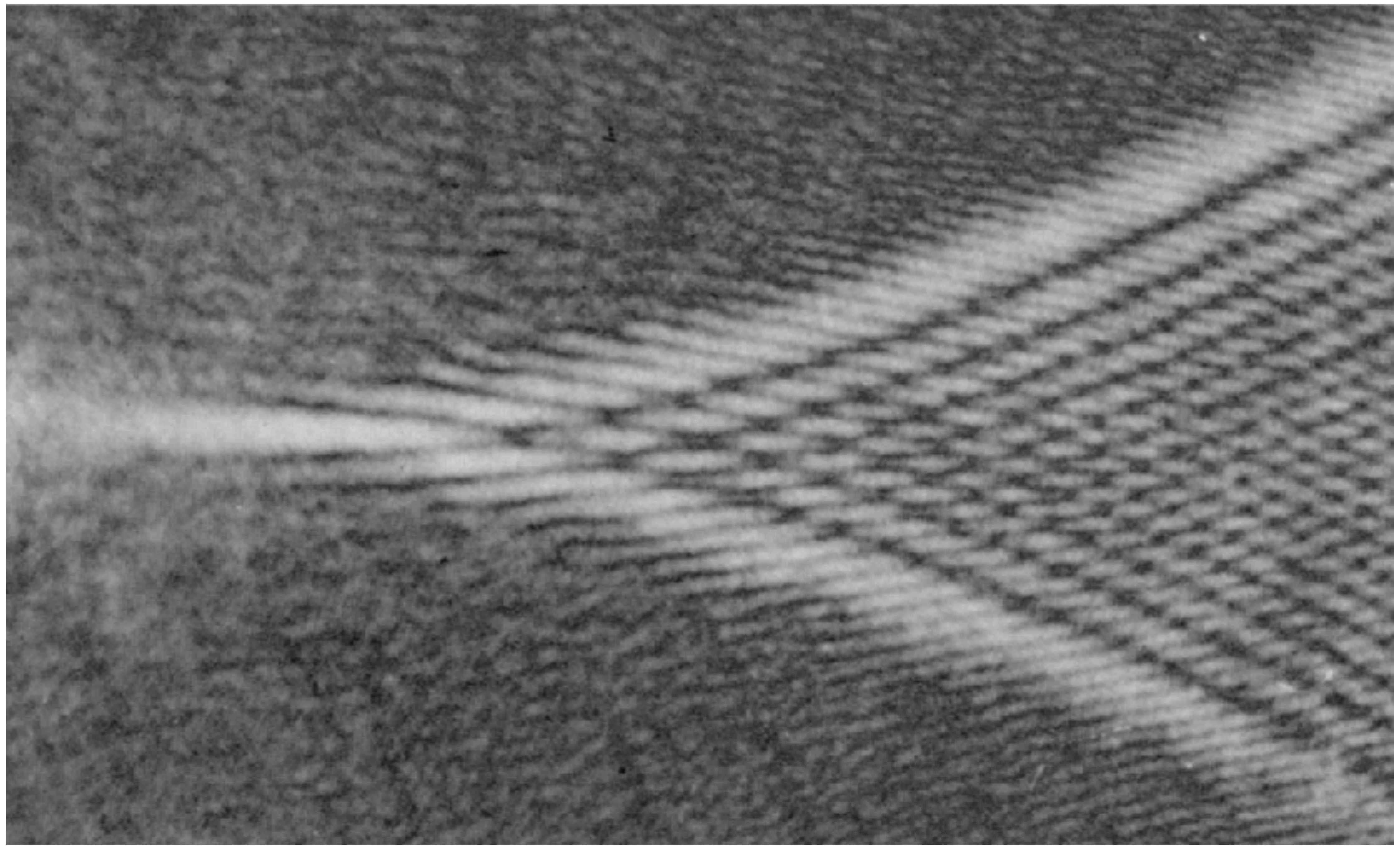
Uhlemann++ PRD 2019
Gough & Uhlemann 2022

Optics analogy

Dark matter

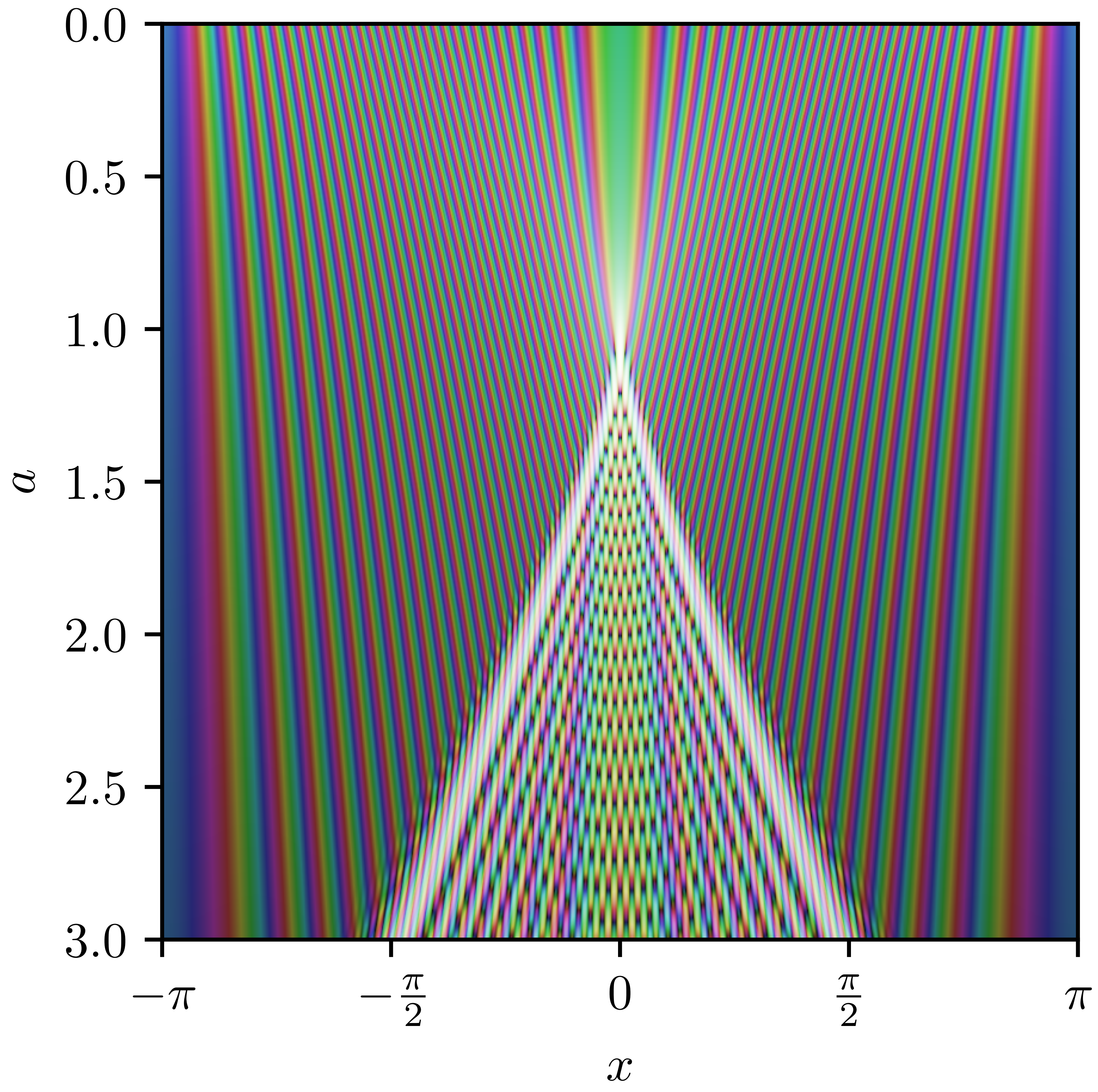


Optics

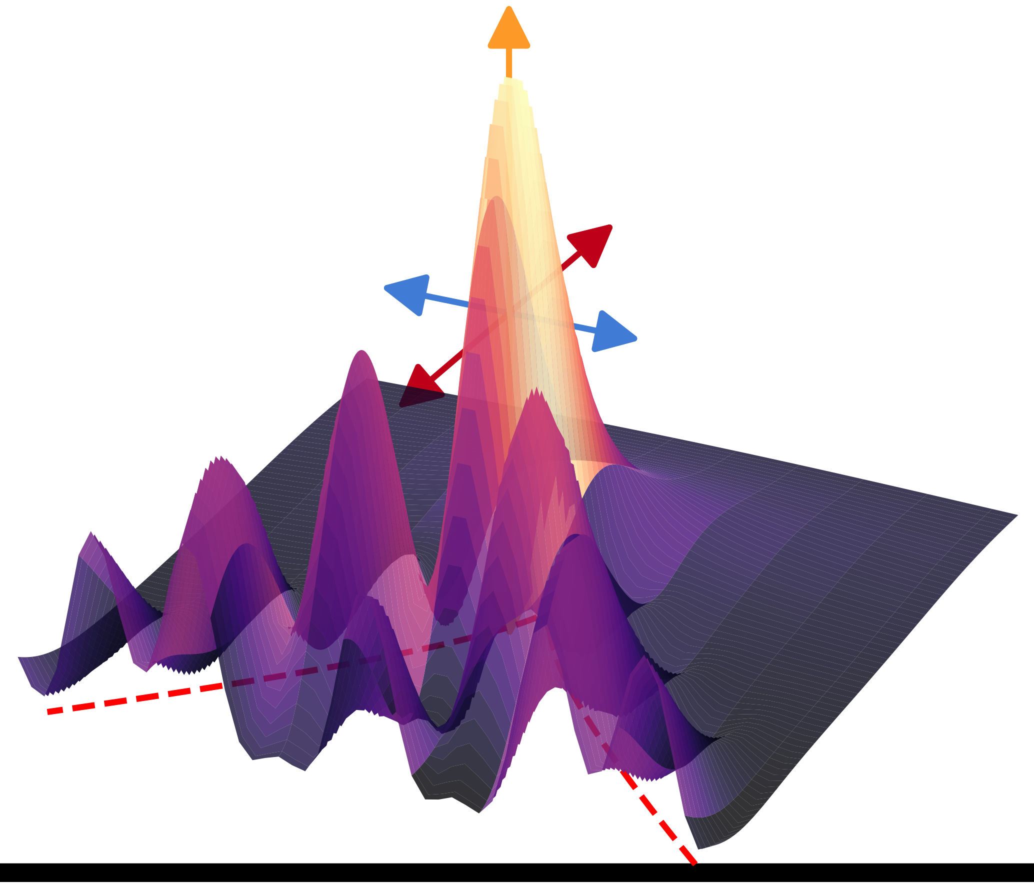


Berry, Nye, Wright `79

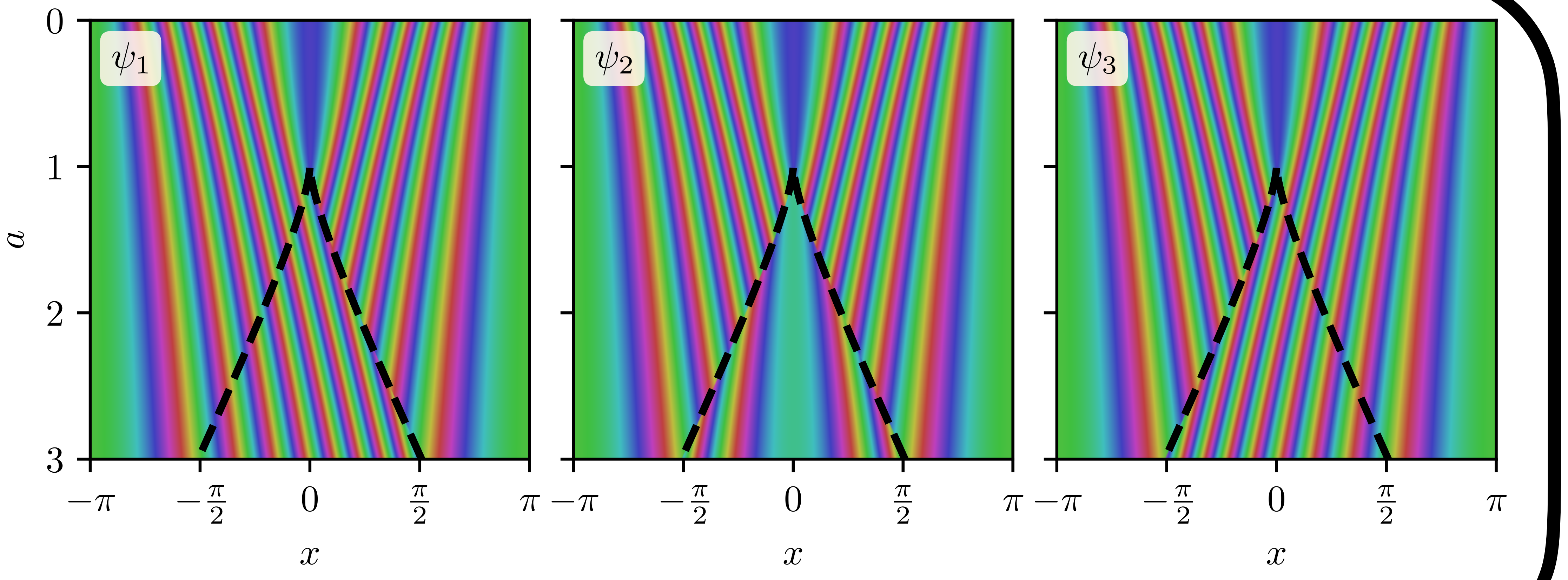
Unweaving the wavefunction



Caustic classification

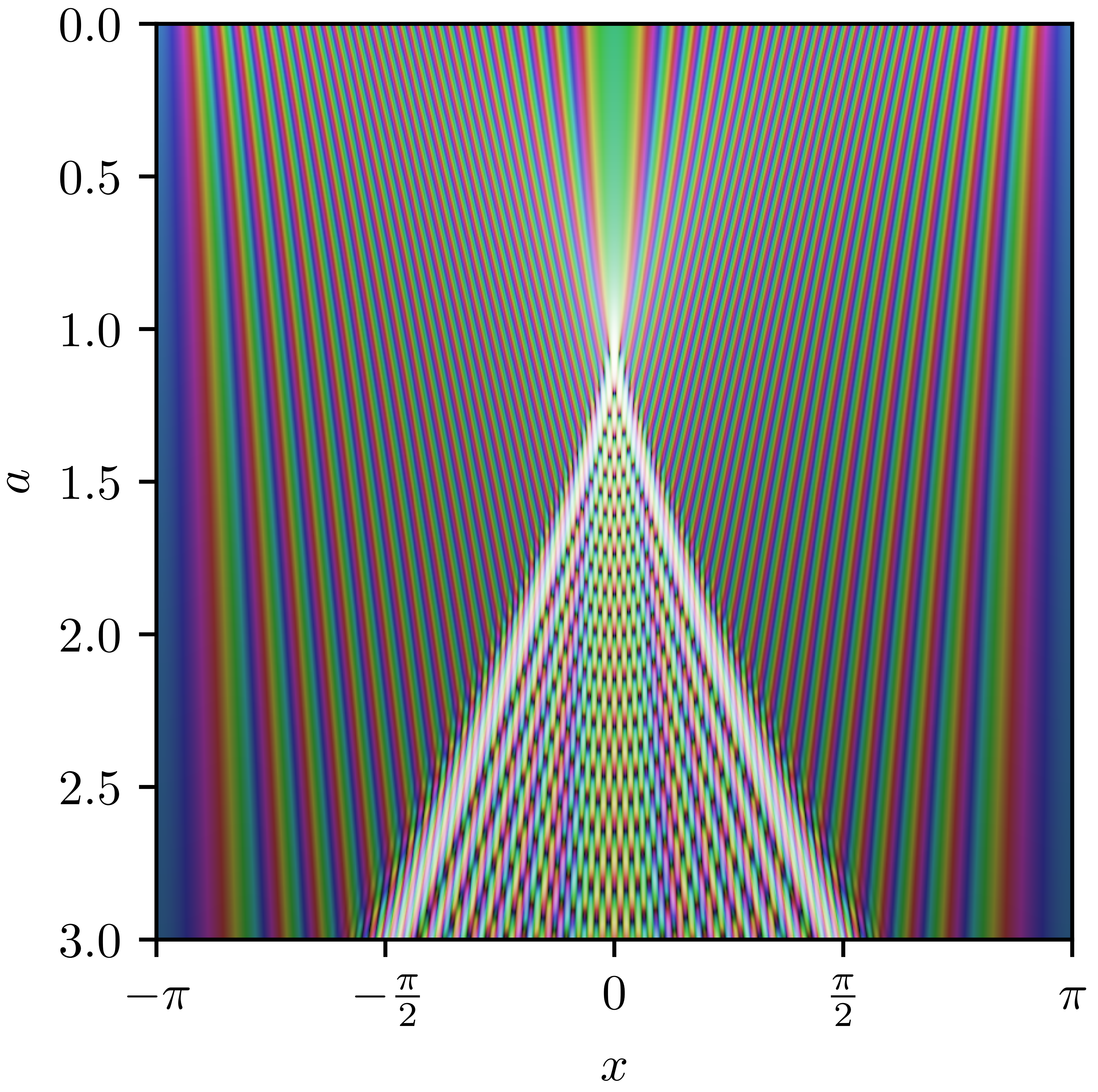


Stream components



Gough & Uhlemann 2022

Unweaving the wavefunction



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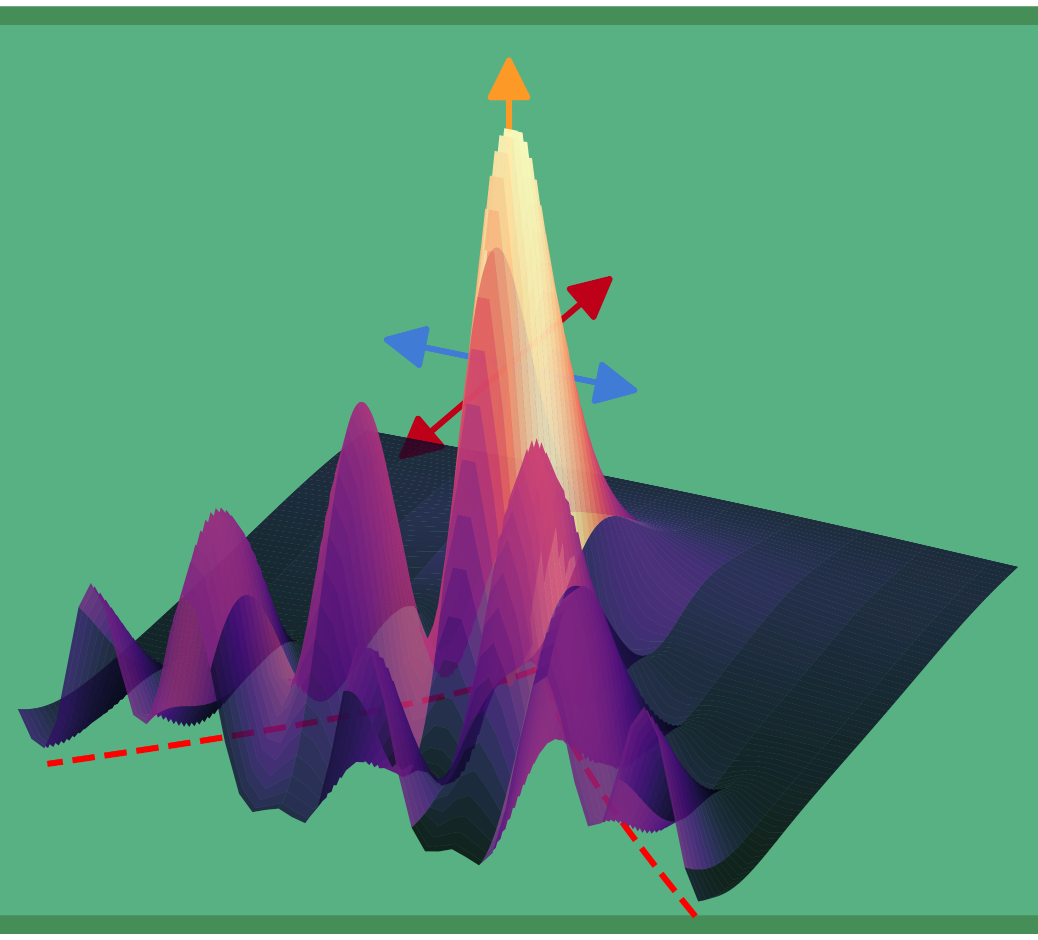
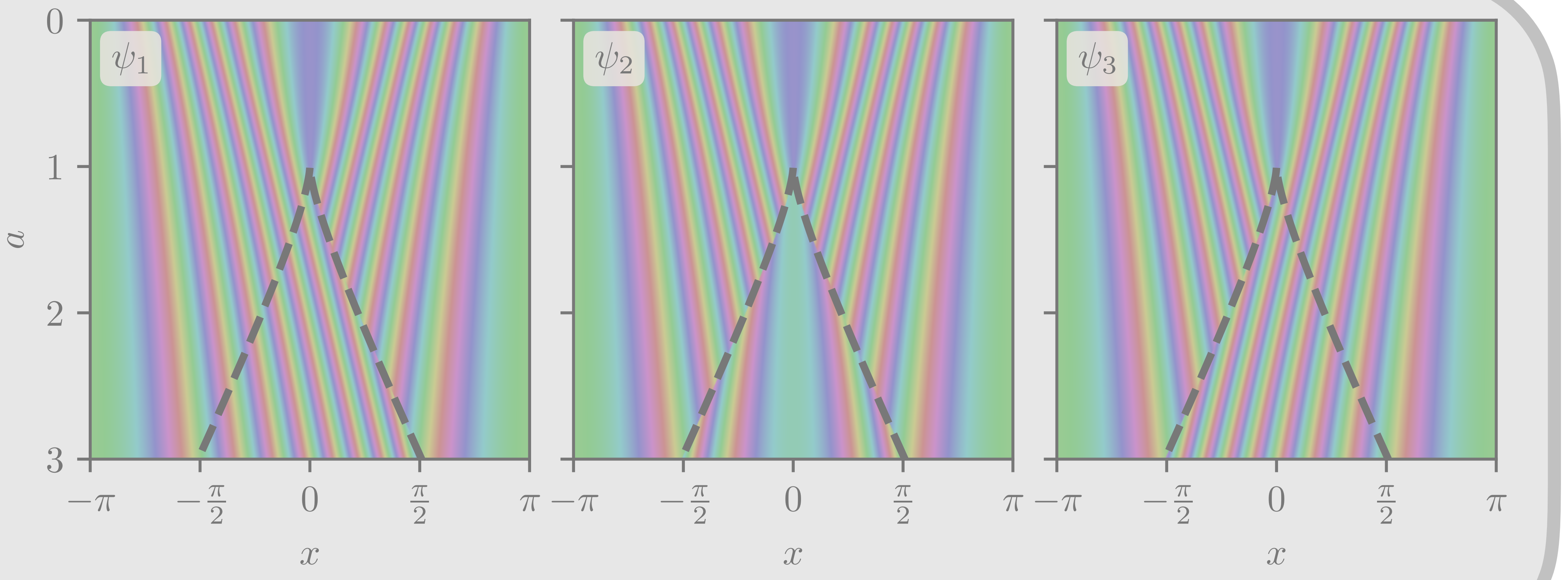


Stream components

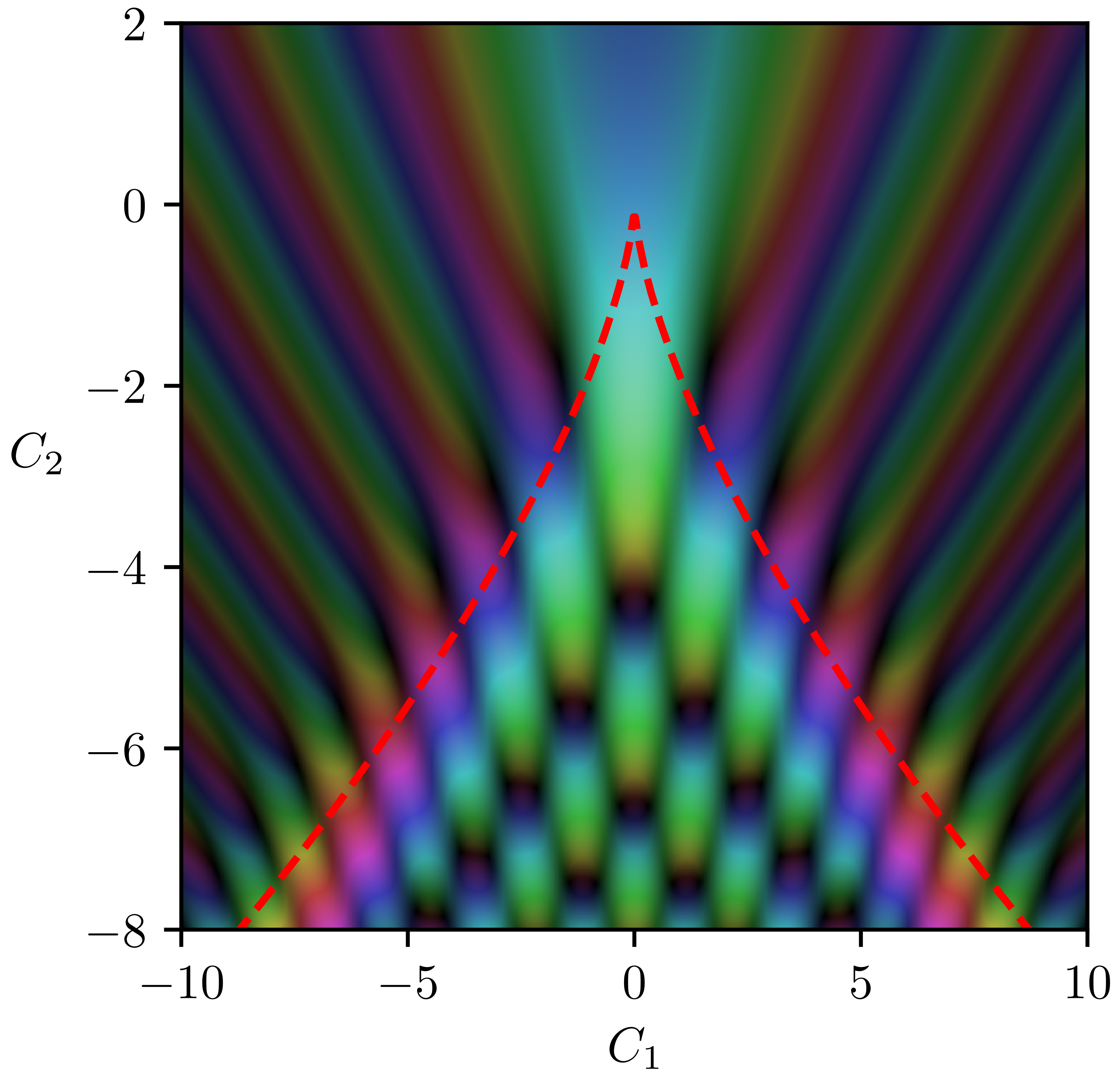


Gough & Uhlemann 2022

Caustic features

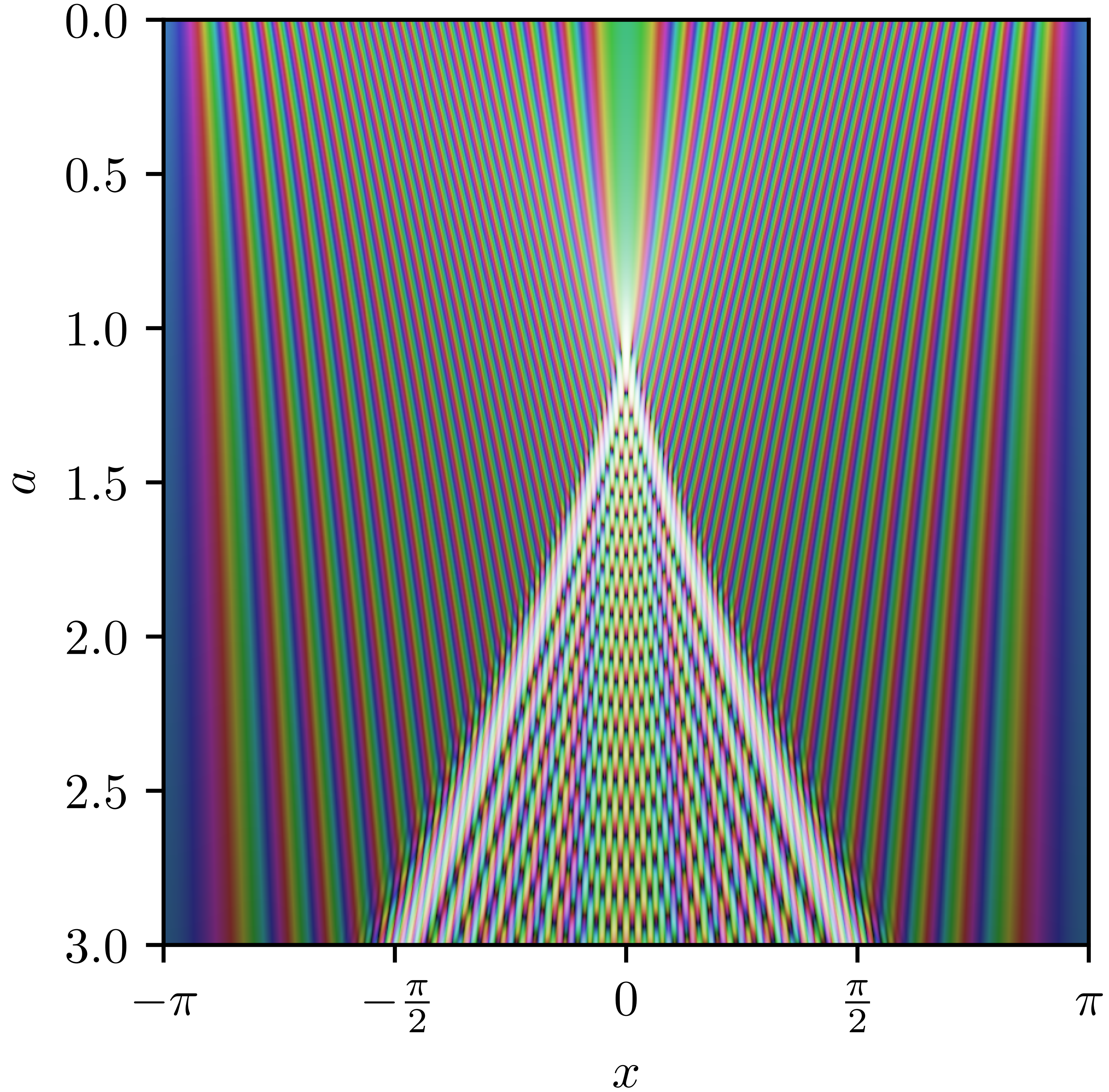
Canonical cusp ($\lambda = 1$)

$$\zeta_{\mathrm{cusp}}(s; C_1, C_2) = \frac{s^4}{4} + C_2\frac{s^2}{2} + C_1 s$$



Wave model ($\hbar = 0.01$)

$$\zeta_{\mathrm{DM}}(q; x, a) = \frac{(x-q)^2}{2a} + \cos(q)$$

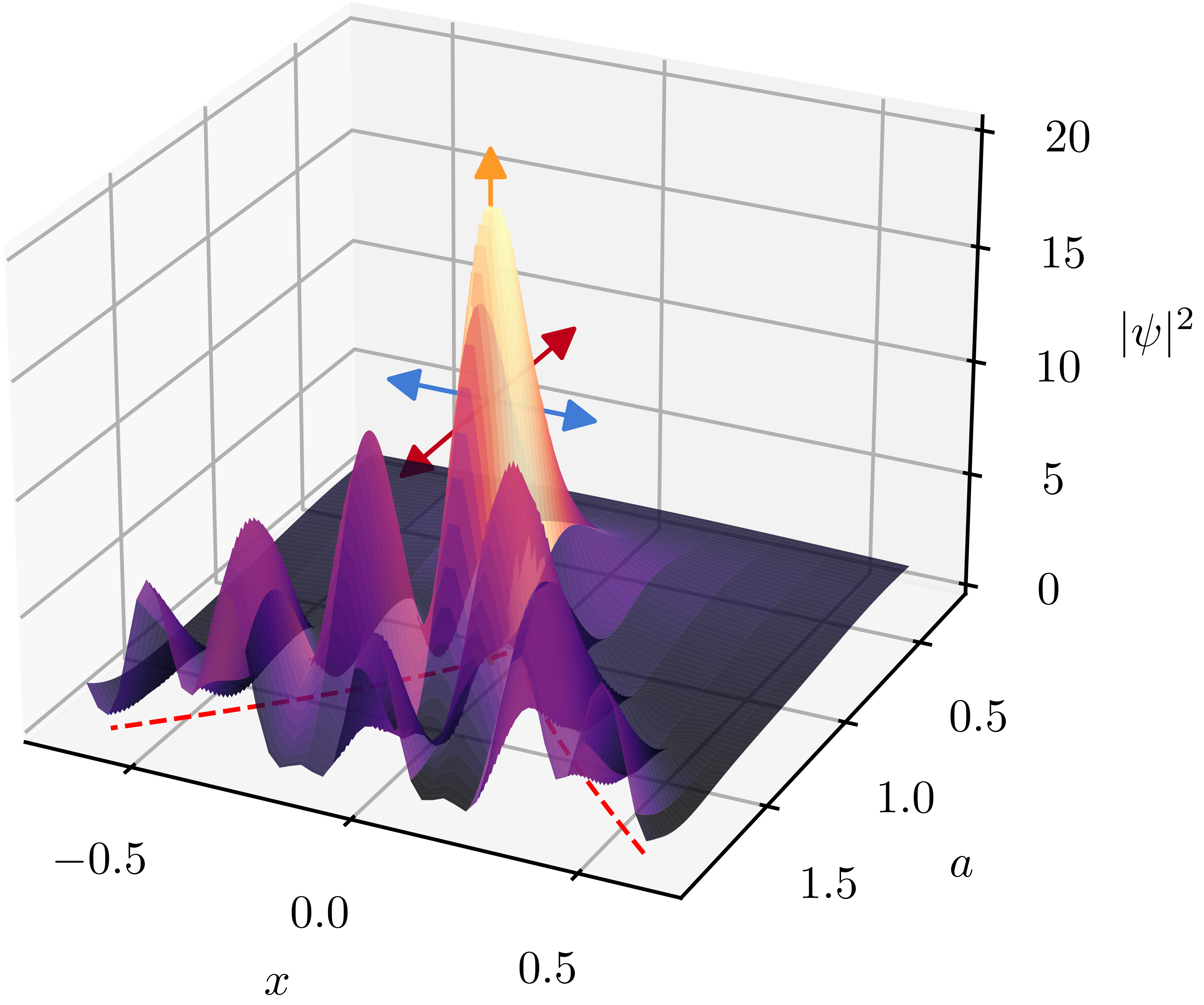


Universal properties

Can always smoothly transform a stable singularity into one of the standard forms

Some properties of wave field preserved by these smooth transformations

- maximum amplitude
- fringe spacing



Gough & Uhlemann 2022

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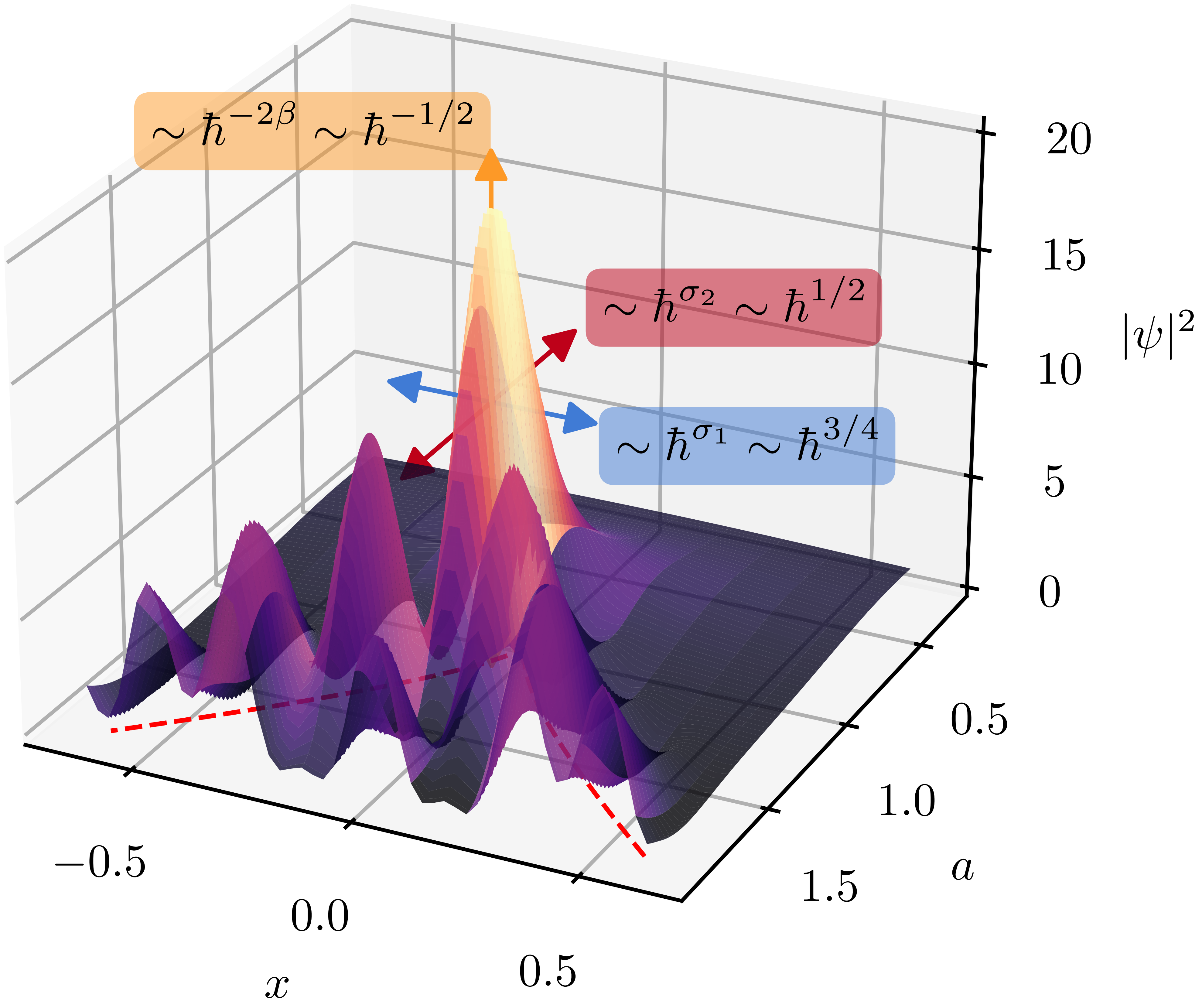
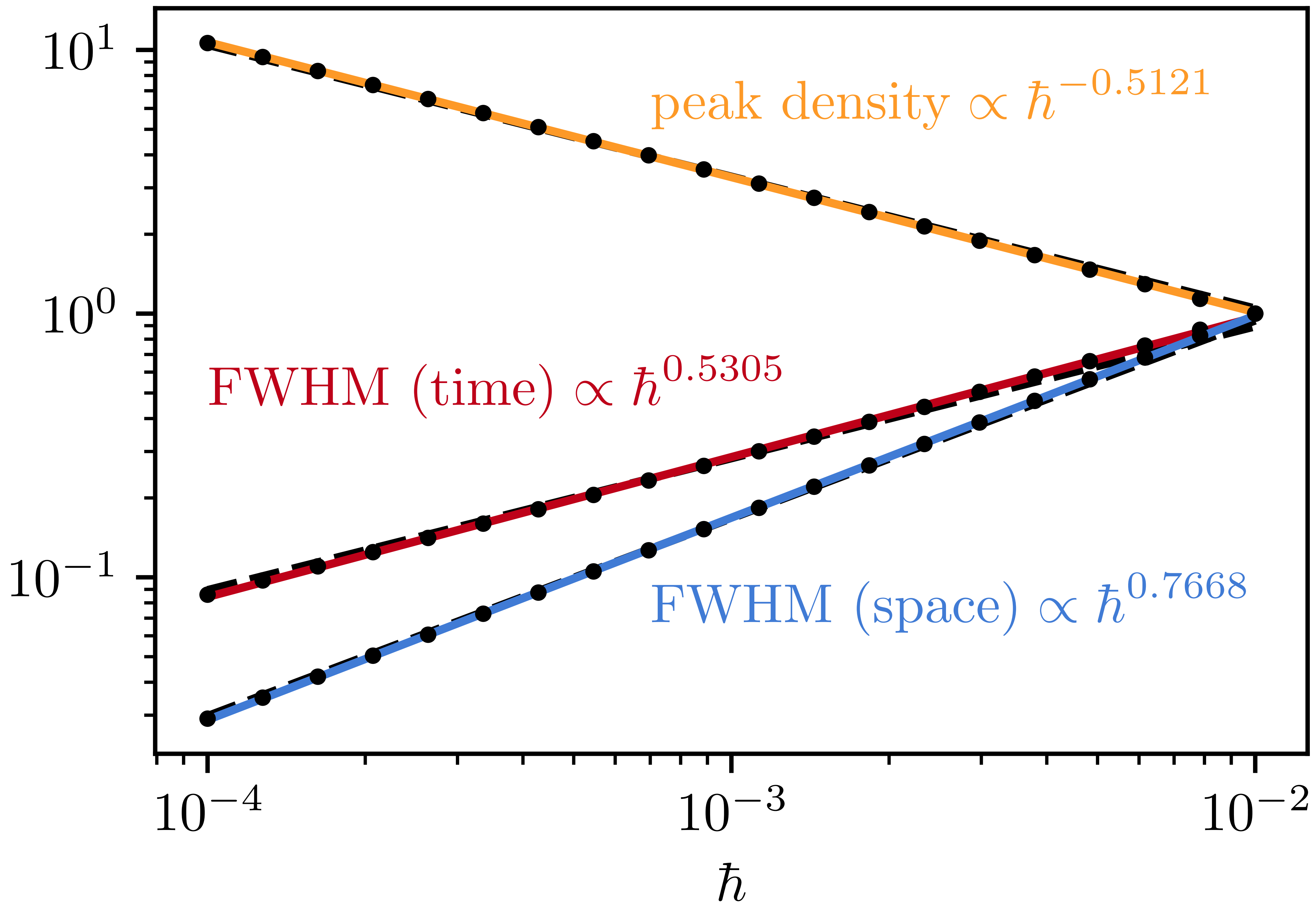
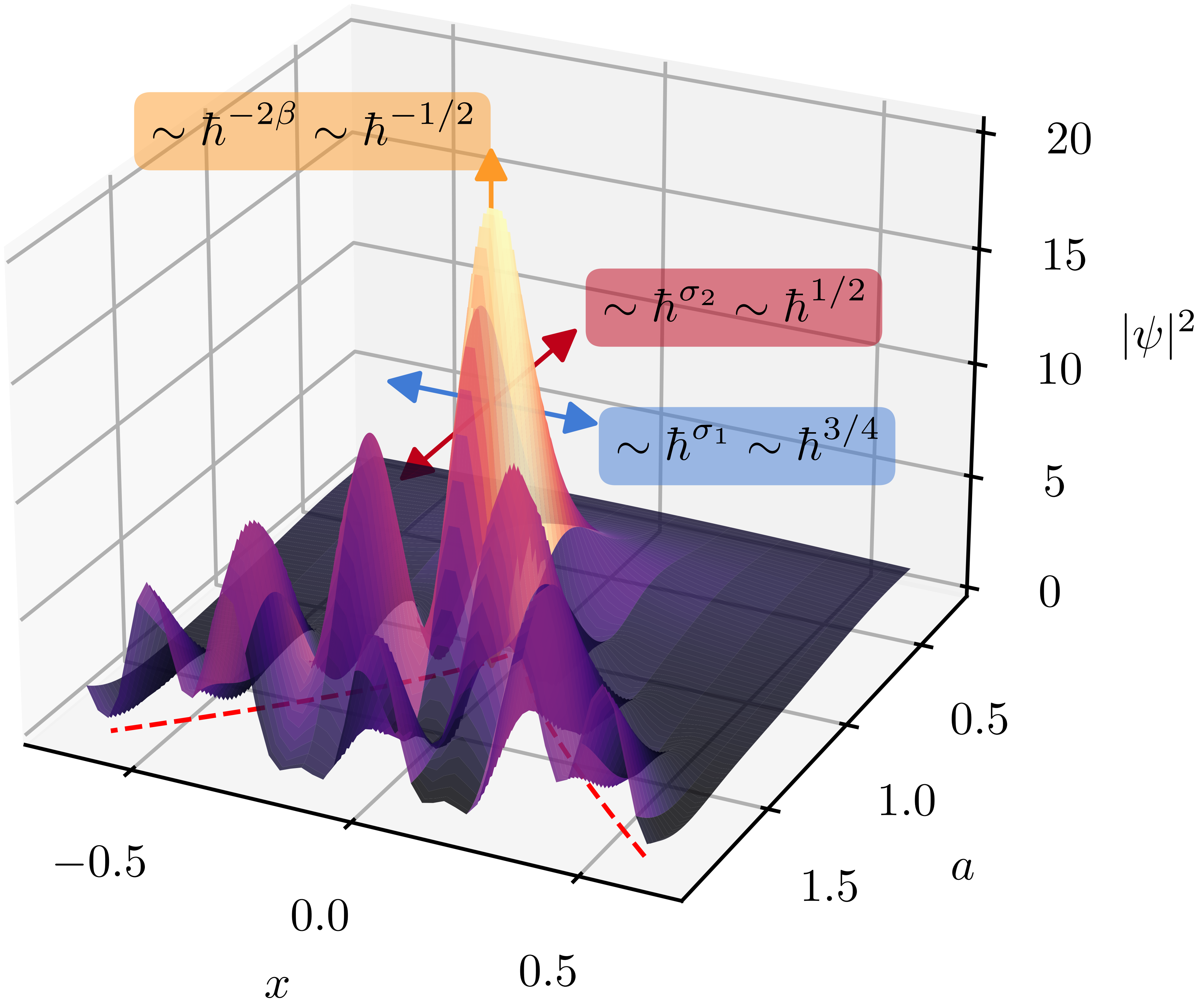


Gough & Uhlemann 2022

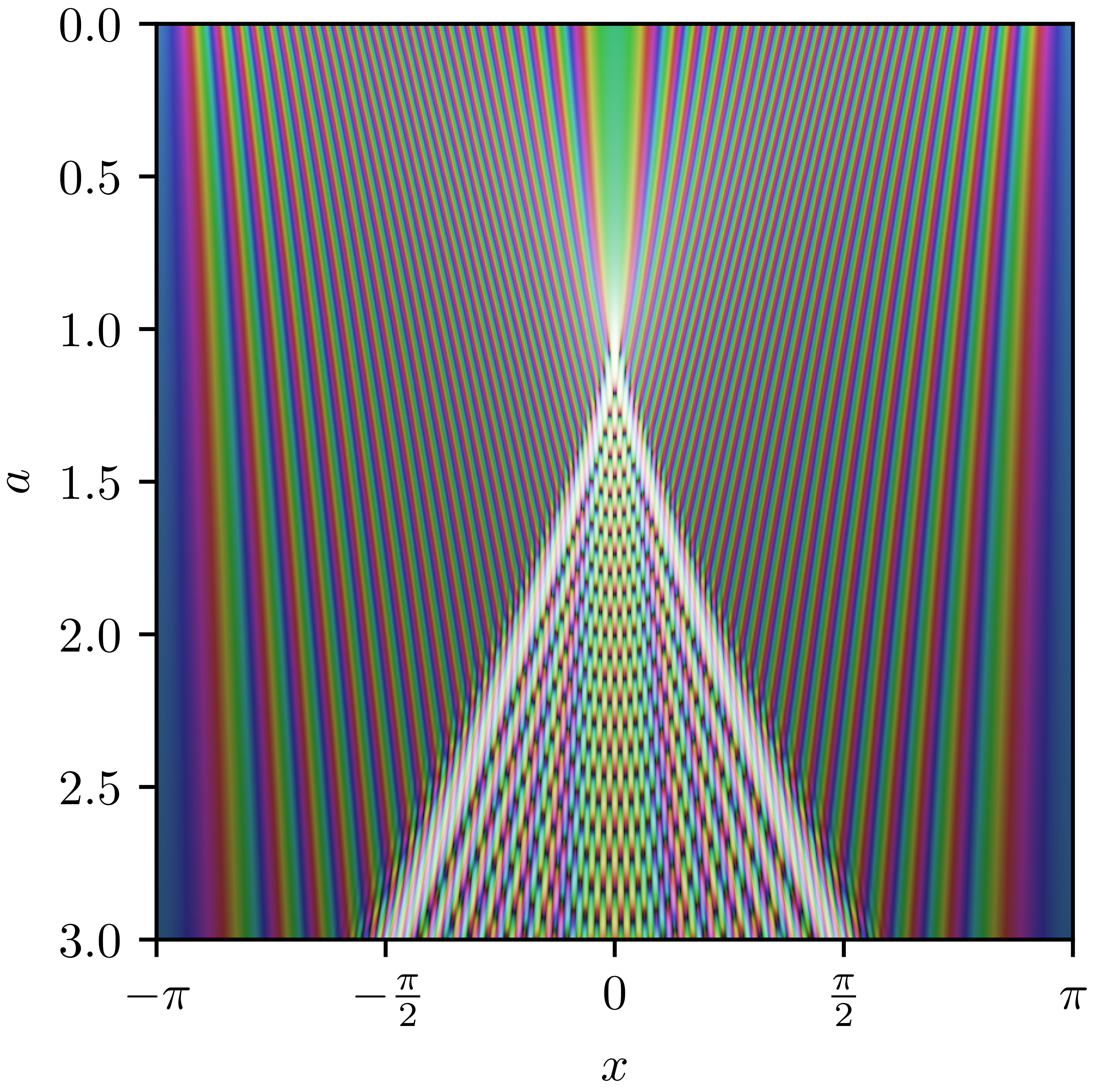
Universal properties



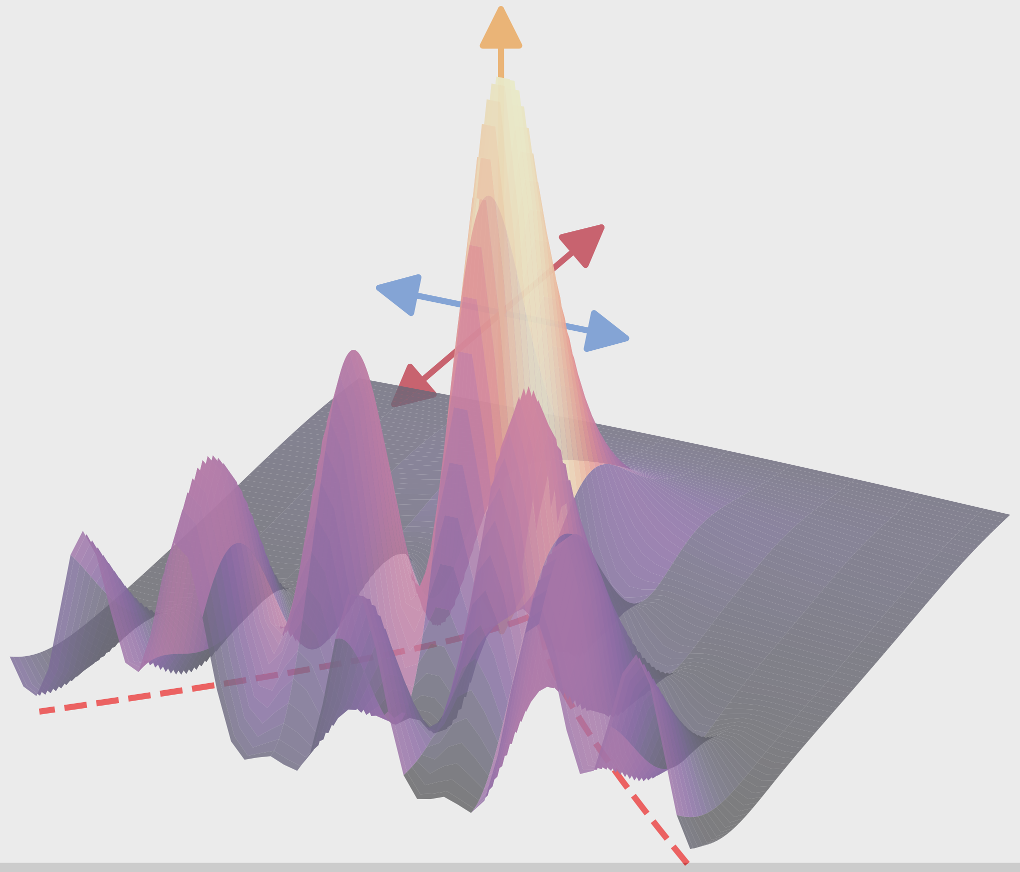


Gough & Uhlemann 2022

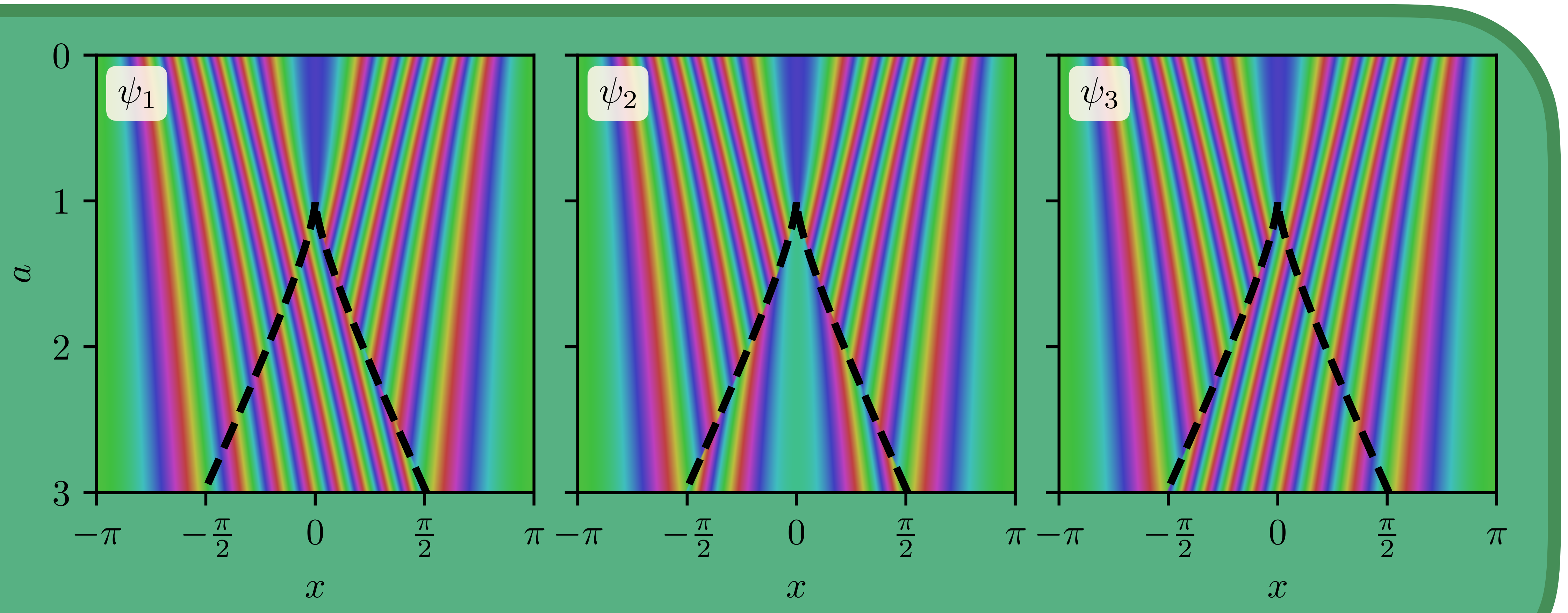
Unravelling the wavefunction



Caustic classification



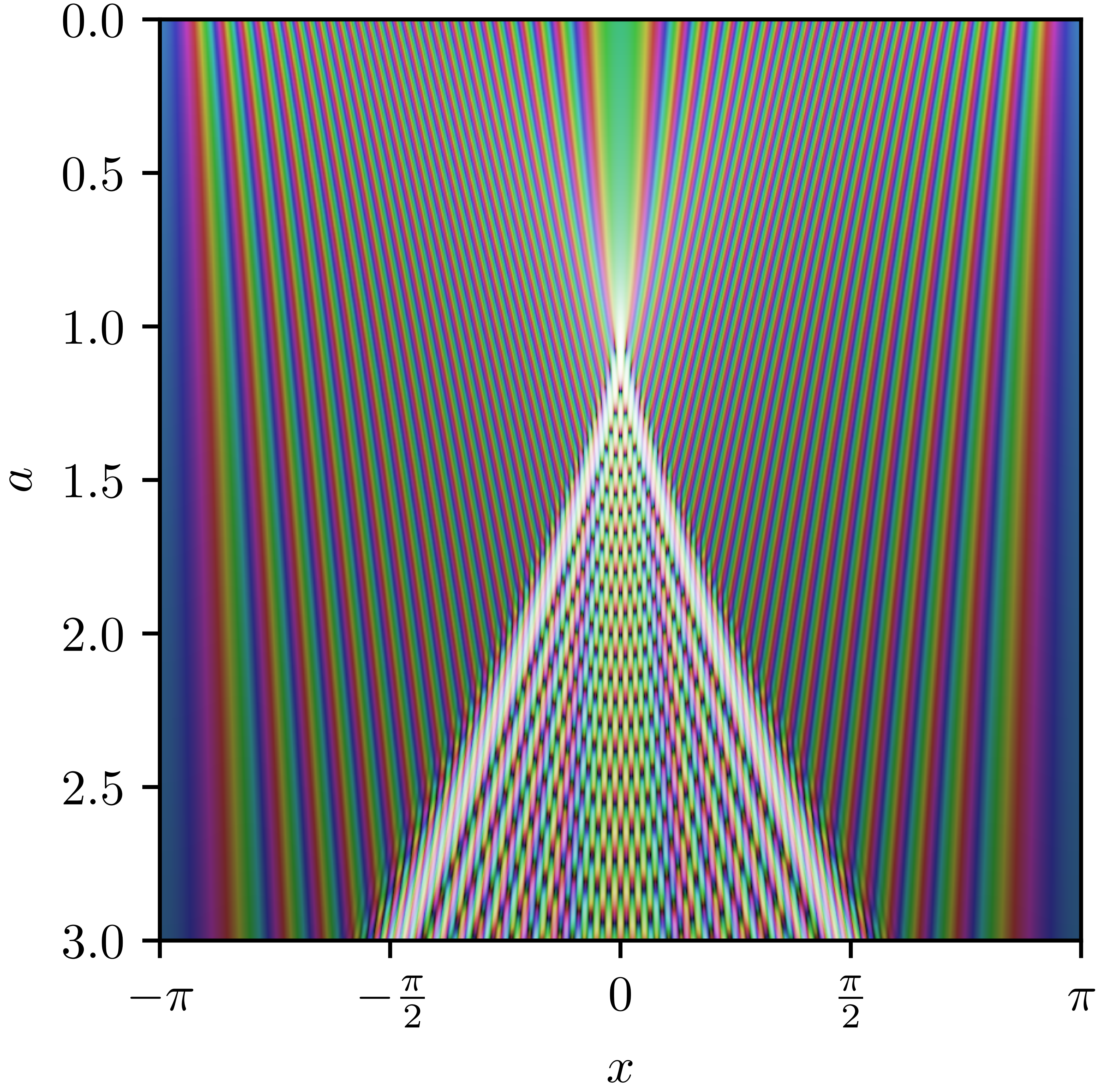
Stream components



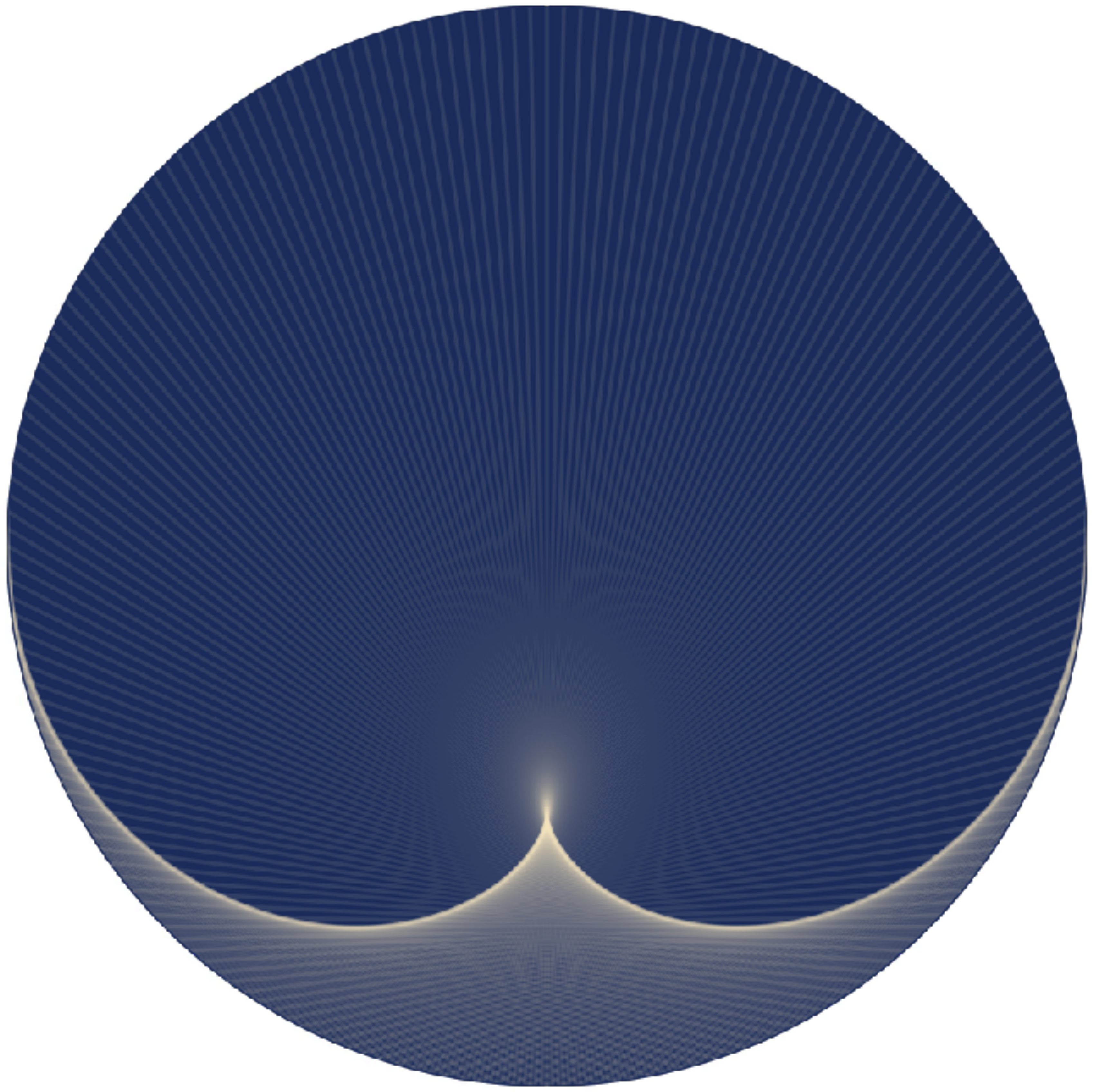
Gough & Uhlemann 2022

Optics analogy

Dark matter

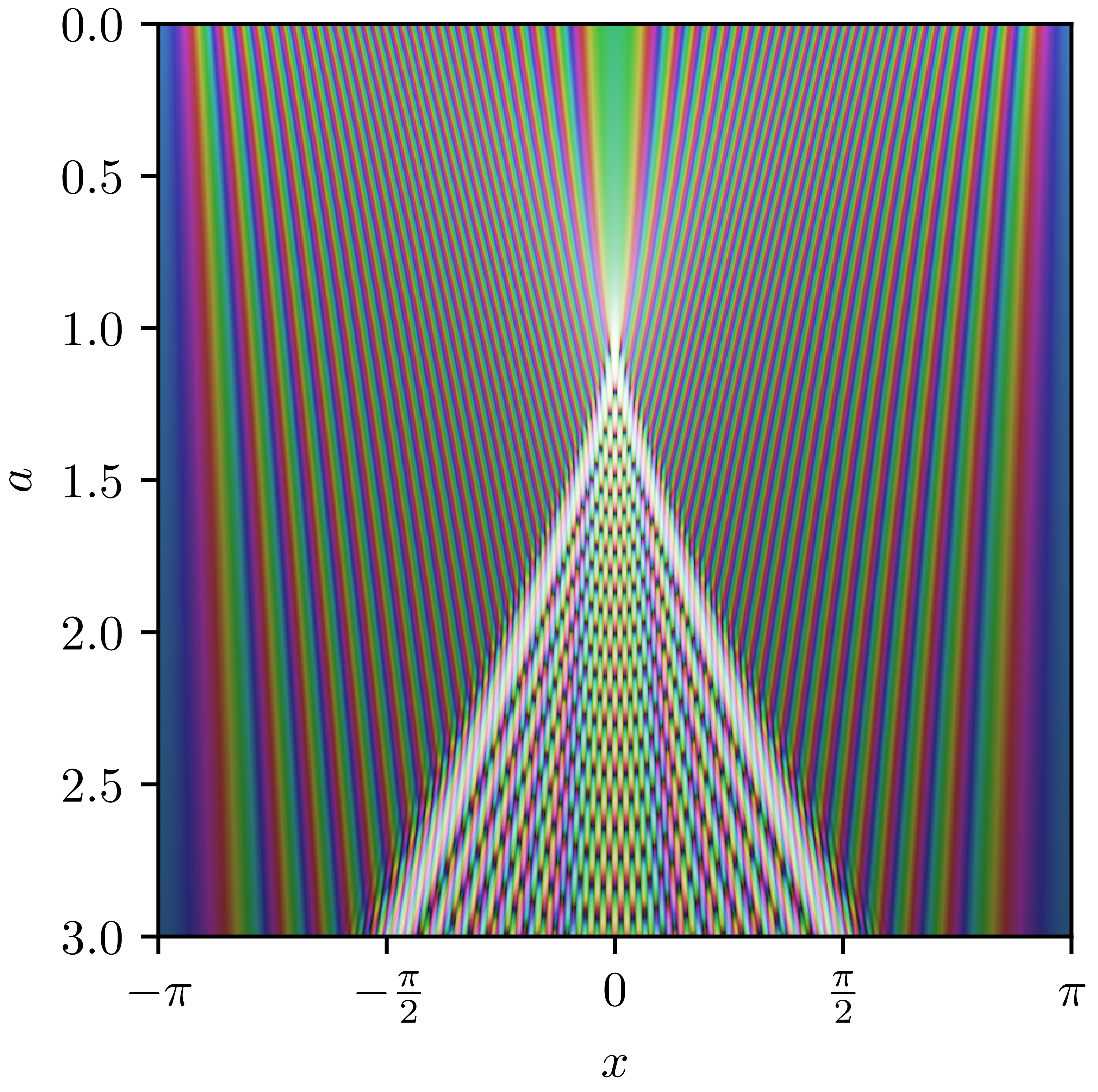


Optics



Optics analogy

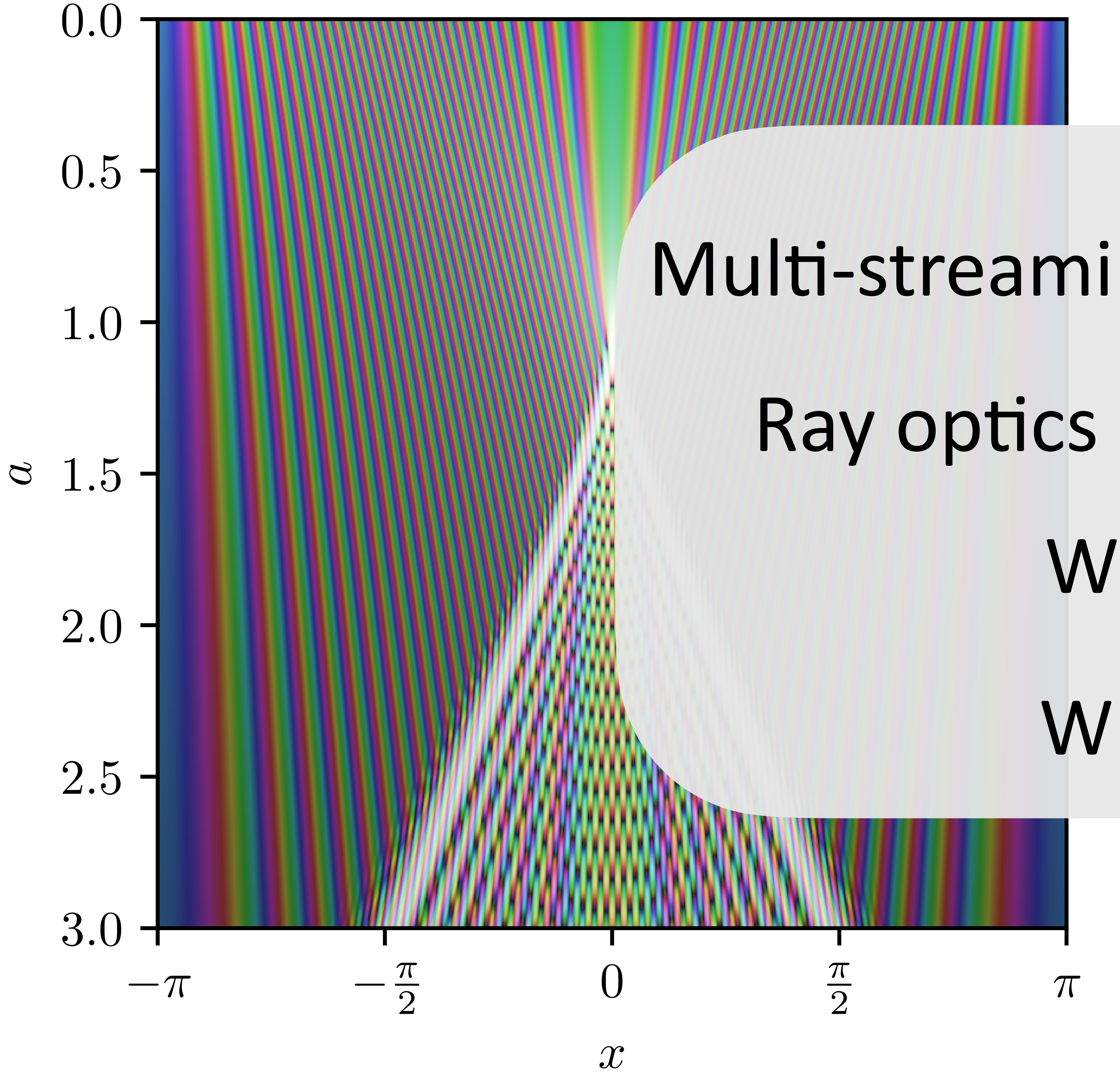
Dark matter



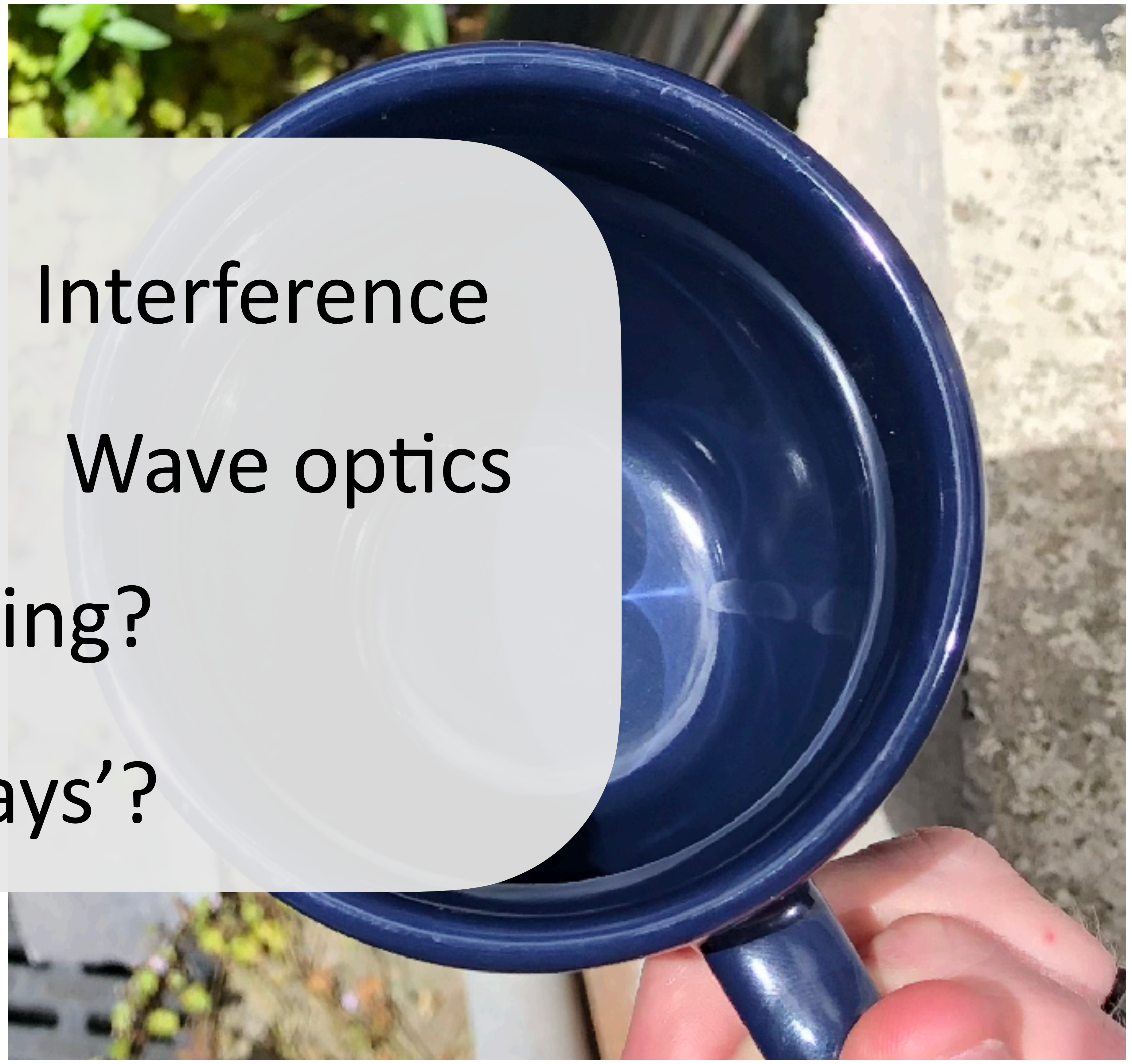
Optics

Optics analogy

Dark matter



Optics



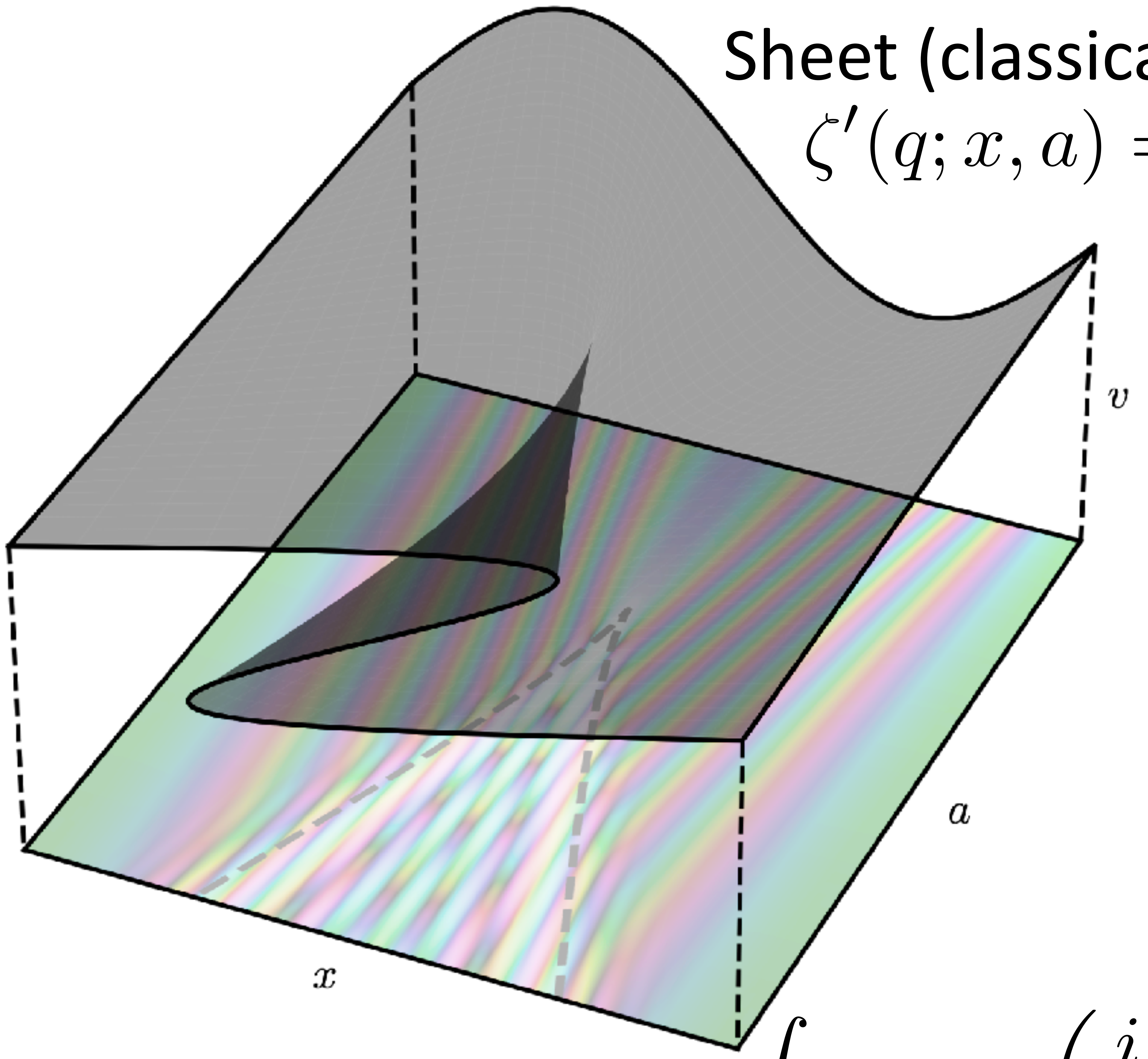
Multi-streaming ↔ Interference

Ray optics ↔ Wave optics

What is interfering?

What are the ‘rays’?

Unweaving interference



Sheet (classical)

$$\zeta'(q; x, a) = 0$$

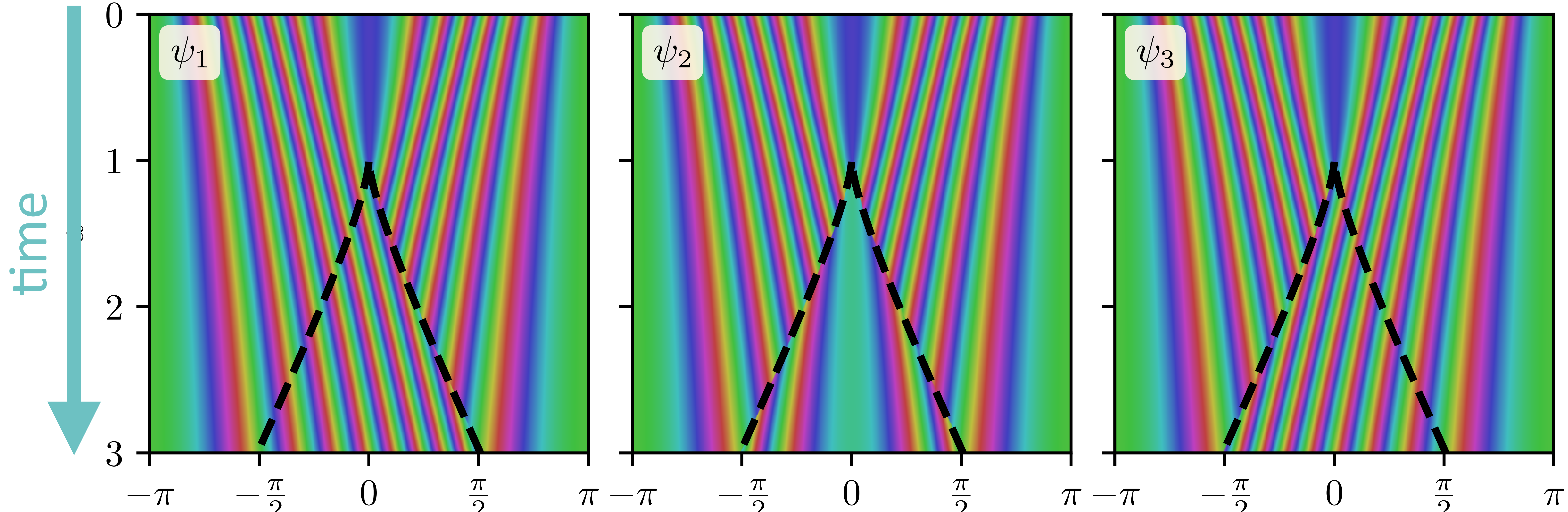
- Idea: calculate contributions from classical trajectories
- where $\zeta'(q) = 0$, oscillations in integral slow down

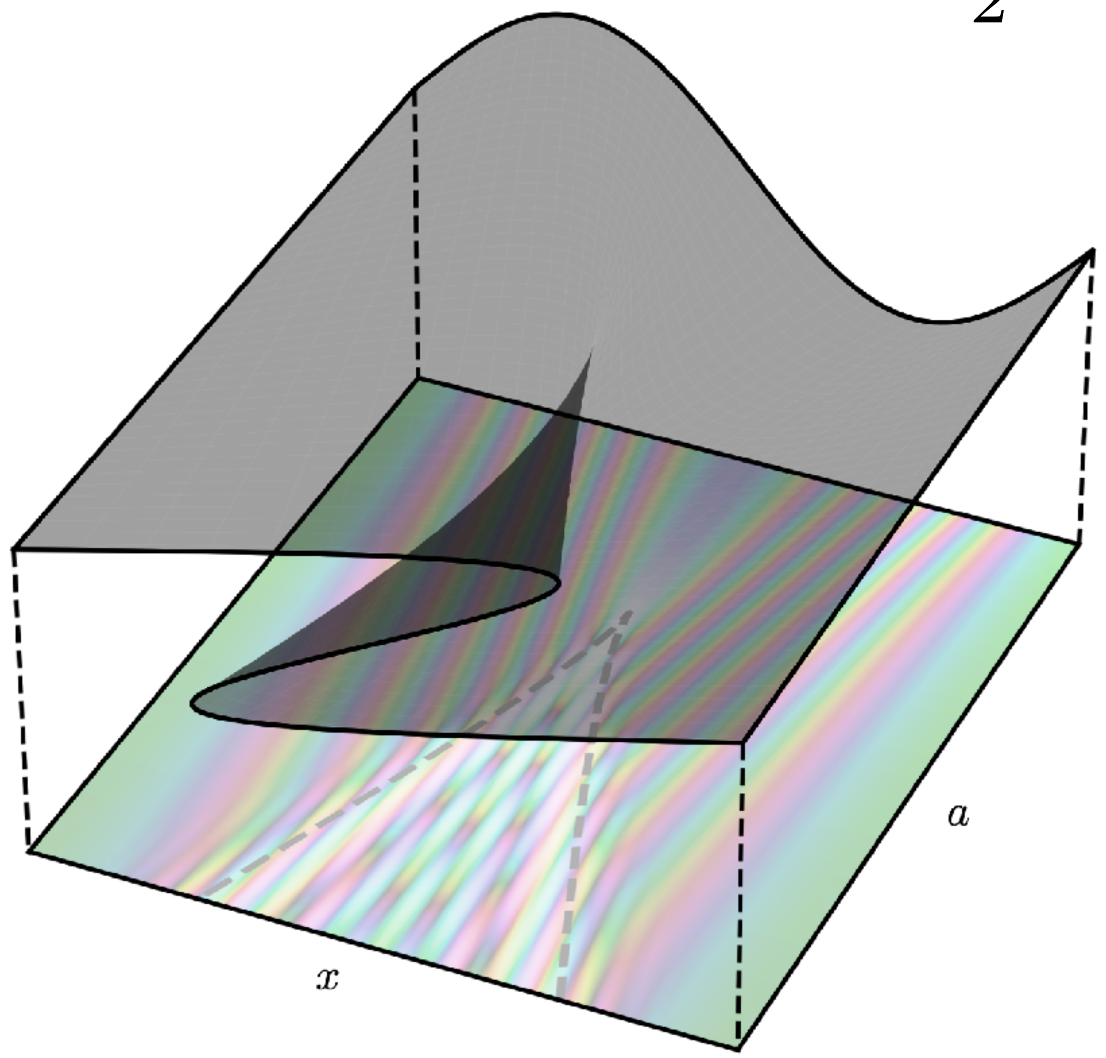
Stationary Phase Approximation

q where $\zeta'(q) = 0$ dominate integral

Full wavefunction $\propto \int \mathrm{d}q \exp\left(\frac{i}{\hbar}\zeta(q; x, a)\right)$

Stream wavefunctions





$$\psi^{\mathrm{SPA}}_{q_*} = \frac{e^{\pm i\pi/4}}{\sqrt{a|\zeta''(q_*)|}} \exp\left(\frac{i}{\hbar}\zeta(q_*)\right)$$

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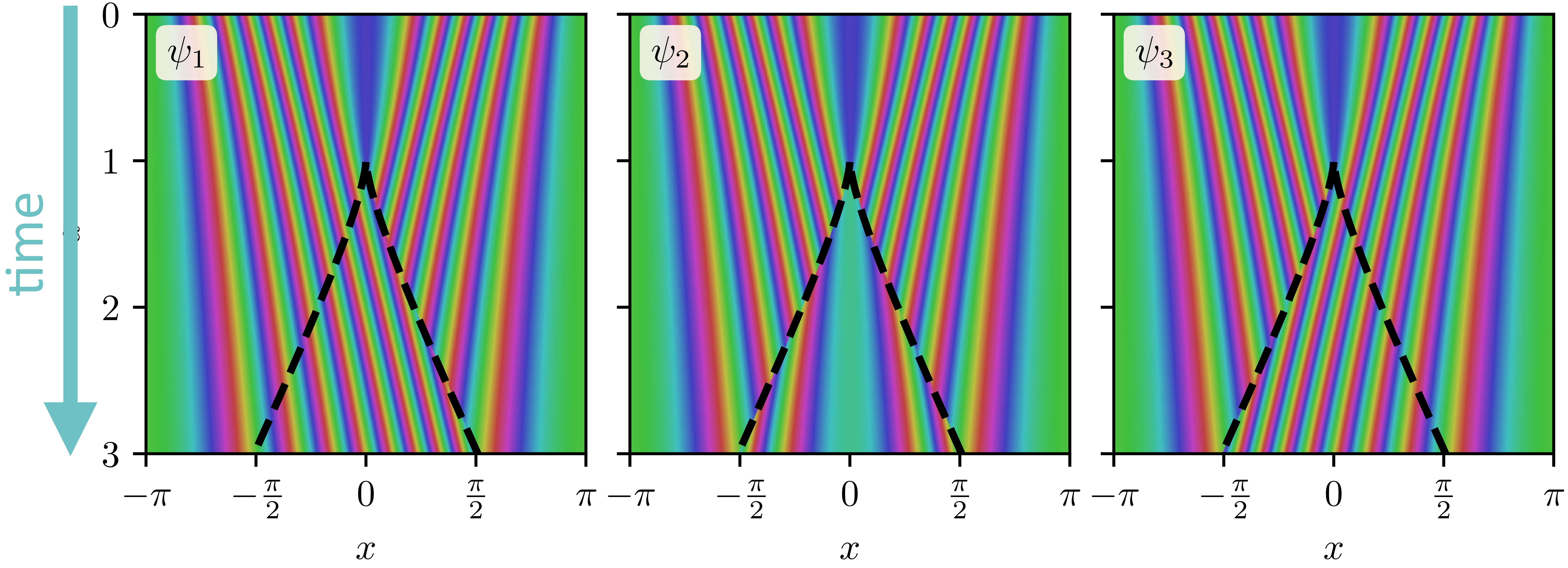
Stream wavefunctions



- Stream splitting without constructing phase space!
- Interference automatically encodes multi-streaming
- Allows us to isolate oscillations and associated observables

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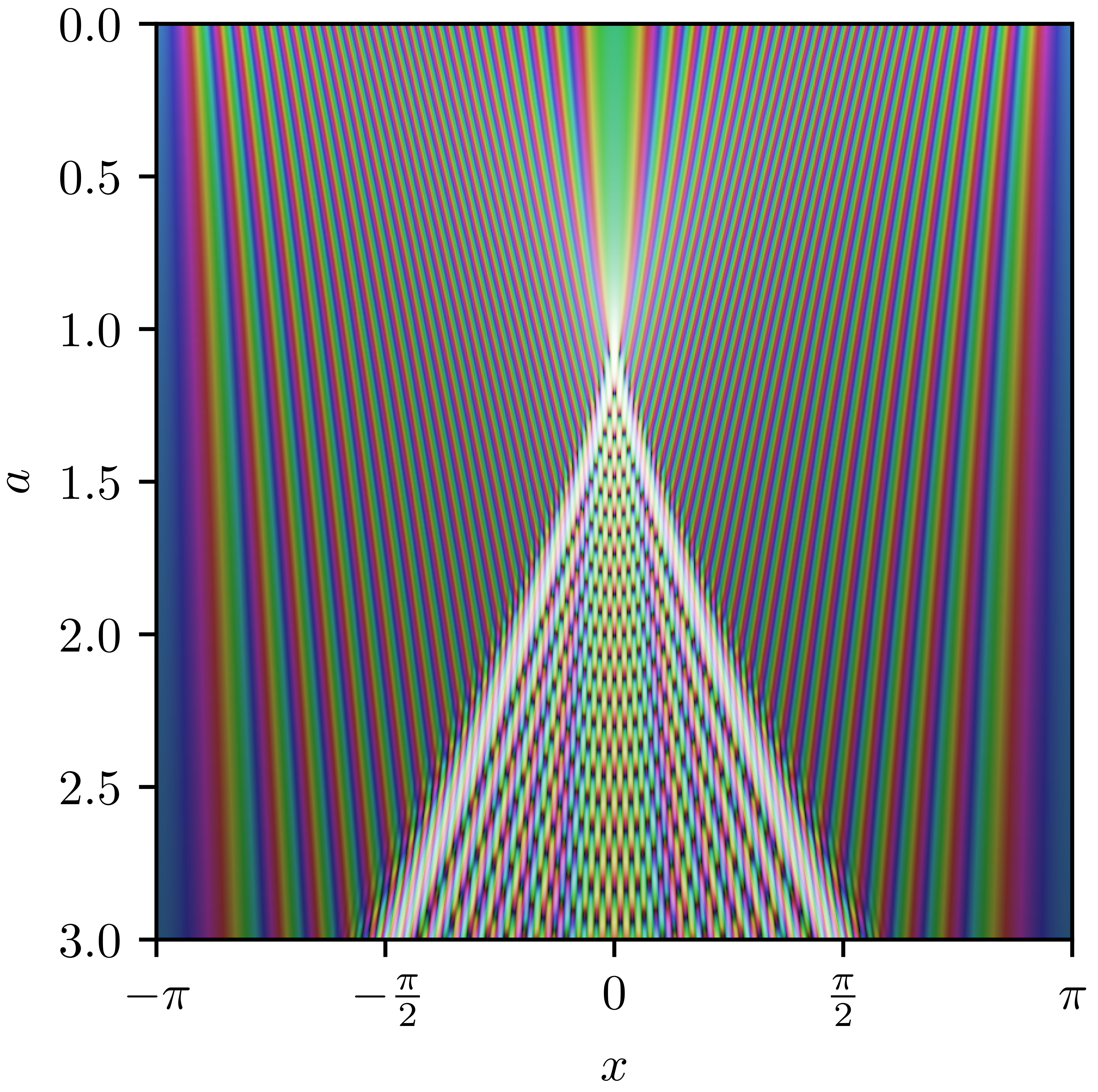
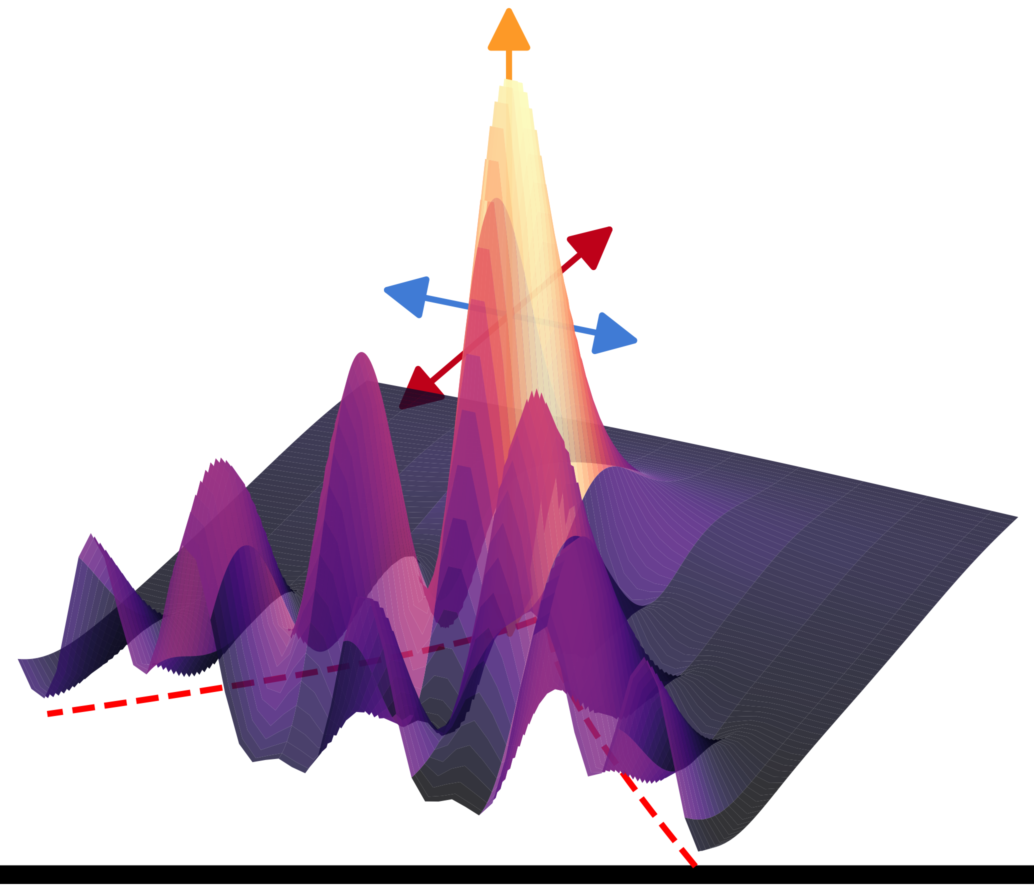
SPA + caustics



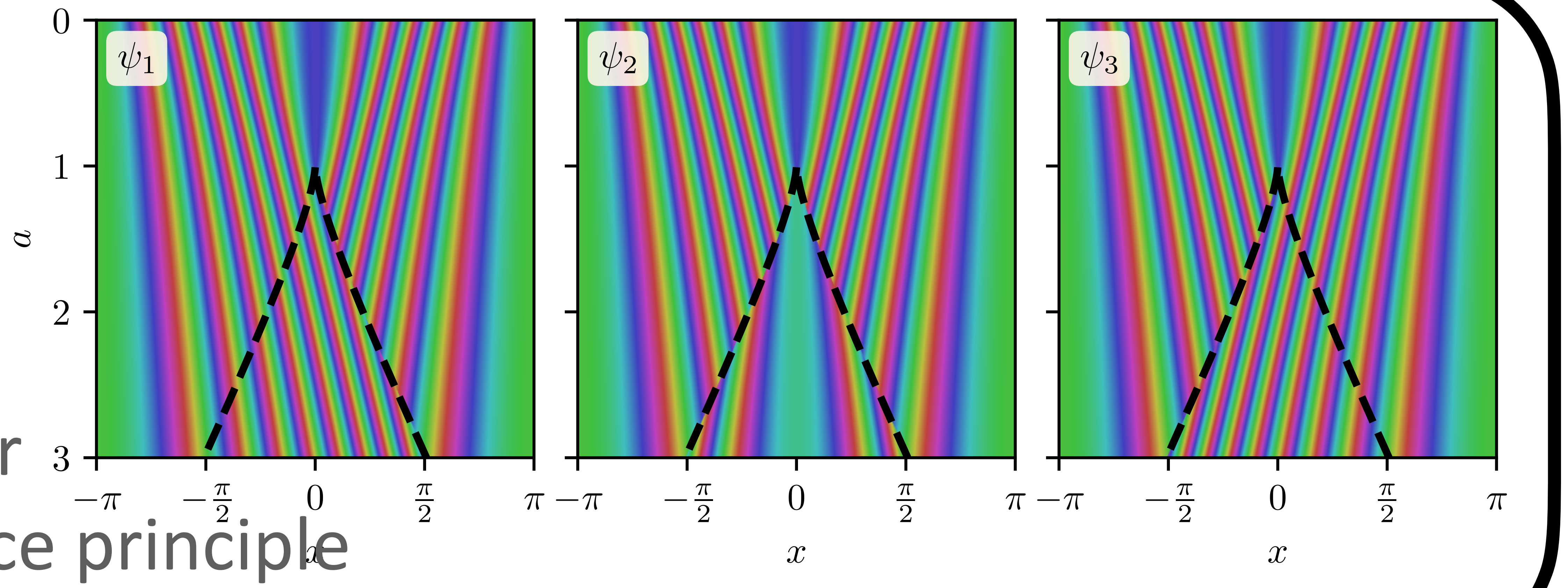
Caustic classification

divergence & fringe scalings

generic classifications



Stream components



fluid behaviour

correspondence principle

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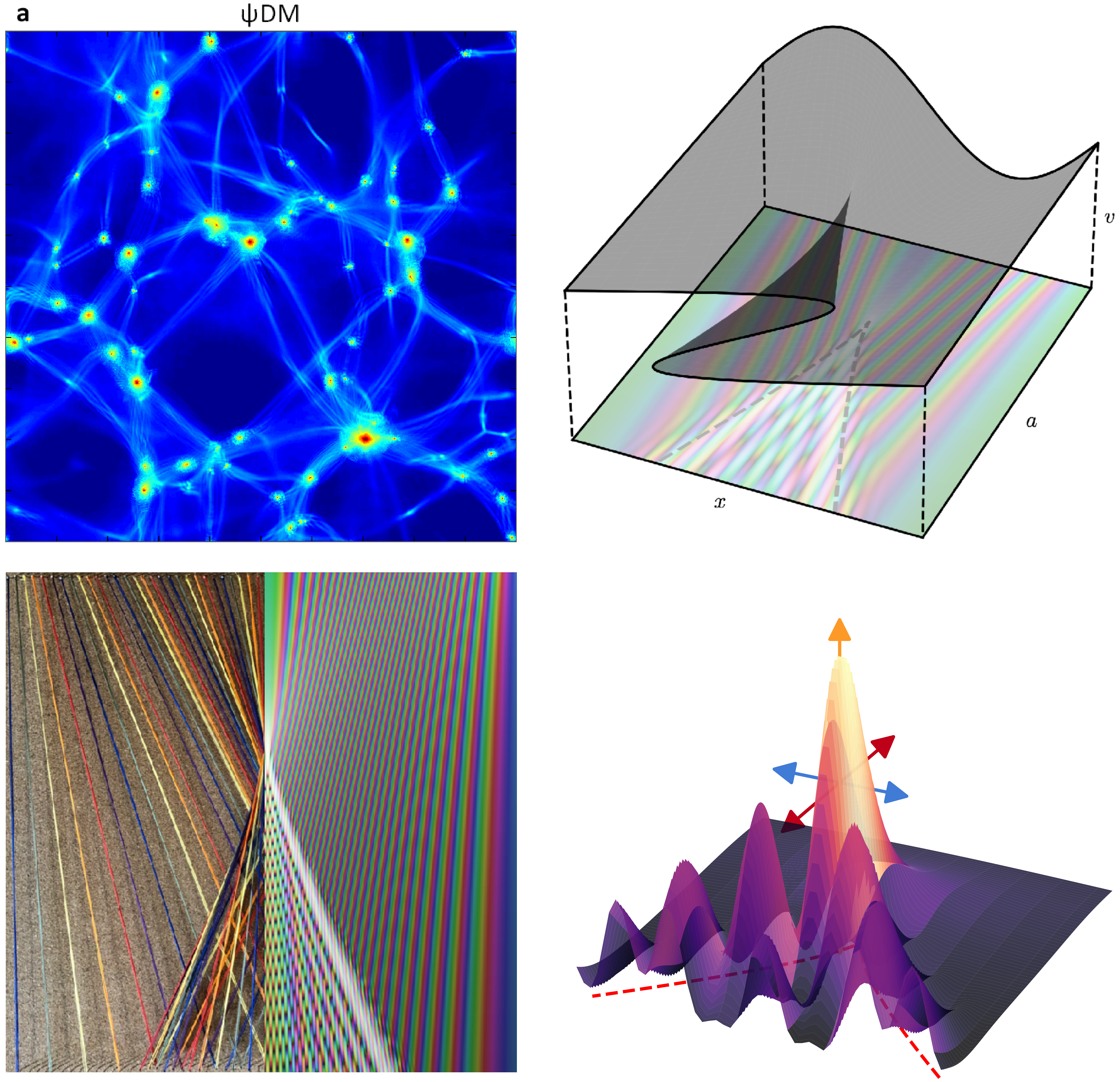
Takeaways

Wave DM presents rich phenomenology, decorating the cosmic web

- caustic structures (fully classified)
- interference ~ multi-streaming
- oscillations/phase jumps ~ beyond perfect fluid + vorticity

Wave models of CDM efficiently capture information beyond fluid models

- prospects for analytic modelling and complementing numerics



arXiv: 2206.11918

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