HEP Summer School 2022 Problems: SM

1. Show that the field strength $F_{\mu\nu}$ may be derived for a U(1) gauge field using the commutator of the covariant derivative D_μ. By definition, for a scalar field ϕ in some representation of a gauge symmetry, the object $D_\mu\phi$ has the same transformation property as ϕ itself. If, under this symmetry, $\phi \to U\phi$, where U is a unitary matrix, then how must D_μ transform? Given this, and the connection with the commutator, how does $F_{\mu\nu}$ transform?

2. In the four fermion operator $\mathcal{L} = \psi^4/\Lambda^2$, where ψ is a fermion, what is the mass and coupling dimension of Λ?

3. Check explicitly that $\mathcal{L}(e)$ is $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant, by applying the infinitessimal transformations of χ_L, e_R, $\vec{W}_\mu$ and B_μ.

4. You will require the Feynman rules and lecture notes from 2019!

Defining $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ show that $\gamma_5^2 = 1$, $\gamma_5^\dagger = \gamma_5$, and $\gamma_5, \gamma_\mu = 0$. Consider a massless fermion with momentum p along the z direction, $p_\mu = (E, 0, 0, E)$. Show that $P_R u(p)$ and $P_L u(p)$ are eigenstates of helicity

$$h = -\frac{\gamma^0\gamma_5 \overrightarrow{\gamma} \cdot p}{E}, \tag{1}$$

with eigenvalues ± 1.

5. There is one Feynman diagram in lowest order electroweak theory for μ^- decay,

$$\mu^-(p) \to \nu_\mu(k) + e^-(p') + \overline{\nu}_e(k'). \tag{2}$$

Draw this diagram and use the electroweak Feynman rules to calculate the spin averaged $|\overline{\mathcal{M}}|^2$ for this decay. To simplify the calculation retain m_μ but set $m_e = 0$. Also, evaluate in the effective "Fermi theory" (see last years notes!) where you leave out the W propagator (set it to $g_{\mu\nu}$) and replace g at the vertices by g/M_W. Why is this a very good approximation for μ^- decay? Does setting $m_\mu = 0$ make any difference?

You are given $\mathrm{Tr}[\gamma_\mu(1-\gamma_5)\not{p}_1\gamma^\nu(1-\gamma_5)\not{p}_2]Tr[\gamma_\mu(1-\gamma_5)\not{p}_3\gamma_\nu(1-\gamma_5)\not{p}_4] = 256(p_1 \cdot p_3)(p_2 \cdot p_4)$.

Write out an expression for $d\Gamma$ (the differential decay rate) in terms of $\not{p}$ and phase space. A tedious phase space integration which you need not attempt then leads to the total μ^- decay rate

$$\Gamma(\mu^-) = \frac{g^4 m_\mu^5}{6144\pi^3 M_W^4} \tag{3}$$

Given $m_\mu = 105.66$ MeV, and the μ_- lifetime

$$\tau(\mu^-)^{\mathrm{exp}} = \frac{1}{\Gamma(\mu^-)} = (2.197138 \pm 0.000065) \times 10^{-6} \text{ sec} \quad . \tag{4}$$

Estimate v, the Higgs vev, in the minimal Standard Model. (In natural units 1 sec $= 1.52 \times 10^{24}$ GeV^{-1}).

6. Use the electroweak Feynman rules to calculate the polarization averaged Z_0 decay width, $\Gamma(Z \to ff), f = e, \nu, q, ...$ Take f massless.

For an external massive spin 1 vector boson with mass M_V you need the Feynman rule for $\epsilon_\mu^{(\lambda)}$, where the $\epsilon_\mu^{(\lambda)}$ is the polarization vector of the vector boson, and the completeness sum over polarizations is

$$\sum_\lambda \epsilon_\mu^{(\lambda)*}\epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_V^2} \tag{5}$$

Suitable choices are ($\overrightarrow{p}$ along the z-axis)

$$\epsilon_\nu^{(\lambda=\pm1)} = \mp(0, 1, \pm i, 0)/\sqrt{2} \tag{6}$$

and

$$\epsilon_\nu^{(\lambda=0)} = \mp(|\overrightarrow{p}|, 0, 0, E)/M_Z \tag{7}$$

One then has

$$\Gamma(Z^0 \to f\overline{f}) = \frac{1}{64\pi^2 M_Z} \int |\overline{\mathcal{M}}|^2 d\Omega \tag{8}$$

Estimate the total Z_0 decay width (take $M_Z = 91$ GeV, $g = 0.65$, $\sin^2(\theta_W) = 0.23$) which should have been observed at LEP. Don't forget three colours for each quark flavour!