Collider Phenomenology

Eleni Vryonidou









STFC school, Oxford 9-16/9/22

Plan for the lectures

- Basics of collider physics
- Basics of QCD
 - DIS and the Parton Model
 - Higher order corrections
 - Asymptotic freedom
 - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

Plan for the lectures

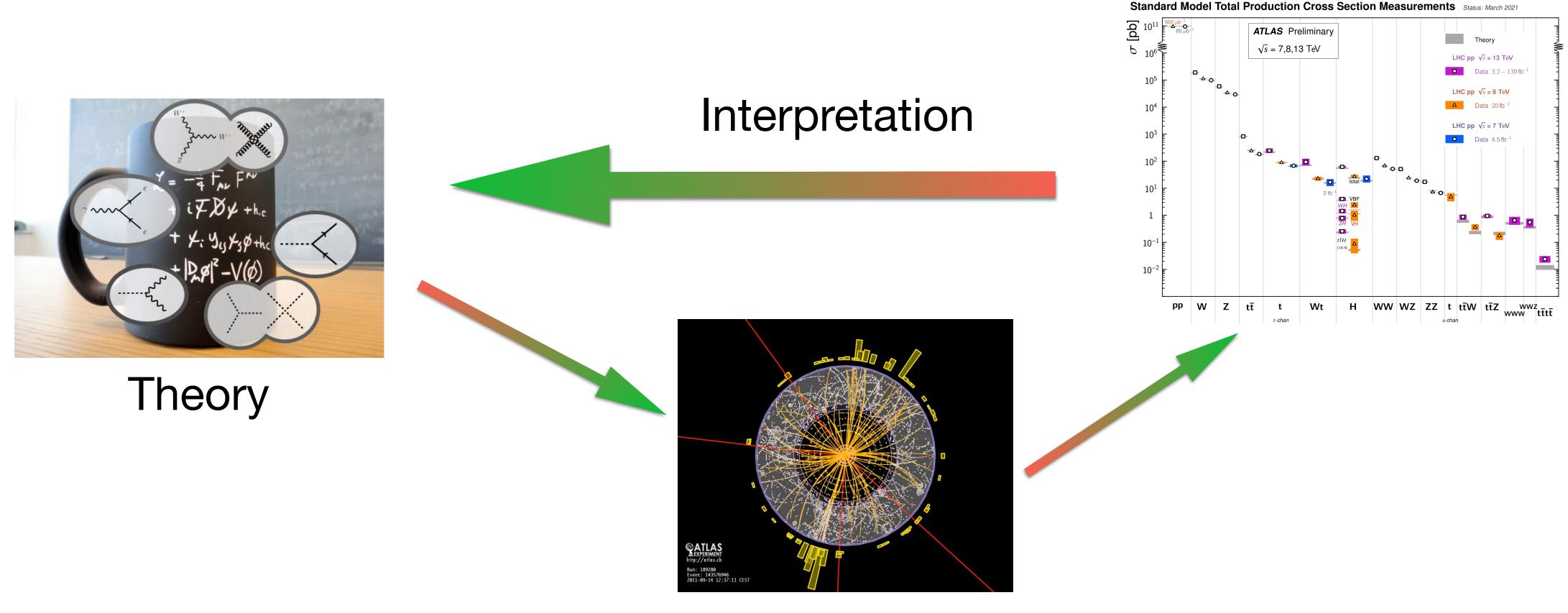
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Basics of collider physics

Goals of collider physics:

- Frest theoretical predictions: Standard Model and New Physics
- Hopefully find the unexpected!

Collider physics



Experiment

Need good control of every step

Historical perspective

Why bother? Because it works!

Collider	When	What particle	Energy	Main Impact
SPS-CERN	1981-1984	pp	600 GeV	W/Z bosons
Tevatron	1983-2011	ppbar	2 TeV	Top quark
LEP-CERN	1989-2000	e+e-	210 GeV	Precision EW
HERA-DESY	1992-2007	ер	320 GeV	QCD/PDFs
BELLE	1999-2010	e+e-	10 GeV	Flavour physics
LHC	2009-Today	pp	7/8/13 TeV	Higgs

Future of collider physics?

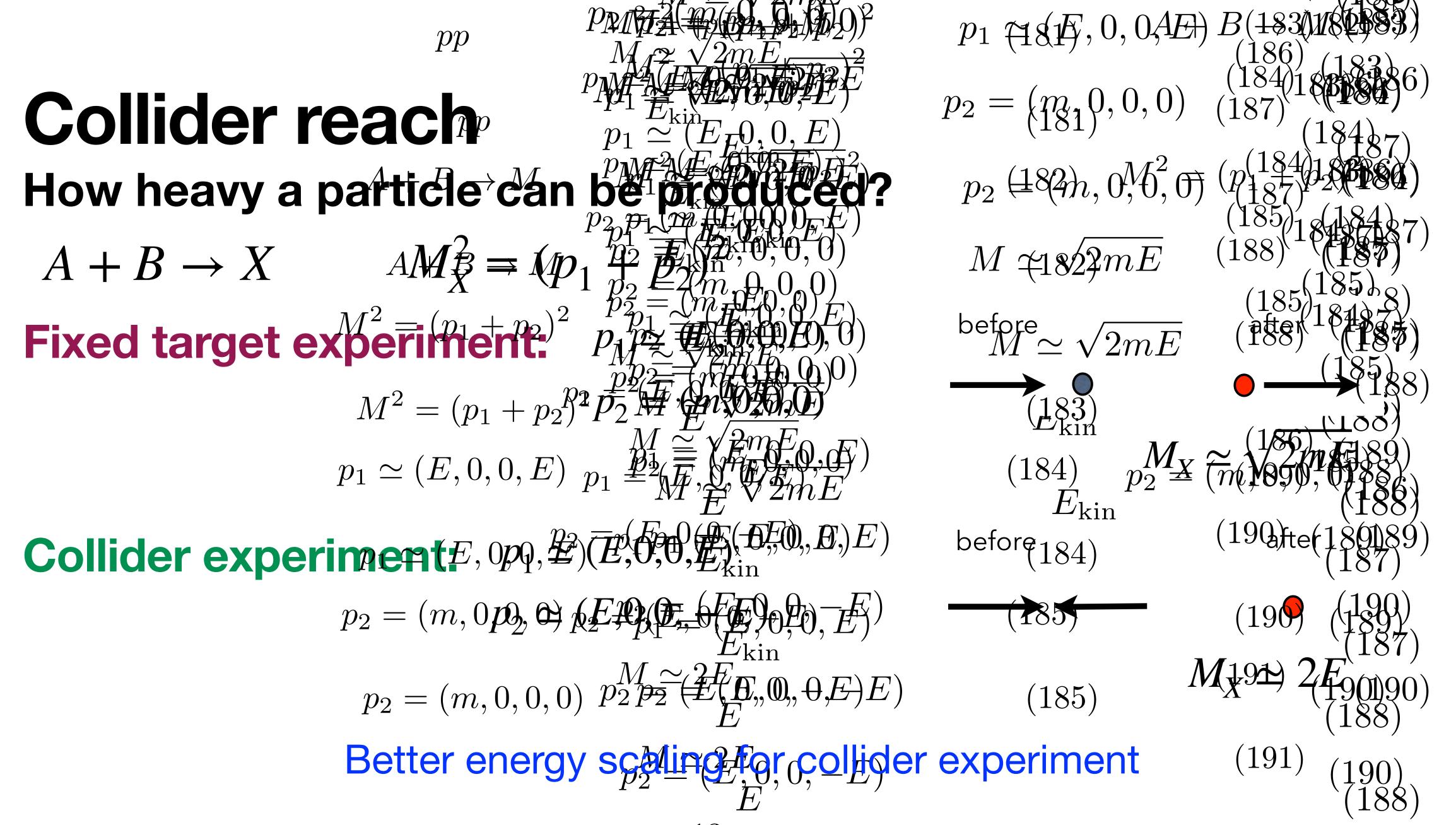












Note: fixed target can benefit from dense target

Collider aspects

Luminosity: rate of particles in colliding bunches

$$\mathcal{L} = \frac{N_1 N_2 f}{A}$$

 $\mathcal{L} = \frac{N_1 N_2 f}{A}$ $N_i \text{ number of particles in bunches}$ f bunch collision rate A transverse bunch area

Integrated Luminosity:
$$L = \int \mathcal{L} dt$$

Number of events for process with cross-section σ : $L\sigma$

LHC luminosity Run II $L = 300 \text{ fb}^{-1}$

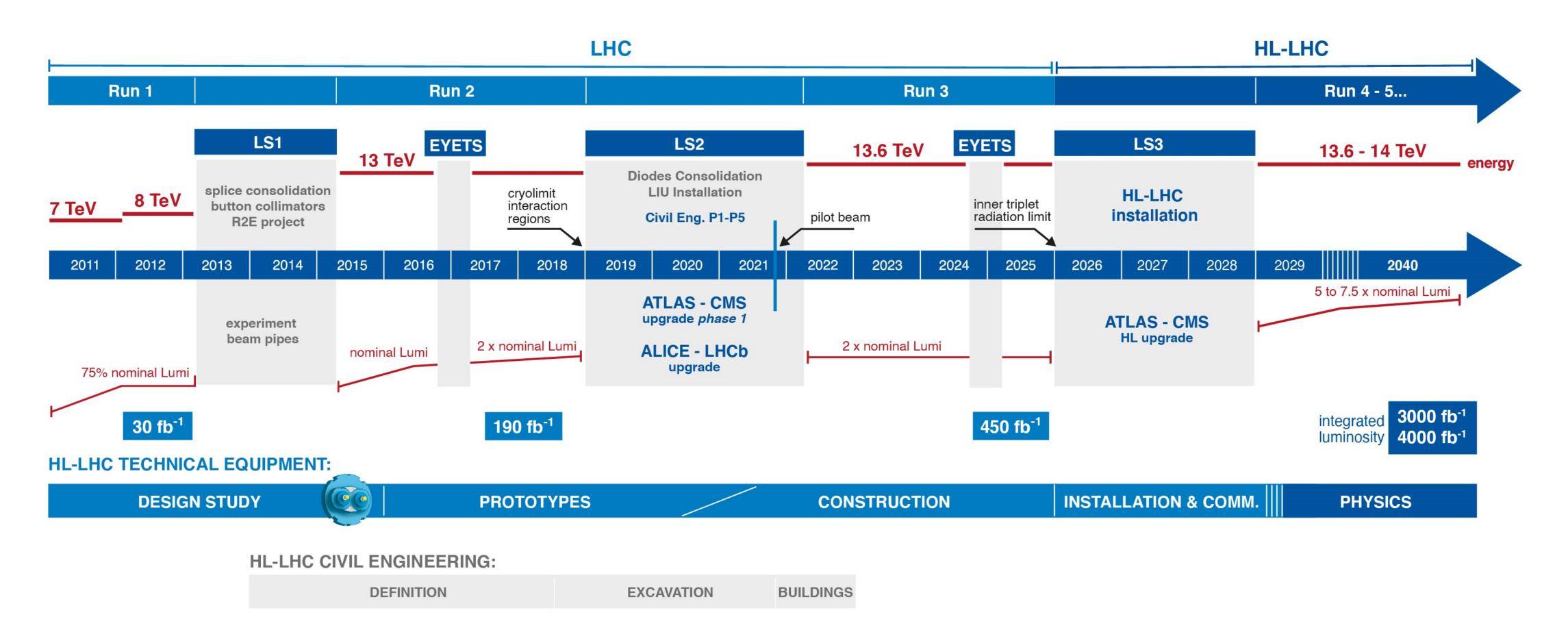
Circular vs linear: circular colliders are compact, but suffer from synchrotron radiation

Lepton vs Hadron: Lepton colliders, all energy available in the collision Hadron colliders, energy available determined by PDFs but can generally reach higher energies

LHC: a hadron collider



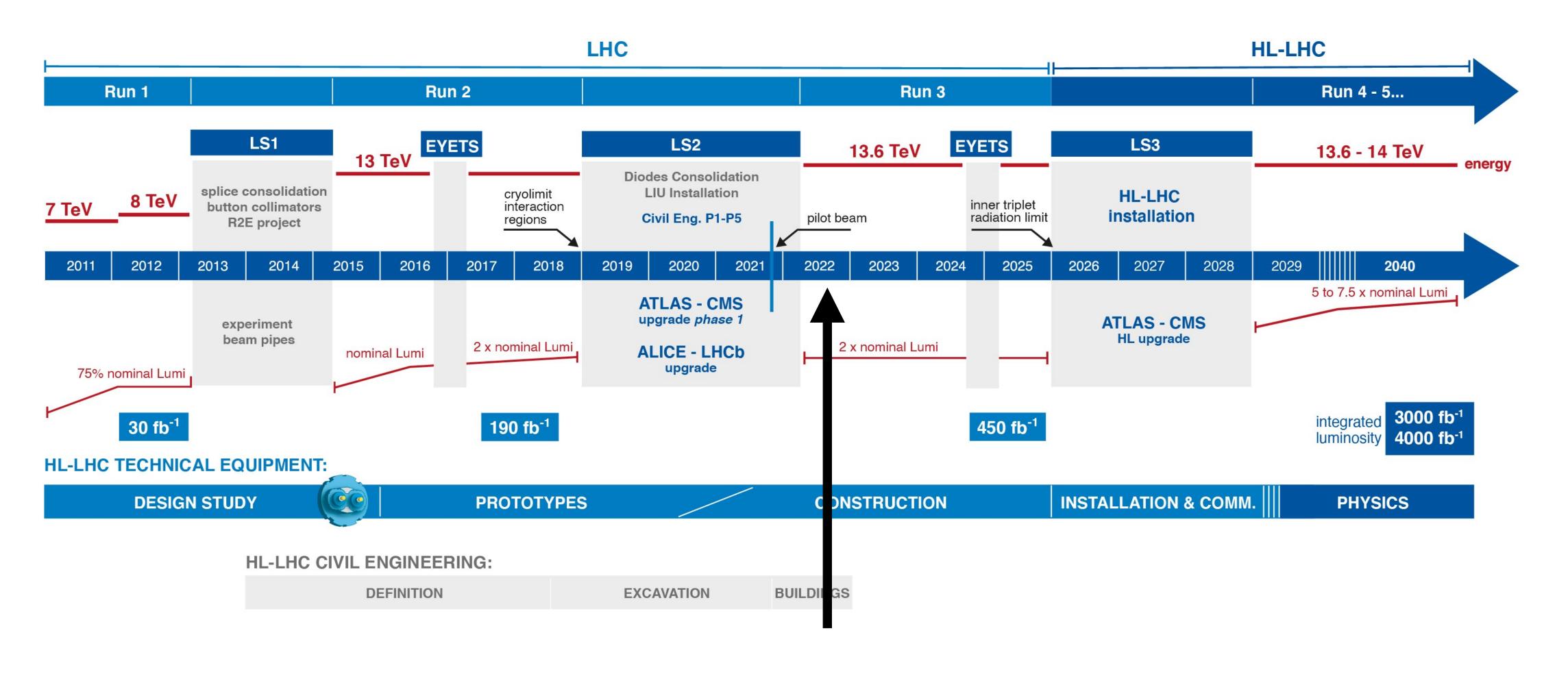




LHC: a hadron collider



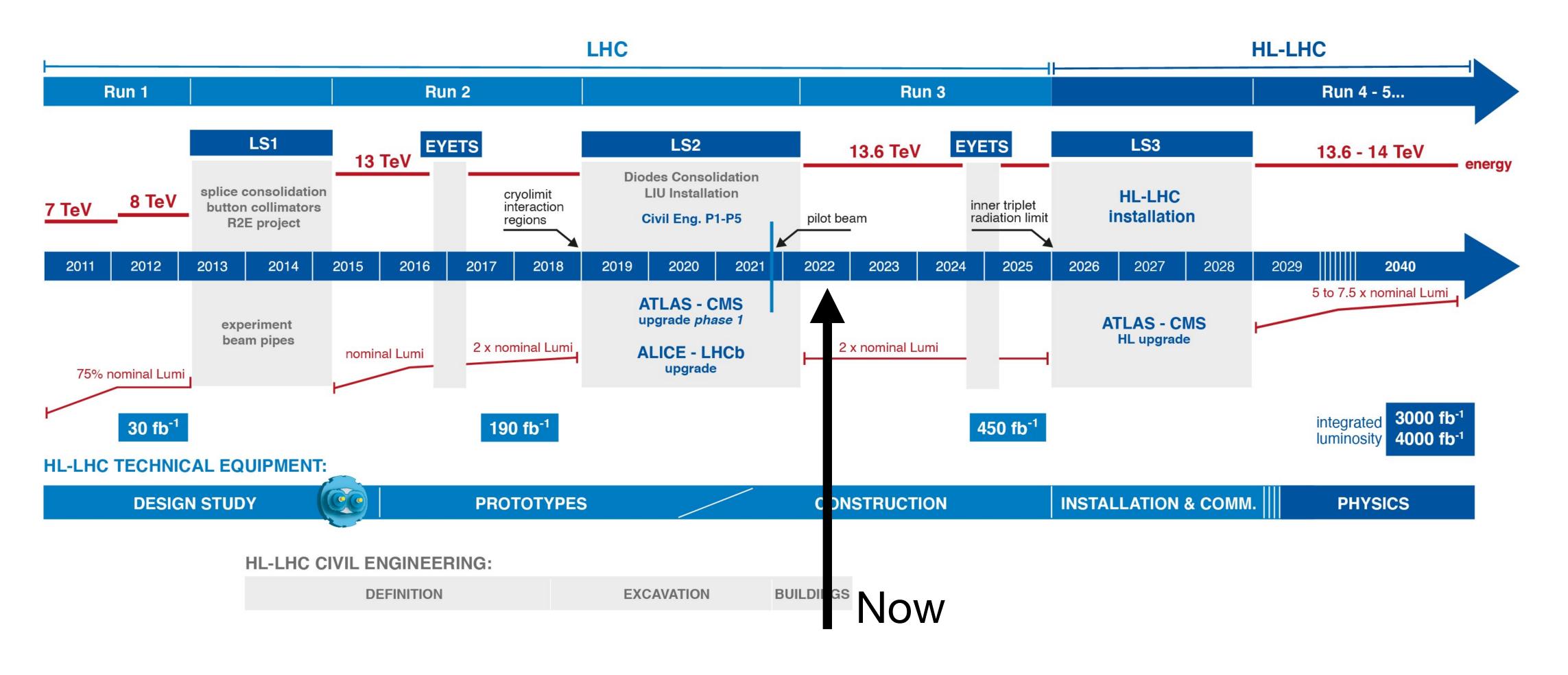




LHC: a hadron collider

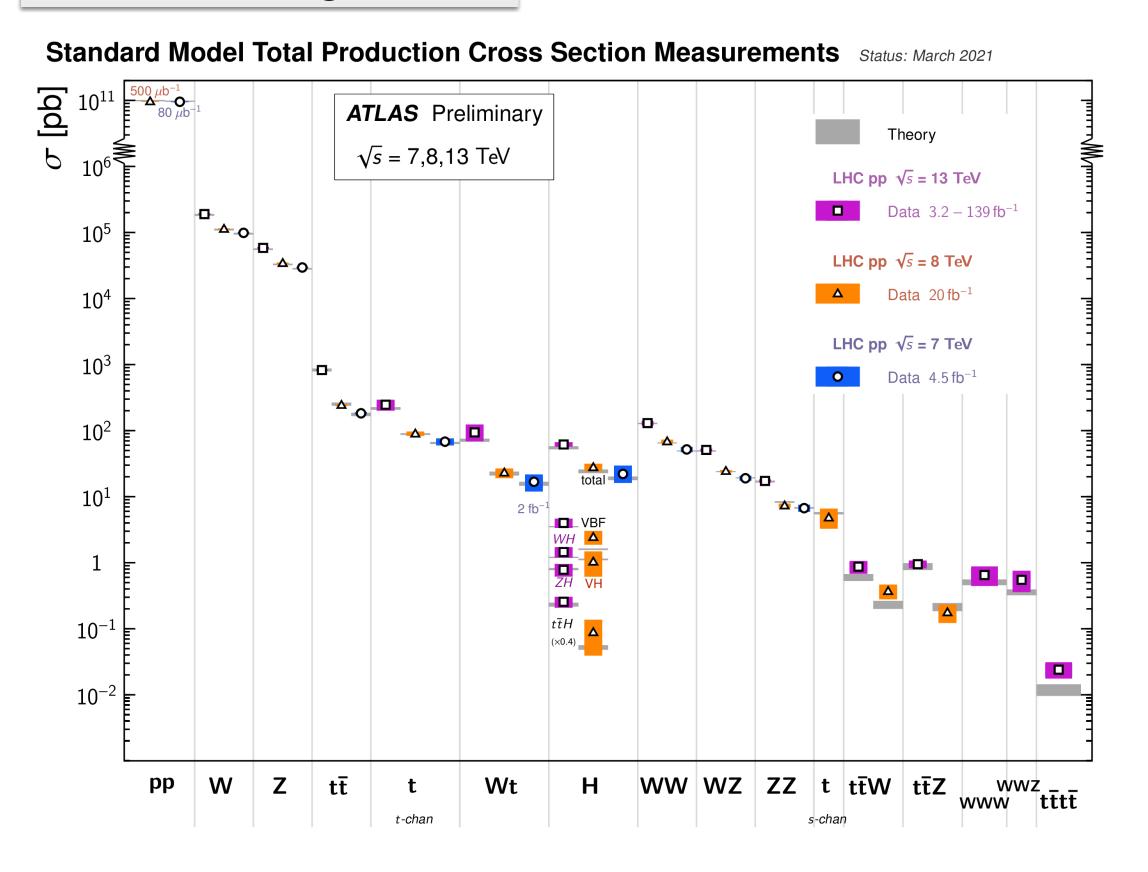




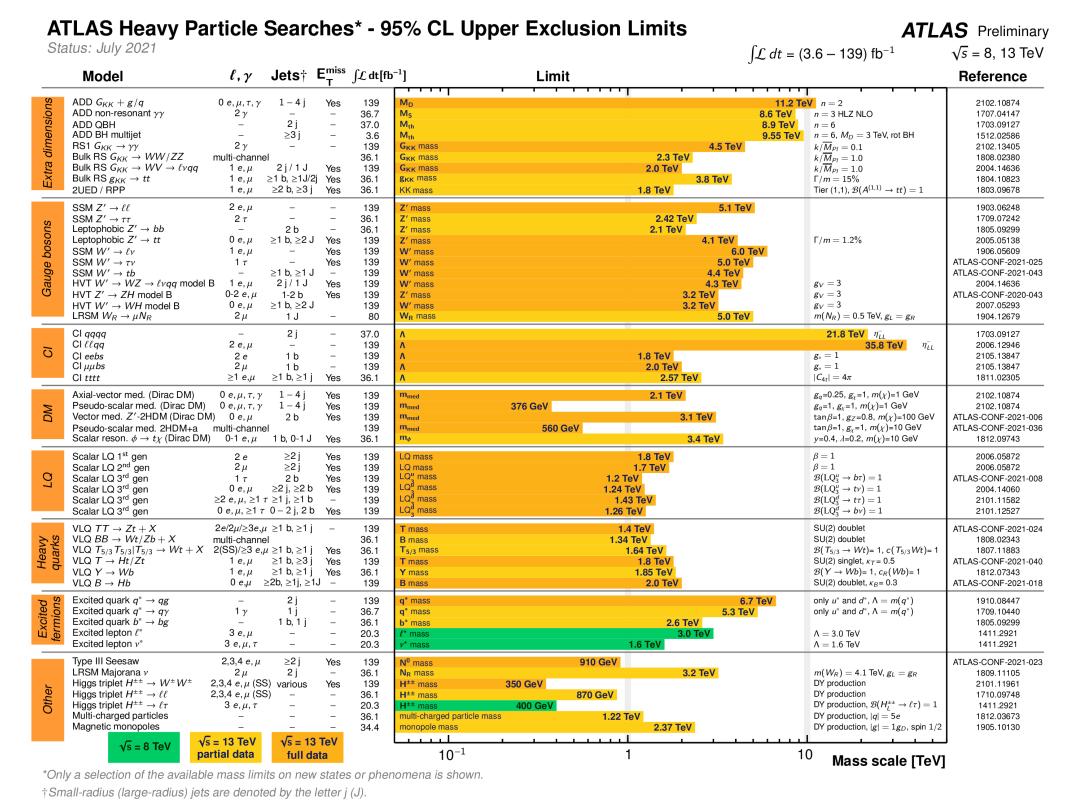


LHC status

Rediscovering the SM



Searching for the unknown



Good agreement with the SM

LHC physics

What's next?

No sign of new physics! Searches for deviations continue

New Physics can be:

Weakly coupled: Small rates means that more Luminosity can help

Exotic: Need new ways to search for it, going beyond standard searches or even beyond high-energy colliders

Heavy: Not enough energy to produce it

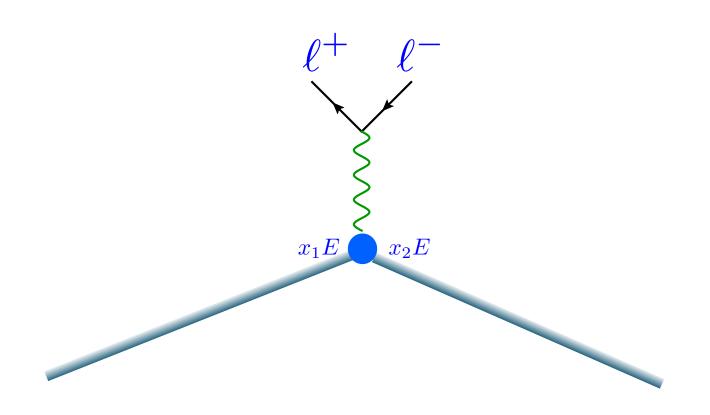
Need indirect searches: SMEFT

What is next for LHC physics

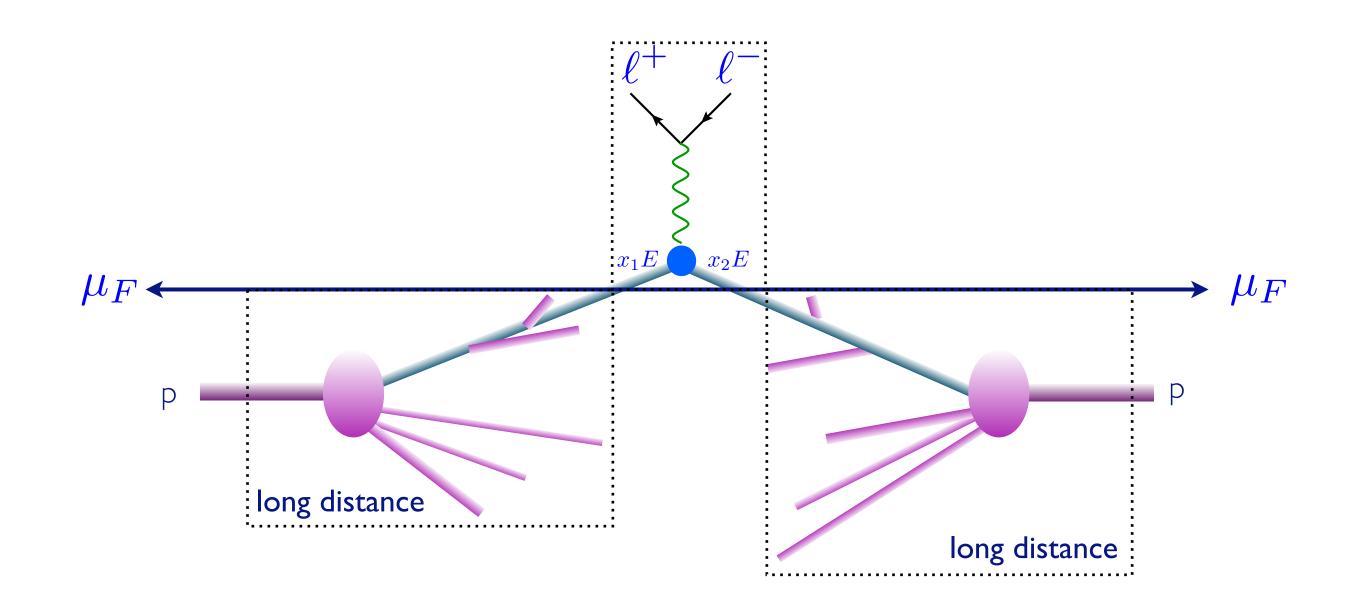
- New Physics is hiding well!
- Need to probe small deviations from the Standard Model using very precise predictions.
- Precise predictions are needed for both the SM and BSM.

In this course we will study the ingredients which enter in theoretical predictions and interpretations of LHC data!

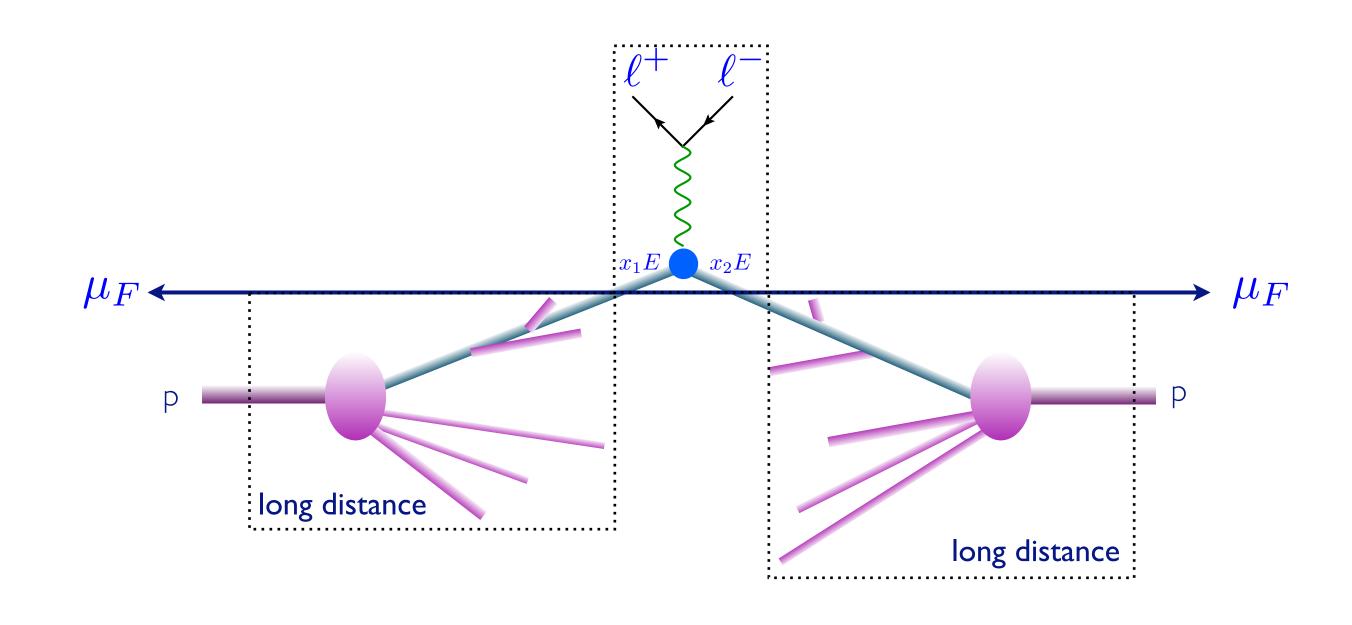
How to compute cross-sections for the LHC



How to compute cross-sections for the LHC



How to compute cross-sections for the LHC



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Master formula for LHC physics

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Important aspect of a Monte Carlo generator

Parton density functions

Universal:

~Probabilities of finding given parton with given momentum in proton

Extracted from data

Parton-level cross section

Subject of huge efforts in the LHC theory community to systematically improve this

Master formula for LHC physics

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

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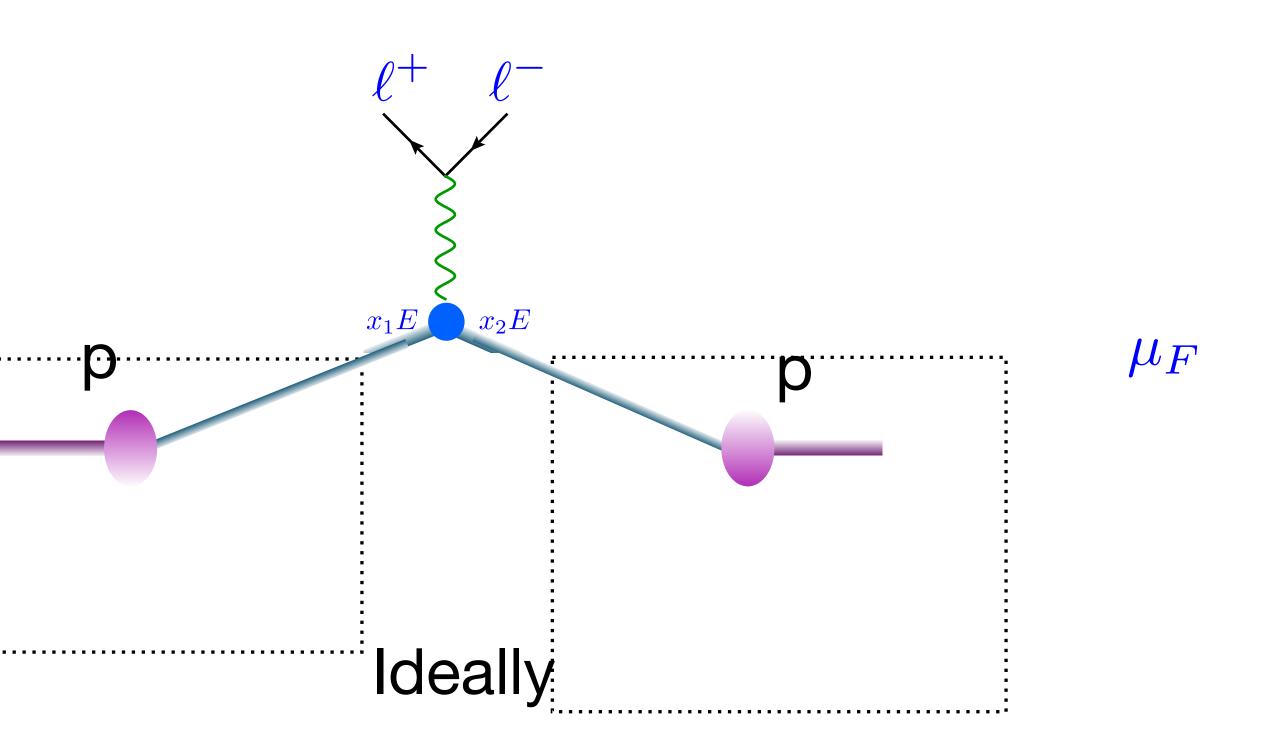
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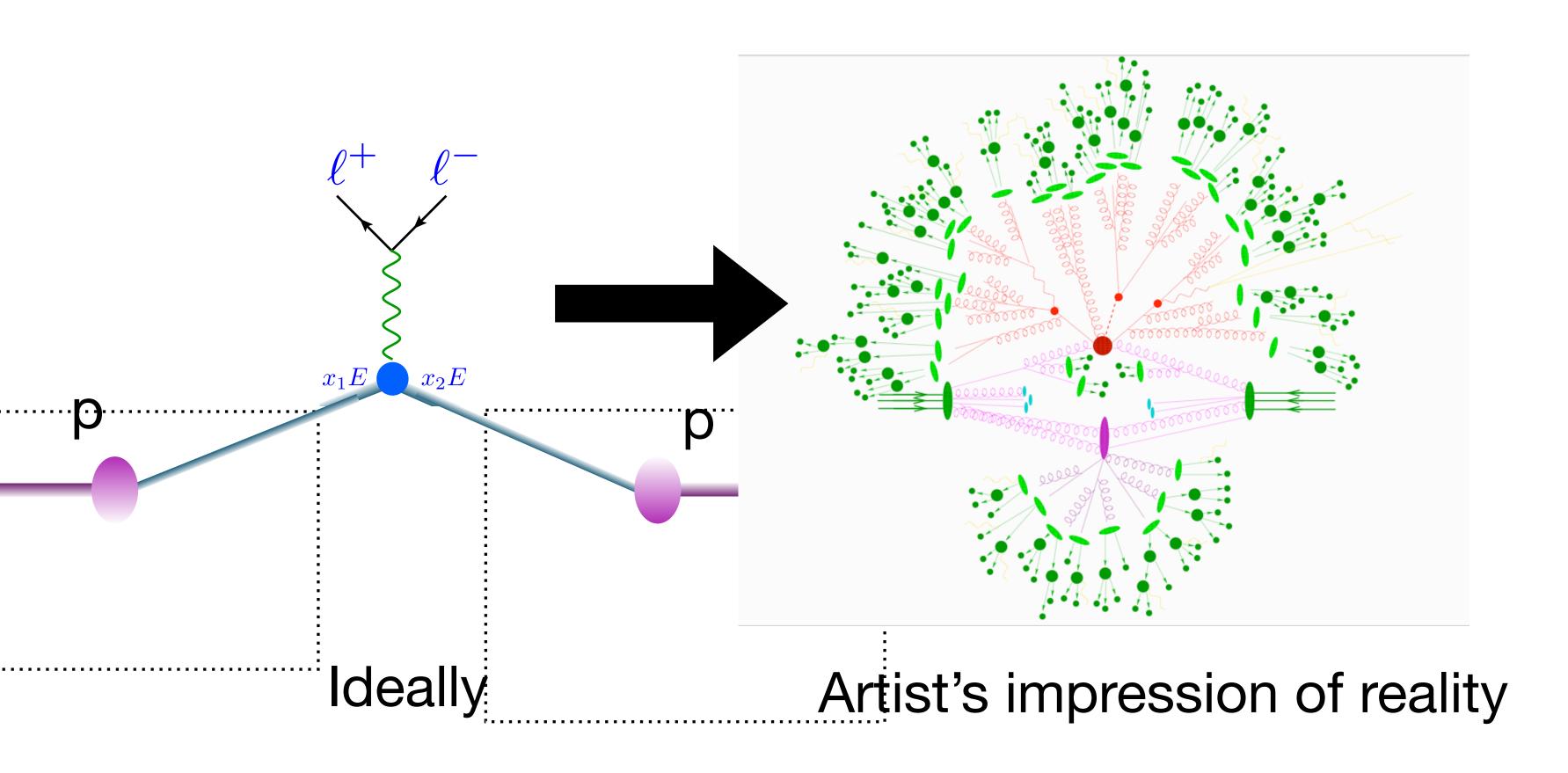
We will study in detail this formula this week!

From the hard scattering to events



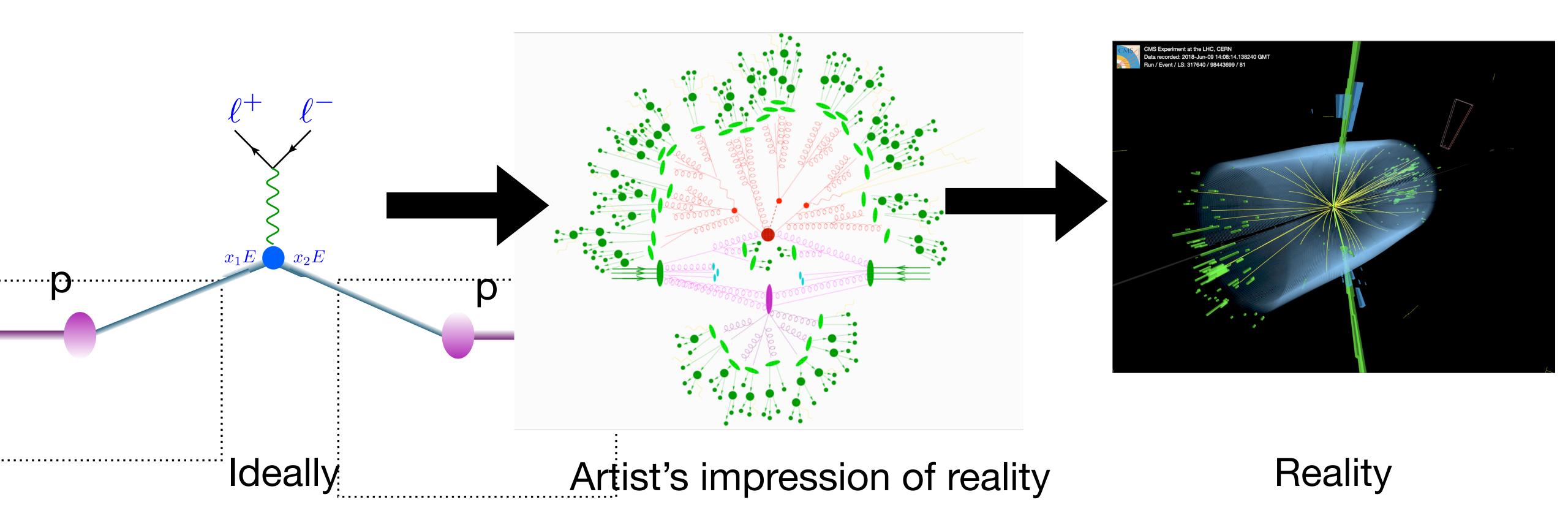
 $x_2d\Phi_{\mathrm{FS}}\,f_a(x_1,\mu_F)f_b(x_2,\mu_F)\,\hat{\sigma}_{ab o X}(\hat{s},\mu_F,\mu_R)$ STFC HEP school 2022

From the hard scattering to events



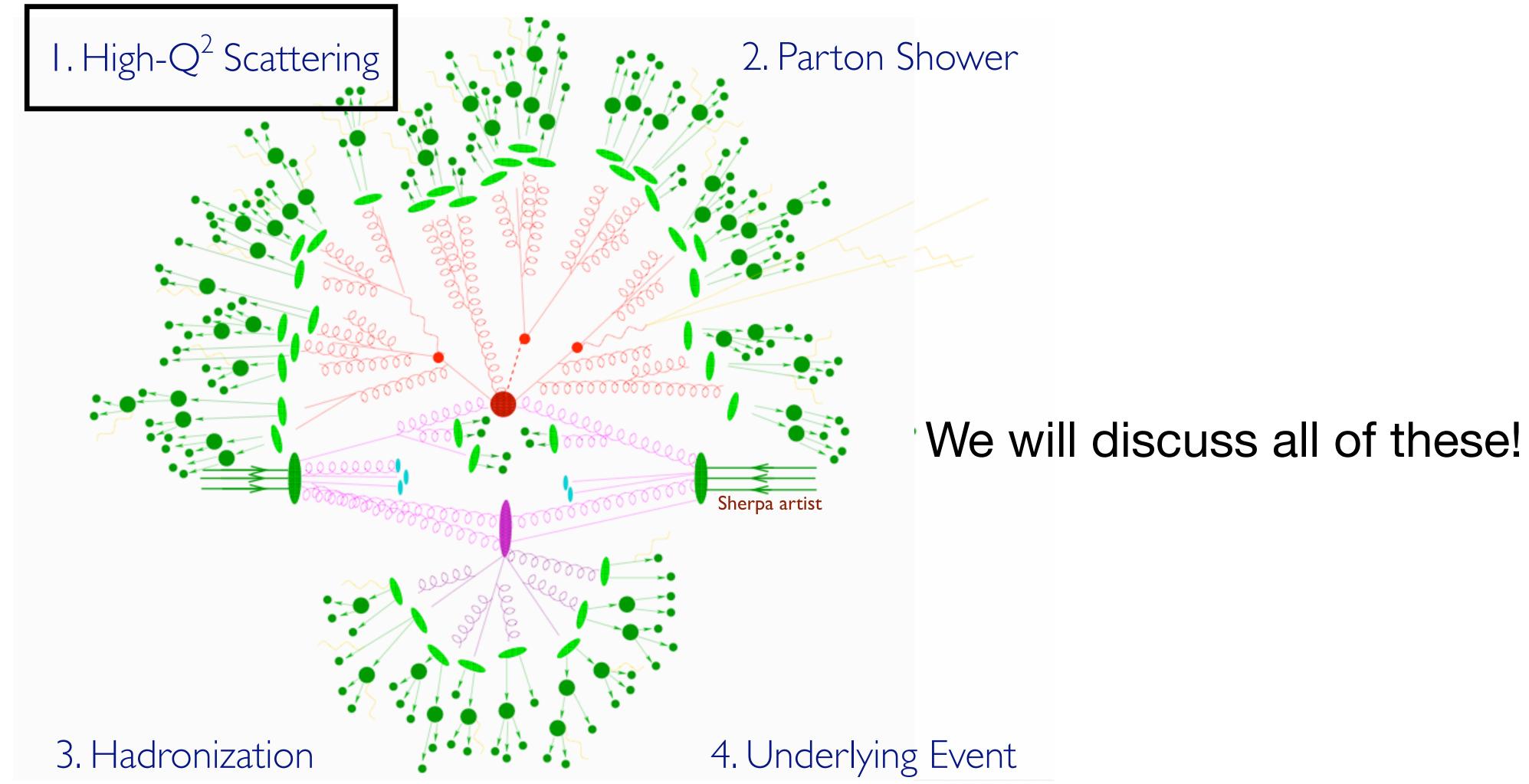
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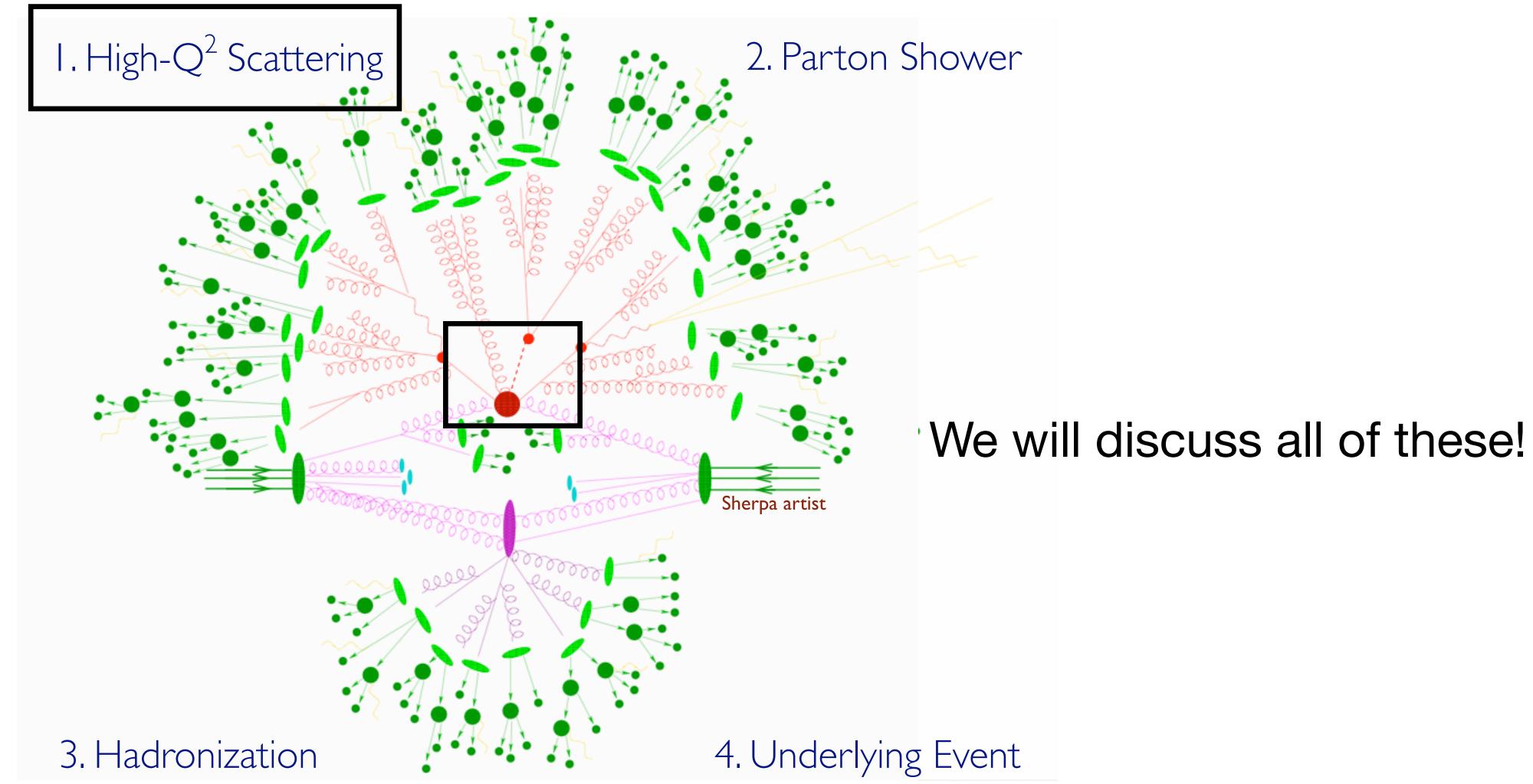
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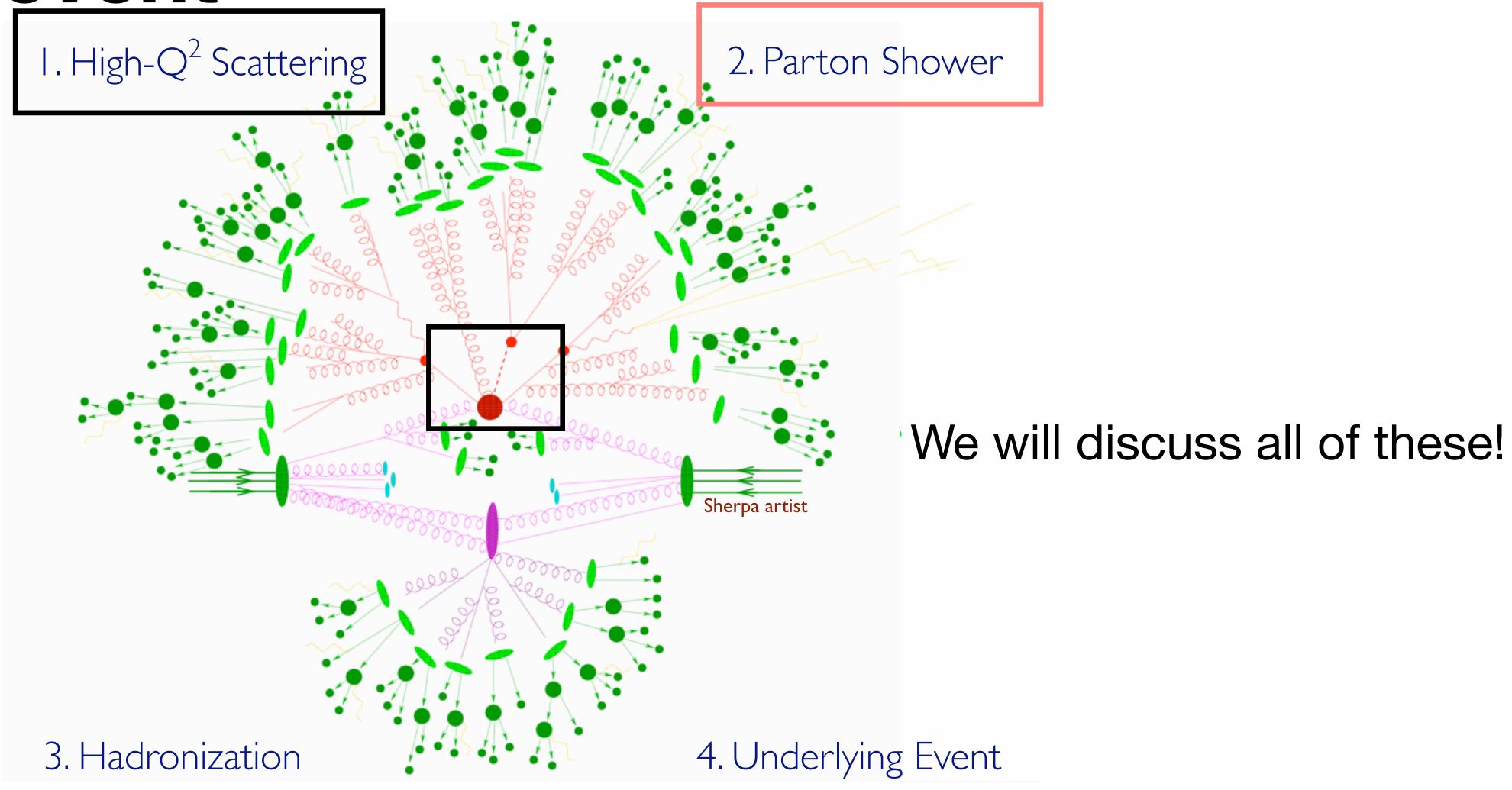


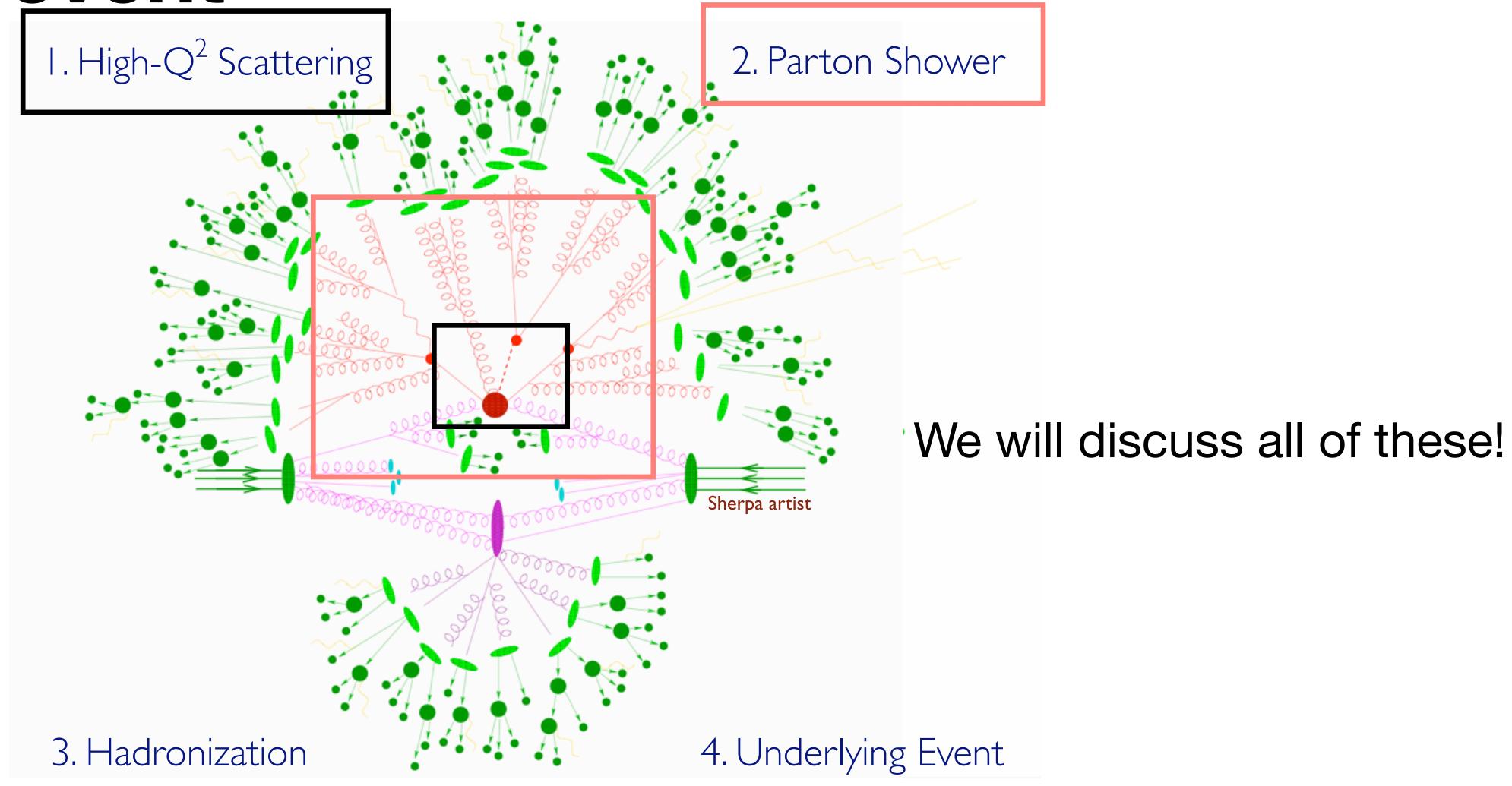
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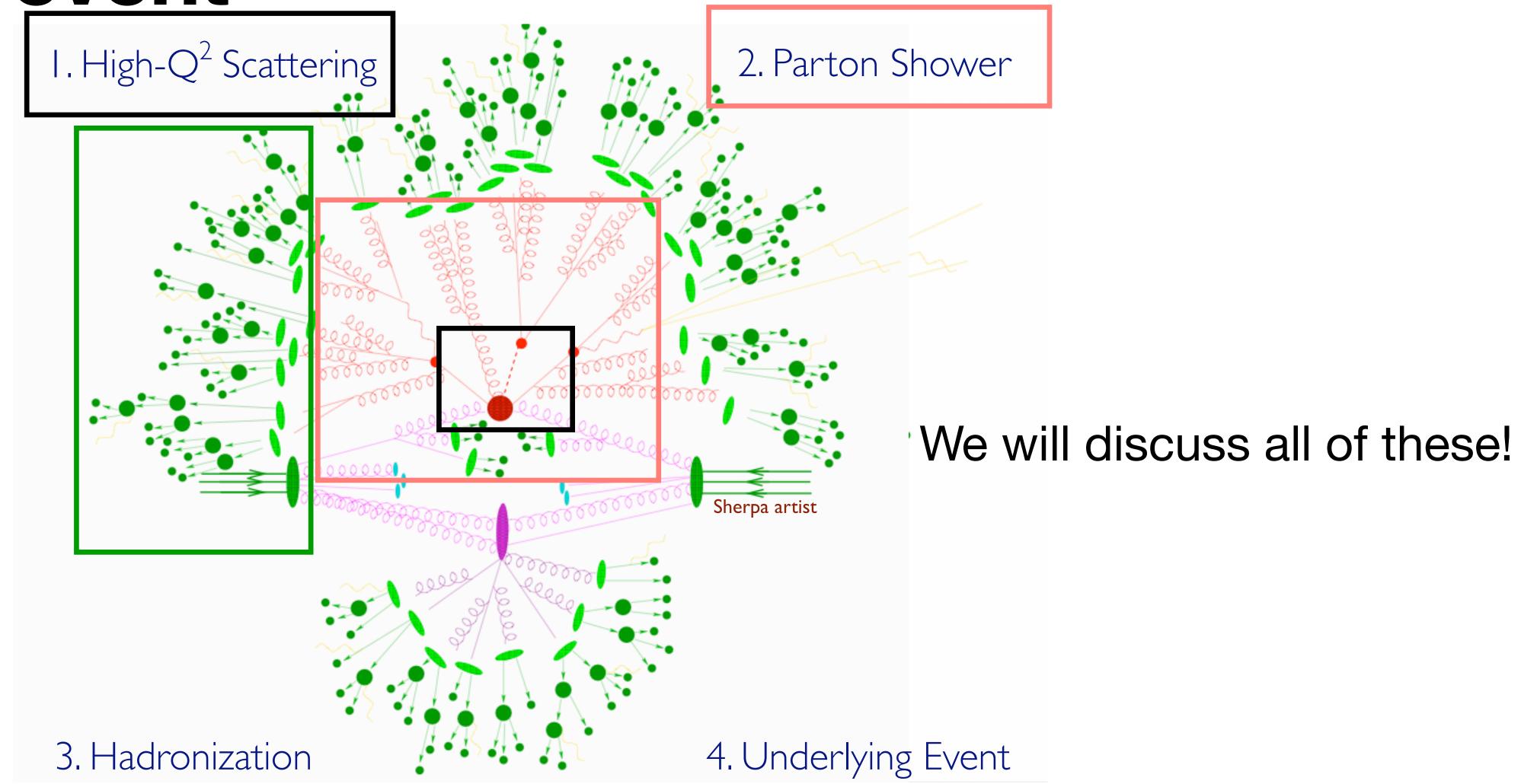


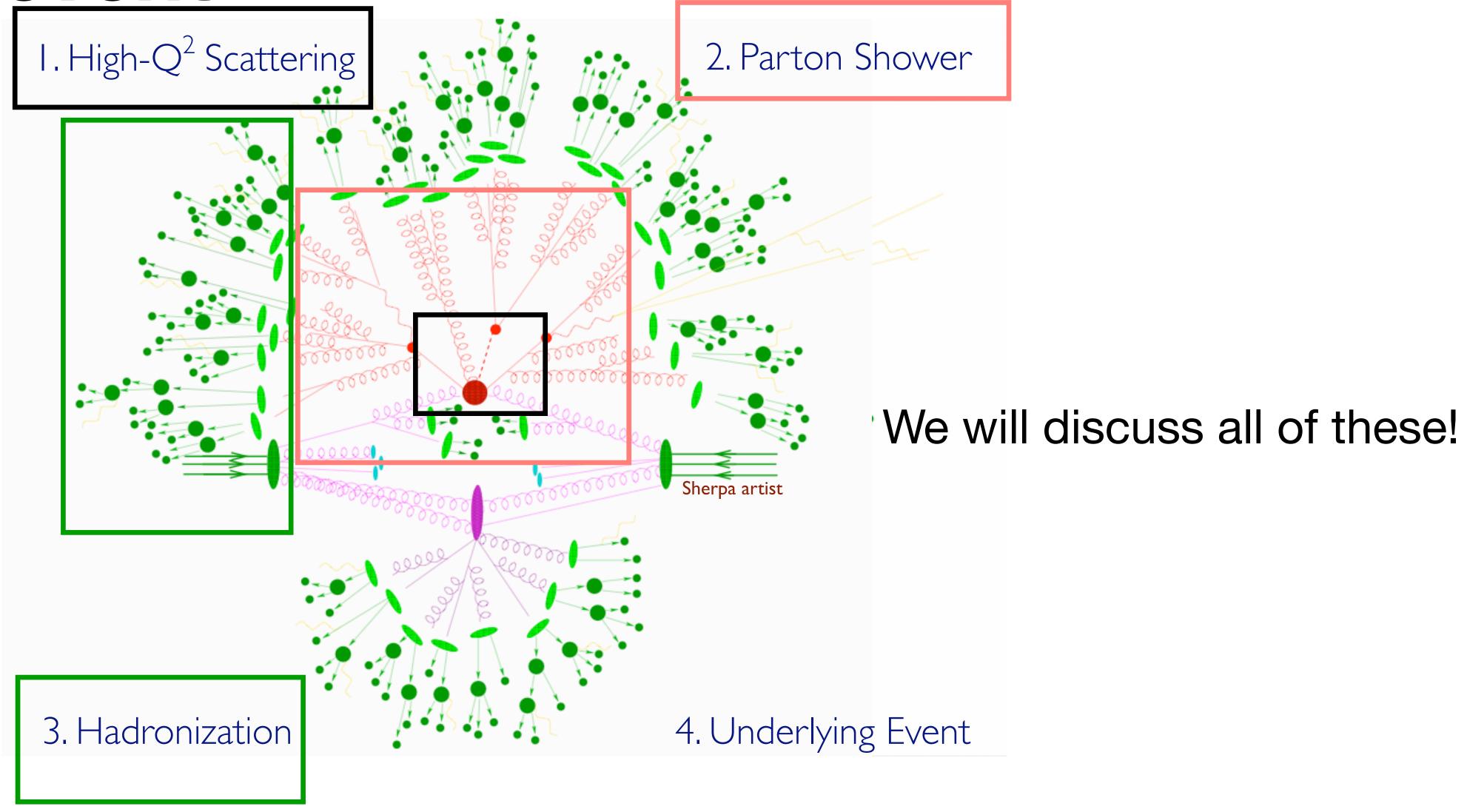


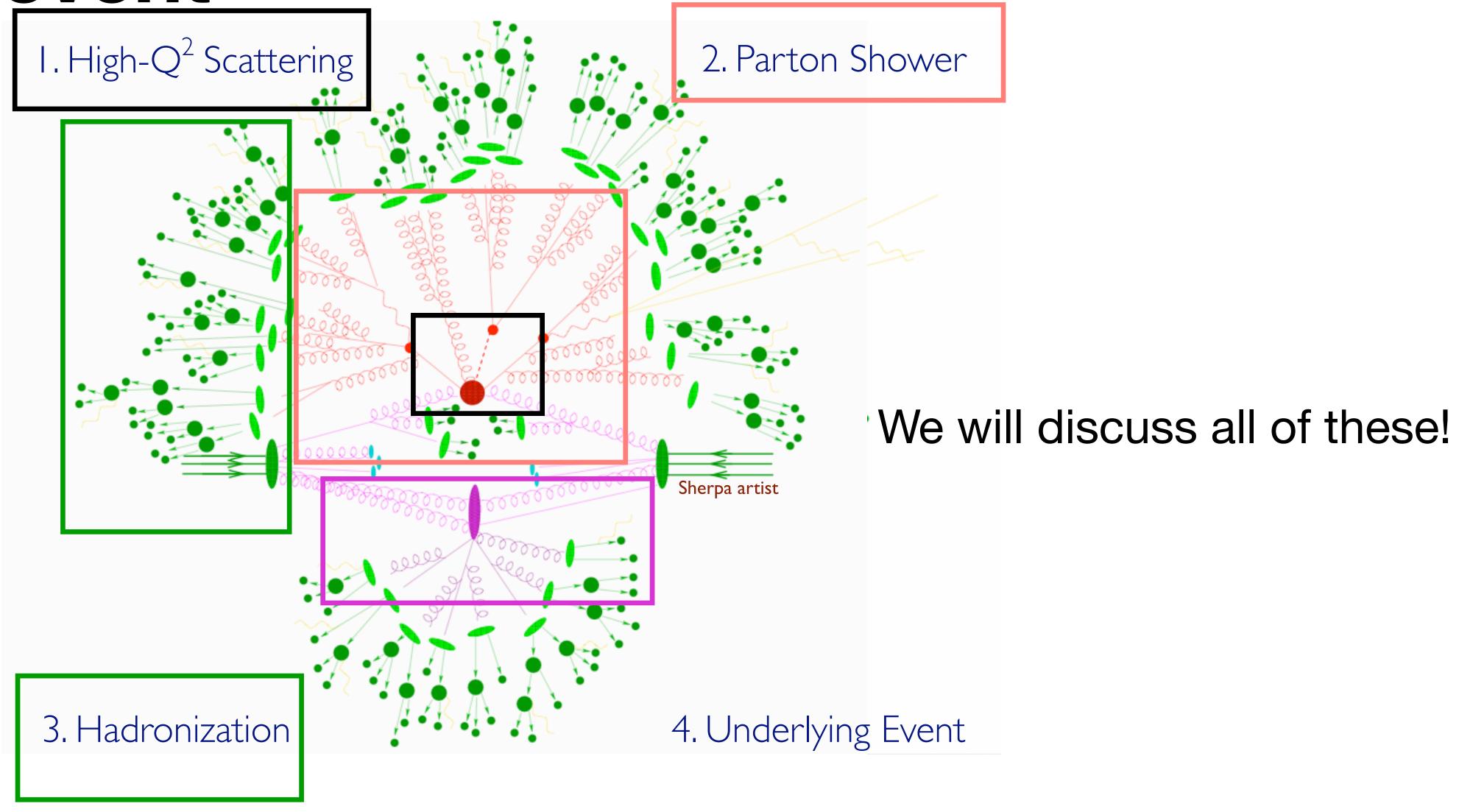














QCD...

LHC is a proton-proton collider:

• colliding particles are proton constituents with are coloured particles QCD plays a crucial role in what we eventually observe in the detectors

Why is QCD "special"? Let's compare it to what we know best: QED

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} |M_{fi}|^2$$

 $|M_{RL}|^2 + |M_{LR}|^2$

From QED to QCD
Example
$$\frac{d\Omega}{d\Omega}R$$
- $\frac{64\pi^{2}}{64\pi^{2}}\frac{(s-m_{Z}^{2})^{2}+m_{Z}^{2}\Gamma_{Z}^{2}}{g_{Z}^{2}}(c_{R}^{\ell})^{-1+\cos\theta}$, $|M_{RL}|^{2}+|M_{LR}|^{2}$
Example $\frac{d\Omega}{d\Omega}R$ - $\frac{64\pi^{2}}{64\pi^{2}}\frac{(s-m_{Z}^{2})^{2}+m_{Z}^{2}\Gamma_{Z}^{2}}{(s-m_{Z}^{2})^{2}+m_{Z}^{2}\Gamma_{Z}^{2}}(c_{L}^{\ell})^{2}(1+\cos\theta)^{2}$
 e^{+}
 VS
 $e^{-\frac{d\sigma_{RL}}{d\Omega}} = \frac{1}{64\pi^{2}}\frac{\ell^{2}S_{L}^{2}}{(s-m_{Z}^{2})^{2}+m_{Z}^{2}\Gamma_{Z}^{2}}(c_{L}^{\ell})^{2}(c_{R}^{\mu})^{2}(1-\cos\theta)^{2}$

Let's compute the matrix-element form $|\mathcal{L}|^2 + |\mathcal{M}_{RL}|^2 \gamma$

Summing and averaging:

$$\sum_{S^2} |M|^2 = \frac{2e^4}{S^2} [t^2 + u^2]$$
 Try this out!

Mandelstam variables: $s = (p_{e+} + p_{e-})^2$ $t = (p_{e+} - p_{\mu+})^2 = -\frac{s}{2}(1 - \cos\theta)$

$$s + t + u = 0$$
 $u = (p_{e+} - p_{\mu-})^2 = -\frac{s}{2}(1 + \cos\theta)$

Eleni Vryonidou

e⁺

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} |M_{fi}|^2$$

 $|M_{RL}|^2 + |M_{LR}|^2$

From QED to QCD
Example
$$\frac{d\Omega}{d\Omega} = \frac{64\pi^2}{64\pi^2} \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \frac{(c_R)^2 (c_R)^2 (1 + \cos \theta)^2}{(c_L^2)^2 (1 + \cos \theta)^2}$$

$$e^+ \frac{\sqrt{Z^0}}{d\Omega} = \frac{1}{64\pi^2} \frac{e^4 Z_L^4}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \frac{(c_L^2)^2 (c_L^2)^2 (c_L^2)^2}{(c_L^2)^2 (c_L^2)^2 (c_L^2)^2} \frac{e^+}{q} \frac{|M_{RR}|^2 + |M_{LL}|^2}{q}$$

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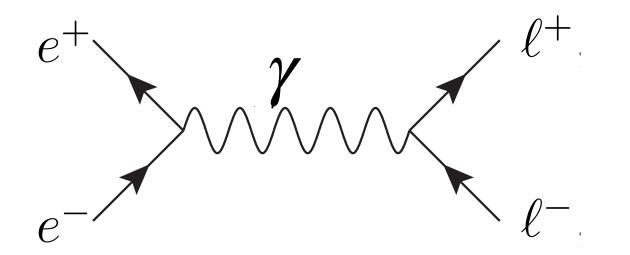
$$s + t + u = 0$$

$$u = (p_{e+} - p_{\mu-})^2 = -\frac{3}{2}(1 + \cos\theta)$$

e⁺

From QED to QCD

Example 1: R-ratio



$$\sum_{\ell=0}^{e^{+}} |M|^{2} = \frac{2e^{4}}{s^{2}} [t^{2} + u^{2}] \qquad \sum_{\ell=0}^{e^{-}} |M|^{2} \propto (1 + \cos^{2}\theta)$$

Cross-section:

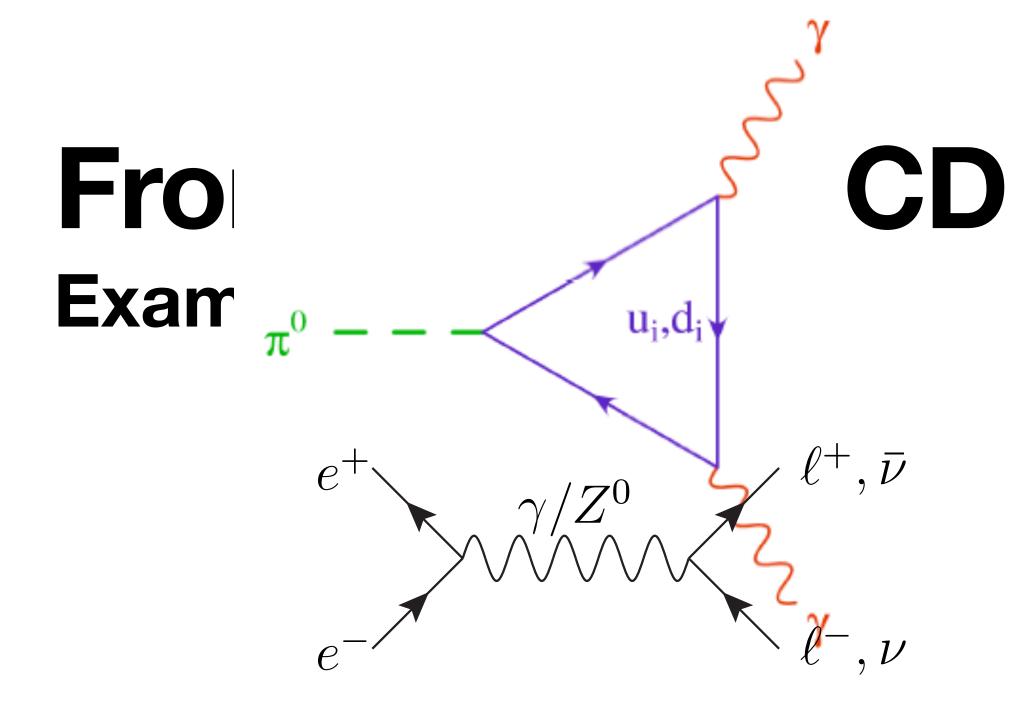
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \bar{\sum} |M|^2$$

2-body phase-space+Momentum conservation

$$d\Omega = d\phi \, d{\rm cos}\theta$$

$$\sigma_{e^+e^-\to\mu^+\mu^-} = \frac{4\pi\alpha^2}{3s}$$

Try this out!



$$\Gamma \sim \sigma (e_c^2 + e_Q^2 + \mu Q_d^2)^2 = \frac{m^2 \pi \alpha^2}{f_{\pi}^2 3s}$$

Difference due to colour!!!

Quark $\frac{\Gamma_E}{a}$ $\frac{7.7}{a}$ $\frac{\pm 0.6}{c}$ be one of $r\bar{r}, g\bar{g}, b\bar{b}$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim N_c \sum_q e_q^2$$
$$= 2(N_c/3) \quad q = u, d, s$$
$$= 3.7(N_c/3) \quad q = u, d, s, c, b$$

Fabio Maltoni

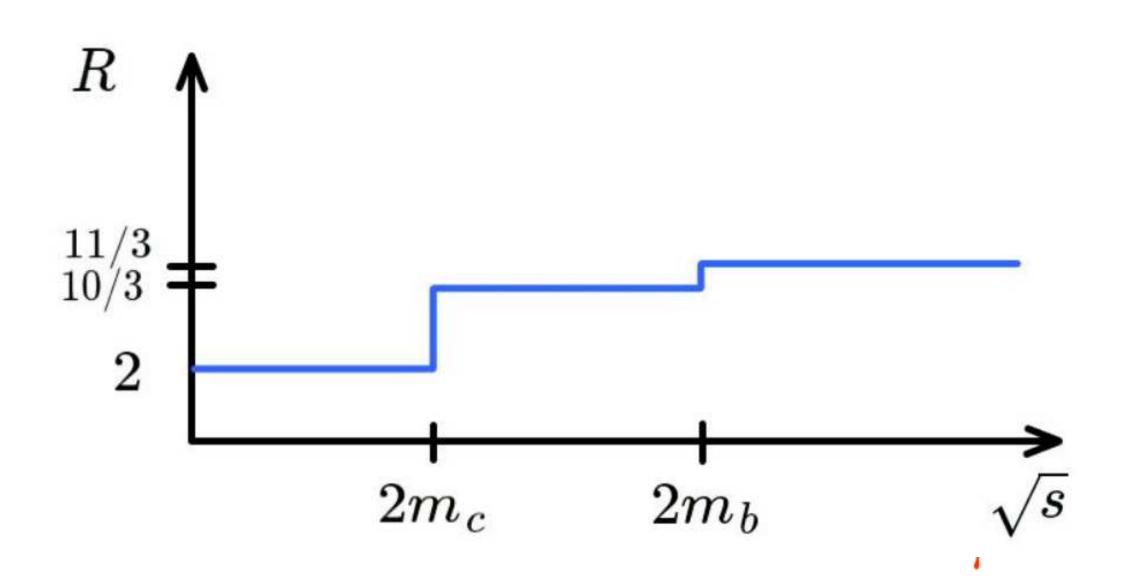
Why did we pick $\mu^+\mu^-$?

Experimental evidence for colour!

From QED to QCD

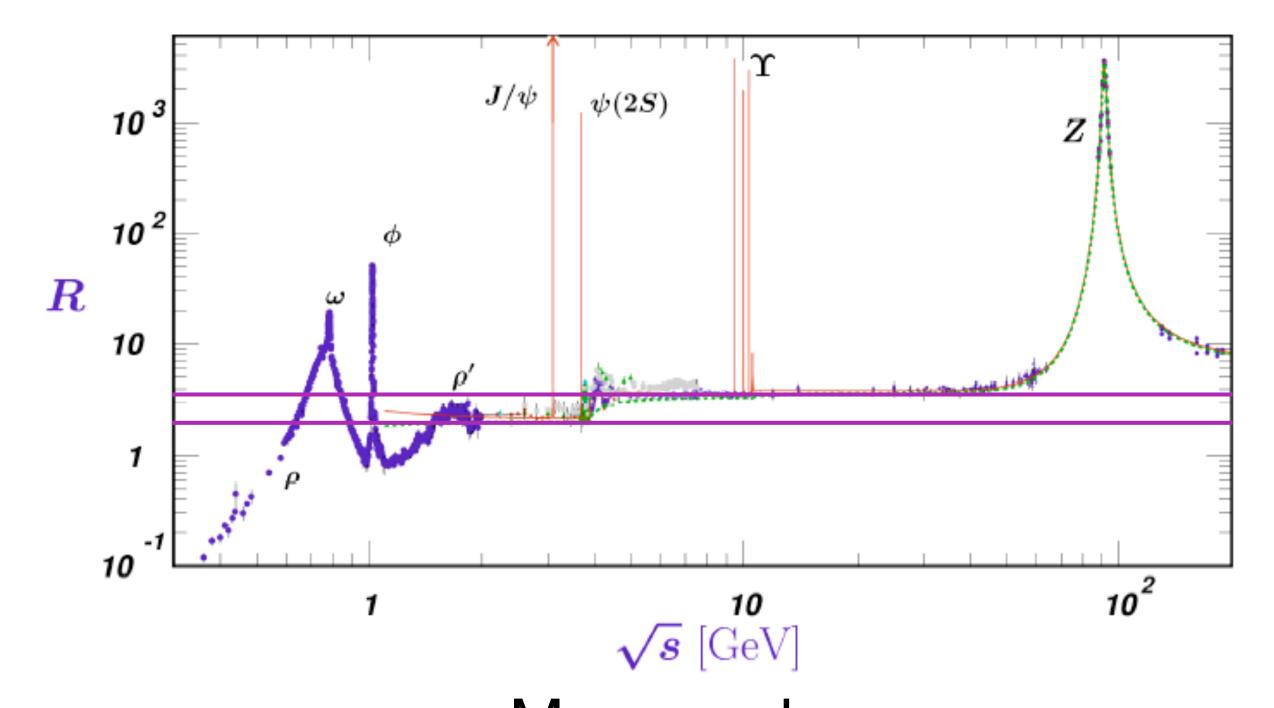
Example 1: R-ratio

R-ratio computation



Expected
$$\Gamma \sim N_c^2 \left[Q_u^2 - Q_d^2\right]^2 \frac{m_\pi^3}{f_\pi^2}$$

$$\Gamma_{TH} = \left(\frac{N_c}{2}\right)^2 7.6 \,\mathrm{eV}$$



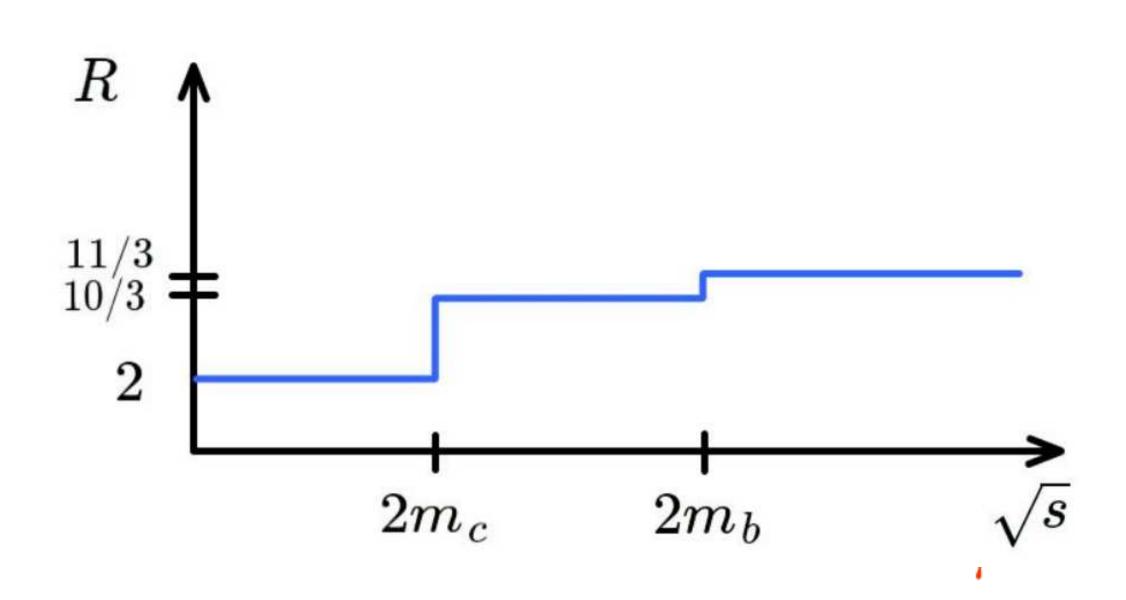
$$R = \frac{\sigma(e^+e^-\text{Measured}_{\rm S})}{\sigma(e^+e^-\to \mu^+\mu^-)} \sim N_c \sum_q e_q^2$$

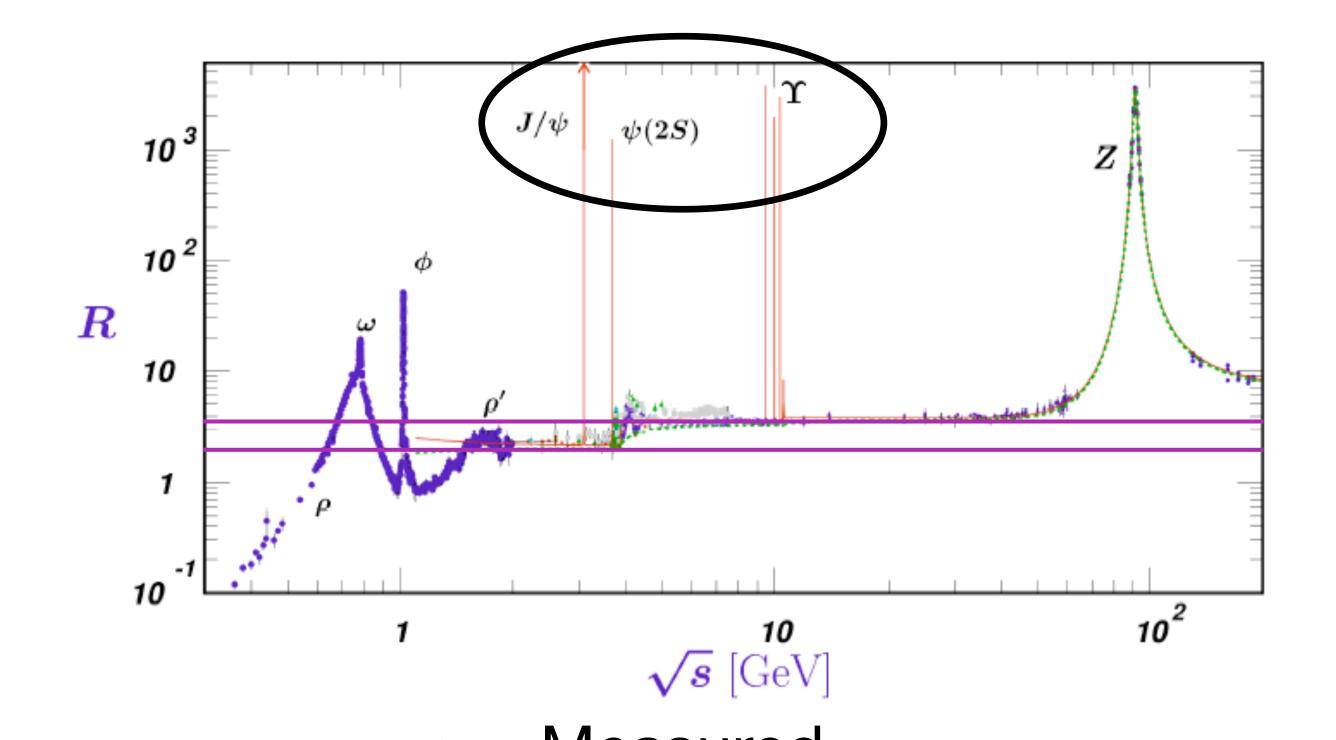
STFC HEP school $\overline{z}\overline{z}2(N_c/3)$ q=u,d,s

From QED to QCD

Example 1: R-ratio

R-ratio computation

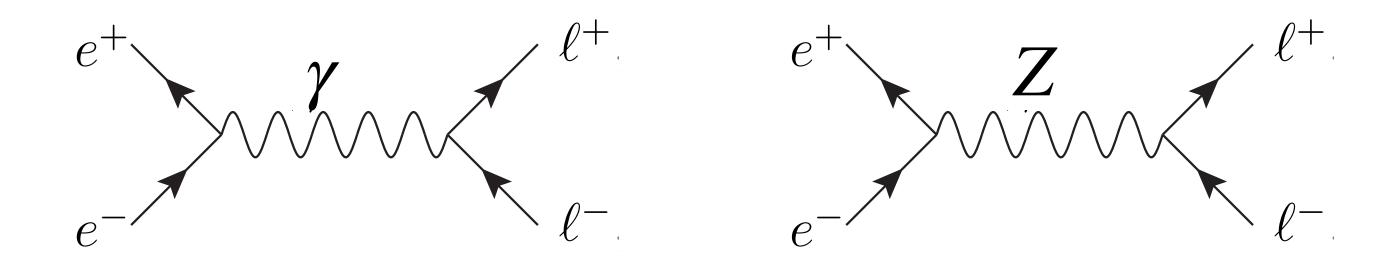




 $\begin{array}{ll} \text{Expected} \\ \Gamma \sim N_c^2 \left[Q_u^2 - Q_d^2\right]^2 \frac{m_\pi^3}{f_c^2} & R = \frac{\sigma(e^+e^-\text{Measured}_{\text{S}})}{\text{Cuarkonium states: very small width pvery long lived states}} \sim N_c \sum_{e_q^2} e_q^2 \\ \Gamma_{TH} = \left(\frac{N_c}{2}\right)^2 7.6 \, \text{eV} & \text{Stfc HEP school } \bar{z}\bar{z}\bar{z}^2 2(N_c/3) & q = u,d,s \end{array}$

A few words about the Z-resonance

Breit -Wigner



Z contribution becomes relevant when $\sqrt{s} \sim M_Z$

We then need both diagrams and their interference

See exercise!

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Z-resonance

Breit-Wigner and Narrow Width Approximation

Z is an unstable particle, we can't simply use $\frac{1}{s-M_Z^2}$

Breit-Wigner propagator: $\frac{1}{s - M_Z^2 + i\Gamma M}$

Narrow width approximation:

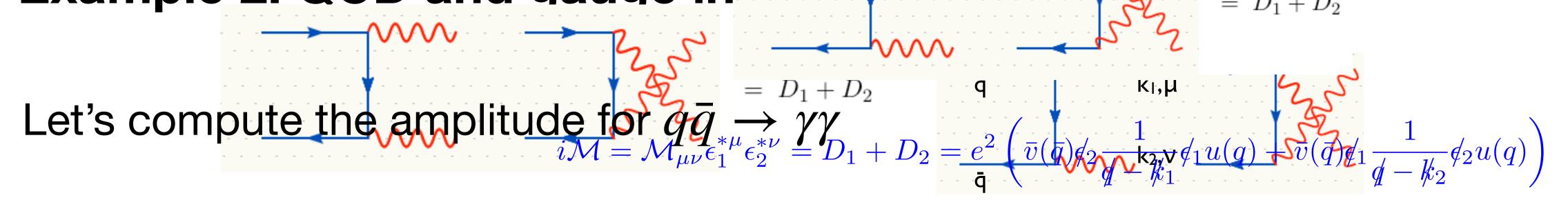
$$\frac{1}{(\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2}\approx \frac{\pi}{M_Z\Gamma_Z}\delta(\hat{s}-M_Z^2) \quad \text{if } \Gamma_Z/M_Z\ll 1$$

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-}\simeq\sigma_{e^+e^-\to Z}\times Br(Z\to\mu^+\mu^-) \text{ with } Br(Z\to\mu^+\mu^-)=\Gamma_{Z\to\mu^+\mu^-}/\Gamma_Z$$

Simplifies computations for particles with narrow width (e.g. Higgs)

From QED to QCD

Example 2: QCD and gauge in $D_1 + D_2 = D_1 + D_2$



$$i\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon_{1}^{*\mu}\epsilon_{2}^{*\nu} = D_{1} + D_{2} = e^{2}\left(\bar{v}(\bar{q})\not\epsilon_{2}\frac{1}{\not q - \not k_{1}}\not\epsilon_{1}u(q) + \bar{v}(\bar{q})\not\epsilon_{1}\frac{1}{\not q - \not k_{2}}\not\epsilon_{2}u(q)\right)_{2} = e^{2}\left(\bar{v}(\bar{q})\not\epsilon_{2}\frac{1}{\not q - \not k_{1}}\not\epsilon_{1}u(q) + \bar{v}(\bar{q})\not\epsilon_{1}\frac{1}{\not q - \not k_{2}}\not\epsilon_{2}u(q)\right)_{2}$$

Gauge invariance requires: $\epsilon_1^{*\mu} k_2^{\nu} \mathcal{M}_{\mu\nu} = \epsilon_2^{*\nu} k_1^{\mu} \mathcal{M}_{\mu\nu} = 0$

$$\begin{split} \epsilon_1^{*\mu} k_2^{\nu} \mathcal{N} & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) (\not k_1 - \not q) \frac{1}{\not k_1 - \not q} \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) (\not k_1 - \not q) \frac{1}{\not k_1 - \not q} \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) (\not k_1 - \not q) \frac{1}{\not k_1 - \not q} \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q - \not q} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\nu} k_1^{*\nu} k_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not e_2 \frac{1}{\not q} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) \not e_2 u(q) \right) & \quad \mathcal{M}_{\mu\nu} k_1^{*\nu} k_1^{*\nu} k_2^{*\nu} k_1^{*\nu} k_2^{*\nu} k_1^{*\nu} k_1^{*\nu} k_2^{*\nu} k_1^{*\nu} k_1$$

Only the sum of the two diagramorkis times invariant. For the amplitude to be gauge invariant it is enough that one of the polarizations is longitudinal. The state of the other

$-v(q)q_2u(q) + v(q)q_2u(q) - 0$

Only the sum of the two diagrams is gauge invariant. For the amplitude to be gauge From Market - Date a plant of the state of the other

Example 2: QCD and gauge novageneralized what twice share done for SU(3). In this case we take the invariant it is enough that one of the polarizations is longitudinal. The state of the other land representation of SU(3), 3 and 3*. Then the gauge boson is irrelevant. killet's try now to generalize what we have done for SU(3). In this case we take the (anti-)quarks to be in the cross test and generalize the like a photon, so we identify the gluon with the octet and generalize the per the other [a,b] the octet and generalize the other [a,b] the octet and generalize the other [a,b] the other [a,b] to [a,b] the other [a,b] to [a,b] the other [a,b] to [a,b] the other [a,

relevant μ ϵ Let's do the same for q

$$t_{ij}^{a}\gamma^{\mu}$$

$$= i f^{abc}t^{c}$$

$$= \lambda f^{abc}t$$

the amplit

inal. The the two

 $[t^a, t^b] = if^{abc}t^c$

We don't get zero anymore!

$$k_{1\mu}M_q^{\mu} = i(-g_s f^{abc} \epsilon_2^{\mu})(-ig_s t_{ij}^c \bar{v}_i(\bar{q})\gamma_{\mu}u_i(q))$$

 $\epsilon_{\text{We ind}}^{*\mu} k_{\text{D}}^{\nu} \mathcal{M}_{\text{we interpret as the normalivertex}} = 0$ times a new 3 gluon vertex: STFC HEP school 20

 $if^{abc}(t^bt^c)_{ij} = \frac{C_A}{2}t^a_{ij}$

Eleni Vryonidou



From QED to QCD $-ig_s^2D_3 = (-ig_st_{ij}^a\bar{v}_i(\bar{q})\gamma^\mu u_j(q)) \times \left(\frac{-i}{p^2}\right) \times \\ = \underbrace{\text{QCD and gauge invariance}}_{ig_s^2D_3 = \left(-ig_st_{ij}^a\bar{v}_i(\bar{q})\gamma^\mu u_j(q)\right) \times \left(\frac{-i}{p^2}\right) \times \\ \text{we missing?}}_{(-p_+k_{j_s^2}Jk_{j_s^2})} \times \underbrace{\left(-p_+k_{j_s^2}Jk_{j_s^2}+\epsilon_{[1-(k_{j_s^2})^{-1}k_{j_s^2}]}^{\rho_1}(k_{j_s^2})^{-1}\mu_j(q)\right) \times \left(\frac{-i}{p^2}\right) \times \left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right) \times \\ = \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right) \times \left(\frac{-i}{p^2}\right) \times \left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right)}_{(-p_+k_{j_s^2}D_3 = 0)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right) \times \left(\frac{-i}{p^2}\right) \times \left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right)}_{(-p_+k_1,k_2)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right) \times \left(\frac{-i}{p^2}\right) \times \left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\rho}(k_2)\right)}_{(-p_+k_1,k_2)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\nu}(k_2)\right)}_{(-p_+k_1,k_2)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\nu}(k_2)\right) \times \left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\nu}(k_2)\right)}_{(-p_+k_1,k_2)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\nu}(k_2)\right)}_{(-p_+k_1,k_2)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1)\epsilon_2^{\nu}(k_2)\right)}_{(-p_+k_1,k_2)} \times \underbrace{\left(-gf^{abc}V_{\mu\nu\rho}(-p,k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_1,k_2)\epsilon_1^{\nu}(k_$

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Fabio Maltoni

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Eleni Vryonidou



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With the above expression we obtain atcomprise the gauge variation of b1+ b2

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STFC HEP school 2022
Fabio Maltoni Eleni Vryonidou



QCD Lagrangian

$$\mathcal{L} = \begin{bmatrix} -\frac{1}{4}F_{\mu\nu}^{a}F_{a}^{\mu\nu} + \sum_{\vec{q}} \bar{\psi}_{i}^{(f)}(i\partial - m_{f})\psi_{i}^{(f)} \\ -\bar{\psi}_{i}^{(f)}(g_{s}t_{ij}^{a}A_{a})\psi_{j}^{(f)} \\ -\bar{\psi}_{i}^{(f)}(g_{s}t_{ij}^{a}A_{a})\psi_{j}^{(f)} \\ \text{Matter} \end{bmatrix} - \begin{bmatrix} \bar{\psi}_{i}^{(f)}(g_{s}t_{ij}^{a}A_{a})\psi_{j}^{(f)} \\ -\bar{\psi}_{i}^{(f)}(g_{s}t_{ij}^{a}A_{a})\psi_{j}^{(f)} \\ \text{Matter} \end{bmatrix}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

$$\operatorname{tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$$

See QCD-QED course!

→ Normalization

Colour algebra



$$\operatorname{Tr}(t^a) = 0$$

$$= 0$$

$$[t^a, t^b] = if^{abc}t^c$$

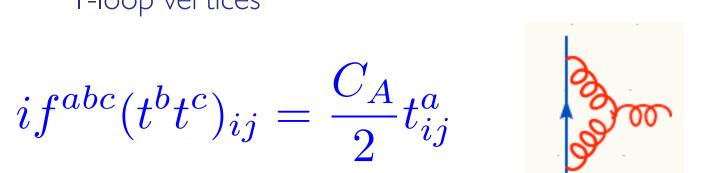
$$Tr(t^a t^b) = T_R \delta^{ab}$$

$$\operatorname{Tr}(t^a t^b) = T_R \delta^{ab}$$
 $_{a} \operatorname{ood} \bigcirc \operatorname{ood}) = \mathsf{T}_{\mathsf{R}} * \operatorname{ood} \bigcirc$

$$[F^a, F^b] = if^{abc}F^c$$

$$(t^a t^a)_{ij} = C_F \delta_{ij}$$

$$= C_F * i$$



$$\sum_{cd} f^{aca} f^{bca}$$

$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$

$$a = C_A *$$

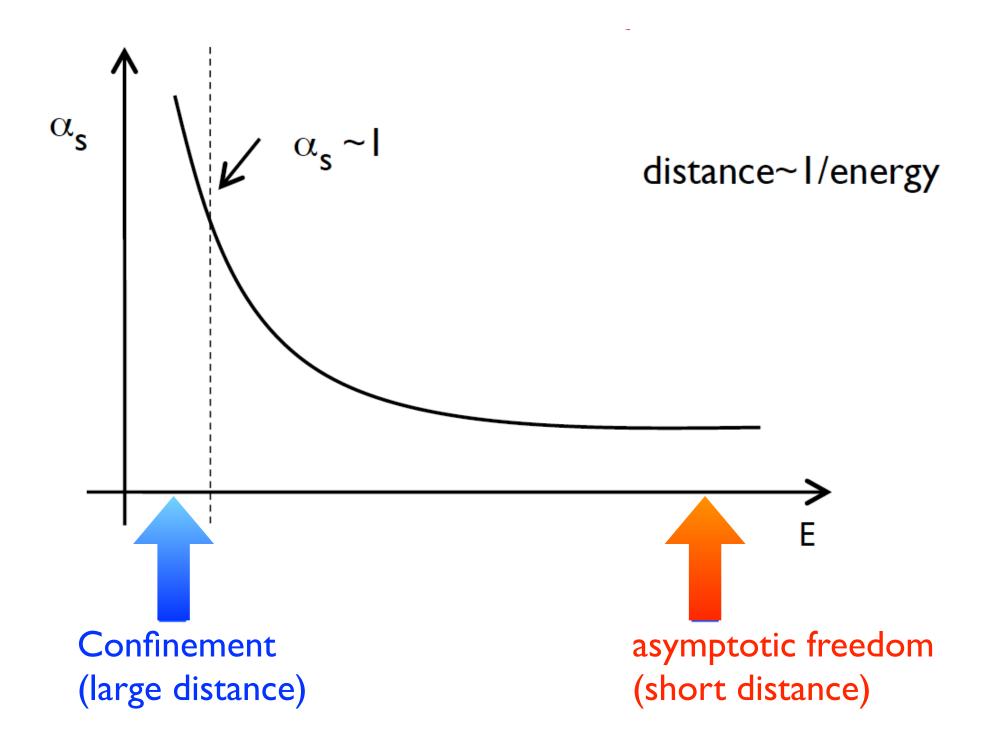
$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t^a_{ij}$$

Can be a bottleneck for higher order computations! People always on the lookout for simplifications! Quite a few computations are done in the large N_c limit.

Properties of QCD

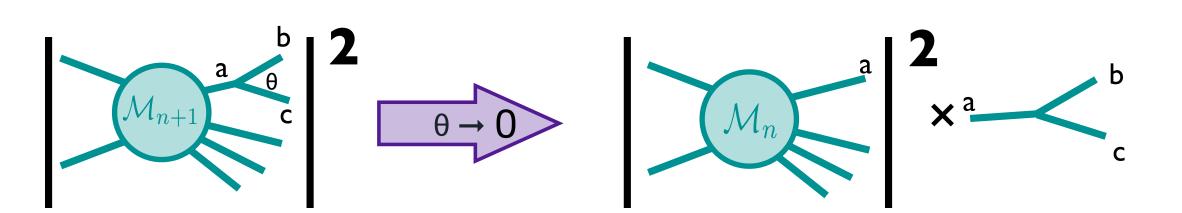
UV: Asymptotic freedom

- Perturbative computations
- Parton model



IR: Universality

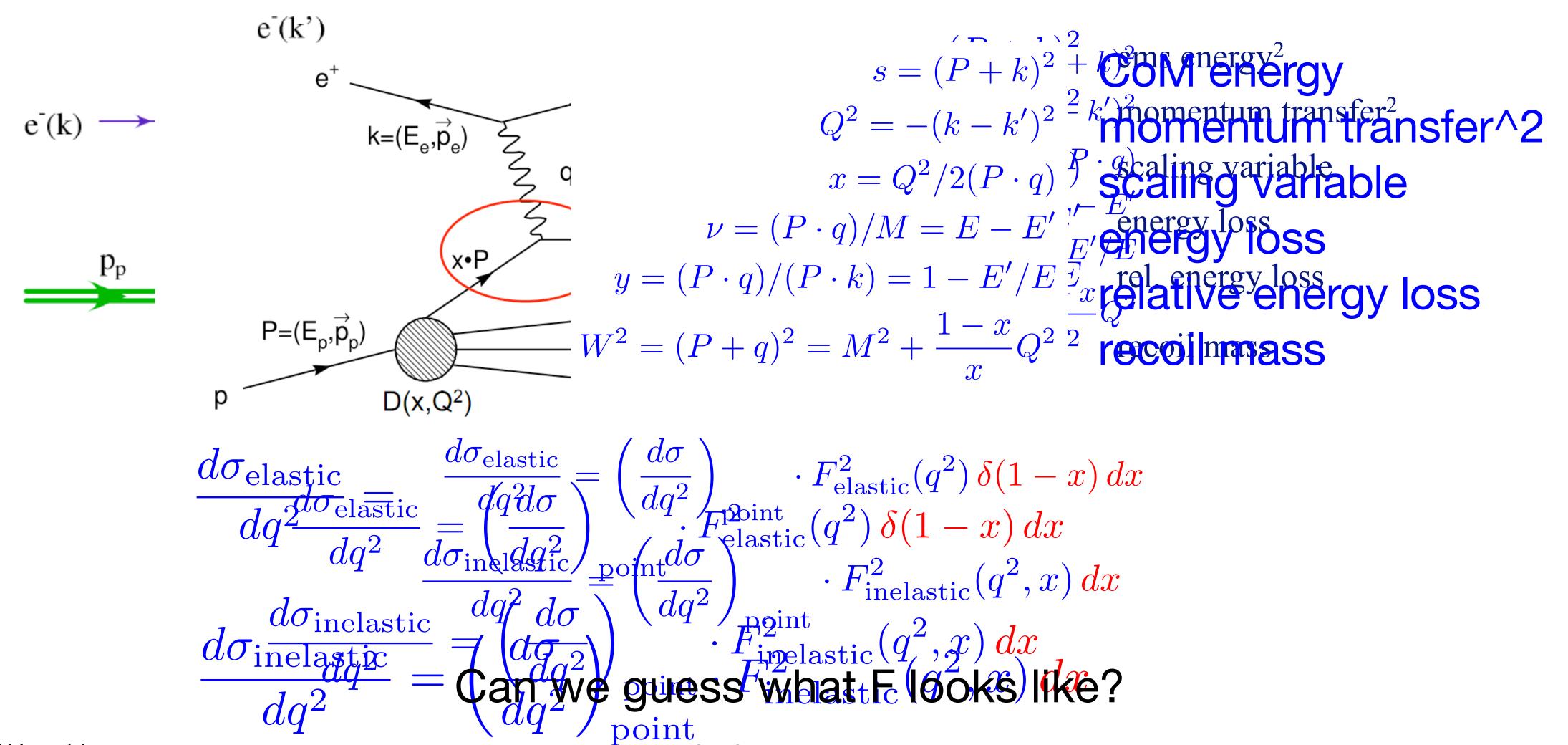
- Collinear Factorisation
- Parton showers



$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_{\mathrm{S}}}{2\pi} P_{a \to bc}(z)$$



Deep Inelastic Scattering



Deep Inelastic scattering

What can $F^2(q^2)$ look like?

1. Proton charge is smoothly distributed (probe penetrates proton like a knife through butter)

$$F_{elastic}^2(q^2) \sim F_{inelastic}^2(q^2, x) \ll 1$$

2. Proton consists of tightly bound charges (quarks hit as single particles, but cannot fly away because tightly bound)

$$F_{elastic}^2(q^2) \sim 1$$
 $F_{inelastic}^2(q^2, x) \ll 1$
 III3. $F_{elastic}^2(q^2) \ll 1$ $F_{inelastic}^2(q^2, x) \sim 1$

Quarks are free particles which fly away without caring about confinement!

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Parton, Model $\frac{1}{4} \text{tr} [k \gamma^{\mu} k' \gamma^{\nu}] \stackrel{\mathbf{Z}}{=} k^{\mu} k'^{\nu} + k' \mu k^{\nu} - g^{\mu\nu} k \cdot k' \\ L^{\mu\nu} = \frac{1}{4} \text{tr} [k \gamma^{\mu} k' \gamma^{\nu}] = k^{\mu} k'^{\nu} + k'^{\mu} k^{\nu} - g^{\mu\nu} k \cdot k'$

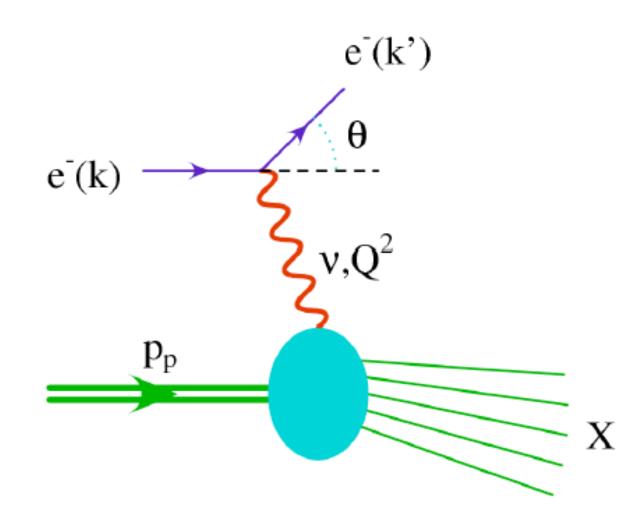
Based on Lorentz and gauge invariance $|\psi_{\gamma}^{\nu}|' = k^{\mu}k'^{\nu} + k'^{\mu}k^{\nu} - q^{\mu\nu}k' \cdot k'$

$$W^{\mu\nu} = \sum_{X} \int d\Phi_X h_{X\mu\nu}$$

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right) F_1(x,Q^2) + \left(p_{\mu} - q_{\mu}\frac{p\cdot q}{q^2}\right) \left(p_{\nu} - q_{\nu}\frac{p\cdot q}{q^2}\right) \frac{1}{p\cdot q} F_2(x,Q^2)$$
 Eleni Vryonidou
$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right) F_1(x,Q^2) + \left(p_{\mu} - q_{\mu}\frac{p\cdot q}{q^2}\right) \left(p_{\nu} - q_{\nu}\frac{p\cdot q}{q^2}\right) \frac{1}{p\cdot q} F_2(x,Q^2)$$

Eleni Vryonidou

Parton Model



$$\sigma^{ep \to eX} = \sum_{X} \frac{1}{4ME} \int d\Phi \frac{1}{4} \sum_{\text{spin}} |\mathcal{M}|^2$$

After a bit of maths (good exercise), we get:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4} \left\{ [1 + (1-y)^2] F_1(x,Q^2) + \frac{1-y}{x} \left[F_2(x,Q^2) - 2xF_1(x,Q^2) \right] \right\}$$

Transverse photon

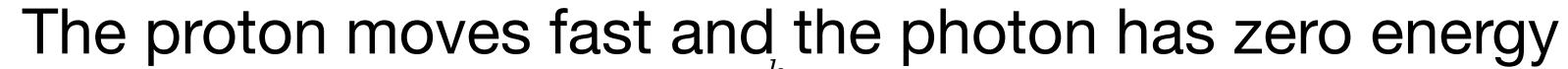
Longitudinal photon

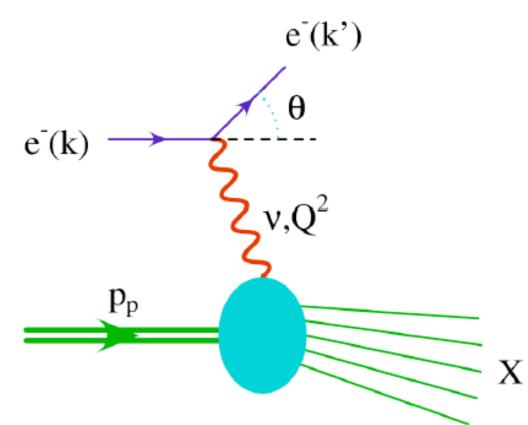
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Parton Model

Breit frame





$$\hat{p} = (E, 0, 0, \xi p)$$

$$q = (0, 0, 0, -Q)$$

$$q = (0, 0, 0, -Q)$$

$$p \equiv \left(\sqrt{\frac{Q^2}{4x^2} + m^2}, \frac{Q}{2x}, \vec{0}_\perp\right) \approx \left(\frac{Q}{2x} + \frac{xm^2}{Q}, \frac{Q}{2x}, \vec{0}_\perp\right)$$

$$q \Rightarrow \left(\sqrt{\frac{Q^2}{4x^2} + m^2}, \frac{Q}{2x}, \vec{0}_\perp\right).$$

$$q \Rightarrow \left(\sqrt{\frac{Q^2}{4x^2} + m^2}, \frac{Q}{2x}, \vec{0}_\perp\right).$$

Rest frame: Proton extent:
$$\Delta x^{+} \sim \Delta x^{-} \sim \frac{1}{m} \sqrt{k'}$$

$$\text{Breit-diparte} \text{Proton extent: } (x, 2) + \frac{1-Q}{\sqrt{m^{2}}} [F_{2}(x, Q_{2}^{2})x^{-2}xF_{1}(x, Q^{2})]$$

Photon extent: $\Delta x^+ \sim 1/Q$

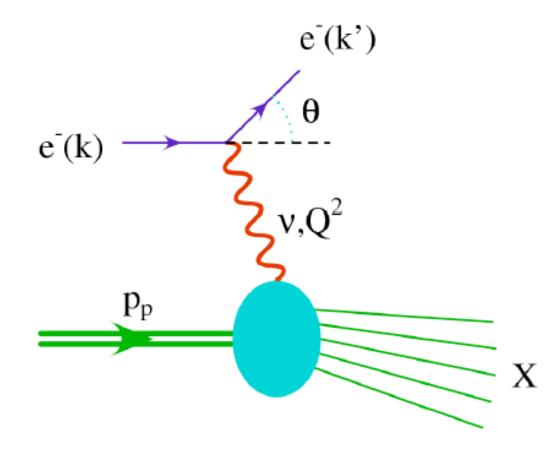
$$(\Delta x^+)_{\text{photon}} \ll (\Delta x^+)_{\text{proton}}$$

The time scale of a typical parton-parton interaction is much larger than the hard interaction time.

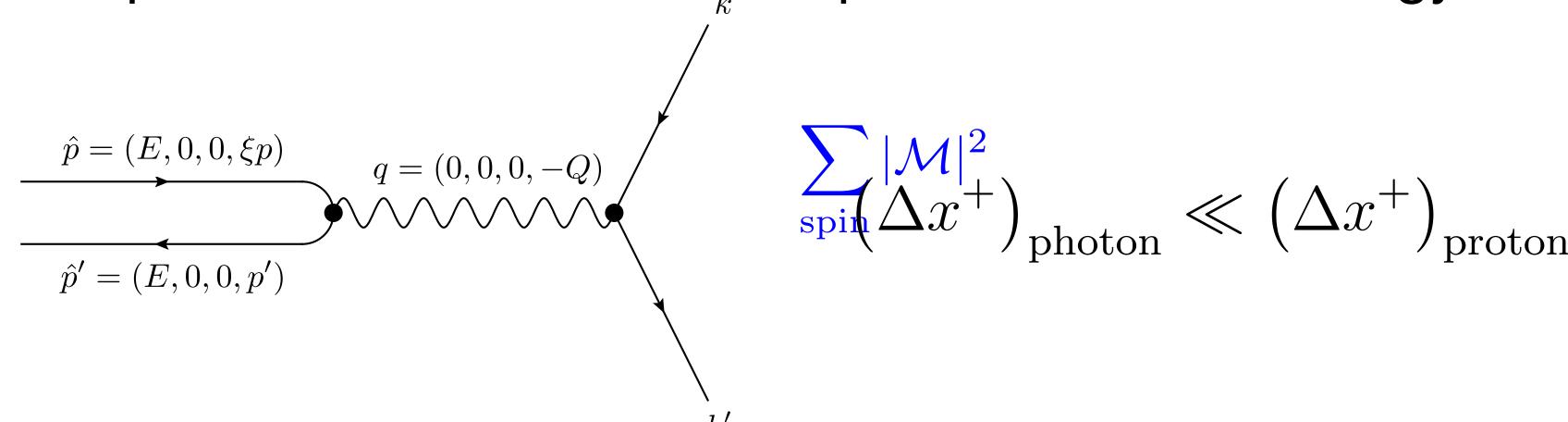


Parton Model

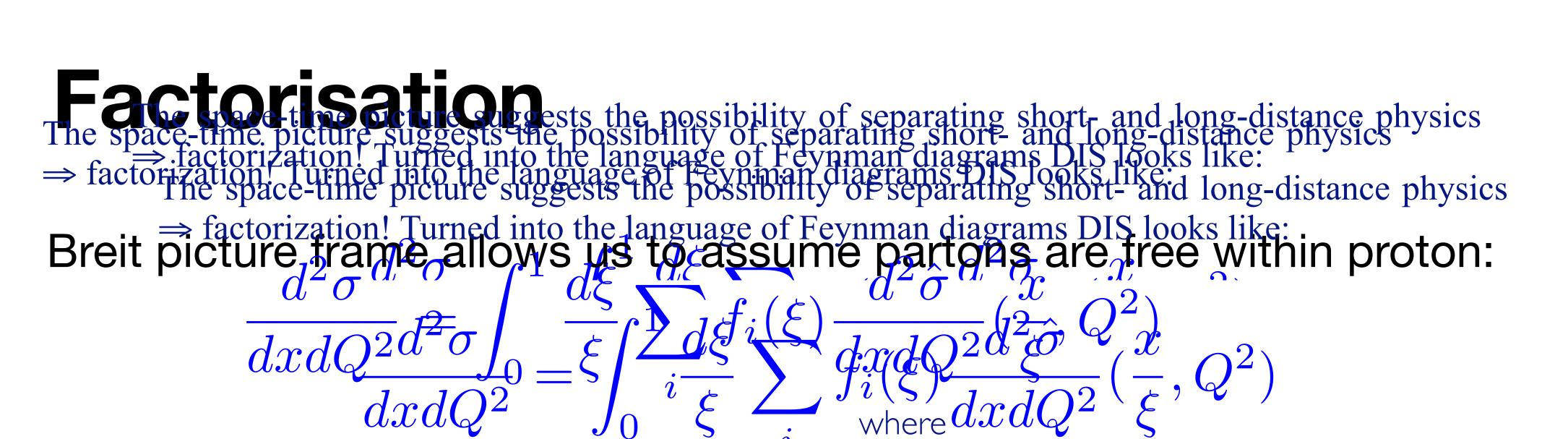
Breit frame

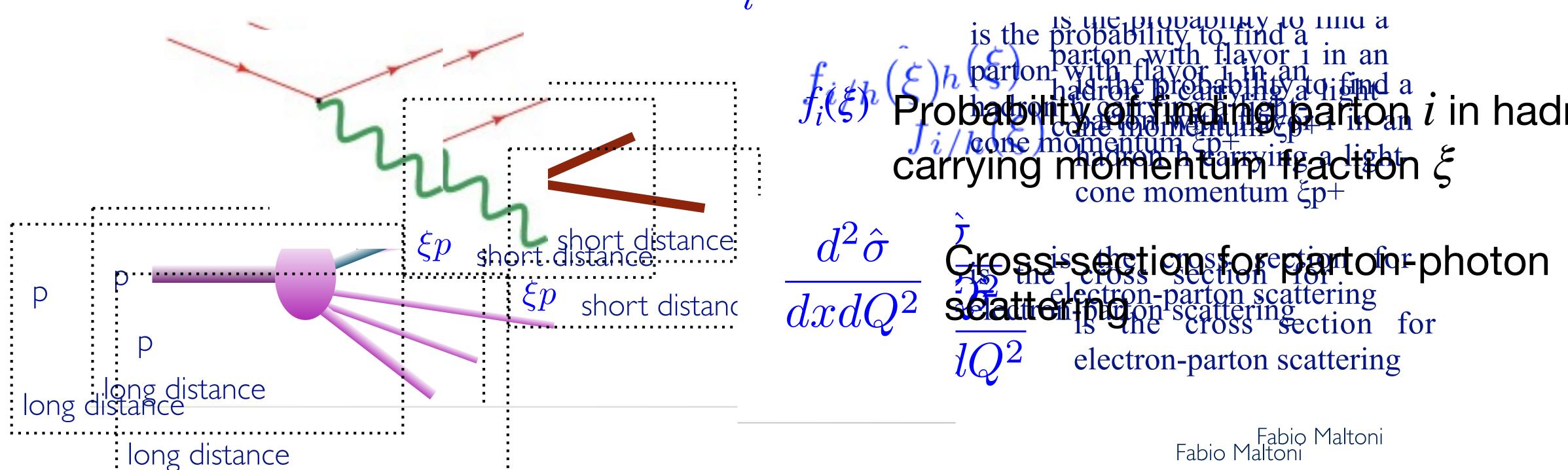


The proton moves fast and, the photon has zero energy



- The time scale of a typical parton-parton interaction is much larger than the hard interaction time. Schematically: in the Breit frame the proton moves very fast towards the photon, and is therefore
- Lorentz contracted to a kind of pancake.
- The photon interaction then takes place on the very short time scale when the photon passes that pancake.
- During the short interaction time, the struck quark thus does not interact with the spectator quarks and can be regarded as a free parton.





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Fabio Maltoni

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DIS cross-section

Comparing our inclusive cross-section:

$$\frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{dx dQ^{2}} \underbrace{\frac{4\pi 4\pi Q^{2}}{\sqrt{4}Q^{2}}}_{Q^{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right] \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right] \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right] \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right] \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right] \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right\}}_{x} \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right\}}_{x} \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) + \frac{1-y}{x} F_{2}(x,Q^{2}) - 2x F_{1}(x,Q^{2}) \right\}}_{x} \right\}}_{x} \\ + \frac{d^{2} \frac{d^{2} \sigma}{dx dQ^{2}}}{\sqrt{4}Q^{2}} \underbrace{\left\{ \left[1 + (1y)^{2} \frac{y}{\sqrt{4}} \right]^{2} F_{1}(x,Q^{2})^{2} + \frac{1-y}{x} F_{2}(x,Q^{2}) + \frac{1-y}{x} F_{2}(x,Q^{2$$

$$\frac{d^{2}d^{2}\sigma}{dx dQ \partial Q^{2}} \int_{0}^{1} \int_{\xi}^{t} \frac{d\xi}{2\pi} \sum_{d^{2}i\sigma} (\xi) \frac{d^{2}\sigma d^{2}\sigma x}{dx dQ^{2}} (\xi) Q^{2} With \frac{d}{d\zeta} \frac{d^{2}\hat{\sigma}}{dQ^{2}dx} = \frac{4\pi\alpha^{2}}{Q^{2}\hat{\sigma}} \frac{1}{2} \left[1 + (1 - y)^{2}\right] e_{q}^{2} \delta(x - \xi)^{2} - \xi$$

$$\frac{d^{2}\sigma d^{2}\sigma}{dx dQ^{2}} \int_{0}^{1} \frac{d\xi}{\xi} \sum_{i} f_{i}(\xi) \frac{d^{2}\sigma}{d\hat{x} dQ^{2}} (\xi, Q^{2})^{2} With \frac{d}{d\zeta} \frac{d^{2}\hat{\sigma}}{dQ^{2}dx} = \frac{4\pi\alpha^{2}}{Q^{2}\hat{\sigma}} \frac{1}{2} \left[1 + (1 - y)^{2}\right] e_{q}^{2} \delta(x - \xi)^{2} - \xi$$

We can express the structure functions as:

$$F_{2}(\mathbf{p}_{2})(\mathbf{p}_{2})(\mathbf{p}_{2})(\mathbf{p}_{3}) = 2xF_{1} = \sum_{i=q,\bar{q}} \int_{0}^{1} d\xi f_{i}(\xi) x e_{q}^{2} \delta(x - \xi) = \sum_{i=q,\bar{q}} e_{q}^{2} x f_{i}(x)$$

DIS cross-section
$$\frac{d^2\hat{\sigma}}{dxdQ^2} = \int_0^\infty \frac{d\xi}{\xi} \sum_i f_i(\xi) \frac{d\hat{\sigma}}{d\hat{x}dQ^2} (\frac{x}{\xi}, Q^2)$$
 $\frac{d^2\hat{\sigma}}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4} \frac{1}{2} \left[1 + (1-y)^2 \right] e_q^2 \delta(x-\xi)$

We can express the structure functions as:

$$F_2(x) = 2xF_1 = \sum_{i=q,\bar{q}} \int_0^1 d\xi f_i(\xi) x e_q^2 \delta(x-\xi) = \sum_{i=q,\bar{q}} e_q^2 x f_i(x)$$

Quarks and anti-quarks enter together.

How can we separate them?

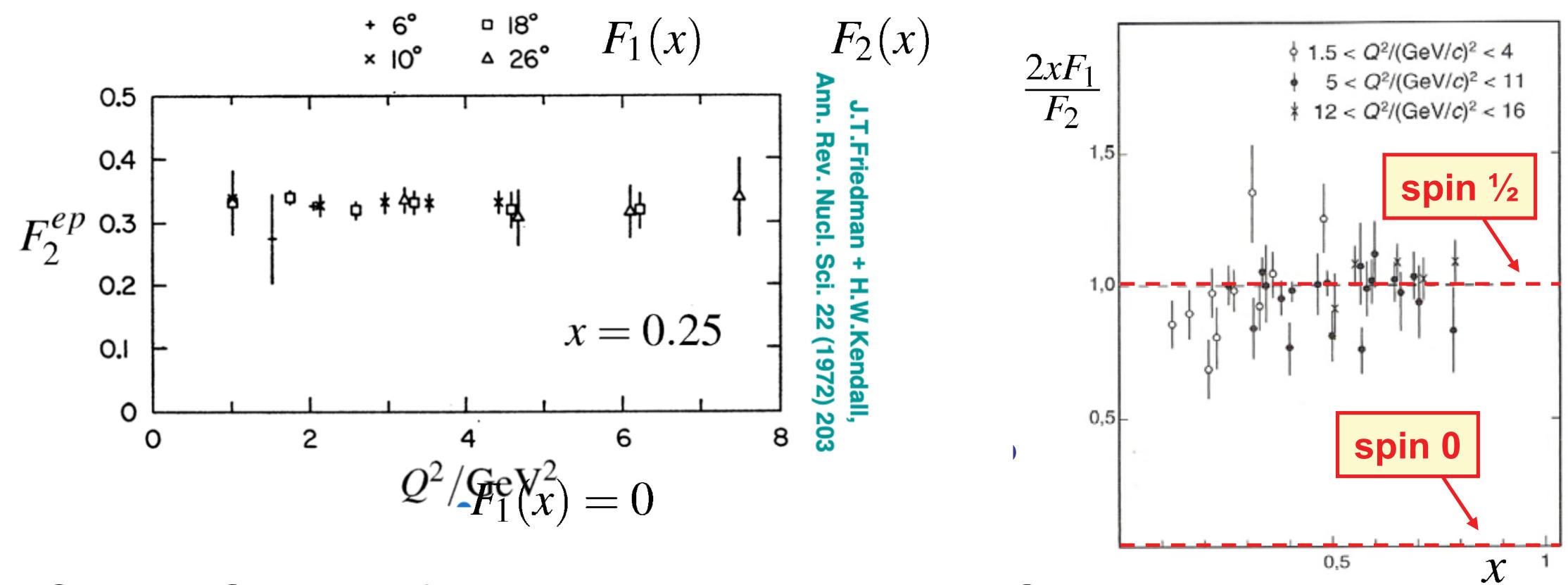
No dependence on Q: Scaling

Fabio Maltoni

 $f_i(x)$ are the parton distribution functions which describe the probabilities of finding specific partons in the proton carrying momentum fraction x

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Scaling and Callan-Gross relation $F_1(x,Q^2) \rightarrow F_1(x)$



Scaling: Structure function does not depend on Q

Quarks are spin-1/2 particles!

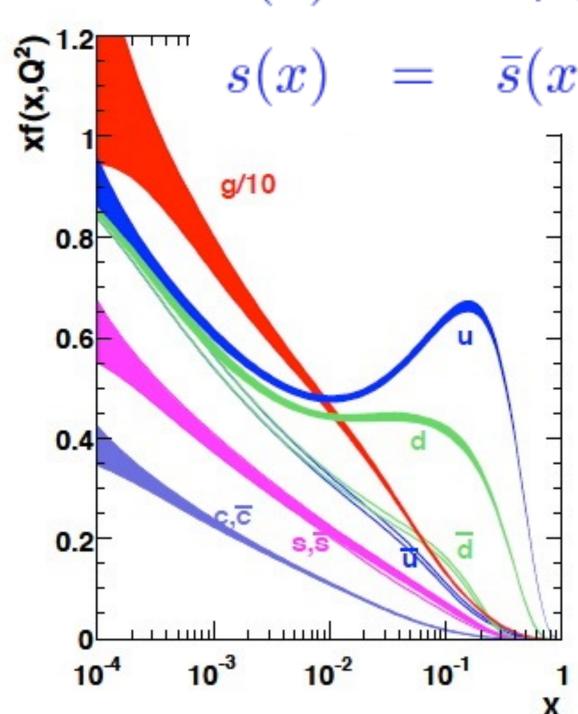
Callan-Gross relation

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Probed at scale Q, sea contains all quarks flavours with mq less than Q. For Q ~1 we expect

Parton distribution functions

$$u(x)=u_V(x)$$
 -Probed at scale Q, sea contains all quarks flavours with images than Q. For Q ~1 we expect $\int_0^x dx \ u_V(x)=2$, $\int_0^x dx \ d_V(x)=1$.



$$s(x)=ar{s}(x)$$
 $u(x)=u_V(x)+ar{u}(x)$ $\int_0^1 dx\ u_V(x)=2\ , \ \int_0^1 dx\ d_V(x)=1$ $\frac{s(x)=ar{s}(x)}{\text{The sea is NOT SU(3) flavor symmetric.}}$

The and
$$\int_q^1 dx \; x[$$
 $\int_0^1 dx \; x[$ $\int_0^1 dx \; x[q(x) + \bar{q}(x)] \simeq 0.5$. Note that there are uncertainty by

The proton of th Although not directly measured in DIS, gluons participate in other hard scattering Processes such as large-pt and prompt photon production. EVIGENCE TOT QIUONS!

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Parton model summary

DIS experiments show that virtual photon scatters off massless, free, point like, spin-1/2 quarks

One can **factorise** the short- and long-distance physics entering this process. Long-distance physics absorbed in PDFs. Short distance physics described by the hard scattering of the parton with the virtual photon. $\sum \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$

Eleni Vryonidou

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Parton model summary

DIS experiments show that virtual photon scatters off massless, free, point like, spin-1/2 quarks

One can **factorise** the short- and long-distance physics entering this process. Long-distance physics absorbed in PDFs. Short distance physics described by the hard scattering of the parton with the virtual photon.

photon. $\sum \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(\hat{s})$

Phase-space integral

Parton density functions

Parton-level cross section

End of Lecture 1

Collider Phenomenology (2)

Eleni Vryonidou









STFC school, Oxford 9-16/9/22

Plan for the lectures

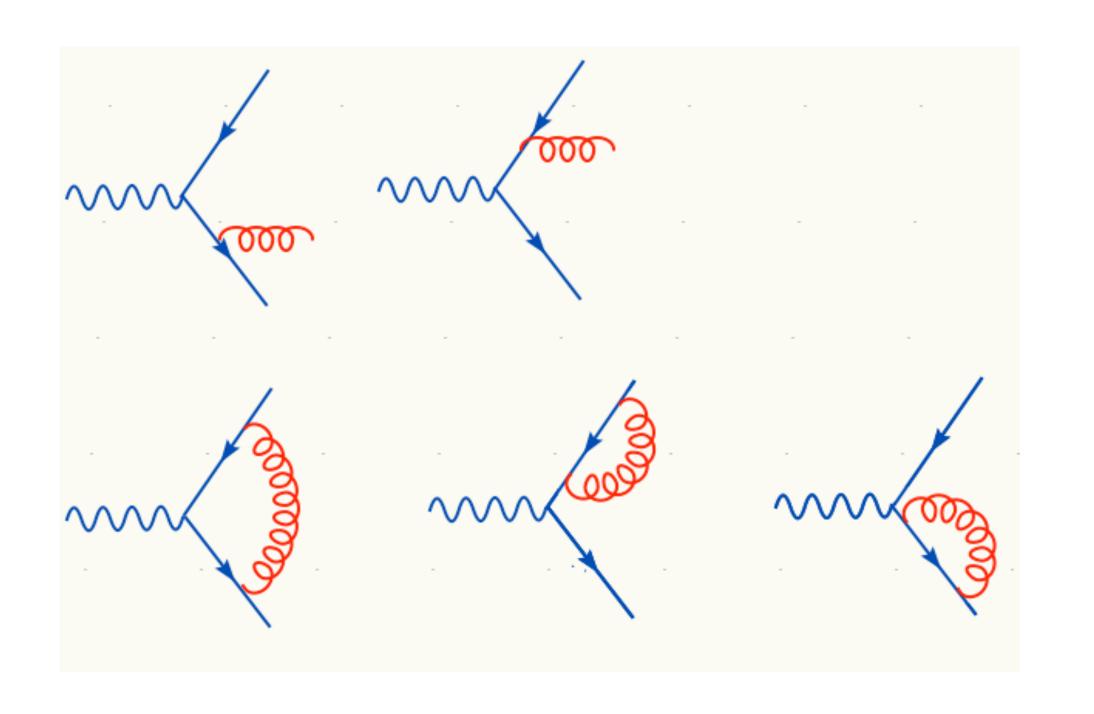
- Basics of collider physics
- Basics of QCD
 - DIS and the Parton Model
 - Higher order corrections
 - Asymptotic freedom
 - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

Plan for the lectures

- Basics of collider physics
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R-ratio@NLO





Real

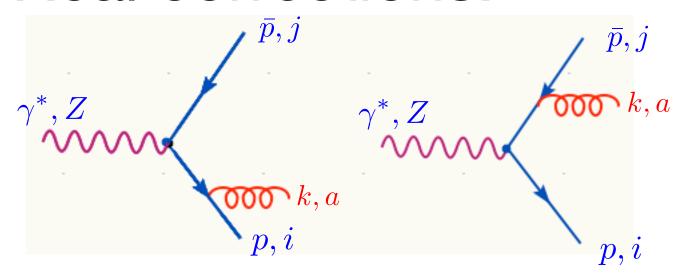
Virtua

$$\sigma_{NLO} = \sigma_{LO} + \int_{R} |M_{real}|^2 d\Phi_3 + \int_{V} 2 \text{Re}(M_0 M_{virt}^*) d\Phi_2$$

$$\sigma^{\text{NLO}} = \int_{R} |M_{real}|^2 d\Phi_3 + \int_{\text{STFC, HEP school 2022}} 2 Re \left(M_0 M_{virt}^*\right) d\Phi_2 = \text{finite!}$$

QCD in the fire k, a k, a k, a R-ratio@NLO

Real corrections:



Real corrections:
$$A = \bar{u}(p) \not\in (-ig_s) \frac{-i}{\not p' + \not k} \Gamma^{\mu} v(\bar{p}) t^a + \bar{u}(p) \Gamma^{\mu} \frac{i}{\not p' + \not k} (-ig_s) \not\in v(\bar{p}) t^a$$

$$= -g_s \left[\frac{\bar{u}(p) \not\in (\not p' + \not k) \Gamma^{\mu} v(\bar{p})}{2p \cdot k} - \frac{\bar{u}(p) \Gamma^{\mu} (\not p' + \not k) \not\in v(\bar{p})}{2\bar{p} \cdot k} \right] t^a$$

$$=-g_s\left[\frac{\bar{u}(p)\cancel{\epsilon}\cancel{(p+\cancel{k})}\Gamma^{\mu}v(\bar{p})}{2p\cdot k}-\frac{\bar{u}(p)\Gamma^{\mu}(\vec{p}+\cancel{k})\cancel{\epsilon}\cancel{v}(\bar{p})}{2\bar{p}\cdot k}\right]t^a$$

 $\begin{array}{l} A = \bar{u}(p) \not\in (-ig_s) \frac{-i}{\sqrt{\mu}} \Gamma^\mu v(\bar{p}) t^a + \bar{u}(p) \Gamma^\mu \frac{i}{\sqrt{\mu}} (2pig_s) \not= v(\bar{p}) t^a - \cos\theta) \\ \text{What are those denominators} \\ = -g_s \left[\frac{\bar{u}(p) \not\in (p'+k') \Gamma^\mu v(\bar{p})}{2p \cdot k} - \frac{\bar{u}(p) \Gamma^\mu (\bar{p}'+k') \not\in v(\bar{p})}{2\bar{p} \cdot k} \right] t^a \\ \text{The phom Kators} 2p_0 k_0 \left(1 - \frac{1}{\cos\theta} \right) \text{COSP} \text{ additions for collinear (cos + \cos\theta) to } \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) A_{Born} \\ \text{emission. By neglecting k in the numerators and using the Dirac equation, the amplitude simplifies} \end{array}$ and factorizes over the Born amplitude:

$$f_{ ext{fies}}^{a} \left(rac{p \cdot \epsilon}{p \cdot k} - rac{ar{p} \cdot \epsilon}{ar{p} \cdot k}
ight) A_{Born}$$

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What happens ψ^a when the gluon is soft $(k_0^{p})^{p}$ or collinear $(\theta \to 0)$ to the quark?

Fabio Maltoni





$$= \bar{u}(p) \not\in (-ig_s) \underbrace{\frac{i}{\not p_j + \not k}} \Gamma^{\mu} v(\bar{p}) t^a + \bar{u}(p) \Gamma^{\mu} \frac{i}{\overline{p}' + \not k} (-ig_s) \not\in v(\bar{p}) t^a \\ = -g_s^* \underbrace{\left[\bar{u}(p) \not\in (\not p' + \not k') \cdot \Gamma_Z^{\mu} v(\bar{p}) \cos \frac{\bar{u}(p) \Gamma^{\mu} (\overline{p}' + \not k') \not\in v(\bar{p})}{2\bar{p} \cdot k} \right]}_{p,i} t^a$$

What happens when the gluon is soft $(k_0) \phi (p \phi)$ linear $(\theta \to 0)$ to the quark?

$$A_{soft} = -g_s t^a egin{pmatrix} p+q & p+q &$$

The denominators $2p \cdot \kappa = p_0 \kappa_{\theta(1 - \cos \theta)}$ give singularties for confine (cos θ - Flagtsorisation of long-wavelength emission. By neglecting k in the numerators and using the Dirac equation, the amplitude simplifies ion from the shortdistance (hard) scattering! $A_{Born} = \bar{u}(p)\Gamma^{\mu}v(p)$

$$A_{soft} = -g_s t^a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) A_{Born}$$

Soft emission factor is universal!

$$p, i$$
 p, i

$$ar{u}(p)$$
 CCD in i the final state i $p+k$ $(-ig_s)$ e $(-ig_s)$ e $(-ig_s)$

$$-g_{s}\begin{bmatrix} \overline{u(p)} \not\in (\cancel{p} + \cancel{k})\Gamma^{\mu}v(\overline{p}) & \overline{u(p)}\Gamma^{\mu}(\overline{p} + \cancel{k})\not\in v(\overline{p}) \\ 2p \cdot k & 2\overline{p} \cdot k \\ 2p \cdot k & \gamma^{*}, Z \\ 2p \cdot k & \gamma^{*}, Z$$

$$A_{soft} = -g_s t^a \underbrace{\left(rac{p \cdot \epsilon}{p \cdot k} - rac{ar{p} \cdot \epsilon}{ar{p} \cdot k}
ight)}_{= -g_s} A_{Born} \, \overline{\not{k}}^{(-ig_s)} A_{Born} \, \overline{\not{k}}^{(-ig_s)} A_{Born}^{(ar{p})} t^a = ar{u}(p) \Gamma^{\mu} v(ar{p})$$

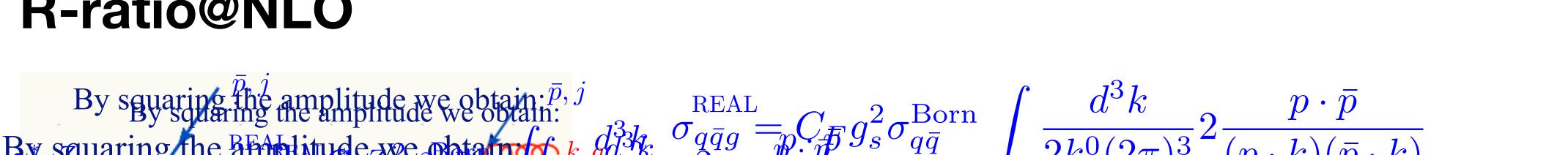
What does that $\overset{2p}{m} \dot{e} \overset{k}{a}$ n for the NL $\overset{2p}{O}$ cross-section?

The denominators
$$2p \cdot k = p_0 k_0 (1 - \cos \theta)$$
 give singularities for collinear $(\cos \theta \to 1)$ or soft $(k_0 \to 0)$ emission. By neglecting k in the and factorizes over the Born and $\sigma_{q\bar{q}g}^{\text{REAL}} = C_F g_s^2 \sigma_{q\bar{q}}^{\text{Born}} \int \frac{d^3k}{2k^0 (2\pi)^3} 2 \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$
$$A_{soft} = -$$

$$= C_F \frac{\alpha_S}{2\pi} \sigma_{q\bar{q}}^{\text{Born}} \int d\cos \theta \frac{dk^0}{k^0} \frac{4}{(1 - \cos \theta)(1 + \cos \theta)}$$

QCD in the final state

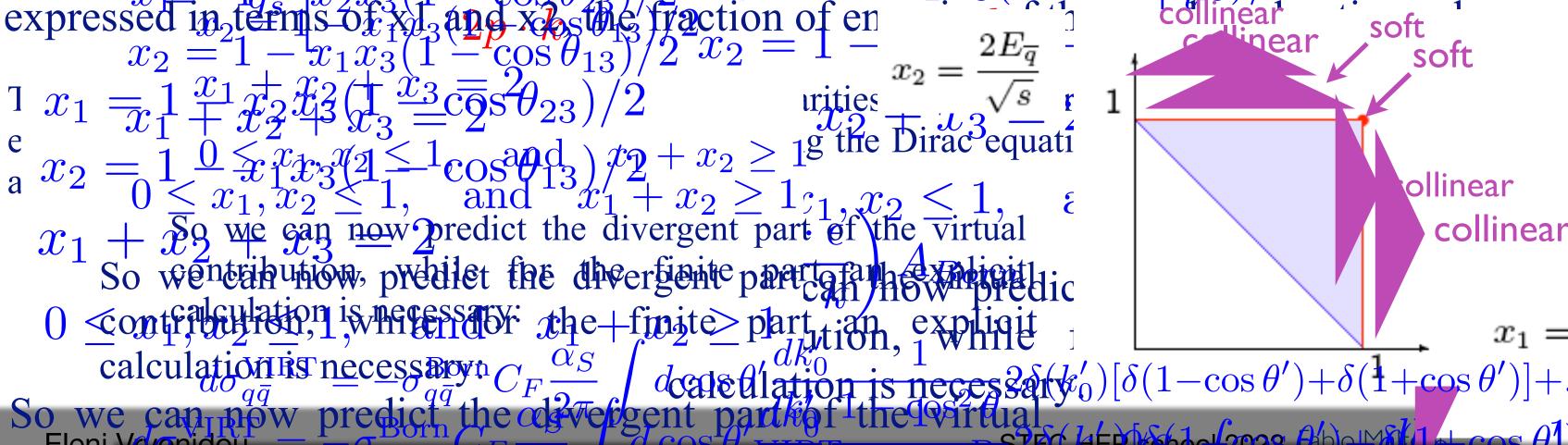
R-ratio@NLO

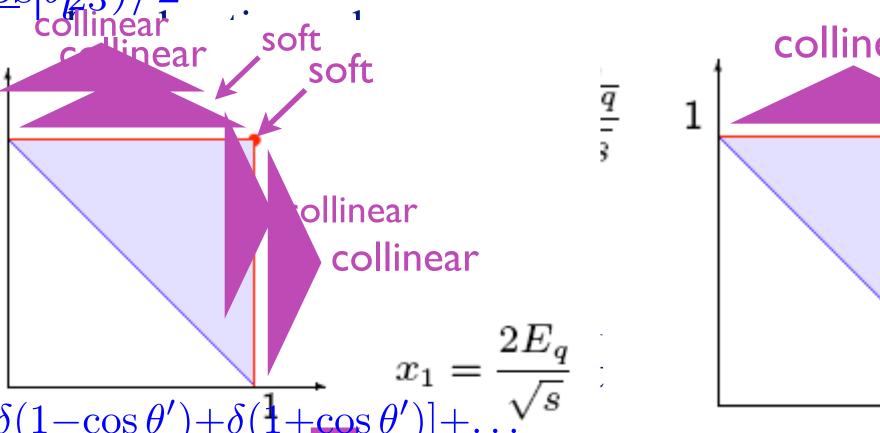


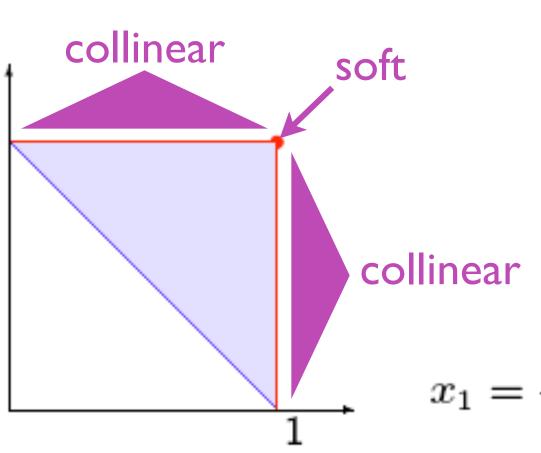
By squaring the amplitude we obtain

Two two this this divergences and a soft one of the work of the work of the first one of the first of the fir

Two collinear divergences and assort one. Very bfter you find the integration over phase space







 $d^{\text{RO}} (1 - \cos\theta) (1 + \cos\theta) \theta)$ redivergences and a soft one. Very often you find the integration of space space terms of x land x, the fraction of energies of the quark and anti-quark: Two collinear divergences and as soft one. Very often you find the integration over phase space expressed in terms of x_1 and x_2 the fraction of en $x_2 = 1 - x_1 x_3 (1 - \cos \theta_{13})/2$ $x_1 = 1 + x_2 x_3 (1 - \cos \theta_{13})/2$ $x_1 = 1 + x_2 x_3 (1 - \cos \theta_{13})/2$ soft Why is $x_1 = x_2 = 1$ the soft $x_2 = \underbrace{1}_{0} \underbrace{5}_{x_1, x_2} \underbrace{x_2, x_3}_{1} \underbrace{5}_{1} \underbrace{cos}_{13} \underbrace{hd}_{13} \underbrace{hd}_{13} \underbrace{hd}_{14} \underbrace{hd}_{14} \underbrace{hd}_{13} \underbrace{hd}_{14} \underbrace{hd}_{$ case? ollinear collinear O contribution in the finite part, an explicit calculation is necessary $C_F \frac{\alpha_S}{q\bar{q}} \int \frac{d\kappa_0}{d\cos\theta'} \frac{1}{k'_0} \frac{1}{1 + \cos\theta'} \frac{2\delta(k'_0)[\delta(1-\cos\theta')+\delta(1+\cos\theta')]+\dots}{2\delta(k'_0)[\delta(1-\cos\theta')+\delta(1+\cos\theta')]+\dots}$ So we can in predict the advergent part of the cost of the contribution, $4\pi^{\frac{1}{2}}$ for α finite part α an α calculation is necessar CF $\int dx_1 dx_2$ $d\sigma_{q\bar{q}}^{\text{VIRT}} = -3s_{q\bar{q}}^{\text{Born}} C_F \frac{2\pi}{2\pi} \int d\cos\theta' \frac{dk(1-k_1)(1-k_1)(1-k_2)}{k'_0} \frac{1-k_1(1-k_2)(1-k_2)}{1-k_2(1-k_2)} [\delta(1-\cos\theta') + \delta(1+\cos\theta')] + \dots$ Integral diverges if $x_1 \to 1$ or $x_2 \to 1$ or $x_1, x_2 \to 1!$ Fabio Maltoni

What happens now?

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IR singularities

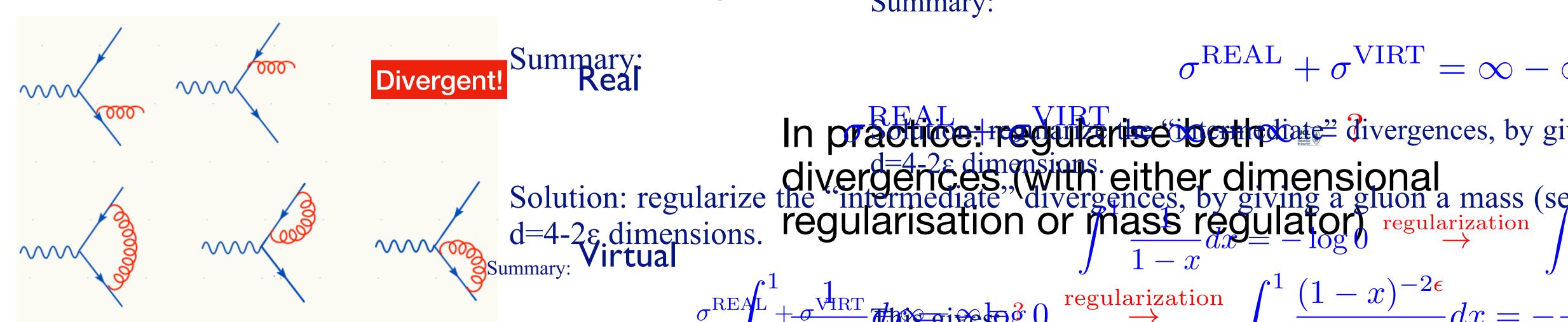
IR singularities arise when a parton is too soft or if two partons are collinear

- Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.
- When distances become comparable to the hadron size of ~1 Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

How do we proceed with our calculation?

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Cancellation of divergences



 $\sigma^{\mathrm{REAL}} + \sigma^{\mathrm{VIRT}} = \infty$ -

In practice: regularise inchiate" divergences, by gi

Solution: regularize the "intermediate" divergences, by giving a gluon a mass (see later) or soing to
$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \frac{3}{\epsilon^2} \right)$$
PS integration

This gives:

$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right)$$

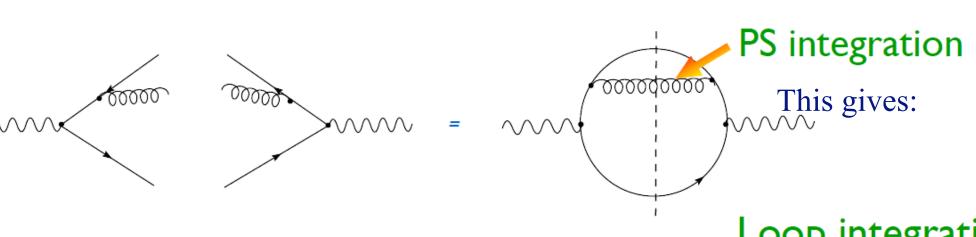
$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \frac{3}{\epsilon^2} \right)$$

$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right)$$

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$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right)$$

$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right)$$



$$x = -\log 0$$
 \rightarrow $\int \frac{1}{1} \frac{dx}{2} \frac{1}{2} \frac{dx}{2} \frac{dx}{2} \frac{1}{2} \frac{dx}{2} \frac{dx}$

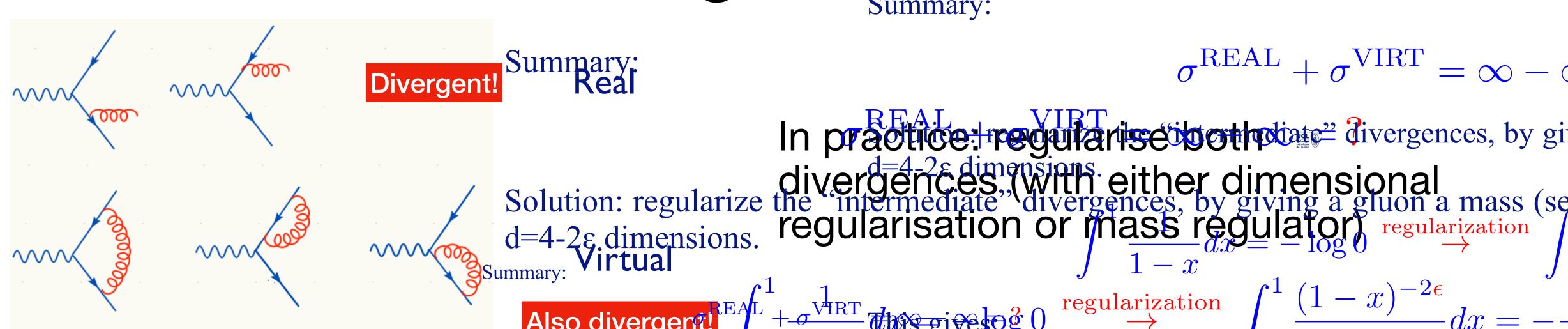
$$\frac{1}{2} \left(\frac{1}{2} + \frac{$$

$$\int_{\text{Coop integration}}^{\text{REAL}} \int_{\sigma^{\text{Born}} C_F}^{\text{Born}} \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \frac{2\pi}{4\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \frac{2\pi}{4\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \frac{2\pi}{4\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \frac{2\pi}{4\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \frac{2\pi}{4\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \frac{2\pi}{4\pi} \left(\frac{2\pi}{4\pi} + \frac{3\pi}{4\pi} + \frac{19\pi}{4\pi} + \frac{19\pi}{4\pi}$$

$$\lim_{\epsilon \to 0} (\sigma^{\text{REAL}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4} \frac{\alpha_S}{\pi} \sigma^{\text{Born}} \quad \exists \quad R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi} \right) 1$$

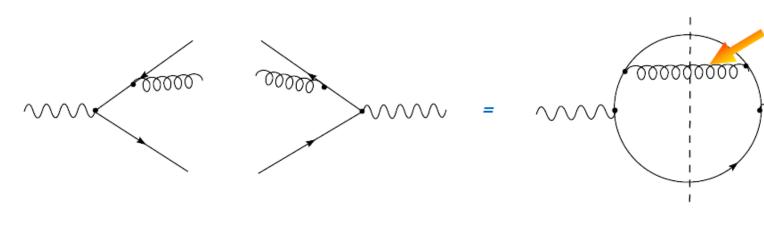
$$R_1 = R_0 \left(1 + \frac{\epsilon}{m}\right)$$
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Cancellation of divergences



$$\sigma^{\text{REAL}} + \sigma^{\text{VIRT}} = \infty -$$

In practice: regularise bottom divergences, by gi



Eleni Vryonidou

$$x = -\log 0$$
 \rightarrow $\int \frac{1}{1} C_{S}$ $\sigma^{\mathrm{REAL}} = \sigma^{\mathrm{Born}} C_{F} \frac{1}{2\pi}$

$$\sigma^{ ext{REAL}} = \sigma^{ ext{Born}} C_F^1 \frac{\partial u_S}{2\pi} \left(\frac{2}{3} + \frac{3}{3} + \frac{19}{3} - \pi^2 \right)$$

$$C_{F_{2}}^{\pi^{2}} \frac{\alpha_{S}}{2\pi} \left(\frac{1}{2} \right)$$

$$\frac{3}{2} = 2\sqrt{12}$$

This gives:
$$2\pi \left(\epsilon^2 + \epsilon \right) = 0$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \alpha_S$$

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$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{19}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{19}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{19}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{19}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{3}{\epsilon} + \frac{3}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

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$$C_F \frac{\alpha_S}{2\pi} \left(\frac{3}{\epsilon} + \frac{3}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

$$C_F \frac{\alpha_S}{2\pi} \left(\frac{3}{\epsilon} - \frac{3}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) \alpha_S$$

$$\lim_{\epsilon \to 0} (\sigma^{\text{REAL}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4} \frac{\alpha_S}{\pi} \sigma^{\text{Born}} \quad \Re \quad R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi} \right) 1$$

$$R_1 = R_0 \left(1 + \frac{\epsilon}{1 + \epsilon} \right)$$
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$$R_0 \left(1 + \frac{1}{2}\right)$$

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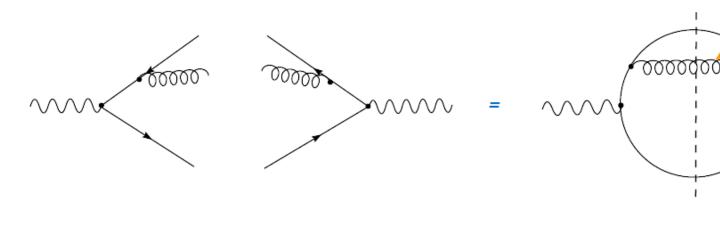
Cancellation of divergences

Divergent! Summary: Real

$$\sigma^{\mathrm{REAL}} + \sigma^{\mathrm{VIRT}} = \infty -$$

In practice: regularise bottom divergences, by gi

Solution: regularize the intermediate divergences, by giving a gluon a mass (see the description of mass regulator) regularization or mass regulator regularization $\frac{d}{1-x}$



$$\sigma^{ ext{REAL}} = \sigma^{ ext{Born}} C_F^{ ext{1}} \frac{\alpha_S}{2\pi}$$

$$\begin{array}{c}
\operatorname{Born} C_{F} \frac{\alpha_{S}}{2} \left(\frac{2}{\epsilon^{2}} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^{2} \right) \alpha_{S} \\
\sigma \operatorname{Born} C_{F} \frac{\alpha_{S}}{2} \left(\frac{2}{\epsilon^{2}} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^{2} \right) \alpha_{S} \\
-\frac{2}{\epsilon} \sigma \operatorname{Born} C_{F} \frac{\alpha_{S}}{2} \left(\frac{2}{\epsilon^{2}} - \frac{3}{\epsilon} - 8 + \pi^{2} \right) 2\pi
\end{array}$$

This gives:
$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) \alpha_S$$
Loop integration
$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) 2\pi \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right) (\sigma^{\text{REAB}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4} + \frac{3}{4} + \frac{19}{2} + \frac{3}{2} + \frac{19}{2} - \frac{3}{\epsilon} - 8 + \pi^2 \right) (\sigma^{\text{REAB}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4} + \frac{3}{4} + \frac{3}{2} + \frac{3}{2} + \frac{19}{2} + \frac{3}{2} + \frac{3}{$$

Loop integration
$$\sigma^{
m V}$$

$$\lim_{\epsilon \to 0} (\sigma^{\text{REAL}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4} \frac{\alpha_S}{\pi} \sigma^{\text{Born}} \quad \exists \quad R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi} \right) \text{ a Finite!}$$

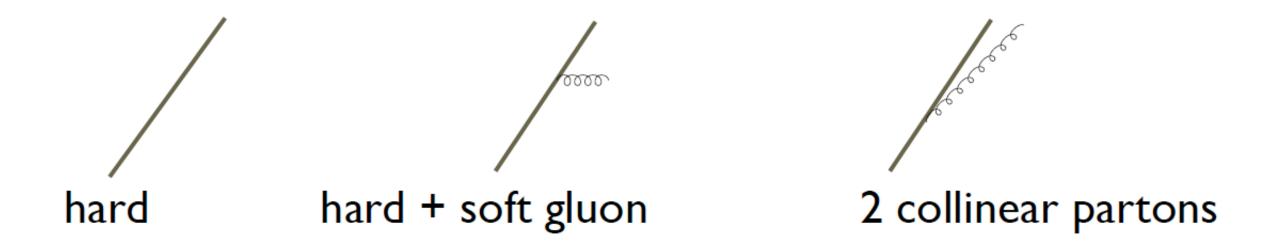
$$R$$
 $R_1=R_0\left(1+rac{lpha_S}{\pi}
ight)$ a**Fir**

$$R_1 = R_0 \left(1 + \frac{\epsilon}{1 + \epsilon}\right)$$
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KLN Theorem

Why does this work?

Kinoshita-Lee-Nauenberg theorem: Infrared singularities in a massless theory cancel out after summing over degenerate (initial and final) states



Physically a hard parton can not be distinguished from a hard parton plus a soft gluon or from two collinear partons with the same energy. They are degenerate states. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual)

Hence, one needs to add all degenerate states to get a physically sound observable

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Infrared safety

How can we make sure IR divergences cancel?

We need to pick observables which are insensitive to soft and collinear radiation. These observables are determined by hard, short-distance physics, with long distance effects suppressed by an inverse power of a large momentum scale.

Schematically for an PR safe observable:

$$\mathcal{O}_{n+1}(k_1, k_2, \dots, k_i, k_j, \dots, k_n) \to \mathcal{O}_n(k_1, k_2, \dots, k_i + k_j, \dots, k_n)$$

whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

Which observables are infrared safe?

energy of the hardest particle in the event

multiplicity of gluons

momentum flow into a cone in rapidity and angle YES

 \blacktriangleright cross-section for producing one gluon with E > E_{min} and θ > θ_{min} NO

▶ jet cross-sections DEPENDS

See exercises!

NO

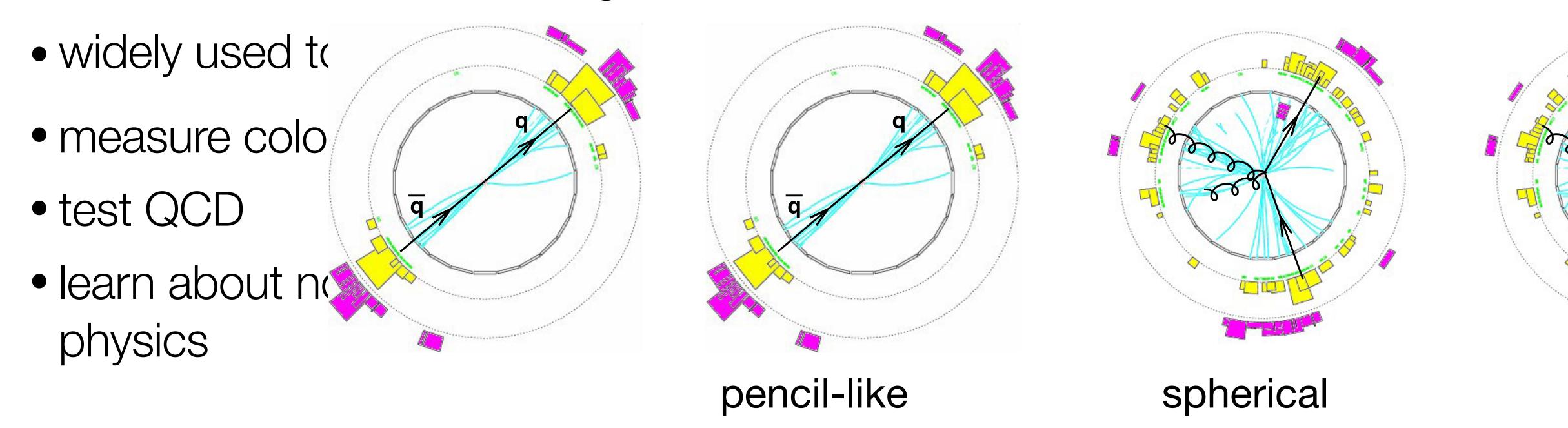
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Event shapes

Event shapes: describe the shape of the event, but are largely insensitive to soft and collinear branching



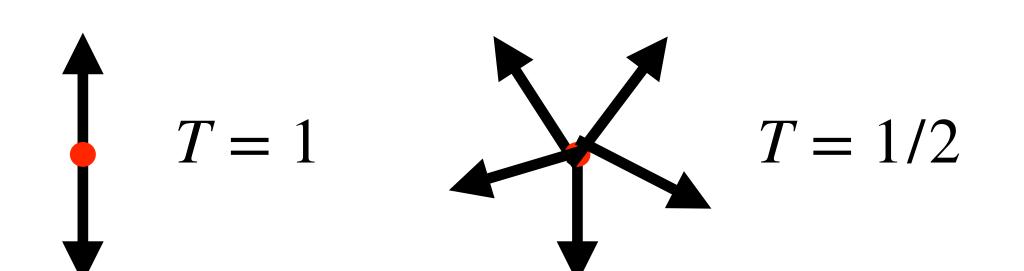
Thrust

Event-shape example

$$T = \max_{\hat{n}} \frac{\sum_{i} |\vec{p_i} \cdot \hat{n}|}{\sum_{i} |\vec{p_i}|}$$

Sum over all final state particles

Find axis *n* which maximises this projection



Noteby: if one of the partons emits a soft or collinear gluon the value of thrust is not changing. IRC safe

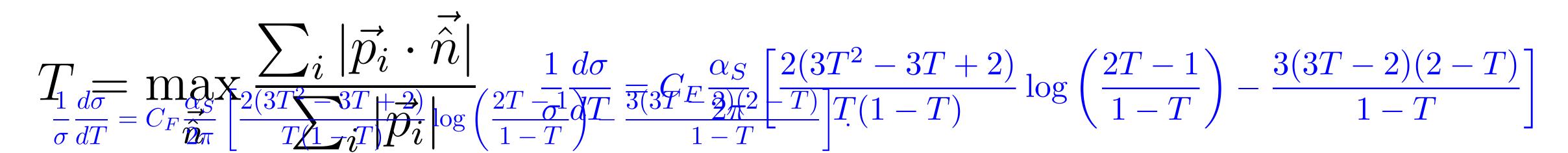
57

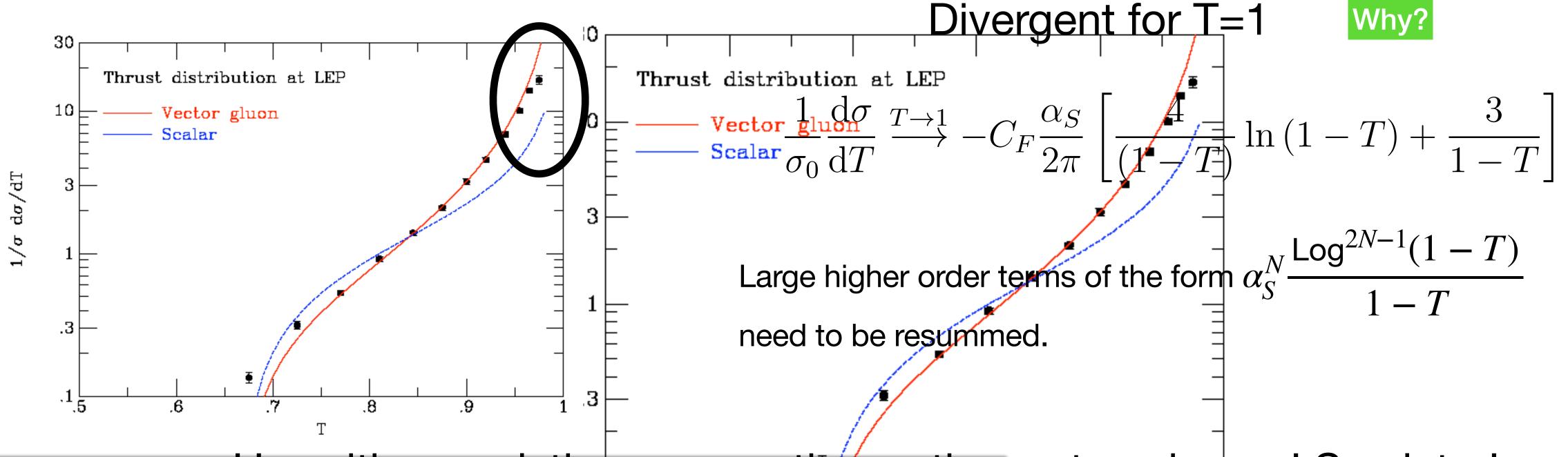
What happens in an $e^+e^- \rightarrow q\bar{q}g$ event?

Thrust

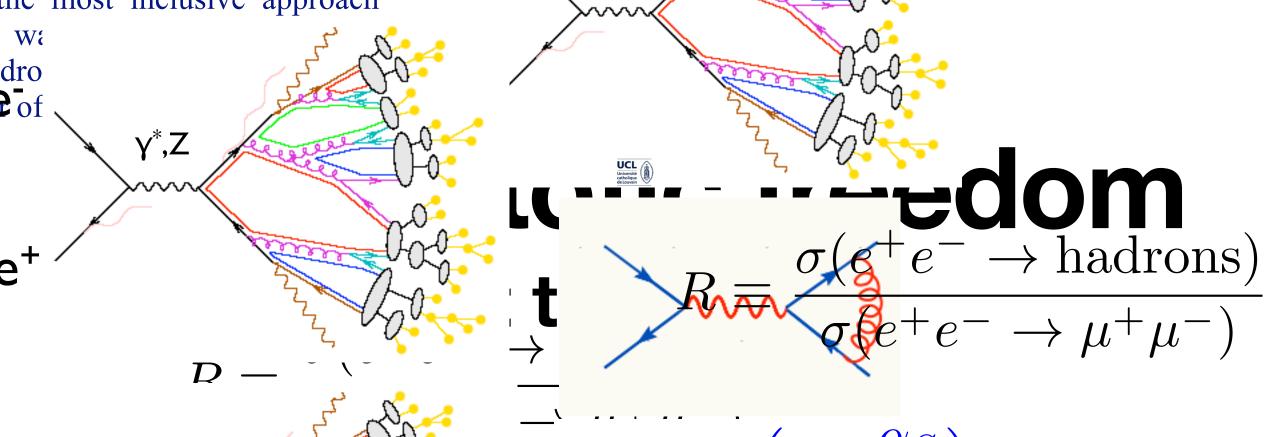
UCL Université catholique

What happens in an $e^+e^- \rightarrow q\bar{q}g$ event?





Use either analytic resummation on the parton shower! See later!



$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi}\right)$$
 No divergences!

What happens at higher orders?

$$R^{(2)} = R^{(0)} \left(1 + \frac{\alpha_S}{\pi} + \left(\frac{\alpha_S}{\pi} \right)^2 \left(c + \pi b_0 \log \left(\frac{M_{\text{UV}}^2}{Q^2} \right) \right) \right) \qquad b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

$$S + \left(\frac{\omega}{V} \right) \left(c + \pi b_0 \log \left(\frac{\omega_V}{Q^2} \right) \right) \right) \qquad b_0 = \frac{12N_c - 4n_f T_R}{12\pi}$$

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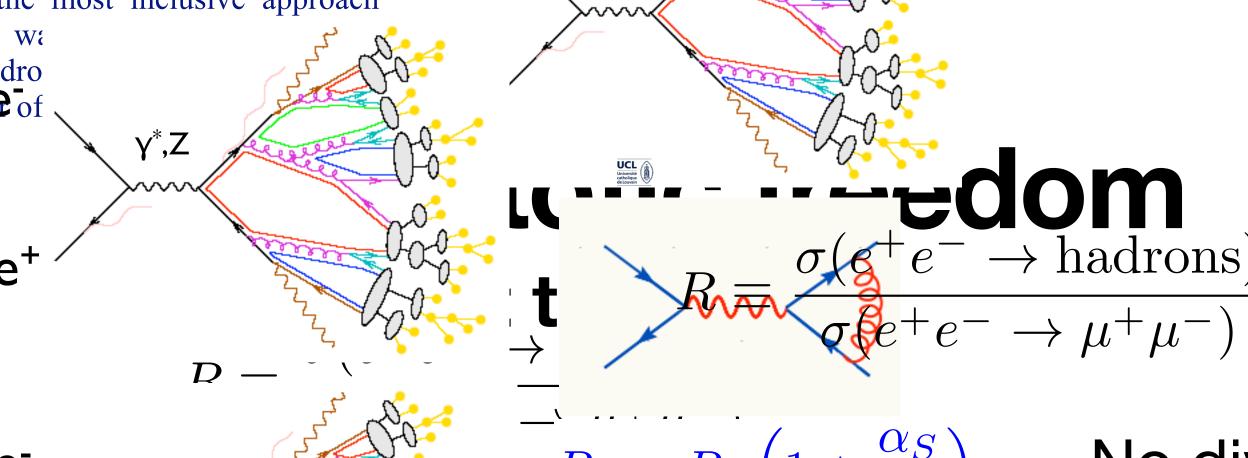
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$$\alpha_{S}(\mu) = \alpha_{S} + b_{0} \log \frac{M^{2}}{\mu^{2}} \alpha_{S}^{2} \qquad \text{Renormalising the} \\ \alpha_{S}(\mu) = \alpha_{S}^{\text{bare}} + b_{0} \log \left(\frac{M_{\text{UV}}^{2}}{\mu^{2}}\right) (\alpha_{S}^{\text{bare}})^{2} \qquad (\alpha_{S}^{\text{pare}})^{2} \qquad \mu^{2} \frac{1}{\mu^{2}} \alpha_{S}(\mu) = -b_{0} \alpha_{S}^{2}(\mu) + \dots \\ R_{2}^{\text{ren}}(\alpha_{S}(\mu), \frac{\mu^{2}}{Q^{2}}) = R_{0} \left(1 + \frac{\alpha_{S}(\mu)}{\pi} + \left[c + \pi b_{0} \log \frac{\mu^{2}}{Q^{2}}\right] \left(\frac{\alpha_{S}(\mu)}{\pi}\right)^{2}\right) \\ (2) \quad \alpha_{S}(\mu) = \alpha_{S} + b_{0} \log \frac{M^{2}}{\mu^{2}} \alpha_{S}^{2} \qquad b_{0} = \frac{11N_{c} - 2n_{f}}{12\pi}$$



$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi}\right)$$
 No divergences!

What happens at higher orders?

$$R^{(2)} = R^{(0)} \left(1 + \frac{\alpha_S}{\pi} + \left(\frac{\alpha_S}{\pi}\right)^2 \left(c + \pi b_0 \log\left(\frac{M_{\text{UV}}^2}{Q^2}\right)\right)\right) \qquad b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

$$S + \left(\frac{\omega}{V}\right) \left(c + \pi b_0 \log\left(\frac{\omega_V}{Q^2}\right)\right) \int_{0}^{\infty} b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

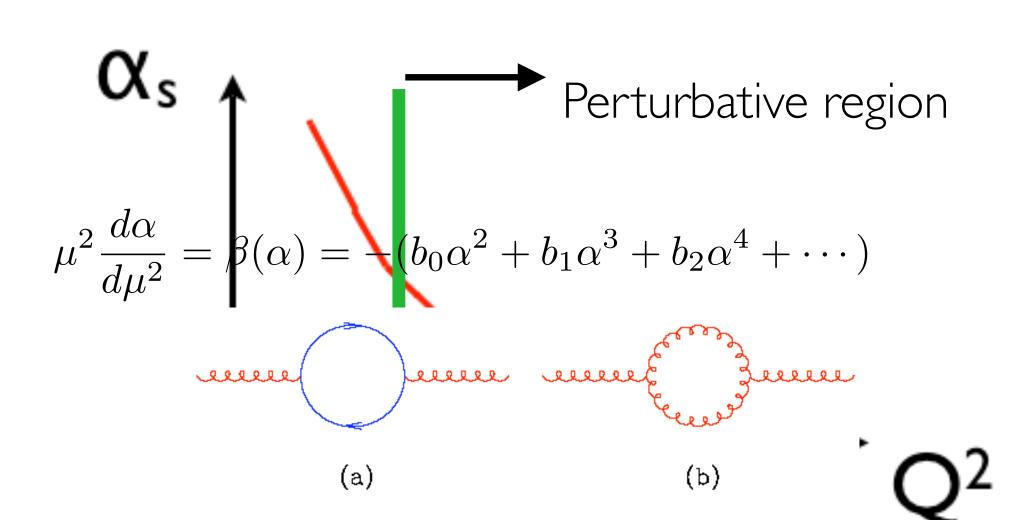
$$VV \text{ divergences don't cancel! We red renormalisation!}$$

$$\alpha_{S}(\mu) = \alpha_{S} + b_{0} \log \frac{M^{2}}{\mu^{2}} \alpha_{S}^{2} \qquad \text{Renormalising the} \\ \alpha_{S}(\mu) = \alpha_{S}^{\text{bare}} + b_{0} \log \left(\frac{M_{\text{UV}}^{2}}{\mu^{2}}\right) (\alpha_{S}^{\text{bare}})^{2} \qquad \mu^{2} \frac{1}{\mu^{2}} \frac{1}{\mu^{2}} = -b_{0} \alpha_{S}^{2}(\mu) + \dots \\ \alpha_{S}(\mu) = \alpha_{S}^{\text{bare}} + b_{0} \log \left(\frac{M_{\text{UV}}^{2}}{\mu^{2}}\right) (\alpha_{S}^{\text{bare}})^{2} \qquad R_{2}^{\text{ren}}(\alpha_{S}(\mu), \frac{\mu^{2}}{Q^{2}}) = R_{0} \left(1 + \frac{\alpha_{S}(\mu)}{\pi} + \left[c + \pi b_{0} \log \frac{\mu^{2}}{Q^{2}}\right] \left(\frac{\alpha_{S}(\mu)}{\pi}\right)^{2}\right) \\ (2) \quad \alpha_{S}(\mu) = \alpha_{S} + b_{0} \log \frac{M^{2}}{M^{2}} \text{ but scale 2 dependent!}$$

Asymptotic freedom

$$\alpha_S(\mu) = \alpha_S + b_0 \log \frac{M^2}{\mu^2} \alpha_S^2$$

$$b_0 = \frac{11N_c - 2n_f}{12\pi}$$



$$b_0 = \frac{11N_c - 2n_f}{12\pi} > 0 \qquad \Rightarrow \beta(\alpha_S) < 0 \qquad \text{in QCD}$$

$$b_0 = -\frac{n_f}{3\pi} > 0 \qquad \Rightarrow \beta(\alpha_{\rm EM}) > 0 \qquad \text{in QED}$$

$$\mu^{2} \frac{d\alpha}{d\mu_{S}^{2}} = \beta(\alpha) \frac{\partial \overline{\alpha}_{S}}{\partial \mu^{2}} = -b_{0} \alpha_{S}^{2} + b_{1} \alpha^{3} + b_{2} \alpha_{S}^{4} + \cdots = \frac{1}{b_{0} \log \frac{\mu}{\Lambda}}$$

$$(2) \quad \alpha_{S}(\mu) \overline{\beta}_{S} = a_{S} + b_{0} \log \frac{M^{2}}{\mu^{2}} \alpha_{S}^{2} \qquad b_{0} = \frac{11N_{c} - 2n_{f}}{4^{12}\pi} > 0$$

$$1 - \log \mu^{2} = -b_{0} \alpha_{S}^{2}(\mu) - b_{1} \alpha_{S}^{3}(\mu) - b_{2} \alpha_{S}^{4}(\mu) + \dots$$

$$a \log \mu^{2} = -b_{0} \alpha_{S}^{2}(\mu) - b_{1} \alpha_{S}^{3}(\mu) = \frac{1}{b_{0} \log \frac{\mu^{2}}{\Lambda^{2}}}$$

$$b_{0} = \frac{\beta(\alpha_{S})}{12\pi} = \mu^{2} \frac{\partial \alpha_{S}}{\partial \mu^{2}} = -b_{0} \alpha_{S}^{2} \qquad \Rightarrow \alpha_{S}(\mu) = \frac{1}{b_{0} \log \frac{\mu^{2}}{\Lambda^{2}}}$$

$$2 - \log \mu^{2} = \frac{12\pi}{\Lambda^{2}}$$

Perturbative region

 $\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \left[1 - \frac{b_1}{b_0^2} \frac{\log \log \mu^2 / \Lambda^2}{\log \mu^2 / \Lambda^2} \right]$

Perturbative region

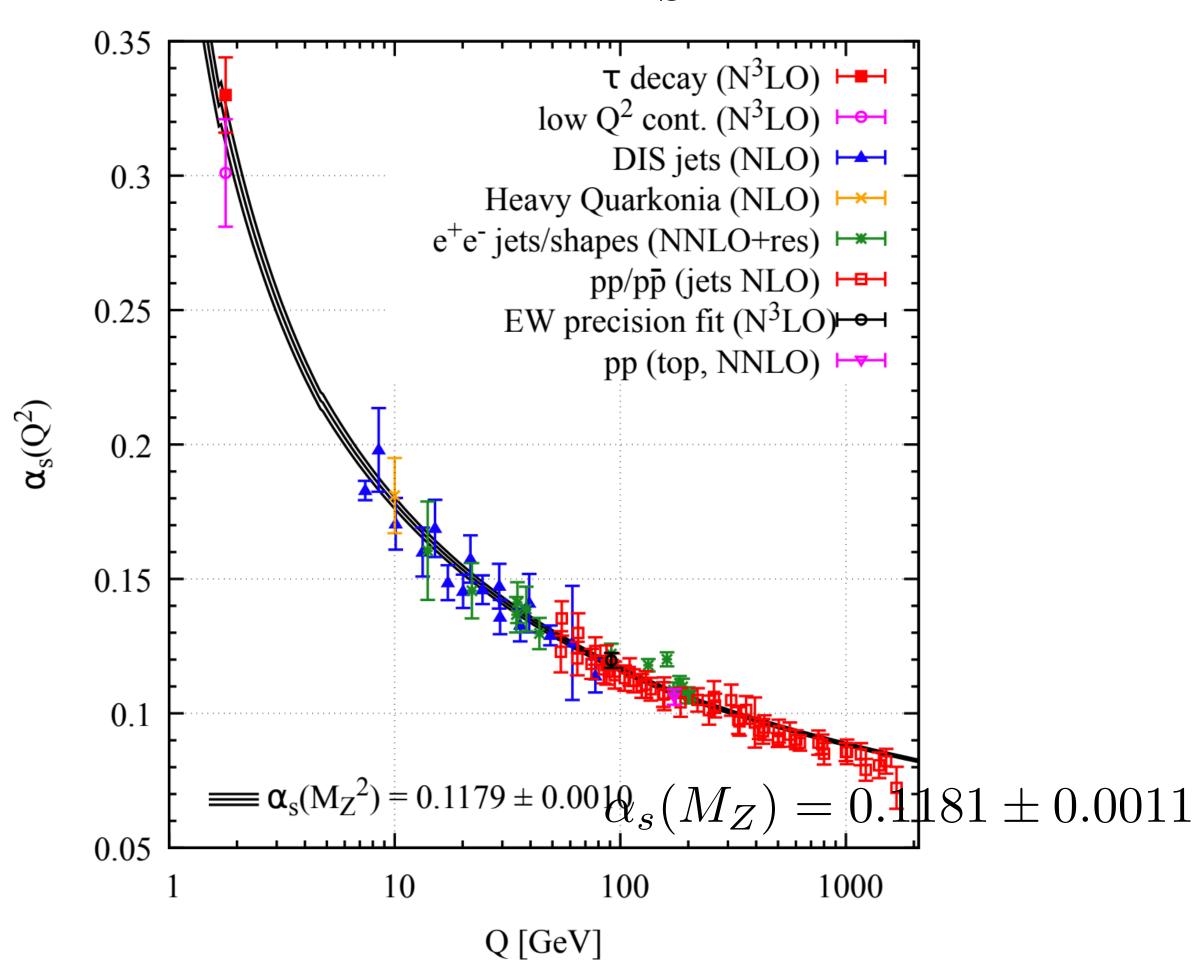
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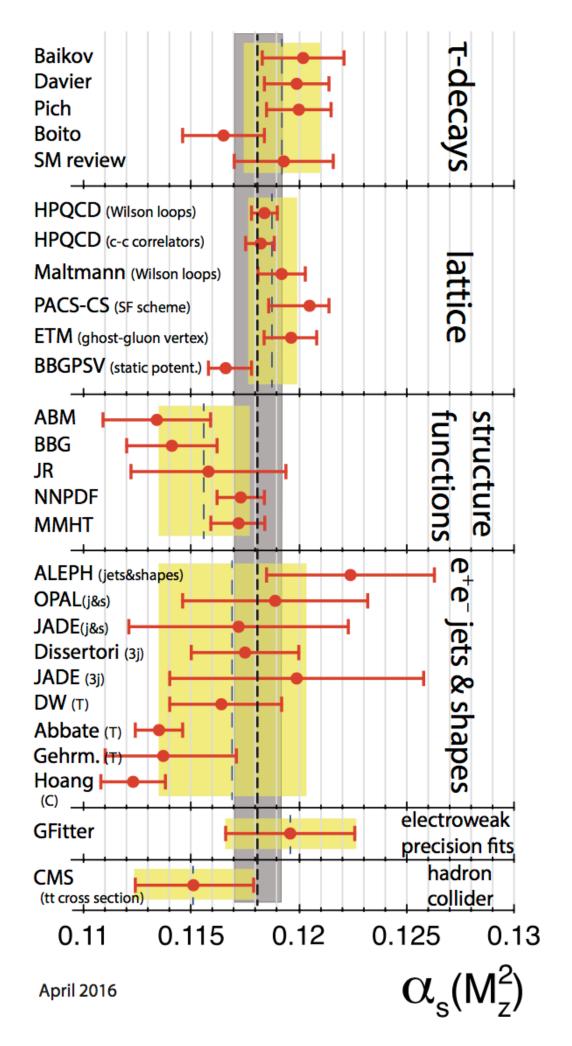
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Eleni Vryonidou

Running of α_s





Many measurements at different scales all leading to very consistent results once evolved to the same reference scale, M_z.

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\downarrow$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \,\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

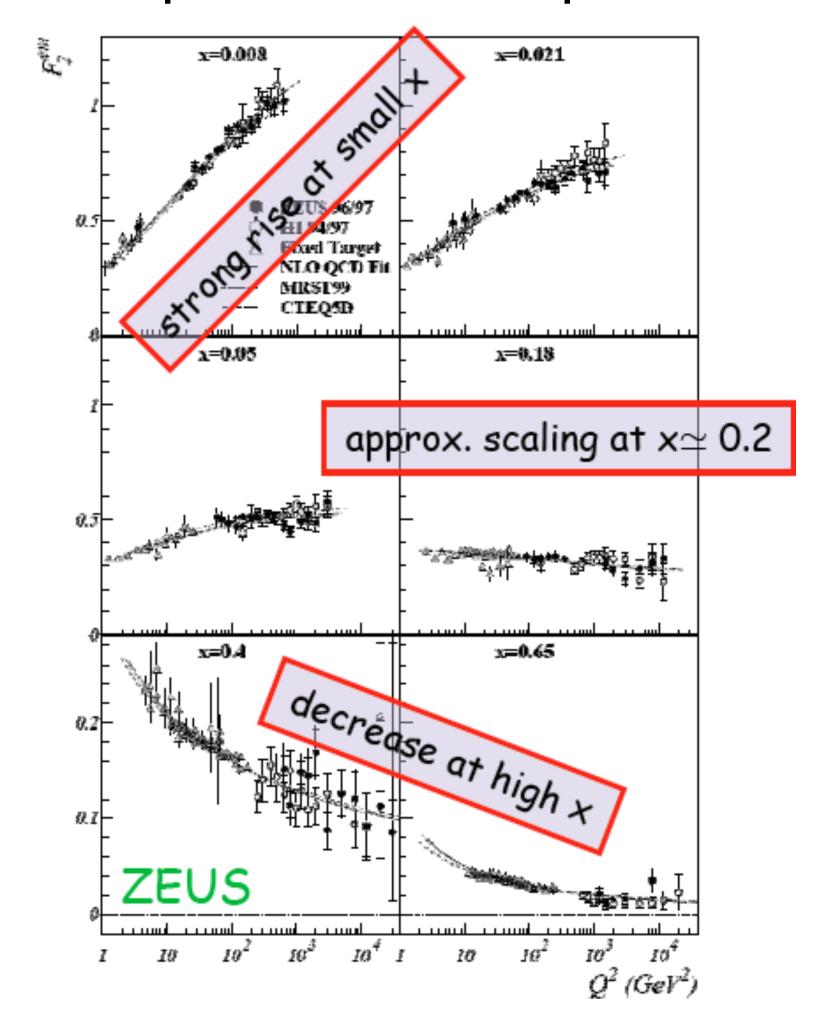
$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\downarrow^{???}$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \,\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

QCD improved parton model

The parton model predicts scaling. Experiment shows:

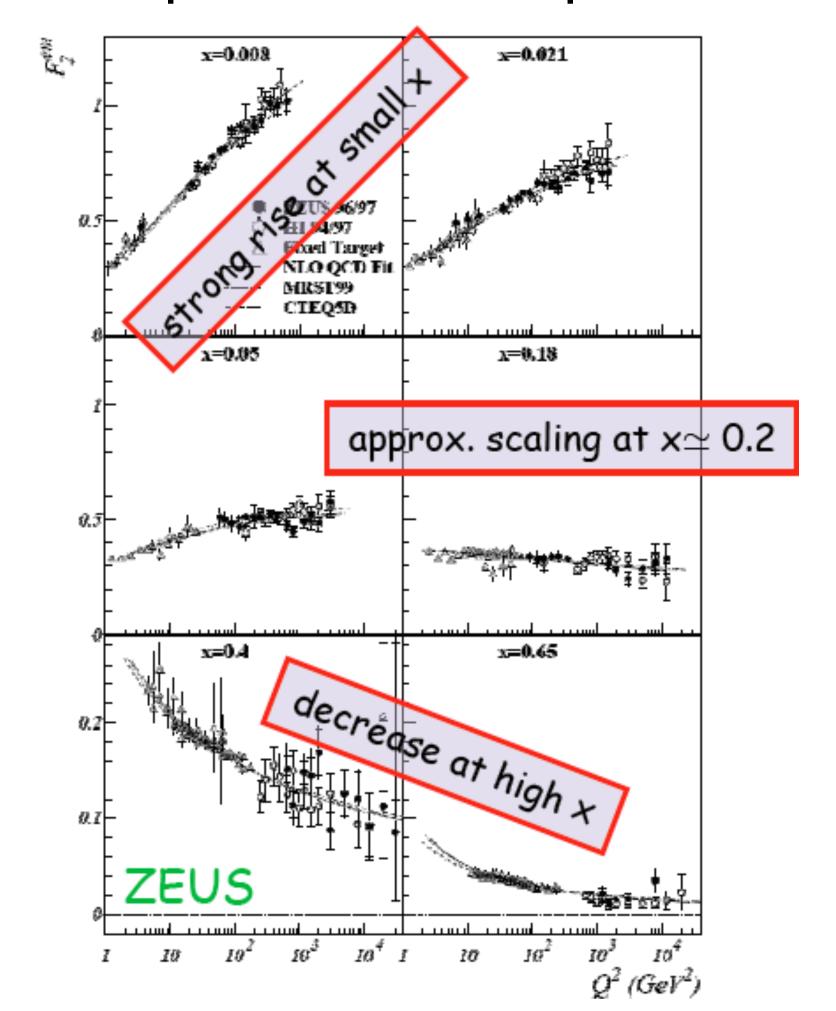


Scaling violation

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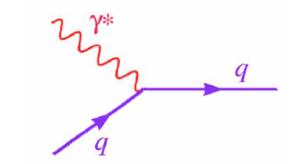
QCD improved parton model

The parton model predicts scaling. Experiment shows:

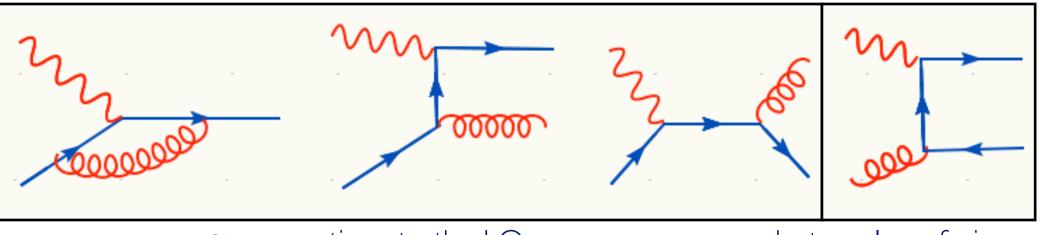


Scaling violation

What are we missing?



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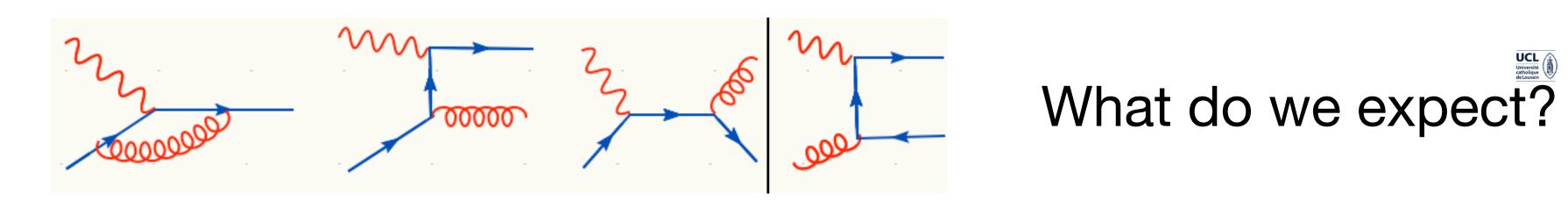


 α_{S} corrections to the LO process

photon-gluon fusion

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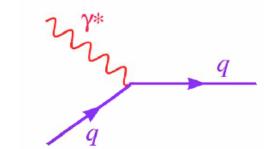
QCD improved p model

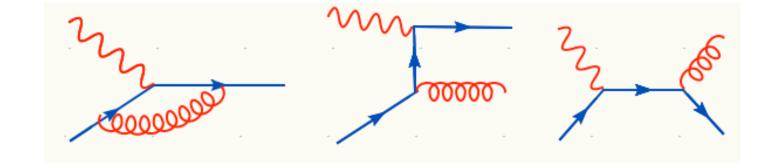


Given the computation of R at NLO, we expect IR divergences

We need to regulate these, and hope that they cancel!

Eleni Vryonidou



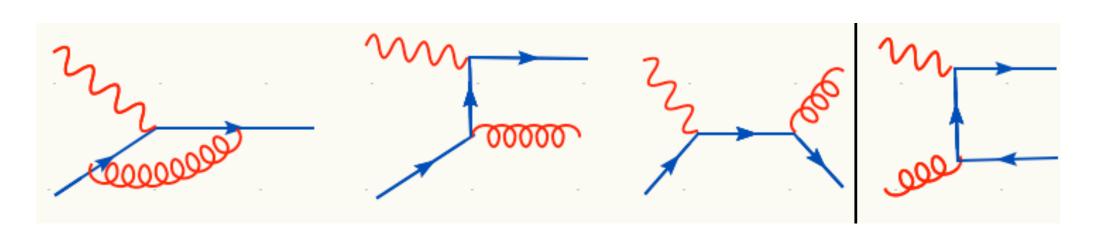


Soft and UV divergences cancel but a collinear divergence arises:

$$\begin{split} \hat{F}_{d\underline{x}}^{\underline{d^2\hat{\sigma}}} & \hat{F}_{\underline{q}}^{\underline{r}} \times \mathcal{E}_{\underline{q}}^{\underline{r}} \times \mathcal{E}_{\underline{q}}^{\underline{r}}$$

$$\hat{F}_{2}^{g} = e_{q}^{2} x [\text{Outoff} \frac{\alpha_{s}}{4\pi} P_{qg} \log \frac{Q^{2}}{m_{g}^{2}} + C_{2}^{g}(x)]$$

QCD improved p model



Soft and UV divergences cancel but a collinear divergence arises:

$$\hat{F}_{2}^{q} = e_{q}^{2}x[\delta(1-x) + \frac{\alpha_{s}}{4\pi}P_{qq}\log\frac{Q^{2}}{m_{g}^{2}} + C_{2}^{q}(x)] \qquad \hat{F}_{2}^{g} = e_{q}^{2}x[0 + \frac{\alpha_{s}}{4\pi}P_{qg}\log\frac{Q^{2}}{m_{g}^{2}} + C_{2}^{g}(x)]$$
Fabric Ut-off

What are functions P_{qq} and P_{qg} ?

Splitting functions $P_{ij}(x)$: they give the probability of parton j splitting into parton i which carries momentum fraction x of the original parton

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$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_{\rm S}}{2\pi} P_{a\to bc}(z)$ Altareli-Ratisi Splitting functions tually a

singular factor, so one will need to make sense precisely of this definition.

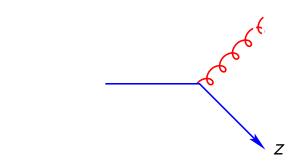
At the leading contribution to the (n+1)-body cross section the Altarelli-Parisi

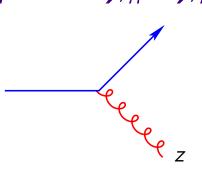
Branching hastagumiversal form given by the Altarelli-Parisi splitting

functions

$$P_{q \to qg}(z) = C_F \left[\frac{1+z^2}{1-z} \right],$$

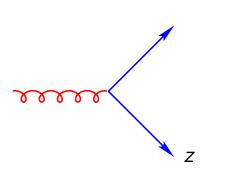
$$P_{q \to qg}(z) = C_F \left[\frac{1+z^2}{1-z} \right], \qquad P_{q \to gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right].$$





$$P_{g \to qq}(z) = T_R \left[z^2 + (1-z)^2 \right],$$

$$P_{g \to qq}(z) = T_R \left[z^2 + (1-z)^2 \right], \qquad P_{g \to gg}(z) = C_A \left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right]$$



$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_{\rm S}}{2\pi} P_{a\to bc}(z)$ Altarelli-Rakisi Splitting functions tually a

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$$P_{q \to qg}(z) = C_F \left[\frac{1+z^2}{1-z} \right], \qquad P_{q \to gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right].$$

$$P_{g \to qq}(z) = T_R \left[z^2 + (1-z)^2 \right], \qquad P_{g \to gg}(z) = C_A \left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right]$$

These functions are universal for each type of splitting



What does this collinear divergence mean?

Residual long-distance physics, not disappearing once real and virtual corrections are added. These appear along with the universal splitting functions.

Can a physical observable be divergent?

No, as the physical observable is the hadronic structure function:

$$F_{2}^{q}(x,Q^{2}) = x \sum_{i=q,\bar{q}} e_{q}^{2} \left[f_{i,0}(x) + \frac{\alpha_{S}}{2\pi} \int_{x}^{1} \frac{d\xi}{2\xi S} f_{i,0}(\xi) d\xi \int_{x}^{q} \frac{Q^{2}}{\xi} + C_{2}^{q}(\frac{x}{\xi}) d\xi \int_{x}^{q} \frac{Q^{2}}{\xi} \int_{x}^{q} \frac{Q^{2}}{$$

We can absorb the dependence on the IR cutoff into the PDF:

$$\begin{split} f_q(x,\mu_f) &\equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}(\frac{x}{\xi}) \log \frac{\mu_f^2}{m_g^2} + z_{qq} \\ f_q(x,\mu_f) &\equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi} \Pr_x \text{Benormalized PBFs!} \frac{x}{\xi} \log \frac{\mu_f^2}{m_g^2} + z_{qq} \end{split}$$
 Eleni Vryonidou

Factorisation



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Structure function is a measurable object and cannot depend on scale at all orders (renormalisation group invariance)

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi} \left[P_{qq}(\frac{x}{\xi}) \log \frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi}) \right] \right]$$

Long distance physics is universally factorised into the PDFs, which now depend on μ_f . PDFs are not calculable in perturbation theory. PDFs are universal, they don't depend on the process.

Factorisation scale μ_f acts as a cut-off, emissions below μ_f are included in the PDFs.



DGLAP

We can't compute PDFs in perturbation theory but we can predict their evolution in scale:

$$\mu^2 \frac{\partial f(\mathbf{x}, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

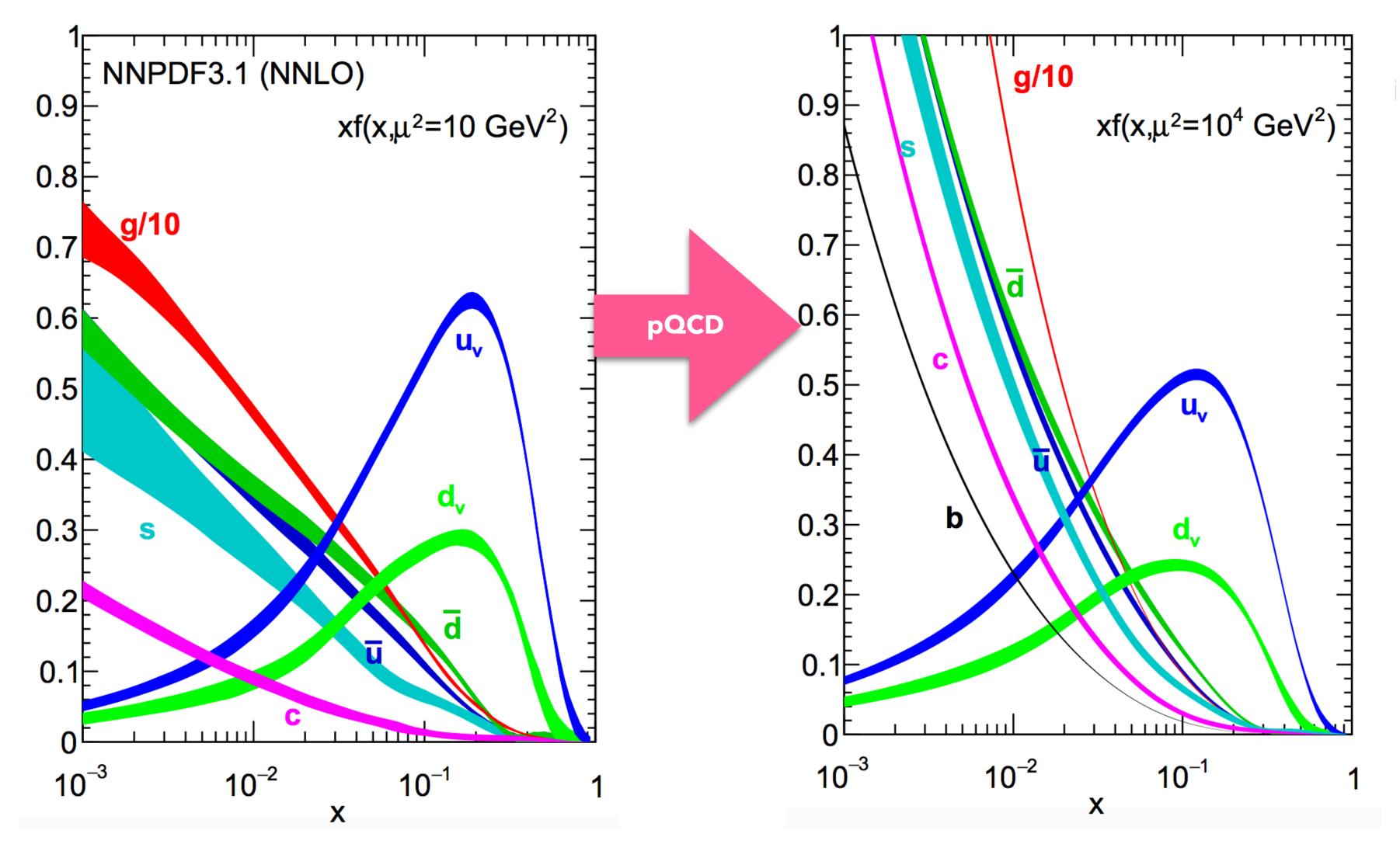
$$P_{ab}(\alpha_S,z) = \frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z) +$$
 Splitting functions improved in perturbation theory!
$$1 \qquad \qquad 1 \qquad \qquad 1$$

NLO Floratos, Ross, Sachrajda; Floratos, Lacaze, Kounnas Gonzalez-Arroyo, Lopez, Yndurain; Curci, Furmanski Petronzio, (1981)

NNLO - Moch, Vermaseren, Vogt, 2004

$$Ji(U,\mu)$$

PDF evolution



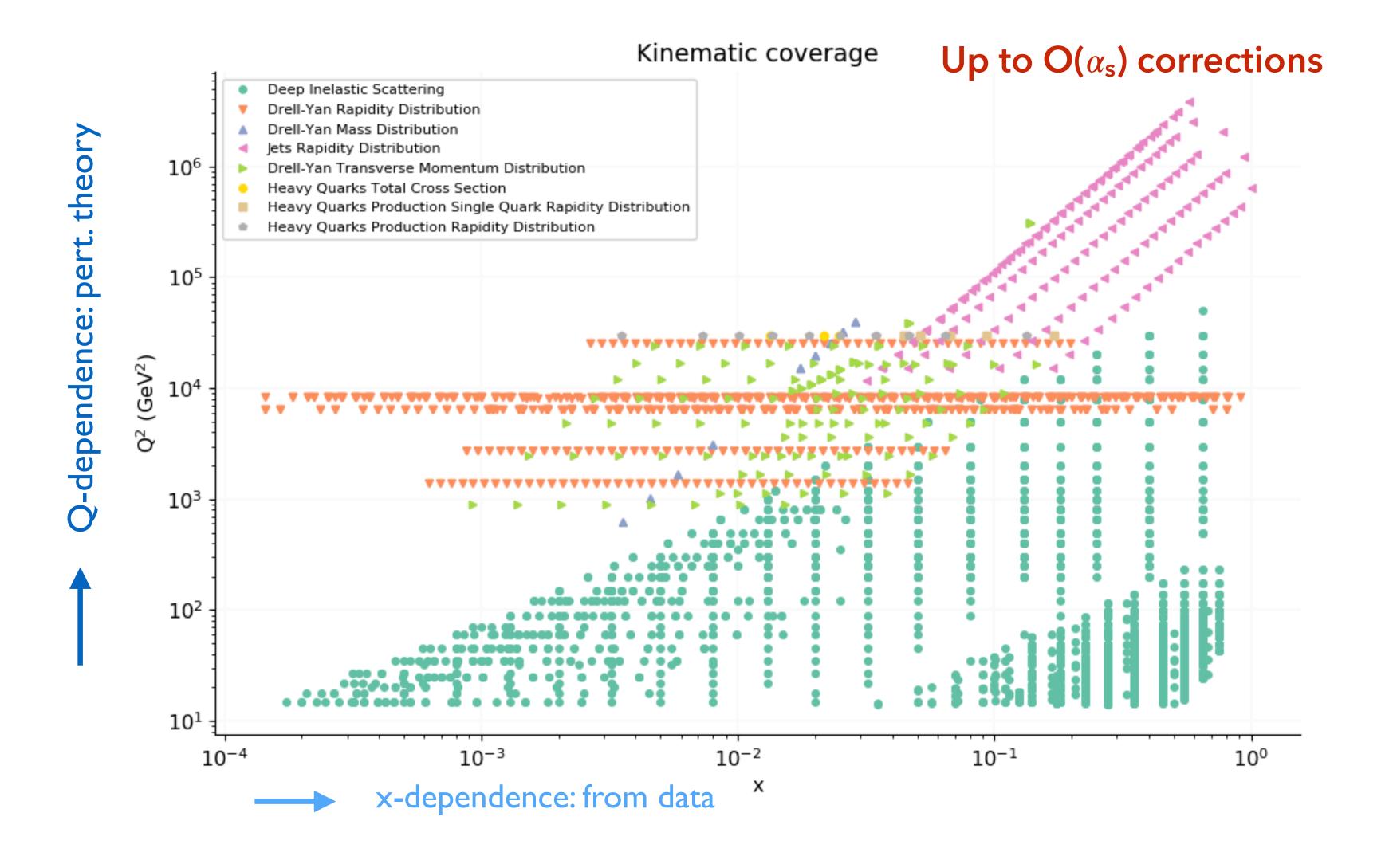
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PDF extraction

We can't compute PDFs in perturbation theory but we can extract them from data, and use DGLAP equations to evolve them to different scales.

- Choose **experimental data** to fit and include all info on correlations **Theory settings**: perturbative order, EW corrections, intrinsic heavy quarks, α_s , quark masses value and scheme
- Choose a starting scale Q₀ where pQCD applies
- Parametrise independent quarks and gluon distributions at the starting scale
- Solve DGLAP equations from initial scale to scales of experimental data and build up observables
- Fit PDFs to data
- Provide PDF error sets to compute PDF uncertainties

Data for PDF determination



LHC kinemat

How can we tell wh

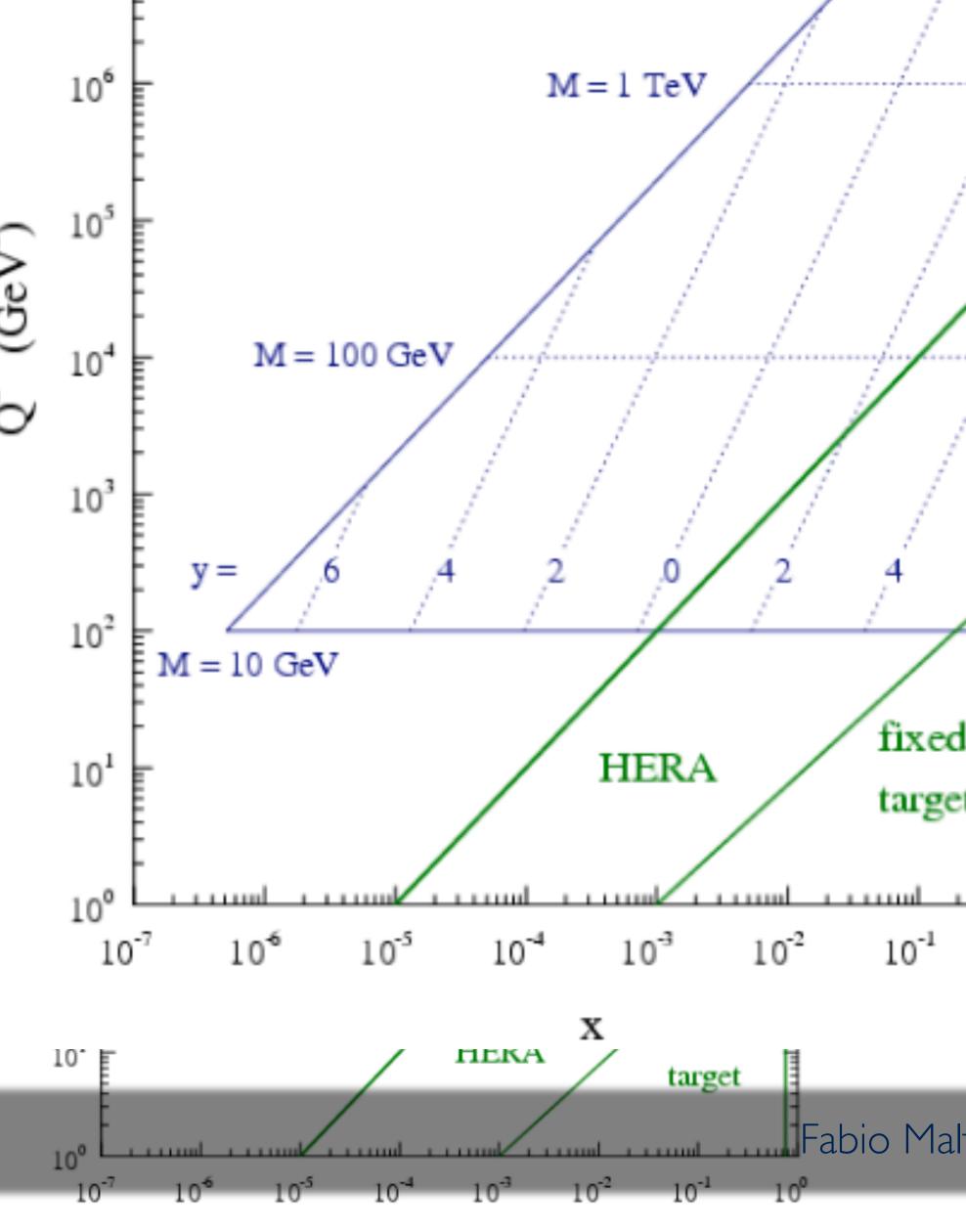
For the production of a par

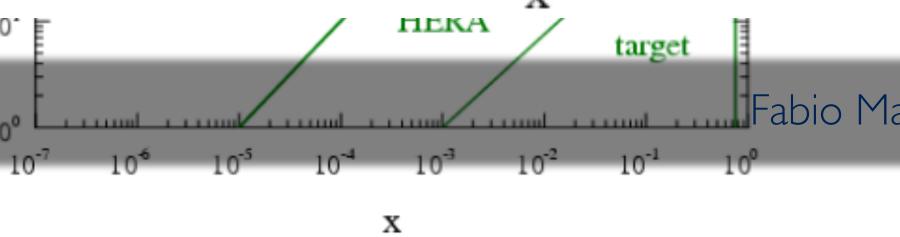
$$M^2 = x_1 x_2 S = x_1 x_2 4 E_{\text{beam}}^2$$

$$y = \frac{1}{2} \log \frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{S}}e^y \quad x_2 = \frac{M}{\sqrt{S}}e^{-y}$$

See exercises!
$$x_2 = \frac{M}{\sqrt{S}}e^{-y}$$

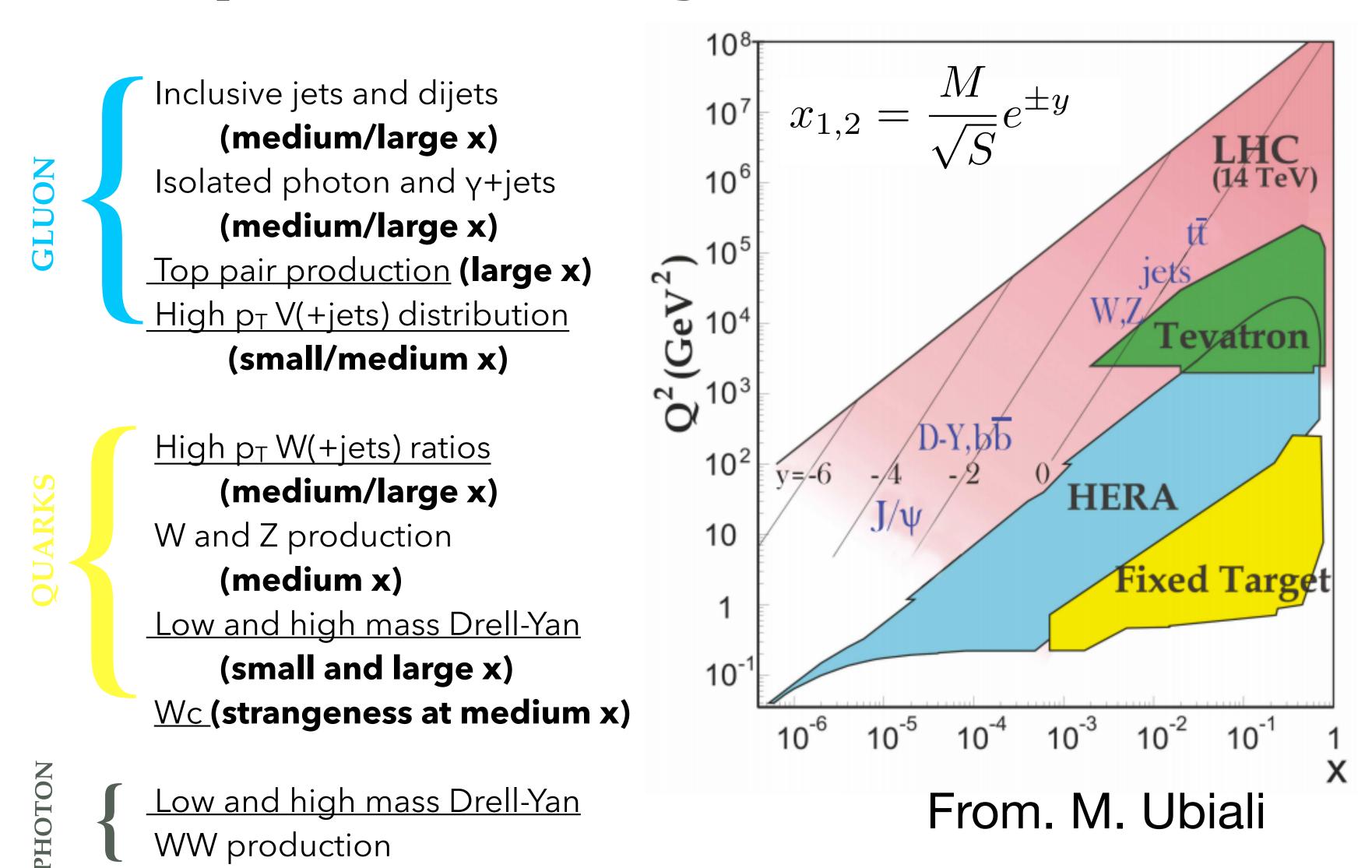




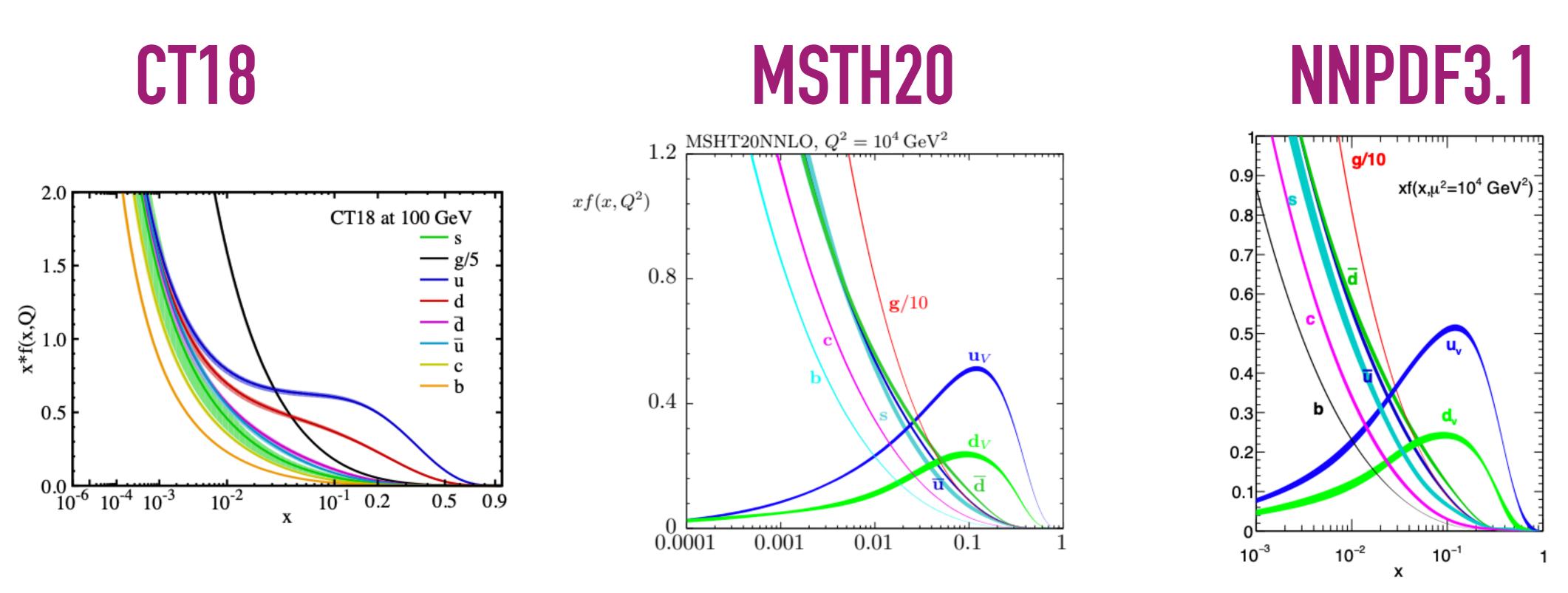
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Data complementarity



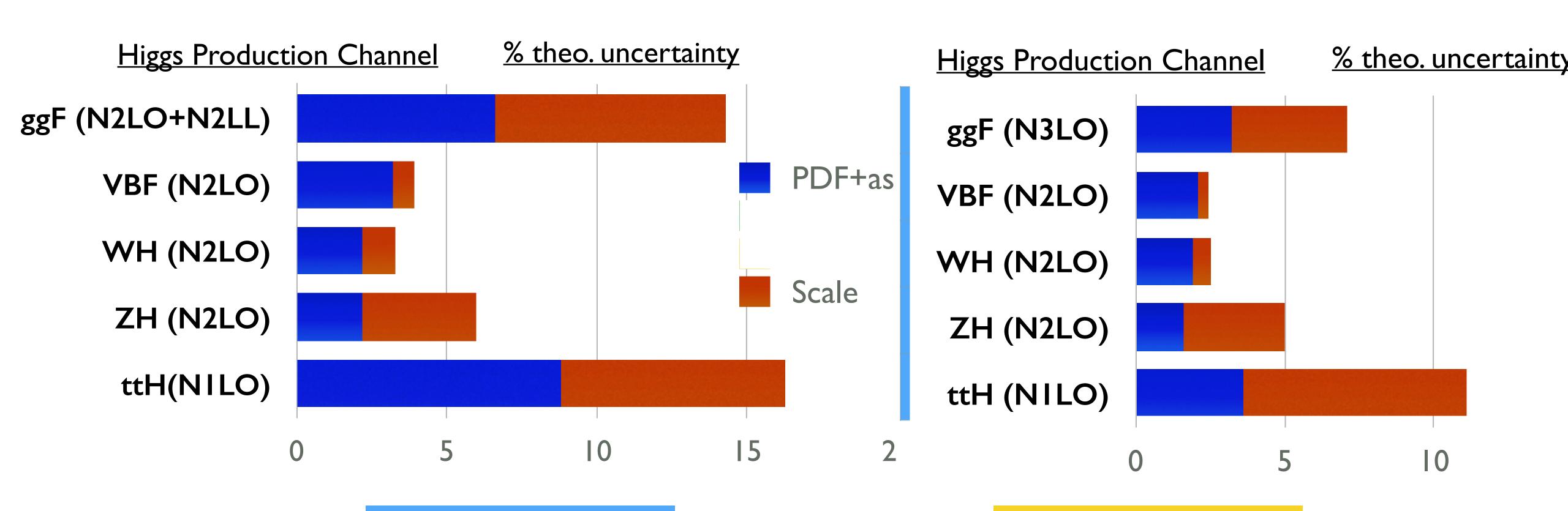
Modern PDFs



Different collaborations, predictions usually computed with different PDFs to extract an uncertainty envelope.

Impact of PDF uncertainties (13)

Yell

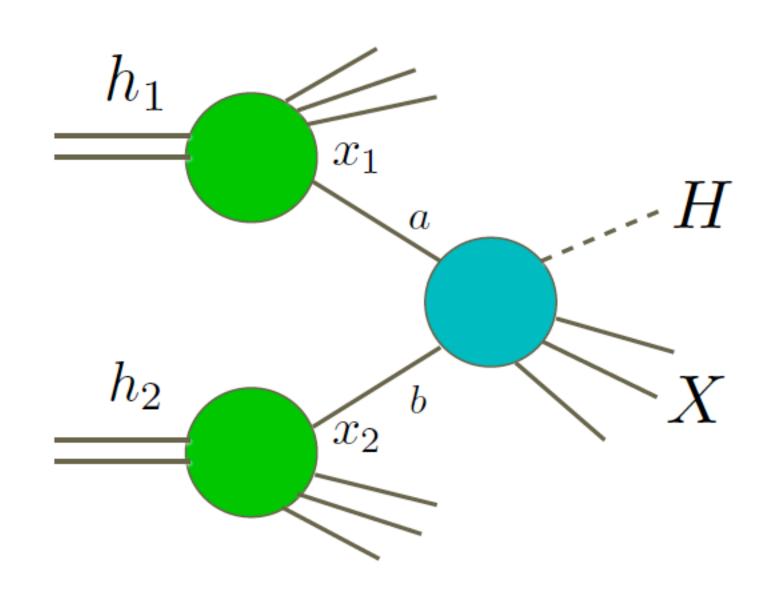


Yellow Report 3 (2013)
limiting factor in the Yellow Report 4 (2016)
accuracy of theoretical predictions
Progress in PDFs!

Reduced (sti PDF uncertai

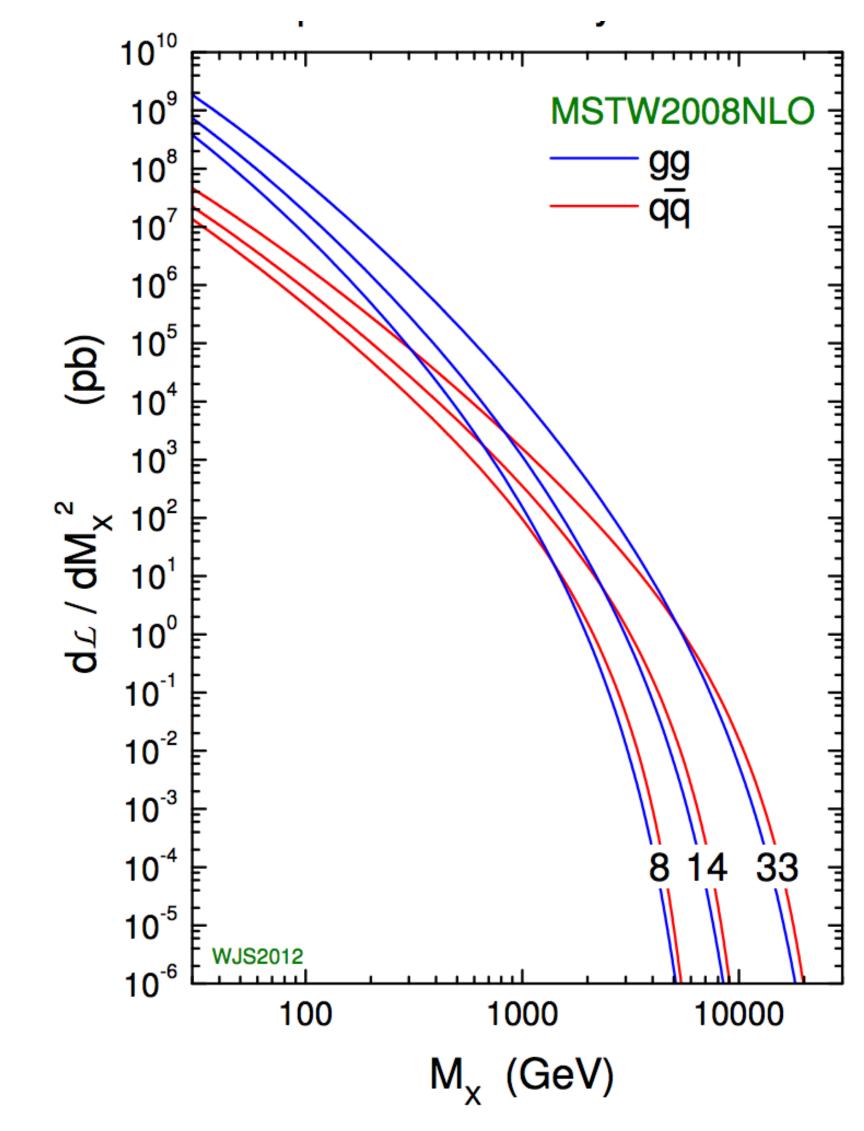
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Parton luminosities and collider reach



$$\sigma(S) = \sum_{i,j} \int d\tau \left[\frac{1}{S} \frac{dL_{ij}}{d\tau} \right] \left[\hat{s} \hat{\sigma}_{ij} \right]$$

$$\tau \frac{dL_{ij}}{d\tau} = \int_0^1 dx_1 dx_2 x_1 f_i(x_1, \mu_F^2) \times x_2 f_j(x_2, \mu_F^2) \delta(\tau - x_1 x_2)$$



 $\Phi_{ab}(M^2)$

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$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\downarrow$$

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$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \,\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s})$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \,\hat{\sigma}(\hat{s}, \mu_R)$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \,\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

End of Lecture 2