STFC HEP summer school 2022

Phenomenology Problems

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1 Basic Kinematics

The rapidity y and pseudo-rapidity η are defined as:

$$y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$$
$$\eta = -\log\left(\tan\left(\frac{\theta}{2}\right)\right)$$

where z is the direction of the colliding beams.

a) Verify that for a particle of mass m

$$E = \sqrt{m^2+p_T^2}\,\cosh y$$
$$p_z = \sqrt{m^2+p_T^2}\,\sinh y$$

with $p_T^2 = p_x^2 + p_y^2$.

b) Prove that $\tanh\eta = \cos\theta$

c) Prove that rapidity equals pseudo-rapidity for a relativistic particle $E \gg m$

d) Prove that the difference of two rapidities is Lorentz invariant.

2 $e^+e^- \to$ hadrons

By studying the shape of the Z-resonance in the R-ratio we can try to see the effect of some parameters in the electro-weak interactions. You can answer the questions qualitatively but you could also plot the function if you have time. Recall from the lectures that:

$$d\sigma\left(f\bar{f} \to f'\bar{f}'\right) = \alpha^2 \frac{\pi}{2s} d(\cos\theta) \Bigg\{ (1+\cos^2\theta)\left(q_f^2 q_{f'}^2 + \frac{g_z^2}{4g_e^2} q_f q_{f'} v_f v_{f'} \chi_1 + \frac{g_z^4}{16 g_e^4}(a_f^2+v_f^2)(a_{f'}^2+v_{f'}^2)\chi_2\right) + \cos\theta\left(\frac{g_z^2}{2g_e^2} q_f q_{f'} v_f v_{f'} \chi_1 + \frac{g_z^4}{2g_e^4} a_f a_{f'} v_f v_{f'} \chi_2\right)\Bigg\}$$

where

$$\frac{g_Z}{g_e} = \frac{1}{\cos\theta_w \sin\theta_w} \qquad \chi_1 = \frac{s(s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \qquad \chi_2 = \frac{s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

	q_f	a_f	v_f
u, c, t	$2/3$	$1/2$	$1/2 - 4/3\sin^2\theta_w$
d, s, b	$-1/3$	$-1/2$	$-1/2 + 2/3\sin^2\theta_w$
e, μ, τ	-1	$-1/2$	$-1/2 + 2\sin^2\theta_w$
ν_e, μ, τ	0	$1/2$	$1/2$

Table 1: EW couplings in the Standard Model.

and

$$g_z^2 = \frac{4\pi\alpha}{\cos^2\theta_w \sin^2\theta_w} \qquad \Gamma_Z \approx \sum_l \Gamma_{Z\to l\bar{l}} + \sum_q N_c \Gamma_{Z\to q\bar{q}} \qquad \Gamma_{Z\to f\bar{f}} = \frac{m_Z \alpha}{12\cos^2\theta_w \sin^2\theta_w}\left(a_f^2 + v_f^2\right)$$

The axial and vector couplings in the Standard Model are given in Table 1.

a) Try to plot $R(\frac{e^+e^-\to\text{hadrons}}{e^+e^-\to\mu^+\mu^-})$ both in the case where both Z and photon exchange are included in both numerator ($e^+e^-\to$ hadrons) and denominator ($e^+e^-\to\mu^+\mu^-$) and in the case where the denominator includes only photon exchange.

b) What happens to the shape of the resonance as you change θ_w?

c) Does the shape of the resonance change if the Z propagator had the opposite sign?

3 Jet Kinematics

At the LHC each beam has an energy of 7 TeV. Two partons collide and produce two jets with negligible mass, transverse momentum p_T and rapidities $y_{3,4}$.

a) Show that

$$x_1 = \frac{p_T}{\sqrt{s}}(e^{y_3} + e^{y_4}), x_2 = \frac{p_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4})$$

b) Show that the invariant mass of the dijet system is

$$M_{JJ} = 2p_T \cosh\left(\frac{y_3 - y_4}{2}\right)$$

and the centre of mass scattering angle is:

$$\cos\theta^* = \tanh\left(\frac{y_3 - y_4}{2}\right)$$

c) Discuss the regions of $x_{1,2}, M_{JJ}$ and θ^* probed with a jet trigger of $p_T > 35$ and $|y_{3,4}| < 3$

4 Event shapes

The thrust is defined as

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|},$$

where the sum is over all the particles, the ith particle has 3-momentum $\vec{p}_i$, and $\vec{n}$ is a unit-vector.

Explain why the value of the thrust is given by

a) 1 for back-to-back configurations,

b) $\frac{1}{2}$ for a perfectly spherical event (i.e. uniform ditribution of momenta).

c) Calculate the minimum possible value of the thrust for a $q\bar{q}g$ state. [Hint: This occurs for the 'Mercedes' configuratio where all the particles have the same energy and the angle between any two particles is 120^0.]

Consider the two event shape variables

$$S_{\text{lin}} = \left(\frac{4}{\pi}\right)^2 \min_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}\right)^2, \qquad S_{\text{quad}} = \frac{3}{2} \min_{\vec{n}} \frac{\sum_i |\vec{p}_i \times \vec{n}|^2}{\sum_i |\vec{p}_i|^2}.$$

d) Determine whether S_{lin} is infrared safe or not.

e) What are the limiting values of S_{lin} for pencil-like (back-to-back) and spherical events?

f) What is the value for the Mercedes configuration?

5 Infrared safety

Are these observables infrared safe at a hadron collider? If not, how would you modify them to make them infrared safe?

a) Partonic center of mass energy (defined as a the invariant mass of the sum of all final state particles in the event).

b) The sum of the energies of all jets with transverse momentum above a given p_T threshold.

c) The invariant mass of all jets in the event.

d) The number of partons.

6 Jet algorithms

Recall the distance measure used by the anti-k_T jet algorithm is given by:

$$d_{i,j} = \min(p_{i,\perp}^{-2}, p_{j,\perp}^{-2})\frac{\Delta R_{ij}^2}{R^2} \ , \quad d_{iB} = p_{i,\perp}^{-2} \ .$$

where $\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$. Consider the clustering of a three particle system with one hard jet $j_H = (p_{H,\perp}, \eta, \phi)$ and two softer jets $j_1 = (p_{1,\perp}, 0, 0)$ and $j_2 = (p_{2,\perp}, \eta_0, 0)$ where $p_{H,\perp} \gg p_{i,\perp}$. You may also take $R = 1$.

a) Find the conditions for the soft jets to be clustered with the hard jet.

b) Show that anti-k_T jets are circular in the $\eta - \phi$ plane.

c) What happens to the clustering when cones of jets 1 and 2 overlap?

d) How does it depend on the relative size of the transverse momentum, $r = \frac{p_{1,\perp}}{p_{2,\perp}}$?

7 EFT and anomalous couplings

The following dimension-6 operators modify the couplings of the top quark to the weak gauge bosons:

$$O_{\varphi Q}^{(3)} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi\right)(\bar{Q}\gamma^\mu \tau^I Q) \tag{1}$$

$$O_{\varphi Q}^{(1)} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{Q}\gamma^\mu Q) \tag{2}$$

$$O_{\varphi t} = i\frac{1}{2}y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\right)(\bar{t}\gamma^\mu t) \tag{3}$$

$$O_{tW} = y_t g_w (\bar{Q}\sigma^{\mu\nu}\tau^I t)\tilde{\varphi}W^I_{\mu\nu} \tag{4}$$

$$O_{tB} = y_t g_Y (\bar{Q}\sigma^{\mu\nu} t)\tilde{\varphi}B_{\mu\nu} \tag{5}$$

a) Explain qualitatatively which particular top couplings will be modified by each operator.

b) Write down the Feynman rules for the ttZ vertex including the impact of 2-fermion operators listed above: $\mathcal{O}_{tW}, \mathcal{O}_{tB}, \mathcal{O}_{\phi t}, \mathcal{O}_{\phi Q}^{(3)}, \mathcal{O}_{\phi Q}^{(1)}$ e.t.c. and compare with the typical anomalous coupling parametrisation of the ttZ vertex.

$$\mathcal{L}_{ttZ} = e\bar{u}(p_t)\left[\gamma^\mu\left(C_{1,V}^Z + \gamma_5 C_{1,A}^Z\right) + \frac{i\sigma^{\mu\nu}q_\nu}{m_Z}\left(C_{2,V}^Z + i\gamma_5 C_{2,A}^Z\right)\right]v(p_{\bar{t}})Z_\mu$$

c) What are the expressions for: $C_{1,V}^Z, C_{1,A}^Z, C_{2,V}^Z$ and $C_{2,A}^Z$ in terms of the dim-6 Wilson coefficients?

d) Use this to explain why there are degeneracies between operators if one only looks at processes involving the ttZ interaction.

8 DGLAP splitting kernels (Optional)

Show that in the collinear limit $p_3 \to z p_{\tilde{13}},\ p_1 \to (1-z) p_{\tilde{13}}$ the matrix element

$$\left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}}, 3_g)|^2 \right\rangle = \frac{4e^2 e_q^2 g_s^2 N_c}{s} C_F \frac{s_{a1}^2 + s_{a2}^2 + s_{b1}^2 + s_{b2}^2}{s_{13} s_{23}}$$

factorizes to

$$|\mathcal{M}_{q\bar{q}g}|^2 \to |\mathcal{M}_{q\bar{q}}|^2 \times \frac{2g_s^2}{s_{13}} \times C_F \frac{1+(1-z)^2}{z}$$

$$\left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}}, 3_g)|^2 \right\rangle \overset{3||1}{\to} \left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, (\tilde{13})_q, 2_{\bar{q}})|^2 \right\rangle \frac{2g_s^2 C_F}{s_{13}} \frac{1+(1-z)^2}{z}$$

with

$$\left\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}})|^2 \right\rangle = 2e^2 e_q^2 N_c \frac{s_{a1}^2 + s_{a2}^2}{s_{ab}}$$

9 Operators and EOMs (Optional)

Show that the two operators in:

$$\mathcal{O}_{gt} = \bar{t}\, T_A \gamma^\mu D^\nu t\, G^A_{\mu\nu}, \tag{6}$$
$$\mathcal{O}_{gQ} = \overline{Q} T_A \gamma^\mu D^\nu Q G^A_{\mu\nu}, \tag{7}$$

can be written as a sum of four fermion operators