Neutrino Physics

History, electroweak interactions and neutrino scattering

Jessica Turner

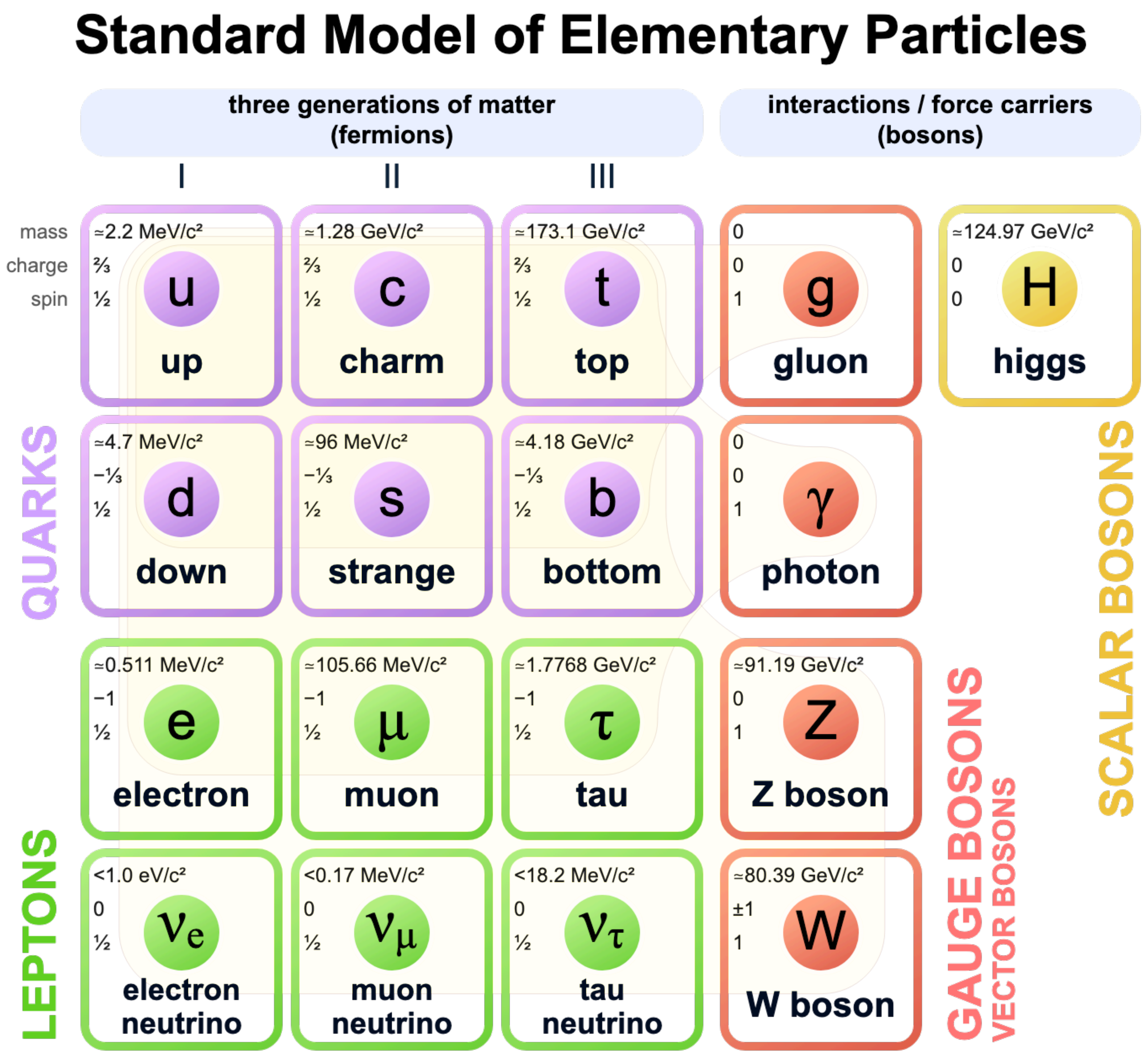
Resources

- Athar & Singh: “The Physics of Neutrino Interactions”
- de Gouvea: “TASI lectures on Neutrino ” available hep-ph/0411274.pdf
- Giunti & Kim: “Fundamentals of Neutrino Physics and Astrophysics”
- Hernandez: “Neutrino Physics ” available hep-ph/1708.01046.pdf
- Pascoli: “Neutrino Physics” available Neutrino Physics
- Thompson: Chapter 13 “Modern Particle Physics”

Outline

- Lecture 1: Historical Overview, Neutrinos in the SM, Neutrino Interactions
- Lecture 2: Neutrino Oscillations and Phenomenology
- Lecture 3: Neutrino Oscillations in Matter
- Lecture 4: Neutrino Masses, Models and Consequences
- Lecture 5: Neutrinos in the Early Universe

Neutrinos in the Standard Model

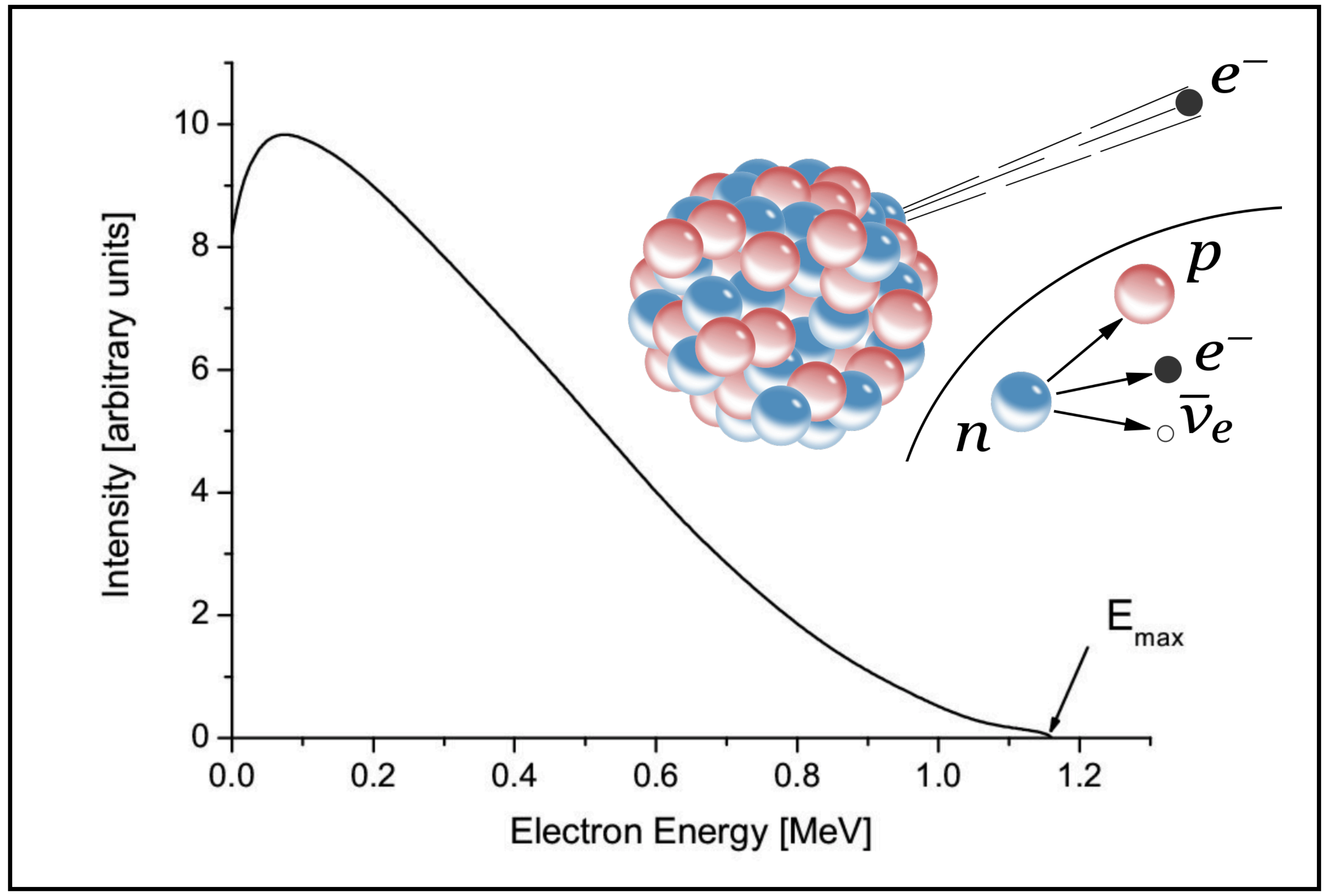


- Neutrinos are electrically neutral fermions which are part of an $SU(2)_L$ doublet
- Neutrinos undergo weak interactions force carriers with the W and Z boson $(m(W^{\pm}) \sim 80\,\mathrm{GeV}, m(Z) \sim 91\,\mathrm{GeV})$
- Neutrinos are very light $m_\nu \lesssim 1\,\mathrm{eV}$

Discoveries of the Neutrino

- 1800s was an extraordinary time for radioactivity discovery: α, β, γ discovered
- α - Helium nucleus, discovered by Rutherford 1899
- γ - electromagnetic radiation (photon) arising from the radioactive decay of atomic nuclei 1900
- β - electron emitted by radioactive nuclei, discovered by Rutherford 1899

$$(A, Z) \to (A, Z+1) + e^- \implies E_e = M(A, Z+1) - M(A, Z)$$

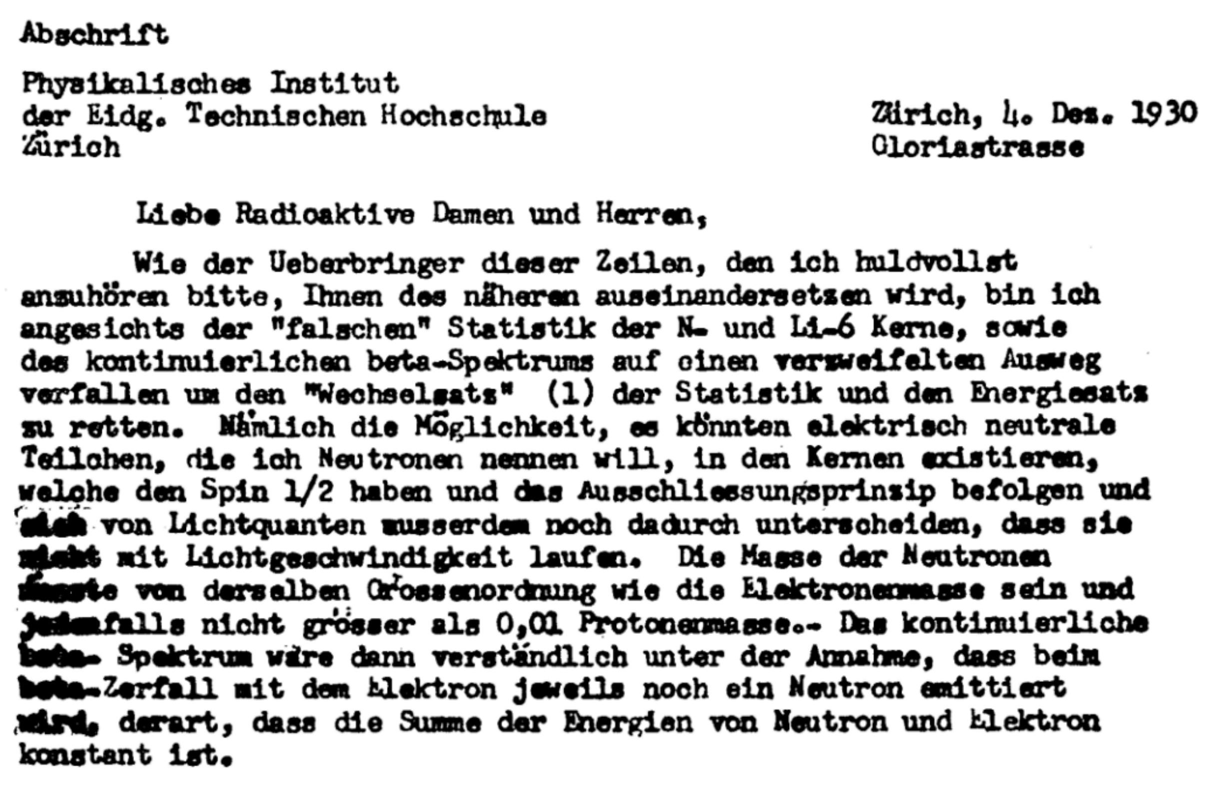


From energy conservation, the electron should have had a fixed energy, not a spectrum which was what was observed.

Discoveries of the Neutrino

- Some scientist thought that energy conservation principle must be violated.

- In his famous letter to "Radioactive Ladies and Gentlemen" Pauli (1930) proposed the existence of a new and yet undiscovered electrically neutral particle which would explain the continuous spectrum observed in beta decay.



Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

$$(A, Z) \rightarrow (A, Z+1) + e^- + \nu$$

- Fermi proposed the new particle should be light, spin 1/2 and electrically neutral. He dubbed this new particle the "neutrino".

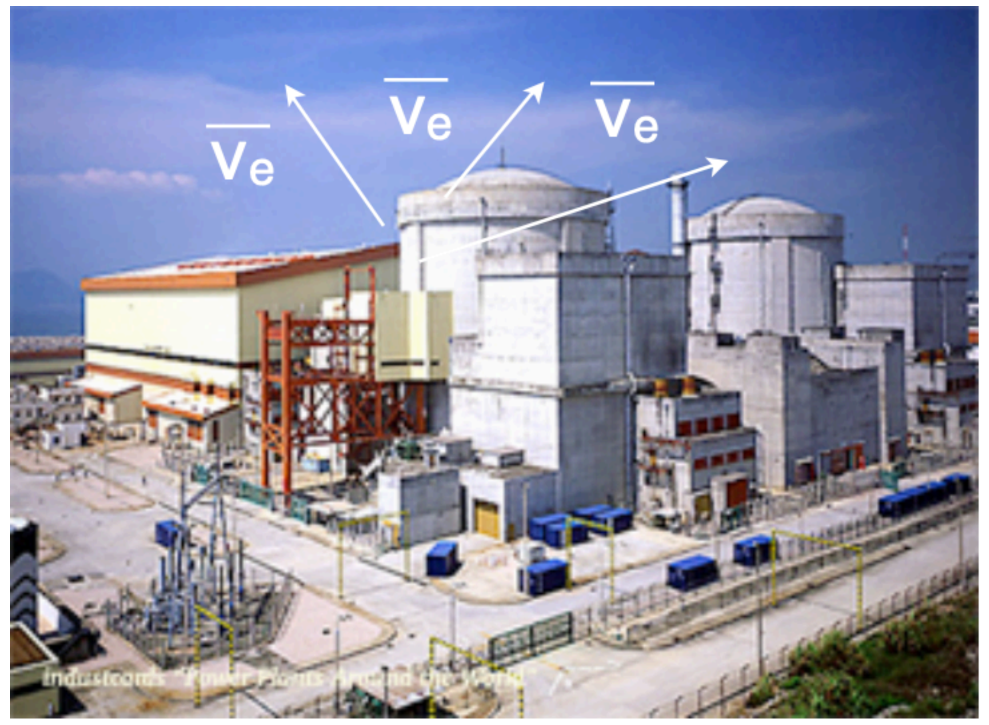
Discoveries of the Neutrino

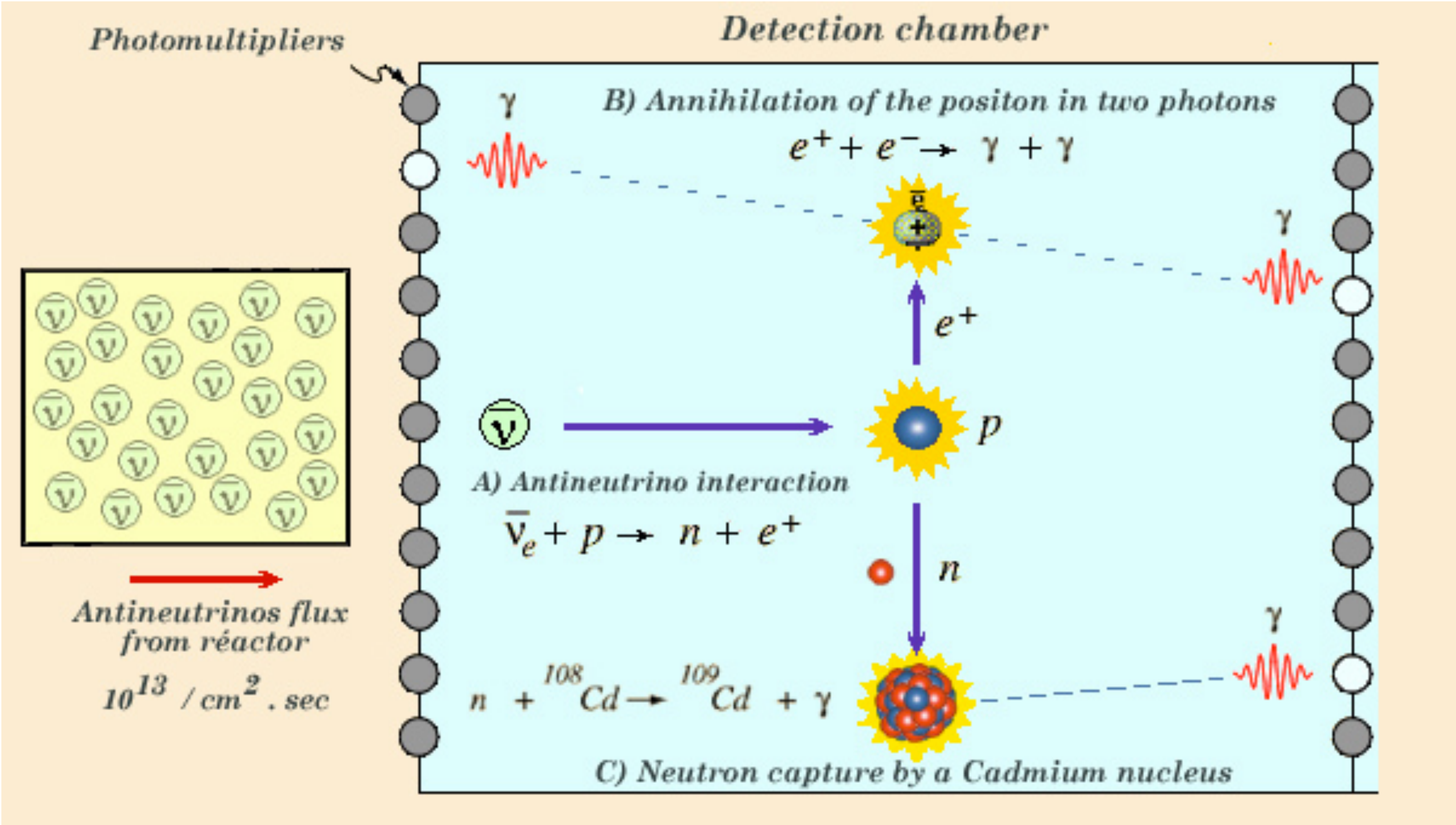
- 1934: Bethe and Peierls showed the interaction cross section of the neutrino should be extremely small $\Longrightarrow$ a neutrino can traverse the Earth without deviation

> *"Today I have done something which no theoretical physicist should ever do in his life: I have predicted something which shall never be detected experimentally!"*

- Walter Baade, had great faith in his experiment colleagues and bet Pauli the neutrino would be discovered. The bet was a crate of champagne.

- 1956: Reines and Cowan placed neutrino detector (vat of water combined with cadmium chloride) in front a fission reactor.





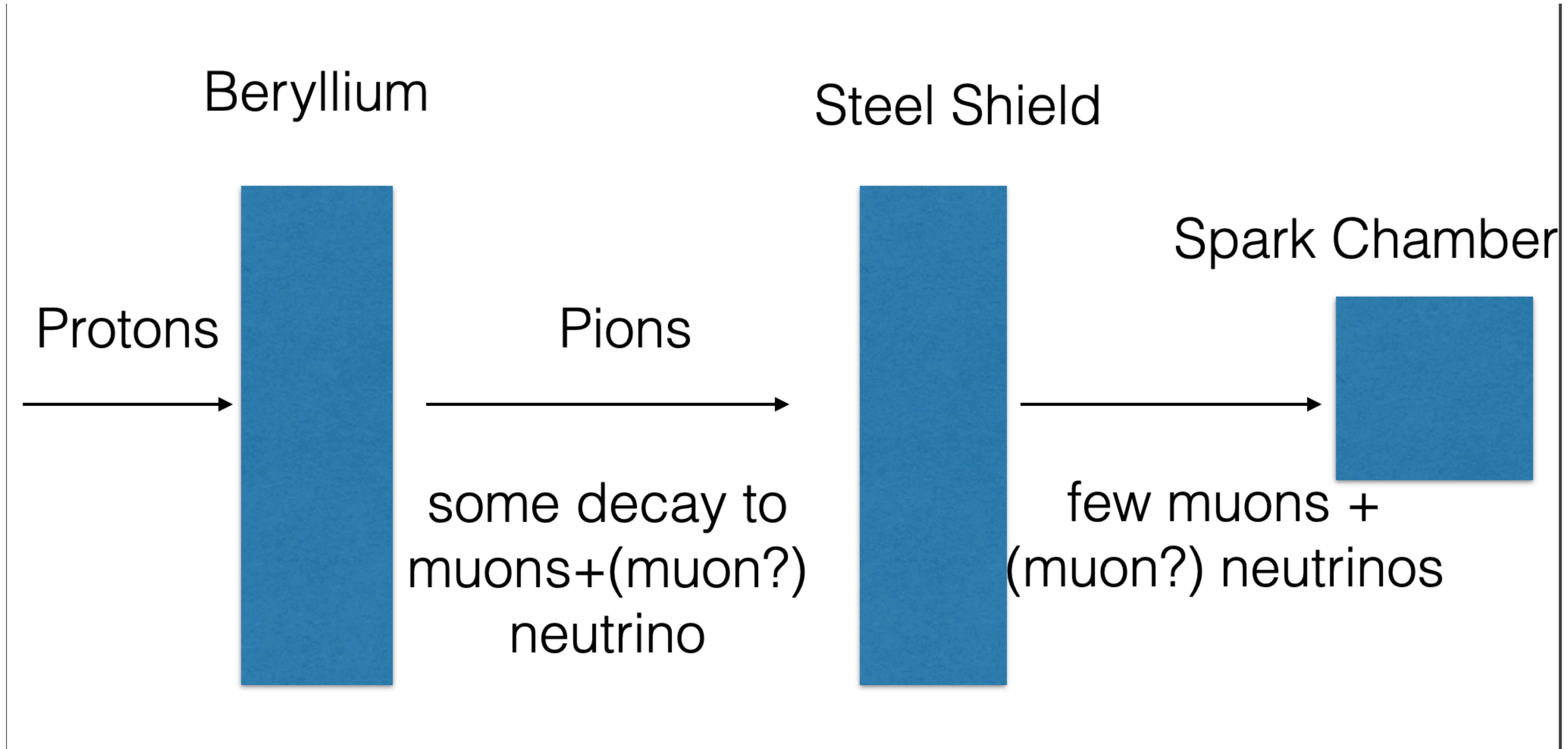
15 msec delay from photon emitted by neutron capture

Discoveries of the Neutrino

- The electron antineutrino had been discovered because the neutrinos coming from fission reactors is emitted together with electrons.

- Muon discovered (cosmic ray showers) in 1936. Was there an associated neutrino?

- 1959 Gaillard, Lederman, Schwartz and Steinberger built a spark chamber (chambers of a stack of metal plates placed in a sealed box filled with a gas such as helium, neon) to detect muon neutrino coming from pion decays



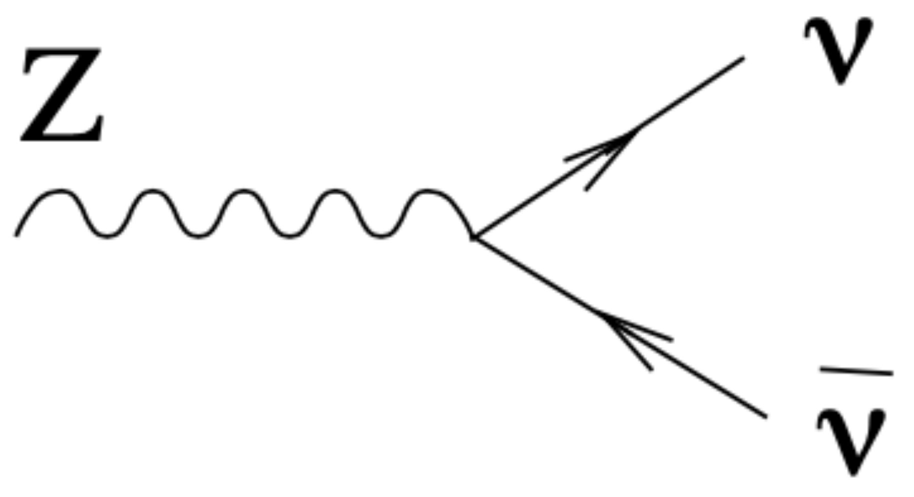
$$\nu_e \equiv \nu_\mu \implies N(\mu^-) = N(e^-)$$

$$36\,\mu^-, 4\,e^-$$

$$36\,\mu^-, 4\,e^- \implies \nu_\mu \not\equiv \nu_e$$

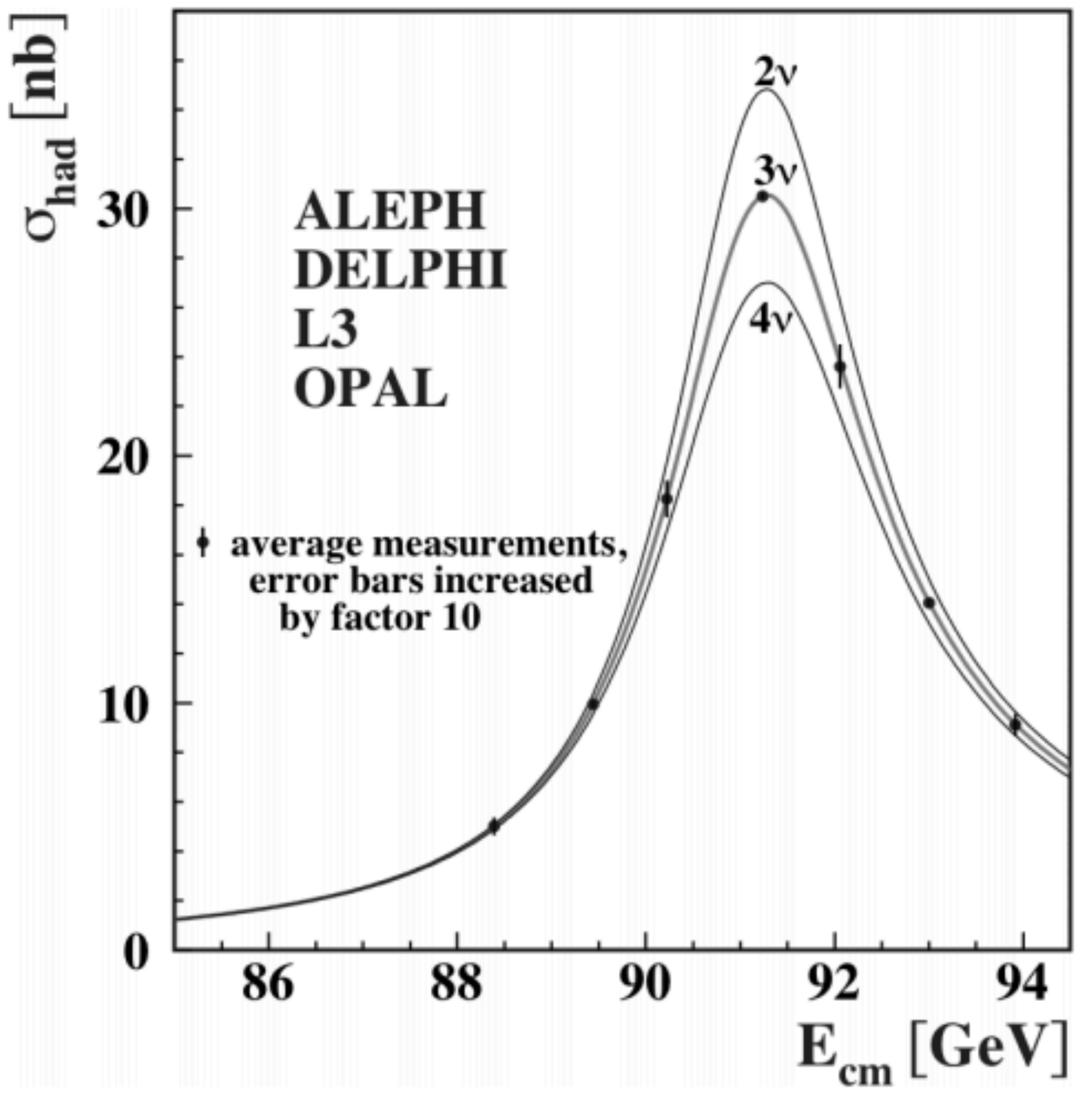
Discoveries of the Neutrino

- neutrinos couple to weak gauge bosons and will modify their decay width
  Z boson width measurement by LEP in 1989 confirmed 3 generations of neutrinos



$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} = \frac{\Gamma_{\rm total} - \Gamma_{\rm visible}}{\Gamma_\nu} = \frac{\Gamma_{\rm total} - \Gamma_{\rm had} - 3\Gamma_{\rm lep}}{\Gamma_\nu}$$

Requires $N_\nu = 2.98 \pm 0.082$

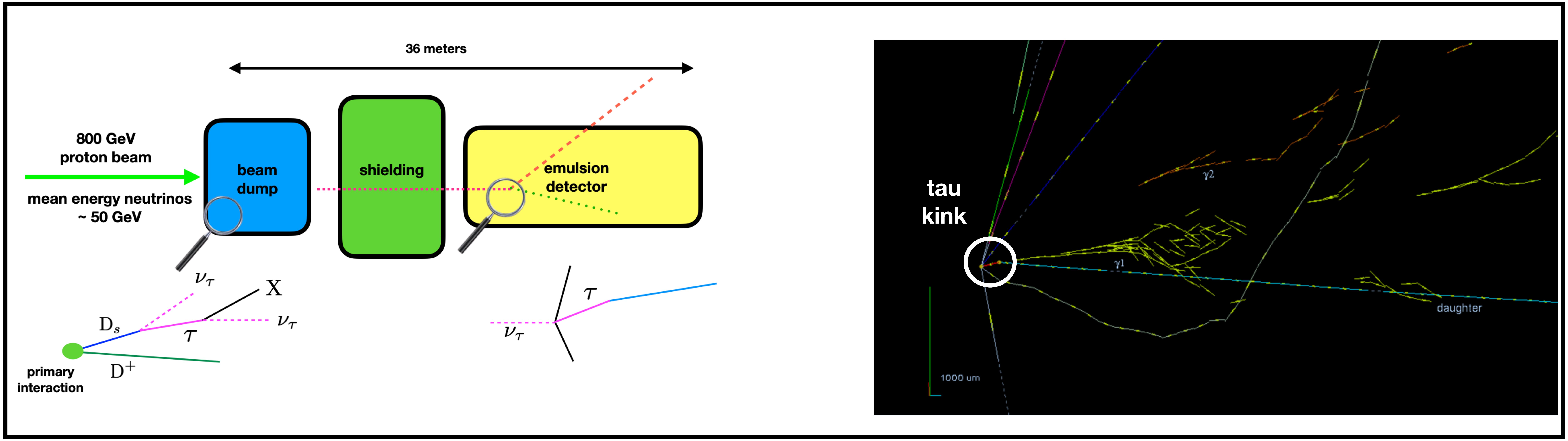


Discoveries of the Neutrino

- Tau lepton was discovered in 1975 by Perls and colleague

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \rightarrow e^\pm + \mu^\pm + 4\nu$$

- Was there a neutrino associated to this new heavy lepton?
- Confirmed in 1997 by the DONUT experiment: they observed 4 tau neutrinos



- OPERA experiment observed about 10 tau neutrinos. These are the least well measured Standard Model particles!!

Neutrinos in the SM

- The Standard Model (SM) gauge group based on the following symmetry:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$$

- We have 3 generations of fermions: generations with identical gauge quantum number but different masses

$$\boxed{Q_{EM} = T_{L_3} + Y}$$

$(1, 2, -\frac{1}{2})$ L_L	$(3, 2, \frac{1}{6})$ Q^i_L	$(1, 1, -1)$ E_R	$(3, 1, \frac{2}{3})$ U^i_R	$(3, 1, -\frac{1}{3})$ D^i_R
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

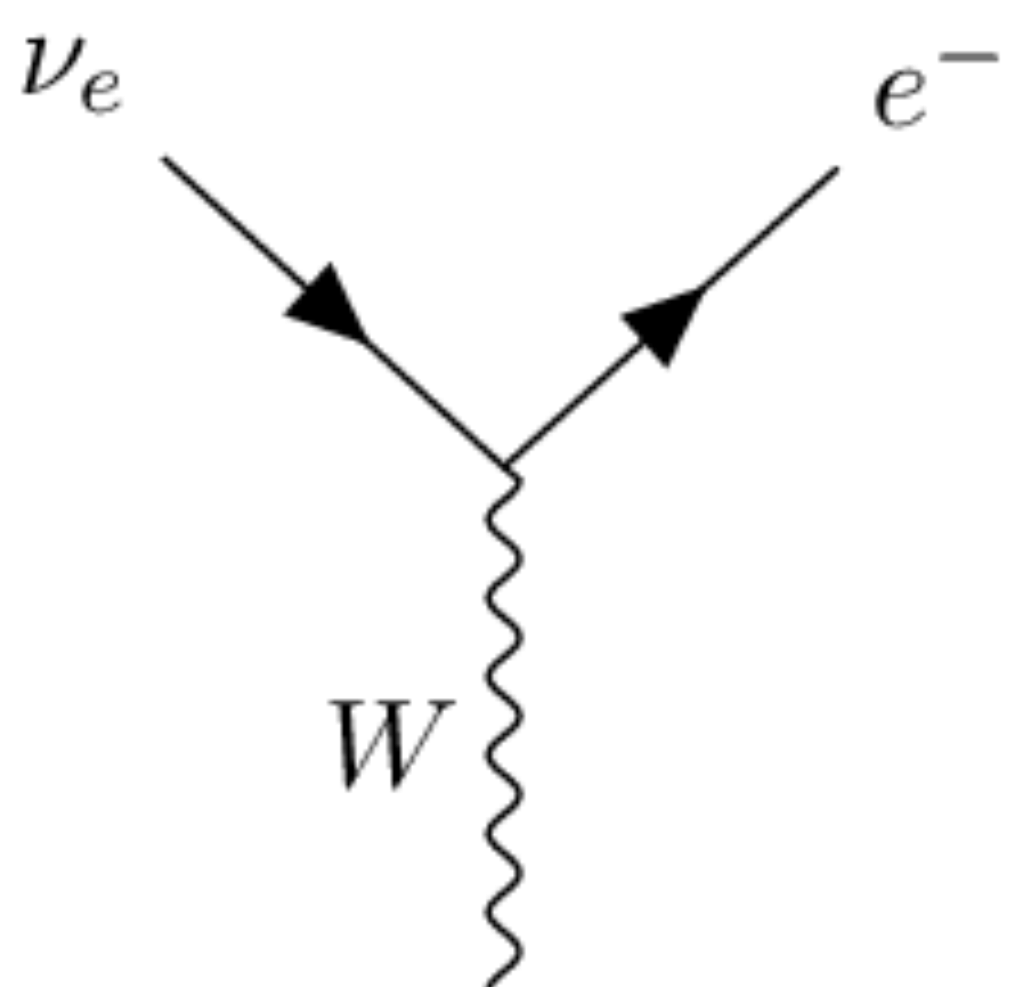
T_{L3} weak component of isospin

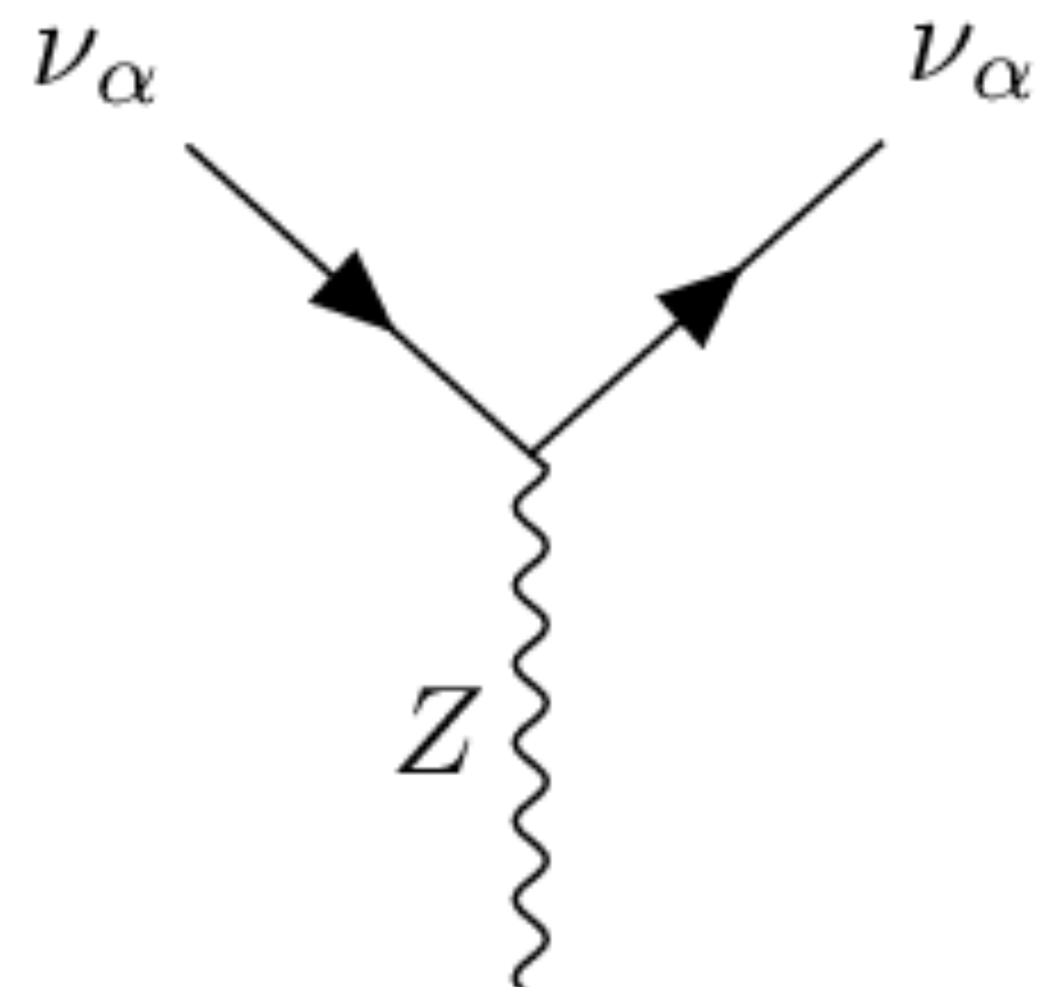
Neutrinos $T_{L3} = 1/2$

Neutrinos have no strong or EM interactions

No right handed neutrinos in the SM

Electroweak Theory for neutrinos





GSW Theory of Leptons

Electroweak Theory for neutrinos

Leptonic $SU(2)_L$ **doublet** interact via their **left-handed** components:

$$\psi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \Psi_L = P_L \Psi \quad P_L = \frac{1}{2}\left(1-\gamma^5\right)$$

Right-handed components are $SU(2)_L$ **singlets**: e_R, ν_R

Build weak interaction Lagrangian for single flavour. Kinetic Term for free leptons:

$$\mathcal{L} = i\bar{\psi}_L \partial\!\!\!/ \psi_L + i\bar{e}_R \partial\!\!\!/ e_R + i\bar{\nu}_R \partial\!\!\!/ \nu_R$$

Add gauge interactions, promote derivative to covariant derivative: $\partial^\mu \rightarrow D^\mu$

$$D^\mu = \partial^\mu + i\frac{g}{2}\vec{\tau}\cdot\vec{W}^\mu + i\frac{g'}{2}YB^\mu$$

Electroweak Theory for neutrinos

$$\mathcal{L} = i\bar{\psi}_L \not{D} \psi_L + i\bar{e}_R \not{D} e_R + i\bar{\nu}_R \not{D} \nu_R \quad (1)$$

Lagrangian is invariant under $SU(2)_L \times U(1)_Y$

$$\begin{aligned} D^\mu \psi_L &= \left(\partial^\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}^\mu - \frac{ig'}{2}B^\mu\right)\psi_L \\ D^\mu e_R &= \left(\partial^\mu - ig' B^\mu\right) e_R \\ D^\mu \nu_R &= \partial^\mu \nu_R \end{aligned} \quad (2)$$

Pauli matrices are generators of $SU(2)_L$

$$\vec{\tau}\cdot\vec{W}^\mu = \sum_{i=1}^{3} \tau^i W^{i\mu} = \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix} \quad (3)$$

Electroweak Theory for neutrinos

Exercise: Substitute (2) into (3) and then into (1) to show **interaction terms** are:

$$\mathcal{L}_{EW} = -\frac{g}{2}\left(\overline{\nu e}_L \bar{e}_L\right)\gamma_\mu \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2}\left(\overline{\nu_e}\bar{e}_L\right)\gamma_\mu B^\mu \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2}\bar{e}_R \gamma_\mu B^\mu e_R$$

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$$= -\frac{g}{2}\left(\bar{\nu}_e \bar{e}_L\right)\gamma_\mu \begin{pmatrix} W^{3\mu} & W^{+\mu} \\ W^{-\mu} & -W^{3\mu} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2}\left(\bar{\nu}_e \bar{e}_L\right)\gamma_\mu B^\mu \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2}\bar{e}_R\gamma_\mu B^\mu e_R$$

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$$= -\frac{g}{\sqrt{2}}\left(\bar{\nu}_{eL}\gamma_\mu W^{\mu+} e_L + \bar{e}_L\gamma_\mu W^{\mu-}\nu_L\right) - \frac{1}{2}\bar{\nu}_{eL}\gamma_\mu\left(gW^{3\mu} - g'B^\mu\right)\nu_L + \frac{1}{2}\bar{e}_L\gamma_\mu\left(gW^{3\mu} + g'B^\mu\right)e_L + \frac{g'}{2}\bar{e}_R\gamma_\mu B^\mu e_R$$

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$$\sqrt{g^2 + g'^2}\, Z^\mu \qquad \frac{\left(g^2 - g'^2\right) Z^\mu + 2gg'A^\mu}{\sqrt{g^2 + g'^2}}$$

Basis Transformation

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}} \qquad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B_\mu \end{pmatrix}$$

Electroweak Theory for neutrinos

Basis Transformation

$$Z_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2+g'^2}} \qquad A_\mu = \frac{g'W^3_\mu + gB_\mu}{\sqrt{g^2+g'^2}} \qquad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B_\mu \end{pmatrix}$$

A_μ remains massless and is the electromagnetic field of the photon

Z_μ is the neutral weak gauge boson which has a mass

$$\mathcal{L}_{EW} = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu(1-\gamma_5)\,eW^+_\mu + \bar{e}\gamma^\mu(1-\gamma_5)\,\nu_e W^-_\mu\right) - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_{eL}\gamma^\mu\nu_{eL}Z_\mu$$
$$+\frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu eA_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right]$$

Charged Current Interactions

$$\mathcal{L}_{EW} = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu_e W_\mu^-\right) - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_{eL}\gamma^\mu\nu_{eL}Z_\mu$$

$$+ \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right]$$

$$\mathcal{L}_{EW}^{CC} = -\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \text{h.c.}$$

$$\left(\frac{g}{2\sqrt{2}}\right)^2\frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}} \qquad G_F = 1.166\times 10^{-5}\,\text{GeV}^{-2}$$

Neutral Current Interactions

$$\mathcal{L}_{EW} = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ + \bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu_e W_\mu^-\right) - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_{eL}\gamma^\mu\nu_{eL}Z_\mu$$

$$+\frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu eA_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right]$$

$$\mathcal{L}_{EW}^{NC} = -\frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_{eL}\gamma^\mu\nu_{eL}Z_\mu$$

$$\mathcal{L}^{EM} = \underbrace{\frac{gg'}{\sqrt{g^2+g'^2}}}_{e}\bar{e}\gamma^\mu eA_\mu$$

Z and photon are different linear combinations of W^3 and B weighted by different factors. We can conveniently parametrise this in terms of the "weak mixing angle" θ_W

$$Z_\mu = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu$$
$$A_\mu = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad M_W = M_Z\cos\theta_W$$

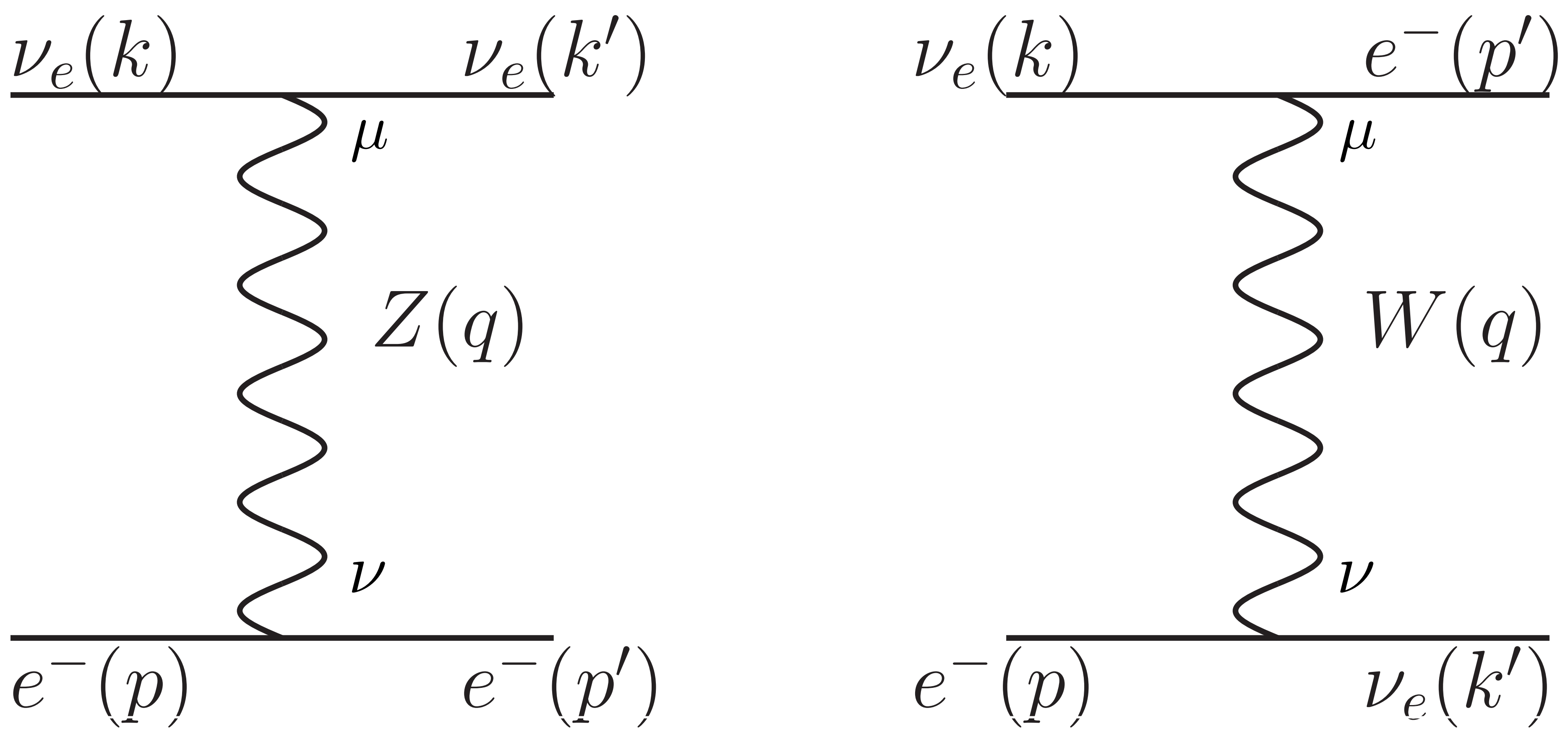
Exercise: rewrite $\mathscr{L}_{EW}$ in terms of weak mixing angle:

$$\mathcal{L}^{\mathrm{em}} = e\bar{e}\gamma^\mu e A_\mu$$
$$\mathcal{L}^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \mathrm{h.c.}\ \right]$$
$$\mathcal{L}^{\mathrm{NC}} = -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e + \bar{e}\gamma^\mu(g_\nu^e - g_A^e\gamma_5)e\right]Z_\mu$$

$$g_V^e = 4\sin^2\theta_W\,, \quad g_A^e = -1\,, \quad \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}, \quad \frac{g}{4\cos\theta_W} = \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Neutrino Interactions - Neutrino Electron Elastic Scattering

$$\nu_e(k) + e^-(p) \rightarrow \nu_e(k') + e^-(p')$$



$\overline{u} \equiv$ outgoing fermion

$u \equiv$ incoming fermion

$$-i\mathcal{M}^{\mathrm{CC}} = \left[\bar{u}\,(p')\,\frac{g}{2\sqrt{2}}\gamma_\mu\,(1-\gamma_5)\,u(k)\right]\left(-\frac{ig^{\mu v}}{M_W^2}\right)\left[\bar{u}\,(k')\,\frac{g}{2\sqrt{2}}\gamma_v\,(1-\gamma_5)\,u(p)\right],$$

$$\Rightarrow \mathcal{M}^{\mathrm{CC}} = \frac{G_F}{\sqrt{2}}\left[\bar{u}\,(p')\,\gamma_\mu\,(1-\gamma_5)\,u(k)\right]\cdot\left[\bar{u}\,(k')\,\gamma^\mu\,(1-\gamma_5)\,u(p)\right],$$

Where we used the replacement:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Neutrino Interactions

$$-i\mathcal{M}_{\mathrm{NC}} = \left[\bar{u}\left(k'\right)\frac{g}{4\cos\theta_W}\gamma_\mu\left(1-\gamma_5\right)u(k)\right] - \frac{ig^{\mu\nu}}{M_Z^2}\left[\bar{u}\left(p'\right)\frac{g}{2\cos\theta_W}\gamma_v\left(\tilde{g}_V^e - \tilde{g}_A^e\gamma_5\right)u(p)\right]$$

$$\Rightarrow \mathcal{M}_{\mathrm{NC}} = \frac{G_F}{\sqrt{2}}\left[\bar{u}\left(k'\right)\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\cdot\left[\bar{u}\left(p'\right)\gamma^\mu\left(\tilde{g}_V^e - \tilde{g}_A^e\gamma_5\right)u(p)\right]$$

$$\tilde{g}_V^e = -\frac{1}{2} + 2\sin^2\theta_W \qquad \tilde{g}_A^e = \frac{1}{2}$$

$$\sum_s u(p,s)\bar{u}(p,s) = \gamma.p + 1_{4\times 4}m$$

$$\sum_s v(p,s)\bar{v}(p,s) = \gamma.p - 1_{4\times 4}m$$

The following Fierz arrangement will be used:

$$(\overline{\Psi}_1\gamma_\mu P_L\Psi_2)(\overline{\Psi}_3\gamma^\mu P_L\Psi_4) = (\overline{\Psi}_1\gamma_\mu P_L\Psi_4)(\overline{\Psi}_3\gamma^\mu P_L\Psi_2)$$

$$\mathcal{M} = \mathcal{M}_{\mathrm{CC}} + \mathcal{M}_{\mathrm{NC}} = \frac{G_F}{\sqrt{2}}\left[\left[\bar{u}\left(p'\right)\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\cdot\left[\bar{u}\left(k'\right)\gamma^\mu\left(1-\gamma_5\right)u(p)\right]\right.$$
$$\left. + \left[\bar{u}\left(k'\right)\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\cdot\left[\bar{u}\left(p'\right)\gamma^\mu\left(\tilde{g}_V^e - \tilde{g}_A^e\gamma_5\right)u(p)\right]\right].$$

The matrix element squared is:

$$\overline{\sum_i}\sum_f |\mathcal{M}|^2 = \overline{\sum_i}\sum_f \left(|\mathcal{M}_{\rm CC}|^2 + \mathcal{M}_{\rm CC}\mathcal{M}^*_{\rm NC} + \mathcal{M}_{\rm NC}\mathcal{M}^*_{\rm CC} + |\mathcal{M}_{\rm NC}|^2\right)$$

$$= 16{\rm G}_F^2\left[\left(g'_V + g'_A\right)^2 (k'\cdot p')(k\cdot p) + \left(g'_V - g'_A\right)^2 (k'\cdot p)(k\cdot p') - m_e^2\left(g'^2_V - g'^2_A\right)(k\cdot k')\right]$$

Average over incoming spins, sum of the spins of the outgoing states $\Longrightarrow$ $1/2 \times 1/2$

The scalar products $\Longrightarrow$ completeness relation, spinor trace & gamma matrices.

We also redefined

$$g'_V = \tilde{g}^e_V + 1 \quad g'_A = \tilde{g}^e_A + 1$$

General expression for $\nu_\mu e^- \to \nu_\mu e^-, \overline{\nu_\mu} e^- \to \overline{\nu_\mu} e^-, \overline{\nu_e} e^- \to \overline{\nu_e} e^-$

$$\overline{\sum_i}\sum_f |\mathcal{M}|^2 = 16G_F^2\left[\alpha\,(k'\cdot p')(k\cdot p) + \beta\,(k'\cdot p)(k\cdot p') - \gamma m_e^2\,(k\cdot k')\right]$$

Where α, β, γ depends on the various couplings of neutrinos/antineutrino to leptons.

Differential cross-section we still need to integrate over phase space of the outgoing states & enforce energy momentum conservation. In the **centre-of-mass frame**:

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{4\pi^2 s} G_F^2 \left[(g'_V + g'_A)^2 \left(\frac{s - m_e^2}{2}\right)^2 + (g'_V - g'_A)^2 \left(\frac{u - m_e^2}{2}\right)^2 + \frac{m_e^2}{2} \left\{ (g'_V)^2 - (g'_A)^2 \right\} t \right]$$

Use the Mandelstam variables:

$$s = (k+p)^2 = k^2 + p^2 + 2(k \cdot p) = m_e^2 + 2(k \cdot p) \implies (k \cdot p) = \frac{s - m_e^2}{2}$$

$$t = (k - k')^2 = -2(k \cdot k') \implies (k \cdot k') = -\frac{t}{2}$$

$$u = (k - p')^2 = k^2 + p'^2 - 2(k \cdot p') = m_e^2 - 2(k \cdot p') \implies (k \cdot p') = -\frac{u - m_e^2}{2}$$

$$s + t + u = k^2 + p^2 + k'^2 + p'^2 = 2m_e^2$$

Experiment	$\frac{\sigma(\nu_e e)}{E_{\nu_e}} (\times 10^{-42} \frac{cm^2}{GeV})$	$\sigma(\bar{\nu}_e e) \times 10^{-46} cm^2$	$\sin^2\theta_W$
Savannah River [354, 408]		7.6 ± 2.2^a	0.25 ± 0.05
(Reactor)		1.86 ± 0.48^b	
Kurchatov (Reactor) [409]		6.8 ± 4.5	0.29 ± 0.10
LAMPF E225 (LAMPF) [407]	$10.0 \pm 1.5 \pm 0.9$		0.249 ± 0.063
LSND [140]	$10.1 \pm 1.1 \pm 1.0$		

1980's LAMPF experiment

$$\sigma(\nu_e e^-) = (3.18 \pm 0.56) \times 10^{-43} \text{cm}^2$$
$$\langle E_\nu \rangle = 31.7\,\text{MeV}$$

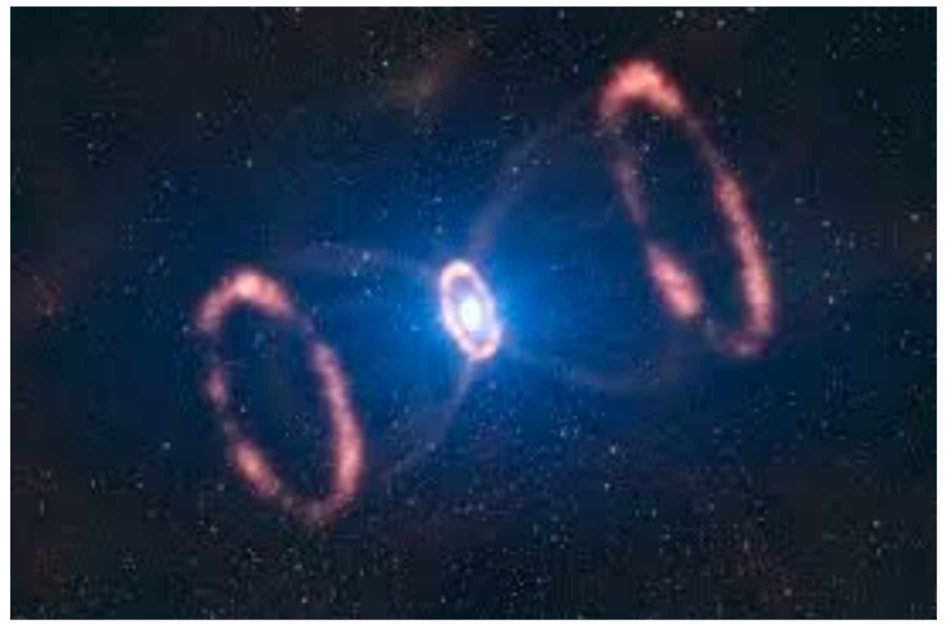
Neutrino Interactions

- By 1990s we knew there were three neutrinos: extremely light, electrically neutral and very weakly interacting → least well understood particle of the SM

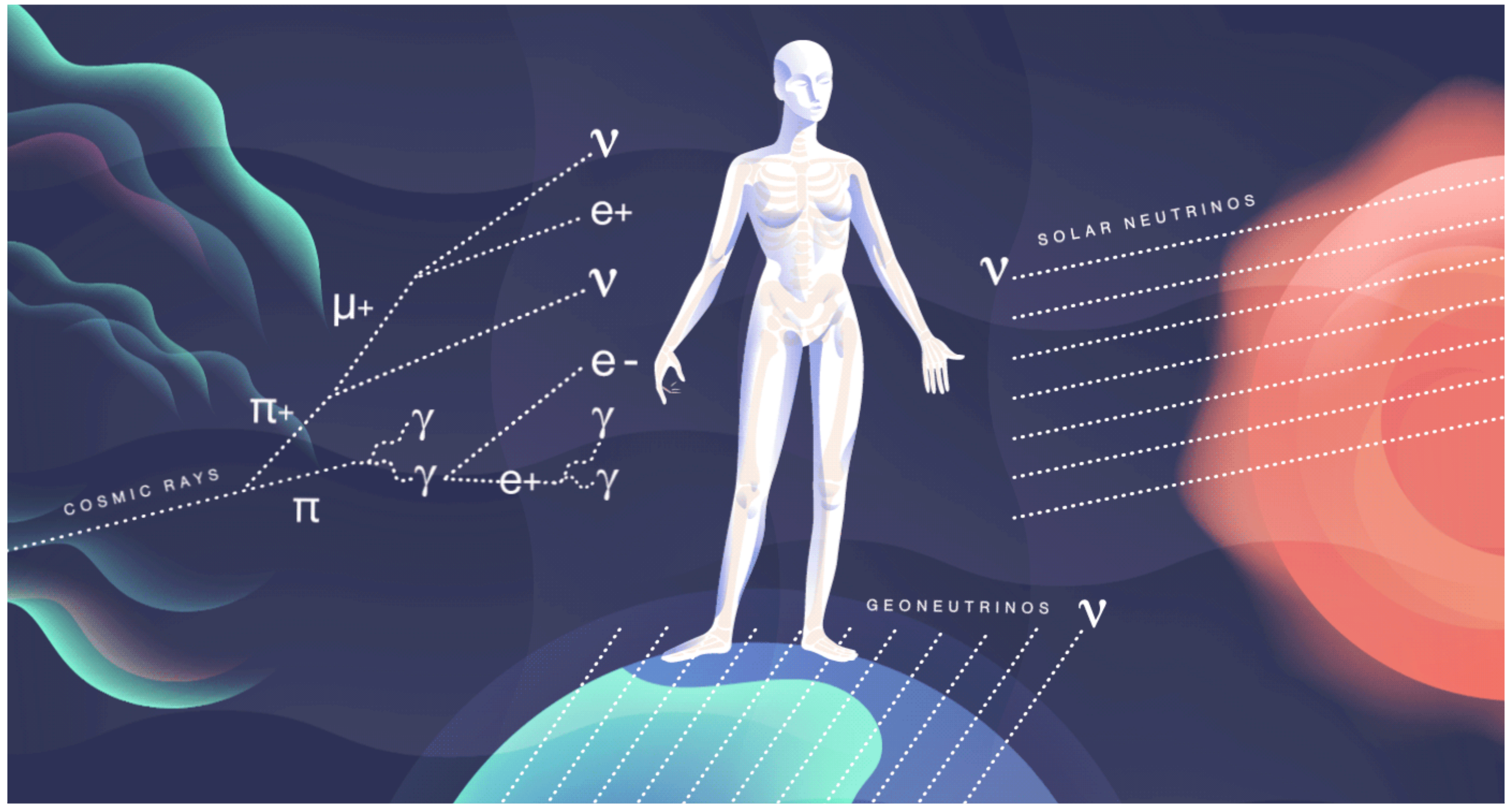
BB: $E_\nu \sim 10^{-4}\,\mathrm{eV}$ $\rho_\nu \sim 330/\mathrm{cm}^3$

Human:
$\Phi_\nu \sim 360 \times 10^6 \nu/\mathrm{day}$

SN1986: $E_\nu \sim \mathrm{MeV}$



atmospheric neutrino:
$E_\nu \sim \mathrm{MeV} - \mathrm{PeV}$
$\Phi_\nu \sim 1\,\nu/\mathrm{cm}^2\mathrm{s}$



Earth: $E_\nu \sim \mathrm{MeV}$

$\Phi_\nu \sim 6 \times 10^6 \nu/\mathrm{cm}^2\mathrm{s}$

$E_\nu \sim \mathrm{MeV}$

$E_\nu \sim \mathrm{GeV}$

Consider atmospheric muon neutrinos with mean energy of a GeV. The interaction cross-section with a proton is

$$\sigma_{\nu p} \sim 10^{-38}\mathrm{cm}^2\,\frac{\mathrm{E}_\nu}{\mathrm{GeV}}$$

How many atmospheric muon neutrinos would interact with you in your lifetime?

$$\Phi = \frac{1\,\nu}{\mathrm{cm}^2\mathrm{sec}} \qquad \langle E_\nu \rangle \sim \mathrm{GeV} \qquad N_{\mathrm{events}} = \sigma \times \Phi \times N_{\mathrm{target}} \times \mathrm{Time}$$

$$1\,\mathrm{kg\,water} \implies 3.3 \times 10^{25}\,\mathrm{H_2O} \implies 1\,\mathrm{kg\,water} \implies 2.6 \times 10^{26}\,\mathrm{protons}$$

Say you're 60 kg and live to a ripe old age

$$N_{\mathrm{target}} \sim 1.5 \times 10^{28}\,\mathrm{protons} \quad \mathrm{Time} = 80\,\mathrm{years} = 2 \times 10^9\,\mathrm{secs}$$

$$N_{\mathrm{events}} \sim 1.5 \times 10^{28} \times 2 \times 10^9 \times 10^{-38} \sim 1!$$

It's likely you will have 1 CC interaction from atmospheric neutrinos your whole life! Not sure why cinema paints neutrinos in such a menacing light (see Alien Covenant and The Core) **Physics point: need huge detectors with long exposures!**

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