Neutrino Physics

Neutrinos and early universe physics

Jessica Turner

Neutrinos in the early Universe

Particles in a thermal bath can be described by their equilibrium Distribution function:

$$f_{\mathrm{eq}} = \frac{1}{\exp\left(\frac{|\vec{p}|-\mu_\nu}{T}\right) \pm 1} \qquad \begin{array}{c}\text{Fermi-Dirac } +\\ \text{Bose-Einstein } -\end{array}$$

Number densities in a thermal bath are

$$\text{relativistic} \quad n_{\mathrm{eq}} \simeq g T^3 \,, \;\; g \equiv \text{ internal d.o.f}$$

$$\text{non-relativistic} \quad n_{\mathrm{eq}} \simeq g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}}$$

The entropy of the thermal bath is

$$s = \frac{2\pi^2}{45} g_* T^3 \qquad g_* = 106.75 \text{ radiation dominated}$$

Neutrino Decoupling

To calculate how a number density of a given species changes over time we must solve Boltzmann Equations

$$\hat{L}[f] = \hat{C}[f]$$

Liouville operator: change in time in the phase space density

Collision operator: number of particle per phase-space volume gained of lost per unit time due to interactions

In a homogeneous & isotopic Universe

$$\hat{L}[f] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}p^2\frac{\partial f}{\partial E}$$

$$g\int \hat{L}[f]\frac{d^3p}{(2\pi)^3} = \frac{dn}{dt} + 3Hn$$

For a two-to-two interaction the collision term is

$$g \int \hat{C}[f] \frac{d^3p}{(2\pi)^3} = -\langle \sigma v \rangle \left(n^2 - n_{\rm eq}^2 \right)$$

Where the cross-section is thermally averaged For a careful derivation see Gelmini and Gondolo, NPB 1991.

For a two-to-two the Boltzmann equation is

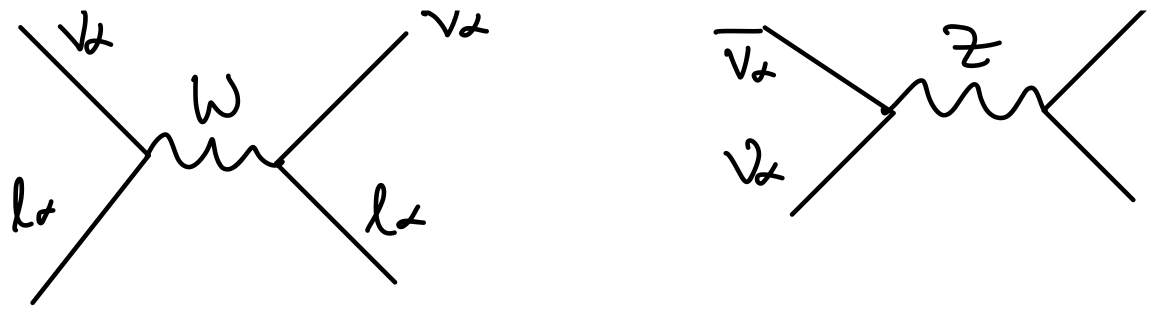
$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm eq}^2 \right)$$

Typically, particles were in thermal equilibrium for T above their mass, if the interactions were fast enough. Interactions such as

$$\nu\bar{\nu} \leftrightarrow e^+e^-, \quad \nu e \leftrightarrow \nu e$$

Keep neutrinos in thermal equilibrium. Neutrinos decouple/drop out of Thermal equilibrium with the plasma when

$$\Gamma \sim H$$



interactions keeping ν's in thermal equilibrium.

$$\sigma \sim \frac{G_F T^2}{4\pi^2}, \quad \Gamma = \langle \sigma n \rangle = \frac{G_F^2 s}{4\pi^2} \times g T^3 = \frac{G_F^2 T^5}{2\pi^2}$$

$$H = \sqrt{\frac{4\pi^3 g^*}{45}} \frac{T^2}{M_{PL}}$$, radiation dominated early Universe, $g_* = 106.75$, $M_{PL} = 1.22 \times 10^{19}$ GeV

ν's begin to decouple / lose thermal contact with plasma when

$$\Gamma = H \Rightarrow \frac{G_F^2 T^5}{2\pi^2} = \frac{17.1\, T^2}{M_{PL}} \Rightarrow T = \left(\frac{17.1 \times 2\pi^2}{M_{PL} \times G_F^2} \right)^{1/3}$$

$$T \sim 6 \times 10^{-3}\ \text{GeV} = 6\ \text{MeV}.$$

Neutrinos decoupled when they were still relativistic but their momentum redshifts over time and now they are non-relativistic and form the CνB (recall in lecture 1 we said there were around 330 cm^{-3}!)

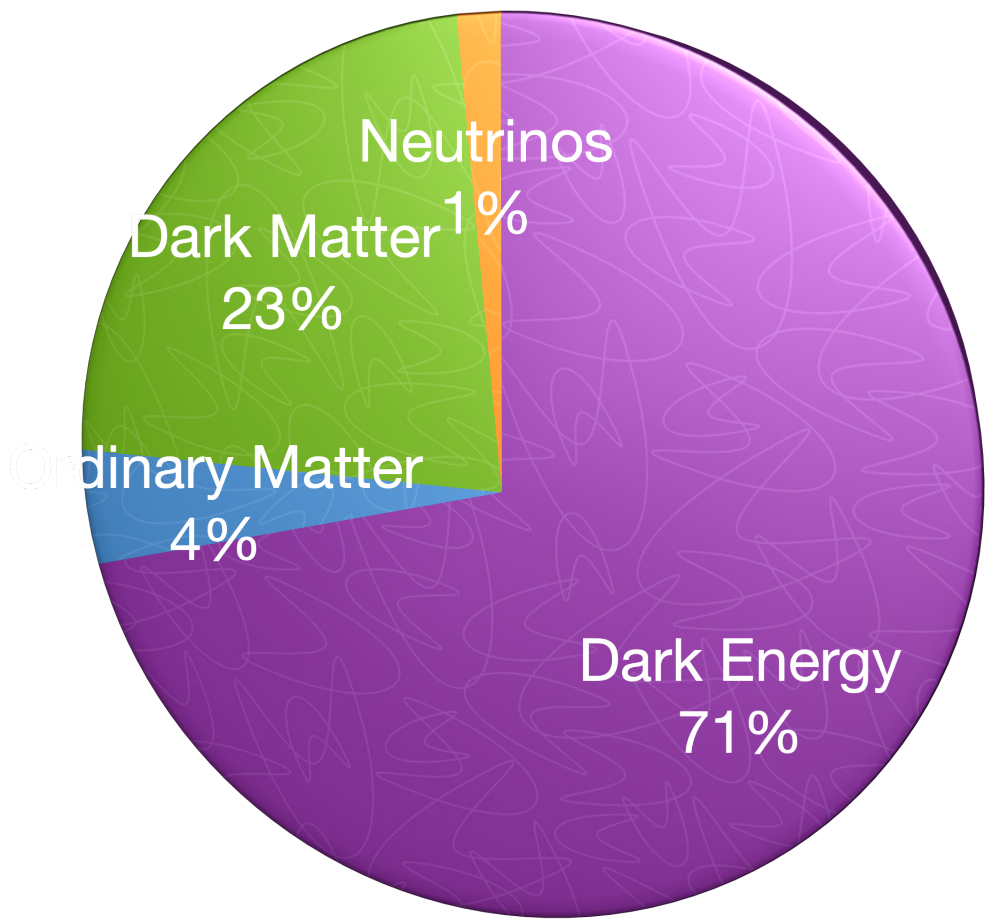
There are a bunch of cold neutrinos permeating the Universe. How do they contribute to the Universe's energy density?

For one flavour $(n(\nu + \bar{\nu}) \sim 110\,\mathrm{cm}^{-3})$

$$\rho_{\mathrm{crit}} = h^2\ 10.54\ \mathrm{keV\,cm}^{-3}$$

In units of critical density

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\mathrm{cr}}} h^2 = \frac{n_\nu m_\nu}{\rho_{\mathrm{cr}}} h^2 \approx \sum_{\mathrm{flavors}} \frac{m_\nu}{94.0\mathrm{eV}}$$



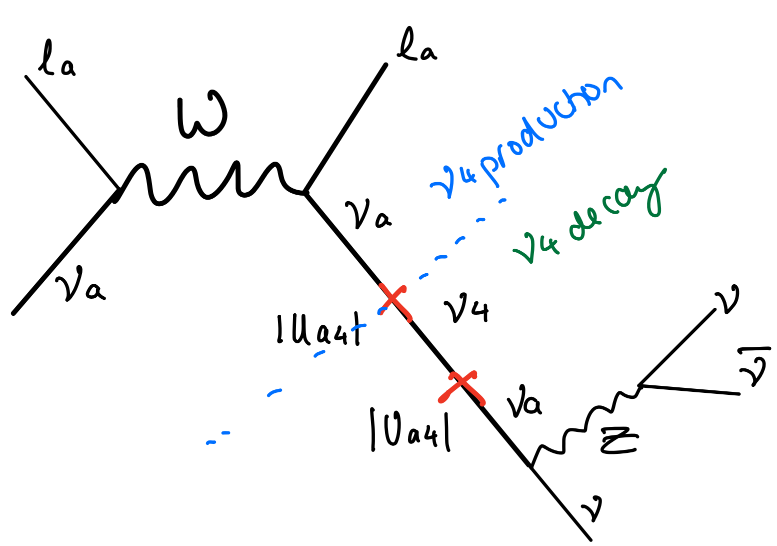
Sterile Neutrinos as DM

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

ν_s does not have Standard Model interactions

ν_4 does have Standard Model interactions because it mixes with the active neutrinos

$$|\nu_4\rangle = \underbrace{U_{s_4}}_{\sim 1} |\nu_s\rangle + U_{a_4} |\nu_a\rangle \quad a = e, \mu, \tau$$



decay rate of $\nu_4 \sim |U_{a4}|^2 G_F^2 |m_4|^5$

for ν_4 to be cosmologically stable and hence DM, $|m_4| \sim$ keV

Sterile Neutrinos as DM

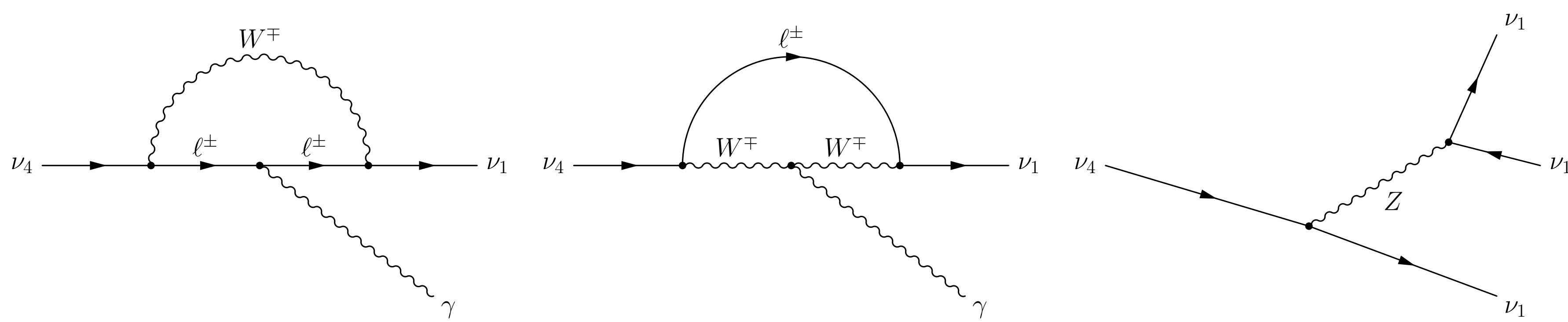
Early in the Universe sterile neutrinos do no exist but are populated via their mixing with the active neutrinos which are produced via weak interactions

After many collisions the sterile neutrino population increases to the abundance of DM we observe today. The collisions which create the sterile stop (fall out of thermal equilibrium) and at this time the sterile abundance “freezes-in”

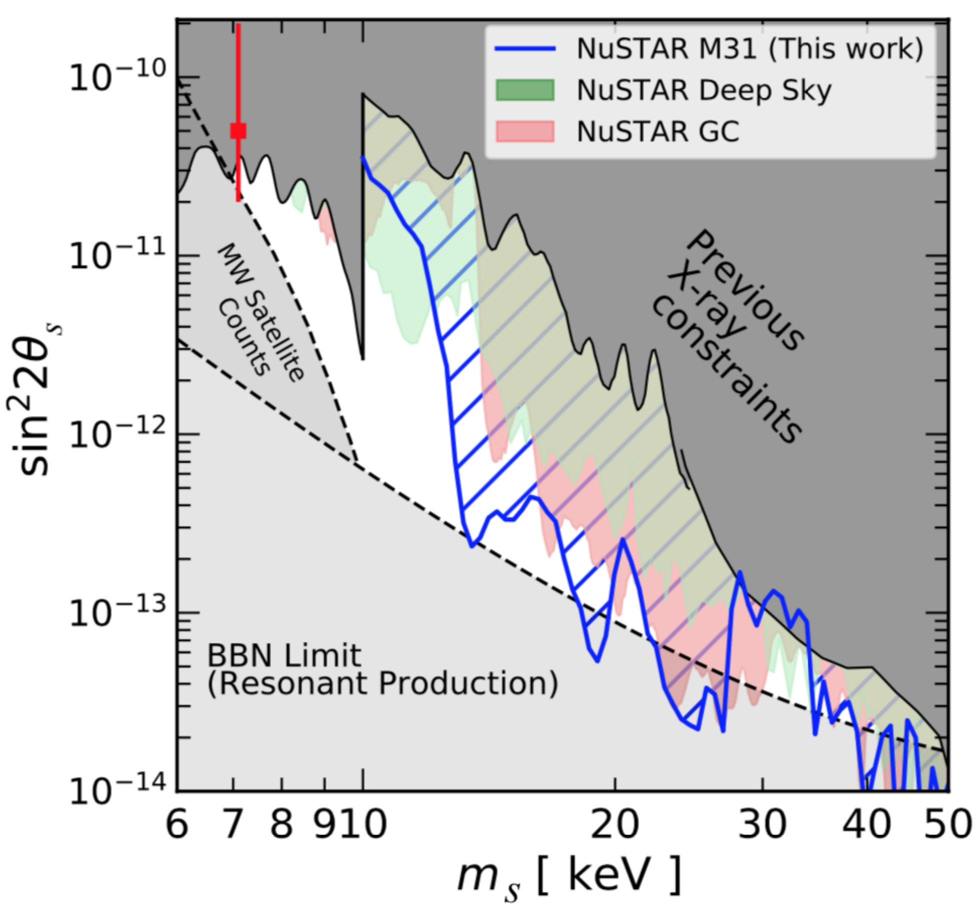
The sterile will decay but on a very long time scale i.e. they can be stable on cosmological timescales.

This requires the sterile mass to be around the **keV scale.**

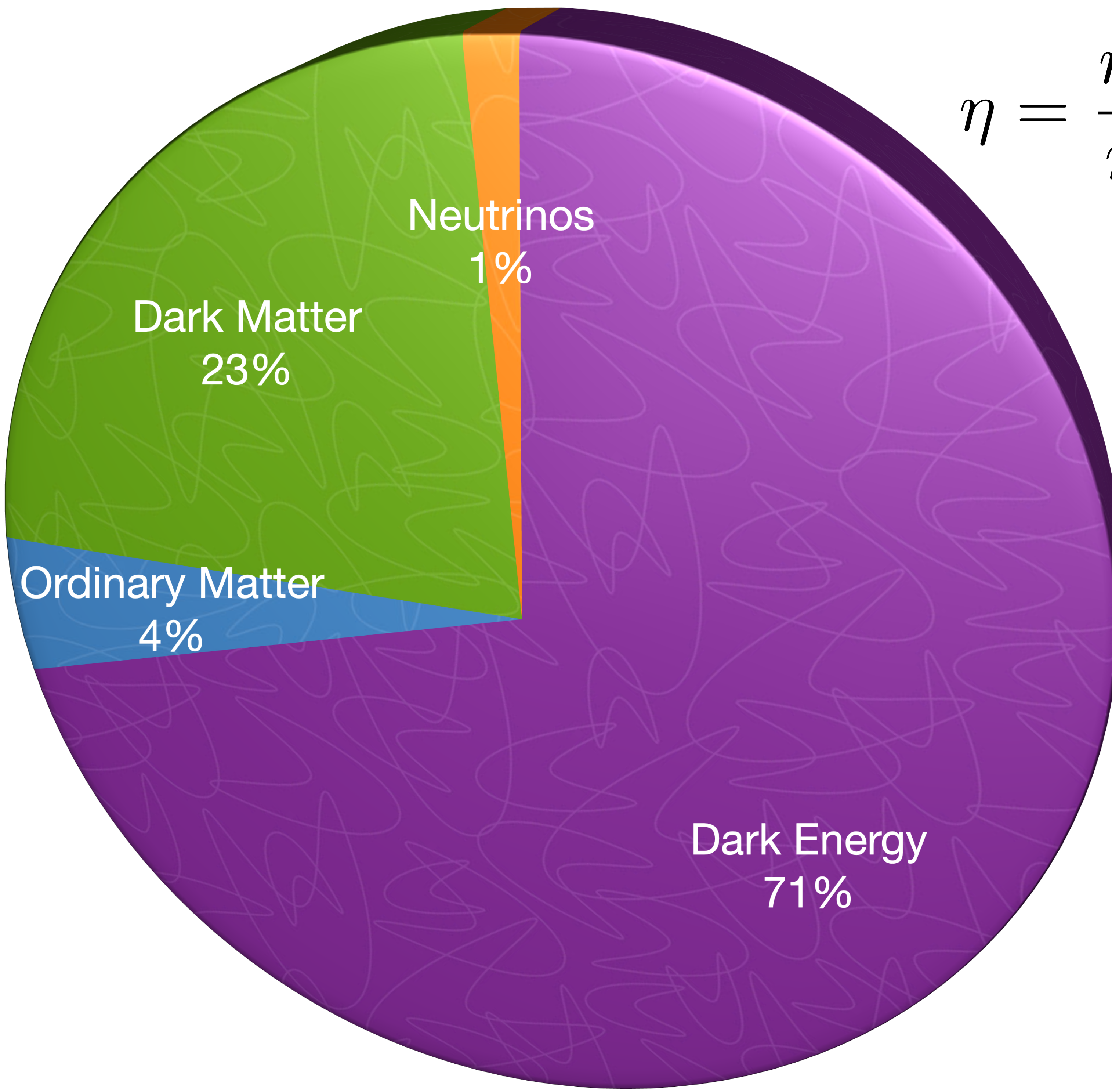
The parameter space for a sterile neutrino as DM is very limited. While they are cosmologically stable they are not absolutely stable and can decay to photons



To enhance their production a lepton asymmetry is needed to cause resonant mixing (see last slide). However, BBN constraints place limits on a preexisting lepton asymmetry



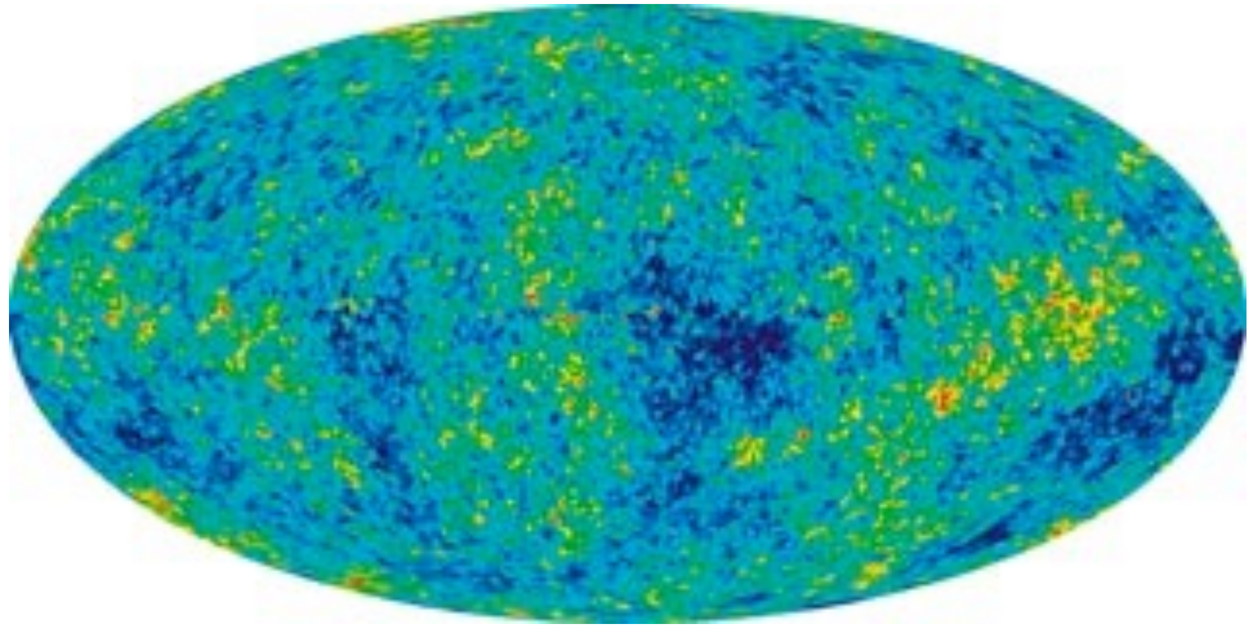
Universe's Energy Budget

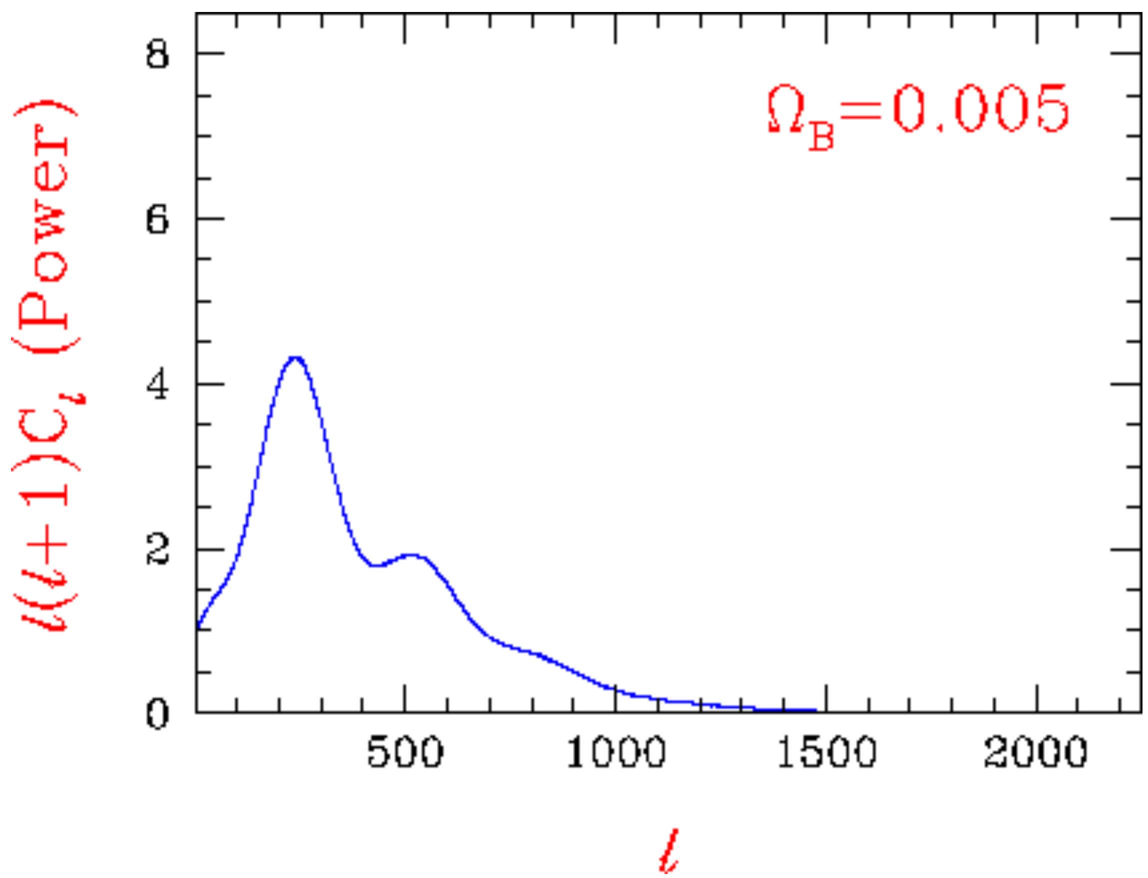


$$\eta = \frac{n_B}{n_\gamma} \sim 6 \times 10^{-10}$$

Cosmic Microwave Background

$T \sim 0.26\,\mathrm{eV} \approx 3000\,\mathrm{K}$



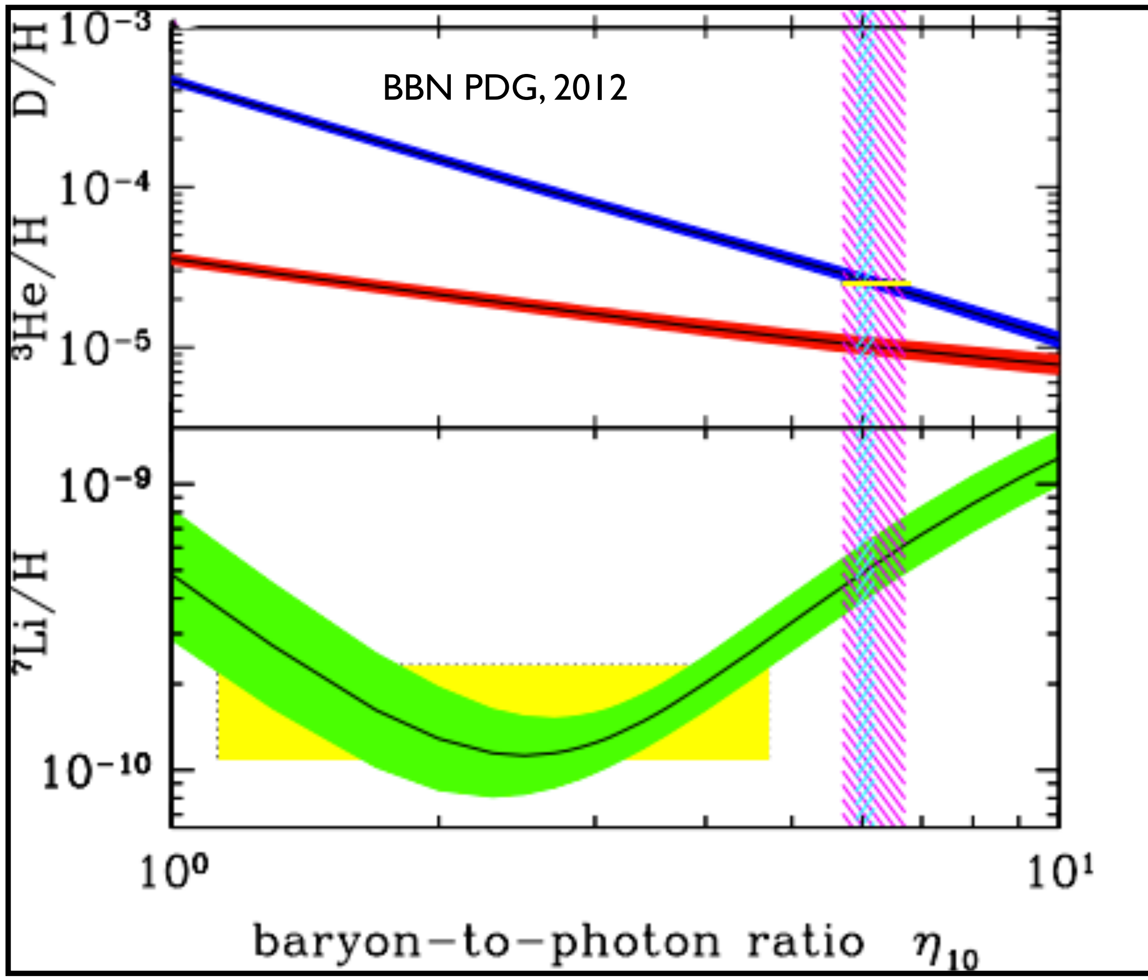


Wayne Hu's website

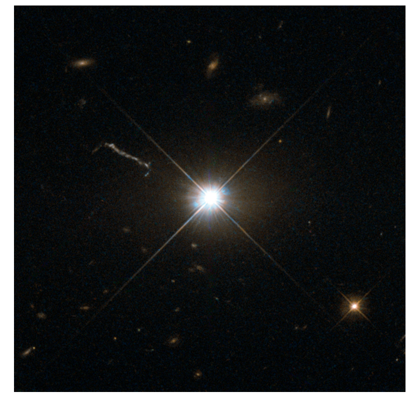
$$\eta_{\mathrm{CMB}} = (6.23 \pm 0.17) \times 10^{-10}$$

Big Bang Nucleosynthesis

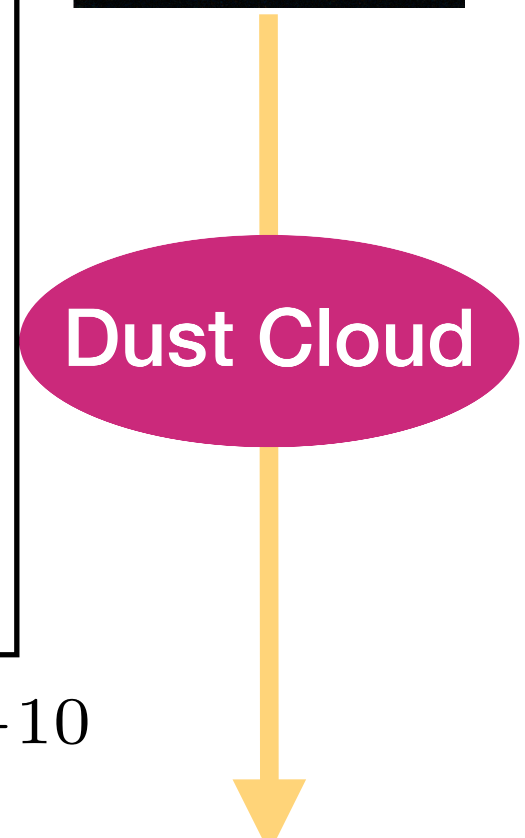
$$T \sim 1\,\mathrm{MeV} \approx 10^9\,\mathrm{K}$$



$$\eta_{\mathrm{BBN}} = (6.08 \pm 0.06) \times 10^{-10}$$



bright quasar 3C 273:
Hubble Space
Telescope



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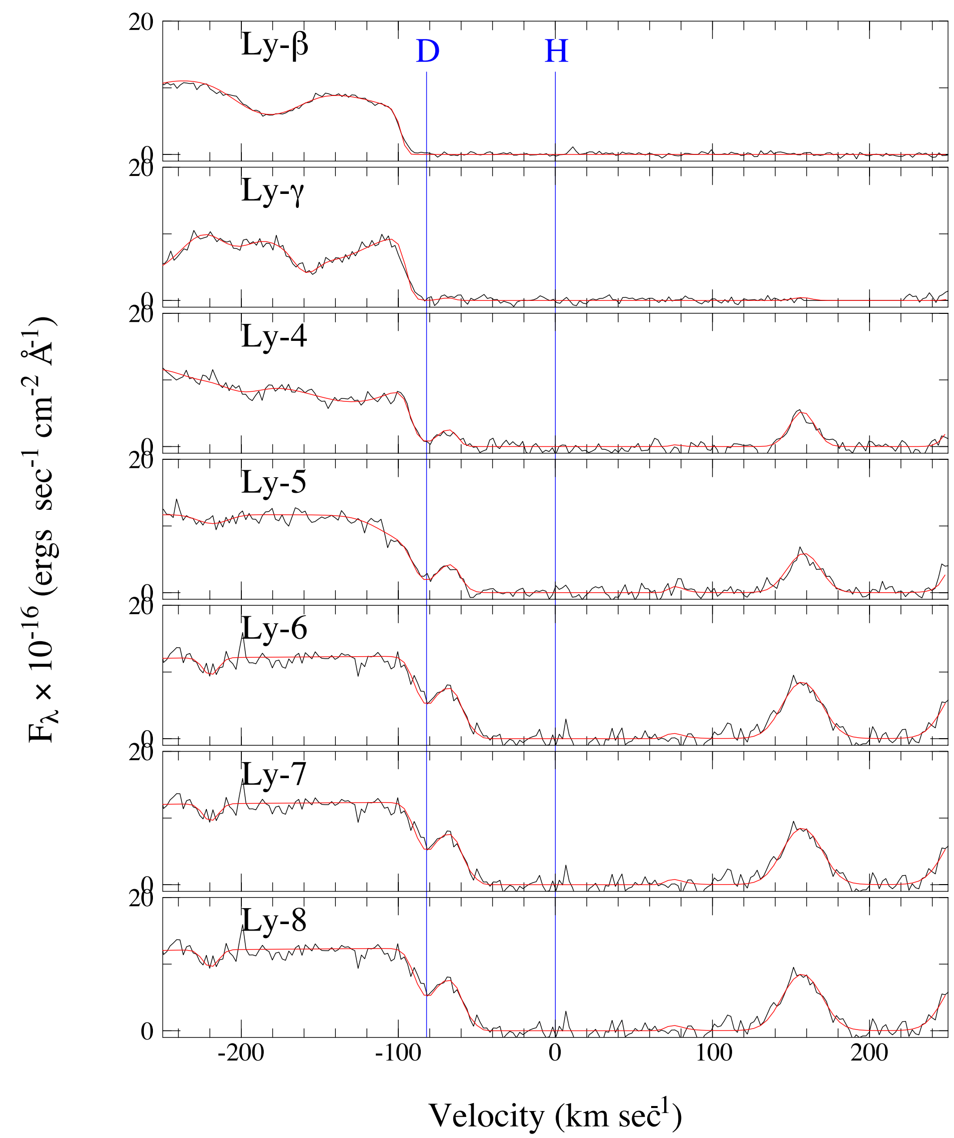
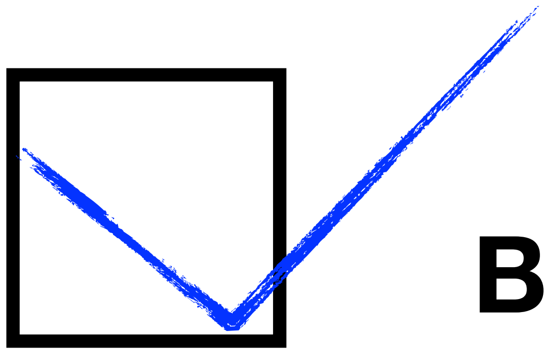
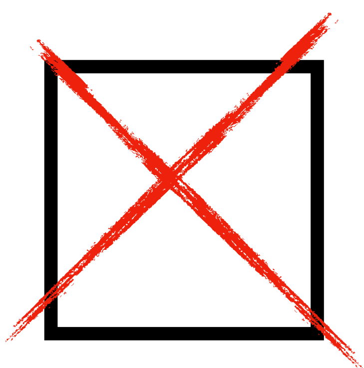


Sakharov's Conditions



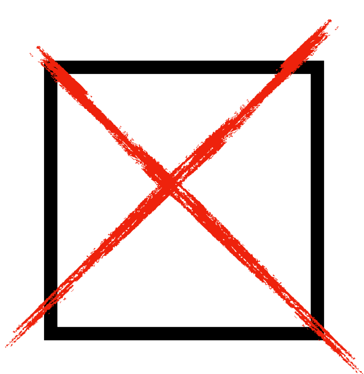
Baryon and Lepton Number Violation

Kuzmin, Rubakov and
Shaposhnikov



Insufficient CP-violation

Gavela, Hernandez, Orloff,
Pene; Huet and Sather



No departure from thermal equilibrium

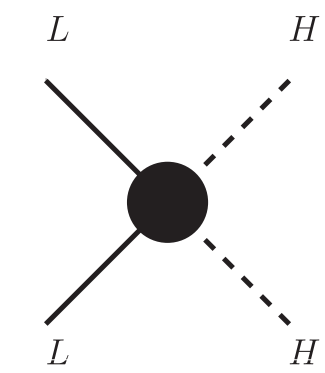
Kajantie, Laine, Rummukainen,
Shaposhnikov

* assumes CPT conserved

SU2L invariant term mass term for neutrinos

Weinberg

$$\mathcal{L}_{d=5} = \frac{\left(Y^T Y\right)_{\alpha\beta}}{\Lambda_{\rm NP}} \left(\overline{L_\alpha} H\right) \left(H^T L^C_\beta\right)$$



Need to form gauge invariant interaction to “complete” the Weinberg operator

$$2 \otimes 2 = 1 \oplus 3$$

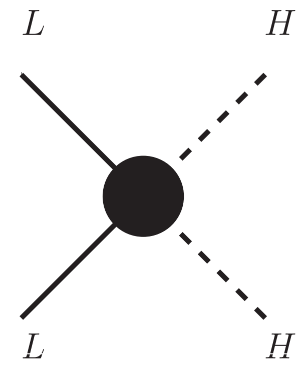
Any pair of fields from {L,H} can be singlet or triplet of SU2L:

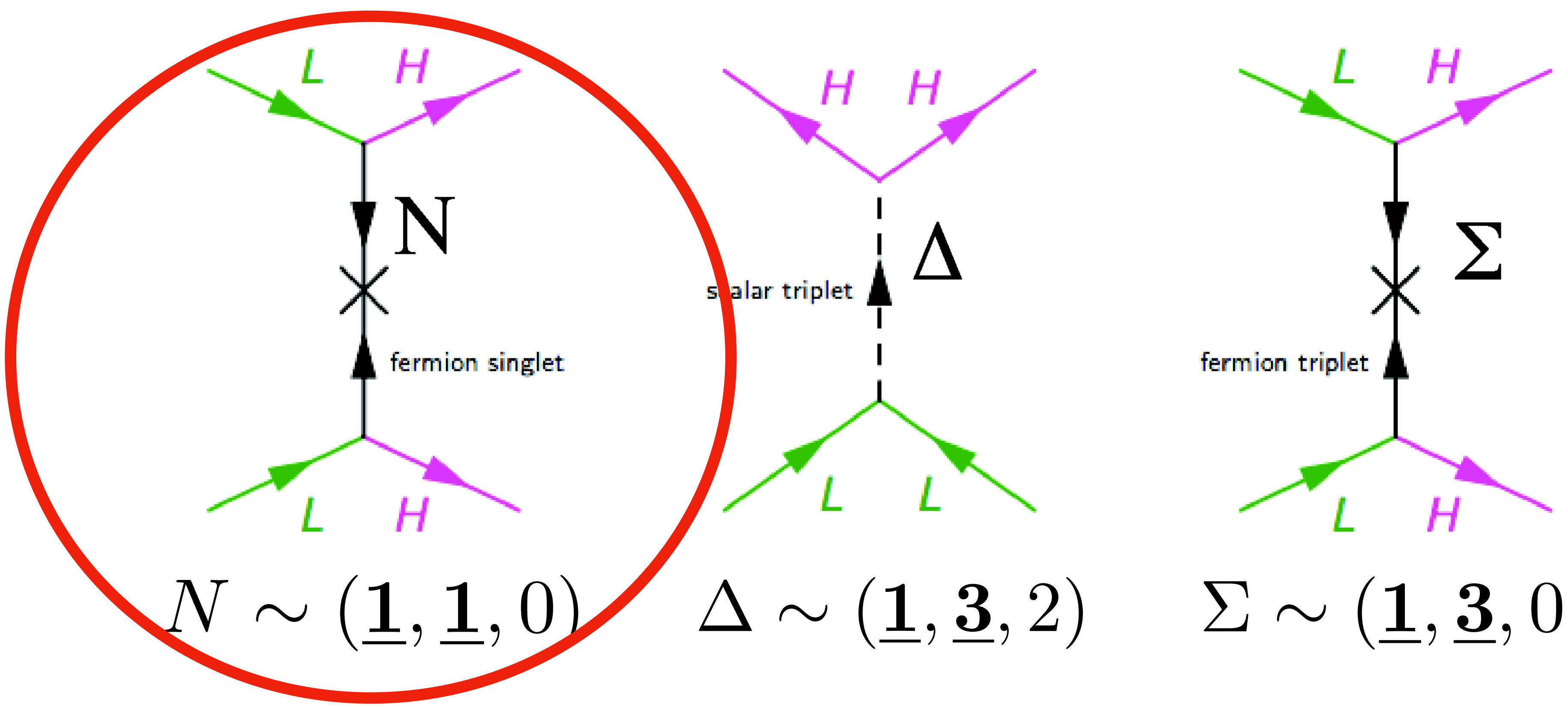
- Type 1: singlet fermion $N \sim (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)$
- Type 2: triplet scalar $\Delta \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 2)$
- Type 3: triplet fermion $\Sigma \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)$

SU2L invariant term mass term for neutrinos

Weinberg

$$\mathcal{L}_{d=5} = \frac{\left(Y^T Y\right)_{\alpha\beta}}{\Lambda_{\rm NP}} \left(\overline{L_\alpha} H\right)\left(H^T L^C_\beta\right)$$

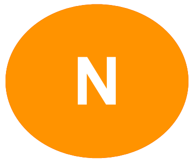




$N \sim (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)$ $\Delta \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 2)$ $\Sigma \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)$

Thermal leptogenesis

Fukugida, Yanagida



Thermal leptogenesis

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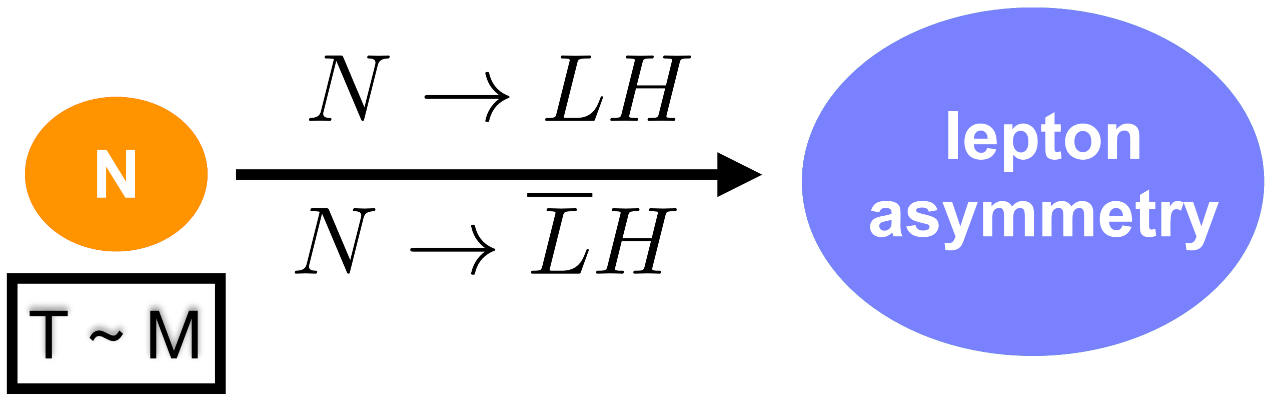
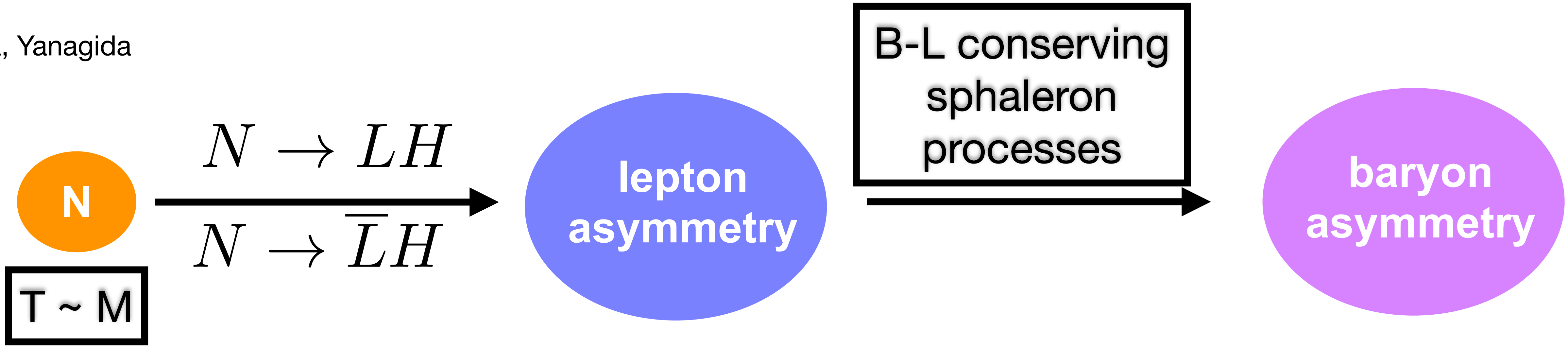


more anti-lepton than leptons
i.e. LN = -1

Thermal leptogenesis

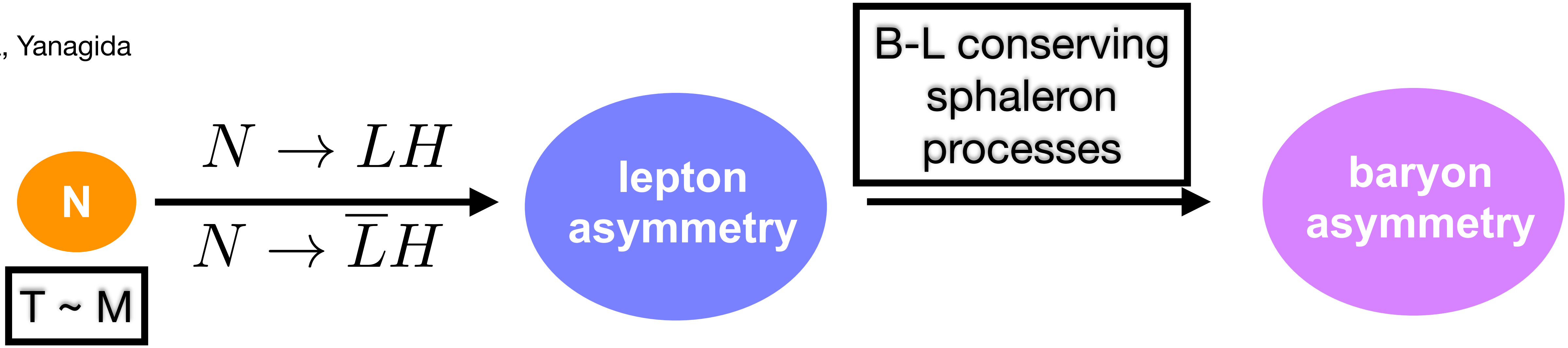
Fukugida, Yanagida



more anti-lepton than leptons
i.e. LN = -1 $\to$ B = + 1

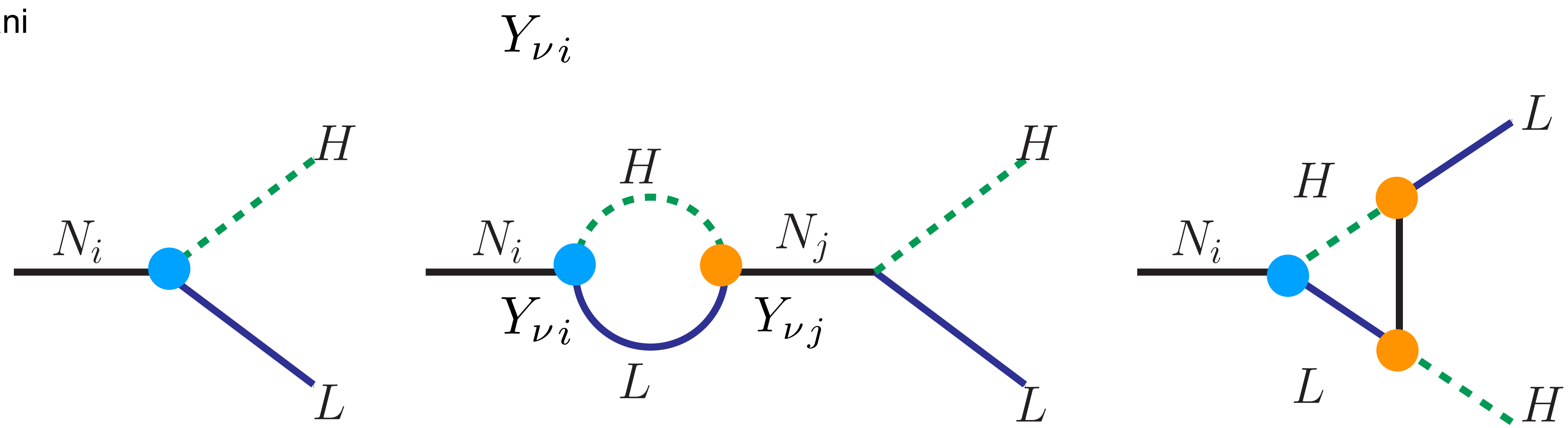
Thermal leptogenesis

Fukugida, Yanagida



Decay asymmetry from interference between tree and loop level diagrams

Covi, Roulet, Vissani

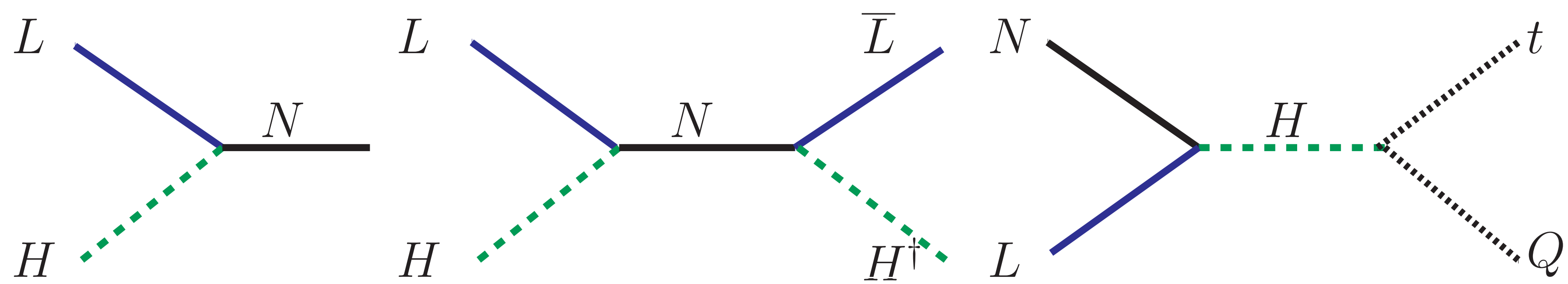


Decay Asymmetry

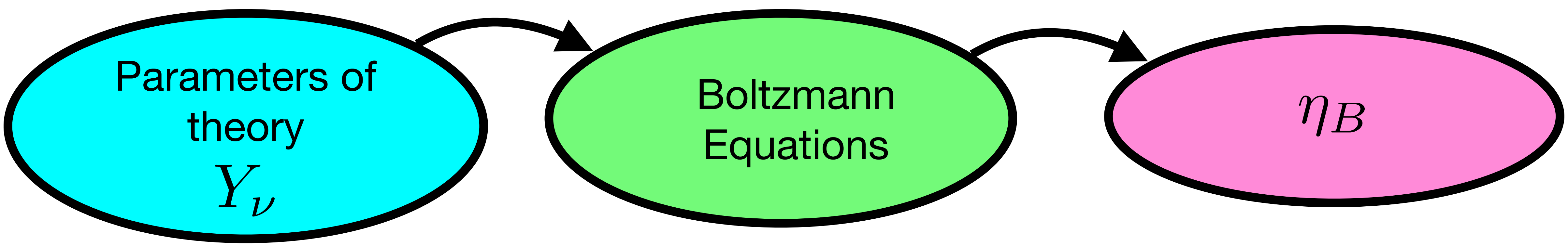
$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i}$$

Thermal leptogenesis

Washout and scattering processes

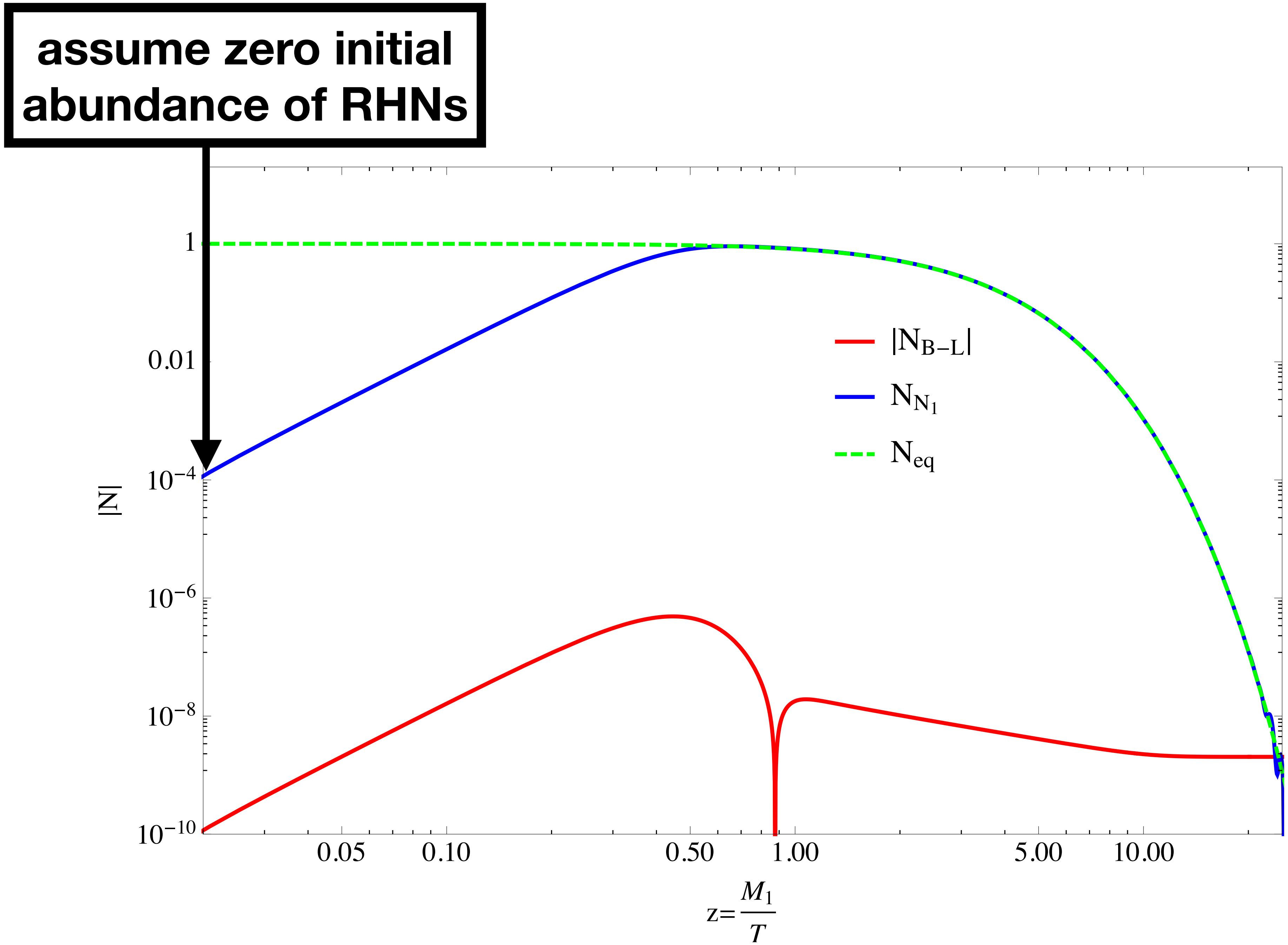


inverse Decays ΔL=2 ΔL=1

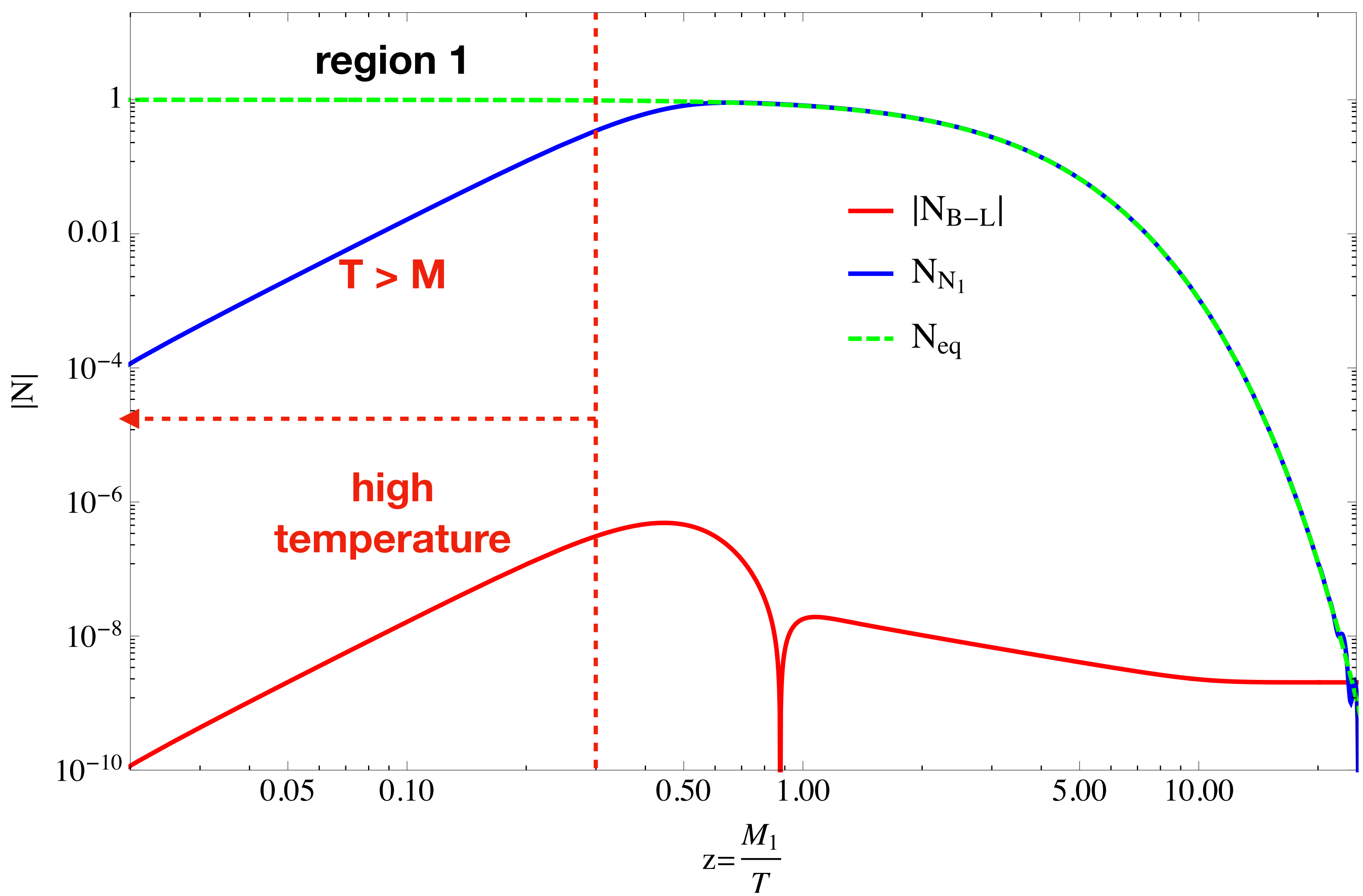


$$\frac{dn_{N_i}}{dz} = -\, D_i(n_{N_i} - n_{N_i}^{\rm eq}),$$

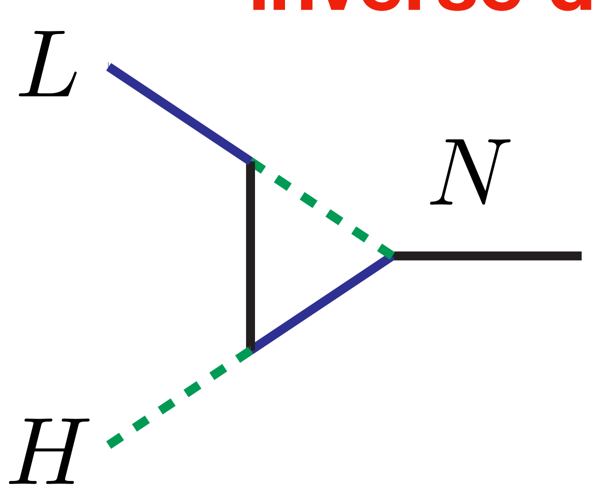
$$\frac{dn_{B-L}}{dz} = \sum_{i=1}^{3} \Big(\overbrace{\epsilon^{(i)} D_i(n_{N_i} - n_{N_i}^{\rm eq})}^{\textbf{source}} - \overbrace{W_i n_{B-L}}^{\textbf{sink}} \Big) \,.$$

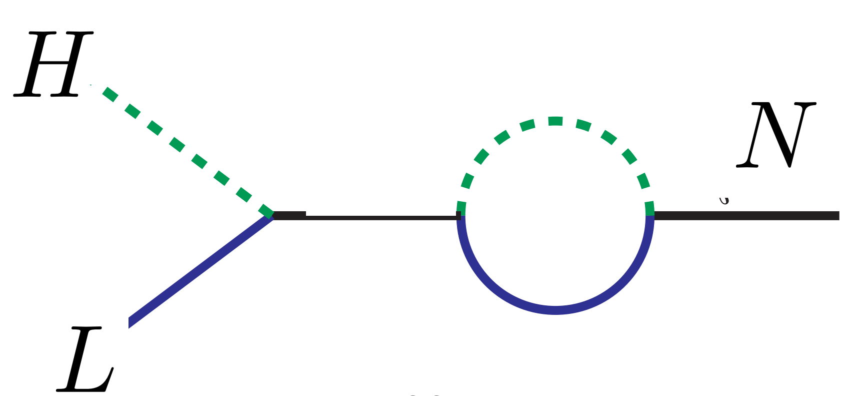


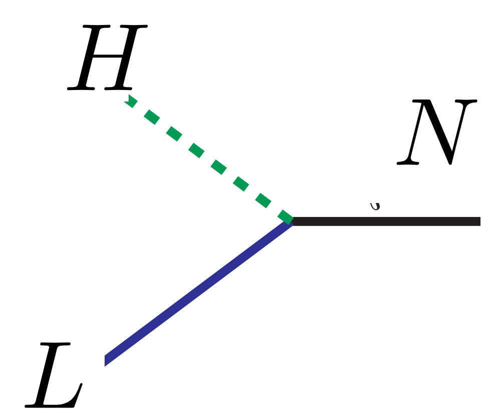
assume zero initial abundance of RHNs
$|N_{B-L}|$
N_{N_1}
N_{eq}
|N|
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0.05
0.10
0.50
1.00
5.00
10.00
$z=\frac{M_1}{T}$

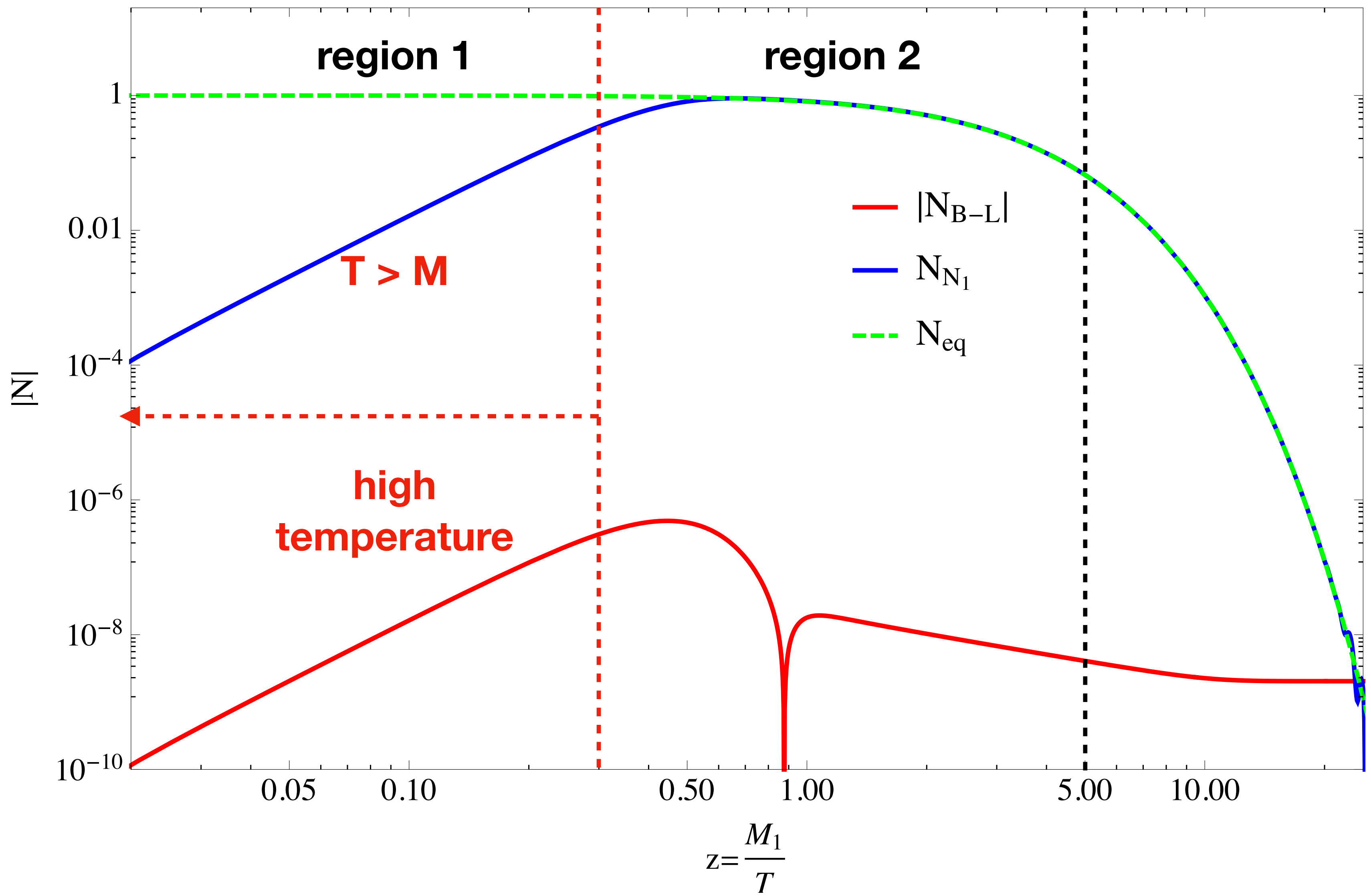


<u>Region 1</u>: leptons and Higgs have enough energy to inverse decay creating a lepton asymmetry

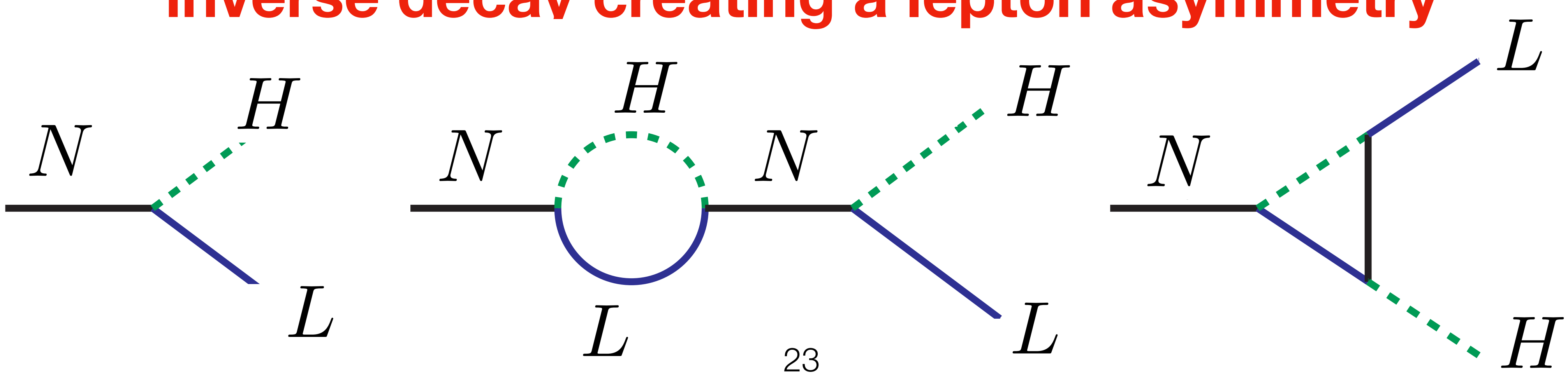


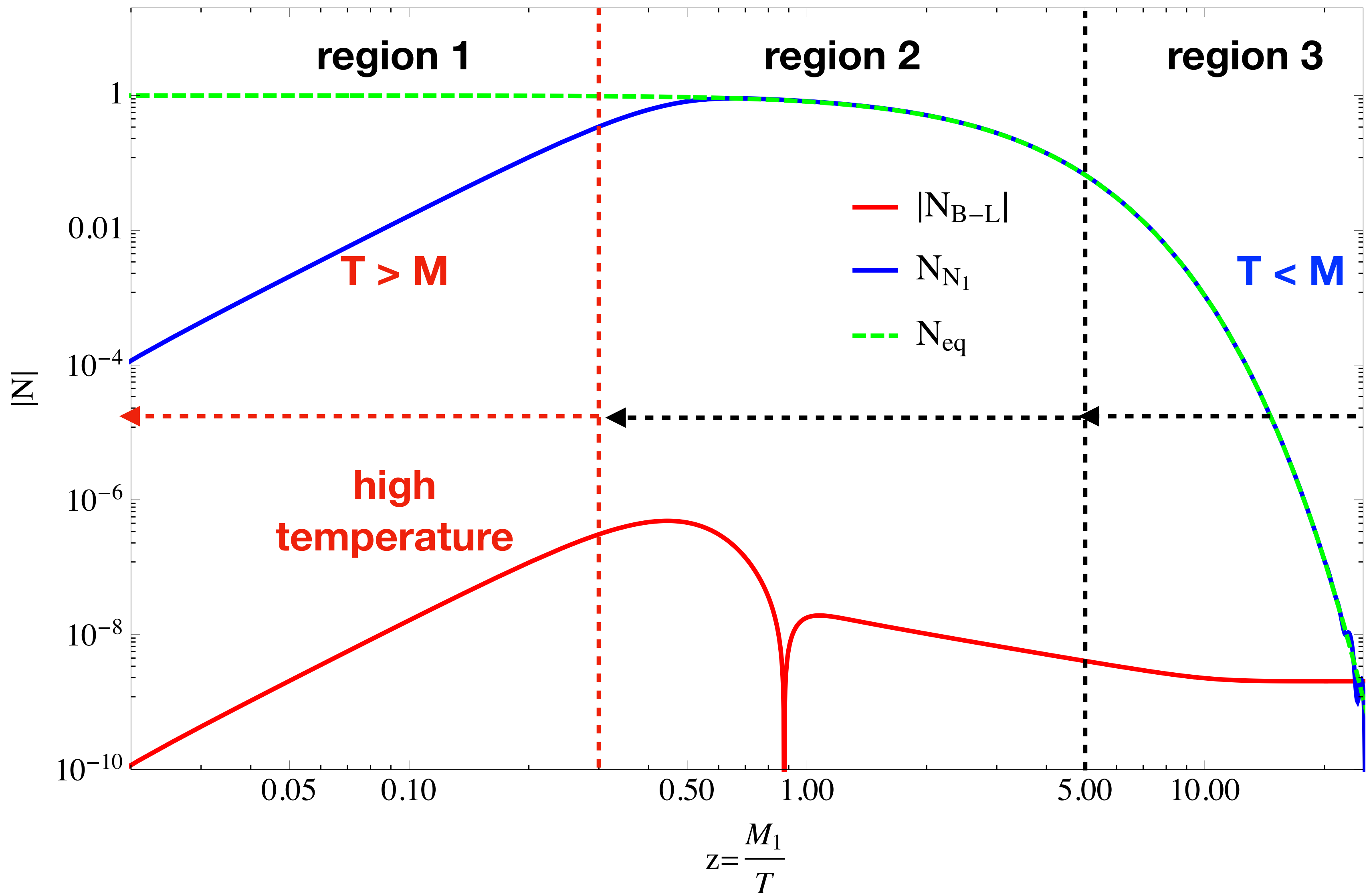






Region 2: leptons and Higgs have enough energy to inverse decay creating a lepton asymmetry





<u>Region 3:</u> At T < M, RHN abundance is depleted. Lepton asymmetry freezes out.

Parameter Space

Casas, Ibarra

$$Y_\nu = \frac{1}{v} U_{\mathrm{PMNS}} \sqrt{m} R^T \sqrt{M}$$

Parameter Space

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low-energy scale: 3 phases, 3 mixing angles and 3 masses

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low-energy scale: 3 phases, 3 mixing angles and 3 masses

high-energy scale: 3 phases, 3 mixing angles and 3 masses

Parameter Space

Casas, Ibarra

$$Y_\nu = \frac{1}{v} U_{\rm PMNS} \sqrt{m} R^T \sqrt{M}$$

low-energy scale: 3 phases, 3 mixing angles and 3 masses

high-energy scale: 3 phases, 3 mixing angles and 3 masses

Without any symmetry constraints 18 parameters in total.

Akmedov, Rubakov, Smirnov, Hernandez, Kekic, Lopez-Pavon, Racker, Salvado,Drewes,Garbcreht, Klaric, Gueter

Leptogenesis via oscillations

Flavour effects can lower the scale

Minimal Leptogenesis

~ eV ~ 0.1 GeV ~50 GeV ~10^5 GeV ~10^7 GeV ~10^{14} GeV

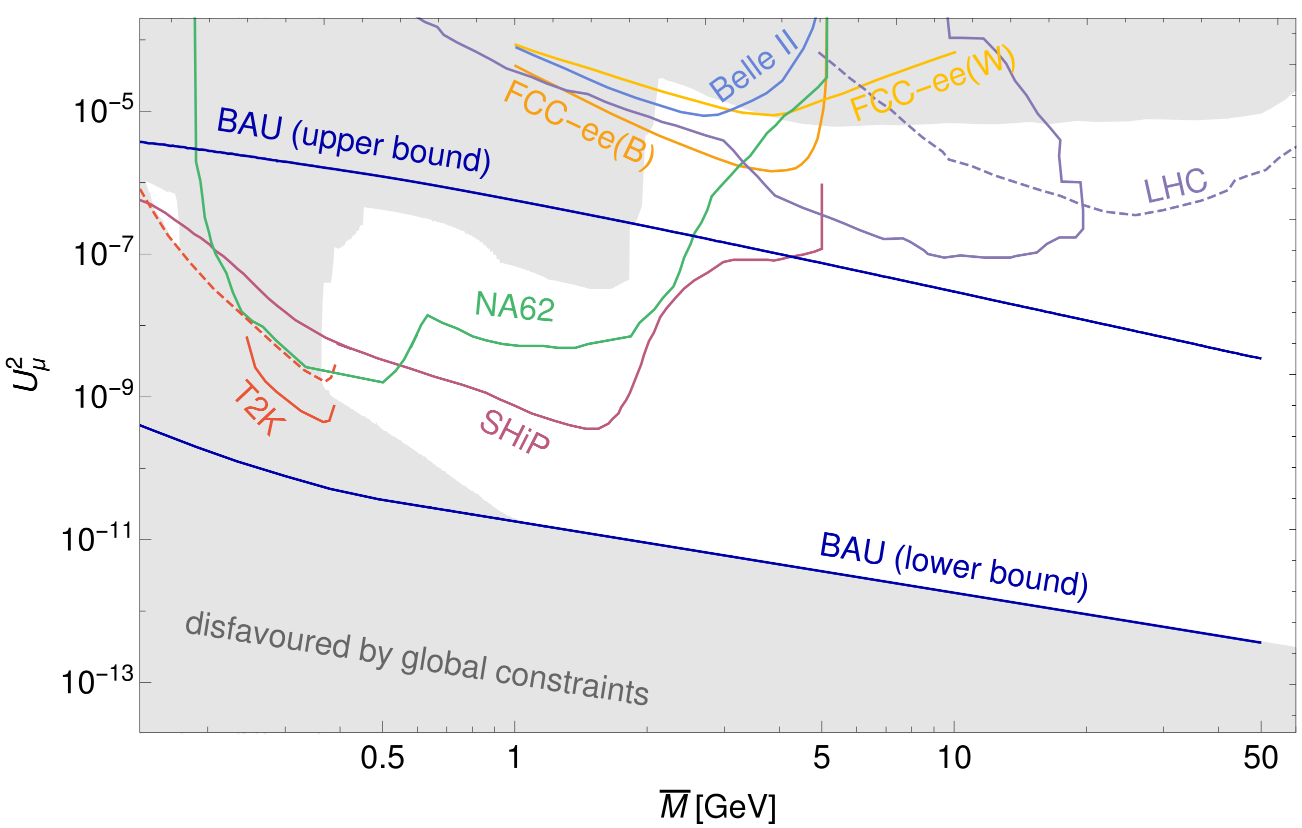
Pilaftsis, Underwood, Millington, Teresi

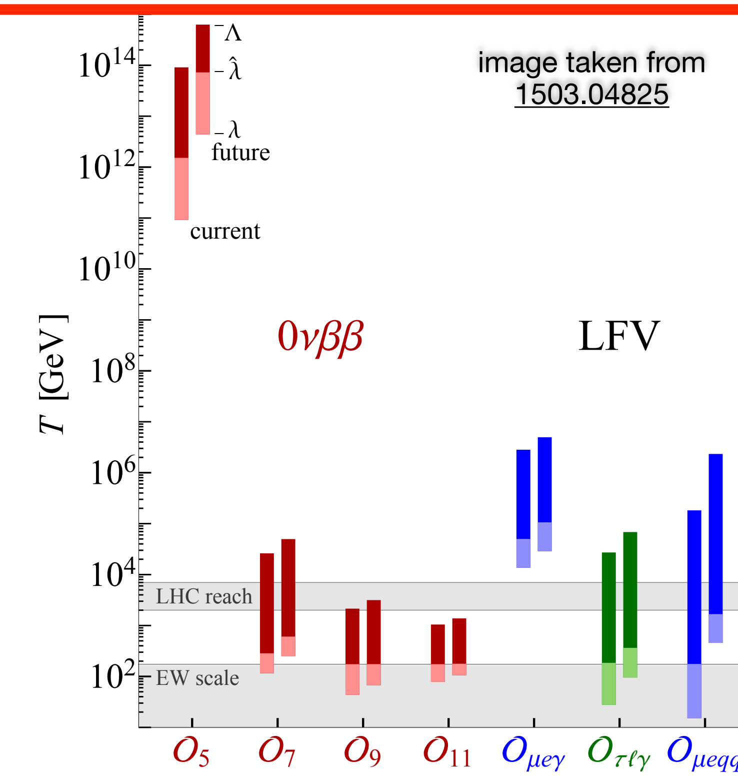
Resonant Leptogenesis

- **small neutrino masses $\Longleftrightarrow$ BAU**
- **minimal: 2RHN**
- **Easily embedded in GUT models**
- **falsifiable**
- **Can induce the EW scale**

- **small neutrino masses $\Longleftrightarrow$ BAU**
- **minimal: 2RHN**
- **neutrino data, NDBD, LFV and LNV, cosmology in meson decays, collider searches**

- **Scale too high can exacerbate Higgs fine tuning**
- **RHNs too heavy to produce**





ULYSSES: Universal LeptogeneSiS Equation Solver

- Thermal and resonant leptogenesis
- Easy parallelisation
- rapid evaluation
- python package

In collaboration with Granelli, Perez-Gonzalez, Moffat & Schulz. Happy for people to add their own plugins

1. Download it: https://github.com/earlyuniverse/ulysses

2. Look in “examples” folder. here are some points in the parameter space: e.g 1N1F.dat

```
m -100
M1 12
M2 13
M3 14
x1 180
y1 1.4
x2 180
y2 11.2
x3 180
y3 11
delta 217
a21 0
a31 0
t23 49.7
t12 33.82
t13 8.610000
```

lightest neutrino mass log10 (eV), here it is set to zero

M1 = 10^{12} GeV, M2 = 10^{13} GeV, M3 = 10^{14} GeV

all other values in degrees

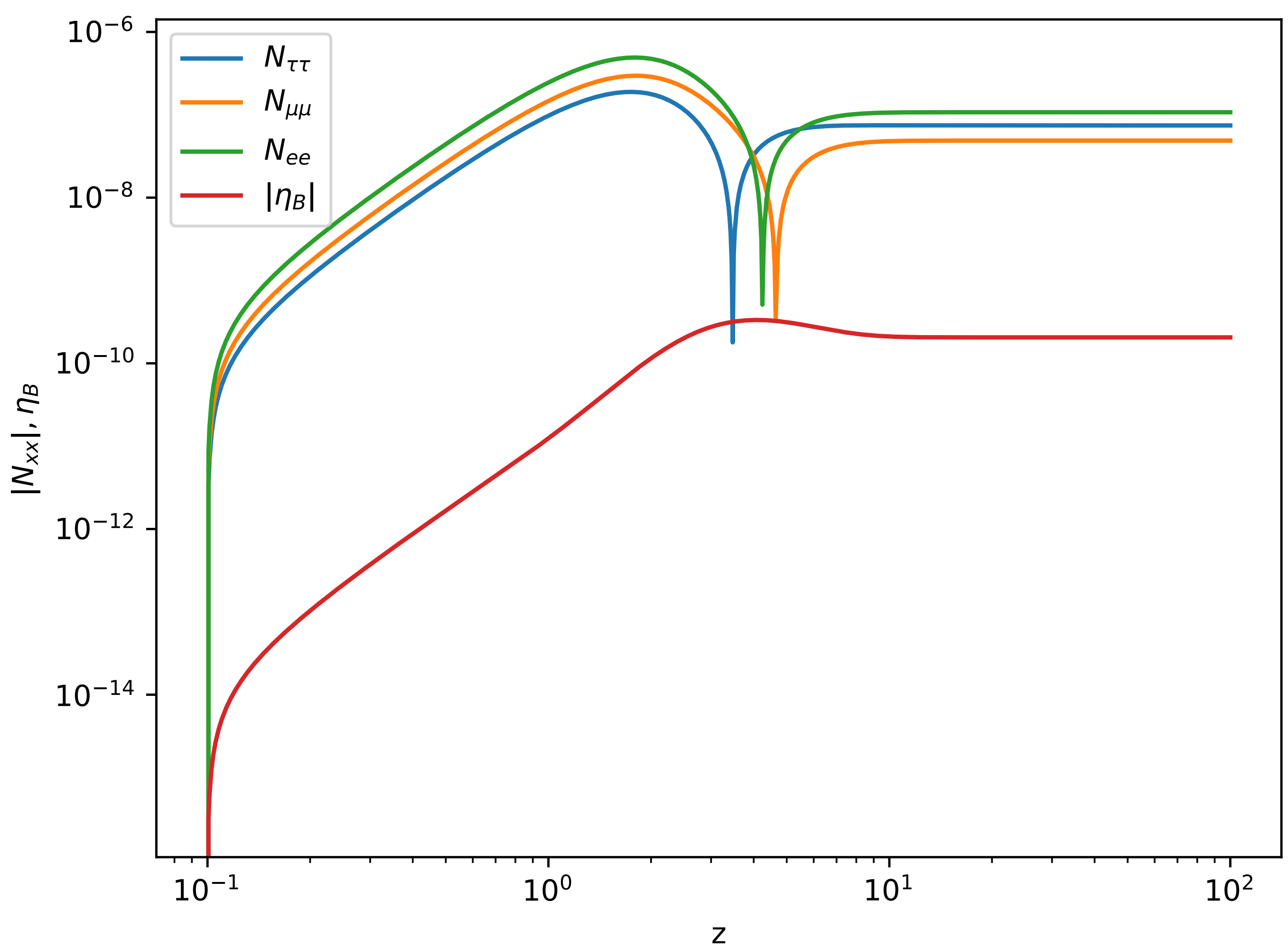
parameter point

```
MBP-1324:examples jessicaturner$ python3 /Users/jessicaturner/Documents/GitHub/ulysses/bin/uls-calc N1F.dat -m 1DME -o test.pdf
[[ 7.22073772e-04+0.00278381j  2.95858166e-02-0.00041085j
  -5.31823063e-02+0.03511161j]
 [-1.26872416e-04-0.00409255j  3.19281069e-02+0.0044666j
   3.09580991e-01-0.00271138j]
 [-1.07595633e-04-0.0070844j  -3.20590369e-02+0.00384186j
   2.69521108e-01+0.00226905j]]
eta_b          -2.0545625617095188e-10
Y_b            -2.9186475226447497e-11
Omega_b h^2    -0.007513698628395756
MBP-1324:examples jessicaturner$
```

yukawa matrix

"model": one RHN decaying including flavour effects

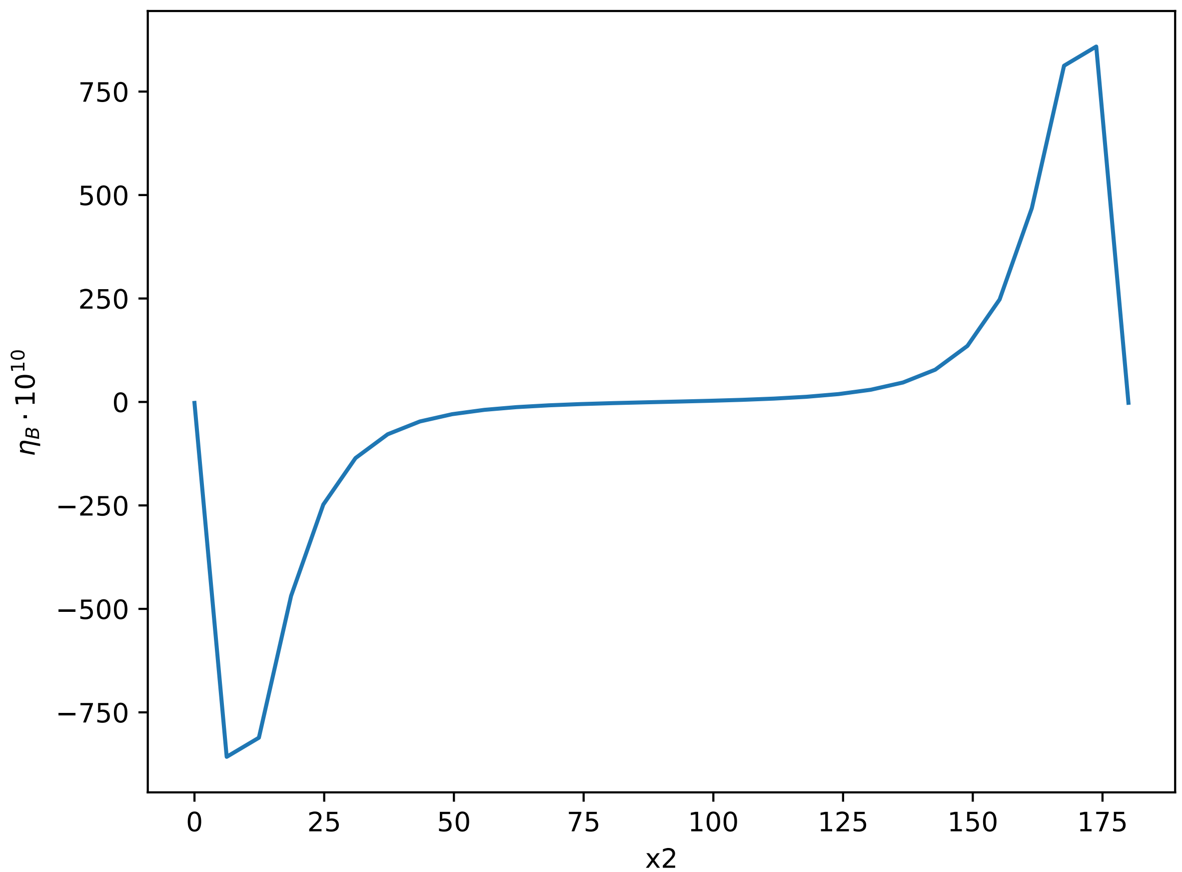
baryon-to-photon ratio



```
m -100
M1 12
M2 13
M3 14
x1 180
y1 1.4
x2 0 180
y2 11.2
x3 180
y3 11
delta 217
a21 0
a31 0
t23 49.7
t12 33.82
t13 8.610000
```

here we choose to scan in “x2” parameter

```
MBP-1324:examples jessicaturner$ python3 /Users/jessicaturner/Documents/GitHub/ulysses/bin/uls-scan 1N1F.dat -m 1DME -o test.pdf
Scanning x2 in [0.0,180.0] for 30 values
MBP-1324:examples jessicaturner$
```



Conclusions

- Thermal leptogenesis is a mechanism that simultaneously explains the smallness of neutrino masses and the excess of matter versus antimatter of our universe

- Leptogenesis assumes active neutrinos are their own anti-particles i.e. that neutrinos are **Majorana fermions.**

- Heavy right-handed neutrinos (RHNs) are introduced via a seesaw mechanism which satisfies Sakharov's three conditions and a lepton asymmetry is generated via the CP-violating and out-of-equilibrium decays of the RHNs.

- The lepton asymmetry is converted via weak sphalerons to a baryon asymmetry.

- Thermal leptogenesis can occur over range of RHN mass scales: 10^6 - 10^{14} GeV. Resonant leptogenesis and leptogenesis via oscillations require smaller RHN masses (TeV and GeV scale respectively).