Neutrino Physics

Neutrino masses and phenomenology

Jessica Turner

Neutrinoless double beta decay

In standard beta decay:

$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$

This arises from the weak decay of a bound d-quark:

$$d \rightarrow u + e^- + \overline{\nu_e}$$

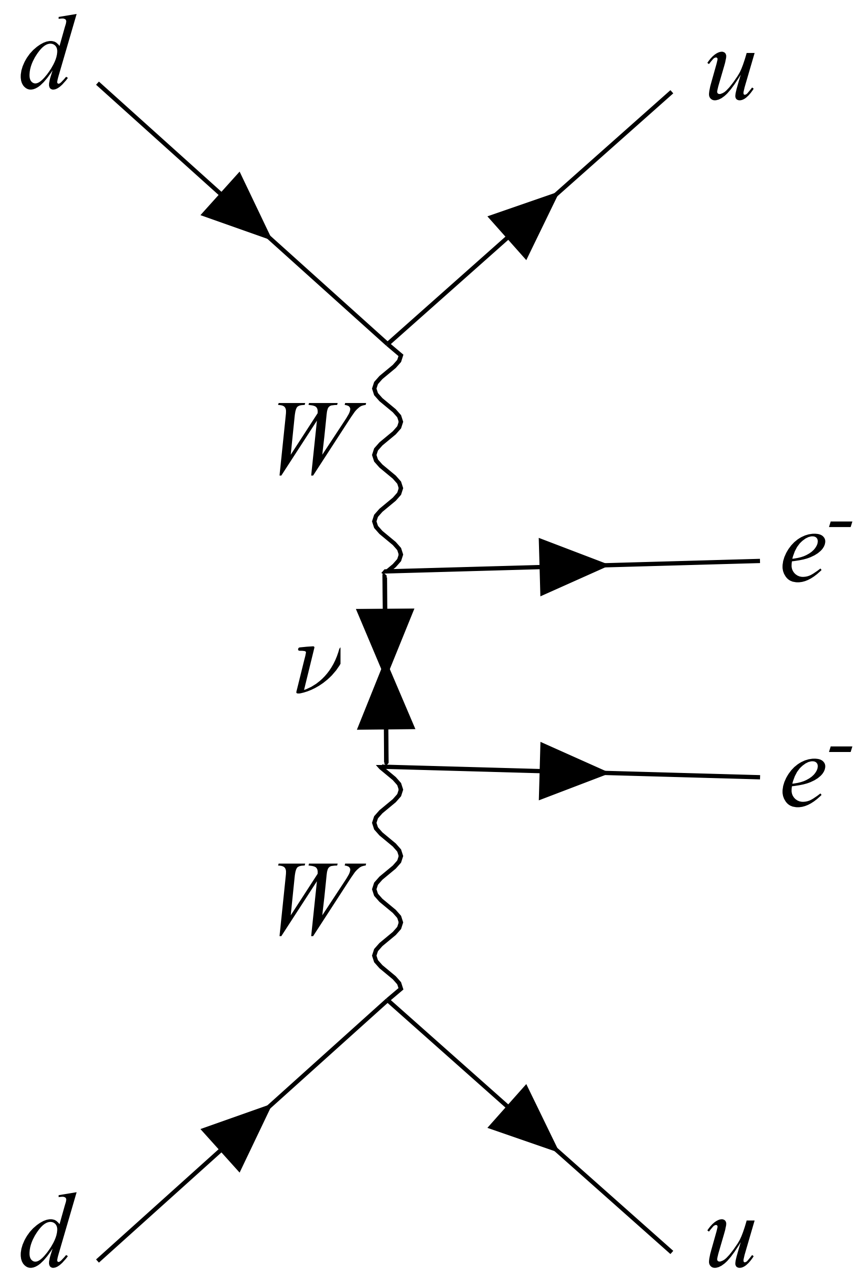
Double beta decay is far more rare (probabilistically need beta decay happen twice simultaneously)

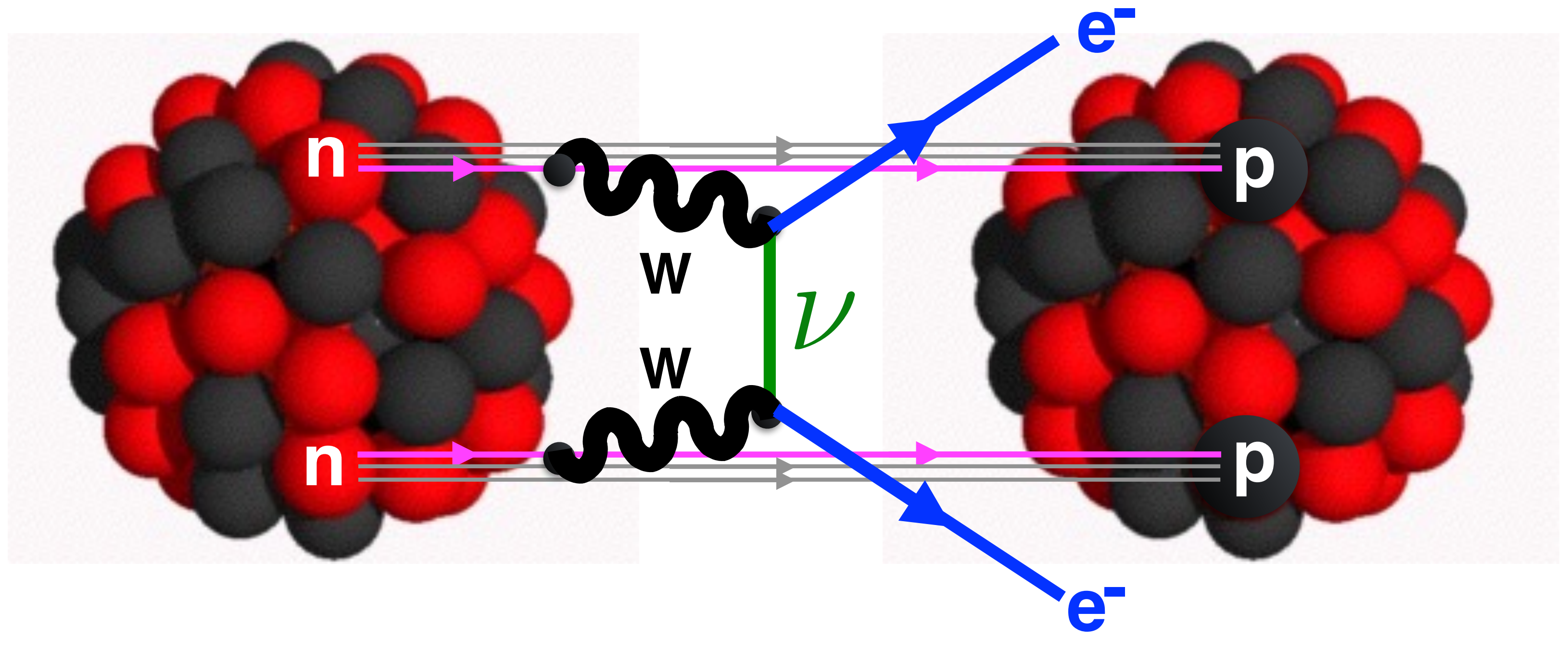
$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

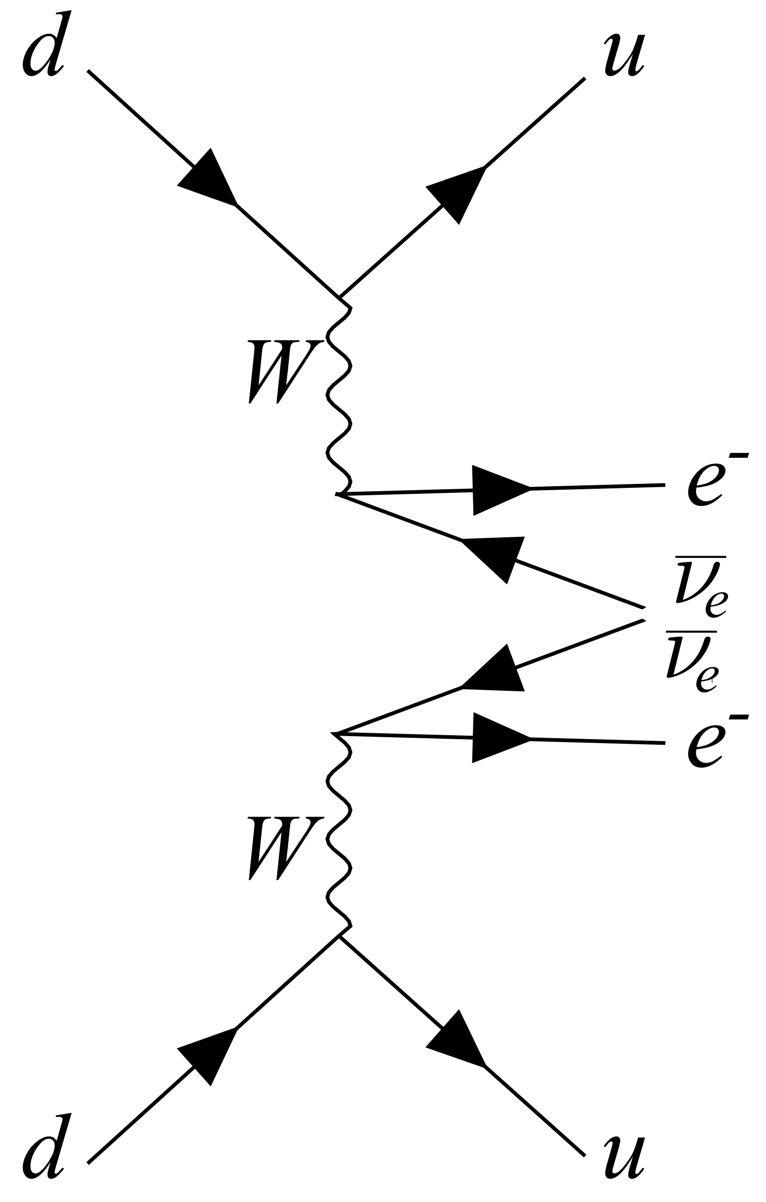
We have 0 leptons before (L=0) and we have 0 leptons after (-2+2=0). This clearly conserved lepton number

Neutrinoless double beta decay

Neutrinoless double beta decay, (A, Z) → (A, Z+2) + 2 e, will test the nature of neutrinos.





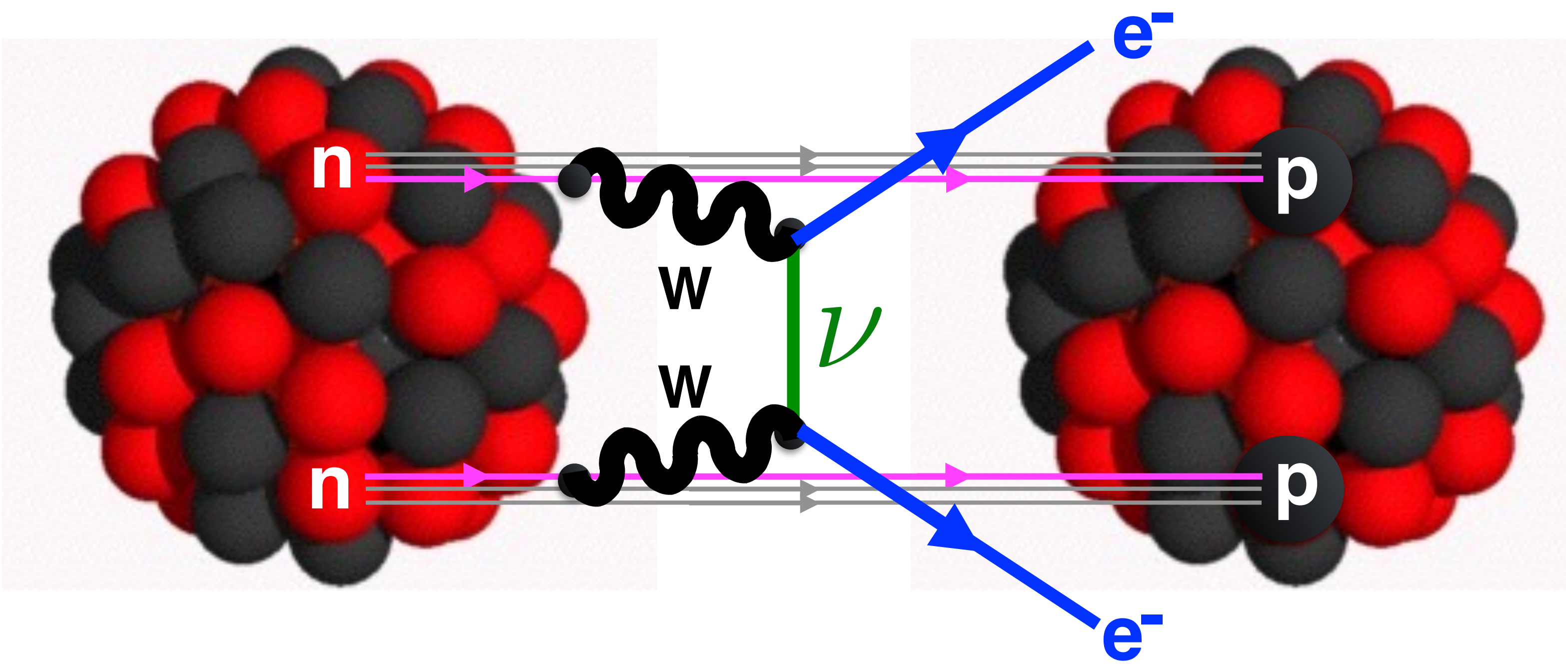


NDBD lepton number violating

double beta decay, lepton number conserving

Massive Majorana neutrinos mediate neutrino less double beta decay which violates lepton number by two units (L=0 before, L=2 after)

Decay Rate



$$\Gamma_{0\nu\beta\beta} = GM\left|m_{\beta\beta}\right|^2$$

$$\begin{aligned} G &= \text{ phase space factor} \\ M &= \text{ nuclear matrix element} \\ m_{\beta\beta} &= \text{ effective majorana mass} \end{aligned}$$

via the effective Majorana mass parameter:

$$\begin{aligned} |m_{\beta\beta}| &= \left|\sum_{i=1}^{3} m_i U_{ei}^2\right| \\ &= \left|m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31}-2\delta)}\right| \end{aligned}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\frac{\alpha_{21}}{2}} & s_{13}e^{i\left(\frac{\alpha_{31}}{2}-\delta\right)} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}e^{i\alpha_{21}/2} - s_{12}s_{23}s_{13}e^{i\delta}e^{i\frac{\alpha_{21}}{2}} & s_{23}c_{13}e^{-i\delta}e^{i\frac{\alpha_{31}}{2}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}e^{i\frac{\alpha_{21}}{2}} - s_{12}c_{23}s_{13}e^{i\delta}e^{i\frac{\alpha_{21}}{2}} & c_{23}c_{13}e^{-i\delta}e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$\boxed{\begin{aligned} m_1 &\simeq 0 & |U_{e1}| &= \cos\theta_{13}\cos\theta_{12} \sim 0.84 \\ m_2 &\simeq \sqrt{\Delta m^2_{21}} & |U_{e2}| &= \cos\theta_{13}\sin\theta_{12} \sim 0.52 \\ m_3 &\simeq \sqrt{\Delta m^2_{31}} & |U_{e3}| &= \sin\theta_{13} \sim 0.1 \end{aligned}}$$

$$|m_{\beta\beta}| = \left| m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31}-2\delta)} \right|$$

- NO ($m_1 \ll m_2 \ll m_3$): $|\langle m_{\beta\beta}\rangle| \sim 1 - 5\,\mathrm{meV}$

$$|m_{\beta\beta}| \simeq \left| \sqrt{\Delta m^2_{21}} \cos^2\theta_{13}\sin^2\theta_{12} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} e^{i(\alpha_{32}-2\delta)} \right|$$

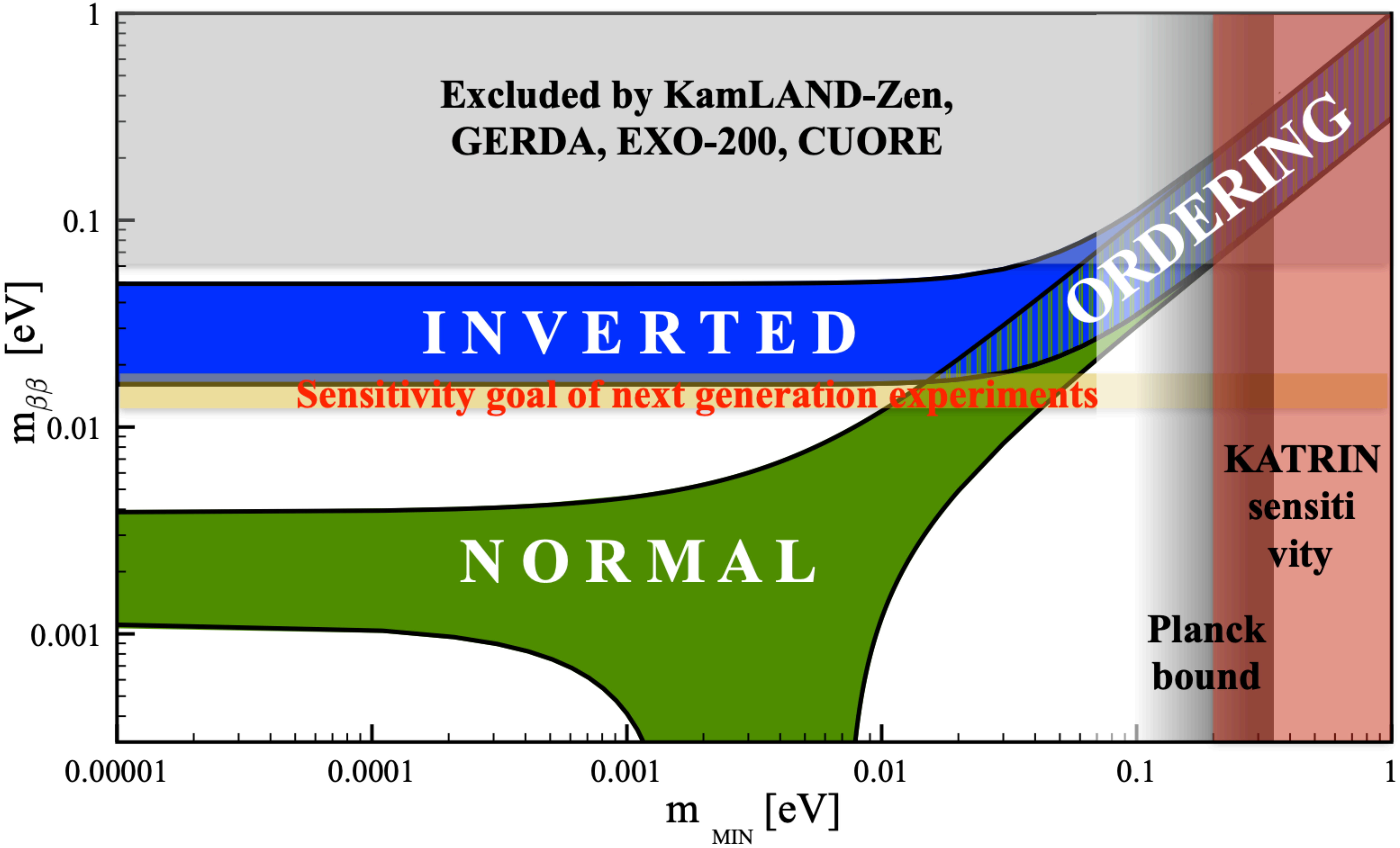
- IH ($m_3 \ll m_1 \sim m_2$): $15\,\mathrm{meV} \lesssim |\langle m_{\beta\beta}\rangle| \lesssim 50\,\mathrm{meV}$

$$\sqrt{\Delta m^2_{31}} \cos 2\theta_{12} \leq |m_{\beta\beta}| \simeq \sqrt{\left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{21}}{2}\right) \Delta m^2_{31}} \leq \sqrt{\Delta m^2_{31}}$$

- QD ($m_1 \sim m_2 \sim m_3$): $44\,\mathrm{meV} \lesssim |\langle m_{\beta\beta}\rangle| \lesssim m_1$

$$|m_{\beta\beta}| \simeq m_0 \left| \left(\cos^2\theta_{12} + \sin^2\theta_{12} e^{i\alpha_{21}}\right) \cos^2\theta_{13} + \sin^2\theta_{13} e^{i\alpha_{31}} \right|$$

Neutrinoless double beta decay



from Silvia Pascoli's lecture notes

Neutrino Masses - Dirac Mass

Introduce a RHN (N) into the SM particle and **assume lepton number is conserved**

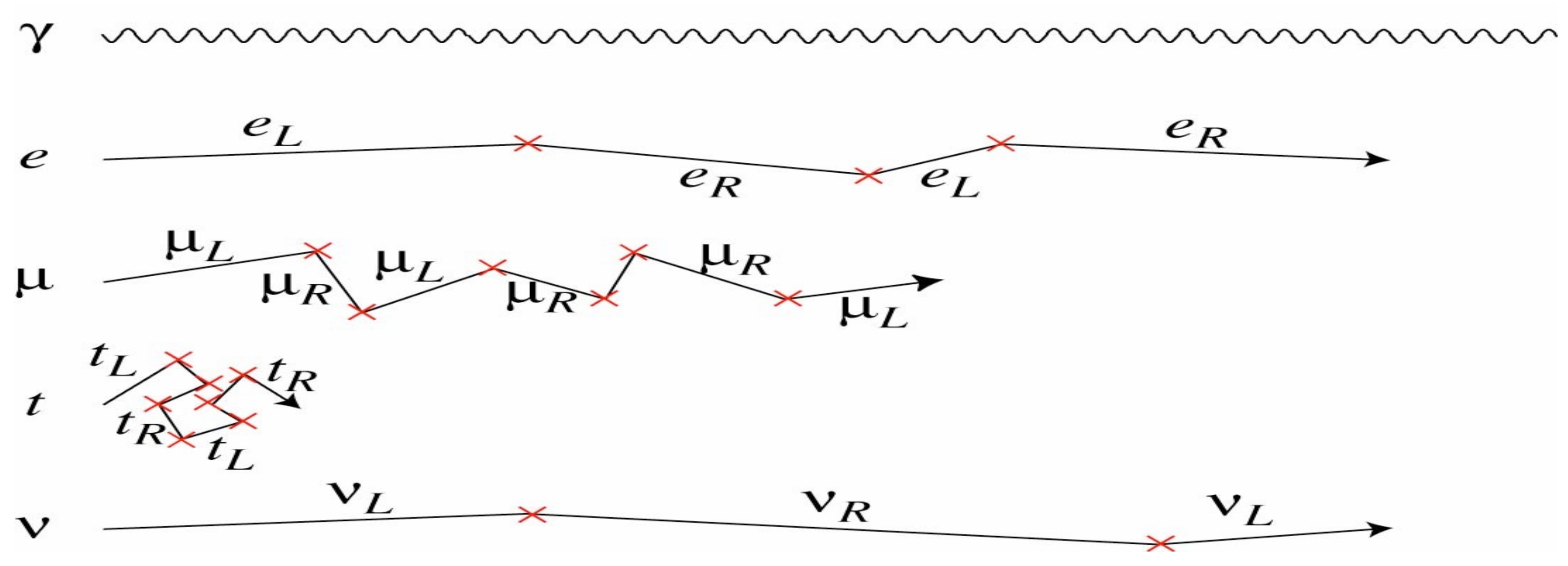
We find neutrinos are Dirac fermions. This term is $SU(2)_L$ invariant

$$\mathcal{L} \supset Y_\nu \overline{L_L} \tilde{H}^\dagger N + \text{ h.c.} \qquad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \tilde{H} = \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix}$$

$$\mathcal{L} \supset Y_\nu \left(\bar{\nu}_L, \bar{\ell}_L\right) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} N + \text{ h.c.}$$

$$\supset Y_\nu \left(\bar{\nu}_L H^{0*} - \bar{\ell}_L H^-\right) N + \text{ h.c.}$$

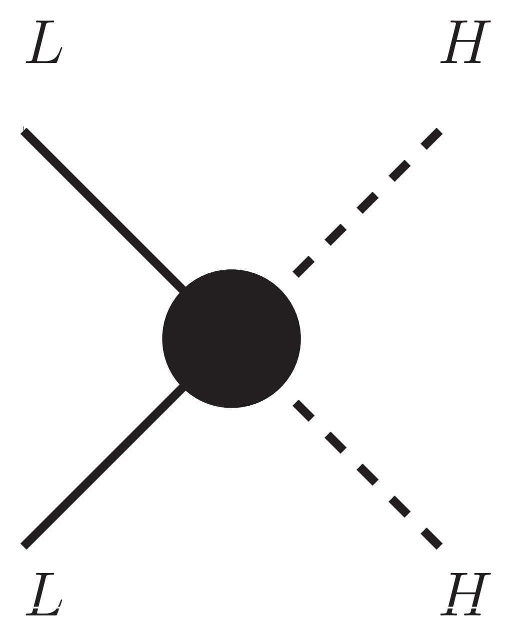
$$\supset \underbrace{\frac{Y_\nu v}{\sqrt{2}}}_{m_\nu} \bar{\nu}_L N + \text{ h.c.}$$



$$Y_\nu \sim \frac{\sqrt{2} m_\nu}{v} \sim \frac{\sqrt{2} \times 0.1\text{eV}}{246\text{GeV}} \sim 5 \times 10^{-13}$$

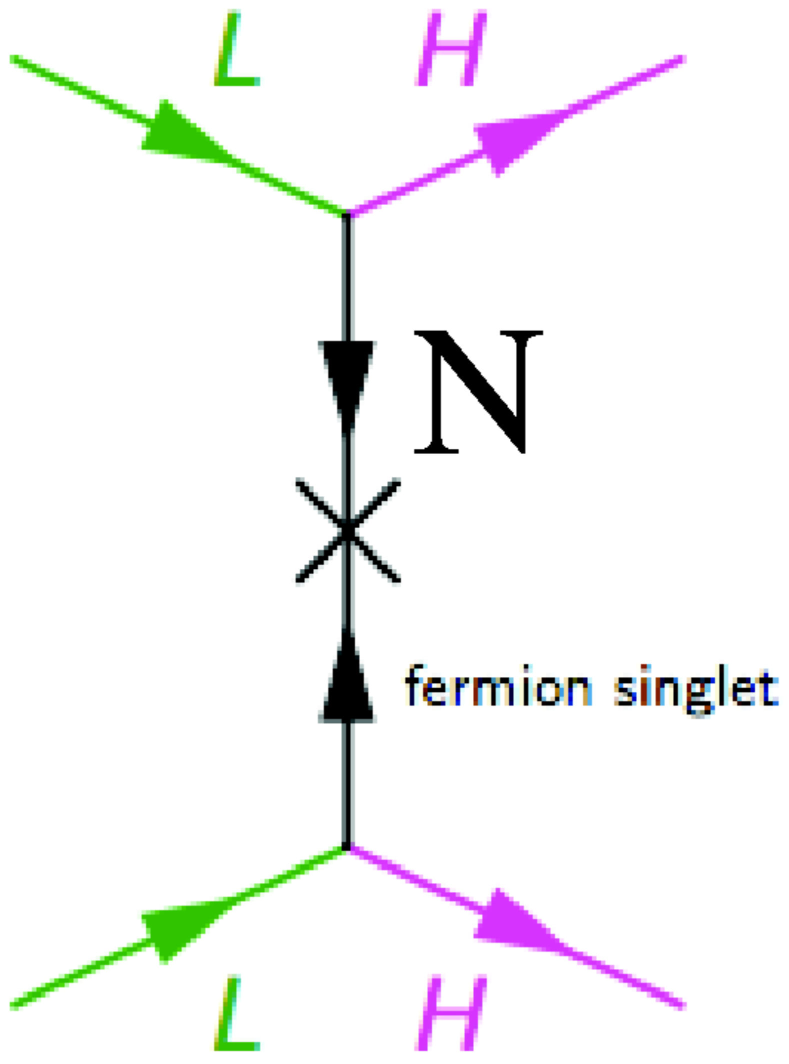
Hypothesise that lepton number is violated and form $\mathrm{SU}(2)_L$ invariant term mass term for neutrinos

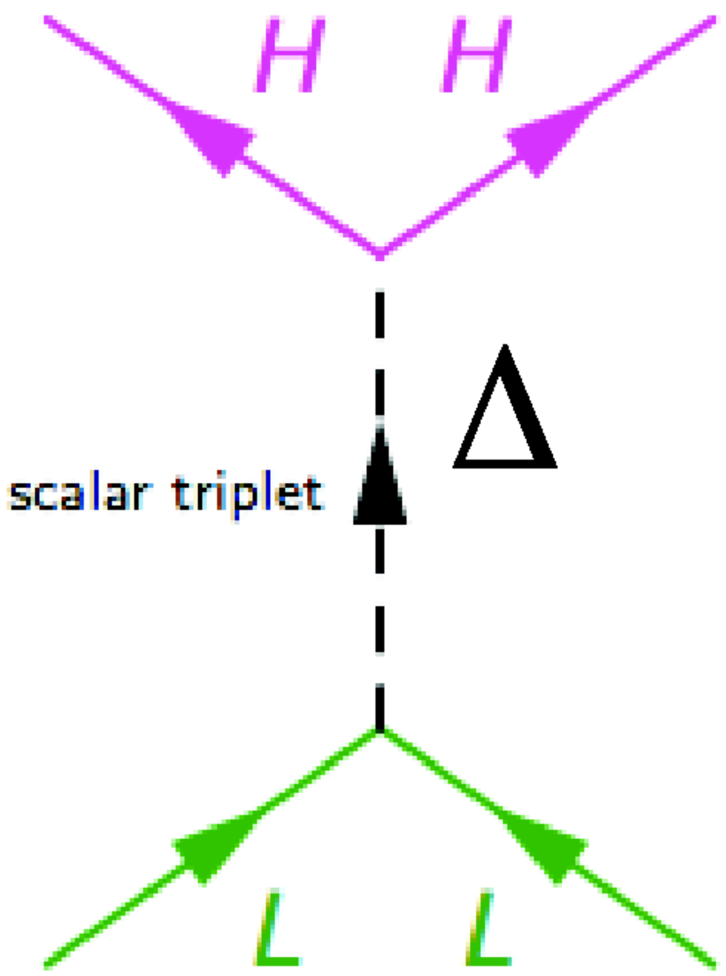
$$\mathcal{L}_{d=5} = \frac{\left(Y^T Y\right)_{\alpha\beta}}{\Lambda_{\mathrm{NP}}} \left(\overline{L_\alpha} H\right)\left(H^T L_\beta^C\right)$$

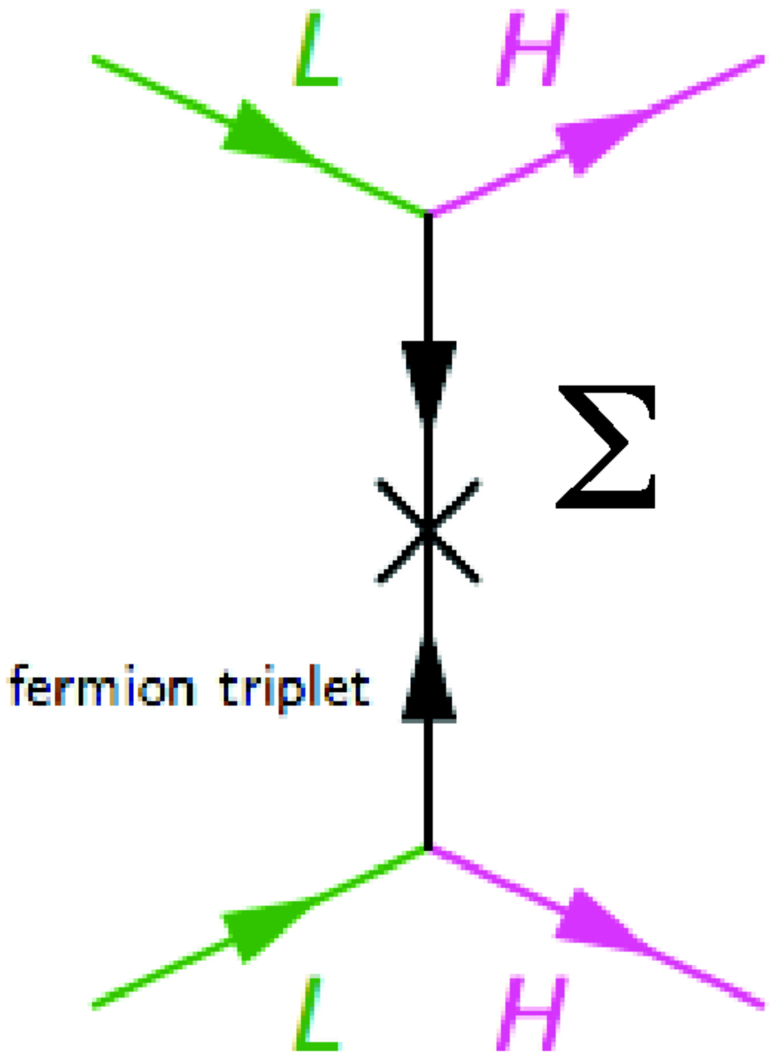


Need to form gauge invariant interaction to "complete" the Weinberg operator

$2 \otimes 2 = 1 \oplus 3$







$N \sim (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)$ $\Delta \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 2)$ $\Sigma \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)$

Type-I Fermionic SM i.e. right-handed neutrino (RHN)

$$\mathcal{L} = \frac{1}{2} Y_\nu \bar{N} L^c H + \frac{1}{2} Y_\nu \bar{L} H N + \frac{1}{2} \bar{N}^c M N + \text{ h.c.}$$

$$\mathcal{L} = \frac{1}{2} \left(\overline{\nu_L} \ \bar{N}^c \right) \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

To find masses need to find eigenvalues of non-diagonal mass matrix:

$$\begin{vmatrix} \lambda & -m_D \\ -m_D & \lambda - M \end{vmatrix} = 0 \implies \lambda^2 - M\lambda - m_D^2 = 0$$

$$\lambda_{1,2} = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} \simeq \begin{matrix} M \\ \frac{M-M}{2} - \frac{4m_D^2}{4M} = -\frac{m_D^2}{M} \end{matrix}$$

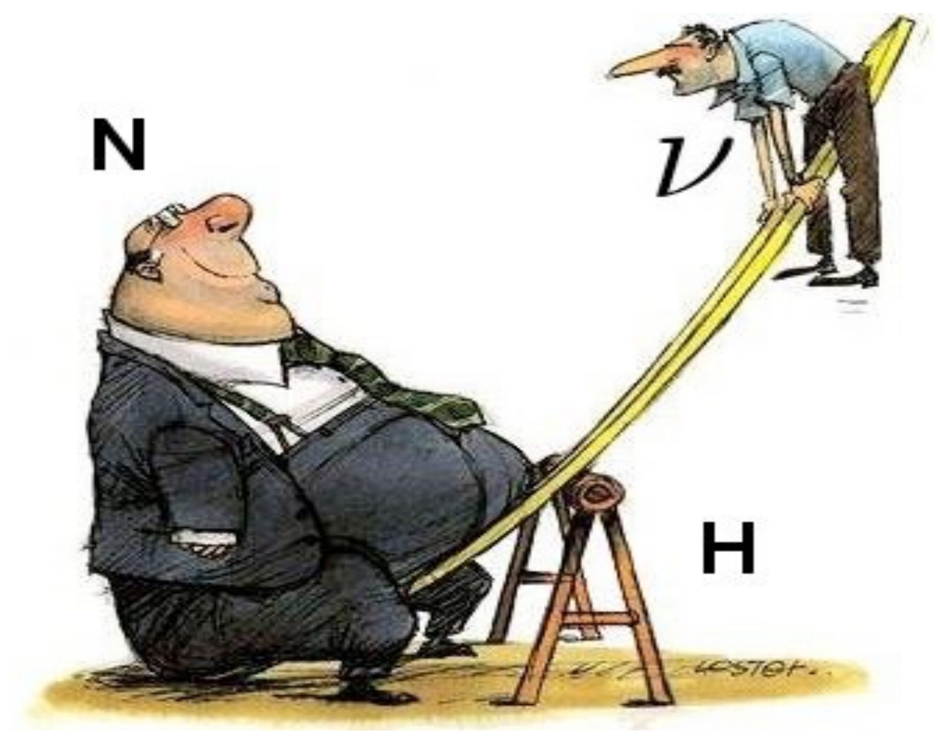
Type-I

One heavy state per one light state but we know from oscillation data there are at least two non-zero neutrino masses $\Longrightarrow$ two non-zero heavy RHN

$$m_\nu \simeq \frac{m_D^2}{M} = \frac{Y_\nu^2 v^2}{M} = \frac{1^2 \times (246\,\mathrm{GeV})^2}{M} \frac{1\mathrm{GeV}^2}{6 \times 10^{14}\mathrm{GeV}} \sim 0.1\mathrm{eV}$$

Mixing between active and very heavy state will occur but you can show (apply unitary matrix to non-diagonal mass matrix) that

$$\tan 2\theta = \frac{2m_D}{M}$$



This mixing is suppressed w.r.t the mass of the heavy RHN.

Heavy RHNs predicted by many Grand Unified Theories $SO(10)$

Type-II Add $SU(2)_L$ triplet scalar $\Delta \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 2)$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad L_\alpha = \begin{pmatrix} \nu_{L,\alpha} \\ e_{L,\alpha} \end{pmatrix} \qquad \Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

Gauge invariant Yukawa potential:

$$\mathcal{L} = f_{\Delta_{ij}} \overline{L_{Li}} \Delta L^C_{L_j} + V(H, \Delta)$$

$$V(H,\Delta) = \lambda |H|^4 - \mu^2 |H|^2 + M_\Delta^2 |\Delta|^2 + \kappa H^T \Delta^\dagger H +$$

Ex: show that minimum occurs at

$$\boxed{\langle H \rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}} \text{ and } \langle \Delta \rangle = \frac{\kappa v^2}{2M_\Delta^2}}$$

$$\Rightarrow m_\nu = f_\Delta \frac{\kappa v^2}{M_\Delta^2}$$

Heavier $\Delta \Longrightarrow$ lighter the neutrino hence "see-saw"

Type-III Add $SU(2)_L$ triplet fermion $\Sigma \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)$

$$\mathcal{L} \supset Y_{ij} \bar{L}_\alpha \sigma H \cdot \overline{\Sigma_j^c} + \frac{1}{2} M_{\Sigma,ij} \overline{\Sigma_\alpha^c} \Sigma_j + \text{ h.c.}$$

$$\overline{\Sigma^c} = \begin{pmatrix} \Sigma^0 & \Sigma^+ \\ \Sigma^- & -\Sigma^0 \end{pmatrix}$$

Again we extract the non-diagonal mass matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_\Sigma \end{pmatrix} \qquad m_D = Yv$$

And find the eigenvalues:

$$m_\nu \simeq -\frac{Y^T Y v^2}{M_\Sigma}$$