Neutrino Physics

Neutrino masses and phenomenology

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Neutrino masses

Weyl fermions any field in an irreducible representation of the Lorentz group will be a Weyl fermion

$$\chi_+ = (1/2, 0) \qquad \chi_- = (0, 1/2)$$

- two component spinors which obey:

$$i\sigma^\mu \partial_\mu \chi_+ = 0 \qquad \sigma^\mu = (1, \sigma)$$

$$i\bar{\sigma}^\mu \partial_\mu \chi_- = 0 \qquad \bar{\sigma}^\mu = (1, -\sigma)$$

- χ_+ and χ_- have different chiralities:

$\chi_+ \Rightarrow$ left-handed

$\chi_- \Rightarrow$ right-handed

Neutrino masses

Dirac fermions

$$(i\not{\partial} - m)\psi = 0 \qquad \psi \equiv \text{four-component spinor}$$

$$\psi = \psi_L + \psi_R = P_L\psi + P_R\psi \qquad P_L = \frac{1-\gamma^5}{2}$$

$$P_R = \frac{1+\gamma^5}{2}$$

In chiral representation, γ_5 diagonal & ψ has the following form:

$$\psi = \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix} \qquad \psi = \psi_L + \psi_R = \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \chi_- \end{pmatrix}$$

Ex in massless limit $(m=0)$ show that two independent Weyl equations are obeyed

Neutrino masses

Majorana fermions Majorana fermions obey the Majorana condition

$$\boxed{\psi = \psi^c = C\bar{\psi}^T}$$

quick check $\gamma^0 = \gamma^{0\dagger}, -\gamma^2 = \gamma^{2*}$

$C^\dagger = (i\gamma^0\gamma^2)^\dagger = -i\gamma^{2\dagger}\gamma^0 = i\gamma^2\gamma^0$

$= -i\gamma^0\gamma^2$

- C is the charge conjugation matrix

$C^* = -i\gamma^0\gamma^{2*} = i\gamma^0\gamma^2$

$$C^{-1}\gamma^\mu C = -(\gamma^\mu)^T$$

$$C^{-1} = C^\dagger = -C^* \Rightarrow C \text{ is an antisymmetric matrix}$$

$\Rightarrow C^\dagger = -C^*$

- C can be defined in any basis but it has the rep $C = i\gamma^0\gamma^2$
- Majorana condition is equivalent to saying a particle is its own anti-particle.

Neutrino masses

$$\psi^c = C\bar{\psi}^T$$

$$\psi = \begin{pmatrix}\chi_+ \\ \chi_-\end{pmatrix} = C\left(\overline{\begin{matrix}\chi_+ \\ \chi_-\end{matrix}}\right)^T = C\left[(\chi_+ \ \chi_-)^* \gamma^0\right]^T$$

$$= C\gamma^0\begin{pmatrix}\chi_+^* \\ \chi_-^*\end{pmatrix} = \frac{i\gamma^0\gamma^2\gamma^0}{C}\begin{pmatrix}\chi_+^* \\ \chi_-^*\end{pmatrix}$$

$\tau \equiv$ Pauli matrices.

$$= -i\gamma^2\begin{pmatrix}\chi_+^* \\ \chi_-^*\end{pmatrix}$$ use chiral representation $$\gamma^2 = \begin{pmatrix}0 & \tau_2 \\ -\tau_2 & 0\end{pmatrix}$$

$$= -i\begin{pmatrix}0 & \tau_2 \\ -\tau_2 & 0\end{pmatrix}\begin{pmatrix}\chi_+^* \\ \chi_-^*\end{pmatrix} = -i\begin{pmatrix}\tau_2\chi_-^* \\ -\tau_2\chi_+^*\end{pmatrix} \Rightarrow \begin{pmatrix}\chi_+ \\ \chi_-\end{pmatrix} = \begin{pmatrix}-i\tau_2\chi_-^* \\ i\tau_2\chi_+^*\end{pmatrix}$$

$$\left.\begin{array}{l}\Rightarrow \chi_+ = -i\tau_2\chi_-^*, \quad \chi_- = i\tau_2\chi_+^* \Rightarrow \\ \chi_- = i\tau_2(i\tau_2^*\chi_-) = -\tau_2\tau_2^*\chi_- = \chi_-\end{array}\right\} \Rightarrow \psi = \begin{pmatrix}-i\tau_2\chi_-^* \\ \chi_-\end{pmatrix}$$

Neutrino masses

⇒ Majorana fermion only has two degrees of freedom ⇒ a single Weyl spinor. You can see this from computing :

$$\psi_L = C\,\overline{\psi_R}^{\,T} \qquad \psi_R = C\,\overline{\psi_L}^{\,T}$$

$$\psi = \psi_L + (\psi_L)^c$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p} \sum_{s=1} \left[C\,\overline{u_s(p)}^{\,T} (a_p^s)^{\dagger} e^{ip\cdot x} + u_s(p)\, a_p^s\, e^{-ip\cdot x} \right]$$

n.b Dirac field would also contain v spinor not just u!

Neutrino masses

Dirac mass

$$m\,\bar\psi\psi = m(\bar\psi_L\psi_R + \bar\psi_R\psi_L)$$

$$m_{\alpha\beta}\,\bar\psi_\alpha\psi_\beta \qquad \alpha,\beta \text{ flavour index.}$$

Majorana mass

$$M\,\bar\psi\psi = M\,\bar\psi_L\psi_R + M\,\bar\psi_R\psi_L$$

$$= M\,\overline{\psi_L^c}\,\psi_L + h.c$$

$$= \frac{1}{2}M_{\alpha\beta}\,\overline{\psi_\alpha^c}\,\psi_\beta$$

$M_{\alpha\beta}$ complex, symmetric matrix ie $M_{\alpha\beta} = M_{\beta\alpha}$.

$$\frac{1}{2}M_{\alpha\beta}\,\overline{\psi_\alpha^c}\,\psi_\beta = \frac{1}{2}M_{\alpha\beta}\,\overline{(C\bar\psi_\alpha^T)}\,\psi_\beta = \frac{1}{2}M_{\alpha\beta}\,\psi_\alpha^T C^{-1}\psi_\beta$$

$$= \frac{1}{2}M_{\alpha\beta}(\psi_\alpha)_a(C^{-1})_{ab}(\psi_\beta)_b$$

$$= -\frac{1}{2}M_{\alpha\beta}(\psi_\beta)_b(C^{-1})_{ab}(\psi_\alpha)_a \quad \leftarrow \text{fermion statistics!}$$

$$= \frac{1}{2}M_{\alpha\beta}(\psi_\beta)_b\,\underline{(C^{-1})_{ba}}\,(\psi_\alpha)_a$$

$$= \frac{1}{2}M_{\beta\alpha}\,\overline{\psi_\alpha^c}\,\psi$$

Neutrino masses

Dirac fermion charged under a U(1) will transform:

$$\psi \to \psi e^{i\theta}$$ (recall how we could rephase the charged leptons to remove 3 phases from U matrix).

<u>Dirac mass</u>

$$m\bar{\psi}\psi \to m\bar{\psi}\psi$$

<u>Majorana mass</u>

$$m\bar{\psi}\psi^c \to m e^{-2i\theta}\bar{\psi}\psi^c$$

majorana mass term violates U(1) symmetry explicitly.

- any particle with a U(1) charge <u>cannot</u> have a Majorana mass.
- accidental symmetries of the SM, importantly L or B-L can be violated in a model with Majorana neutrinos.

Neutrino masses

To go from flavor to mass basis we must diagonalize the non-diagonal mass matrix. For a complex symmetric matrix (Majorana mass matrix) this requires some care as it is not a "normal" matrix i.e. it does not commute with hermitian conjugate.
We recap on SVD (singular value decomposition).

M square complex matrix $\exists$ SVD factorisation s.t

$$M = U D V^\dagger$$

$D \equiv$ diagonal matrix with REAL entries. $D_{ii} > 0 \quad \forall i$.

U, V unitary i.e. $U^\dagger U = U U^\dagger = \mathbb{1}$.
$V^\dagger V = V V^\dagger = \mathbb{1}$.

Neutrino masses

We can compute U & V by diagonalising the hermitian matrix:

$$U^\dagger (MM^\dagger) U = D^2 \qquad V^\dagger (M^\dagger M) V = D^2.$$

If M is complex, symmetric $\Rightarrow$ relationship between U & V

$$M = U^T D U$$

$$U^\dagger U = U U^\dagger = \mathbb{1}$$ Takagi factorisation.

Neutrino masses

Application to mass matrices

Dirac mass matrix $M_{\alpha\beta}$ is an arbitrary complex matrix:

$$\mathcal{L}_m = M_{\alpha\beta}\, \bar{\psi}_L^{\alpha}\, \psi_R^{\beta}$$

we can act on left & right fields independently

$$\psi_L^{\alpha} \to U_{\alpha j}\, \psi_L^{j} \qquad \psi_R^{\alpha} \to V_{\alpha j}\, \psi_R^{j}$$

unitary matrix

also unitary

$$\mathcal{L}_m = (U^{\dagger} m\, V)_{ij}\, \bar{\psi}_L^{i}\, \psi_R^{j}$$
$$= D_{ii}\, \bar{\psi}^{i}\, \psi_R^{i}$$

For majorana mass, $M_{\alpha\beta}$, we don't have the freedom to rotate the two fields in the mass term independently:

$$\mathcal{L}_m = M_{\alpha\beta}\, \bar{\psi}_{\alpha}^{c}\, \psi_{\beta}$$
$$= (U^{T} M U)_{ij}\, \bar{\psi}_i^{c}\, \psi_j$$
$$= D_{ii}\, \bar{\psi}_i^{c}\, \psi_j$$