4th Workshop / Thinkstart on $\gamma^* \to \ell\ell$ (Durham)

Massification of real corrections

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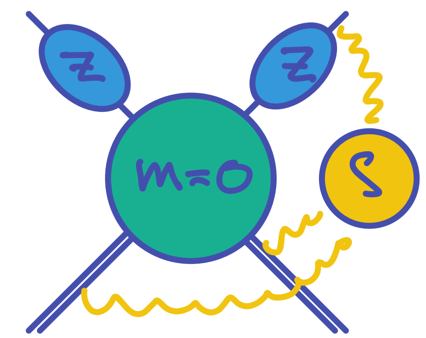
Massification [Becher, Melnikov 07; McMule 18]

@ 2loop — extension to heavy external states

hierarchy

$$p_j^2 = m_j^2 \sim \lambda^2 \ll Q^2$$

$$p_j \sim (1, \vec{e}_j \beta_j)\,, \quad \beta_j = 1 - \mathcal{O}(\lambda^2)$$



factorisation

process indep. — 1+ fermion loops

$$\mathcal{A}_n = \prod_j Z(m_j)\, S(Q^2, \{m_j\})\, \mathcal{A}_n(Q^2, \{m_j = 0\}) + \mathcal{O}(\lambda)$$

SCET

high-energy coll. modes — hard scattering operator — soft modes

$$\mathcal{L}_n = \sum_j \underbrace{\mathcal{L}_{SCET}(\psi_j, A_j^\mu)}_{\nexists \text{ soft-coll IA (decoup. trsf.)}} + C_n \underbrace{\mathcal{O}_n(\{\psi_j\}, \{A_j^\mu\}, A_s^\mu)}_{\prod_j \mathcal{O}_n^j(\psi_j, A_j^\mu)\, \mathcal{O}_n^s(A_s^\mu)}$$

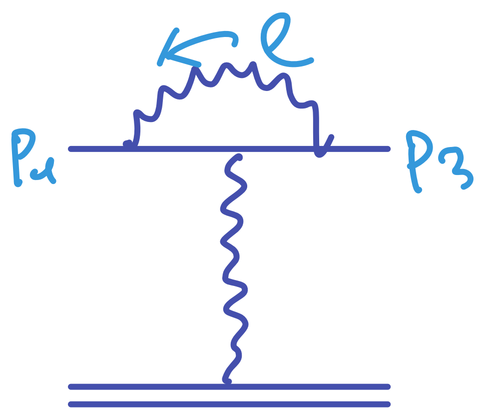
$$\rightsquigarrow \mathcal{A}_n \sim \langle 0 | \psi_1 \cdots \psi_n e^{iS_n} | 0 \rangle$$

$$\sim \prod_j \underbrace{\langle 0 | \psi_j \mathcal{O}_n^j e^{iS^j_{SCET}} | 0 \rangle}_{Z}\; \underbrace{\langle 0 | \mathcal{O}_n^s | 0 \rangle}_{S}\; \underbrace{C_n}_{\mathcal{A}_n(\{m_j = 0\})}$$

Method of regions

[Beneke, Smirnov 98]

{ convenient tool
  to extract Z



expand in small param. @ integrand

$$\rightsquigarrow \quad A^{(1)} \sim \int d\ell \, \frac{1}{[\ell^2][\ell^2 + 2\ell\cdot p_1][\ell^2 + 2\ell\cdot p_3]}$$

light-cone basis

$$\{ e_j = (1, \vec{e}_j), \ \bar{e}_j = (1, -\vec{e}_j) \}$$

decomposition into small/large comp.

$$p_j = \underbrace{(e_j\cdot p_j)\,\bar{e}_j}_{p_j^+} + \underbrace{(\bar{e}_j\cdot p_j)\,e_j}_{p_j^-} + p_j^\perp \sim (\lambda^2, 1, \lambda)$$

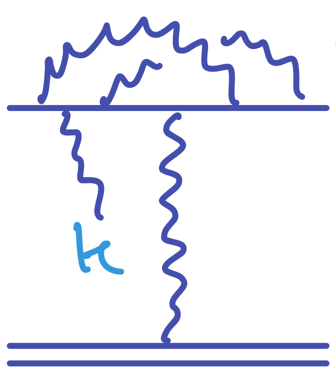
$A^{(1)}(m=0)$ $\quad (p_j^-)^2 = 0$ $\quad 2Z^{(1)}A^{(0)}(m=0)$

$$\rightsquigarrow \quad A^{(1)} \sim \int d\ell \, \frac{1}{[\ell^2][\ell^2 + 2\ell\cdot p_1^-][\ell^2 + 2\ell\cdot p_3^-]} + (\ell \sim p_1) + (\ell \sim p_3)$$

$$\sim Z(m)\, Z(m)\, A(m=0)$$

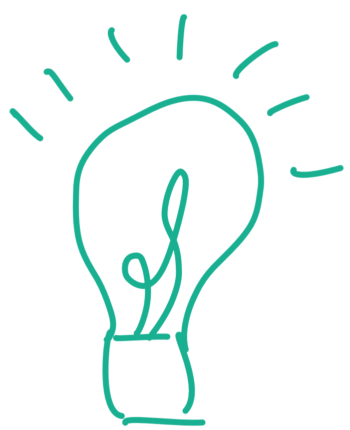
$Z(m) = 1 + Z^{(1)} + \mathcal{O}(\alpha^2)$ $\qquad A(m=0) = A^{(0)} + A^{(1)} + \mathcal{O}(\alpha^3)$

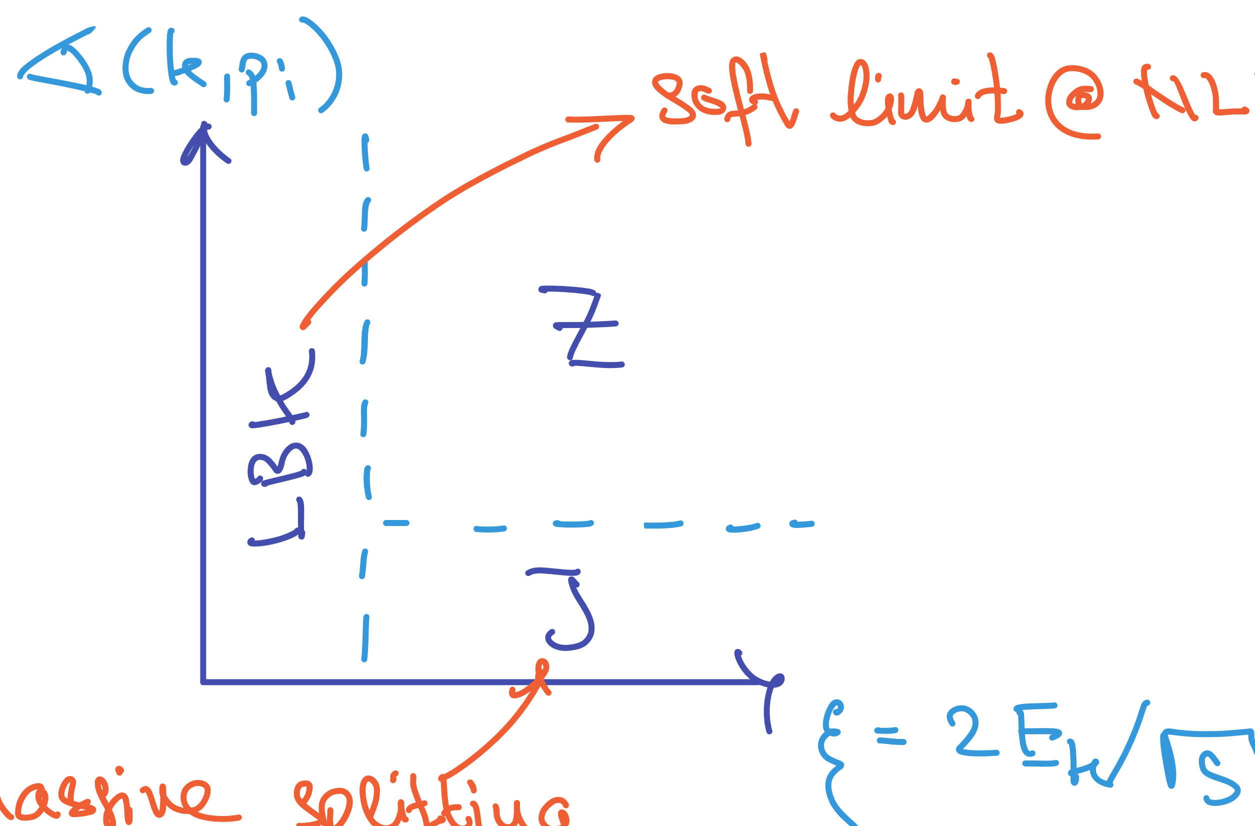
Massification of real corrections



only known for $m=0$

$\rightsquigarrow$ $\exists$ breakdown of massified approx. for $k \to 0$ (soft) and $k\cdot p_i \sim m^2$ (collinear)





soft limit @ NLP ($\rightsquigarrow$ nts stabilisation)

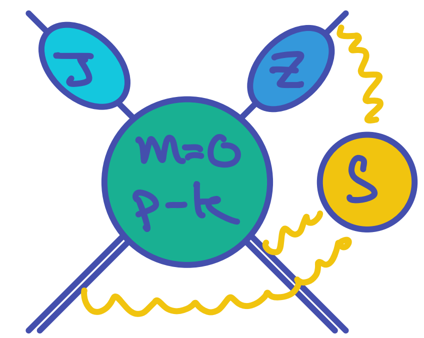
massive splitting function

universal structure of radiative QED amplitudes

$\rightsquigarrow$ [McMule 21]

@ one loop

Collinear factorisation



hierarchy $k\cdot p_i \sim p_i^2 \sim m^2 \sim \lambda_c^2 \ll Q^2$

factorisation

$$A_{n+1} \sim \langle 0|\psi_1 \cdots \psi_i A^\mu_{i\ldots} \psi_n\, e^{iS_{n+1}}|0\rangle$$

$$\sim \langle 0|\psi_i A^\mu_i\, \mathcal{O}^i_{n+1}\, e^{iS^i_{SCET}}|0\rangle \prod_{j\neq i} Z\; S\; C_{n+1}$$

universal splitting function J
≙ $(\ell \sim p_i \sim k)$ in HOR

$A_n(p\pm k, m=0)$

m→0 limit

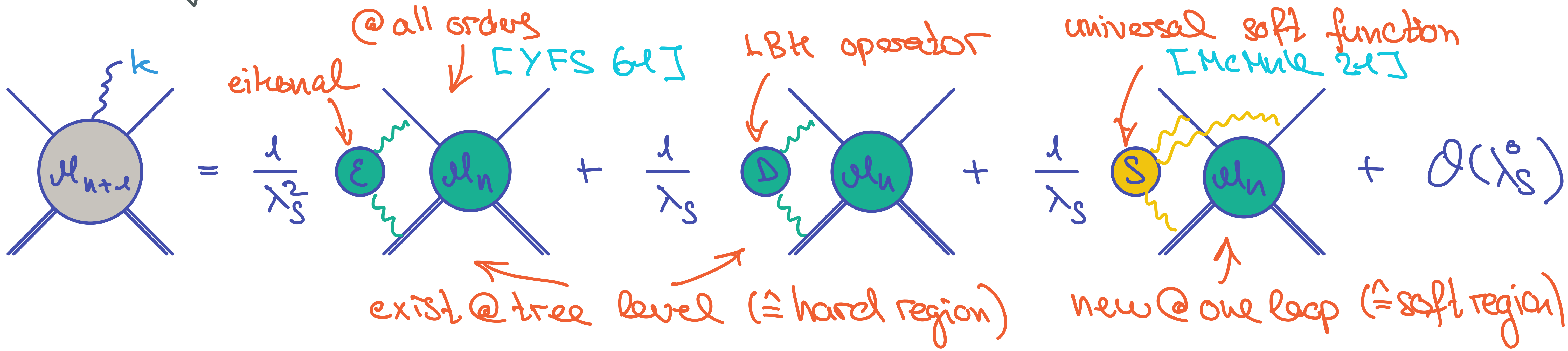
$$J(y,m) \overset{m\to 0}{=} Z(m)\, \bar{J}(y) + \mathcal{O}(m)$$

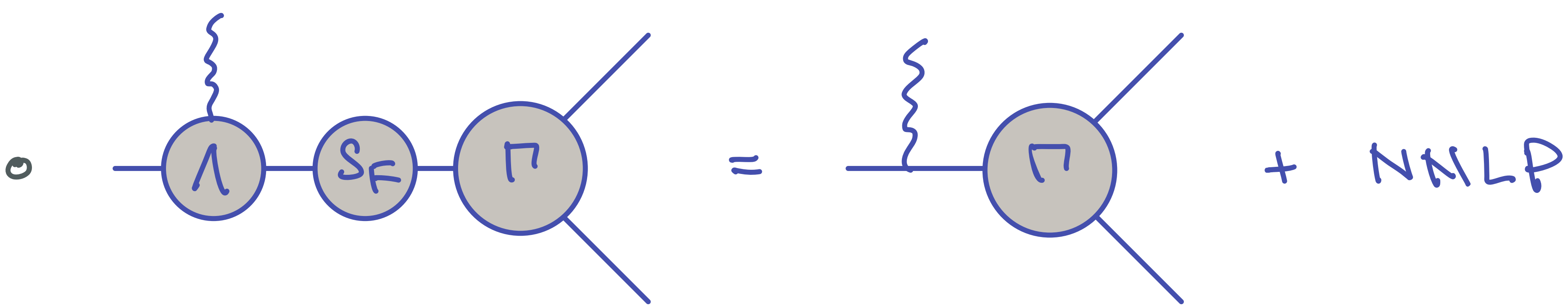
massless splitting function
[Bern, Dixon, Kosower 04; Badger, Glover 04]

@ two loop

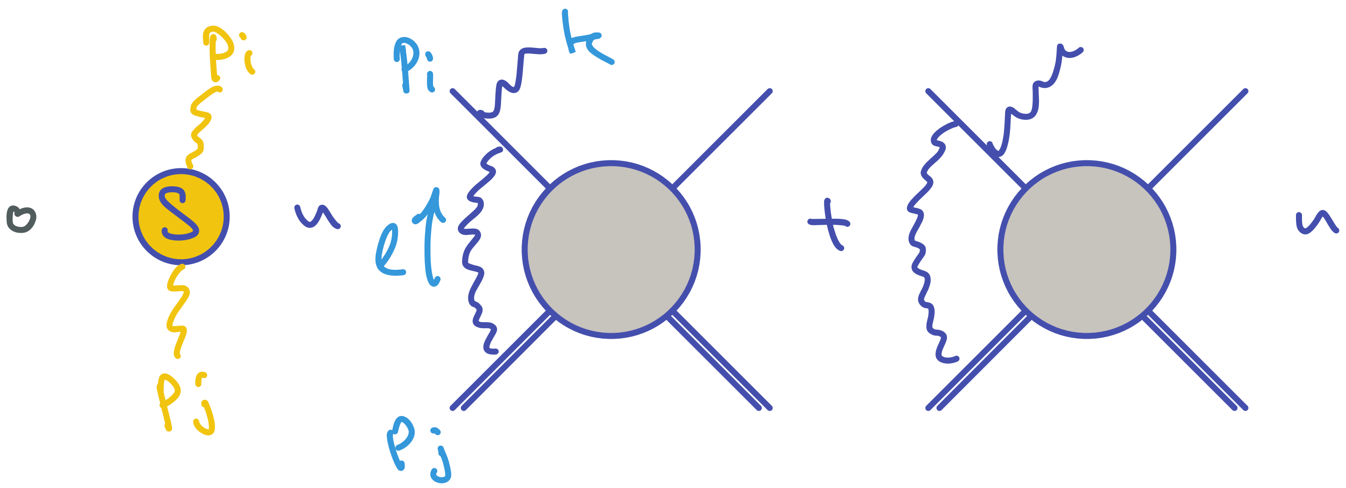
LBK theorem [Low 58; Burnett, Kroll 68]

hierarchy $k \sim \lambda_s \ll Q^2$ } no coll. scale $\rightsquigarrow$ hard & soft d.o.f. (HQET)





} holds @ one loop for spin-summed sq. amp.



$$\sim p_i \cdot p_j \, I_1(k, p_i) + m_j^2 \, k \cdot p_i \, I_2(k, p_i, p_j)$$

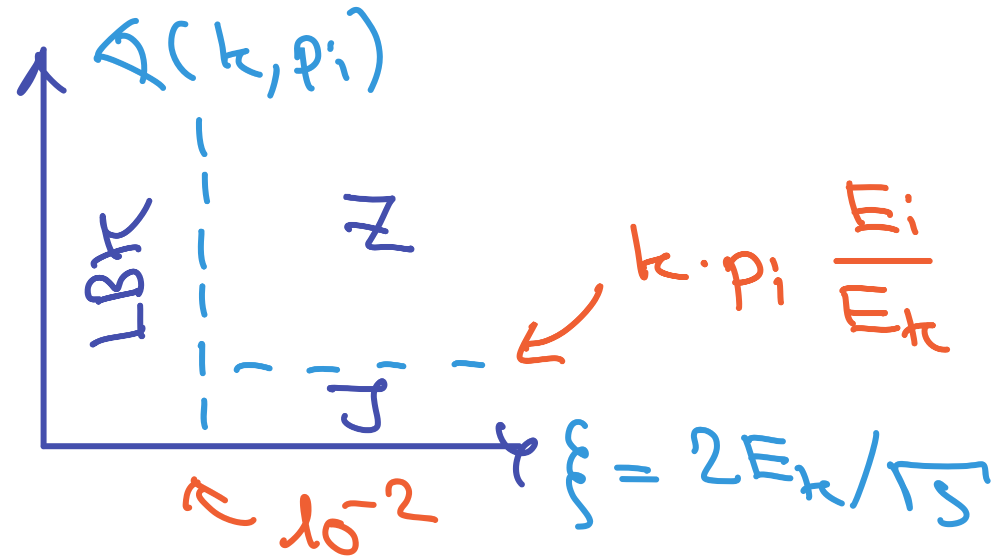
$$I_1: \int d\ell \, \frac{1}{[\ell^2][\ell \cdot p_i - k \cdot p_i]}$$

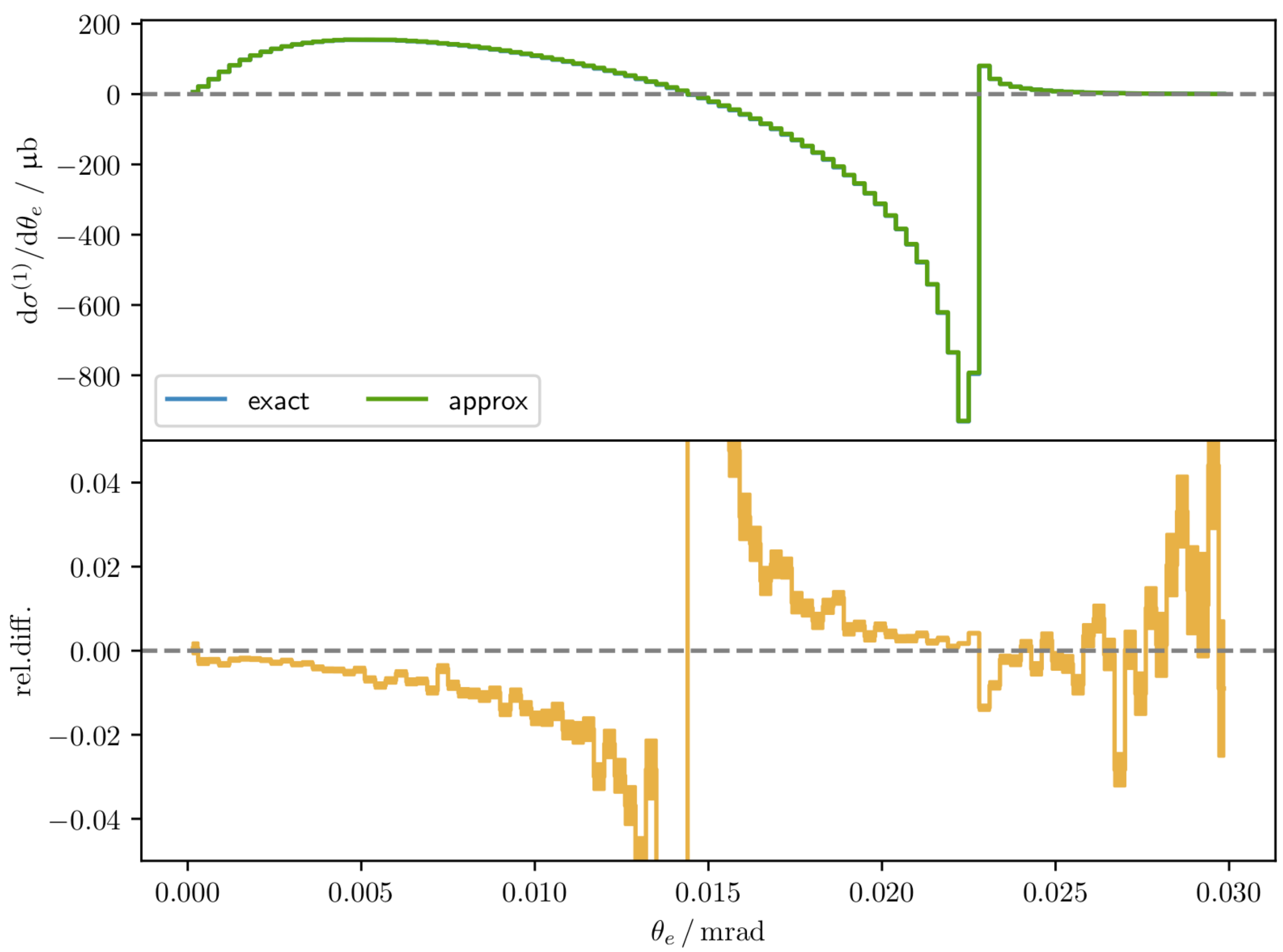
$$I_2: \int d\ell \, \frac{1}{[\ell^2][-\ell \cdot p_j][\ell \cdot p_i - k \cdot p_i]}$$

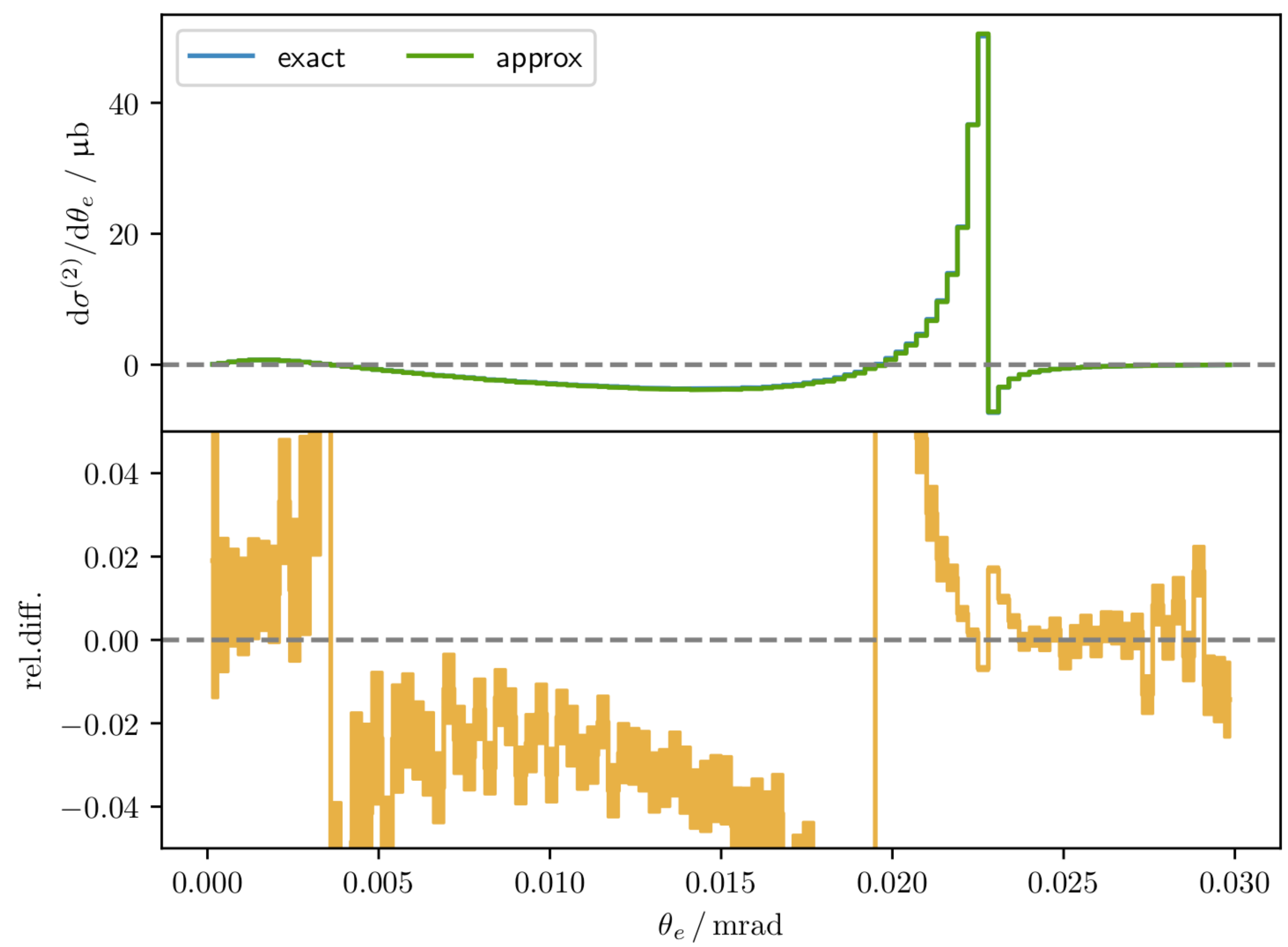
Test @ NLO/NNLO w/ electronic corrections

MUonE observable $E_\mu^{\text{beam}} = 150\,\text{GeV}, \quad E_e \gtrsim 1\,\text{GeV}, \quad \theta_\mu \gtrsim 0.3\,\text{mrad}$

switch parameters







Conclusion / outlook

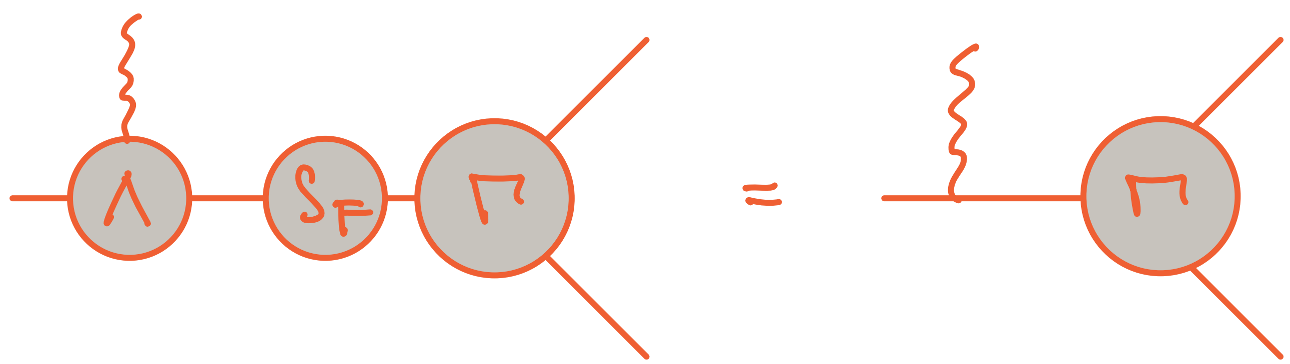
massify massless real-virtual-virtual amplitude

$\rightsquigarrow$ LBK thm. + massive splitting function

$\rightsquigarrow$ accuracy @ NLO/NNLO $\sim$ 5 %

requirements

- extend LBK theorem to two loop

↳ 

\+ NNLP { true @ two loop (beyond ?)

- calculate splitting function @ two loop

↳ two loop / two scale problem $\rightsquigarrow$ DEQ

collinear anomaly $\rightsquigarrow$ analytic regularisation