4$^{\text{th}}$ Workstop / Thinkstart

Towards N^{3}LO for $\gamma^* \to \ell\bar\ell$

Welcome and motivation

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A warm welcome to IPPP!

- you should have received a program and information
- please give us an estimate for your travel expenses & sign up for dinner(s)
- conference dinner on Thursday, paid for by grant
- unofficial dinners on Tue/Wed/Fri
- lunch and coffee just outside, keep discussing!
- sign up to Slack if you haven't already

- this is not a workshop but a workstop
- we are here to work and start thinking, not to read emails
- ideally have a compare-and-contrast write-up
- 4th edition, previous installations in Zurich and at GGI
  - 2016: new collaborative understanding of regularisation schemes
    [To d or not to d, 2017]
  - 2019: progress report and future planning of the MUonE Theory Initiative
    [Theory for muon-electron scattering @ 10 ppm, 2020]
  - 2019: four-dimensional regularisation at NNLO (GGI)
    [May the four be with you, 2020]

we are not here to give talks

- each session is 4h and about one topic
- we start with $\mathcal{O}(1\mathrm{h})$ of joint talks
- these will overrun and that's fine
- please be honest!

we are here to work!

- after 1h coffee will be delivered
- discussion from the talk will evolve into a blackboard discussion (with tea and snacks)
- playing with code is encouraged (after the talks)!
- please don't just read your emails!

- with we FKS^ℓ have a subtraction scheme for N^{3}LO in QED ($\to$ assembly session)
- ... but not matrix elements ($\to$ afternoon & tomorrow)
- can we find the missing matrix elements?
- I think so, at least for $\gamma^* \to \ell\bar{\ell}$
  - VVV: [Fael, Lange, Schönwald, Steinhauser 22]
  - RVV: integrals known for $m_\ell = 0$ (three-jet @ NNLO)
  - RRV: OpenLoops + dirty tricks
  - RRR: “trivial”
- we need to forge these efforts into a coherent Initiviate (as we did for MUonE)
- what do we need and who can provide it?

- measure HVP of $(g-2)_\mu$ using t-channel μ-e scattering at 10ppm

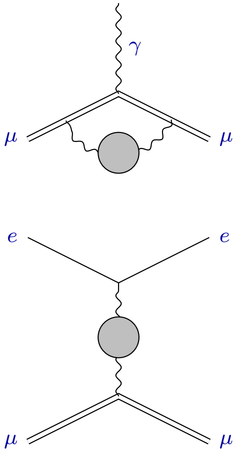
$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x(1-x)\underbrace{\Delta\alpha_{\text{had}}\Big(\frac{x^2}{x-1}m_\mu^2\Big)}_{\propto \mathrm{d}\sigma/\mathrm{d}t}$$

[Carloni Calame et al. 2015, Abbiendi et al 2016]

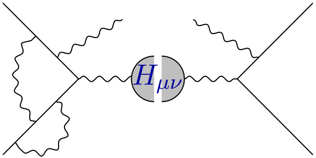
- proposed experiment @ CERN's M2

  [Matteuzzi et al 2019 (LOI)]

- NNLO is complete
- ... but most likely not enough
- dominant N^{3}LO $e \to e\gamma^*$



- HVP is also measured using $ee \to \mathrm{hadrons}$
- theory precision needed as well
  [STRONG2020 Virtual Workshop 2021]
- hadronic side: nothing we (this group) can do
- leptonic side: here we can help!
- calculate $ee \to \gamma^*$ and $ee \to \gamma^*\gamma^*$ as well as possible
- let's focus on $ee \to \gamma^*$ at N^{3}LO (but keep in mind the other as well)



- FKS^ℓ: subtraction scheme at ℓ-loop

$$\sigma_{n+m} \sim \int \mathrm{d}\Phi_n \int \mathrm{d}\Phi_m \Big(\mathcal{M}^{(\ell),f}_{n+m} - \theta(E_\gamma < E_\gamma^{\text{cut}}) \mathcal{E} \times \mathcal{M}^{(\ell),f}_{n+m-1} \Big)$$

- massification: SCET-ish trick to expand matrix element in $m \ll Q^2$ (two-loop)

$$\mathcal{M}(m) = \mathcal{S} \times \sqrt{Z_1} \times \cdots \times \sqrt{Z_n} \times \mathcal{M}(0) + \mathcal{O}(m^2/Q^2)$$

- NTS stabilisation: expand real matrix element for small $E_\gamma^2 \ll Q^2, m^2$ (one-loop)

$$\mathcal{M}_{n+1} = \frac{1}{E_\gamma^2} \mathcal{E} \times \mathcal{M}_n + \frac{1}{E_\gamma} \mathcal{D}\Big[\mathcal{M}_n\Big]$$

- jettification: expand real matrix element for small angle and masses (one-loop)

$$\mathcal{M}(m) = \mathcal{S} \times \sqrt{J_1} \times \cdots \times \sqrt{Z_n} \times \mathcal{M}(0) + \mathcal{O}(\{p_1^2, p_i \cdot p_\gamma\}/Q^2)$$

VVV

- can we use FLSS' result directly?
- what about n_f (or hadronic) corrections? Hyperspherical...?
- does it make sense to try massification (cross check / semi-analytic)?

RVV

- what do we need to do to use the known integrals from $\gamma^* \to q\bar{q}g$?
- how fast / stable an expression can we get?
- we can do things massively?
- OpenLoops?

RRV

- is OpenLoops fast / stable enough?
- can we switch to quad precision point-wise?
  (stability system for massive QED or side-step it entirely?)
- can we use NTS stabilisation?

assembly & dirty tricks

- jettification / NTS stabilisation at two-loop?
- how do we massify real correction?
- how does it influence stability & speed requirements?
- what implementation problems do we expect?

VVV

- VVV result
- n_f
- massification?

RVV

- using integrals
- speed/stability
- $m_\ell > 0$
- OpenLoops

RRV

- OpenLoops
- stability system
- NTS

assembly

- jettification / NTS
- massify real?
- requirements?
- ??