Another Perspective on Dark Matter Alternatives

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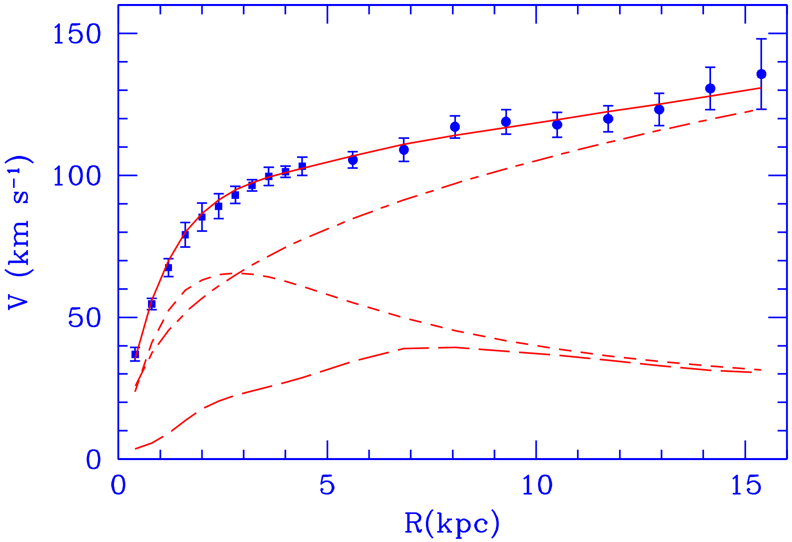
Introduction

<u>The Dark Matter Problem:</u> Using General Relativity (w/ Newtonian as the weak field limit) as the theory of gravity, at galactic scale and up we observe mismatch between visible matter and the dynamics (i.e. things move faster than expected).

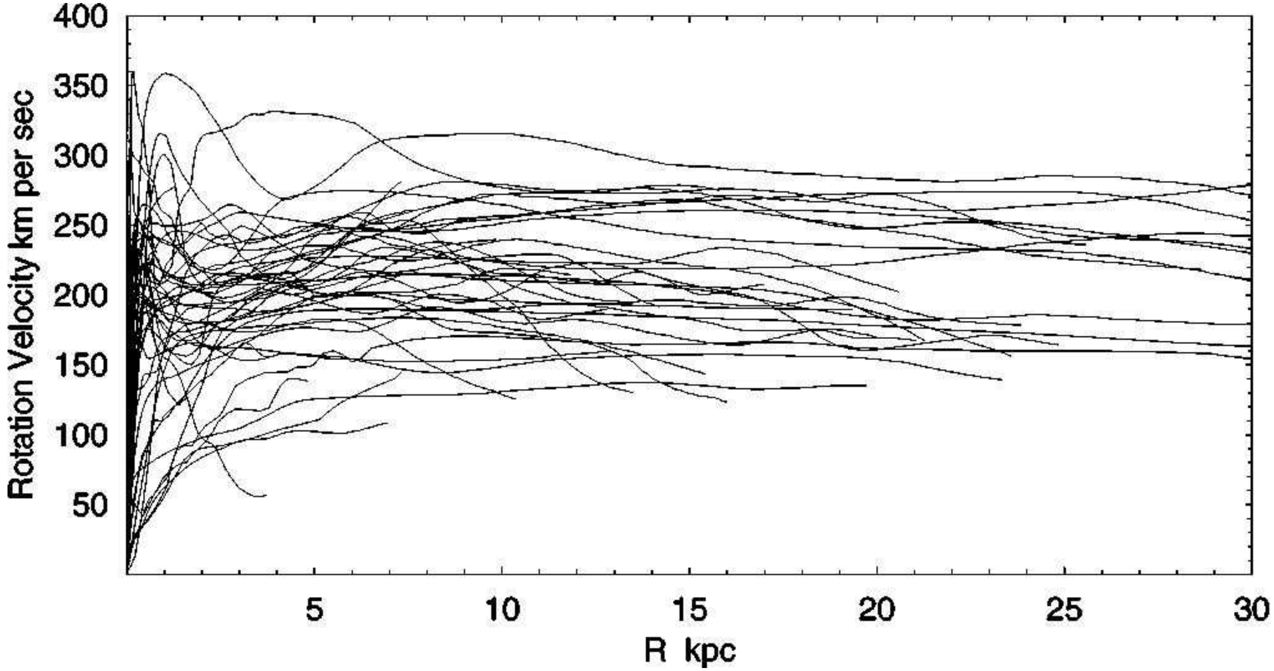
Spiral galaxy M33.



Rotation curve.

$$\frac{v^2}{r} = G_N \frac{M(r)}{r^2}$$

Generic phenomena for spiral galaxies:



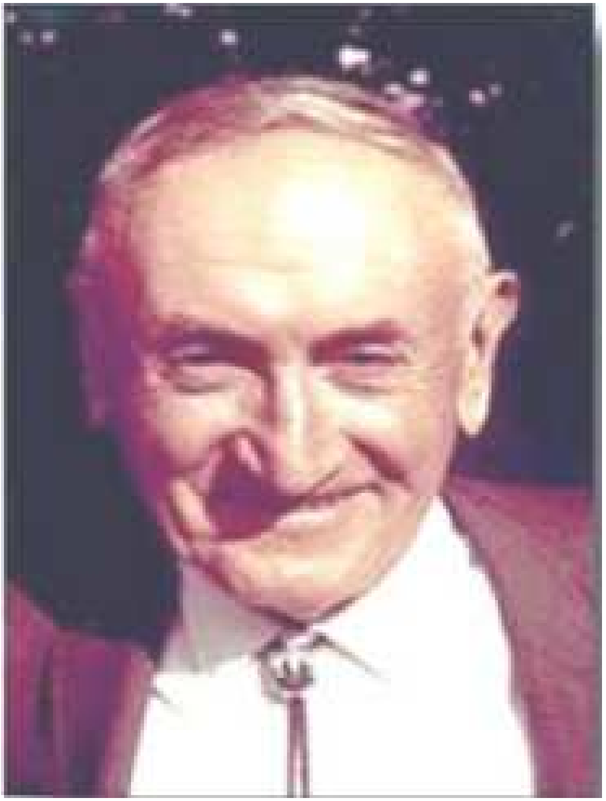
$$a_c = \frac{v^2}{r} = G_N \frac{M(r)}{r^2}$$

Is it be dark matter? (Zwicky, 1933) $\longrightarrow$ $\boxed{M(r)}$

or something else,

MOND? (Milgrom, 1983) $\longrightarrow$ $\boxed{=}$

Dark Matter or MOND?

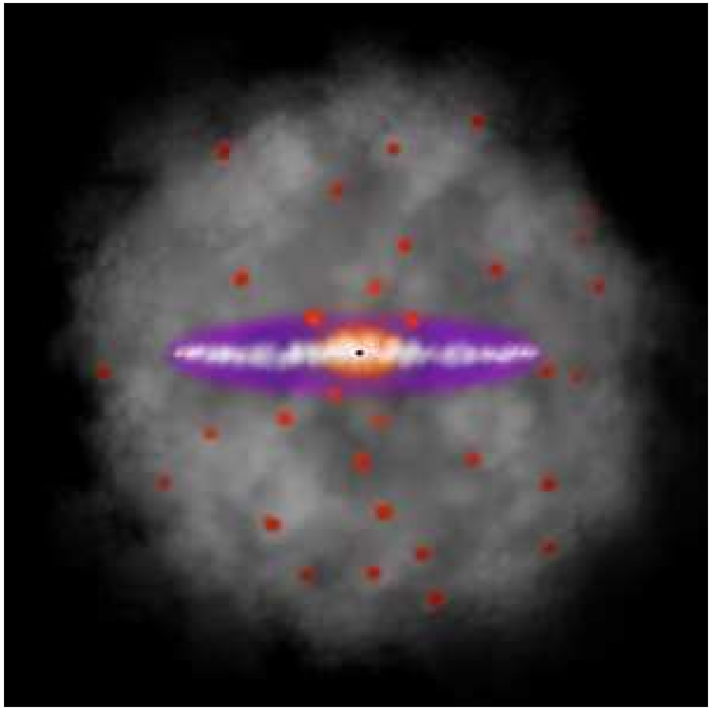


Zwicky

Milgrom

Dark Matter Halo

- Disk stability - more stable with invisible halo (Ostriker & Peebles 1973).
- Halo dark matter can support galaxy formation (Frenk and White).



chandra.harvard.edu

NFW profile:

$$\frac{\rho(r)}{\rho_{\rm crit}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

MOND

(Modified Newtonian Dynamics)

Instead of $\mathbf{a} = -\nabla\phi$, Milgrom suggests

$$\mu(\mathbf{a}/a_0)\mathbf{a} = -\nabla\phi$$

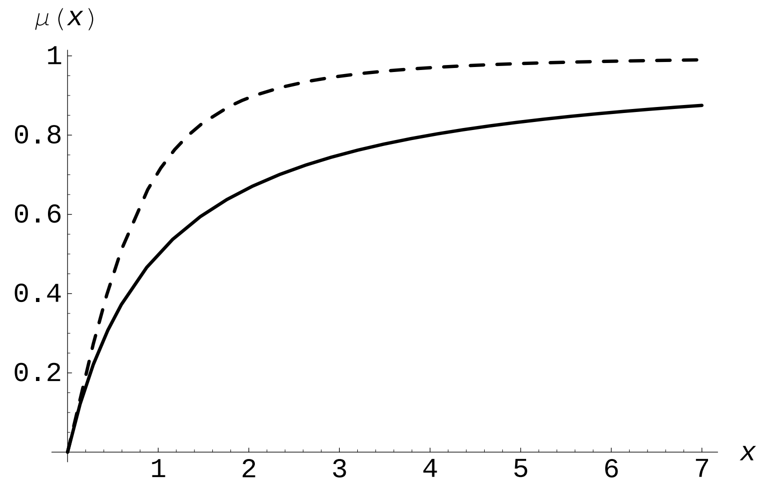
where $a_0 \simeq 10^{-10}$ m/s, and

$$\mu(x) = \begin{cases} x & \text{for} \quad x < 1 \\ 1 & \text{for} \quad x \gg 1 \end{cases}$$

e.g.

$\mu(x) = x/(1+x)$ (solid)

$\mu(x) = x/\sqrt{1+x^2}$ (dash)



We still have

$$|\mathbf{a}| = \frac{v^2}{r} \qquad \text{and} \qquad |\nabla\phi| = \frac{GM}{r^2}$$

Then for small $|\mathbf{a}|$:

$$\frac{v^4}{r^2} \simeq a_0 \frac{G_N M}{r^2}$$

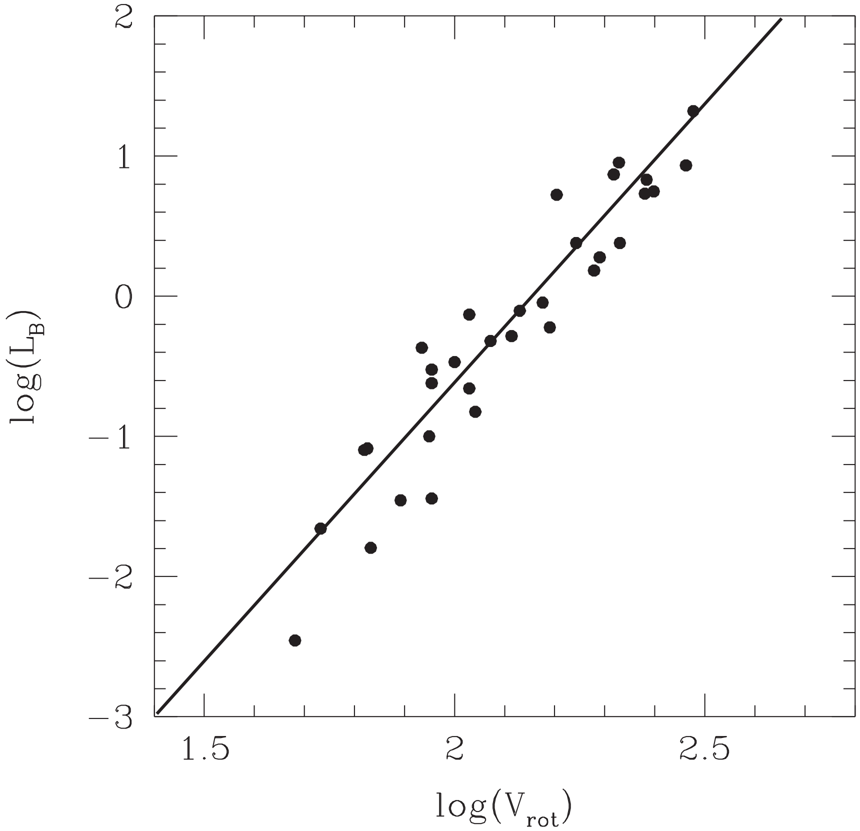
or

$$M(r) \simeq (G_N a_0)^{-1} v(r)^4$$

for large r.

Tully - Fisher

Tully-Fisher: Luminosity $\propto v_{\mathrm{flat}}^{k}$



(Bekenstein)

With $M \propto L \Rightarrow$ MOND agrees (with k=4).

MOND Fits

$$v_{Ni} = \sqrt{G_N M_i(r)/r}$$

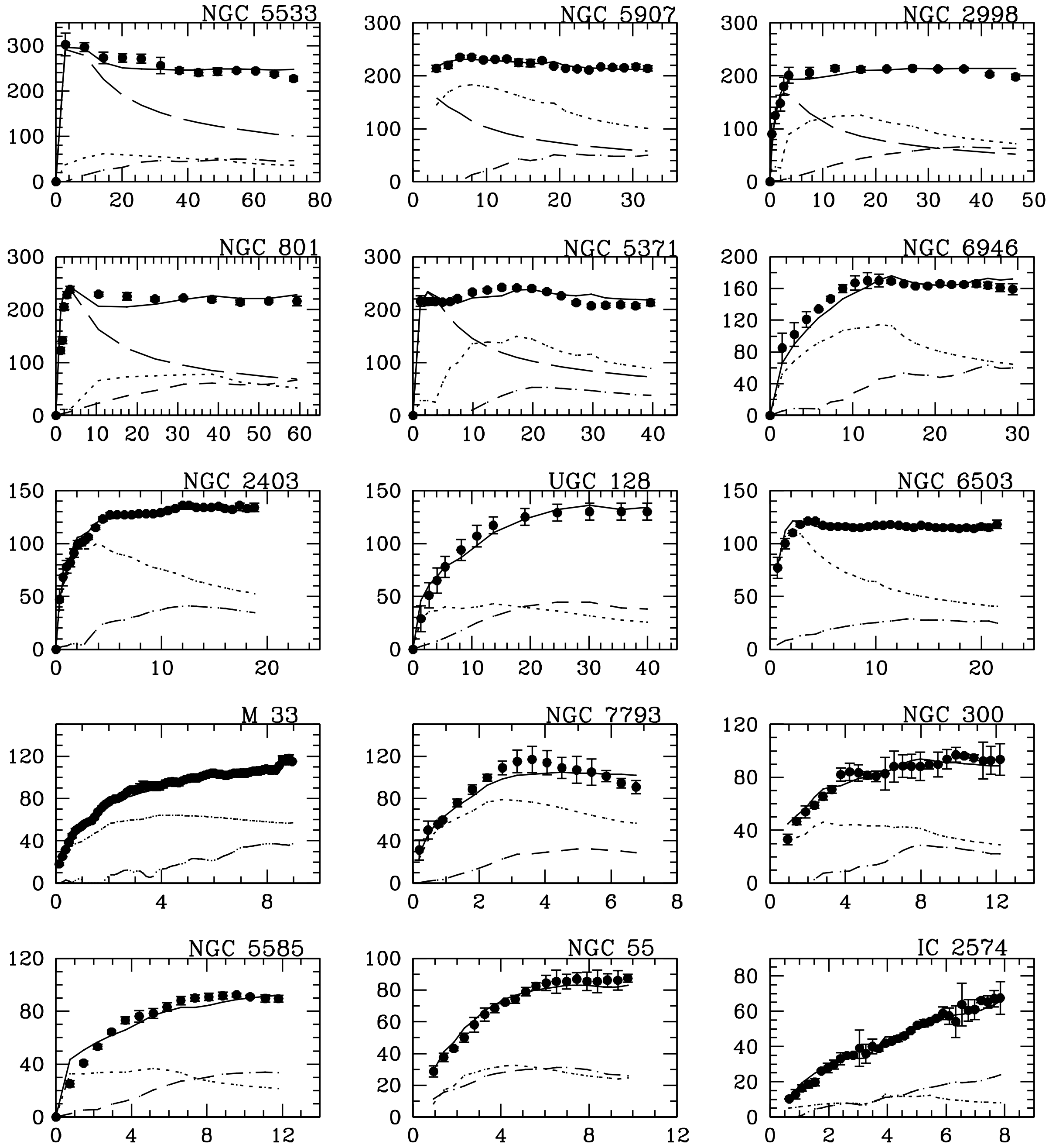
$$M(r) = \sum_i M_i = \frac{r}{G_N} \sum_i v_{Ni}^2$$

So (at large r)

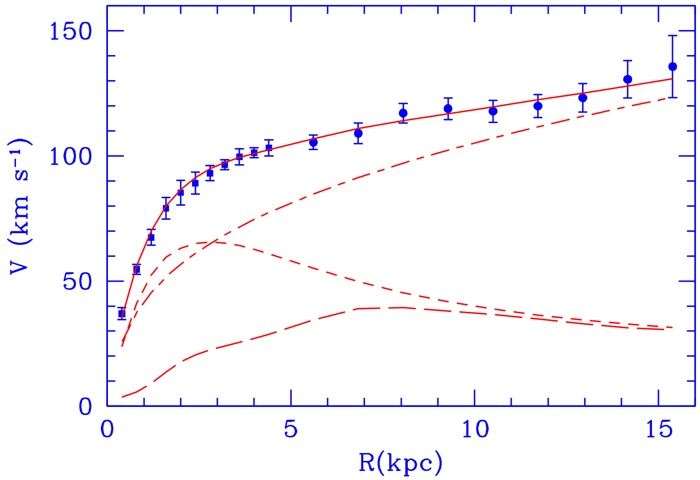
$$v_{\mathrm{MOND}} = (a_0 G_N M(r))^{1/4} = \left(a_0 r \sum_i v_{Ni}^2 \right)^{1/4}$$

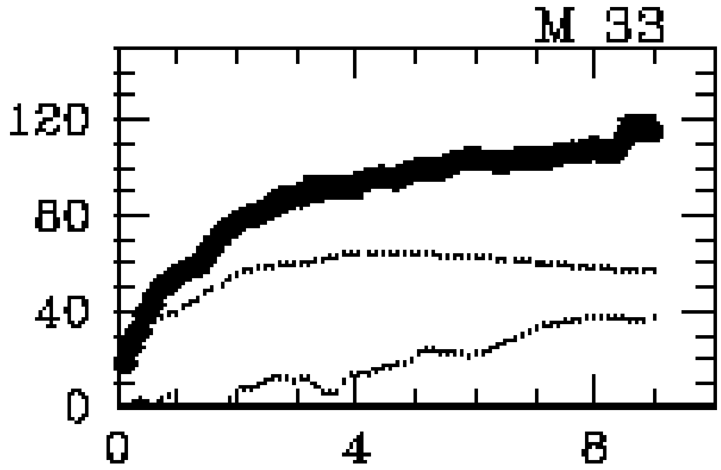
Best value for fit

$$a_0 \simeq 1.2 \times 10^{-10} \mathrm{ms}^{-2}$$


NGC 5533
NGC 5907
NGC 2998
NGC 801
NGC 5371
NGC 6946
NGC 2403
UGC 128
NGC 6503
M 33
NGC 7793
NGC 300
NGC 5585
NGC 55
IC 2574

A Closer Look





DM fit
Corbelli & Salucci

MOND fit
Sanders & McGaugh

$$v^2 = \sum_i v_i^2 + v_{DM}^2$$

or

$$v_{\rm MOND} = \left(a_0 r \sum_i v_{Ni}^2\right)^{1/4}$$

Some Theoretical Isues with MOND

- Conservation of momentum and kinetic energy. Consider binary system with one member has $|\mathbf{a}| < a_0$ but not both. Then $d/dt(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) \neq 0$.
- More than one source of gravity

$$\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2$$

$|\mathbf{a}_1| > a_0$, $|\mathbf{a}_2| < a_0$, then

$$\mathbf{a}_1 = -\nabla\Phi_1 \qquad |\mathbf{a}_2|\mathbf{a}_2 = -a_0\nabla\Phi_2$$

$$\Phi \neq \Phi_1 + \Phi_2$$

- Flat forever? Once outside galaxy, v becomes constant.

AQUAL

(Aquadratic Lagrangian - Bekenstein & Milgrom 1984)

$$L = -\int \left[\frac{a_0^2}{8\pi G_N} F\left(\frac{|\nabla\Phi|^2}{a_0^2} \right) + \rho\Phi \right] d^3x$$

Newton's limit: $F(X) \to X$. General solution:

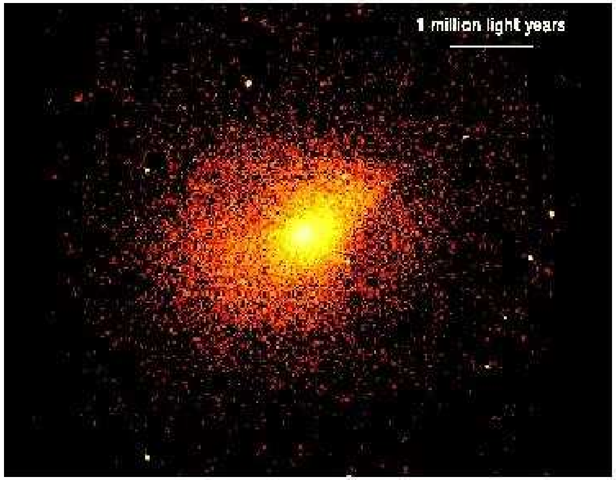
$$\nabla \cdot (\mu(|\nabla\Phi/a_0|)\nabla\Phi) = 4\pi G_N \rho$$

where $\mu(\sqrt{X}) \equiv F'(X)$. So MOND as modification of gravity, $\mathbf{a} = -\nabla\Phi$ with

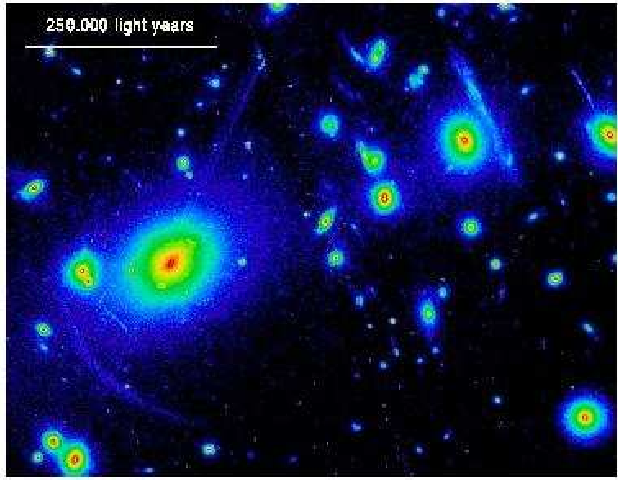
$$\mu(|\nabla\Phi/a_0)\nabla\Phi = \nabla\Phi_N + \nabla \times h$$

MOND's limit: $\nabla \times h \to 0$.
AQUAL satisfy momentum, angular momentum and energy conservation.

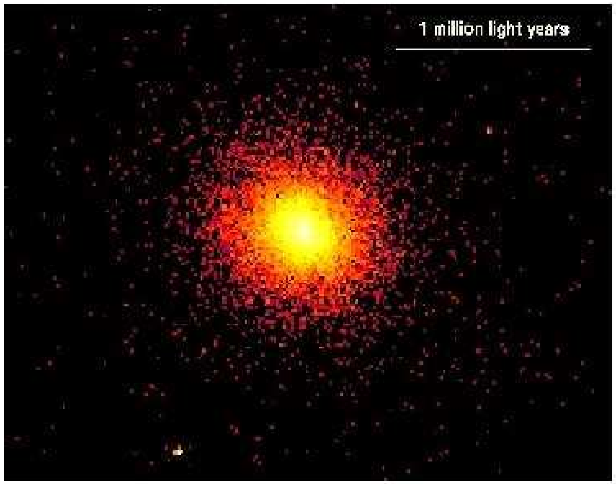
Gravitational Lensing



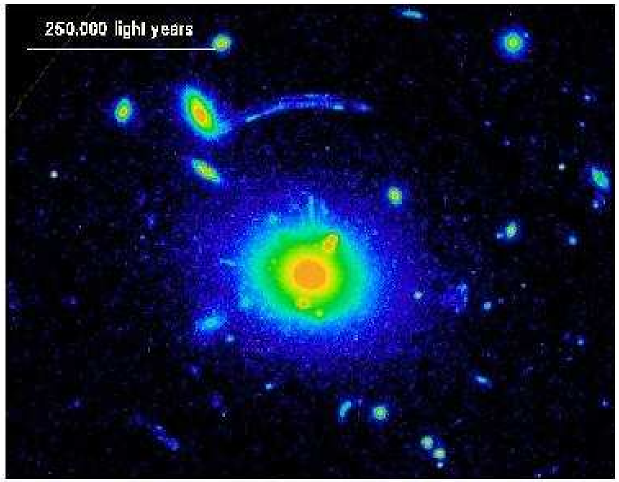
Abell 2390: Chandra (ACIS)



Abell 2390: HST (WFPC2)



MS2137.3-2353: Chandra (ACIS)



MS2137.3-2353: HST (WFPC2)

RAQUAL

(Relativistic AQUAL)

$$\text{Replace} \qquad g_{\alpha\beta} \longrightarrow \tilde{g}_{\alpha\beta} = e^{2\psi/c^2} g_{\alpha\beta}$$

where $g_{\alpha\beta}$ is the GR metric. From AQUAL

$$|\nabla\Phi|^2 \rightarrow g^{\alpha\beta}\partial_\alpha\psi\partial_\beta\psi \qquad d^3x \rightarrow (-g)^{1/2} d^3x$$

In the weak limit:

$$\Phi \simeq \Phi_N + \psi$$

RAQUAL still cannot deal with gavitational lensing. - Maxwell equation is conformally invariant.

TeVeS

(Tensor-Vector-Scalar theory of gravity - Bekenstein 2004)

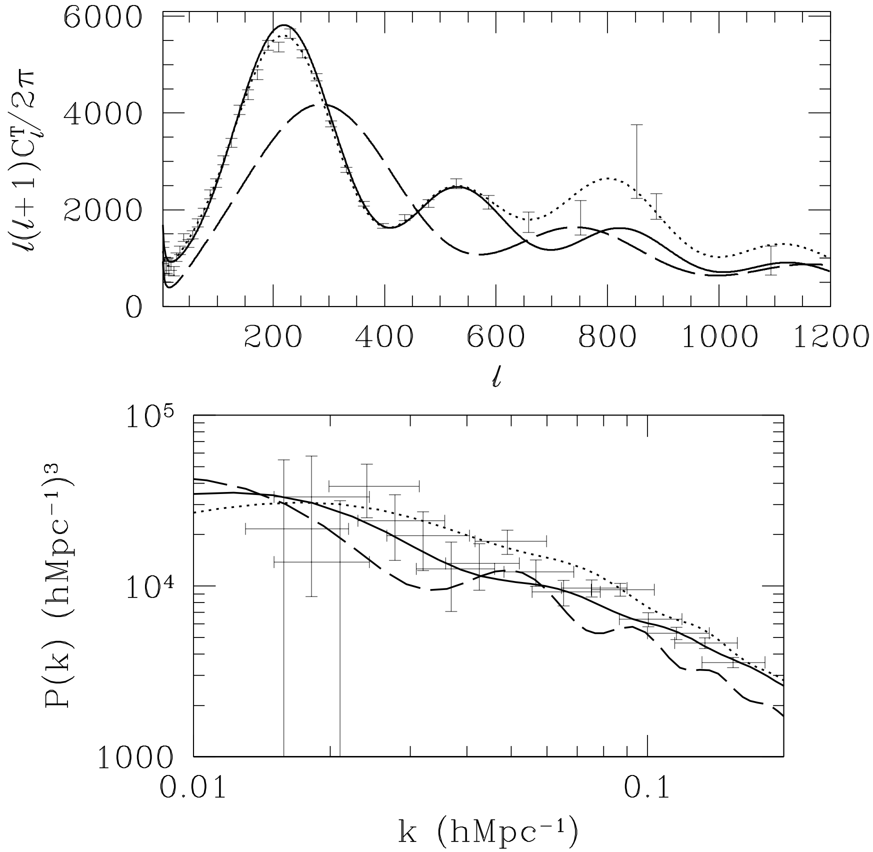
$$\tilde{g}_{\alpha\beta} = e^{-2\phi/c^2} g_{\alpha\beta} - (e^{2\phi/c^2} - e^{-2\phi/c^2})\mathcal{U}_\alpha \mathcal{U}_\beta$$

etc, etc, then

$$\Phi = \left[(1 - K/2)^{-1} e^{-2\phi_c/c^2}\right] \Phi_N + \phi$$

Line element

$$d\tilde{s}^2 = -(1 + 2\Phi/c^2)dt^2 + (1 - 2\Phi/c^2)(dx^2 + dy^2 + dz^2)$$



dotted = ΛCDM
solid = TeVeS $\Omega_\Lambda = 0.78$, $\Omega_\nu = 0.17$, $\Omega_B = 0.05$
dashed = TeVeS $\Omega_\Lambda = 0.95$, $\Omega_B = 0.05$

Skordis, Mota, Ferreira, Boehm (2006)

MOG

(Modified Gravity - Moffat 2004)

Formerly known as STVG = Scalar Tensor Vector Gravity

MOG action

$$S = \int (\mathcal{L}_G + \mathcal{L}_\phi + \mathcal{L}_S)\sqrt{-g}d^4x + S_M$$

where

$$\mathcal{L}_\phi = -\omega \left[\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi_\mu\phi^\mu + V_\phi(\phi)\right]$$

where $B_{\mu\nu} \equiv \partial_{[\mu}\phi_{\nu]}$; and

$$\begin{aligned}\mathcal{L}_S &= -\frac{1}{G}\left[\frac{1}{2}g^{\mu\nu}\left(\frac{\nabla_\mu G\nabla_\nu G}{G^2} - \frac{\nabla_\mu \mu\nabla_\nu \mu}{\mu^2} + \nabla_\mu\omega\nabla_\nu\omega\right)\right. \\ &\left. -\frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2} - V_\omega(\omega)\right]\end{aligned}$$

Modified Einstein equation:

$$\begin{aligned} & R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 8\pi G \omega \left(B_{\mu\kappa} B^{\kappa}_{\nu} - \frac{1}{4} g_{\mu\nu} B_{\kappa\lambda} B^{\kappa\lambda} \right) \\ & + 8\pi G \omega \mu^2 \phi_\mu \phi_\nu \\ = \ & 8\pi G T_{\mu\nu} \end{aligned}$$

Centripetal acceleration:

$$a = -\frac{GM}{r^2} + \beta\kappa\omega(1 + \mu r)\frac{e^{-\mu r}}{r^2}$$

Modified Einstein equation:

$$
\begin{aligned}
& R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 8\pi G\omega \left(B_{\mu\kappa} B^{\kappa}_{\nu} - \frac{1}{4} g_{\mu\nu} B_{\kappa\lambda} B^{\kappa\lambda} \right) \\
& + 8\pi G\omega\mu^2 \phi_\mu \phi_\nu \\
= \quad & 8\pi G T_{\mu\nu}
\end{aligned}
$$

Centripetal acceleration:

$$
a = -\frac{GM}{r^2} + \beta\kappa\omega(1 + \mu r)\frac{e^{-\mu r}}{r^2}
$$

Could this be dark matter profile?

“TeVeS gets caught on caustics”

Contaldi, Wiseman, Withers
arXiv: 0802.1215

ETG

(Extended Theories of Gravity)

See review by Capozziello & Francaviglia, 2007.

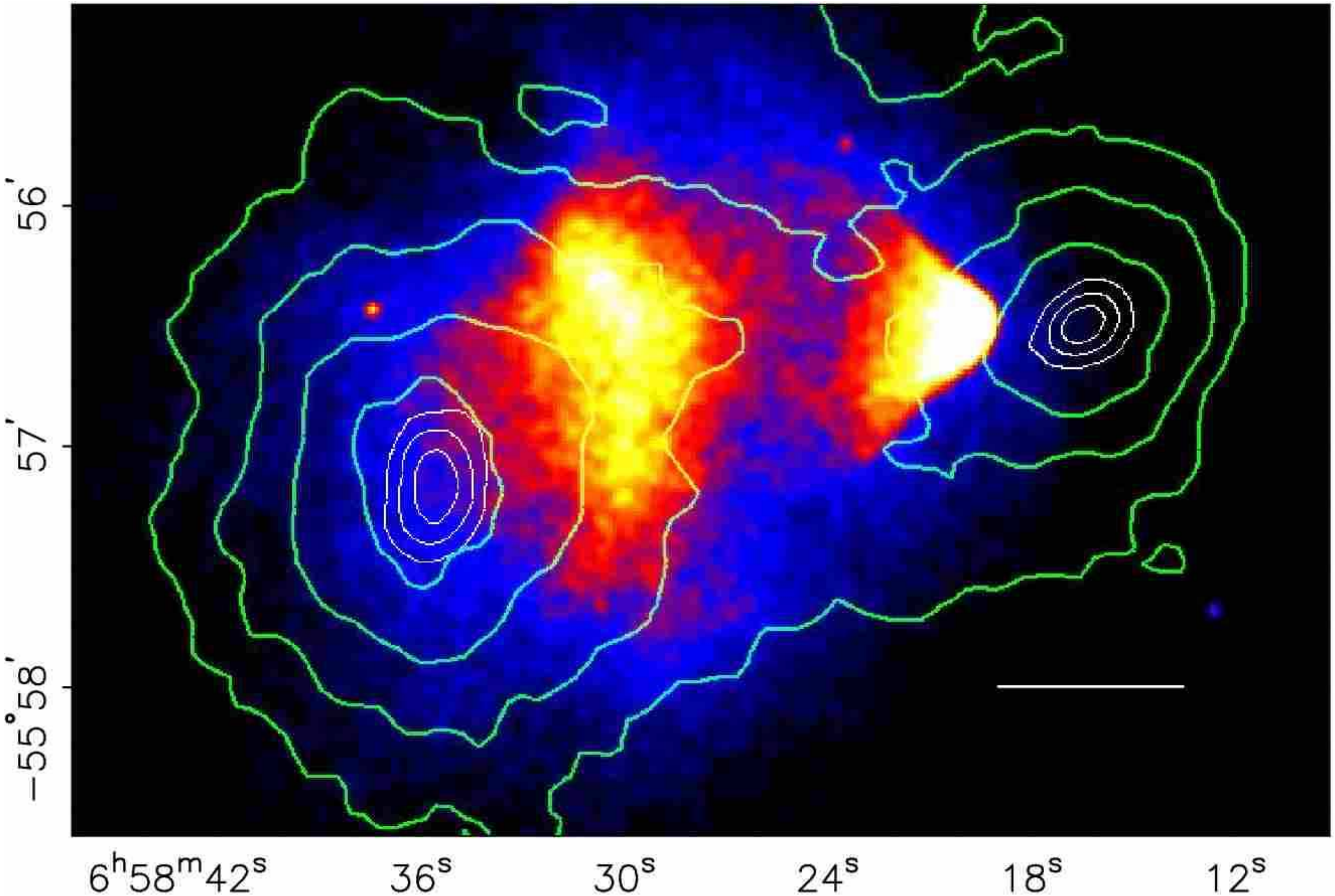
$$S = \int d^4x \sqrt{-g} \left[F(R, \Box R, \Box^2 R, \ldots, \Box^k R, \phi) - \frac{\epsilon}{2} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu} \right] + S_M$$

Simplest model - $f(R)$ gravity:

$$F = f(R) \qquad\qquad \epsilon = 0$$

...

The Bullet Cluster



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Misleading Coincidence?

Milgrom's law: DM effect (or deviation form Newtonian dynamics) appears when $a \sim 10^{-10}\mathrm{m/s}^2 \sim cH_0$.

Kaplinghat and Turner (2002), show by using formation theory connecting cosmological parameters and dark matter halo profile that indeed at the point when dark matter gravity begins to dominate

$$a_{\rm DM} \equiv \frac{G_N M(r_{\rm DM})}{r_{\rm DM}^2} \sim O(1) cH_0$$

so argue that this is just numerical coincidence.

So, what do you think?