



# The inclusion of theory errors in PDF fitting.

The NNPDF4.0MHOU PDFs set

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QCD@LHC2023

04/09/2023



# Outline.



## INTRODUCTION

- What are theory errors?
- How can we estimate them?
- Why is it relevant to include them in a PDF fit?



## METHODOLOGY AND VALIDATION

- How can we include MHOU in a PDF fit?
- Can we validate our estimation?



## RESULTS

- What is the impact on the PDFs at NLO and NNLO?
- What is the impact on the phenomenology?

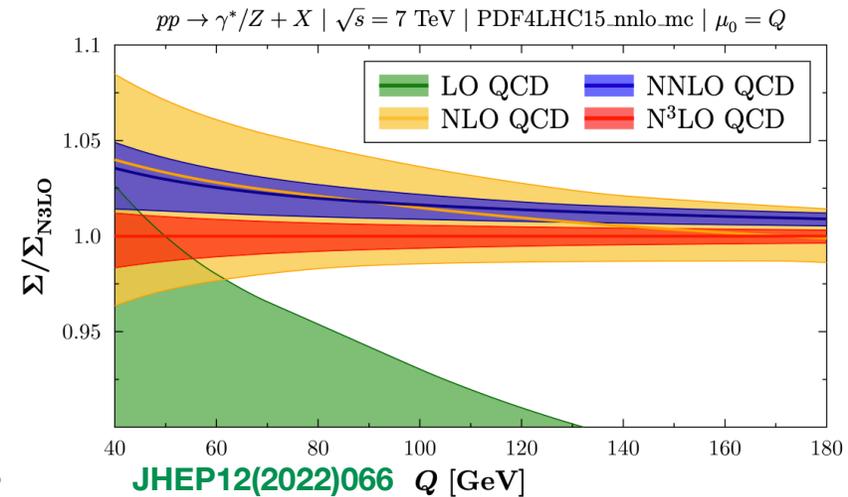
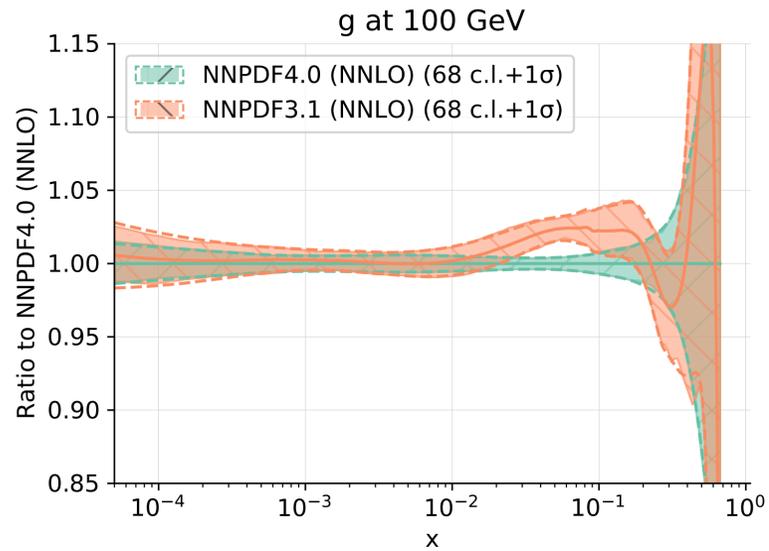
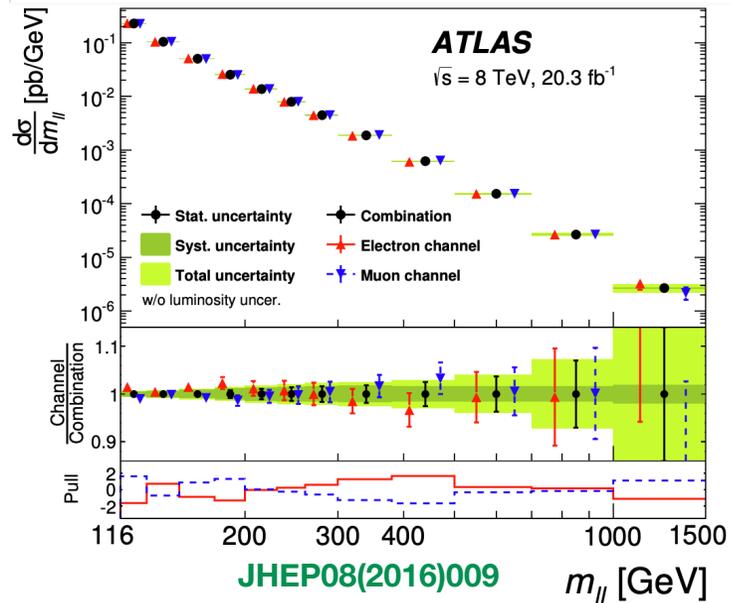
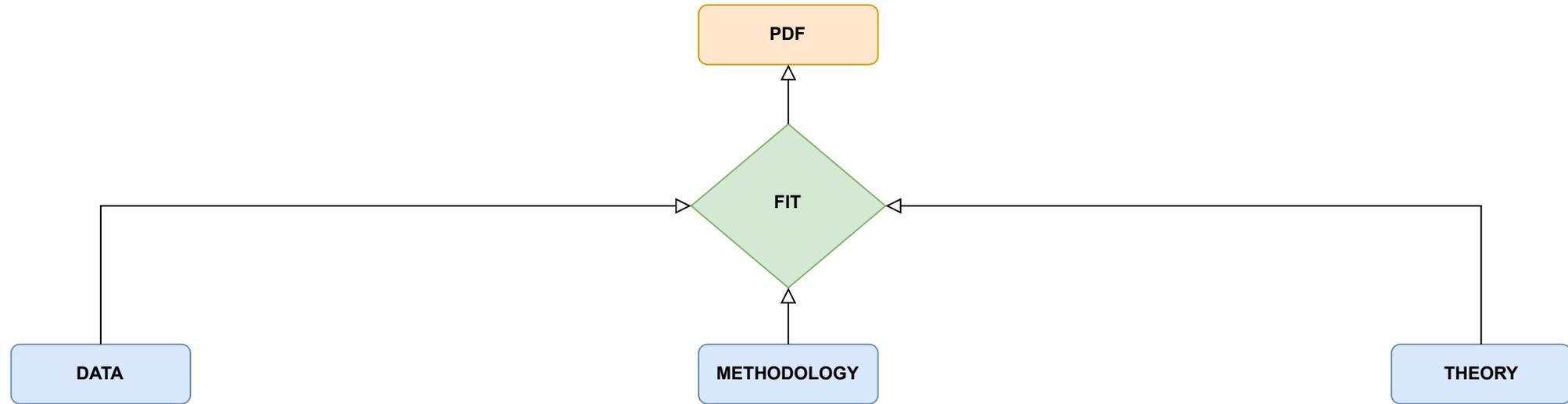


## INTRODUCTION

- What are theory errors?
- How can we estimate them?
- Why is it relevant to include them in a PDF fit?

*“We are not strangers, only the introduction is missing”  
(Jesus Apolinaris)*

# Motivation.



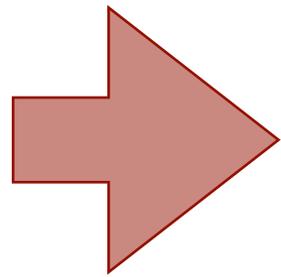
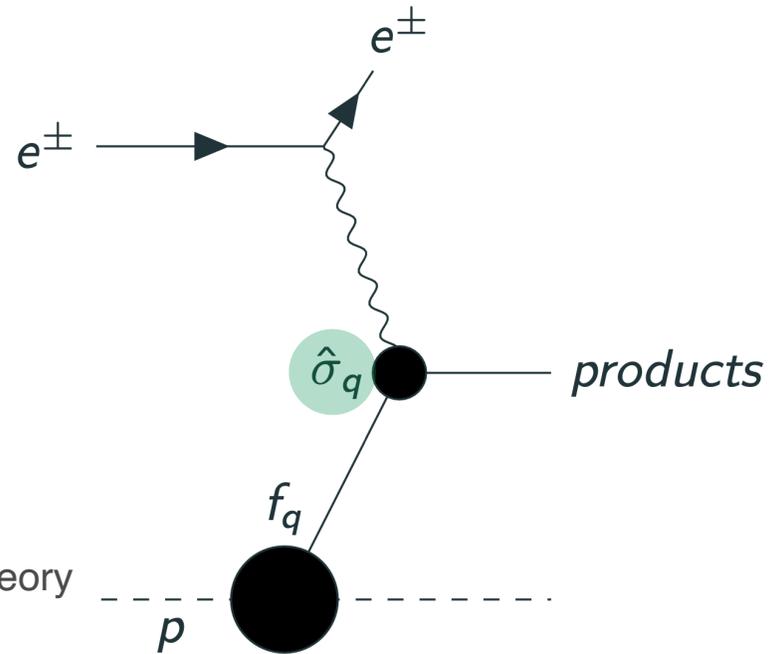
# Definition of the problem.

$$F(Q) = \hat{\sigma}(Q) \otimes U(Q, Q_0) \otimes f(Q_0)$$

- **Partonic cross sections** are computed in perturbation theory
- **DGLAP evolution operator** evolves the PDFs from  $Q_0$  to  $Q$
- **Anomalous dimensions** inside DGLAP operator are computed in perturbation theory

$$\hat{\sigma}^{NLO} = \hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \mathcal{O}(\alpha_s^2)$$
$$\gamma^{NLO} = \alpha_s \gamma^{(0)} + \alpha_s^2 \gamma^{(1)} + \mathcal{O}(\alpha_s^3)$$

## Deep Inelastic Scattering (DIS)



**MHOU**  
(Missing Higher Order Uncertainties)

How can we estimate them?

# Theory errors: estimation.

## Scale Variations

$$\bar{F}^{NLO}(\mu_f = \kappa_f Q, \mu_r = \kappa_r Q) - F^{NLO}(\mu_f = Q, \mu_r = Q) = \mathcal{O}(NNLO)$$

### Factorization scale

Estimates **MHOU** of anomalous dimensions

$$U^{NLO}(Q, Q_0) \rightarrow \bar{U}^{NLO}(Q, Q_0, \kappa_f)$$

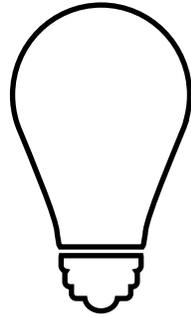
### Renormalization scale

Estimates **MHOU** of partonic cross sections

$$\hat{\sigma}^{NLO}(Q) \rightarrow \bar{\sigma}(Q, \kappa_r)$$



$\kappa_f, \kappa_r \in (0.5, 2.0)$  is the most common choice



## METHODOLOGY AND VALIDATION

- How can we include MHOU in a PDF fit?
- Can we validate our estimation?

*“Truth has nothing to do with the conclusion, and everything to do with the methodology”  
(Stefan Molyneux)*

# MHOU in a PDF fit: the *theory covmat*.



→ Minimization of a *loss function* depending on

Also, for a NNPDF fit...

→ Fit through a *Neural Network*

→ Delivery of a Montecarlo sample of fits on *replicas*



## FIT WITHOUT THEORY ERRORS

$$\chi^2 \propto (D_i - T_i) C_{ij}^{-1} (D_j - T_j)$$

$$\text{Pseudodata replica} \propto C$$

## FIT WITH THEORY ERRORS

$$\chi^2 \propto (D_i - T_i) (C + S)_{ij}^{-1} (D_j - T_j)$$

$$\text{Pseudodata replica} \propto C + S$$

# MHOU in a PDF fit: the *theory covmat*.

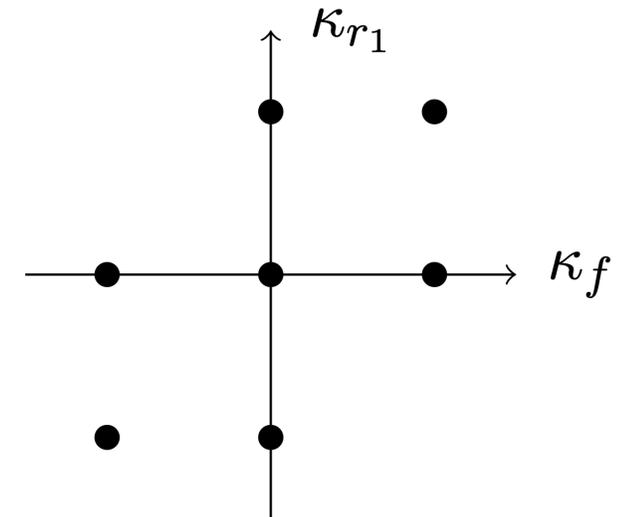
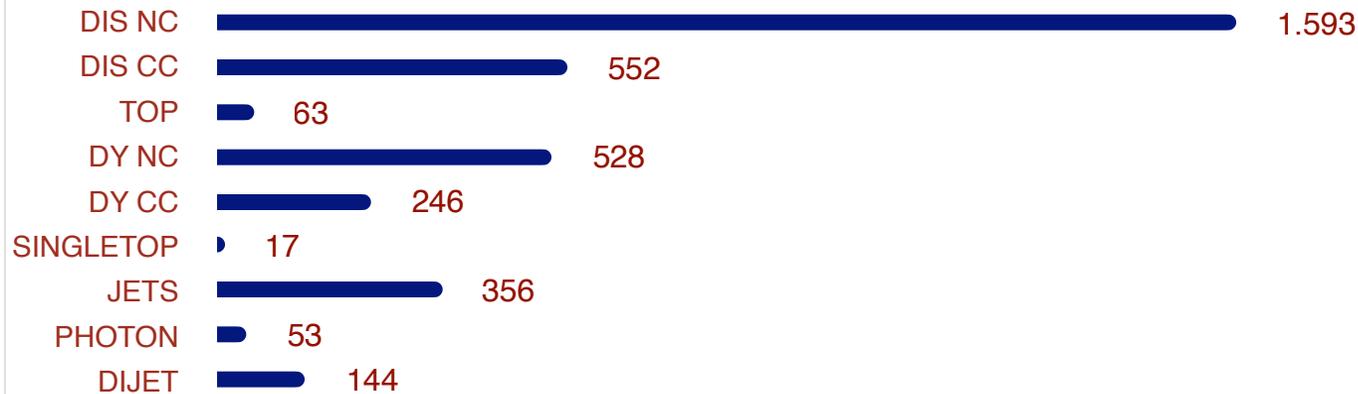


$$S_{ij} = n_m \sum_{V_m} \left( \bar{F}(\kappa_f, \kappa_{r_a}) - F \right)_{i_a} \left( \bar{F}(\kappa_f, \kappa_{r_b}) - F \right)_{j_b}$$

How to construct it

→ Factorization scale **correlates** all the points

→ Renormalization scale **correlates** points belonging to the same process



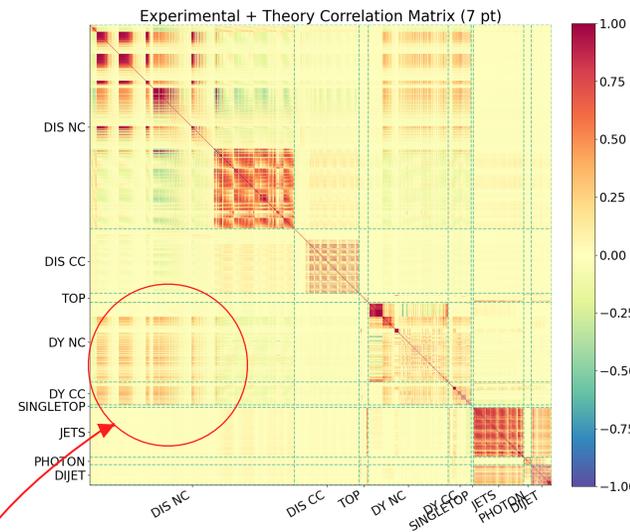
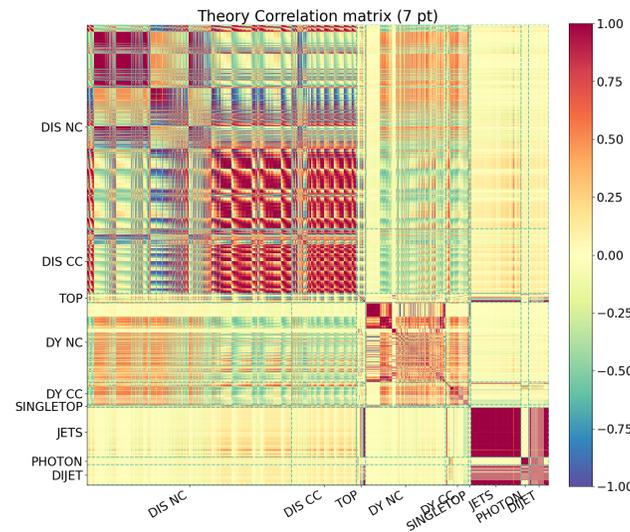
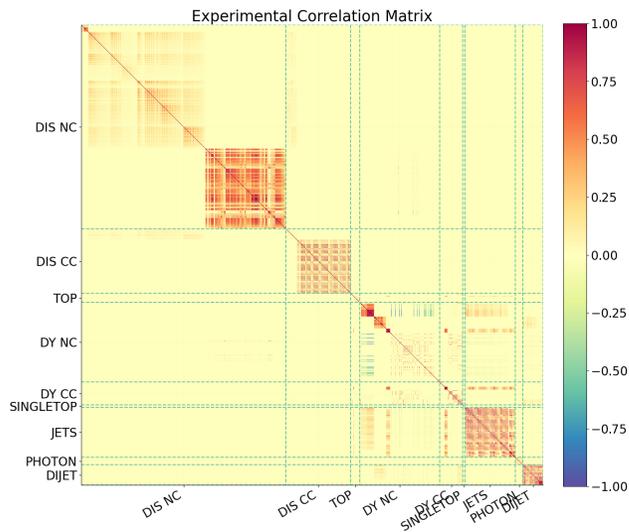
# How do they look like?

↓  
 $C$

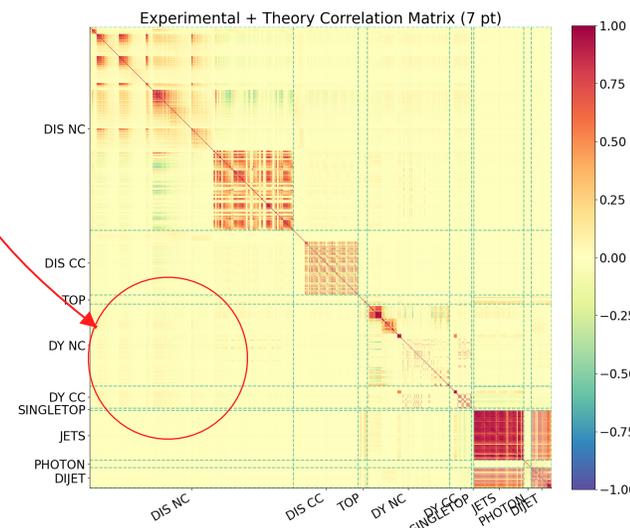
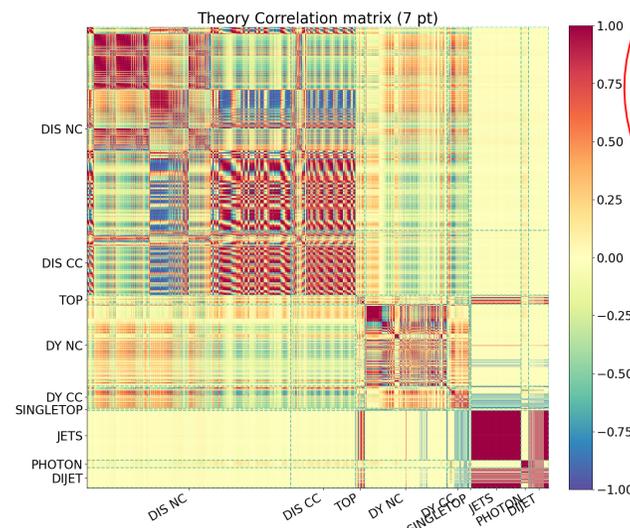
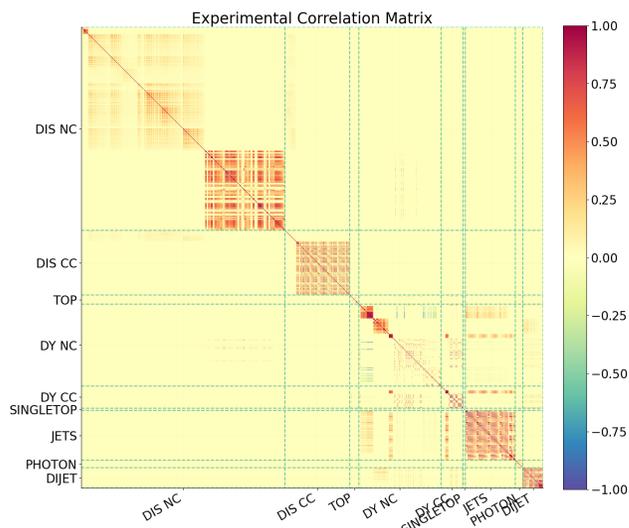
↓  
 $S$

↓  
 $C + S$

→  $NLO$

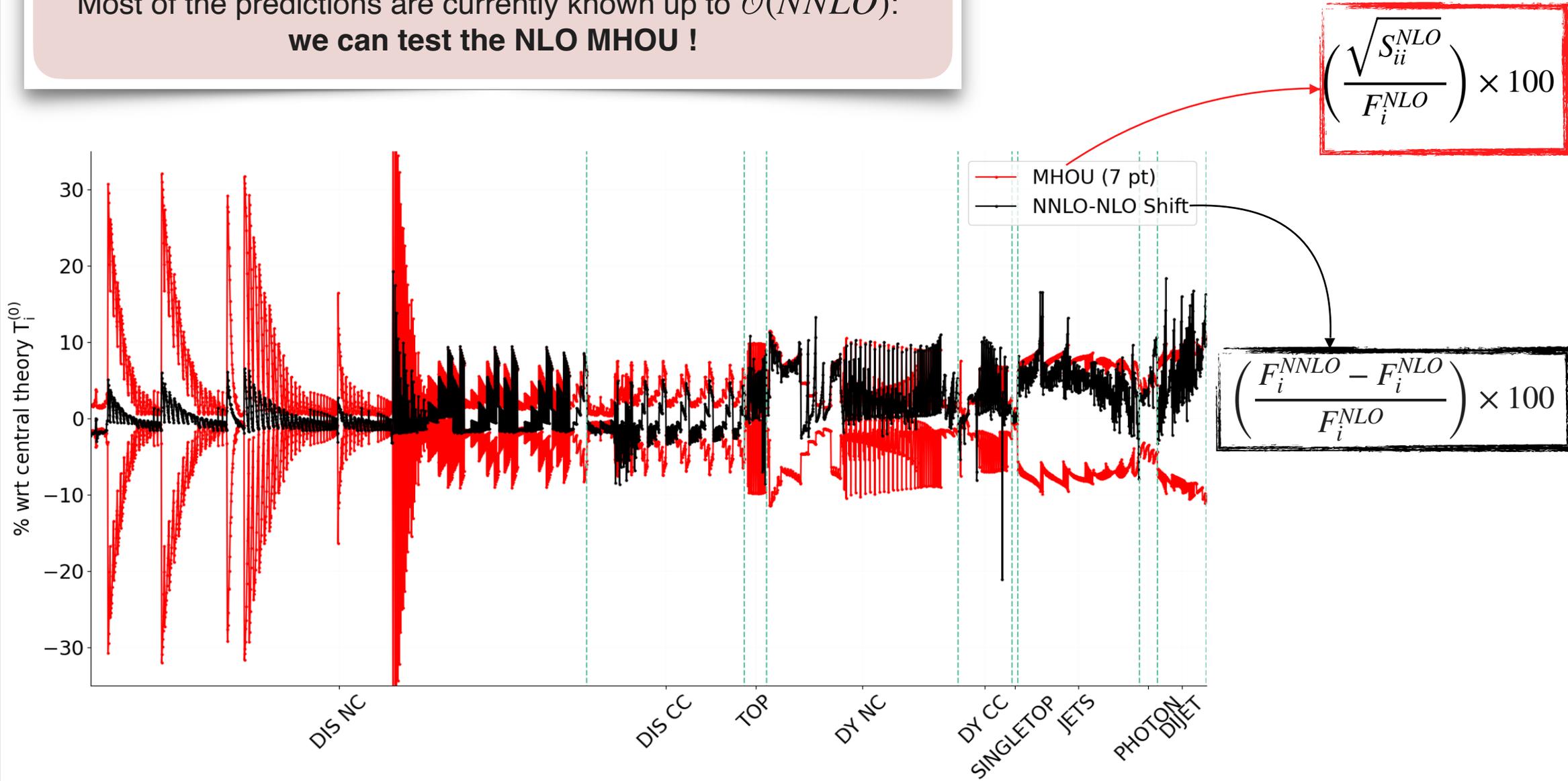


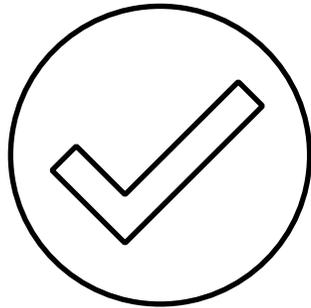
→  $NNLO$



# Validation: is it reproducing the known MHOUs?

Most of the predictions are currently known up to  $\mathcal{O}(NNLO)$ :  
we can test the NLO MHOUs !





## RESULTS

- What is the impact on the PDFs at NLO and NNLO?
- What is the impact on the phenomenology?

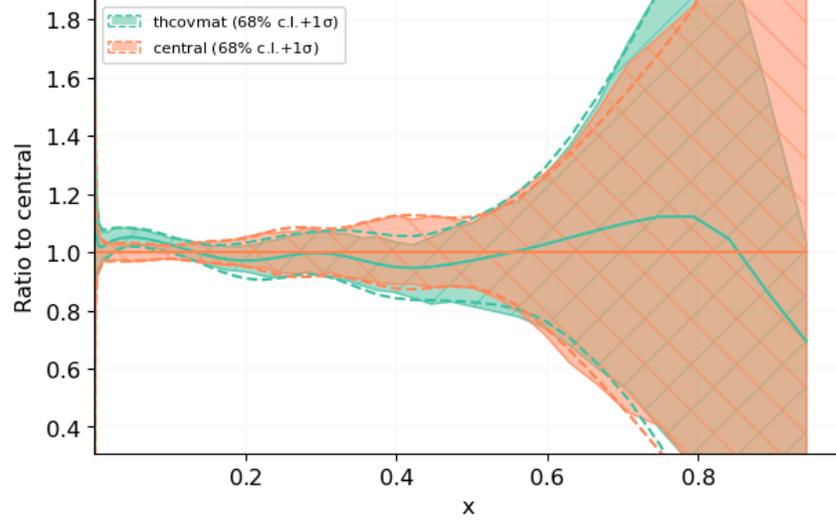
*“We're always, by the way, in fundamental physics, always trying to investigate those things in which we don't understand the conclusions. After we've checked them enough, we're okay”*

*(Richard P. Feynman)*

# PDF central values change: NLO.

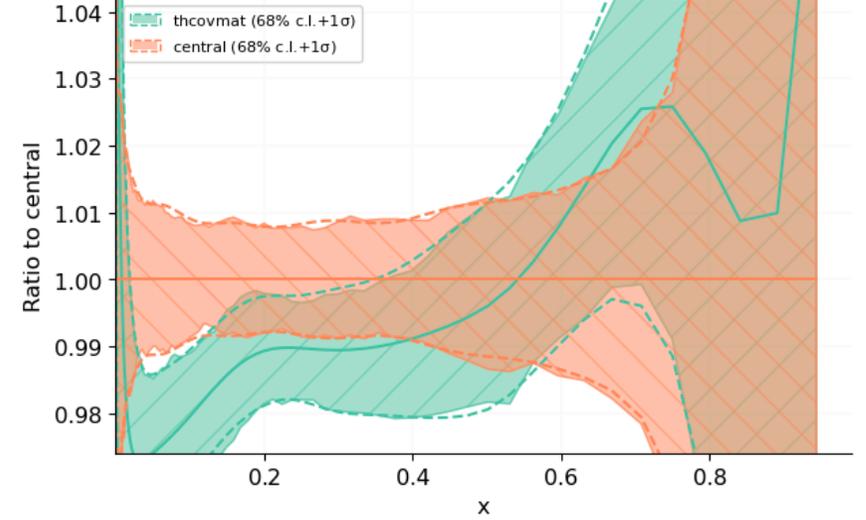
$g$

$g$  at 1.651 GeV



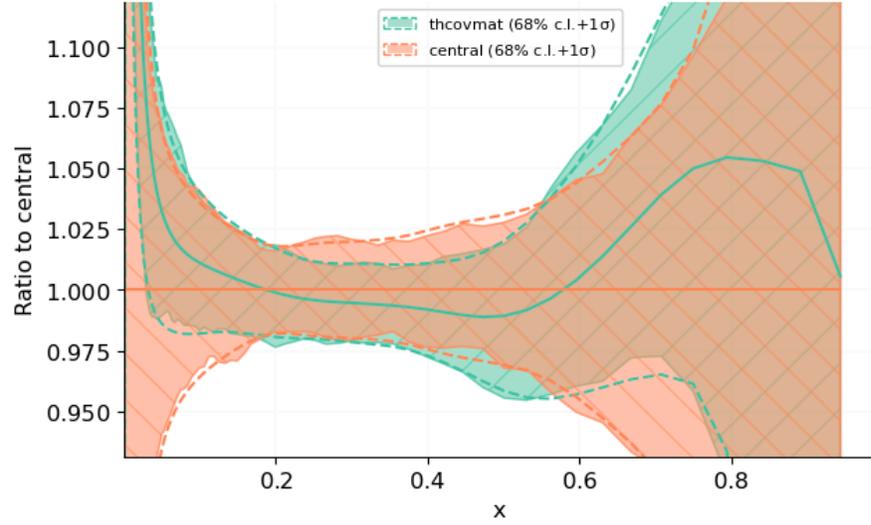
$\Sigma$

$\Sigma$  at 1.651 GeV



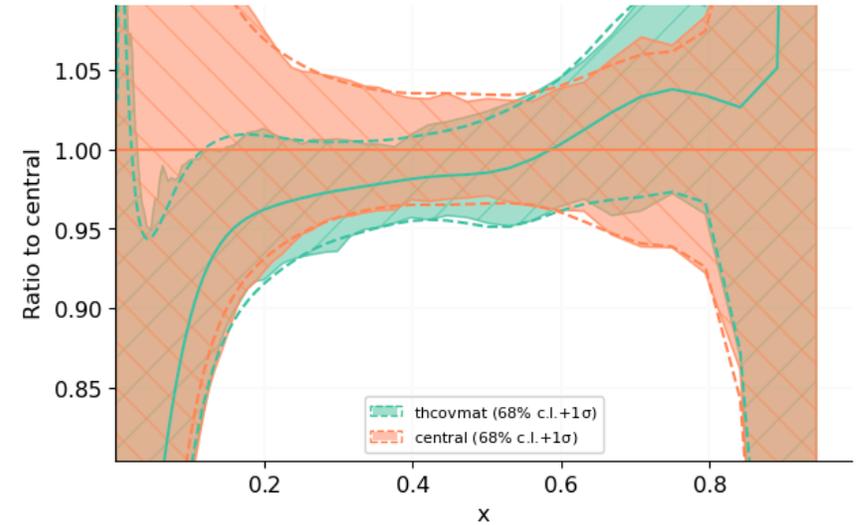
$V$

$V$  at 1.651 GeV



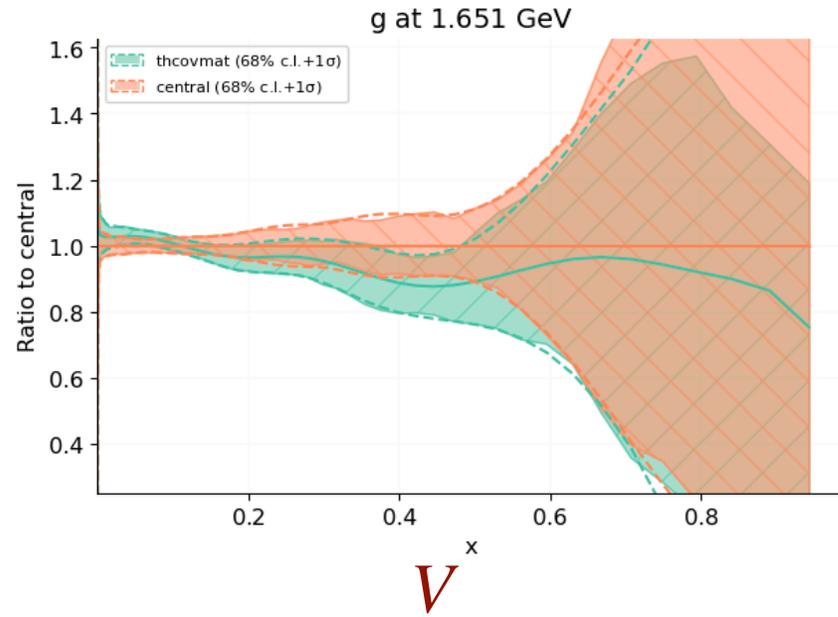
$T3$

$T3$  at 1.651 GeV

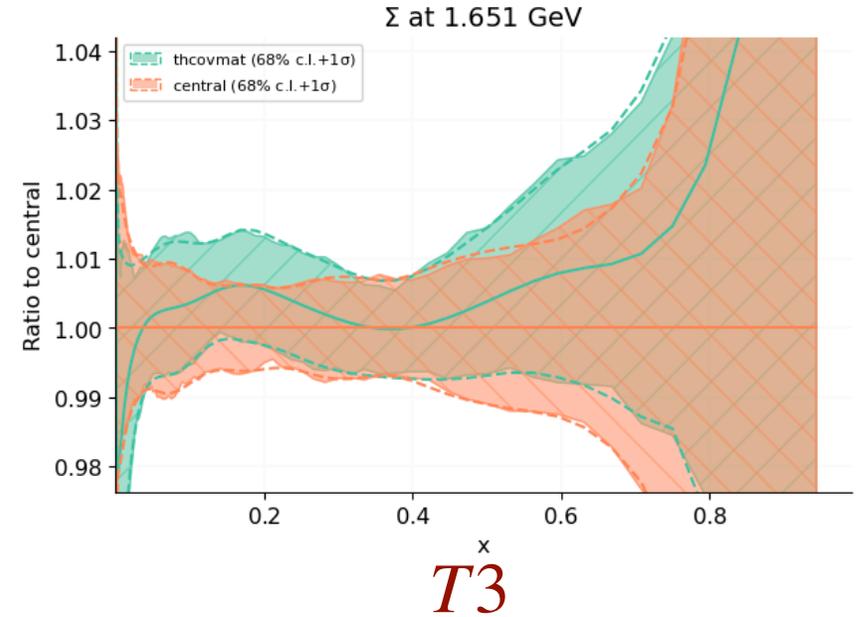


# PDF central values change: NNLO.

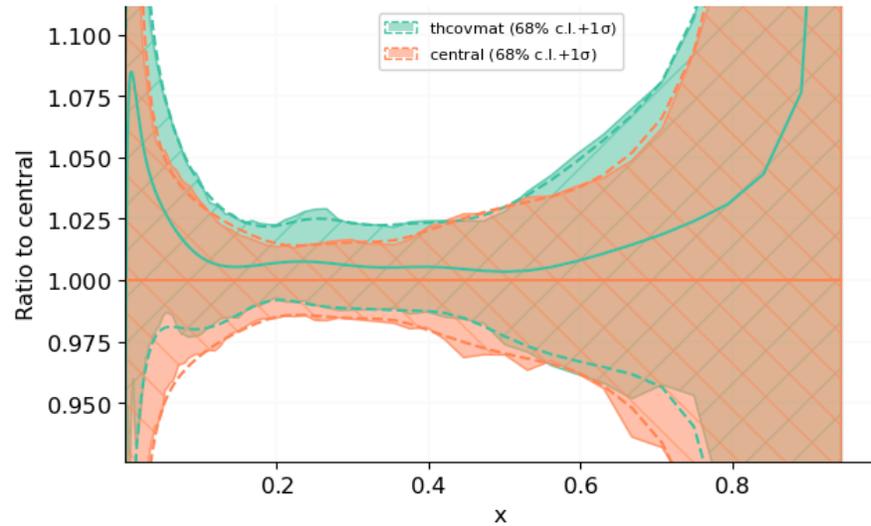
$g$



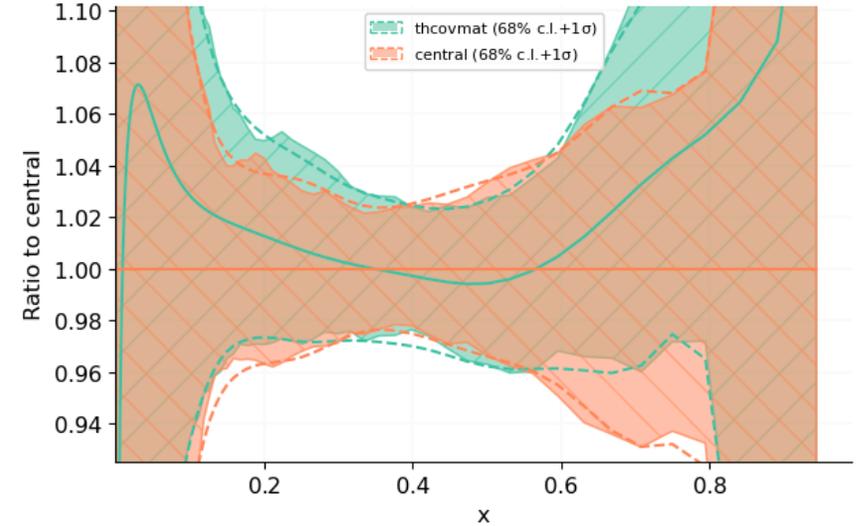
$\Sigma$



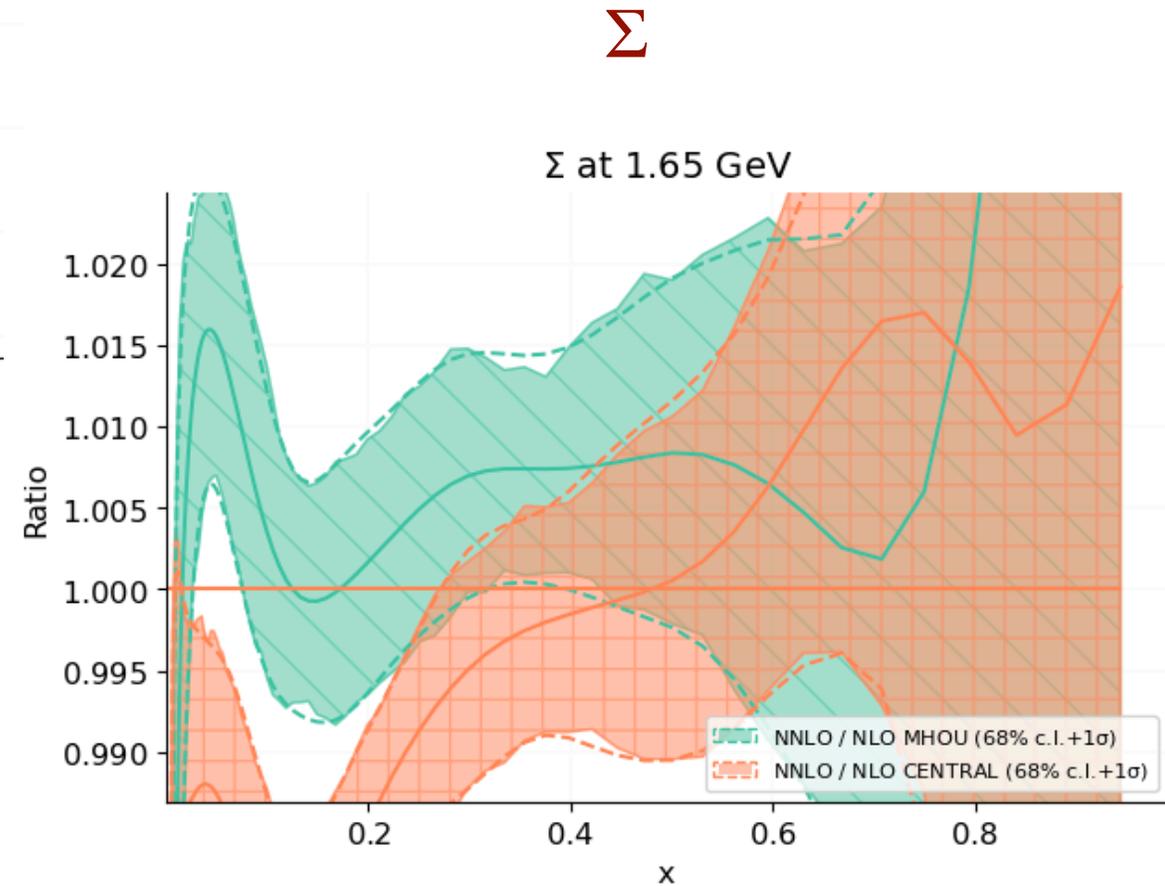
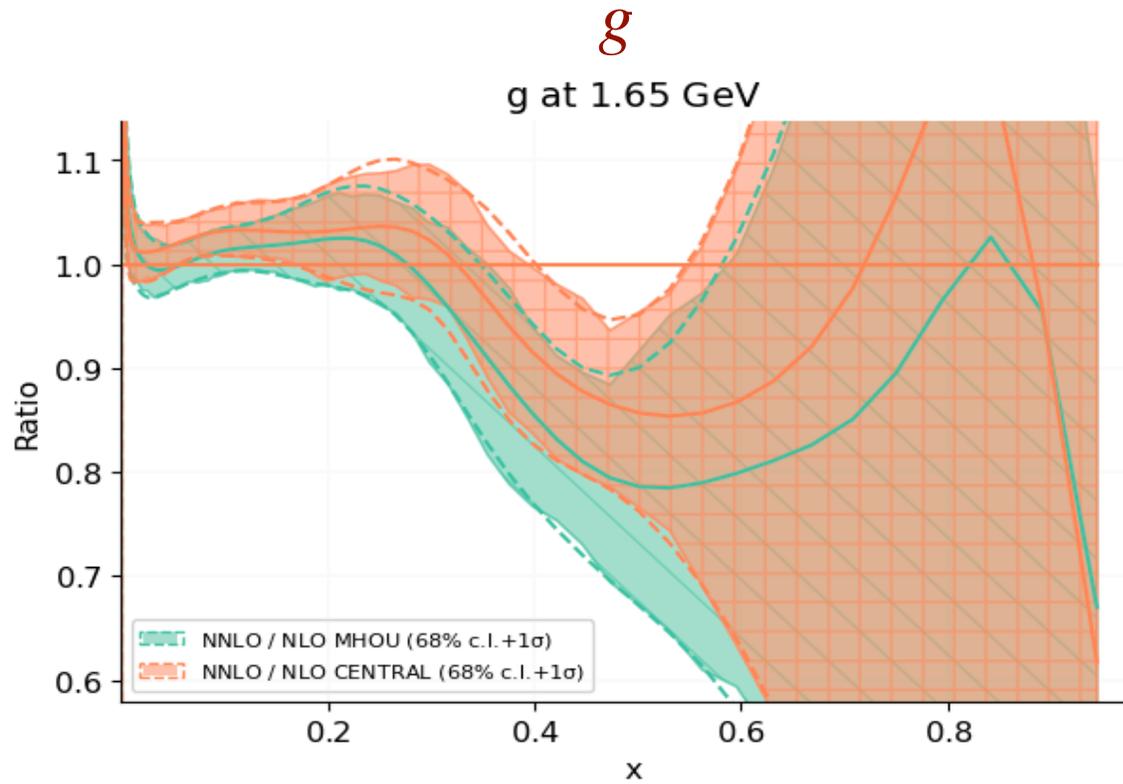
V at 1.651 GeV



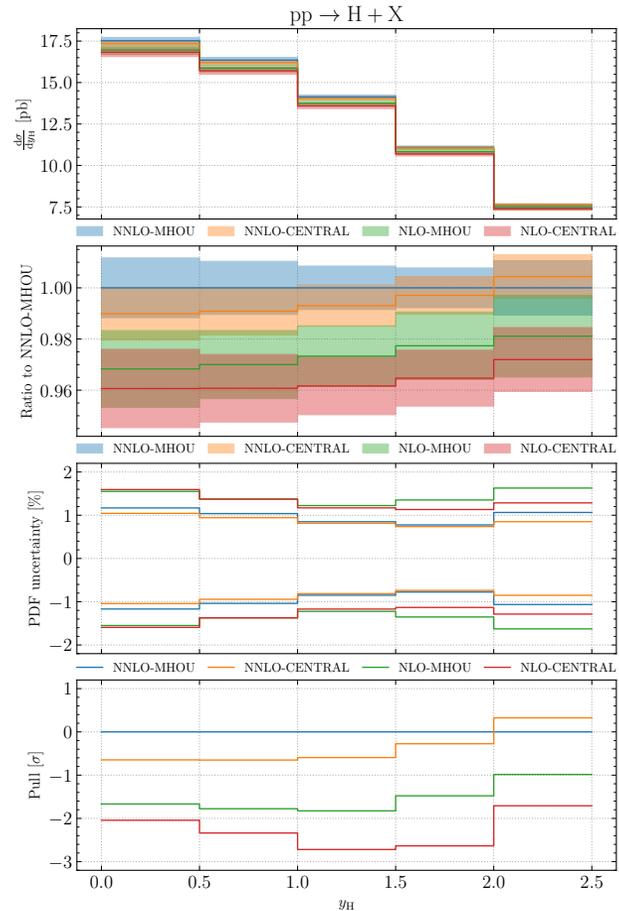
T3 at 1.651 GeV



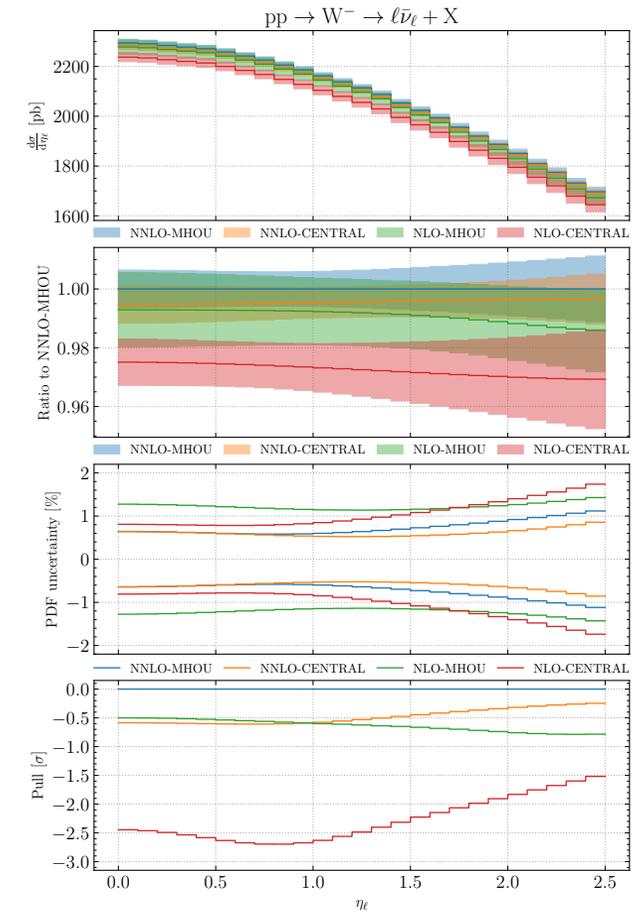
# The perturbative convergence : NLO vs NNLO.



# PDFs comparison in phenomenology.



NLO-MHOU is closer to NNLO than NLO-CENTRAL!



Using these MHOU PDF sets still requires the addition of the usual MHOU in the **partonic cross sections**

# Conclusions.

- Thanks to *scale variations* it is possible to estimate MHOUs while, thanks to the theory covmat formalism, it is possible to include such estimation in a PDF fit
- Including MHOUs in a PDF fit is necessary to have faithful uncertainties and central values
- The perturbative convergence from NLO to NNLO improves once theory errors are accounted for

Thanks for your attention!