

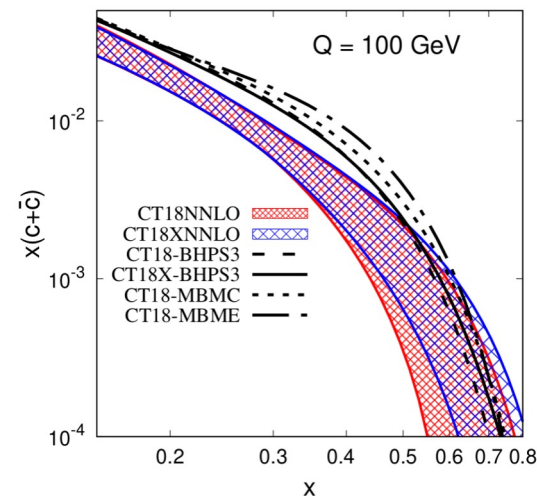
# CT18 FC: heavy quarks in CTEQ-TEA QCD analyses

Tim Hobbs, ANL

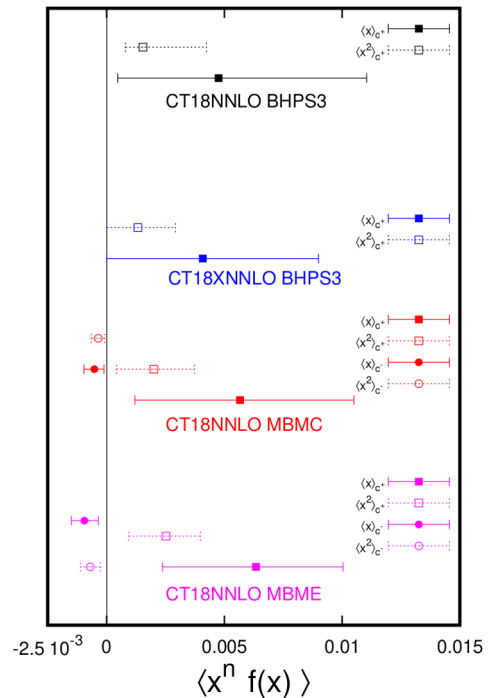


recently published:

[PLB 843 \(2023\) 137975](#)



Nonperturbative charm moments  $Q_0 = 1.27$  GeV  
Intervals of  $\Delta\chi^2 < 10$



with M. Guzzi, K. Xie, J. Huston,  
P. Nadolsky, C.-P. Yuan

and members of the  
[CTEQ-TEA](#) (Tung Et. Al.) working group

see also: [1707.00657](#)  
and [2205.10444](#)

# References

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HQs, charm in the proton have extensive literature...

## Heavy quarks in pQCD calculations

1. ACOT: Aivazis, Collins, Olness, Tung; PRD50 (1994) 3102-3138
2. NC DIS, NNLO: Guzzi, Nadolsky, Lai, Yuan; PRD86 (2012) 053005
3. CC DIS, NNLO: Gao, TJH, Nadolsky, Sun, Yuan; PRD105 (2022) 1, L011503

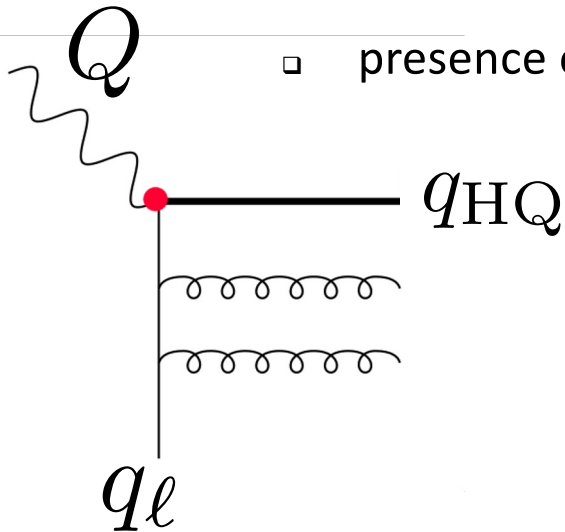
## Intrinsic charm from nonperturbative methods and models:

1. BHPS: Brodsky, Hoyer, Peterson, Sakai, PLB 93 (1980) 451
2. Meson-Baryon models (MBM): TJH, Londergan, Melnitchouk, PRD 89 (2014) 074008
3. Light-front WF models: TJH, Alberg, Miller, PRD 96 (2017) 7, 074023

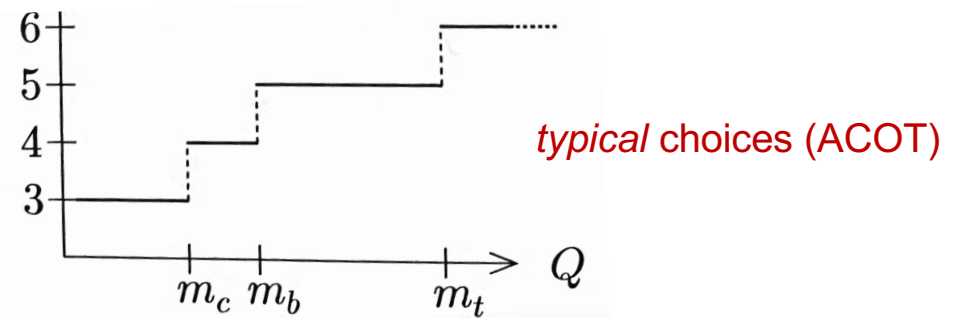
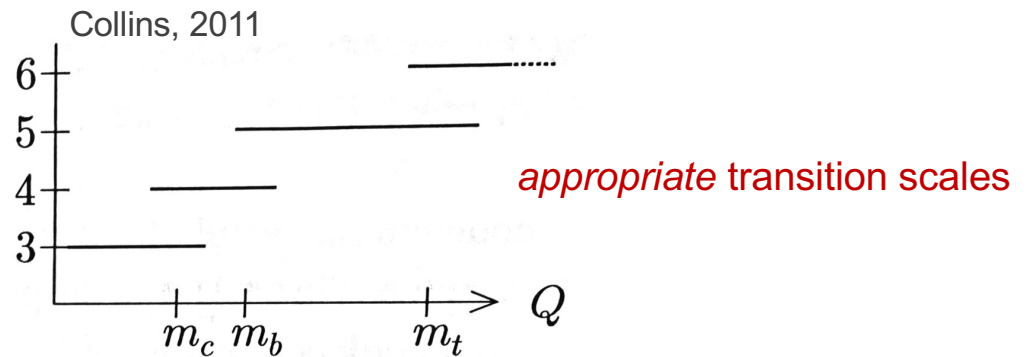
## QCD analyses of nonperturbative charm

1. P. Jimenez-Delgado, TJH, Londergan, Melnitchouk, PRL114 (2015) 8, 082002; NLO fit; first analysis with EMC data
2. T.-J. Hou et al., JHEP 02 (2018) 059; QCD factorization with the NP charm and CT14 IC NNLO pheno analysis
- 3. M. Guzzi, TJH, K. Xie, et al., arXiv:2211.01387; **new CT18 FC analysis with LHC Run-1 and 2 data**

# QCD analyses traverse many scales, $Q$ ; variable flavor #



- presence of HQ introduces an additional effective scale into hard process



$$Q \gg m_{\text{HQ}}$$

- perturbatively-generated HQ  
PDFs resum large logs,  
 $\sim \ln(Q^2/m_{\text{HQ}}^2)$

→ number of active parton flavors is a scheme-dependent choice

- PDF fits typically assume a *variable flavor number* scheme with assumed number of active flavors; usually  $n_F = 5$

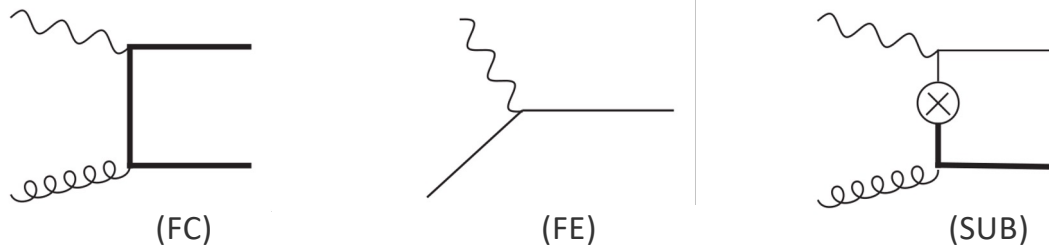
# general-mass schemes: S-ACOT- $\chi$

variable flavor-number scheme to interpolate between ZM and FFN regimes: **ACOT**

Aivazis, Collins, Olness, Tung; PRD50 (1994) 3085-3118

→ systematic approach to incorporating HQ mass dependence

introduce subtraction term(s) to eliminate double counting between FC/FE contributions:



$$\left\{ \begin{array}{l} Q \gtrsim M_Q \implies (\text{SUB}) \approx (\text{FE}) \text{ such that } n_f = 3 \text{ FC dominates} \\ Q \gg M_Q \implies (\text{SUB}) \approx (\text{FC}) \text{ such that } n_f = 4 \text{ FE dominates} \end{array} \right.$$

$$\chi(x, Q, M_Q) = x \left( 1 + \frac{1}{Q^2} \left[ \sum_{\text{F.S.}} M_Q \right]^2 \right)$$

“simplified” ACOT (S-ACOT): neglect full HQ mass dependence in FE graphs

**[CT default]** S-ACOT- $\chi$ : smooth HQ thresholds, approx. HQ mass dependence:  $C_i(x) \rightarrow C_i(\chi)$

requires careful tracking of diagrams to organize calculation correctly

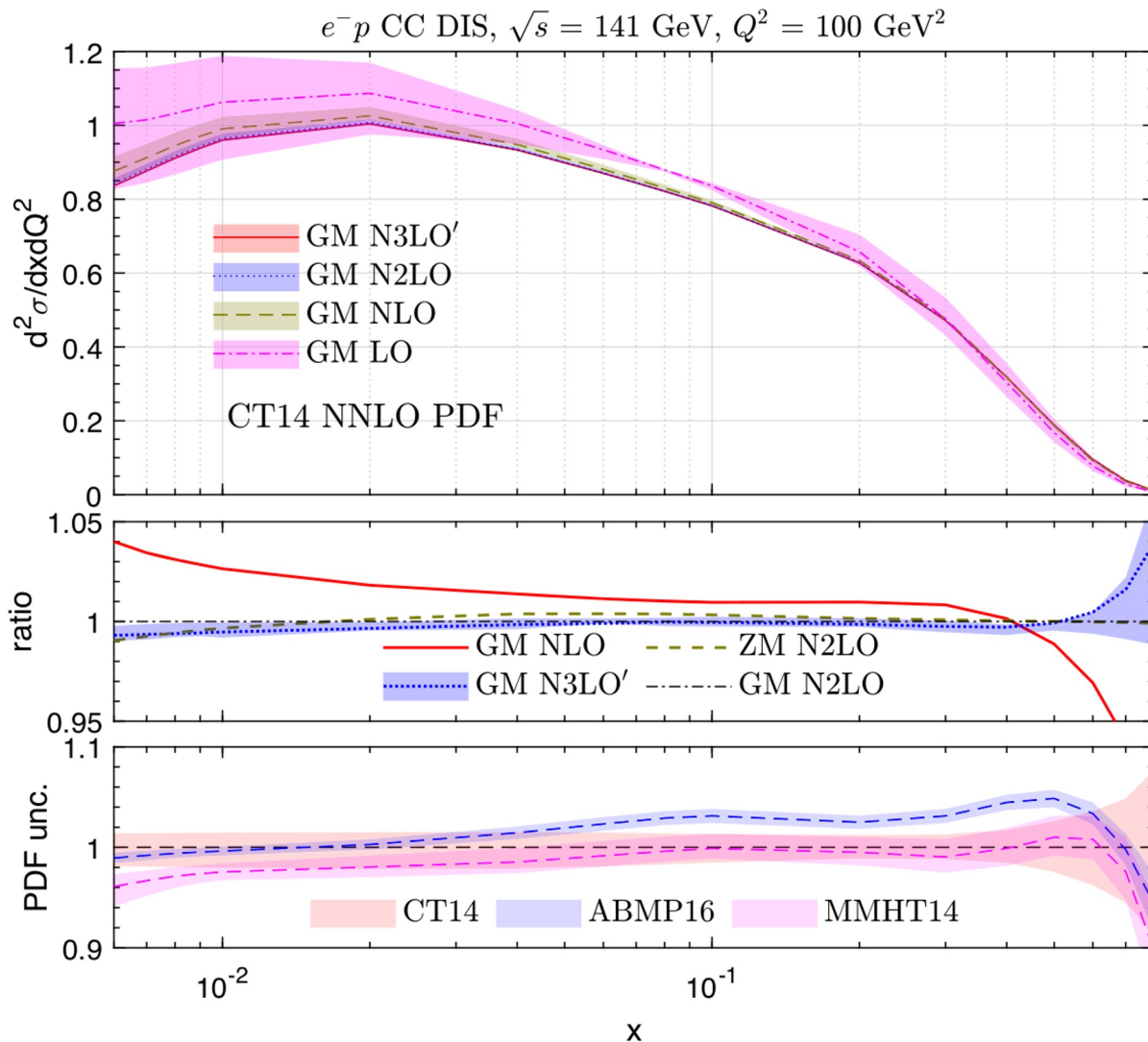
# implications for PDF extractions at precise measurements

NNLO formalism generalizes to other DIS processes, e.g., CC DIS

PRD105 (2022) 1, L011503

→ Inclusive Reactions Study (YR7.1.1): CC DIS – including positron beam – may access  $d, s$  PDFs

arXiv: 2103.05419



(high-energy EIC collisions)

→ for N<sup>3</sup>LO', scale variations generally contained to  $\lesssim 0.5 - 1\%$

→ strong perturbative convergence

significantly smaller than PDF-driven uncertainties, which can be as large as  $\approx 2\%$

control over **subtle QCD theory** for heavy quarks may be important for PDF extractions at EIC, other expts

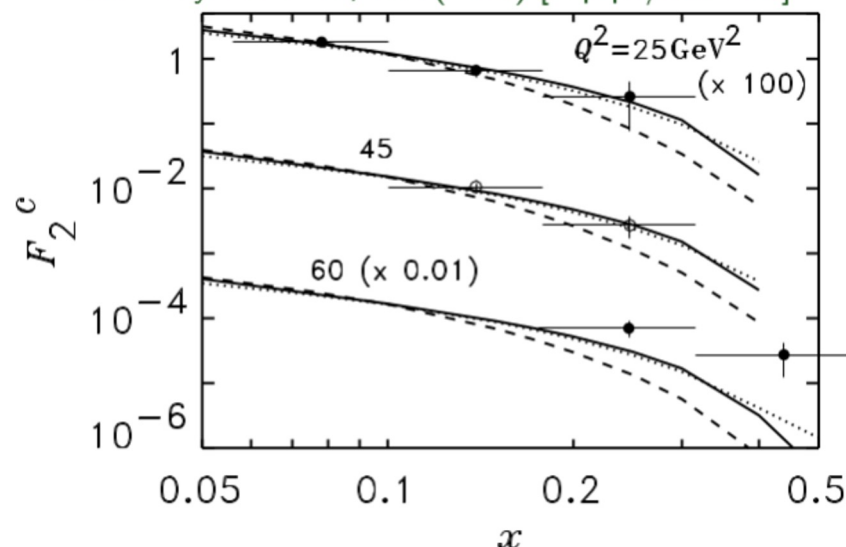
# possible subtlety: nonperturbative heavy-quark PDFs --- ‘intrinsic charm’

$$F_i = C_i \otimes f_{c/p}$$

might also explore *nonperturbative* charm; *i.e.*, not radiatively generated,

$$c(x, Q = m_c) = c^{\text{IC}}(x) \neq 0$$

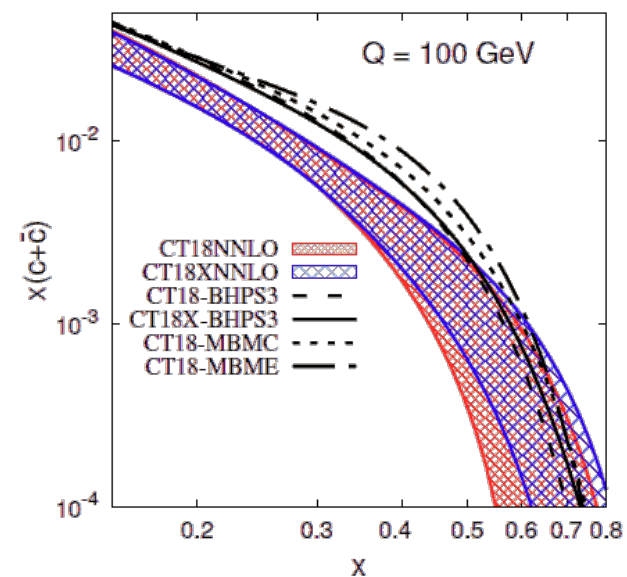
F. M. Steffens, W. Melnitchouk and A. W. Thomas, Eur. Phys. J. C 11, 673 (1999) [hep-ph/9903441].



but can global PDF fits constrain intrinsic charm?

“fitted charm” is a more direct term to describe the charm PDF found in the global QCD fit

analog: the fitted charm mass



# PDF fits may include a fitted charm PDF

fitted charm = “higher-twist charm”  
+ other (possibly not universal)  
higher  $O(\alpha_s)$ , higher-power terms

QCD factorization theorem for DIS structure function  $F(x, Q)$  [Collins, 1998]:

All  $\alpha_s$  orders: 
$$F(x, Q) = \sum_{a=0}^{N_f} \int_x^1 \frac{d\xi}{\xi} \mathcal{C}_a \left( \frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_c}{\mu}; \alpha(\mu) \right) f_{a/p}(\xi, \mu) + \mathcal{O}(\Lambda^2/m_c^2, \Lambda^2/Q^2)$$

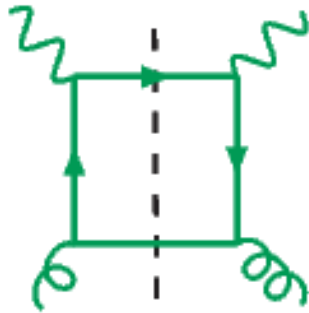
PDF fits implement this formula only up to (N)NLO ( $N_{ord} = 1$  or  $2$ ):

PDF fits: 
$$F(x, Q)^{[\text{trunc}]} = \sum_{a=0}^{N_f} \int_x^1 \frac{d\xi}{\xi} \mathcal{C}_a^{(N_{ord})} \left( \frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_c}{\mu}; \alpha(\mu) \right) f_{a/p}^{(N_{ord})}(\xi, \mu)$$

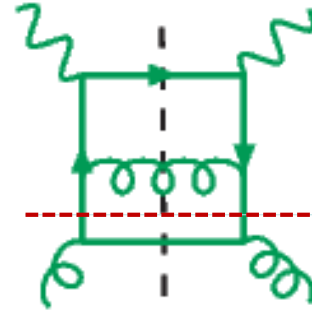
leading-power charm PDF component cancels at  $Q \approx m_c$ , up to a higher order  
fitted charm component may potentially **absorb missing terms** of orders  $\alpha_s^p$  with  $p > N_{ord}$ , or  $\Lambda^2/m_c^2$ , or  $\Lambda^2/Q^2$

# A twist-4 contribution in HERA DIS charm production (⊂ “intrinsic charm”)

Twist-2  
 $\gamma^* g \rightarrow c\bar{c}$



Order  $\alpha_s(Q)$

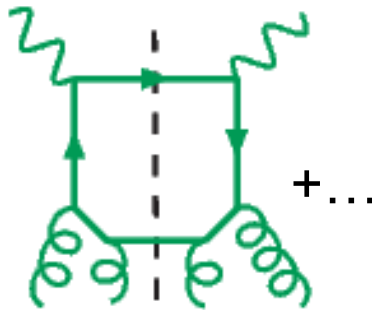


$\alpha_s^2(Q) \cdot \ln(Q^2/m_c^2)$

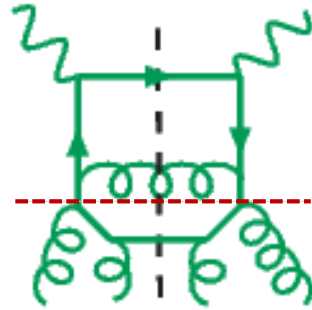


A ladder; must be resummed in  $c(x, Q)$  in the  $N_f = 4$  scheme at  $Q^2 \gg m_c^2$ ; e.g., in the ACOT scheme

Twist-4  
 $\gamma^*(gg) \rightarrow c\bar{c}$



+...



$\alpha_s^3(Q) \cdot (\Lambda^2/m_c^2) \ln(Q^2/m_c^2)$



The ladder subgraphs can be resummed as a part of  $c(x, Q)$  in the  $N_f = 4$  scheme at  $Q^2 \gg m_c^2 > \Lambda^2$ ;

$\Lambda \lesssim 1 \text{ GeV}$

$\alpha_s^2(Q) \cdot (\Lambda^2/Q^2)$   
or  $\alpha_s^2(Q) \cdot (\Lambda^2/m_c^2)$

Collins, PRD58 (1998) 094002



Can be of order ~10% of the twist-2  $\alpha_s^2$  term

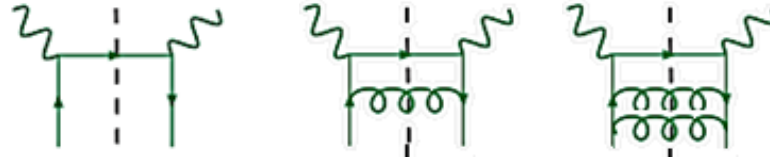
contributes to the boundary condition for  $c(x, Q_0)$  at  $Q_0 \approx m_c$ ;

obeys twist-2 DGLAP equations.



# fitted charm contributions, practical implementation in CT18

Keep only  $c_{h,h} \otimes f_h$ :  
 Discard  $c_{h,gg}^{(k)} \otimes f_{gg}$ , etc.



In the absence of full computation, we (and other groups) make the simplest approximation:

$$F_{FC}(x, Q_0) = [c_{h,h} \otimes f_{c/p}^{FC}](x, Q_0)$$

$c_{h,h}$  is the **twist-2 charm DIS coefficient function** introduced to factorize the twist-4 ladder terms; defined according to the SACOT-MPS scheme

Start with  $N_f = 3$  at  $\mu_0 = m_c - \epsilon$ , evolve to  $\mu > \mu_0$  by incrementing  $N_f$  to 4 and 5

FC is compatible with any version of the ACOT scheme (cf. arXiv:1707.00657).

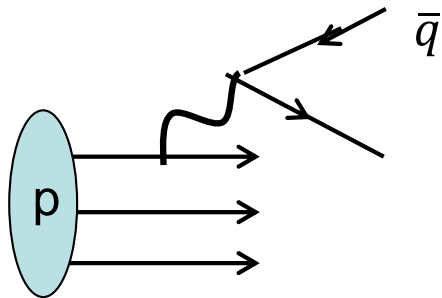
Flavor-excitation coefficient functions of these schemes differ by terms of  $O(m_c^2/Q^2)$ . Their overall differences are of  $O(\Lambda^2/Q^2)$ , i.e., within the accuracy of the factorization theorem.

# In nonperturbative models:

naturally formulated in language of **proton wave function**

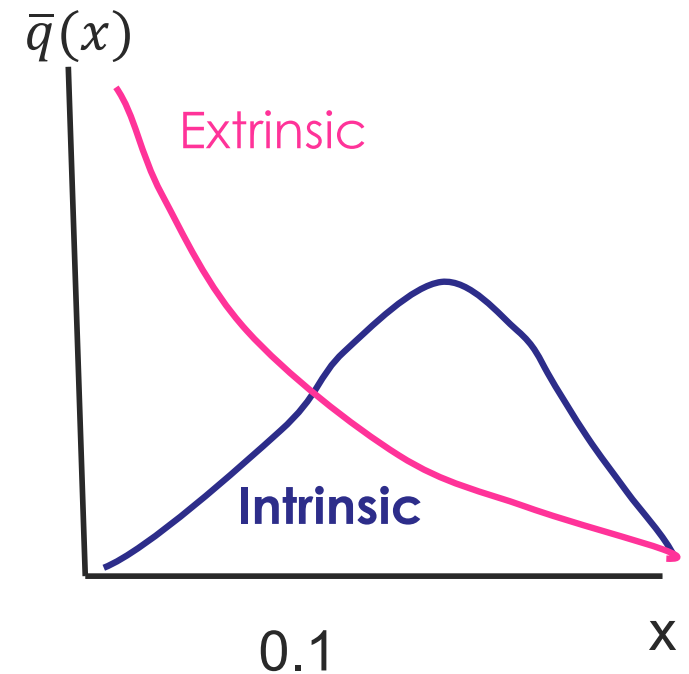
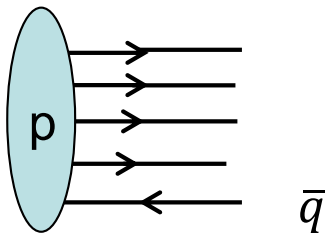
## → “Extrinsic” sea

[maps onto leading-power sea production from light flavors]

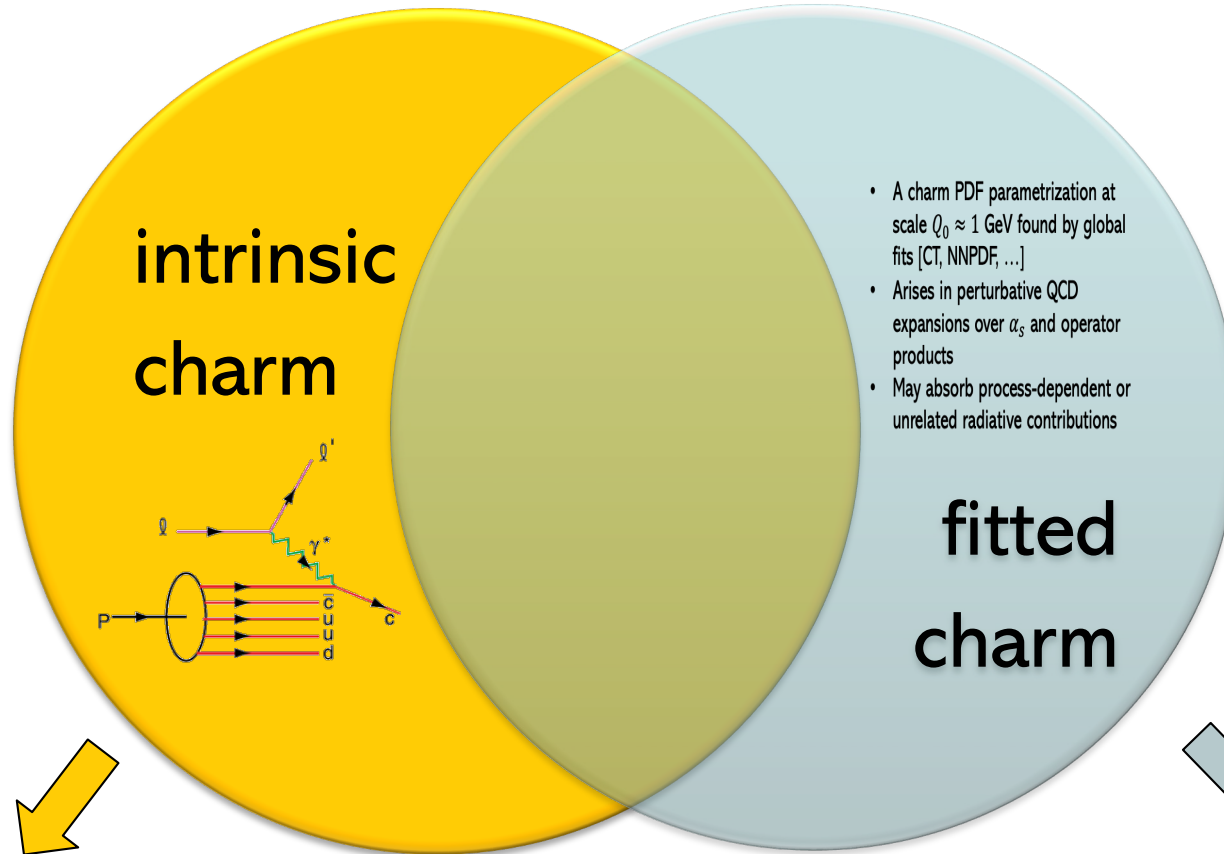


## → “Intrinsic” sea

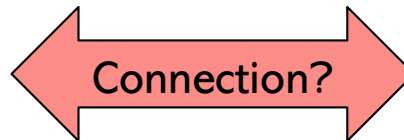
[excited Fock nonpert. states;  
...beyond leading-power production]



# challenging to formulate a rigorous definition of intrinsic charm



- The concept of nonperturbative methods
- Can refer to a component of the hadronic Fock state or the type of the hard process
- Predicts a typical enhancement of the charm PDF at  $x \gtrsim 0.2$

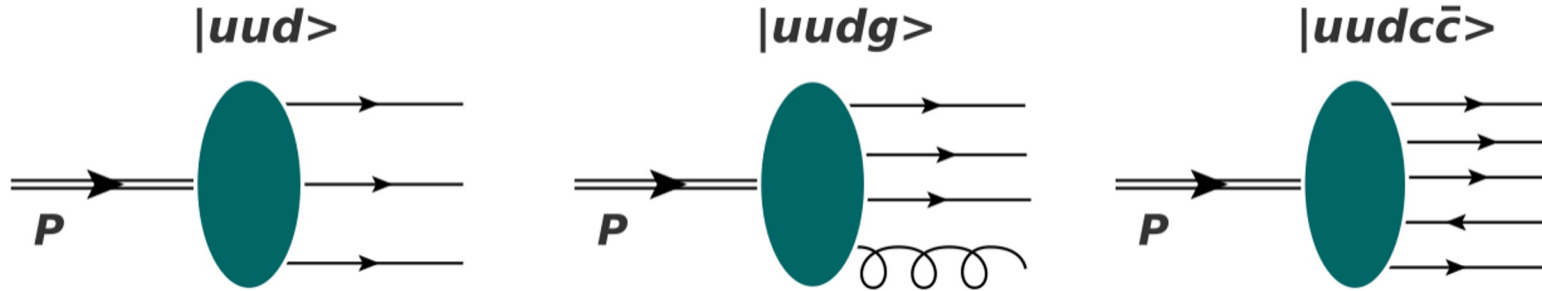


- A charm PDF parametrization at scale  $Q_0 \approx 1$  GeV found by global fits [CT, NNPDF, ...]
- Arises in perturbative QCD expansions over  $\alpha_s$  and operator products
- May absorb process-dependent or unrelated radiative contributions

# nonperturbative QCD can generate a low-scale charm PDF

## Fock expansion

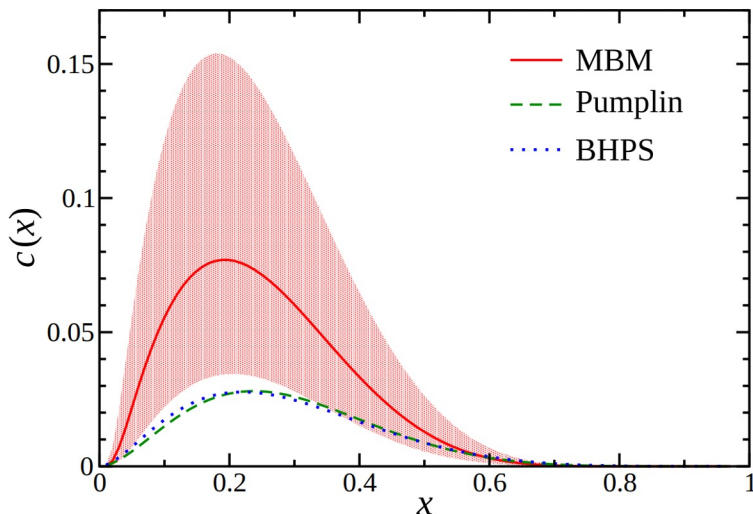
Brodsky, Hoyer, Peterson, Sakai (BHPS); Phys. Lett. **B93** (1980) 451.



IC PDF: transition matrix element,  $|\text{proton}\rangle \rightarrow |uudc\bar{c}\rangle \rightarrow$  old-fashioned PT; **scalar field theory**

$$P(p \rightarrow uudc\bar{c}) \sim \left[ M^2 - \sum_{i=1}^5 \frac{k_{\perp i}^2 + m_i^2}{x_i} \right]^{-2} \quad (\text{'BHPS3': full mass dependence})$$

$$m_c = m_{\bar{c}} \implies c^{\text{BHPS}}(x) = \bar{c}^{\text{BHPS}}(x)$$



$\rightarrow$  more complex models: meson-baryon model (MBM); produce charm-anticharm asymmetry

TJH, Londergan, Melnitchouk, PRD89, 074008 (2014).

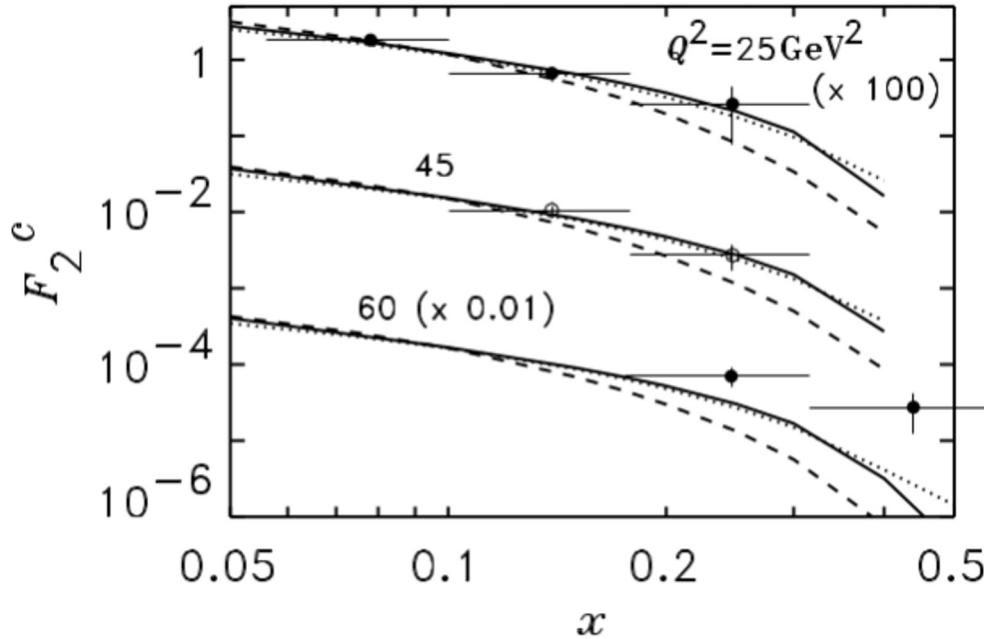
$\rightarrow$  generically yields valence-like shape; governed by charm, hadronic mass scales

# few expts with ‘smoking gun’ sensitivity to FC; but **EMC data (?)**

historically, charm structure function data,  $F_2^{c\bar{c}}$ , from EMC were suggestive

J. J. Aubert *et al.* (EMC), NPB213 (1983) 31–64.

F. M. Steffens, W. Melnitchouk and A. W. Thomas, Eur. Phys. J. C 11, 673 (1999) [hep-ph/9903441].



- hint of high- $x$  excess in select  $Q^2$  bins
- data were analyzed only at LO
- show anomalous  $Q^2$  dependence
- EMC data fit poorly in CT14 IC study

**we do not include EMC in CT18 FC**

CT14 IC, arXiv: 1707.00657.

Candidate NNLO PDF fits	$\chi^2/N_{\text{pts}}$			
	All Experiments	HERA inc. DIS	HERA $c\bar{c}$ SIDIS	EMC $c\bar{c}$ SIDIS
CT14 + EMC (weight=0), no IC	1.10	1.02	1.26	3.48
CT14 + EMC (weight=10), no IC	1.14	1.06	1.18	2.32
CT14 + EMC in BHPS model	1.11	1.02	1.25	2.94
CT14 + EMC in SEA model	1.12	1.02	1.28	3.46

# FC at LHC: $Z+c$ suggested as sensitive probe

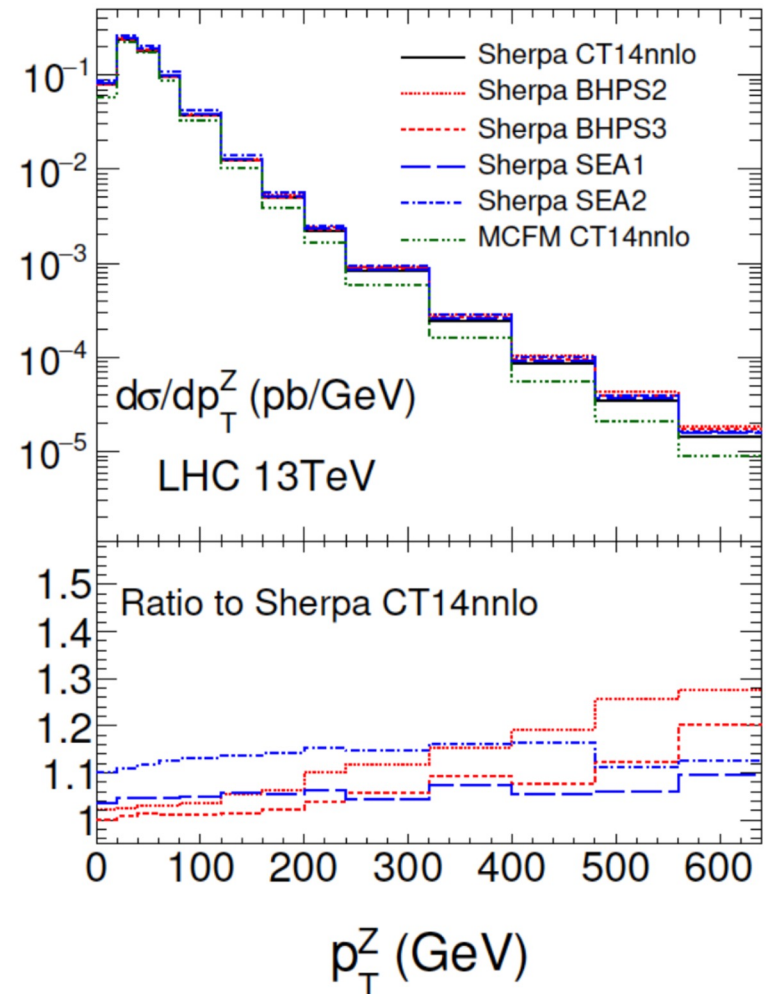
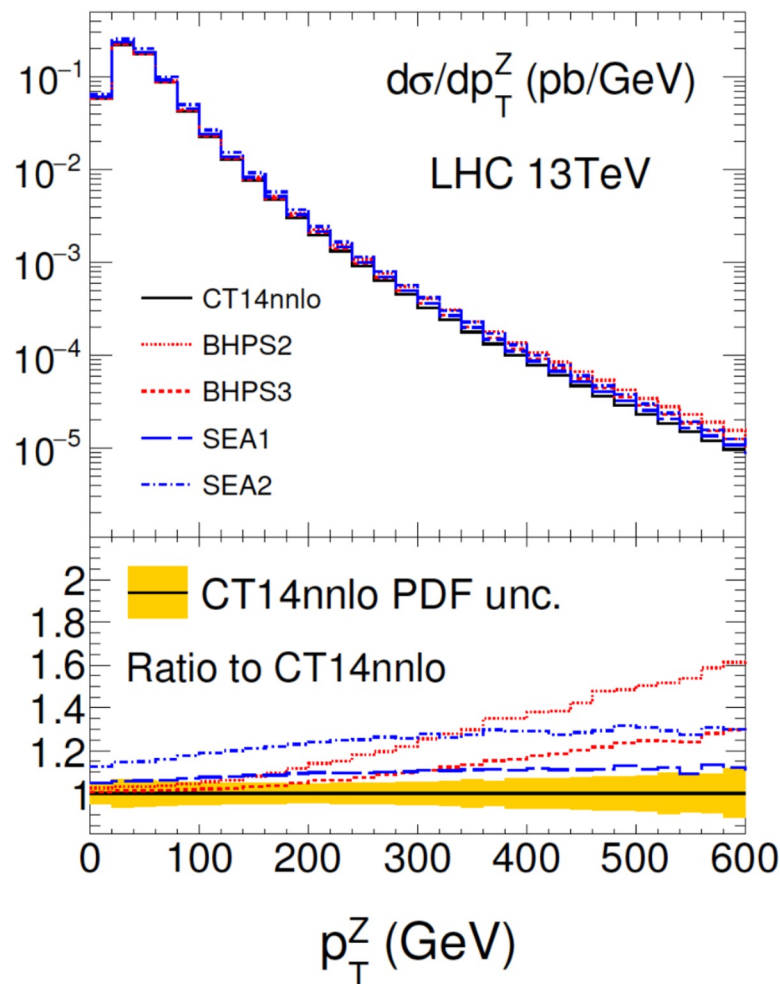
T. Boettcher, P. Ilten, M. Williams, 1512.06666; Bailas, Goncalves, 1512.06007

$p_T$  spectra, rapidity distributions nominally sensitive to high- $x$  charm PDF

→ parton-shower effects can dampen high- $p_T$  tails

## $Z+c$ NLO LHC 13 TeV

[Hou et al., arXiv:1707.00657]



# Z+c at LHCb: intriguing new data; need theory development

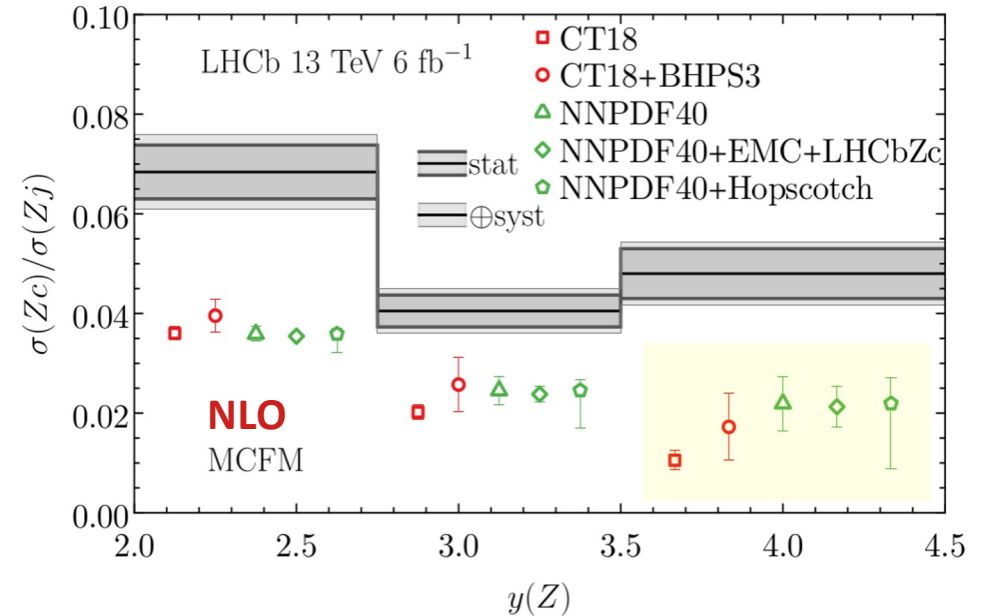
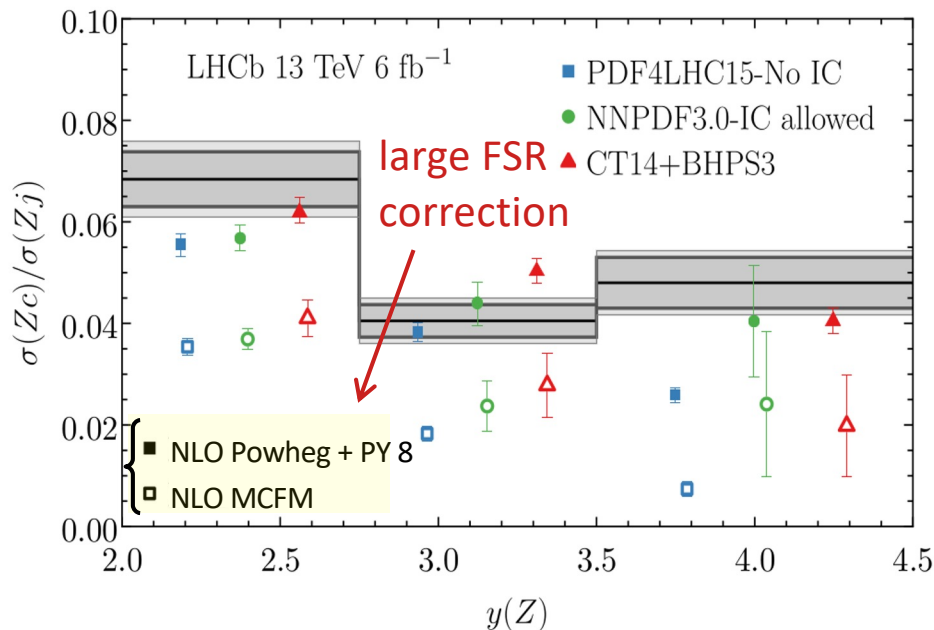
2022 LHCb 13 TeV data: (Z+c) / (Z+jet) ratios; 3 rapidity bins

R. Aaij, *et al.* (LHCb); arXiv: 2109.08084.

- FC slightly enhances ratio; not enough to improve agreement with data

→ meanwhile, significant theory uncertainties

$$x \sim \frac{Q}{\sqrt{s}} \exp y$$



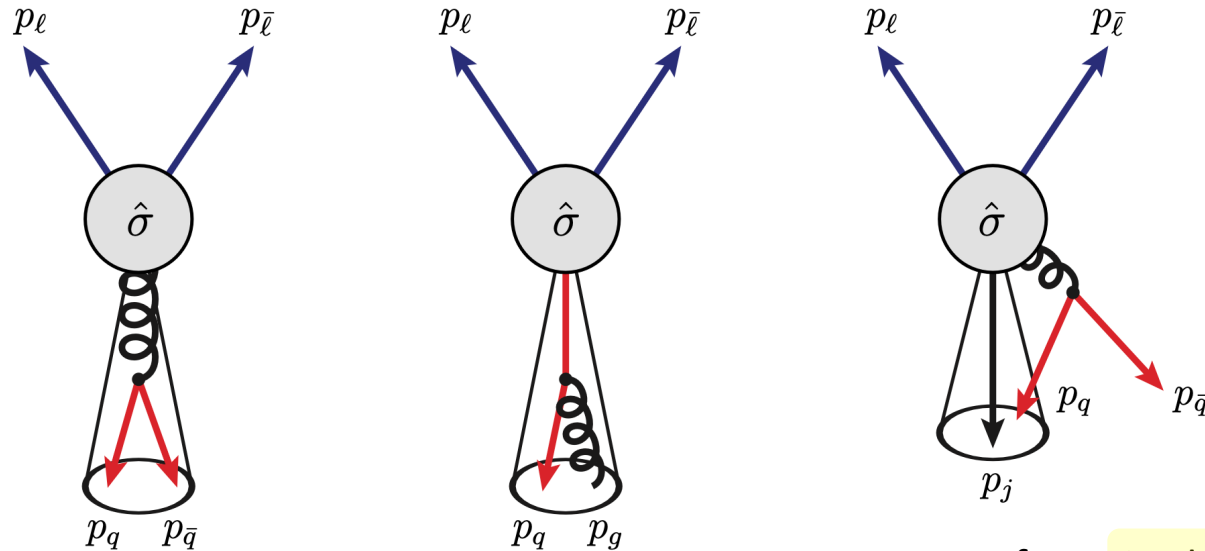
→ calculated **NLO** cross-section ratio similarly depends on showering, hadronization

NNLO calculations recently available, but not implemented in PDF fits

R. Gauld, *et al.*; arXiv: 2005.03016; [2302.12844](https://arxiv.org/abs/2302.12844) M. Czakon, *et al.*; arXiv: 2011.01011.

# Z+c at LHCb: intriguing new data; need theory development

- in addition to NNLO corrections, IRC safety poses challenges to jet algorithms:



from Gauld et al., 2302.12844

- multi-parton interactions (MPI) can represent a significant correction
  - ~10% effect on  $(Z+c)/(Z+jet)$ , especially for  $y(Z) > 3.5$ ; large uncertainty
- massless fixed-order calculation affected by divergences

theory improvements would help guarantee interpretation for PDF extractions

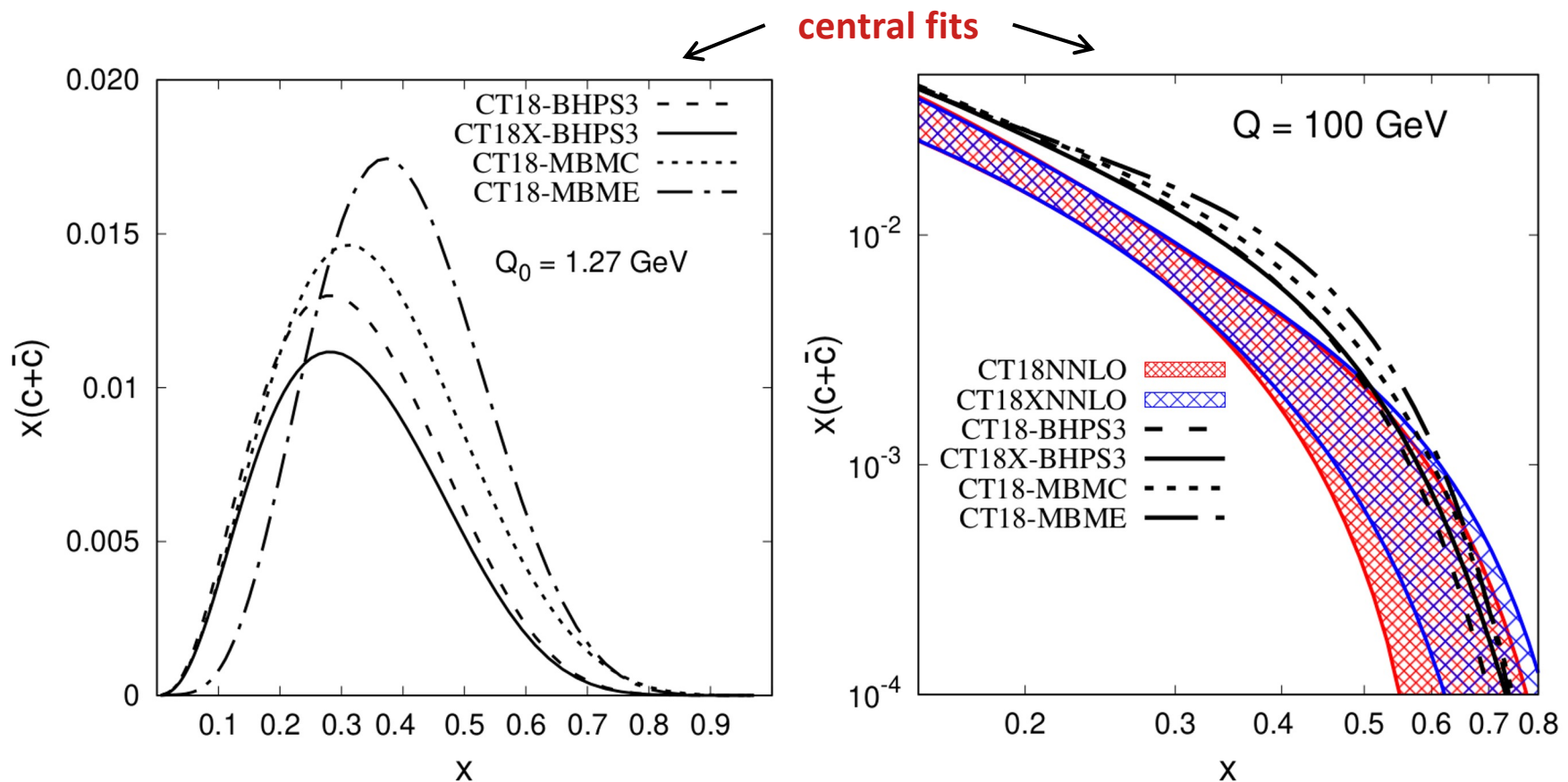


# CT18 FC: fit charm against baseline CT data set

FC scenarios traverse range of high- $x$  behaviors from IC models

- fit implementation of BHPS from CT14IC (BHPS3) on CT18 or CT18X (NNLO)
- fit two MBMs: MBMC (confining), MBME (effective mass) on CT18

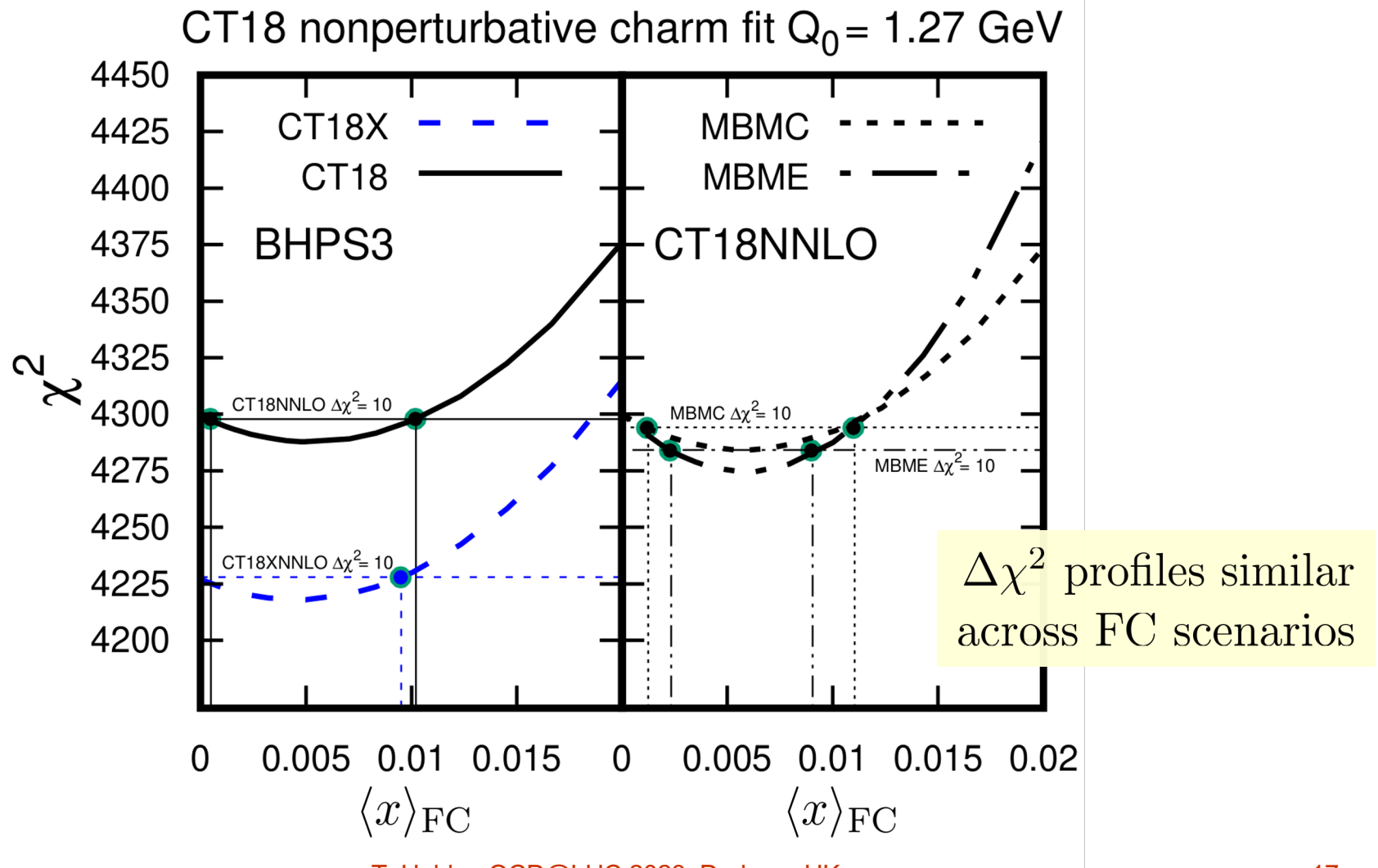
investigate constraints from newer LHC data in CT18



signal for FC in CT18 study, but with shallower  $\Delta\chi^2$  than CT14 IC

FC uncertainty quantified by normalization via  $\langle x \rangle_{\text{FC}}$  for each input IC model

→  $\langle x \rangle_{\text{FC}} \approx 0.5\%$  ( $\Delta\chi^2 \gtrsim -25$ ) vs.  $\langle x \rangle_{\text{FC}} \approx 0.8-1\%$  ( $\Delta\chi^2 \gtrsim -40$ ) **CT14 IC**



# FC PDF moments as F.o.M.

moments of the FC PDFs often used to characterize magnitude, asymmetry

$$\langle x^n \rangle_{c^\pm} = \int_0^1 dx x^n (c \pm \bar{c})[x, Q]$$

$$\langle x \rangle_{\text{FC}} \equiv \langle x \rangle_{c^+} [Q_0 = 1.27 \text{ GeV}] \quad \dots \text{at NNLO.}$$

$$= 0.0048^{+0.0063}_{-0.0043} \left( {}^{+0.0090}_{-0.0048} \right), \text{ CT18 (BHPS3)}$$

$$= 0.0041^{+0.0049}_{-0.0041} \left( {}^{+0.0091}_{-0.0041} \right), \text{ CT18X (BHPS3)}$$

$$= 0.0057^{+0.0048}_{-0.0045} \left( {}^{+0.0084}_{-0.0057} \right), \text{ CT18 (MBMC)}$$

$$= 0.0061^{+0.0030}_{-0.0038} \left( {}^{+0.0064}_{-0.0061} \right), \text{ CT18 (MBME)}$$

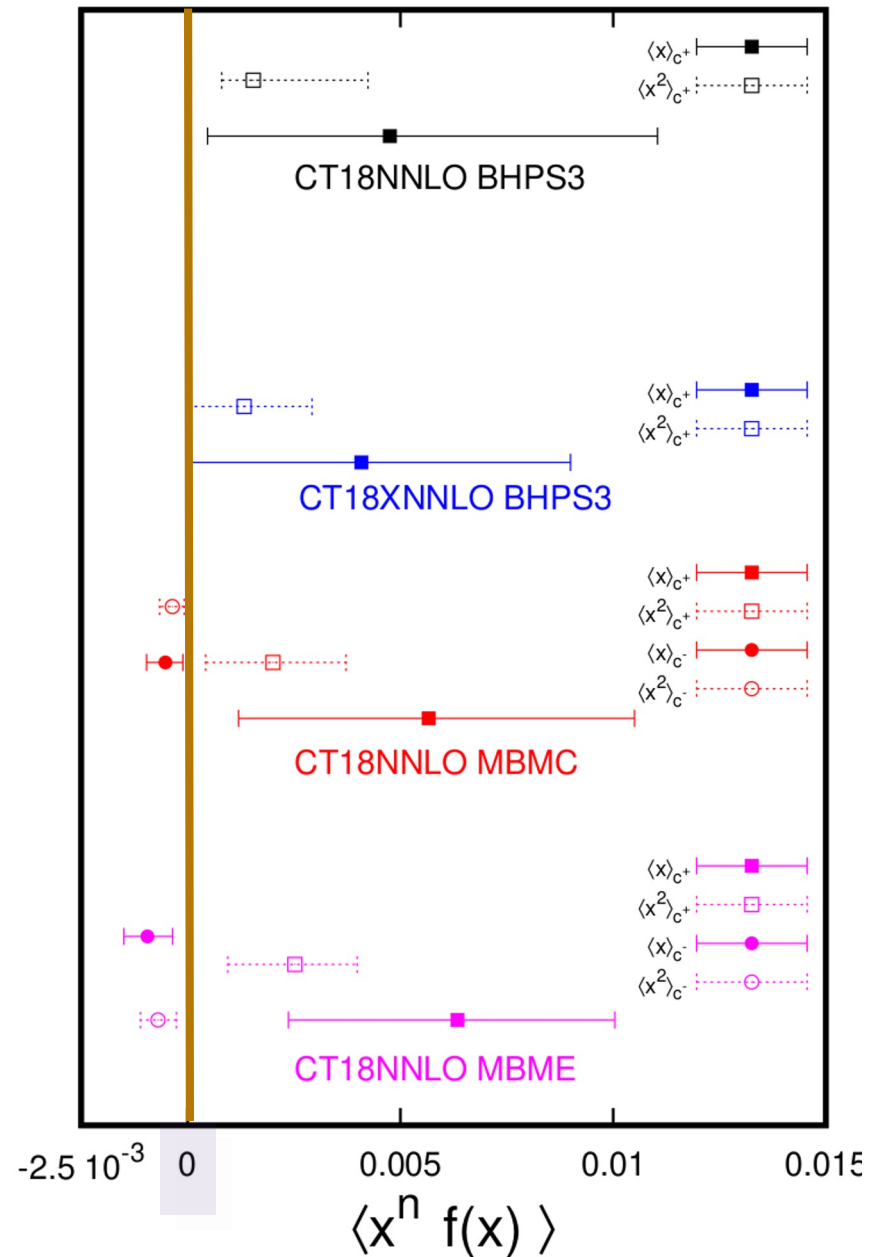
$$\Delta\chi^2 \leq 10$$

(restrictive tolerance)

$$\Delta\chi^2 \leq 30$$

(~CT standard tolerance)

Nonperturbative charm moments  $Q_0 = 1.27 \text{ GeV}$   
Intervals of  $\Delta\chi^2 < 10$



# FC PDF moments as F.o.M.

**even restrictive uncertainties give moments consistent with zero**

→ broaden further for default CT tol.

→ lattice may give  $\langle x \rangle_{c^+}$ ,  $\langle x^2 \rangle_{c^-}$

$$\langle x \rangle_{\text{FC}} \equiv \langle x \rangle_{c^+} [Q_0 = 1.27 \text{ GeV}]$$

$$= 0.0048^{+0.0063}_{-0.0043} \left( {}^{+0.0090}_{-0.0048} \right), \text{ CT18 (BHPS3)}$$

$$= 0.0041^{+0.0049}_{-0.0041} \left( {}^{+0.0091}_{-0.0041} \right), \text{ CT18X (BHPS3)}$$

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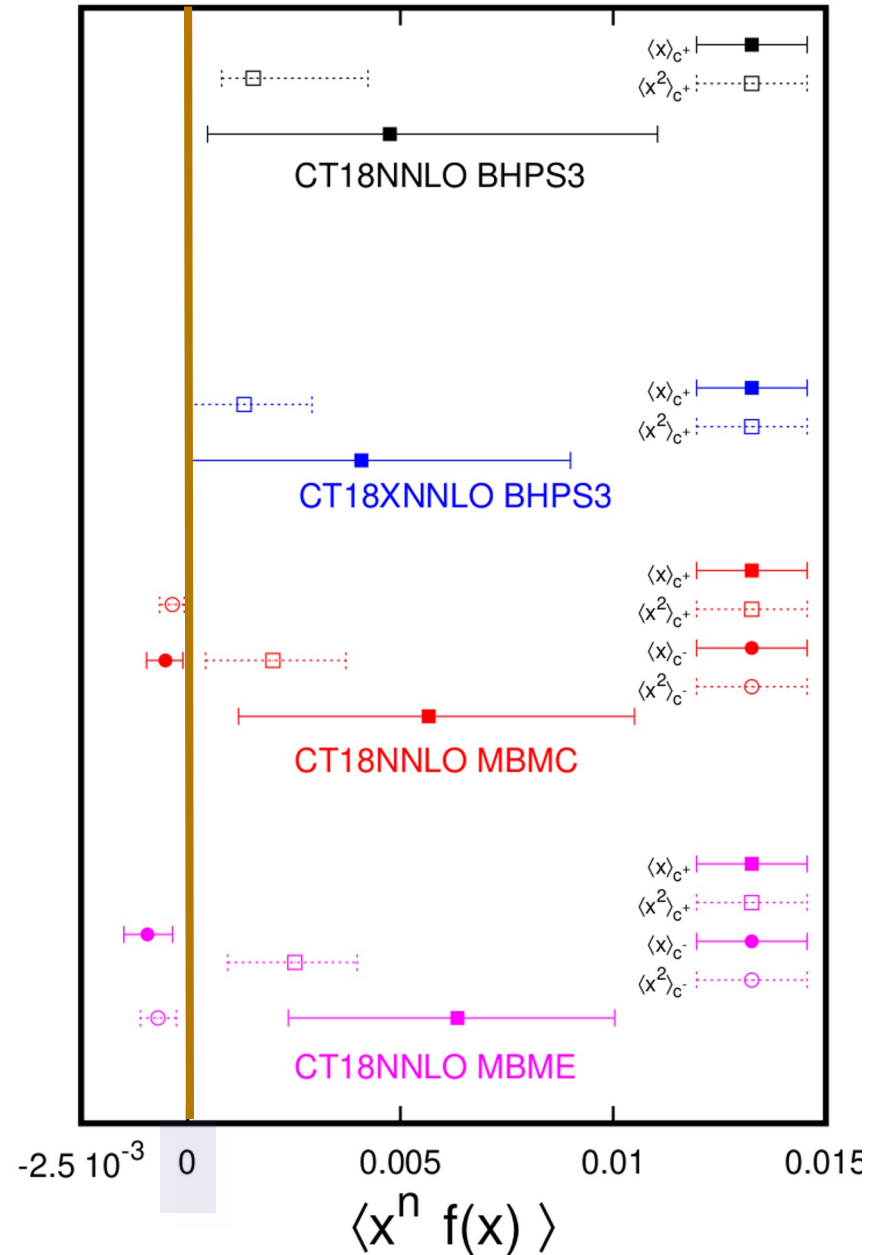
$$\Delta\chi^2 \leq 10$$

$$\Delta\chi^2 \leq 30$$

(restrictive tolerance)

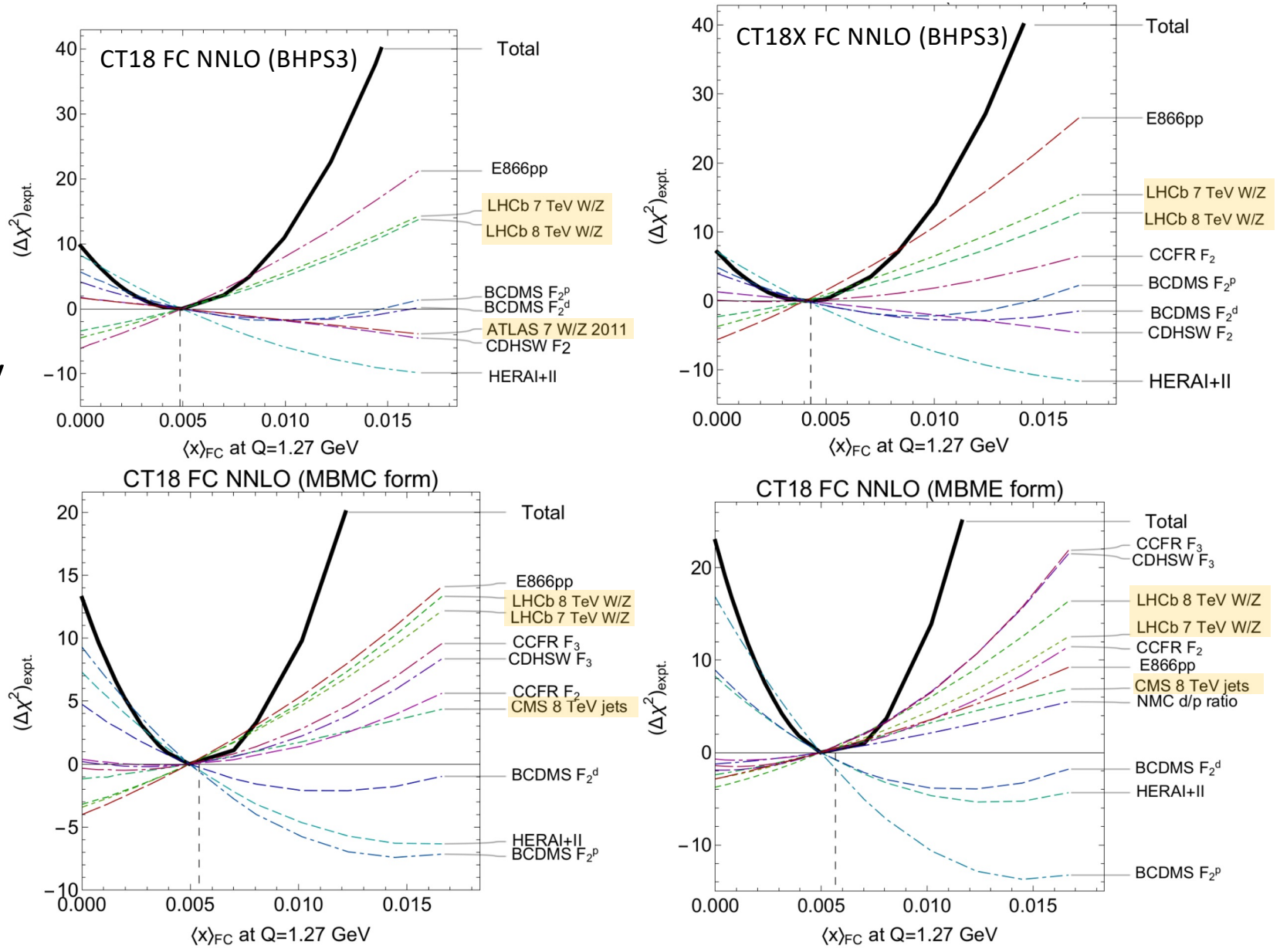
(~CT standard tolerance)

Nonperturbative charm moments  $Q_0 = 1.27 \text{ GeV}$   
Intervals of  $\Delta\chi^2 < 10$



# data pull opposingly on $\langle x \rangle_{FC}$ ; depend on FC scenario, enhancing error

Run-1 LHC  
expts generally  
prefer smaller  
magnitudes of  
FC

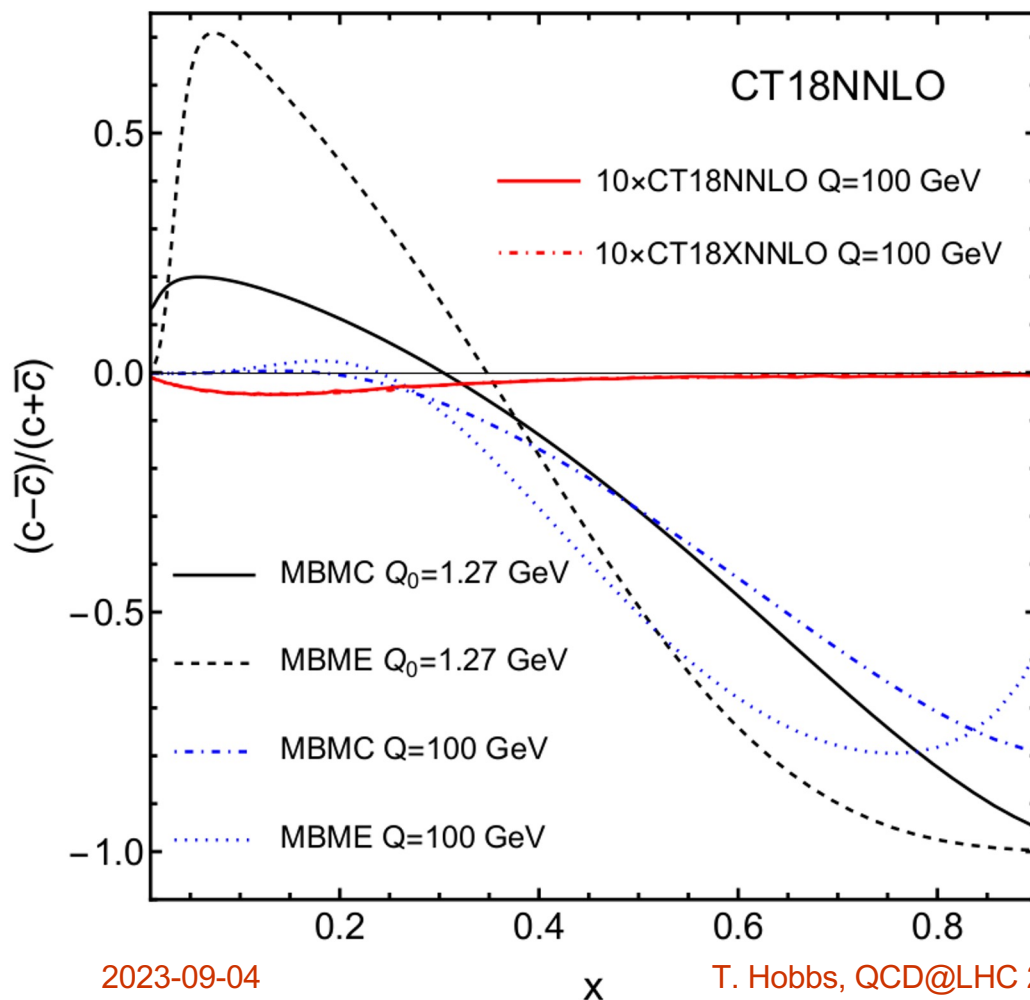


# possible charm-anticharm asymmetries

pQCD only very weakly breaks  $c = \bar{c}$  through HO corrections

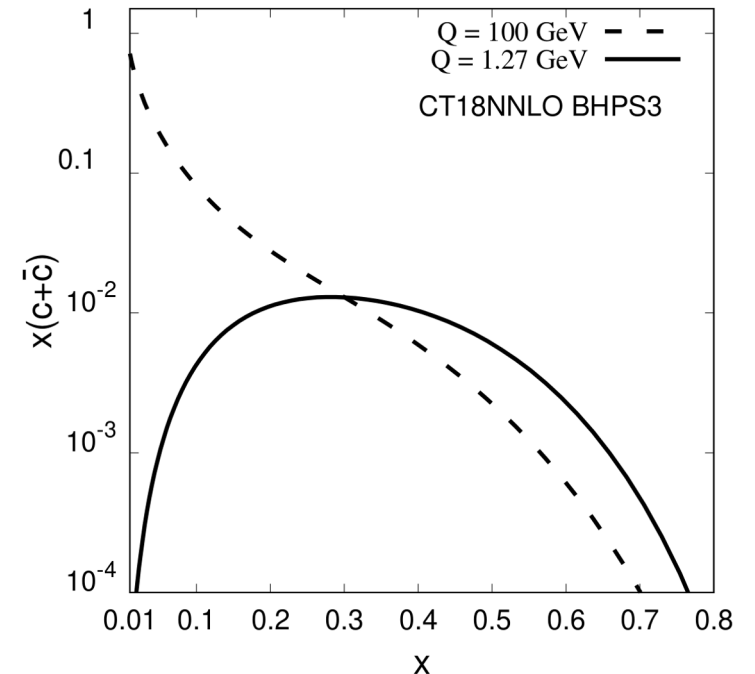
- large(r) charm asymmetry would signal nonpert dynamics, IC
- MBM breaks  $c = \bar{c}$  through hadronic interactions

TJH, Londergan, Melnitchouk, PRD89, 074008 (2014).



consider two MBM models as **examples** (not predictions)

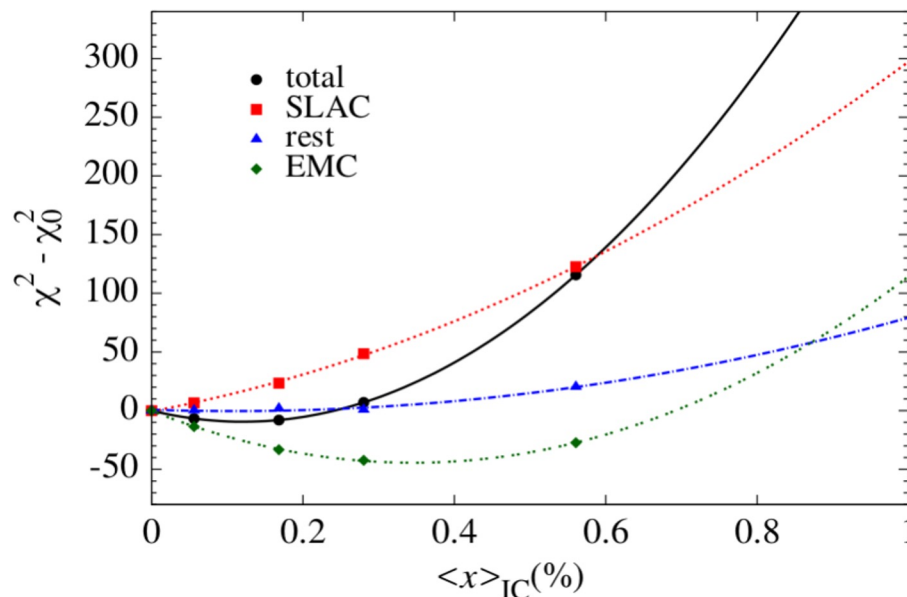
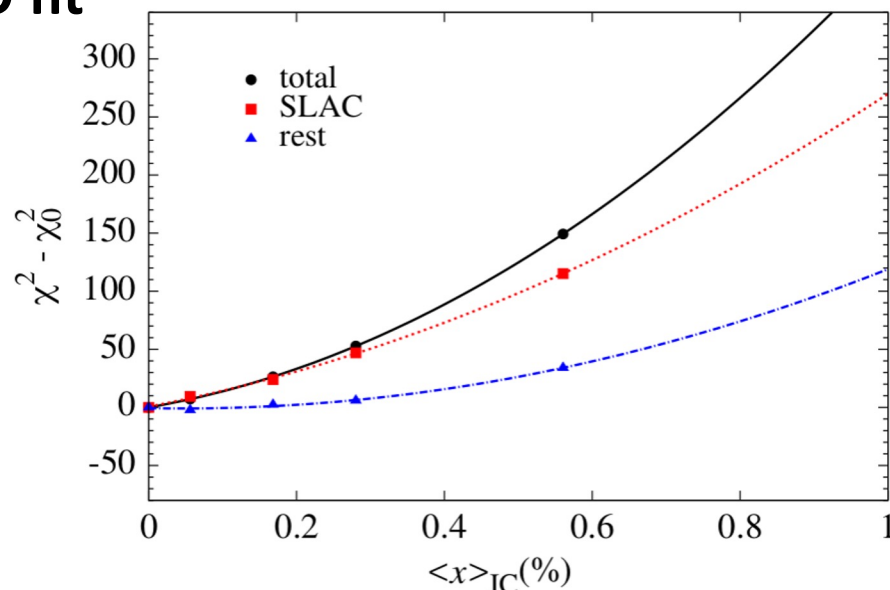
- asym. small but ratio (left) can be bigger; will be hard to extract from data



re: other studies → first full fit with EMC data; no signal for ‘IC’

P. Jimenez-Delgado, TJH, J. T. Londergan and W. Melnitchouk; PRL 114, no. 8, 082002 (2015).

## NLO fit



‘SLAC + REST’ ⇒  $\langle x \rangle_{IC} < 0.1\%$ ; at  $5\sigma$  !

‘REST’ only ⇒  $\langle x \rangle_{IC} < 0.1\%$ ; at  $1\sigma$

EMC alone:  $\langle x \rangle_{IC} = 0.3 - 0.4\%$

+ **SLAC**/‘**REST**’:  $\langle x \rangle_{IC} = 0.13 \pm 0.04\%$

as in CT, can be related to hardness of gluon PDF,

$x \sim 0.1$

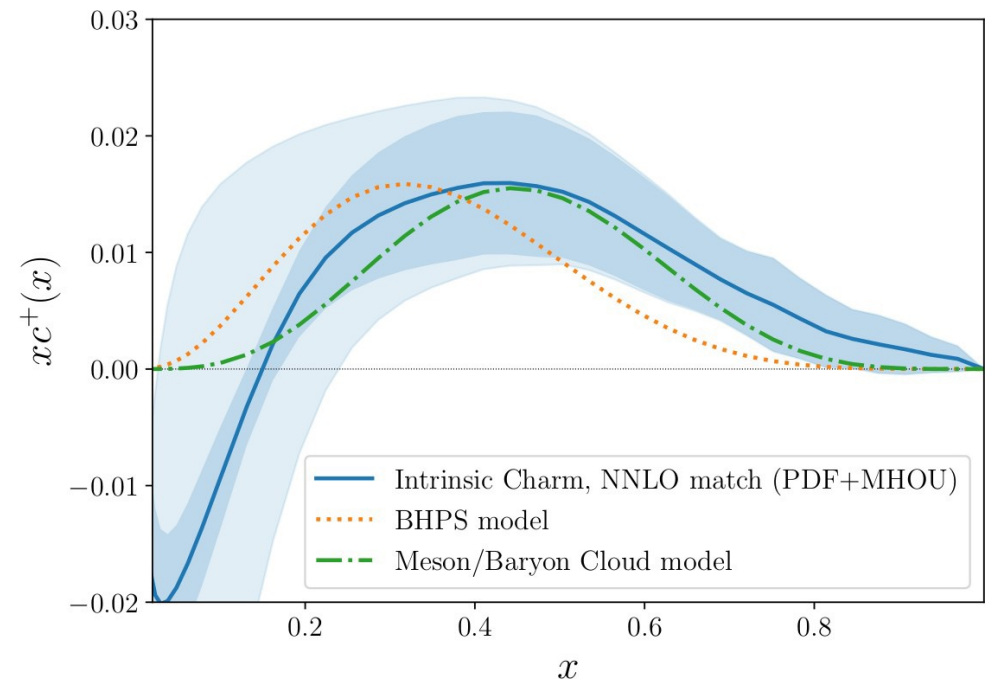
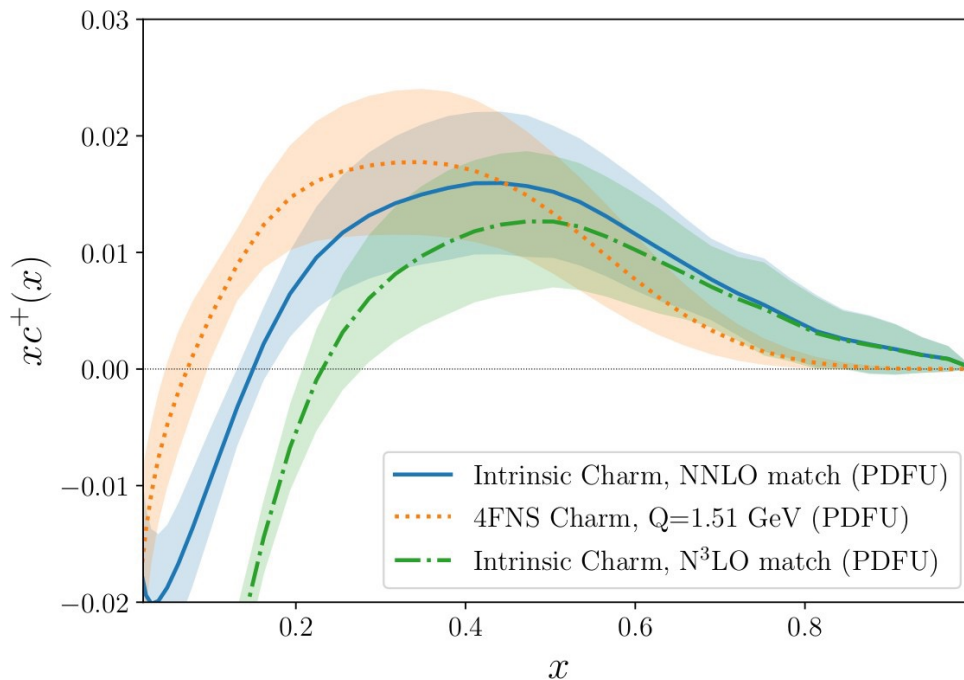
included SLAC DIS; used more restrictive tolerance criterion,  $\Delta\chi^2 = 1$

...but  $F_2^{c\bar{c}}$  poorly fit —  $\chi^2 \sim 4.3$  per datum!

# recent NNPDF IC analysis

NNPDF, Nature 608 (2022) 7923, 483.

- NNPDF have recently claimed  $3\sigma$  evidence for ‘IC’
  - based on local ( $x$ -dependent) deviation of FC PDF from the ‘no-FC’ scenario
  - implies crucial dependence on size and shape of PDF uncertainty



→ **Two classes of uncertainties need further scrutiny:**

1. Missing HO unc (MHOU): N3LO in DIS, etc.; N2LO in Z+c production

$$\langle x \rangle_{\text{FC}} = 0.62 \pm 0.28\% \text{ without MHOU} \quad \langle x \rangle_{\text{FC}} = 0.62 \pm 0.61\% \text{ with MHOU}$$

2. Parametrization sampling uncertainty (underestimation of PDFU)



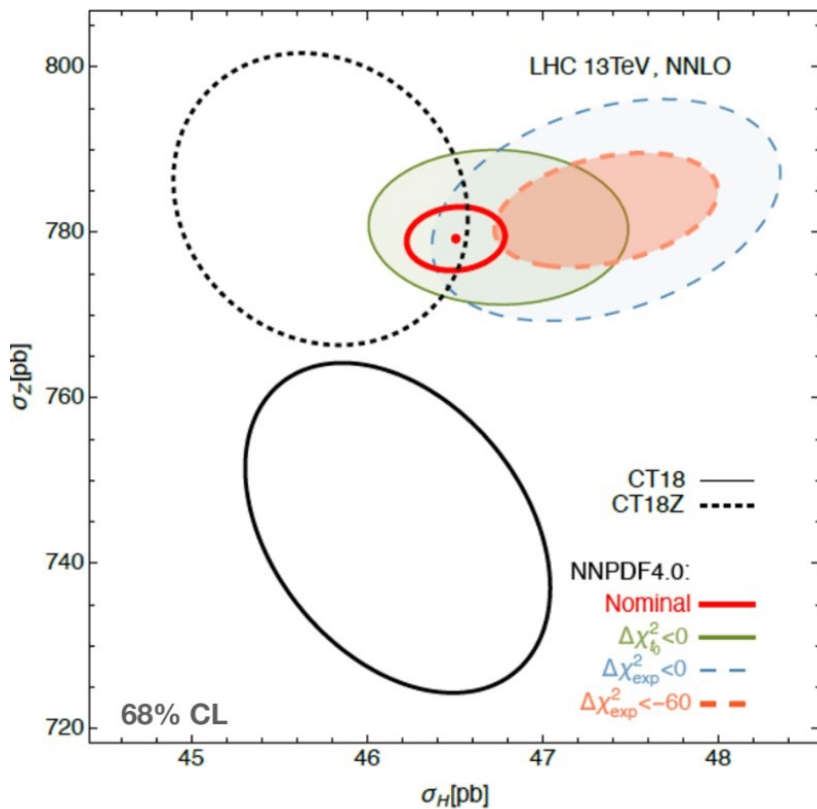
# MC uncertainties closely connected to sampling strategy

- default replica-training in MC studies may omit nominally acceptable solutions

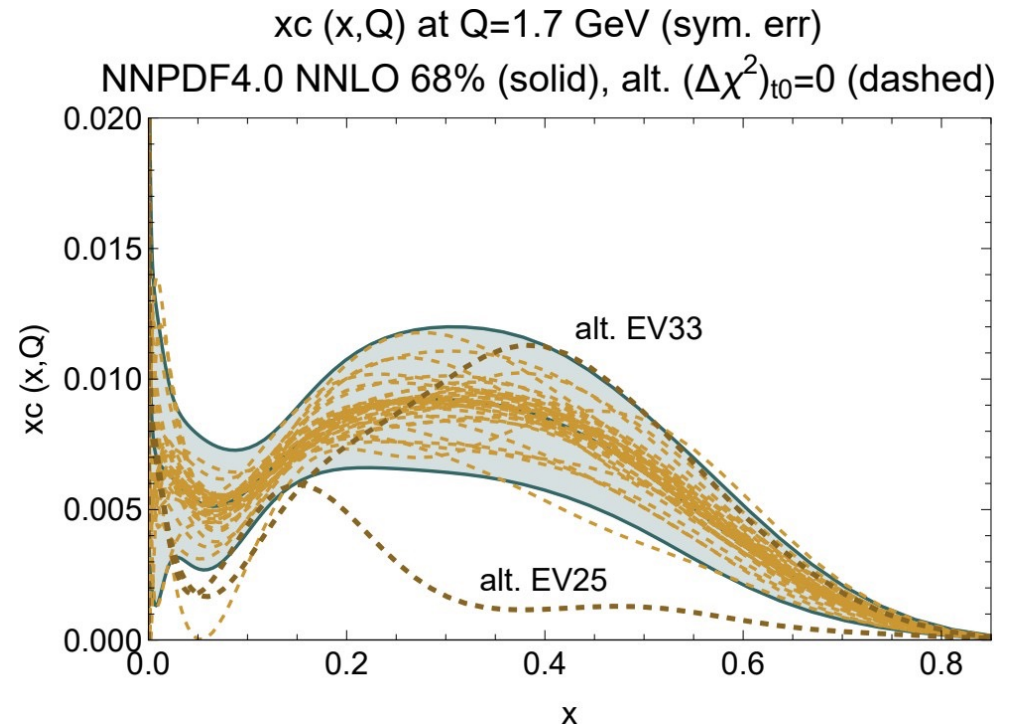
Courtoy *et al.*, PRD107 (2023) 3, 034008.

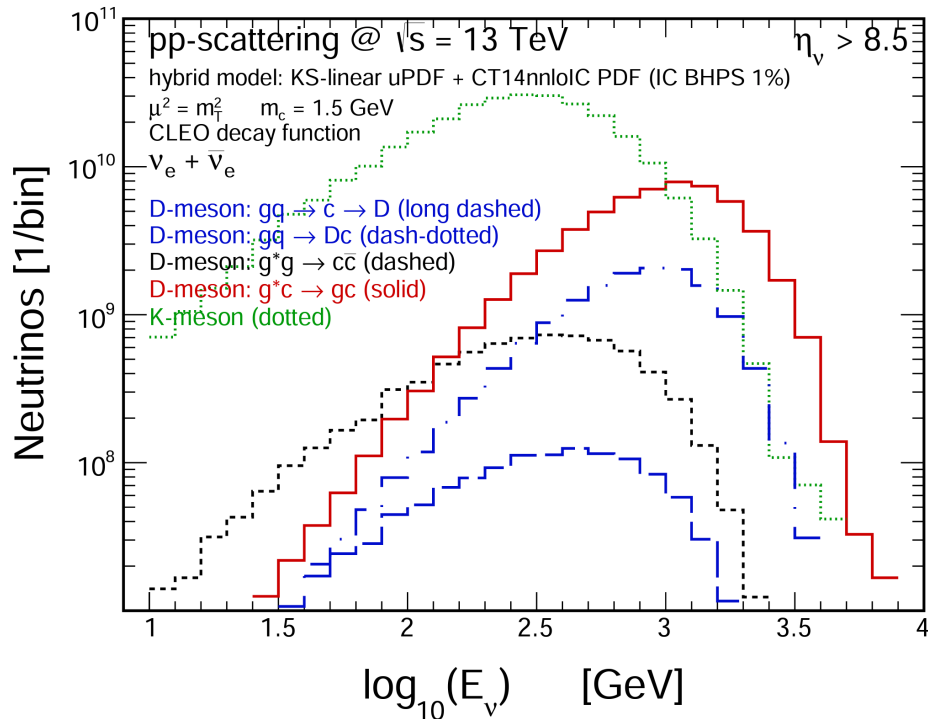
see also, NNPDF response: 2211.12961.

- broader sampling with the public NNPDF4.0 code impacts PDF errors of cross sections



- substantially broadens high- $x$  FC error



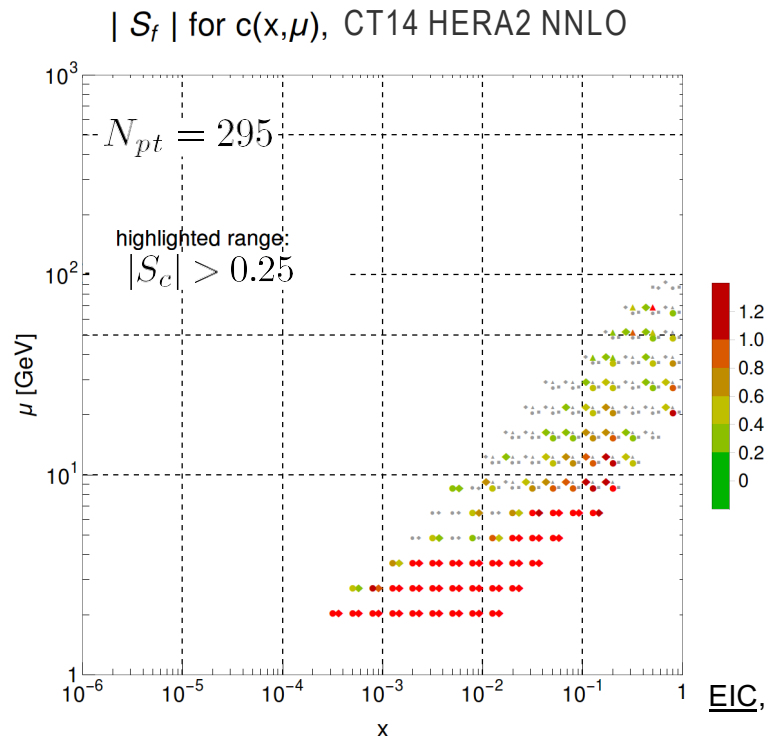


## future data will inform FC

- experiments like  $Z$ +charm, FPF, EIC + lattice QCD may potentially constrain FC scenarios

possible FC sensitivity at CERN FPF:  $k_T$  factorization-based study

(FPF Whitepaper, 2203.05090)



- enhanced FC momentum implied by EMC data  $\rightarrow$  small high- $x$  effects in structure function; need high precision

future experiments are expected to have precise sensitivity to the few- GeV, high- $x$  region where FC signals are to be expected

# outlook

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- size, shape of nonpert charm remains **indeterminate**
  - theoretical ambiguities in relation between FC/IC unresolved
  - need more sensitive data; FC currently consistent with zero

concordance with enlarged error estimates:  $\langle x \rangle_{\text{FC}} \sim 0.5\%$ , well below evidence-level

- need more NNLO+ and better showering calculations (*e.g.*, for  $Z+c$ )
- further progress in quantifying and estimating PDF uncertainties

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opportunities to improve knowledge of FC:

- promising experiments at LHC, FPF; EIC
- lattice data on key charm PDF moments; quasi-PDFs
- direct benchmarking of FC among PDF fitting groups