



Istituto Nazionale di Fisica Nucleare



UNIVERSITÀ  
DEGLI STUDI  
DI MILANO



# Including QED corrections in PDFs fits

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**The NNPDF4.0QED PDFs set**

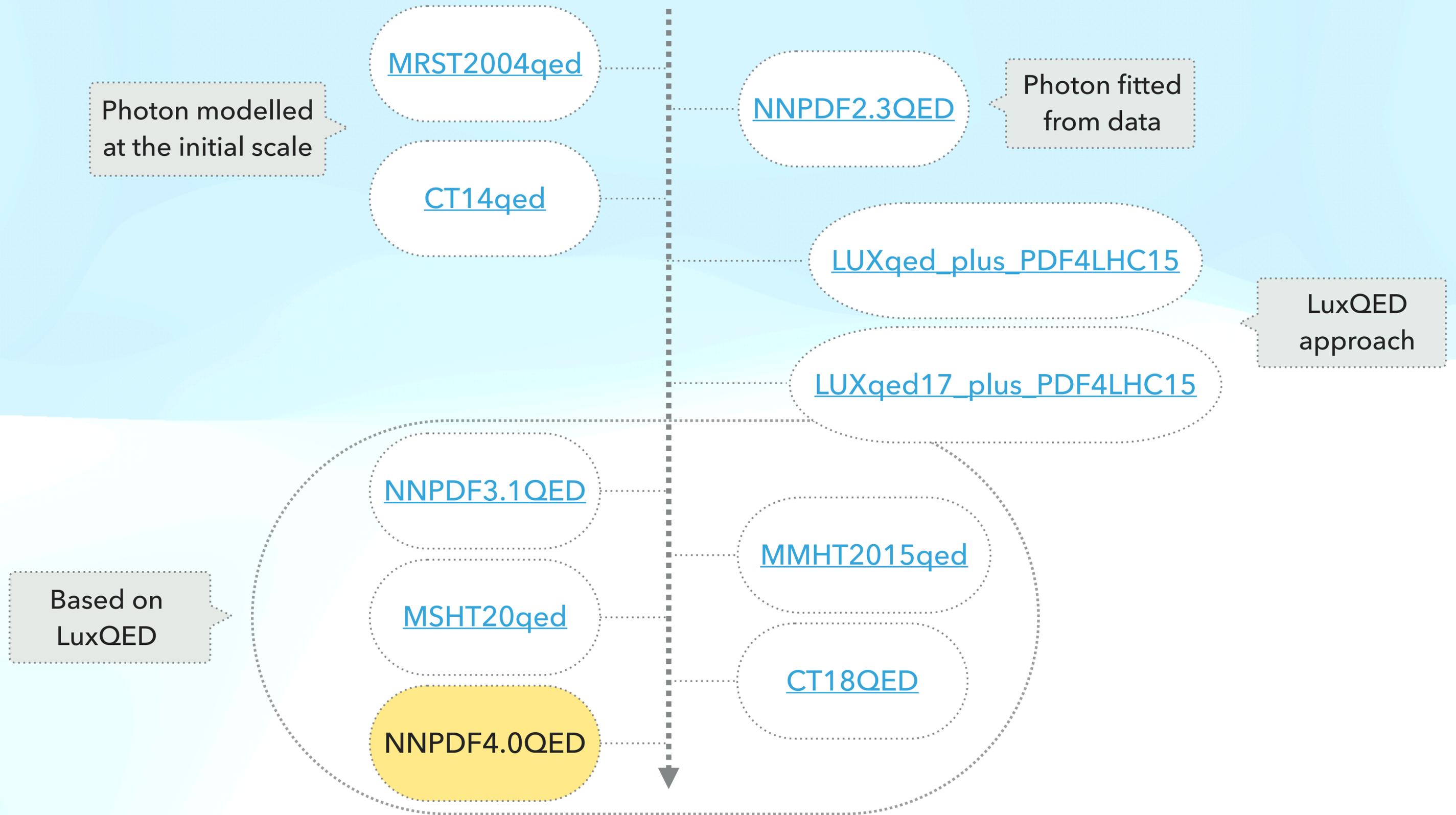
**Niccolò Laurenti, on behalf of the NNPDF collaboration, QCD@LHC23**

# Summary

- Introduction
- Adding QED to a PDF fit
- Results
- Impact on phenomenology
- Conclusions

# INTRODUCTION

# Introduction: QED fits done in the past



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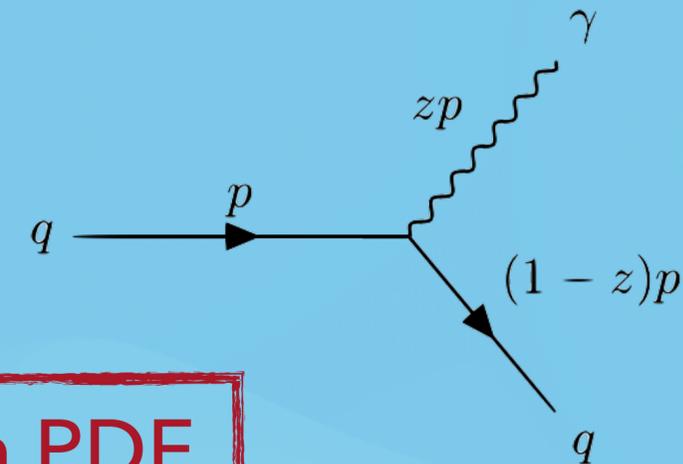
# ADDING QED TO A PDF FIT

$$\alpha \sim \mathcal{O}(\alpha_s^2) \sim \mathcal{O}(1\%)$$

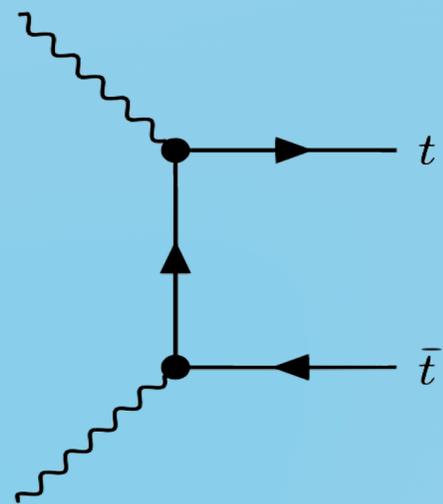
percent correction

quarks can emit photons

we get a photon PDF



For some processes we can't neglect photon induced contributions



starts at  $\mathcal{O}(\alpha_s^0)$

In some kinematical regions QED corrections are important

see [phenomenology section](#)

$$\mu^2 \frac{d}{d\mu^2} f_i(x, \mu^2) = \sum_{j=q, \bar{q}, g, \gamma} \int_x^1 \frac{dz}{z} P_{ij} \left( \frac{x}{z}, \alpha_s(\mu^2), \alpha(\mu^2) \right) f_j(z, \mu^2) \quad i = q, \bar{q}, g, \gamma$$

$$P_{ij}(\alpha_s, \alpha) = P_{ij}(\alpha_s) + \tilde{P}_{ij}(\alpha_s, \alpha)$$

pure QED and QCD ⊗ QED terms

$$\alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)} + \dots$$

pure QCD terms

$$\alpha P_{ij}^{(0,1)} + \alpha_s \alpha P_{ij}^{(1,1)} + \alpha^2 P_{ij}^{(0,2)} + \dots$$

Gluon couples in the same way to all quarks

Photon couples in different ways to up-like and down-like quarks  
More difficult to diagonalize (see next slide)

## Solving the system

### Pure QCD case

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} g \\ \Sigma \end{pmatrix} = \begin{pmatrix} P_{gg} & P_{gq} \\ P_{qg} & P_{qq} \end{pmatrix} \otimes \begin{pmatrix} g \\ \Sigma \end{pmatrix}$$

$$\mu^2 \frac{d}{d\mu^2} V = P_{ns,v} \otimes V$$

$$\mu^2 \frac{d}{d\mu^2} f_{ns,\pm} = P_{ns,\pm} \otimes f_{ns,\pm}$$

$$\Sigma = \sum_{i=1}^{n_f} q_i^+ \quad V = \sum_{i=1}^{n_f} q_i^- \quad f_{ns,\pm} = \begin{cases} u^\pm - d^\pm \\ u^\pm + d^\pm - 2s^\pm \\ u^\pm + d^\pm + s^\pm - 3c^\pm \\ u^\pm + d^\pm + s^\pm + c^\pm - 4b^\pm \\ u^\pm + d^\pm + s^\pm + c^\pm + b^\pm - 5t^\pm \end{cases}$$

$$q^\pm = q \pm \bar{q}$$

### QCD $\otimes$ QED case

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} g \\ \gamma \\ \Sigma \\ \Sigma_\Delta \end{pmatrix} = \mathbf{P}_s \otimes \begin{pmatrix} g \\ \gamma \\ \Sigma \\ \Sigma_\Delta \end{pmatrix}$$

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} V \\ V_\Delta \end{pmatrix} = \mathbf{P}_v \otimes \begin{pmatrix} V \\ V_\Delta \end{pmatrix}$$

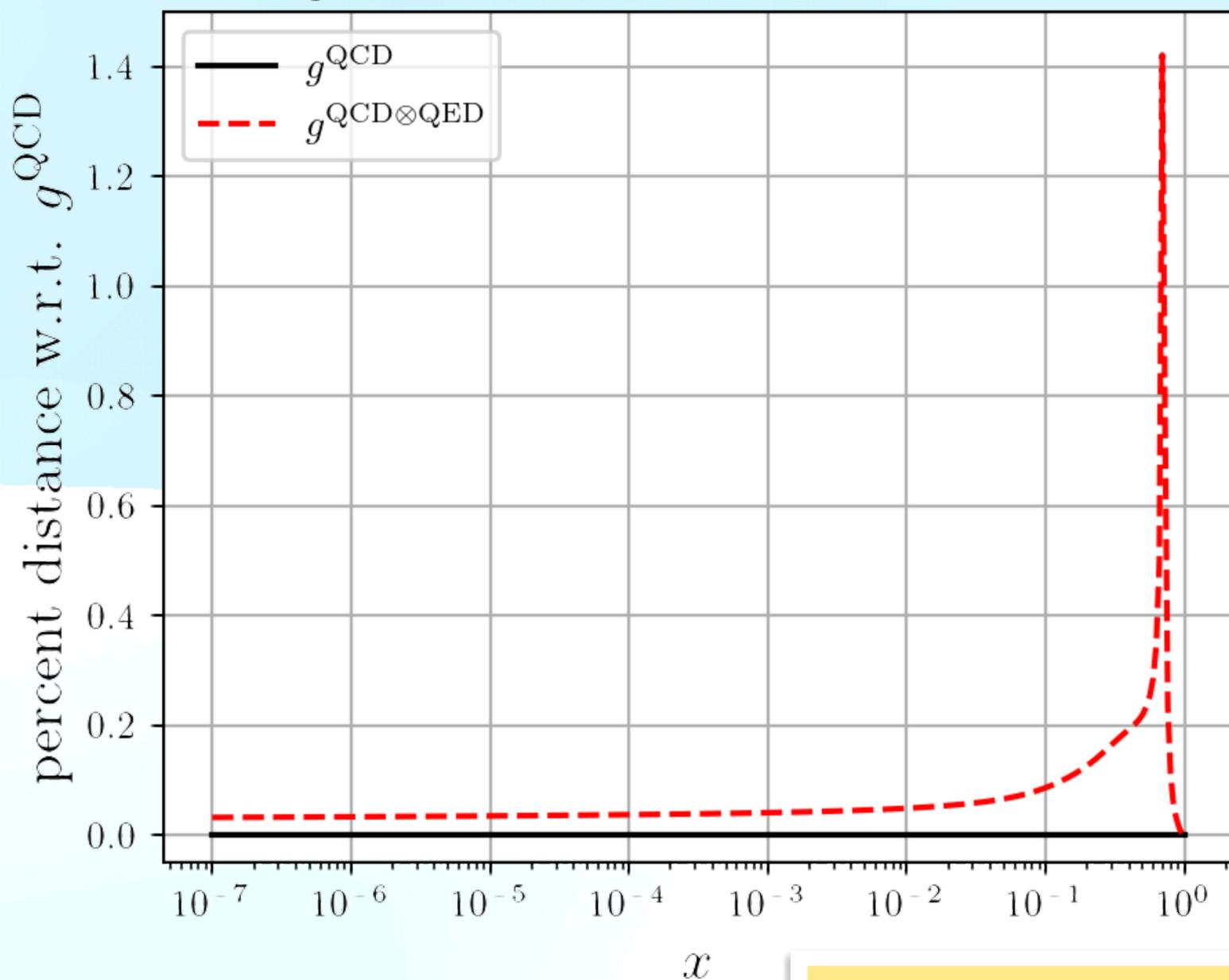
$$\mu^2 \frac{d}{d\mu^2} f_{ns,\pm}^{ul/d} = \left( P_{ns,\pm} + \tilde{P}_{ns,\pm}^{ul/d} \right) \otimes f_{ns,\pm}^{ul/d}$$

$$f_{ns,\pm}^u = \begin{cases} u^\pm - c^\pm \\ u^\pm + c^\pm - 2t^\pm \end{cases} \quad f_{ns,\pm}^d = \begin{cases} d^\pm - s^\pm \\ d^\pm + s^\pm - 2b^\pm \end{cases}$$

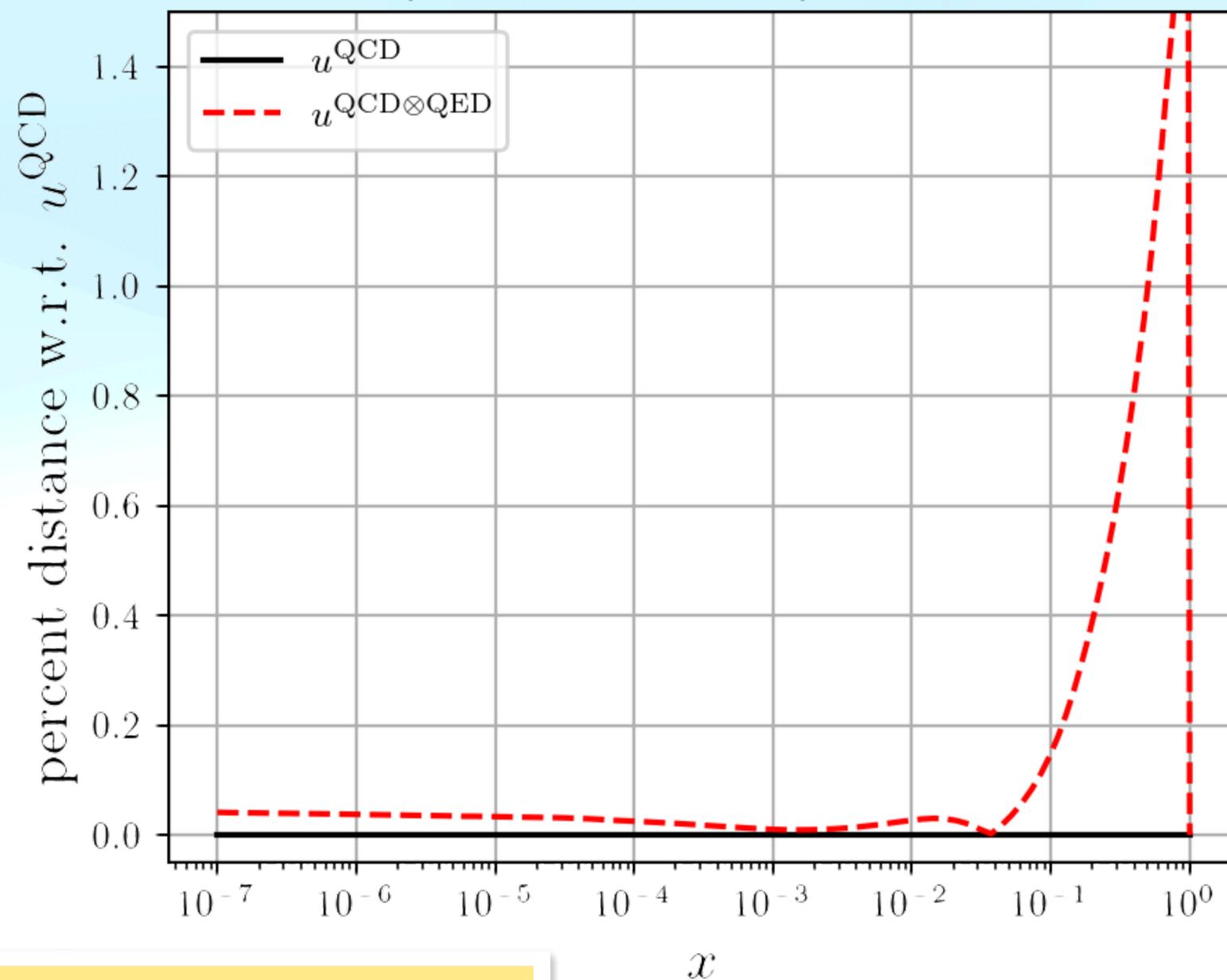
$$\Sigma_\Delta = \frac{n_d}{n_u} \sum_{i=1}^{n_u} u_i^+ - \sum_{i=1}^{n_d} d_i^+ \quad V_\Delta = \frac{n_d}{n_u} \sum_{i=1}^{n_u} u_i^- - \sum_{i=1}^{n_d} d_i^-$$

## Numerical results

$g, Q_0=5 \text{ GeV} \rightarrow Q=100 \text{ GeV}$



$u, Q_0=5 \text{ GeV} \rightarrow Q=100 \text{ GeV}$



small correction for quark and gluon...but needed for the evolution of the photon!

\* Photon PDF is obtained from LuxQED approach [[Manohar, Nason, Salam, Zanderighi, 2016](#)]

$$x\gamma(x, \mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{m_p^2 x^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[ \left( zP_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L(x/z, Q^2) \right] - \alpha^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\}$$

\*  $\gamma$  modifies the momentum sum rules:

$$\int_0^1 dx x (\Sigma(x, Q^2) + g(x, Q^2) + \gamma(x, Q^2)) = 1$$

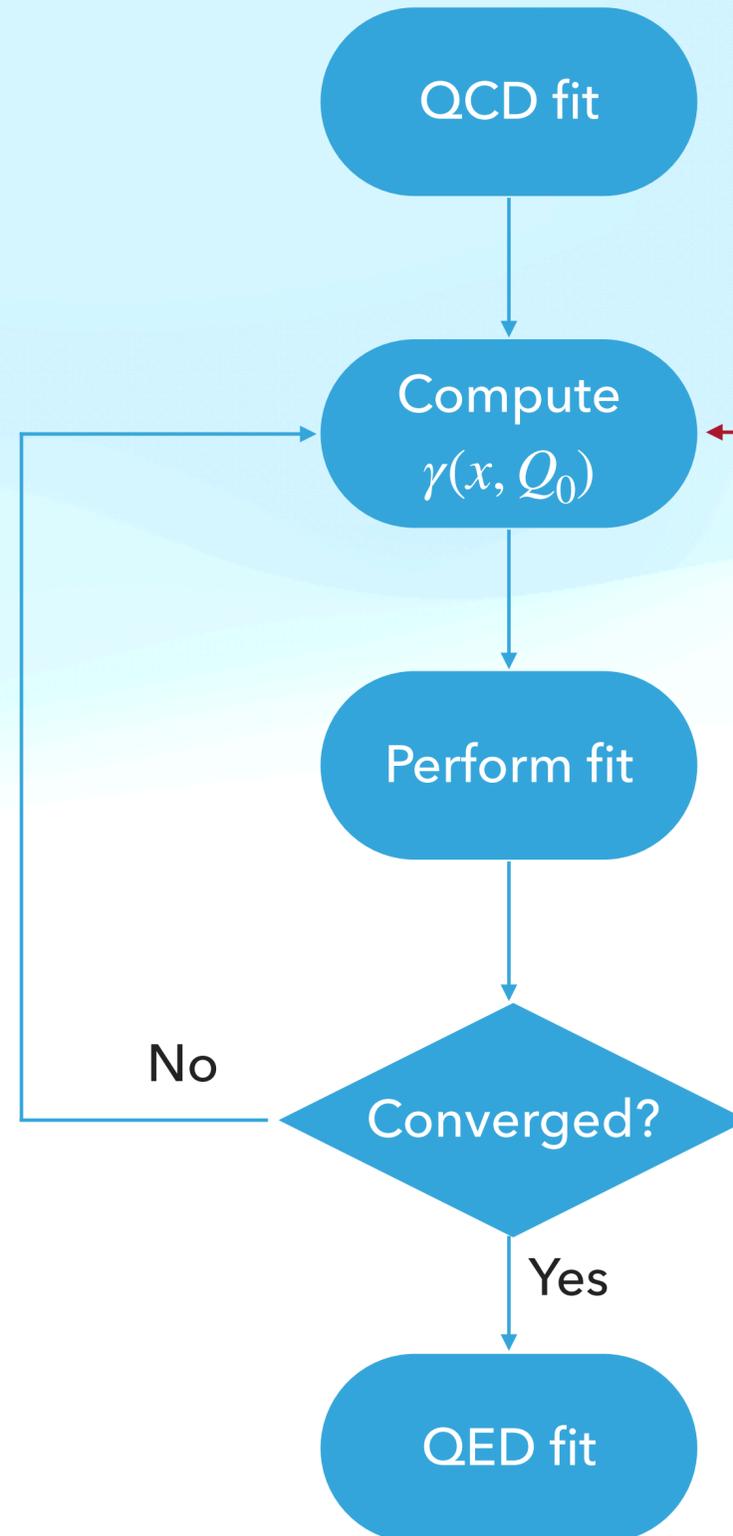
$$F_{2,L} = \sum_i C_{2,L,i} \otimes f_i$$

$\gamma$  depends on the other PDFs!!

LuxQED formula gives a constraint between  $\gamma$  and the other PDFs



**Such constraint is implemented iteratively**



Computed at 100 GeV and evolved back to fitting scale with DGLAP [Manohar, Nason, Salam, Zanderighi, 2017]

# RESULTS OF THE FIT

No photon-initiated contributions in theory predictions

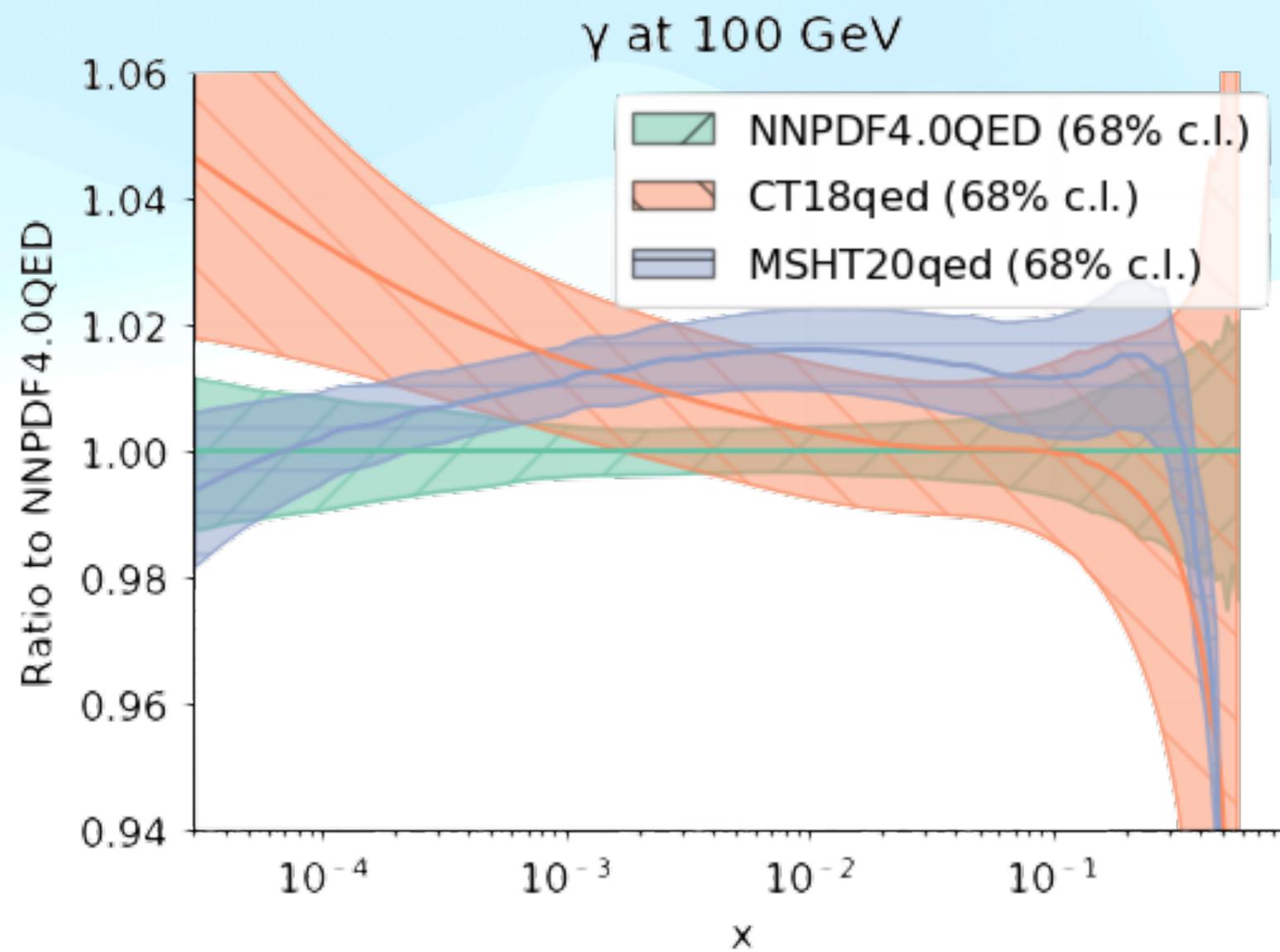
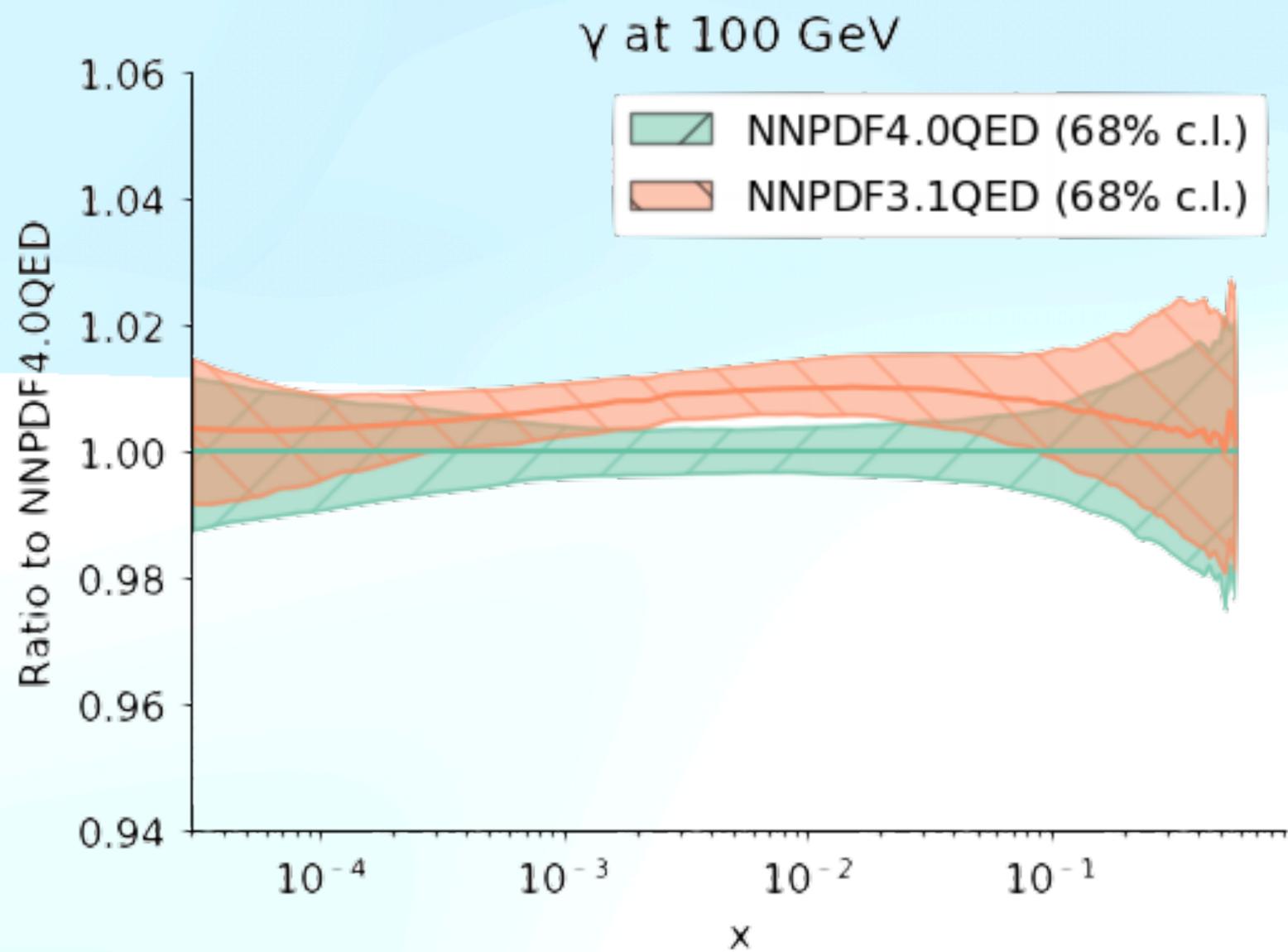
Chosen dataset cuts points in which they are important

QED corrections enter in the fit through momentum sum rule and DGLAP evolution

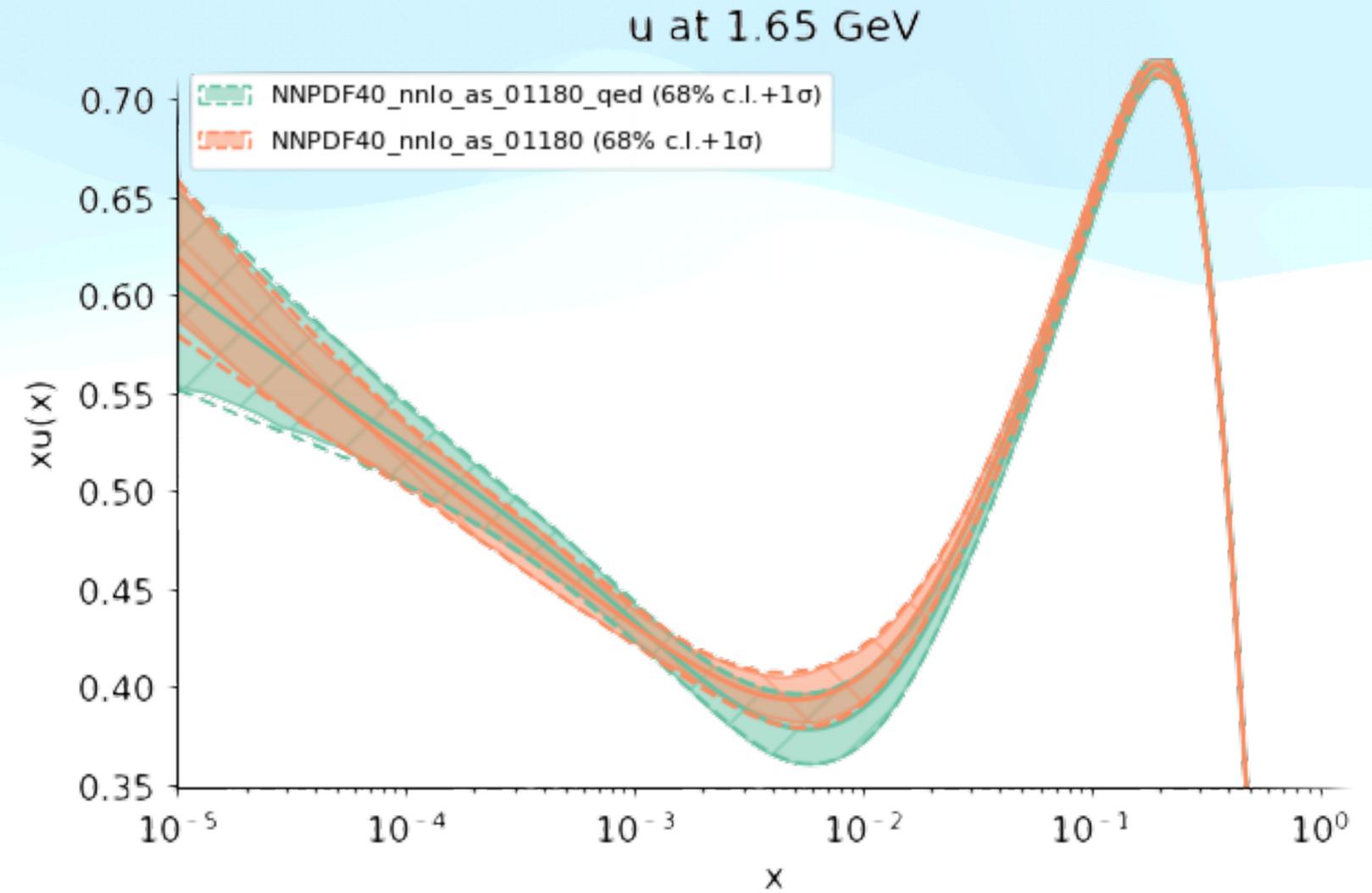
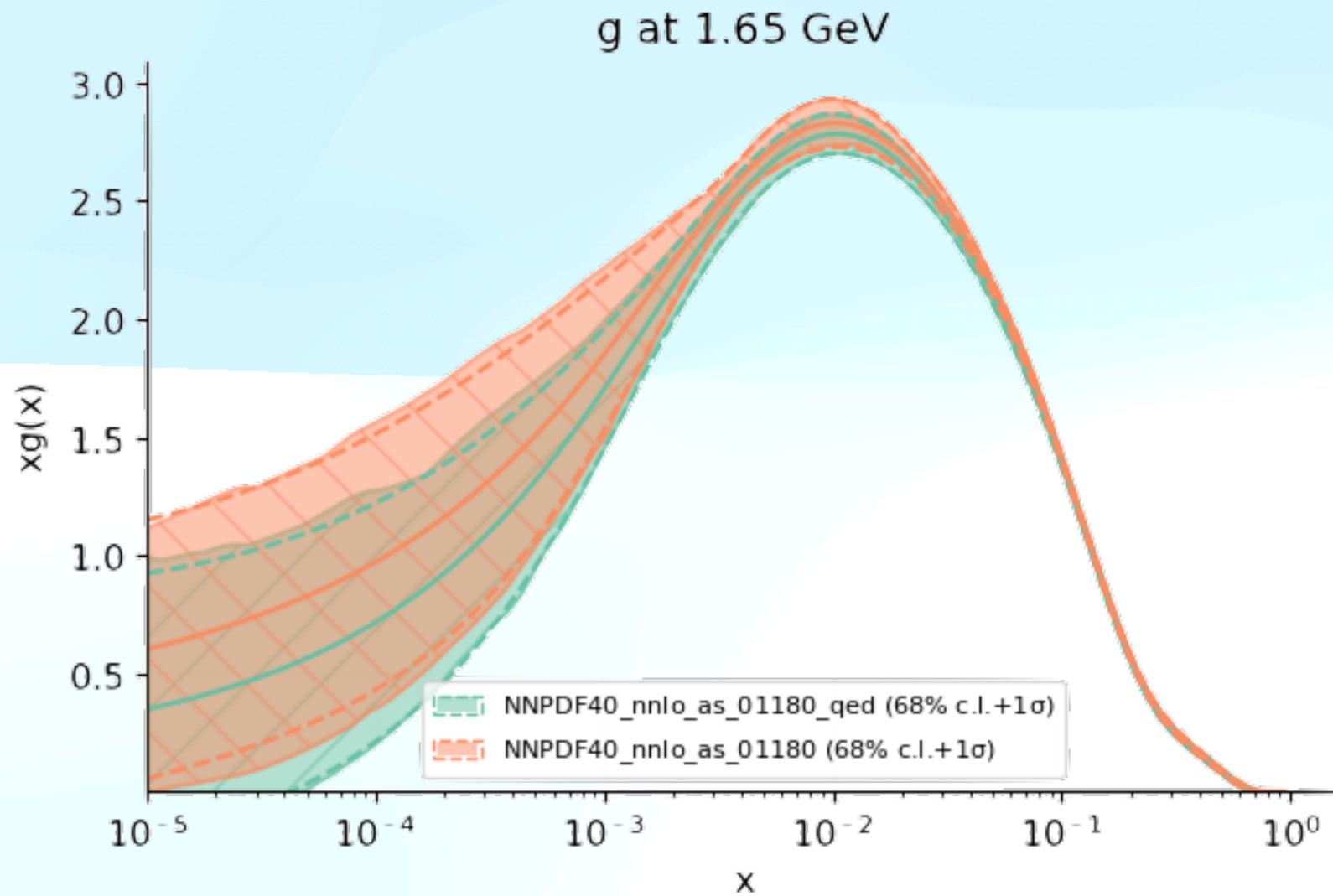
### **Future development**

Add photon-initiated contributions and add data points in which they are important

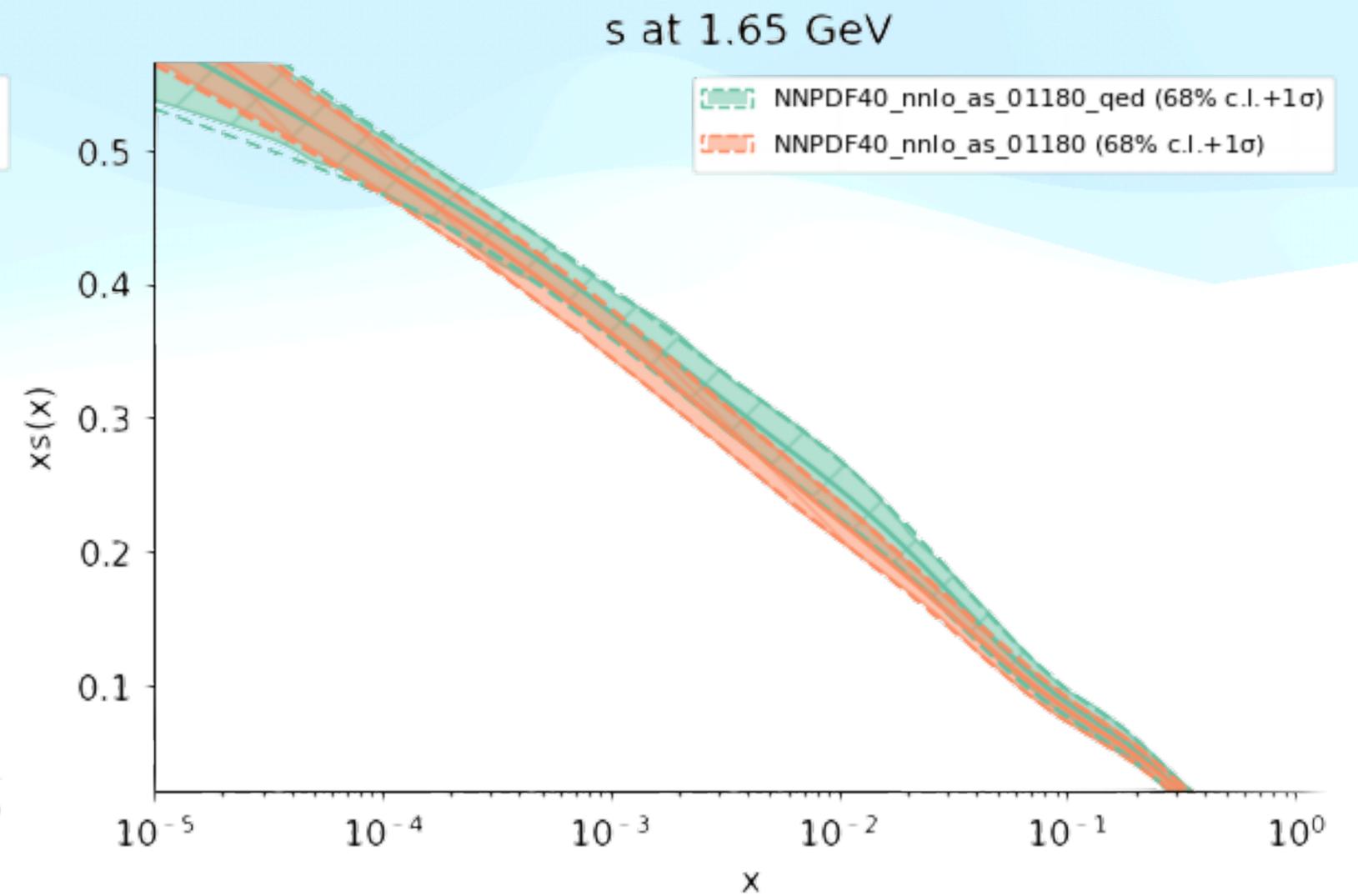
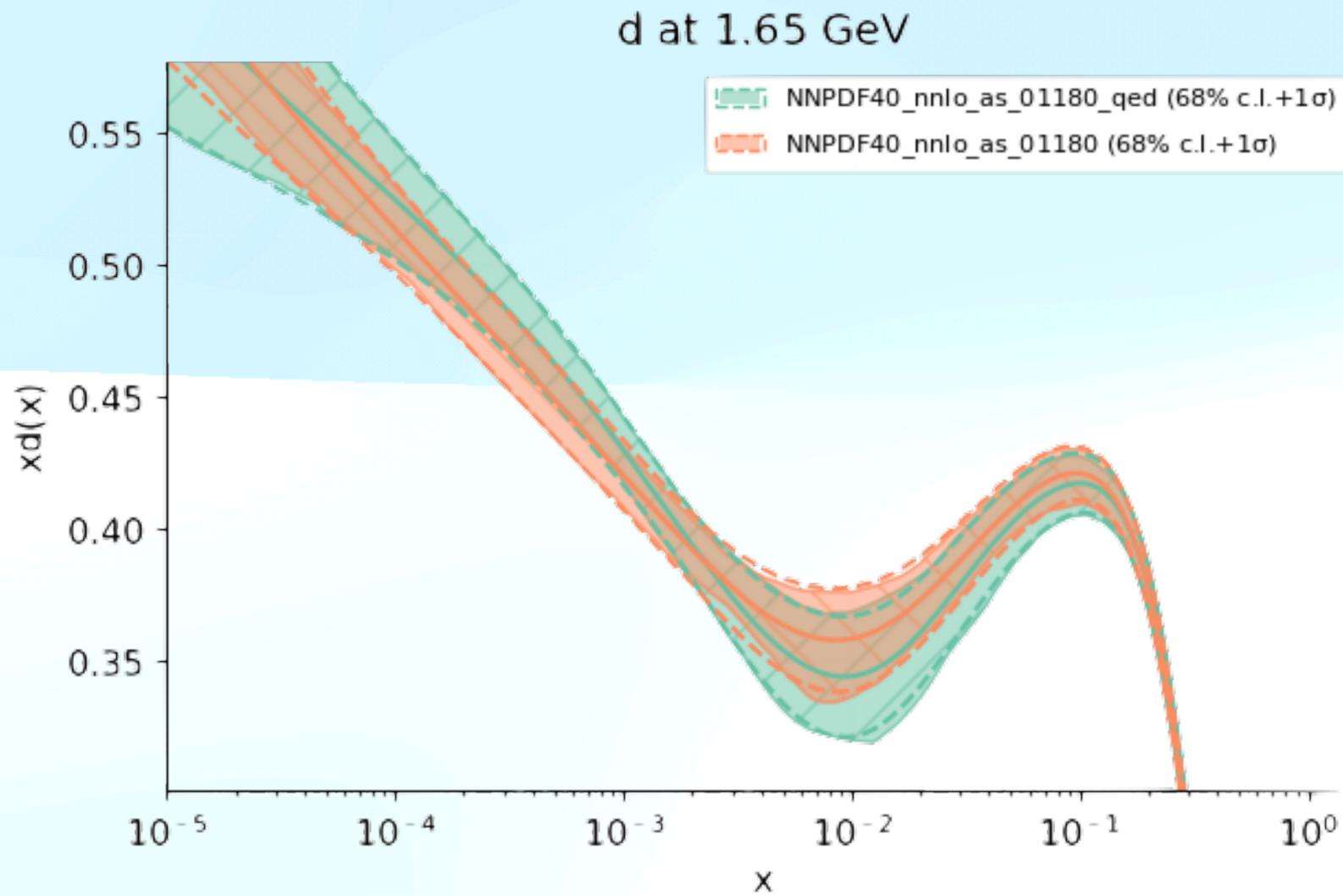
## Comparison with other QED fits

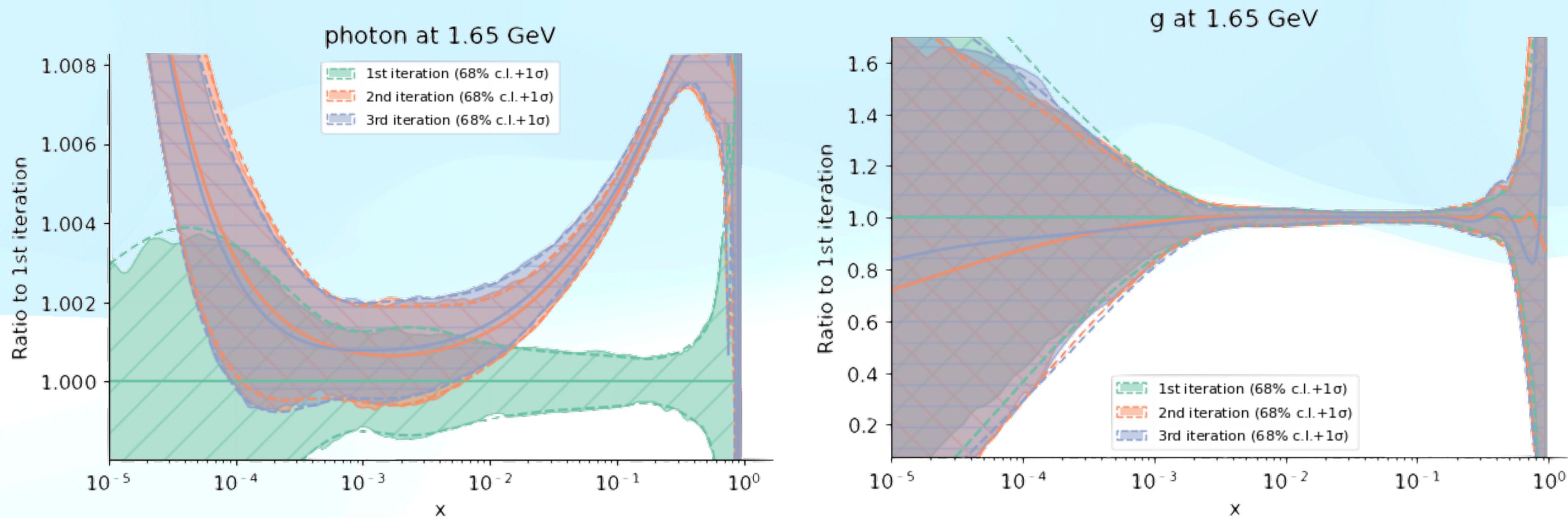


## Comparison with NNPDF4.0



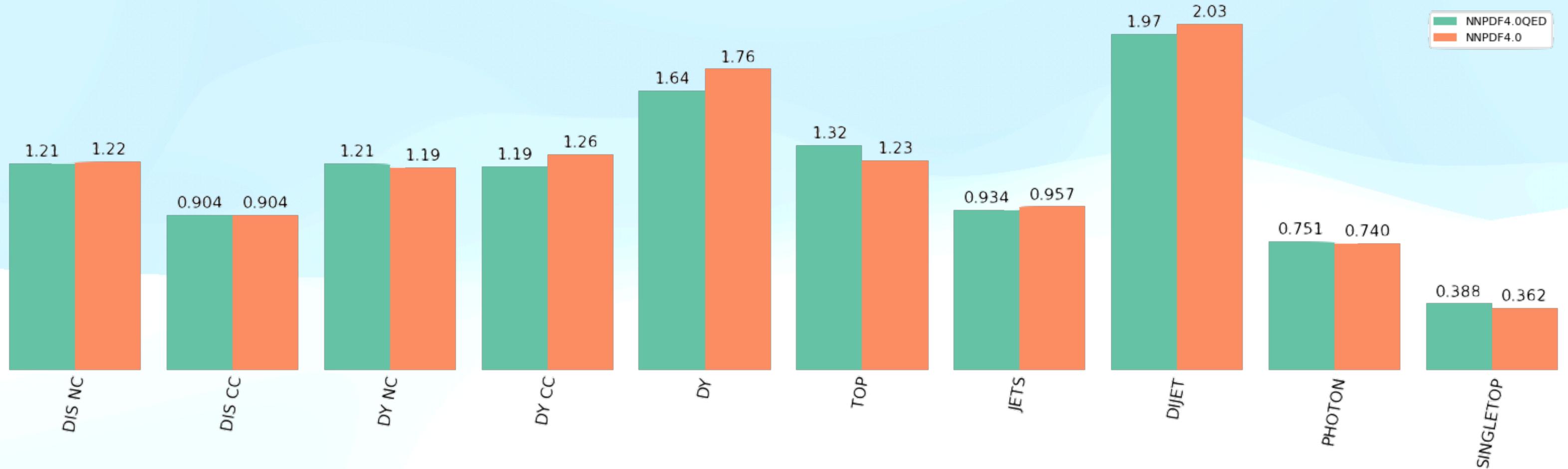
## Comparison with NNPDF4.0

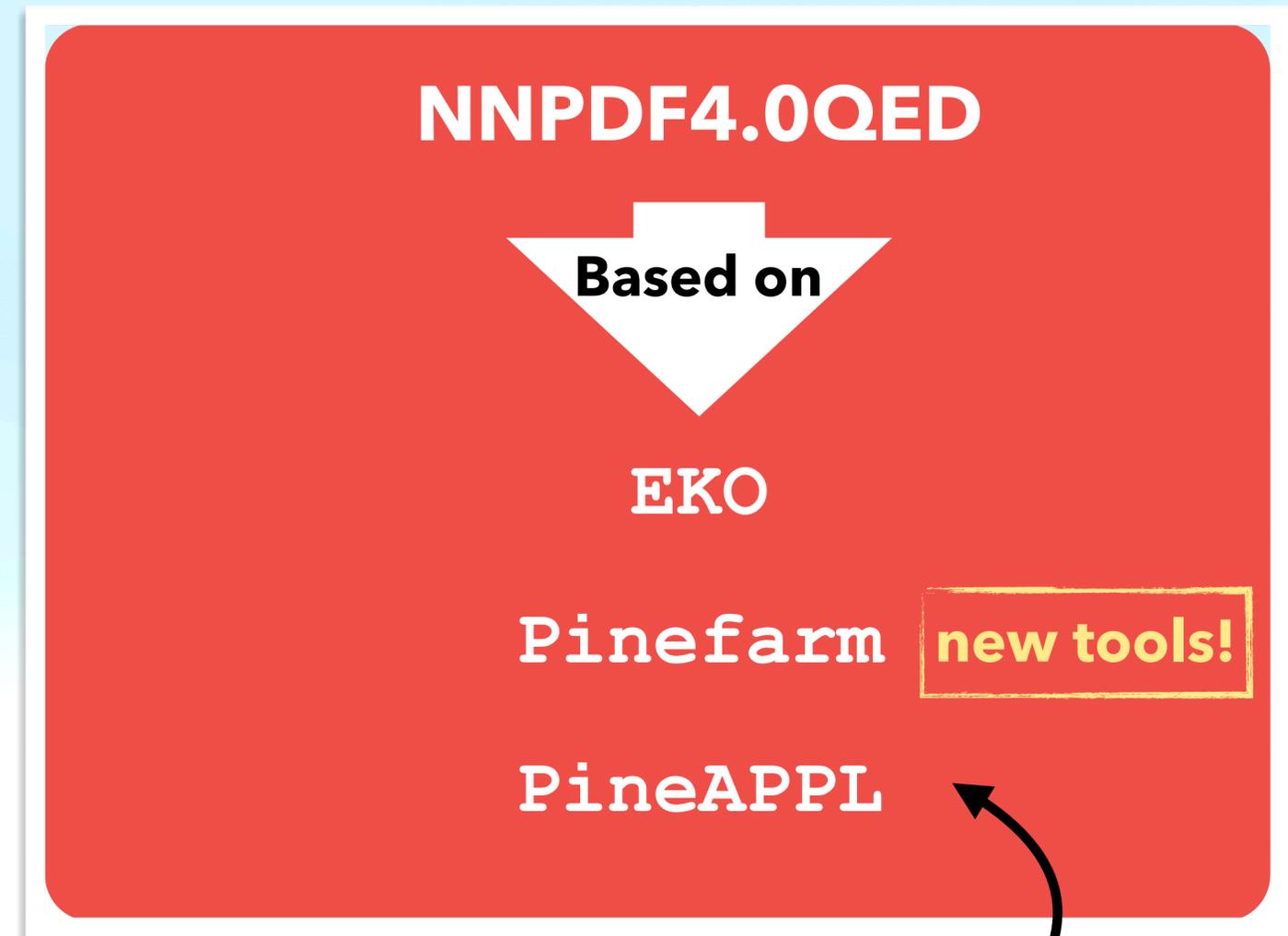
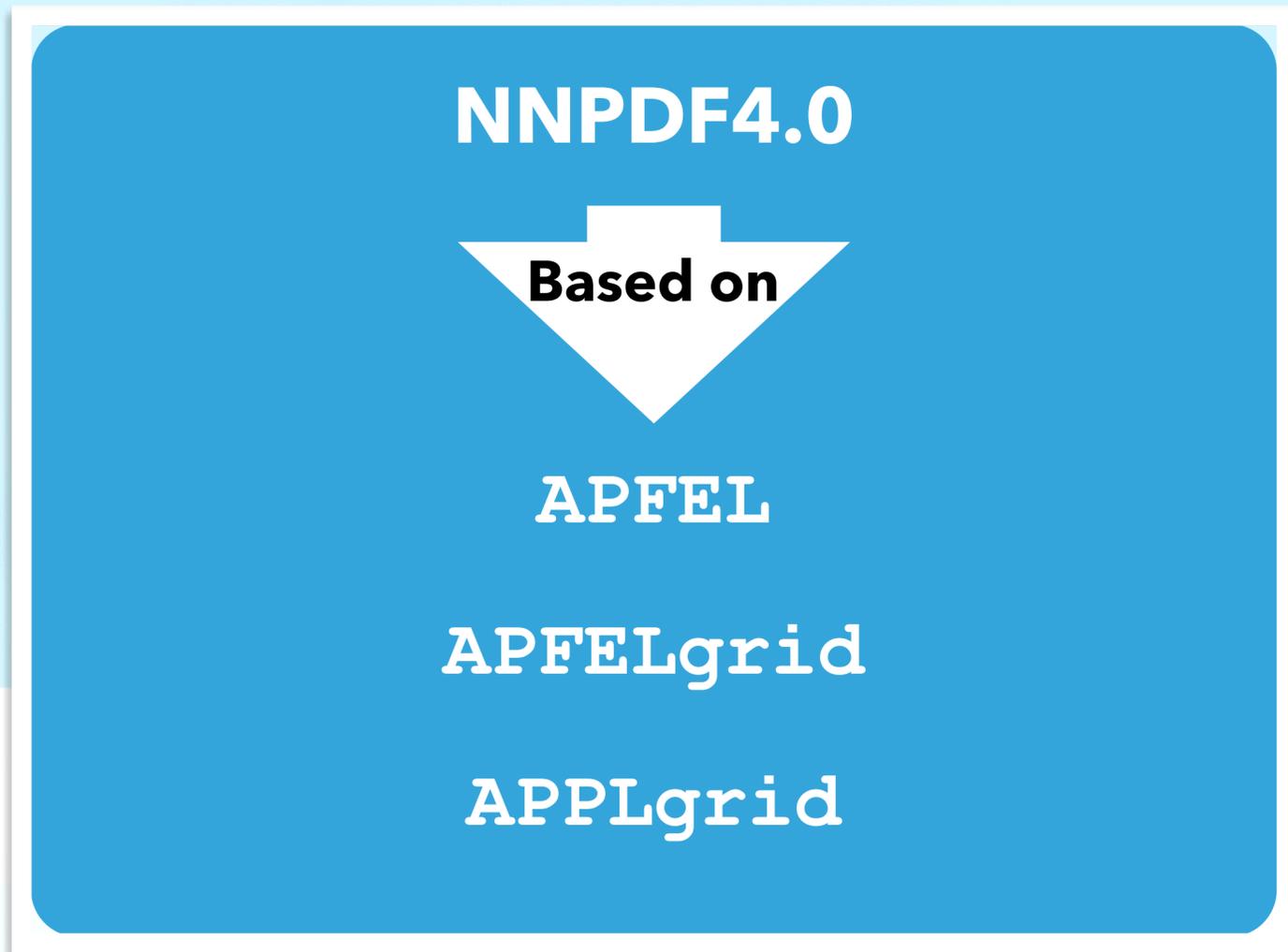




**Two iterations are enough  
for the fit to converge!**

$\chi^2$  by processes:



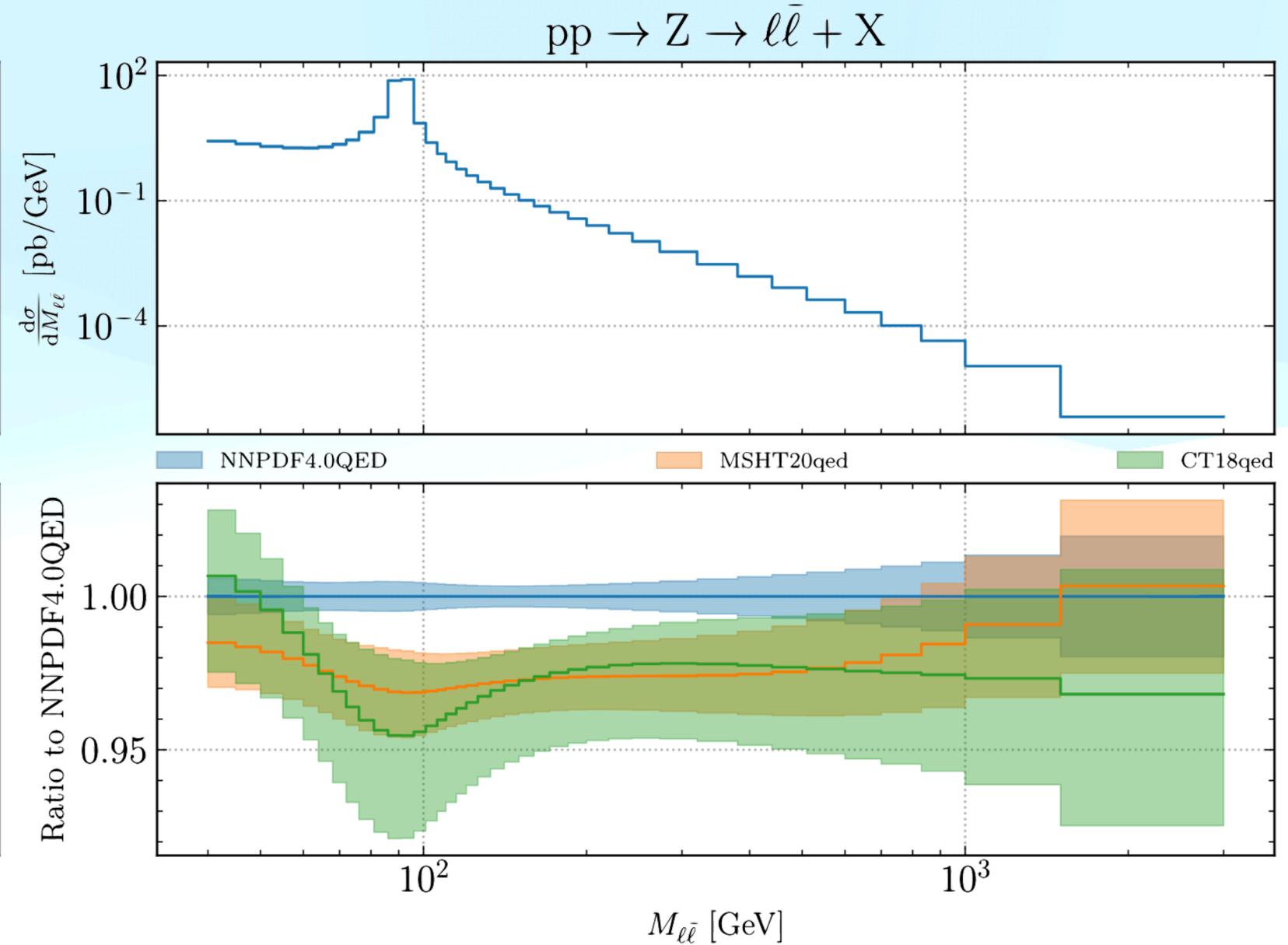
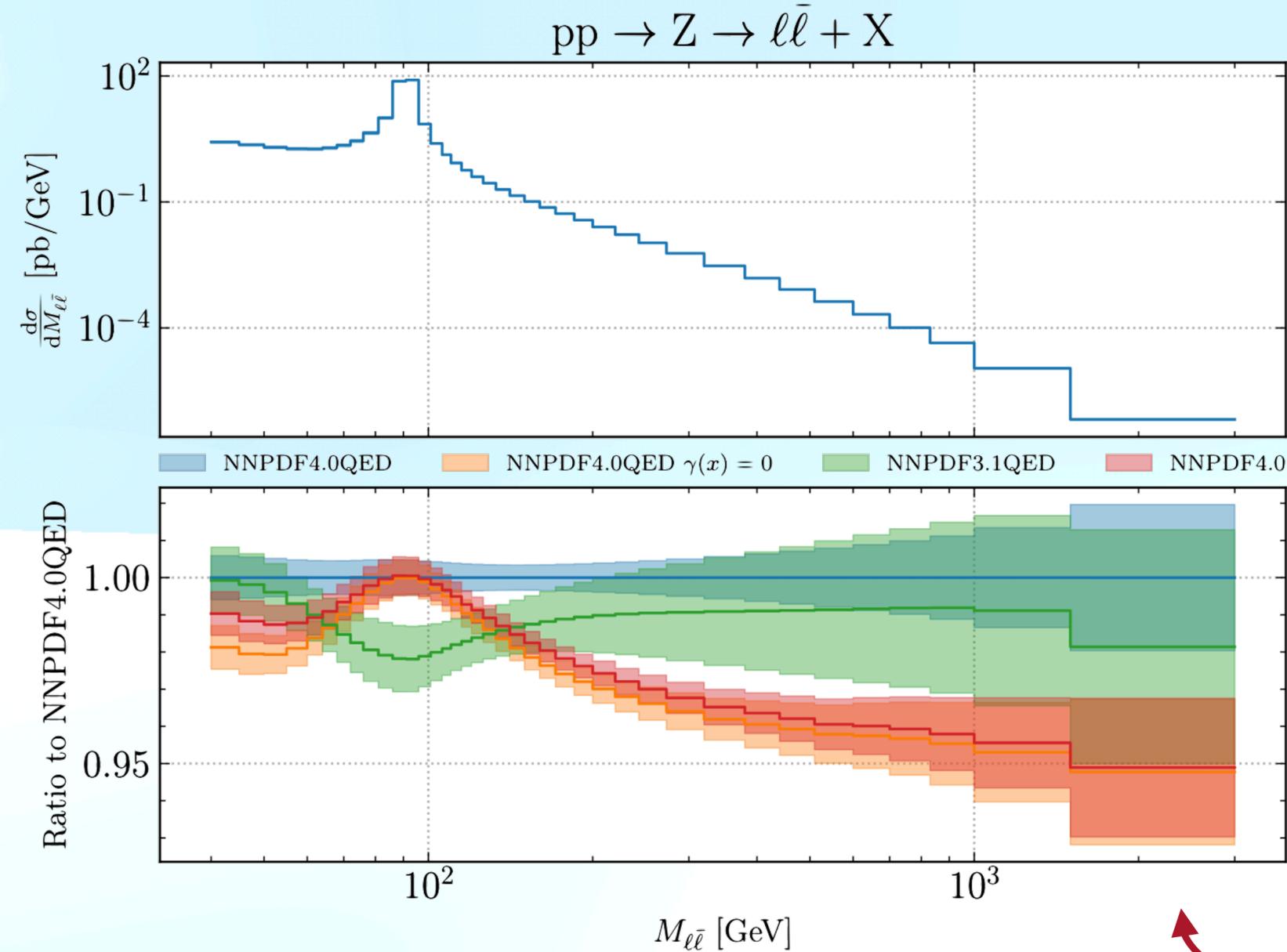


**It will allow photon induced contributions in the theory predictions!**

See ["Pineline: Industrialization of High-Energy Theory Predictions"](#) by A. Barontini

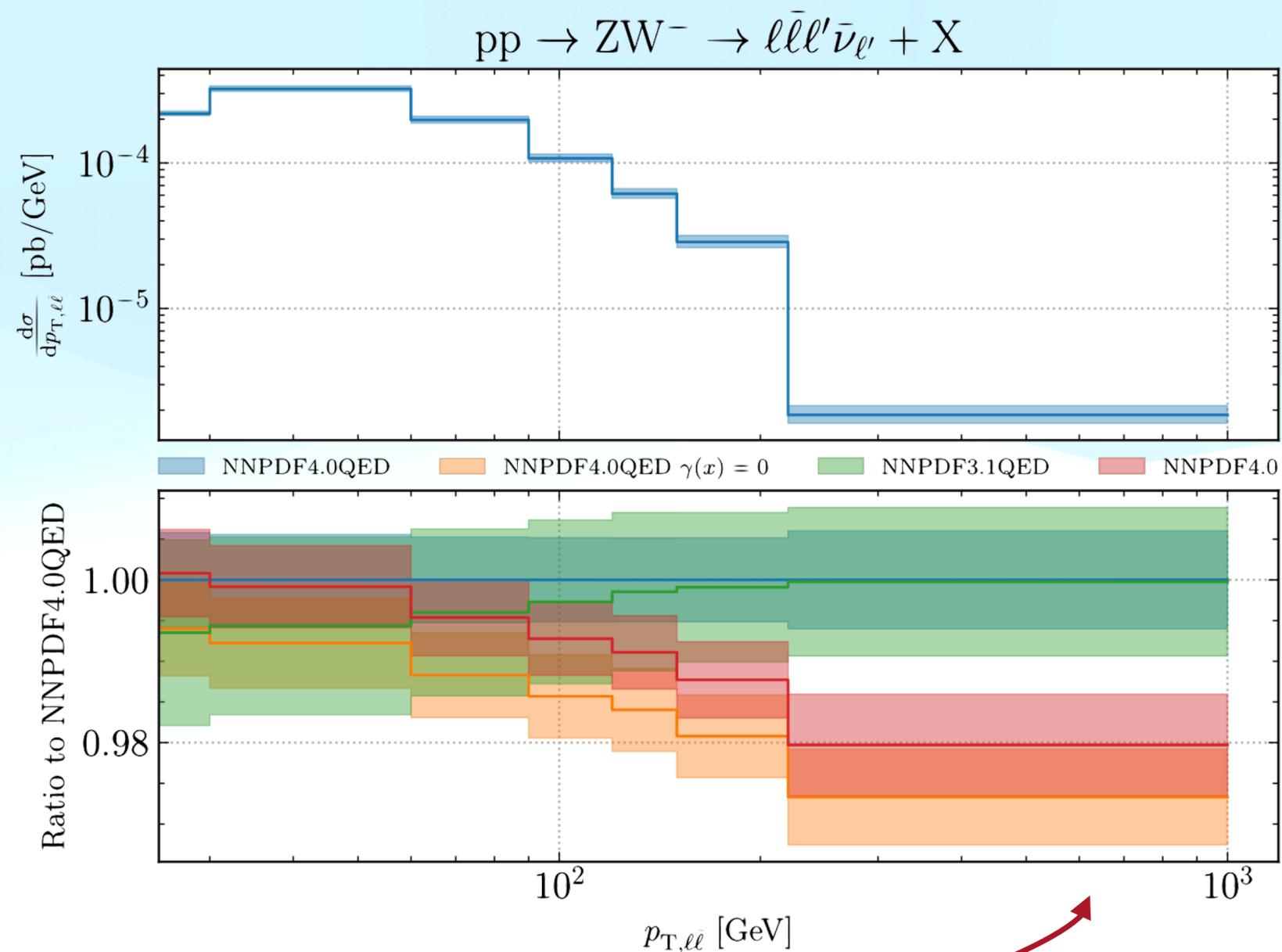
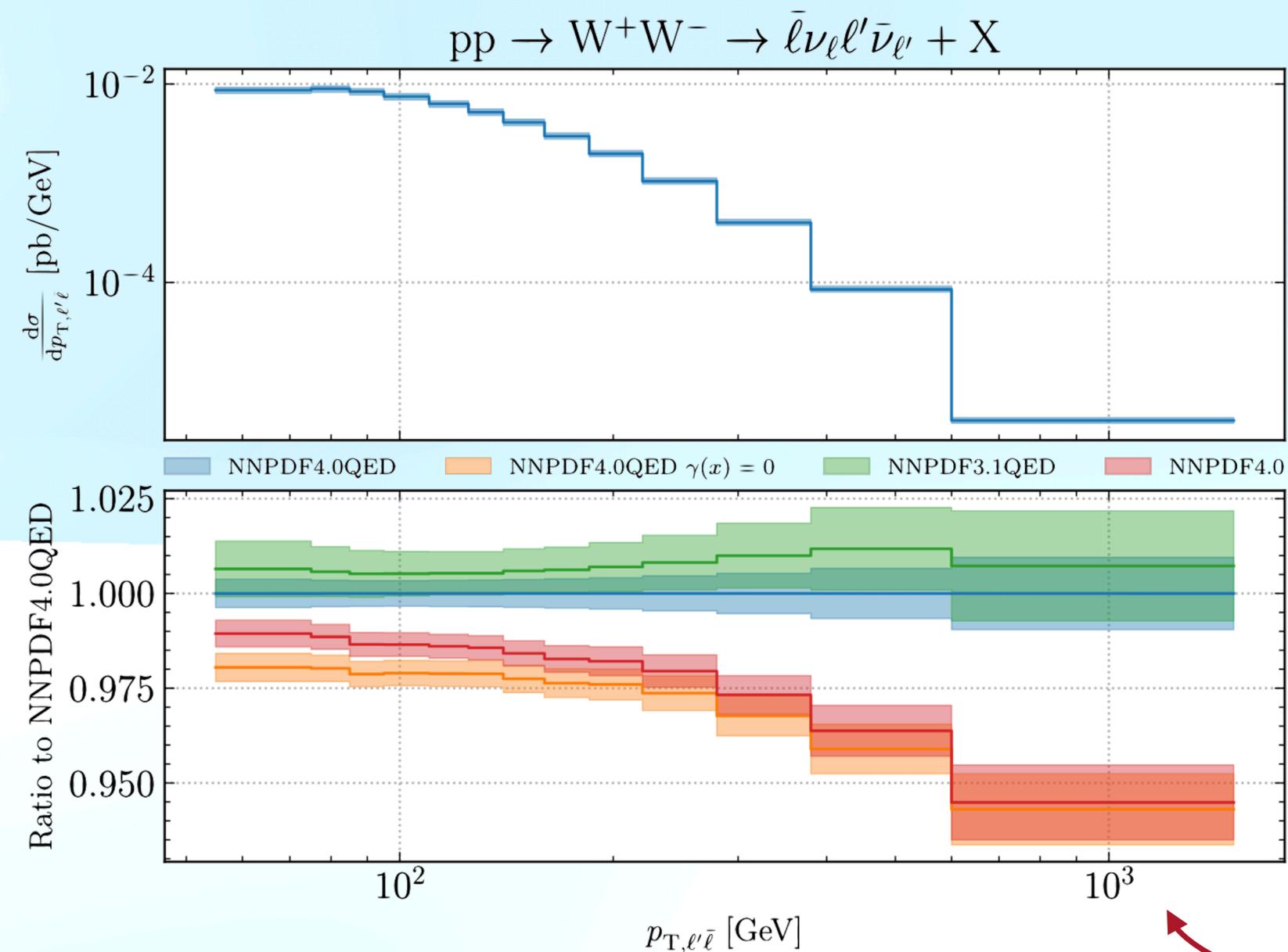
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# IMPACT ON PHENOMENOLOGY



In this section photon-induced channels have been included

In the high invariant mass region QED corrections are not negligible!



In the high  $p_T$  region QED corrections are not negligible!

# **SUMMARY AND CONCLUSIONS**

- We can add QED corrections to PDF fitting, getting a photon PDF
- The photon PDF is compatible with the most recent QED fits
- Quarks and gluon are almost unchanged (the photon PDF is small)
- There are processes in which photon initiated contributions are not negligible
- Soon we will be able to add such processes in our theory predictions

**Thanks for your attention!!**

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# BACKUP SLIDES

# Solving DGLAP

## Singlet and Valence sectors

$$\mathbf{P}_s = \begin{pmatrix} P_{gg} + \tilde{P}_{gg} & \tilde{P}_{g\gamma} & P_{gq} + \langle \tilde{P}_{gq} \rangle & \nu_u \tilde{P}_{g\Delta q} \\ \tilde{P}_{\gamma g} & \tilde{P}_{\gamma\gamma} & \langle \tilde{P}_{\gamma q} \rangle & \nu_u \tilde{P}_{\gamma\Delta q} \\ 2n_f(P_{qg} + \langle \tilde{P}_{qg} \rangle) & 2n_f \langle \tilde{P}_{q\gamma} \rangle & P_{qq} + \langle \tilde{P}_q^{\text{ns},+} \rangle + \langle e_q^2 \rangle^2 \tilde{P}_{ps} & \nu_u \tilde{P}_{\Delta q}^{\text{ns},+} + \nu_u e_{\Delta q}^2 \langle e_q^2 \rangle \tilde{P}_{ps} \\ 2n_f \nu_d \tilde{P}_{\Delta qg} & 2n_f \nu_d \tilde{P}_{\Delta q\gamma} & \nu_d \tilde{P}_{\Delta q}^{\text{ns},+} + \nu_d e_{\Delta q}^2 \langle e_q^2 \rangle \tilde{P}_{ps} & P_{\text{ns},+} + \{ \tilde{P}_q^{\text{ns},+} \} + \nu_u \nu_d (e_{\Delta q}^2)^2 \tilde{P}_{ps} \end{pmatrix}$$

$$\mathbf{P}_v = \begin{pmatrix} P_{\text{ns},V} + \langle \tilde{P}_q^{\text{ns},-} \rangle & \nu_u \tilde{P}_{\Delta q}^{\text{ns},-} \\ \nu_d \tilde{P}_{\Delta q}^{\text{ns},-} & P_{\text{ns},-} + \{ \tilde{P}_q^{\text{ns},-} \} \end{pmatrix}$$

$$\nu_{uld} = \frac{n_{uld}}{n_f}, \quad \langle \tilde{P}_q^{\text{ns},\pm} \rangle = \nu_u \tilde{P}_u^{\text{ns},\pm} + \nu_d \tilde{P}_d^{\text{ns},\pm},$$

$$\{ \tilde{P}_q^{\text{ns},\pm} \} = \nu_d \tilde{P}_u^{\text{ns},\pm} + \nu_u \tilde{P}_d^{\text{ns},\pm}, \quad \tilde{P}_{\Delta q}^{\text{ns},\pm} = \tilde{P}_u^{\text{ns},\pm} - \tilde{P}_d^{\text{ns},\pm}$$

# Solving DGLAP

## Solution of the non-diagonal sectors

$$\mathbf{E}_S(\mu^2 \leftarrow \mu_0^2) = \mathcal{P} \exp \left( - \int_{\log \mu_0^2}^{\log \mu^2} \gamma_S(\alpha_s(\mu'^2), \alpha(\mu'^2)) d \log \mu'^2 \right) \simeq \prod_{k=0}^{n-1} \mathbf{E}_S(\mu^{2(k+1)} \leftarrow \mu^{2(k)})$$

$\mu^{2(n)} = \mu^2$

$$\gamma(N) = - \int_0^1 dz z^{N-1} P(z)$$

Solved in Mellin space

$$\mathbf{E}_S(\mu^{2(k+1)} \leftarrow \mu^{2(k)}) = \exp \left( - \gamma_S(\alpha_s(\mu^{2(k+1/2)}), \alpha(\mu^{2(k+1/2)})) \Delta \log \mu^{2(k)} \right)$$

$$\log \mu^{2(k+1/2)} = \frac{\log \mu^{2(k+1)} + \log \mu^{2(k)}}{2}$$

$$\Delta \log \mu^{2(k)} = \log \mu^{2(k+1)} - \log \mu^{2(k)}$$

# Computation of the photon

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Why the LuxQED formula is used at 100 GeV?

LuxQED neglects higher twist corrections

$$\mathcal{O}\left(\frac{\Lambda}{\mu}\right)$$

For low  $\mu$ , the integral is dominated by low  $Q^2$  structure functions  
**non-perturbative!**