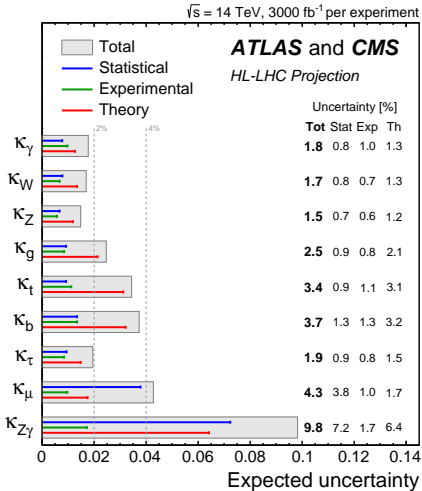
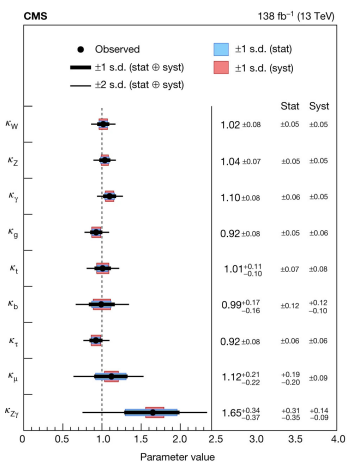


# Precision Higgs Calculations

Matthias Steinhauser | QCD@LHC 2023, Durham, UK, Sep. 4-8, 2023

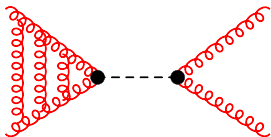
TTP KARLSRUHE

# Higgs couplings

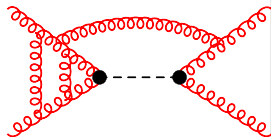
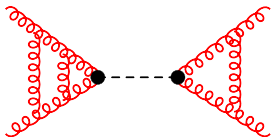


$$gg \rightarrow H$$

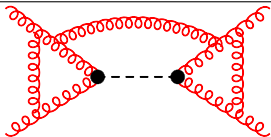
# Higgs production to N<sup>3</sup>LO



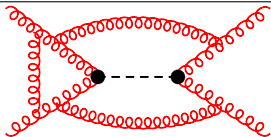
[Baikov,Chetyrkin,Smirnov,Smirnov,  
Steinhauser'09],  
[Gehrmann,Glover,Huber,Ikizlerli,  
Studerus'10]; [Lee,Smirnov'10]



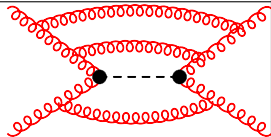
[Duhr,Gehrmann'13], [Li,Zhu'13],  
[Dulat,Mistlberger'14],  
[Duhr,Gehrmann,Jaquier'14]



[Anastasiou,Duhr,Dulat,Herzog,  
Mistlberger'13], [Kilgore'13]

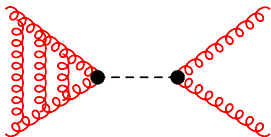


[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,  
Herzog,Mistlberger'14],  
[Li,von Manteuffel,Schabinger,Zhu'14]



[Anastasiou,Duhr,Dulat,Mistlberger'13]

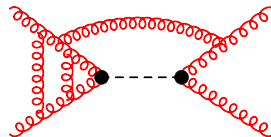
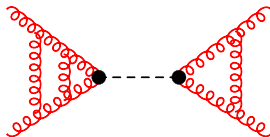
# Higgs production to N<sup>3</sup>LO



[Baikov,Chetyrkin,Smirnov,Smirnov,  
Steinhauser'09],

[Gehrmann,Clevers,Huber,Kizileva,

St

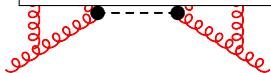


[Duhr,Gehrmann'13], [Li,Zhu'13],  
[Dulat,Mistlberger'14],

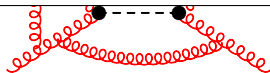
**N<sup>3</sup>LO:** [Anastasiou,Duhr,Dulat,Herzog,Mistlberger'15]

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16]

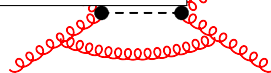
[Mistlberger'18]



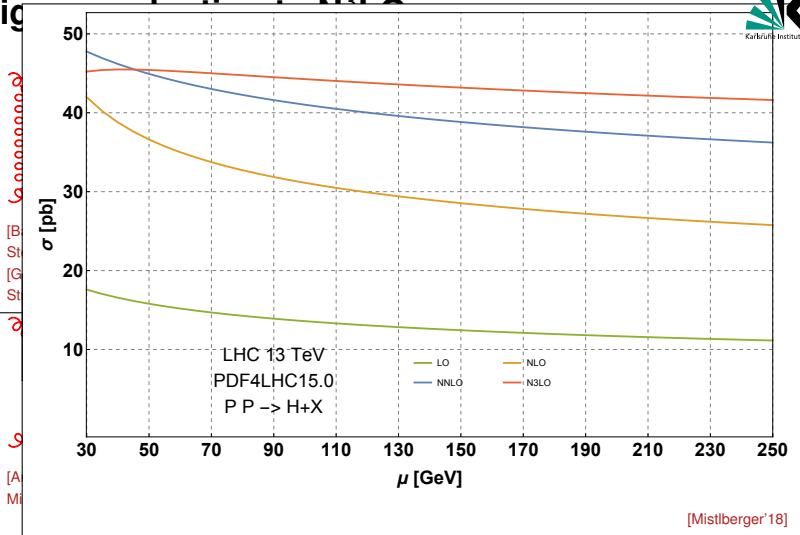
[Anastasiou,Duhr,Dulat,Herzog,  
Mistlberger'13], [Kilgore'13]



[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,  
Herzog,Mistlberger'14],  
[Li,von Manteuffel,Schabinger,Zhu'14]



[Anastasiou,Duhr,Dulat,Mistlberger'13]



[B  
St  
[G  
St  
[A  
Mi

[Mistlberger'13],  
 ]  
 [Mistlberger'13]

[Mistlberger'18]

# Remaining uncertainties

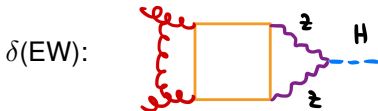
[Anastasioua,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16]

$\delta(\text{scale})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
+0.21%	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$
-2.37%				

# Remaining uncertainties

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16]

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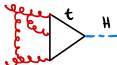
[Anastasiou,Boughezal,Petriello'09; Bonetti,Melnikov,Tancredi'16'18; Hirschi et al.'19; Anastasiou et al. '18; Bonetti et al.'20; . . . ; Becchetti,Bonciani,Del Duca,Hirschi,Moriello,Schweitzer'20]



[Anastasioua,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16]

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-2.37%				

$\delta(1/m_t)$ : finite  $m_t$  terms at NNLO

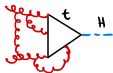


[Czakon,Harlander,Klappert,Niggetiedt'21]

massive 3-loop form factor:

[Davies,Gröber,Maier,Rauh,Steinhauser'19; Harlander,Prausa,Usovitsch'19; Cakon,Niggetiedt'20]

4-loop form factor



available for  $m_H \ll m_t$  [Davies,Herren,Steinhauser'19]

(3 expansion terms; fast convergence)

1st step at N<sup>3</sup>LO beyond the  $m_t \rightarrow \infty$  limit

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger'16]

$\delta(\text{scale})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
+0.21%	$\pm 1.16\%$	<del><math>\pm 1\%</math></del>	$\pm 0.83\%$	<del><math>\pm 1\%</math></del>
-2.37%				

$\delta(t, b, c)$ : finite  $m_b$  effects in  $t - b$  interference



effective expansion parameter:  $\alpha_s \log^2 \frac{m_H^2}{m_b^2} \sim 4$

[Melnikov, Penin'16; Liu, Penin'17; ... Caolo et al.'18; ...; Anastasiou, Penin'20; Liu, Neubert, Schnubel, Wang'22]

all-order sub-leading logarithmic approximation:  $\alpha_s^n \log^{2n-1} (m_H^2/m_b^2)$

⇒ estimate of the bottom quark mediated contribution beyond NLO:  
 $-0.34... + 0.08 \text{ pb}$

⇒ **factor 2 reduction** [Anastasiou, Penin'20]

# Remaining uncertainties

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16]

$\delta(\text{scale})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
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-2.37%			→ reduced	

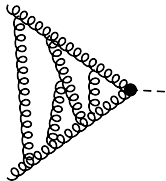
[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistberger'16]

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-2.37%			→ reduced	

- N<sup>4</sup>LO ? ⇔ 1st steps
- 4-loop  $Hgg$  form factor

[Lee,von Manteuffel,Schabinger,Smirnov,Smirnov,Steinhauser'22]

- soft-virtual approximation: [Das,Moch,Vogt'20]



# Remaining uncertainties

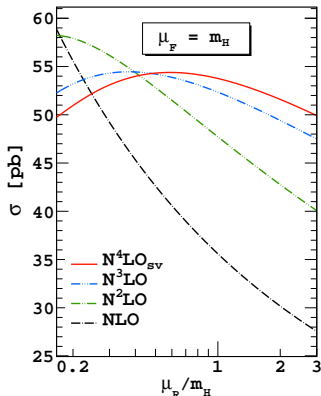
[Anastasioua,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16]

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[Lee,von Manteuffel,Schabinger,Smirnov,Smirnov,Steinhauser'22]

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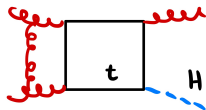
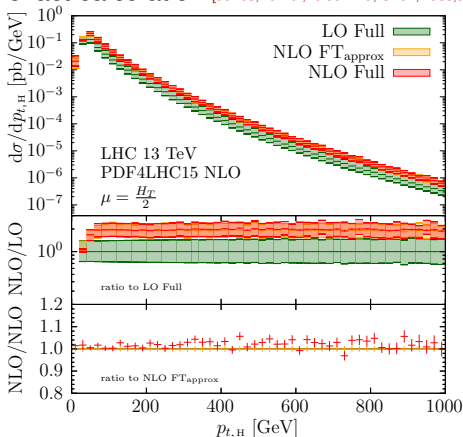




$H + \text{jet}$

# $gg \rightarrow Hg$ at NLO

- small  $m_t$  expansion [Kudashkin,Lindert,Melnikov,Wever'18; Neumann'18]
- exact calculation [Jones,Kerner,Luisoni'18; Chen,Huss,Jones,Kerner,Lang,Lindert,Zhang'21]



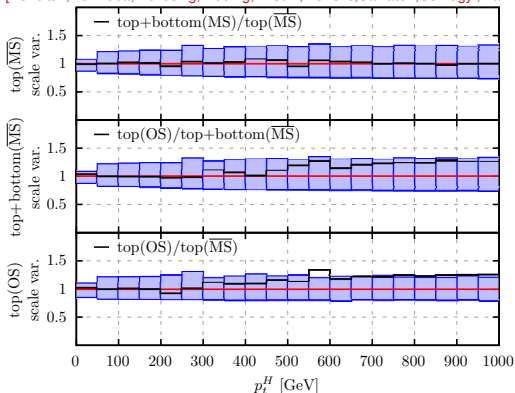
- EW corrections  $\sim \lambda$  [Gao,Shen,Wang,Yang,Zhou'23]
- full NLO EW correction in large- $m_t$  expansion [Davies,Schönwald,Steinhauser,Zhang'23]



# $gg \rightarrow Hg$ at NLO

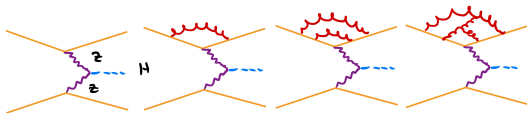
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- exact calculation [Jones,Kerner,Luisoni'18; Chen,Huss,Jones,Kerner,Lang,Lindert,Zhang'21]
- $m_t$  and  $m_b$  dependence; study of renormalization schemes

[Bonciani,Del Duca,Frellesvig,Hidding,Hirschi,Moriello,Salvatori,Somogyi,Tramontano'22]

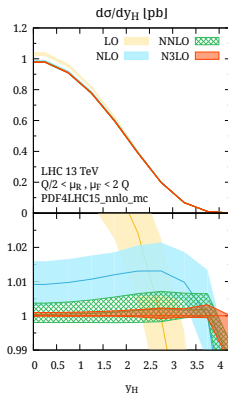
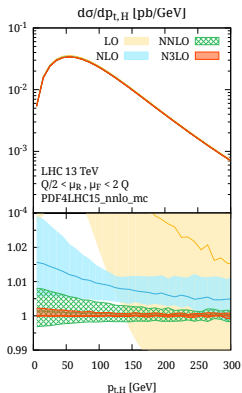
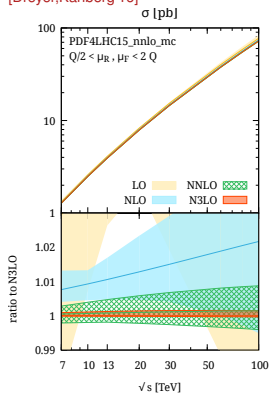


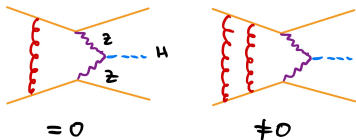
# VBF

# Factorizable



[Dreyer, Karlberg '16]



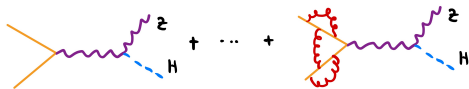


- $1/N_c^2$  suppressed
- but:  $\pi^2$  enhancement [Liu,Melnikov,Penin'19] (see also [Dreyer,Karlberg,Tencredi'22])
- [Asteriadis,Brønnum-Hansen,Melnikov'23]: real-virtual and double-real non-factorizable contribution
- [Long,Melnikov,Quarroz'23]: leading power correction to the eikonal limit

⇒ talk by Christian Brønnum-Hansen

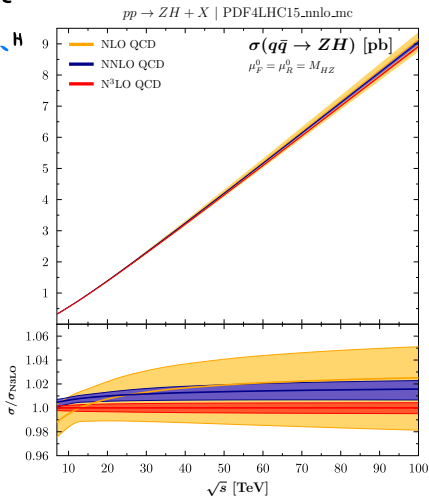
$$H + V$$

# $q\bar{q}$ channel to N<sup>3</sup>LO

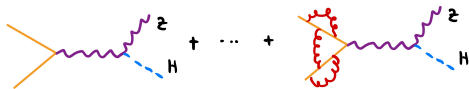


## Inclusive N<sup>3</sup>LO

[Baglio, Duhr, Mistlberger'22]



# $q\bar{q}$ channel to $N^3\text{LO}$



- Inclusive  $N^3\text{LO}$

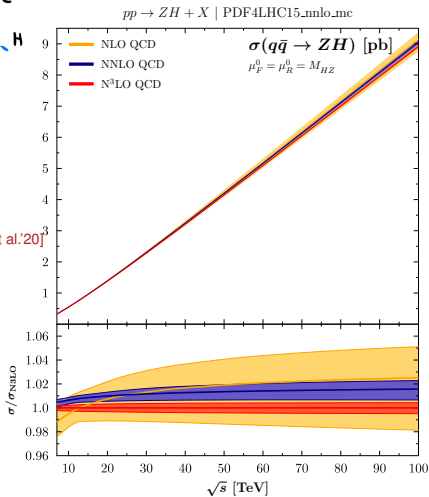
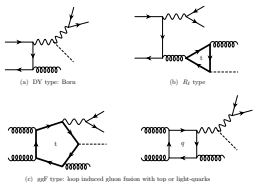
[Baglio,Duhr,Mistlberger'22]

- NNLO QCD +  $H$  decays

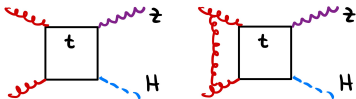
[Ferrera et al'17; Caola et al.'17; Gauld et al. '19; Behring et al.'20]

- $VH$  + jet to  $\alpha_s^3$

[Gauld,Gehrmann-De Ridder,Glover,Huss,Majer'21]



# $gg \rightarrow ZH$

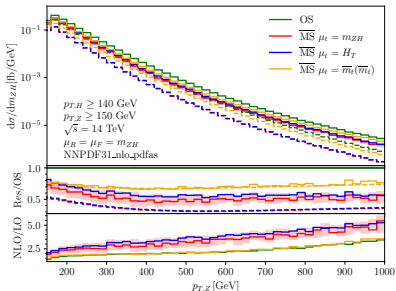
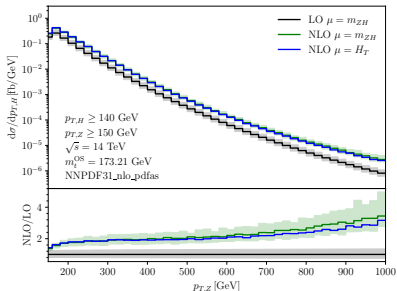


- [Davies,Mishima,Steinhauser'20]: high-energy expansion ( $s, t \gg m_t^2 \gg m_Z^2, m_H^2$ )
- [Bellafronte,Degrassi,Giardino,Gröber,Vitti'22]: small  $p_T$  expansion
- [Chen,Heinrich,Jones,Kerner,Klappert,Schlenk'20] numerical calculation ( $s, t, m_t, m_Z, m_H$ )
- [Wang,Xu,Xu,Yang'21]: expansion in  $m_Z$  and  $m_H$ ; analytic and numeric methods for the remaining integrals
- [Chen,Davies,Heinrich,Jones,Kerner,Mishima,Schlenk,Steinhauser'22]:  
numerical calculation for  $p_T < 150$  GeV  
⊗ high-energy expansion for  $p_T > 150$  GeV  
⇒ avoid costly numerical evaluation in large parts of phase space
- [Degrassi,Gröber,Vitti,Zhao'22]: small  $p_T$  ⊗ high-energy expansion



# $gg \rightarrow ZH$

[Chen,Davies,Heinrich,Jones,Kerner,Mishima,Schlenk,Steinhauser'22]

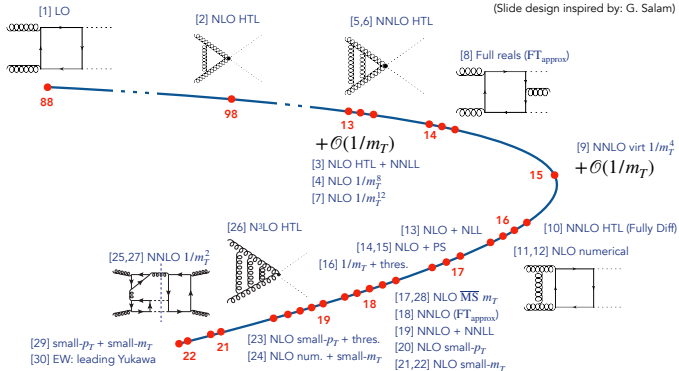


- large  $K$  factors (slightly tamed by cuts on soft  $Z$  or  $H$ )
- large  $m_t$  renormalization scheme dependence

*HH*

# HH: Theory History

(Slide design inspired by: G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22;

[slide from Stephen Jones]

- scales:  $s$ ,  $t$ ,  $m_t$ ,  $m_H$

- Purely numerical calculations

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke'16;

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke'16;

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher'18]

- Combination of analytic high-energy and numerical calculation  
(needed for smaller phase-space)

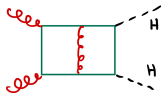
[Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann'19]

- Combination of low-order high-energy and “ $p_T$  expansion”

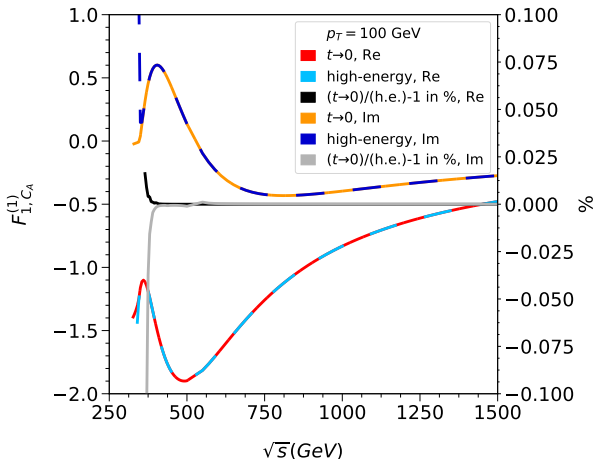
[Bellafronte, Degrandi, Giardino, Gröber, Vitti'22]

- Combination of high-order high-energy and “ $t$  expansion”

[Davies, Mishima, Steinhauser'23]

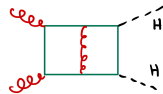


# Combination of high-order high-energy and “ $t$ expansion”

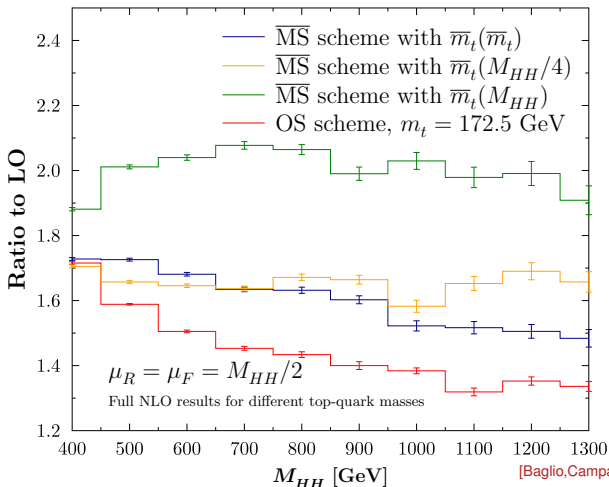


[Davies,Mishima,Schönwald,Steinhauser'23]

# Renormalization scheme dependence



$gg \rightarrow HH$  at NLO QCD |  $\sqrt{s} = 13$  TeV | PDF4LHC15

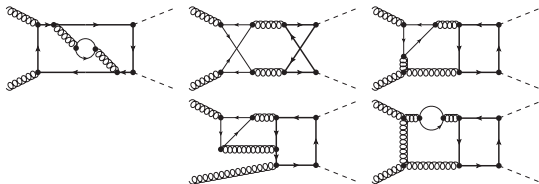


⇒ NNLO needed !?

[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]

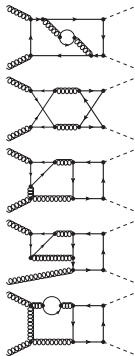
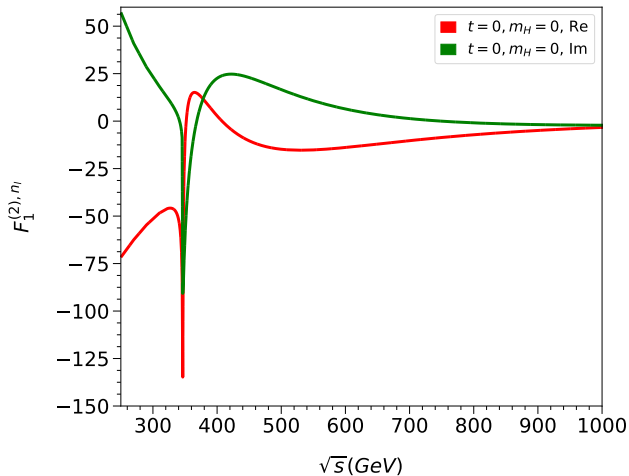
# Can we go to 3 loops?

- promising:  $t \rightarrow 0$  expansion
- fast convergence
- covers important region of phase space
- result would be useful for studying renormalization scheme dependence
- 1st step: fermionic corrections for  $t = 0$ ,  $m_H = 0$



# 3-loop $n_l$ for $t = 0, m_H = 0$

[Davies,Schönwald,Steinhauser'23]





- NNLO  $t\bar{t}H$  production [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini'23]  
⇨ talk by Simone Devoto
- VBF for HH: [Dreyer, Karlberg'18; Dreyer, Karlberg, Tancredi'20; Dreyer, Karlberg, Lang, Pellen'20]
- $pp \rightarrow H + \geq 1 \text{ jet}$ , resummed  $\log(s/p_T^2)$   
[Andersen, Hassan, Maier, Paltrinieri, Papaefstathiou, Smillie'22]
- $m_b$  effects to  $p_{TH}$  spectrum [Pietrulewicz, Stahlhofen'23]
- $q_T$  distribution of  $q\bar{q} \rightarrow H$ ,  $q = s, c, b$ , to  $N^3LL' + aN^3LO$   
[Gal, von Kuk, Lim, Tackmann'23]
- $gg \rightarrow H \rightarrow \gamma\gamma \otimes \gamma\gamma \leftarrow gg$  [Bargiela, Buccioni, Caola, Devoto, von Manteuffel, Tancredi'22]
- next-to-soft resummations [Ravindran, Sankar, Tiwari'22, ...]
- Complete NLO EW corrections to  $gg \rightarrow HH$  for large  $m_t$  [Davies et al. 23]
- $H$  decays
- $\Gamma_H$
- BSM, SMEFT, ...
- PDFs ⇨ talk by Lucian Harland-Lang
- ...

- Many complicated calculations within a short time
- (Complete) **NLO** is default  
(Towards) **NNLO** where necessary  
Sometimes also **N<sup>3</sup>LO**
- Many **innovative** tools for computing Feynman integrals
- Theory will be ready for HL-LHC