GENERALIZATION OF THE NESTED SOFT-COLLINEAR SUBTRACTION METHOD FOR NNLO QCD CALCULATIONS

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In collaboration with:
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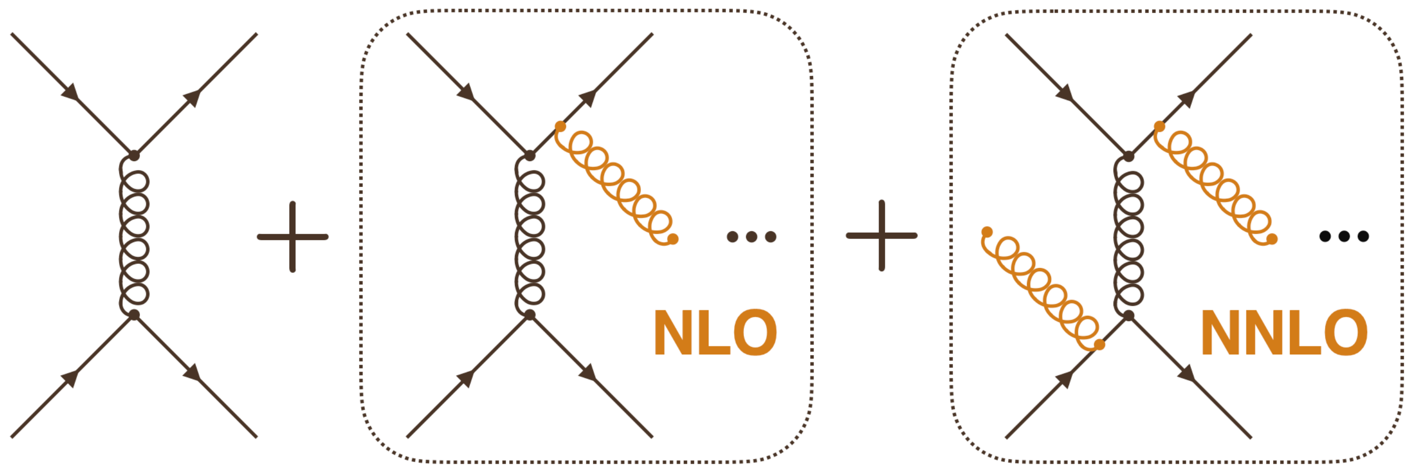
PROBLEMS AND SOLUTIONS

- In collider physics we need to compute differential partonic cross section through **fixed-order perturbation theory**



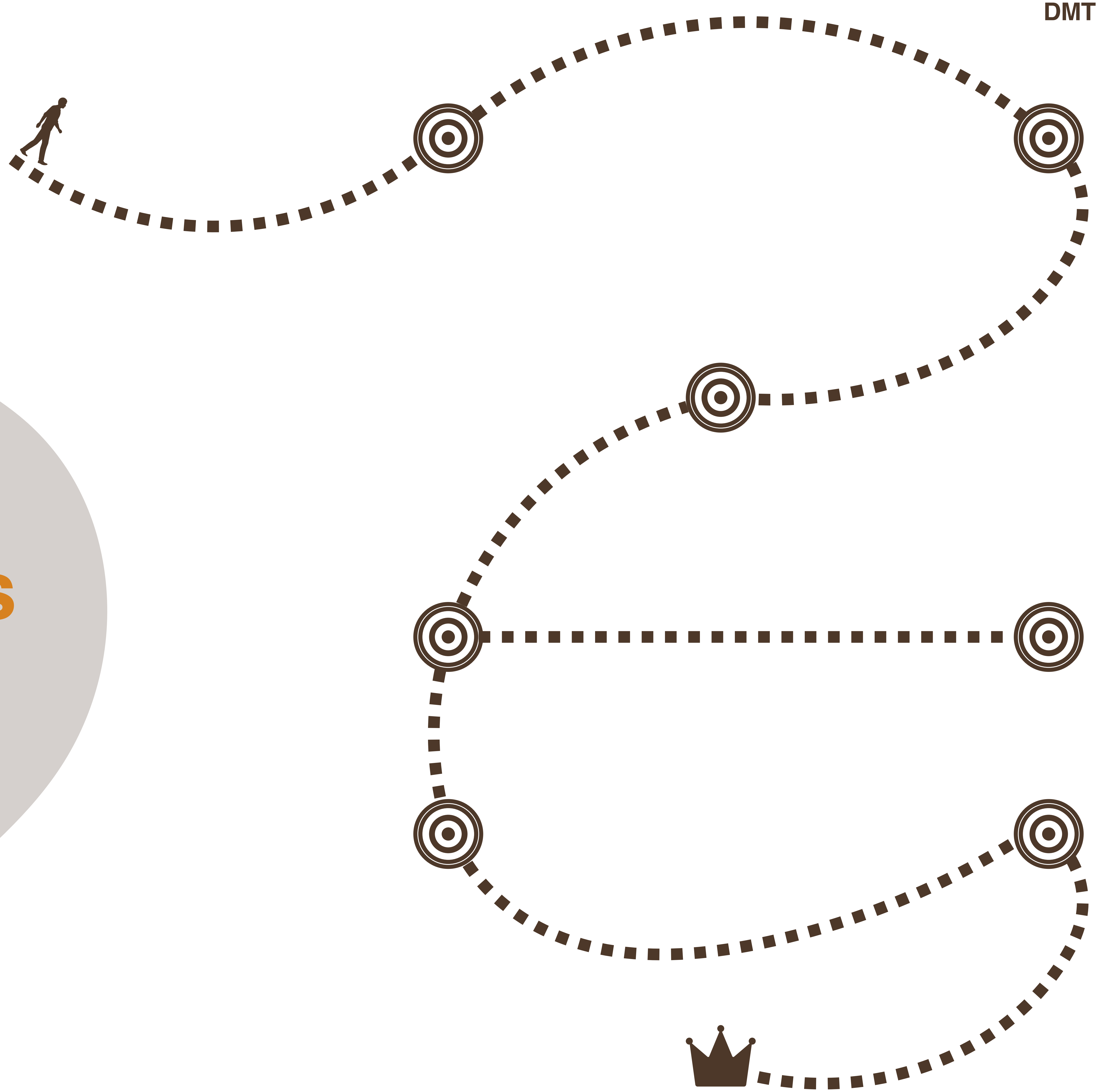
- Two main difficulties: **IR singularities**, arising from real and virtual radiation, and **multi-loop amplitude** calculations

- *About IR singularities*: they are unphysical and require specific methods to arrive at a finite physical result. Among those methods, we focus on **SUBTRACTION SCHEMES**

- Some of the many available schemes:

Analytic Sector Subtraction [Magnea et al. 1806.09570, ...]
Antenna [Gehermann-De Ridder et al. 0505111, ...]
ColorfullNNLO [Del Duca et al. 1603.08927, ...]
STRIPPER [Czakon 1005.0274, ...]
Geometric IR subtraction [Herzog 1804.07949, ...]
Unsubtraction [Sborlini et al. 1608.01584, ...]
Universal Factorization [Anastasiou et al. 2008.12293, ...]
FDR [Pittau 1208.5457, ...]
Nested Soft-Collinear Subtraction (NSC) [Caola et al. 1702.01352, ...]

WHY WE STUDY
$P + P \longrightarrow X + N$ gluons
AT NNLO



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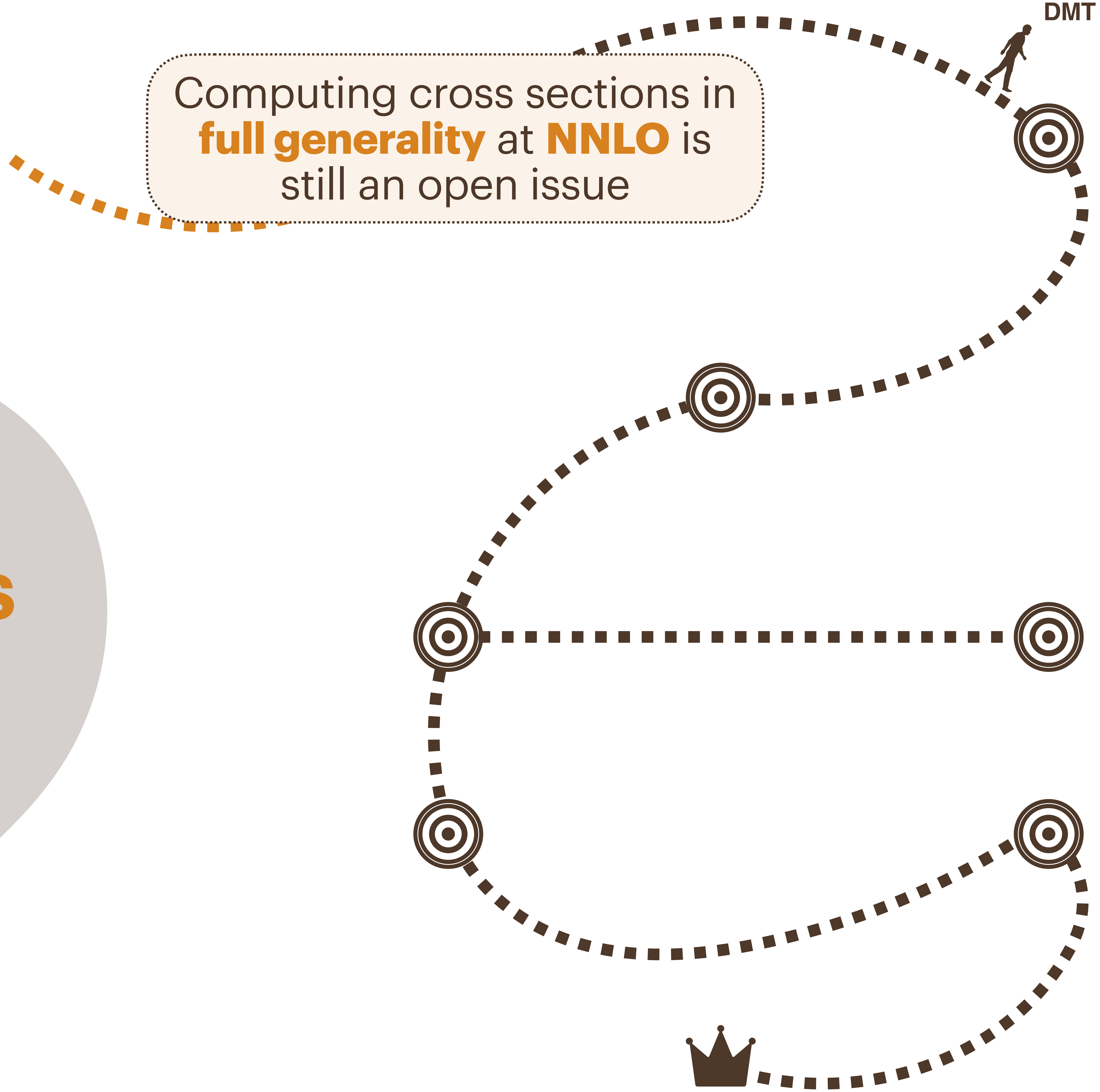
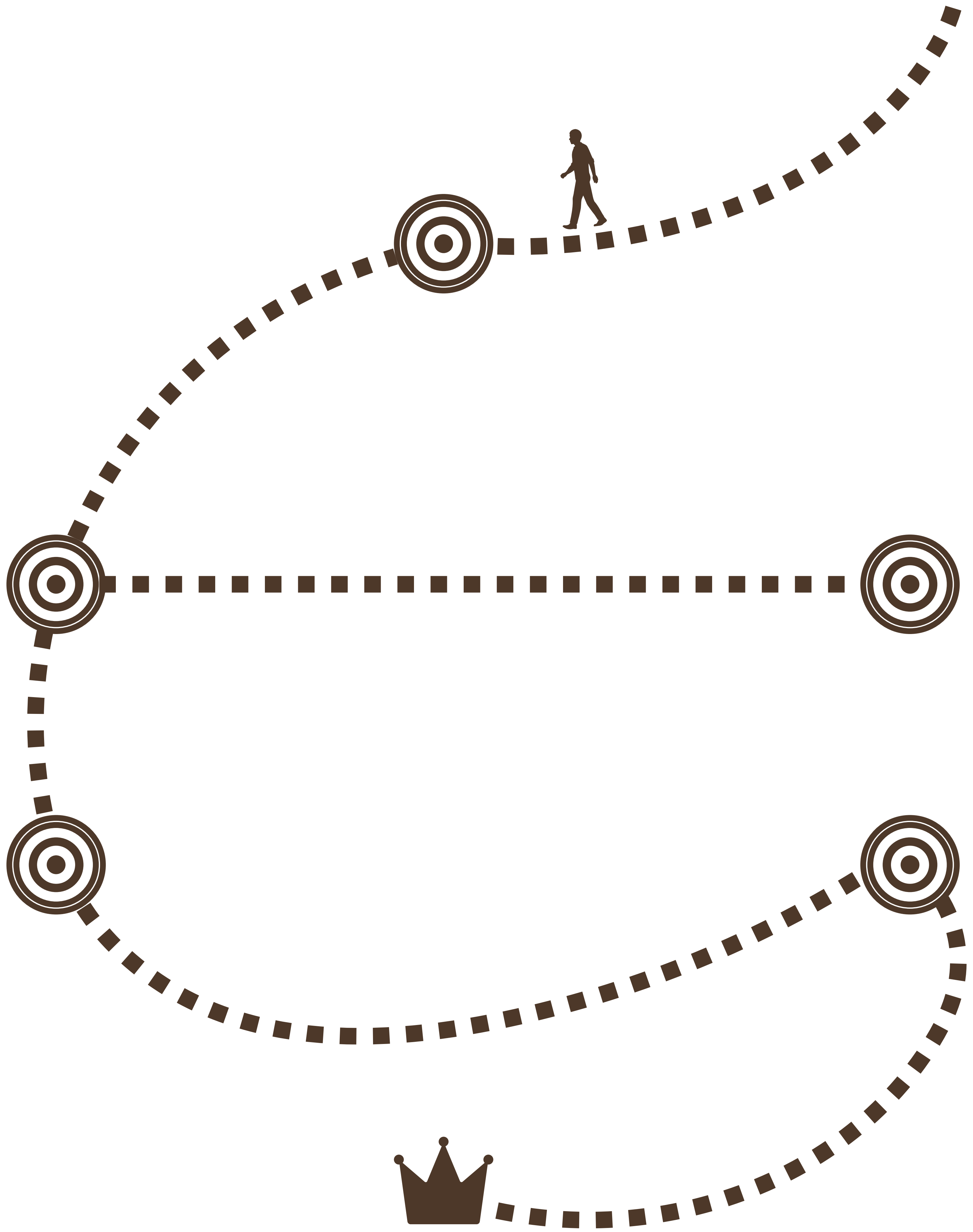


WHY WE STUDY P + P → X + N gluons AT NNLO

Computing cross sections in **full generality** at **NNLO** is still an open issue

Up to now **NSC** has only been applied to **simple** processes



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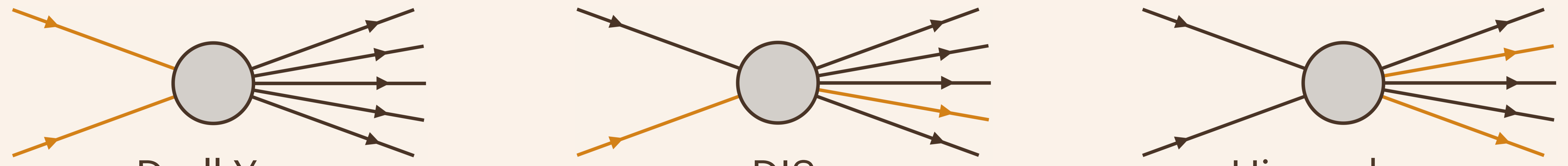

Drell-Yan [Caola, Melnikov, Röntsch '19] | DIS [Asteriadis, Caola, Melnikov, Röntsch '19] | Higgs decay [Caola, Melnikov, Röntsch '19]

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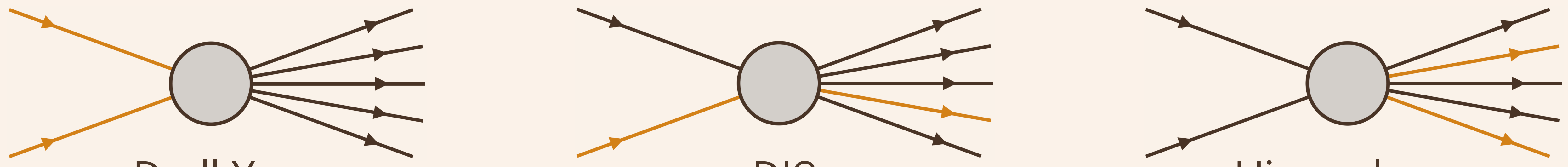

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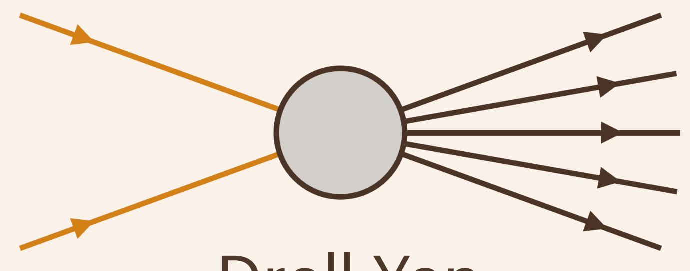
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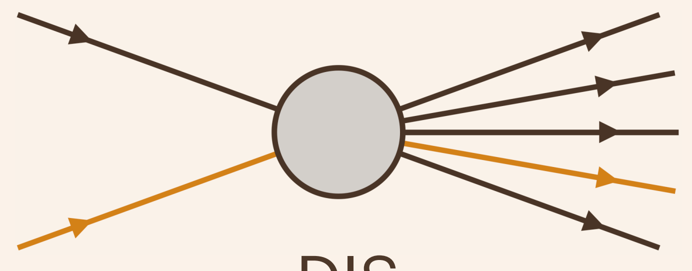
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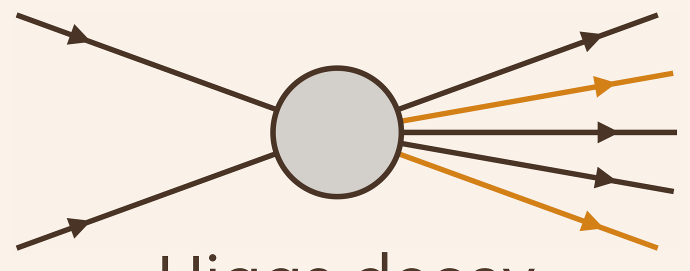
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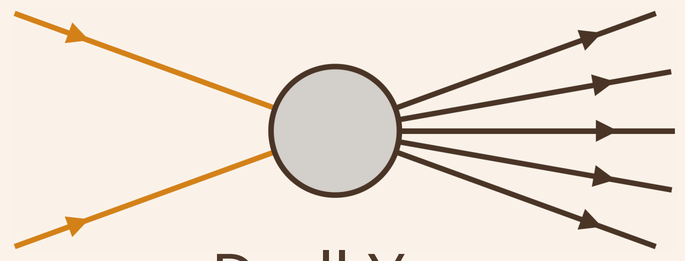

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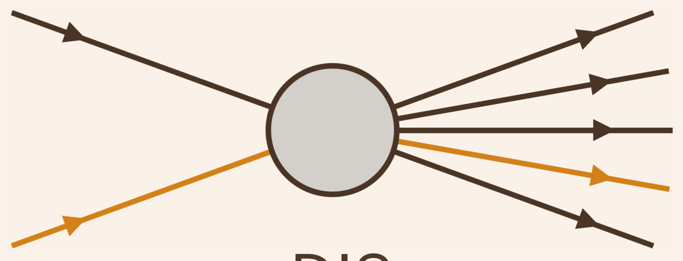

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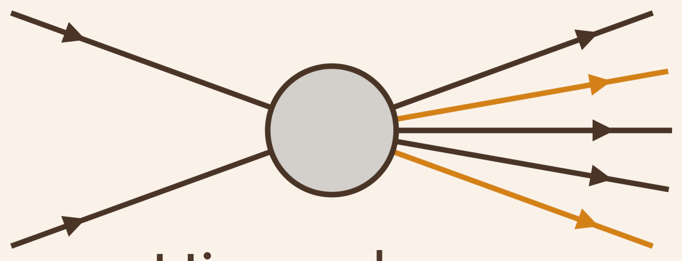

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What is a good prototype of the problem?
P + P → X + N gluons

This talk!
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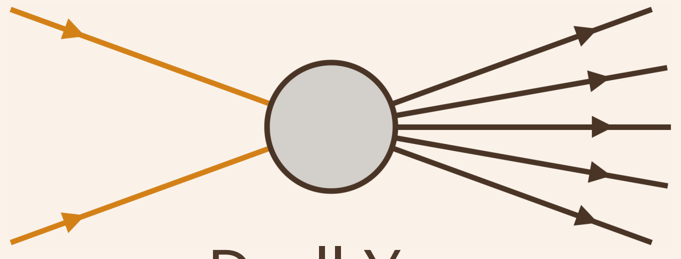

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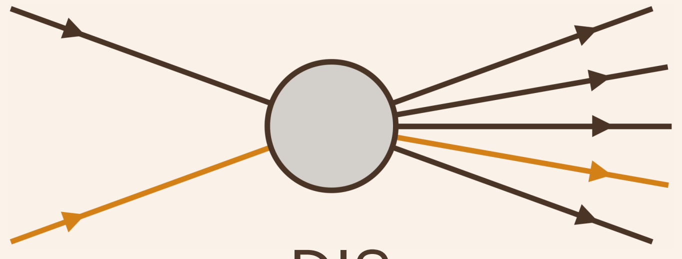

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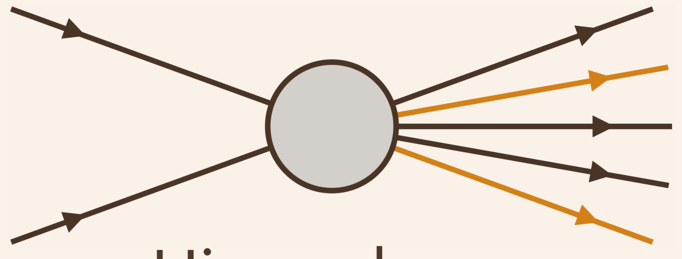

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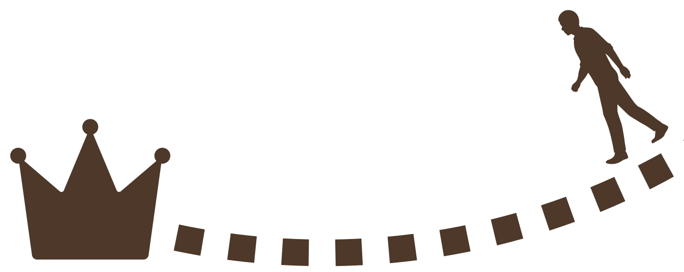
Remaining bottleneck?
double-loop amplitudes



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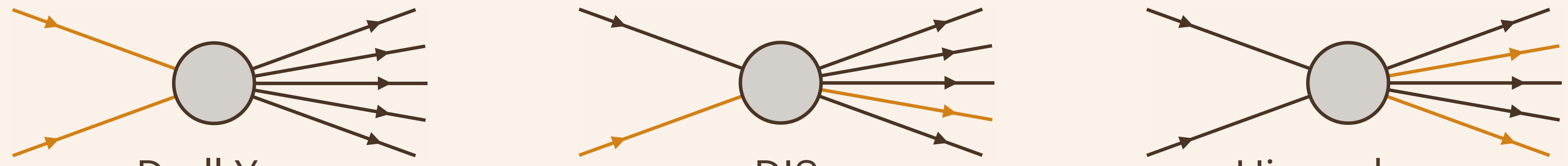

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Remaining bottleneck? **double-loop** amplitudes

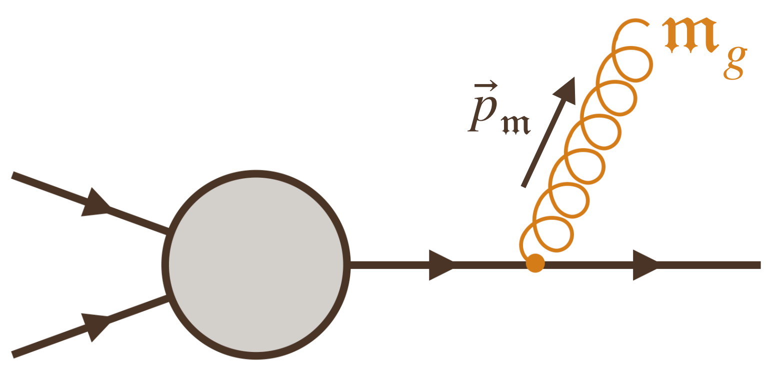
<< If someone gives me the finite part of the double-loop amplitude of any kind of process, then I can give back the analytical expression of the whole partonic cross section. >>

HOW THE **NSC** WORKS?

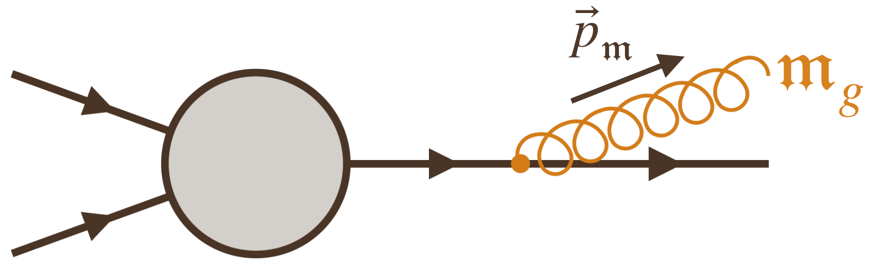
$$\int |\mathcal{M}|^2 F_J \, \mathrm{d}^{(d)}\phi = \int \left[|\mathcal{M}|^2 F_J - K \right] \mathrm{d}^{(d)}\phi + \int K \, \mathrm{d}^{(d)}\phi$$

- fully **local**
- fully **analytic**

Problem of **OVERLAPPING SOFT** and **COLLINEAR** emissions

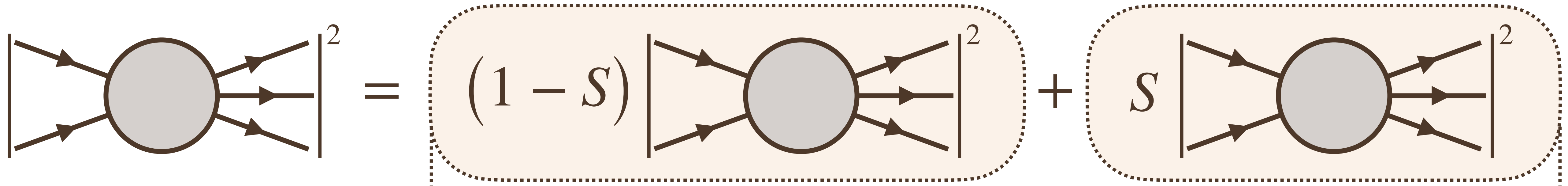


Soft emission
$S_{\mathfrak{m}}: |\vec{p}_{\mathfrak{m}}| \to 0$



Collinear emission
$C_{i\mathfrak{m}}: \theta_{i\mathfrak{m}} \to 0$

At **NLO** we start by regularizing soft divergences (see FKS)



Soft-regulated
still contains collinear divergences

Soft-counterterm
provides the formula of the soft poles

The **soft-regulated** term then needs a similar treatment for **collinear divergences**: all the singular configurations can be separated out

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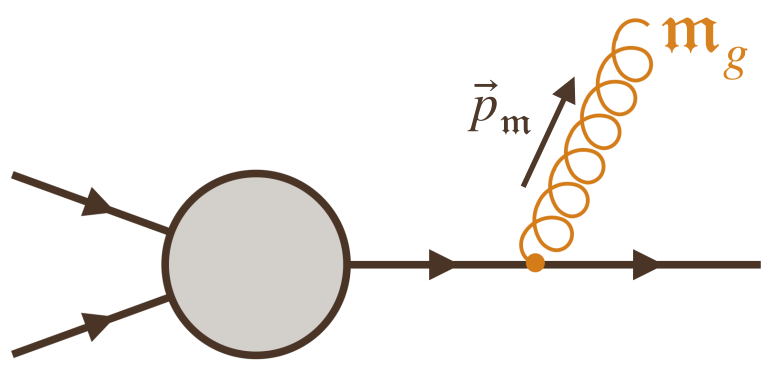

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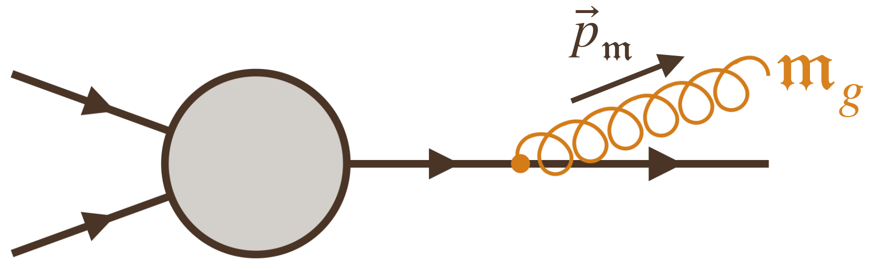

Collinear emission
$C_{i\mathfrak{m}}: \theta_{i\mathfrak{m}} \to 0$

At **NNLO** we follow the same idea of **separating out divergences**

- start from **double-soft** regularization
- regularize also **single-soft** divergences

} The cross section is now soft-regularized

- at this point we have to regularize **collinear** divergences ($C_{i\mathfrak{m}}$, $C_{j\mathfrak{n}}C_{i\mathfrak{m}}$, $C_{i\mathfrak{m}\mathfrak{n}}$) ⇒ we avoid overlapping thanks to **PARTITIONING** and **SECTORING** [Czakon 1005.0274]

RECURRING OPERATORS AT NLO

Virtual corrections $\mathrm{d}\hat{\sigma}^{\mathrm{V}}$: the IR content of virtual amplitudes is known [Catani '98]. Through the operator

$$\bar{I}_1(\epsilon) = \frac{1}{2}\sum_{i\neq j}^{Np} \frac{\mathcal{V}_i^{\mathrm{sing}}(\epsilon)}{\boldsymbol{T}_i^2}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\left(\frac{\mu^2}{2p_i\cdot p_j}\right)^{\epsilon} e^{i\pi\lambda_{ij}\epsilon}$$

$$\mathcal{V}_i^{\mathrm{sing}}(\epsilon) = \frac{\boldsymbol{T}_i^2}{\epsilon^2} + \frac{\gamma_i}{\epsilon}$$

$$N_p = N + 2$$

the divergent part of $\mathrm{d}\hat{\sigma}^{\mathrm{V}}$ can be written as

$$\boldsymbol{I}_{\mathrm{V}}(\boldsymbol{\epsilon}) = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)$$

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Virtual corrections $d\hat{\sigma}^{V}$: the IR content of virtual amplitudes is known [Catani '98]. Through the operator

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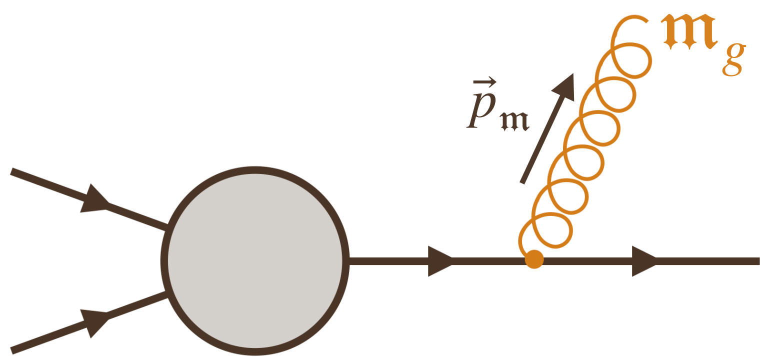
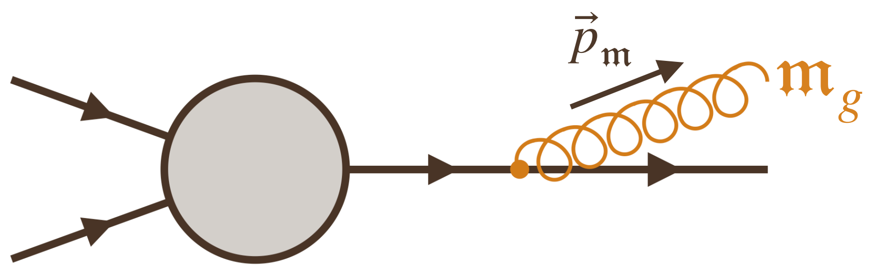
the divergent part of $d\hat{\sigma}^{V}$ can be written as

$$\boldsymbol{I}_{\rm V}(\boldsymbol{\epsilon}) = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)$$

Real corrections $d\hat{\sigma}^{R}$: we would like something similar



Soft emission
$S_{\mathfrak{m}}$: $|\vec{p}_{\mathfrak{m}}| \to 0$



Collinear emission
$C_{i\mathfrak{m}}$: $\theta_{i\mathfrak{m}} \to 0$

Making use of **NSC** (FKS at NLO) to regularize this divergences we obtain [Caola, Melnikov, Röntsch '17]

$$d\hat{\sigma}^{R} = \left\langle S_{\mathfrak{m}} F_{\rm LM}(\mathfrak{m})\right\rangle + \sum_{i=1}^{N_p}\left\langle \bar{S}_{\mathfrak{m}} C_{i\mathfrak{m}} \Delta^{(\mathfrak{m})} F_{\rm LM}(\mathfrak{m})\right\rangle + \left\langle \mathcal{O}_{\rm NLO}\,\Delta^{(\mathfrak{m})} F_{\rm LM}(\mathfrak{m})\right\rangle$$

Soft term
$[S_{\mathfrak{m}}: E_{\mathfrak{m}} \to 0]$

Hard-Collinear term
$[C_{i\mathfrak{m}}: \theta_{i\mathfrak{m}} \to 0]$

RECURRING OPERATORS AT NLO

It turns out that the **soft term** can be written by means of an **operator** that, at least in principle, is very **close to $I_V(\epsilon)$**:

$$I_S(\epsilon) = -\frac{(2E_{max}/\mu)^{-2\epsilon}}{\epsilon^2}\sum_{i\neq j}^{N_p}\eta_{ij}^{-\epsilon}K_{ij}\,(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)$$

$$\eta_{ij} = (1-\cos\theta_{ij})/2$$

$$K_{ij} \sim \eta_{ij}^{1+\epsilon}\,{}_2F_1(1,1,1-\epsilon,1-\eta_{ij})$$

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- Combination of $I_V(\epsilon)+I_S(\epsilon)$: not only does it **vanishes** the pole $\mathcal{O}(\epsilon^{-2})$, but it makes the pole $\mathcal{O}(\epsilon^{-1})$ free of **color-correlations**

$I_{V,S}(\epsilon) \sim \boldsymbol{T}_i\cdot\boldsymbol{T}_j$ $\quad$ $\boldsymbol{T}_i =$ matrices in color space

$N_p < 4 \Rightarrow d\hat{\sigma}^{NLO} \sim \frac{C_{A,F}}{\epsilon}\langle M_0|M_0\rangle$ — **NO** color-correlations

$N_p \geq 4 \Rightarrow d\hat{\sigma}^{NLO} \sim \frac{1}{\epsilon}\langle M_0|\boldsymbol{T}_i\cdot\boldsymbol{T}_j|M_0\rangle$ — **YES** color-correlations

This result for $I_V(\epsilon)+I_S(\epsilon)$ is trivially **dependent** on the **number of gluons** in the final state

RECURRING OPERATORS AT NLO

What about the **hard-collinear term**? Some parts vanish against the DGLAP contribution, the remaining part **can be collected** within the following **Catani-like operator**

$$I_{\mathrm{C}}(\epsilon) = \sum_{i=1}^{N_p} \frac{\Gamma_{i,f_i}}{\epsilon}$$

$$\Gamma_{a,f_a} = \left[\left(\frac{2E_a}{\mu}\right)^{-2\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\right] \left[\gamma_{f_a} + C_{f_a} \frac{1-e^{-2\epsilon L_a}}{\epsilon}\right], \qquad a = 1,2$$

$$\Gamma_{i,f_i} = \left[\left(\frac{2E_i}{\mu}\right)^{-2\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\right] \gamma^{22}_{z,g\to gg}(\epsilon, L_i), \qquad i \in [3,N_p]$$

Once more the definition **depends** in a trivial way on N_p

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Once more the definition **depends** in a trivial way on N_p

$I_{\mathrm{C}}(\epsilon)$ cancels perfectly the pole $\mathcal{O}(\epsilon^{-1})$ left by $I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon)$. It is thus natural to introduce the **total operator**

$$I_{\mathrm{T}}(\epsilon) = I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) + I_{\mathrm{C}}(\epsilon)$$

- pole free
- fully general w.r.t. N_p

In this way the final result for the NLO fits in a line:

$$\mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = [\alpha_s]\left\langle I_{\mathrm{T}}(\epsilon)\cdot F_{\mathrm{LM}}\right\rangle + [\alpha_s]\left[\left\langle P^{\mathrm{NLO}}_{aa}\otimes F_{\mathrm{LM}}\right\rangle + \left\langle F_{\mathrm{LM}}\otimes P^{\mathrm{NLO}}_{aa}\right\rangle\right] + \left\langle F^{\mathrm{fin}}_{\mathrm{LV}}\right\rangle + \left\langle \mathcal{O}_{\mathrm{NLO}}\,\Delta^{(\mathfrak{m})}F_{\mathrm{LM}}(\mathfrak{m})\right\rangle$$

[Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, 2309.xxxxx]

WHAT HAPPENS AT NNLO?

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}^{\mathrm{VV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RR}} + \mathrm{d}\hat{\sigma}^{\mathrm{pdf}}$$

Double-Virtual, Real-Virtual, Double-Real, PDFs Renor.

Consider for instance $\mathrm{d}\hat{\sigma}^{\mathrm{VV}}$ $\Rightarrow$ it depends **quadratically** on $\bar{I}_1(\epsilon)$ and $\bar{I}_1^\dagger(\epsilon)$

$$\Rightarrow \bar{I}_1, \bar{I}_1^\dagger \sim \boldsymbol{T}_i \cdot \boldsymbol{T}_j$$

$$\Rightarrow \mathrm{d}\hat{\sigma}^{\mathrm{VV}} \sim (\boldsymbol{T}_i \cdot \boldsymbol{T}_j) \cdot (\boldsymbol{T}_k \cdot \boldsymbol{T}_l)$$

double color-correlations

We expect the **same** to happen for $\mathrm{d}\hat{\sigma}^{\mathrm{RV}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{RR}}$. Dealing with such double-color correlated terms (**DCC**) in general makes the **structure of the poles very complicated**

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$\Downarrow$

The strategy: **isolate DCC** in $\mathrm{d}\hat{\sigma}^{\mathrm{RV}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{RR}}$ and then **combine** them with **those** contained within $\mathrm{d}\hat{\sigma}^{\mathrm{VV}}$

$\Downarrow$

The goal: **assemble** all these **DCC** into an expression that we expect to be **quadratic** in $I_{\mathrm{T}}(\epsilon)$

WHAT HAPPENS AT NNLO?

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, to appear]

$$Y_{\rm VV} = \frac{[\alpha_s]^2}{2} \left\langle M_0 \middle| \bar{I}_1^2 + (\bar{I}_1^\dagger)^2 + 2\bar{I}_1^\dagger \bar{I}_1 \middle| M_0 \right\rangle + \dots$$

$$Y_{\rm RR}^{\rm (ss)} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_{\rm S}^2 | M_0 \rangle + \dots$$

$$Y_{\rm RR}^{\rm (shc)} = [\alpha_s]^2 \langle M_0 | I_{\rm S} I_{\rm C} | M_0 \rangle + \dots$$

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WHAT HAPPENS AT NNLO?

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, to appear]

$$Y_{\rm VV} = \frac{[\alpha_s]^2}{2}\left\langle M_0 \left| \bar{I}_1^2 + (\bar{I}_1^\dagger)^2 + 2\bar{I}_1^\dagger \bar{I}_1 \right| M_0 \right\rangle + \dots$$

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Once combined, these objects return

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2}\Big\langle M_0\Big|\big[I_{\rm V} + I_{\rm S} + I_{\rm C}\big]^2\Big|M_0\Big\rangle + \dots \equiv \langle M_0\,|\,I_{\rm T}^2\,|\,M_0\rangle + \dots$$

WHAT HAPPENS AT NNLO?

The benefits of introducing these Catani-like operators:

- the problem of **double color-correlated poles disappears**, since everything is written in terms of $I_{\mathrm{T}}^{2}(\epsilon)$, which is $\mathcal{O}(\epsilon^{0})$
- the **definition** of $I_{\mathrm{T}}(\epsilon)$ depends trivially on N_p so the result we got is **fully general w.r.t. the number of final state gluons**
- We **do not explicitly calculate** the individual sub-blocks of the process. Instead, we write each of these in terms of $I_{\mathrm{V}}(\epsilon)$, $I_{\mathrm{S}}(\epsilon)$ and $I_{\mathrm{C}}(\epsilon)$, then recombine them to get $I_{\mathrm{T}}(\epsilon)$. The **cancellation of the poles** takes place **automatically**

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WHAT HAPPENS AT NNLO?

TRIPLE-POLES known in the literature (for $N_p \geq 4$):

From $d\hat{\sigma}^{VV}$

$$H_2(\epsilon) = \frac{i f_{abc}}{384\epsilon}(\gamma_0^{\text{cusp}})^2 \sum_{(i,j,k)}^{N_p} T_i^a T_j^b T_k^c \log\frac{-s_{ij}}{-s_{jk}} \log\frac{-s_{jk}}{-s_{ki}} \log\frac{-s_{ki}}{-s_{ij}}$$
$$- \frac{i f_{abc}}{128\epsilon}\gamma_0^{\text{cusp}} \sum_{(i,j,k)}^{N_p} T_i^a T_j^b T_k^c \left(\frac{\gamma_0^i}{C_{f_i}} - \frac{\gamma_0^j}{C_{f_j}}\right) \log\frac{-s_{ij}}{-s_{jk}} \log\frac{-s_{ki}}{-s_{ij}}$$
$$+ \frac{\boldsymbol{\Gamma}_1}{16\epsilon} - \frac{\gamma_1^{\text{cusp}}\boldsymbol{\Gamma}_0}{64\epsilon} - \frac{\pi^2\beta_0\Gamma_0'}{128\epsilon}$$

From $d\hat{\sigma}^{RV}$

$$S_{\text{m}}^{\text{tri}}\,\text{RV} \sim \sum_{(i,j,k)} \frac{s_{ij}}{s_{im}s_{jm}}\left(\frac{s_{jk}}{s_{jm}s_{km}}\right)^{\epsilon} T_i^a T_j^b T_k^c$$

$\mathcal{O}(\epsilon^{-2})$

⊖

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Need to add **other contributions**. But **where** do they come from?

If $N_p \geq 4$

$$[\bar{I}_1, \bar{I}_1^\dagger] \neq 0$$
$$[\bar{I}_1^\dagger, \bar{I}_S] \neq 0 \;\rightarrow\; f_{abc}T_i^aT_j^bT_k^c$$
$$[\bar{I}_1, \bar{I}_S] \neq 0$$

$\Rightarrow$ Combining the commutators

$$I^{\mathrm{tri}} = \frac{1}{2}\left[I_V + I_S, \bar{I}_1 - \bar{I}_1^\dagger\right] - \frac{1}{4}\left[I_V, \bar{I}_1 - \bar{I}_1^\dagger\right]$$

Once combined with the other triples, this **cancels out** all the **triple-poles**

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0\right|\left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}}\right]^2\left|M_0\right\rangle + \ldots \equiv \langle M_0 | I_{\mathrm{T}}^2 | M_0\rangle + \ldots$$

CONCLUSIONS AND OUTLOOK

1. We find **recurring building blocks**, i.e. $I_V(\epsilon)$, $I_S(\epsilon)$, $I_C(\epsilon)$ and $I_T(\epsilon)$, which let us solve the problem of color-correlated poles
2. The **procedure** is (almost) entirely **process independent**
3. The cancellation of the poles is **analytical** and takes place automatically for N_p **gluons**
4. *Work in progress*: next step is a generalization to **asymmetric initial state** and **arbitrary final state**
5. *Outlook*: application of the method to **pheno-studies**

MANY THANKS FOR YOUR ATTENTION

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