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Der Wissenschaftsfonds.

# Prospects for $\alpha_s$ measurement at the LHC using soft drop jet mass

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DESY

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arXiv 2210.04901

QCD@LHC 2023



**UK Research  
and Innovation**

# Motivation

Fine structure constant:  $\alpha = 7.297\ 352\ 5693(11) \times 10^{-3}$

PDG

# Motivation

Strong coupling constant:  $\alpha_s(m_Z) = 0.1179 \pm 0.0010$

PDG, World average

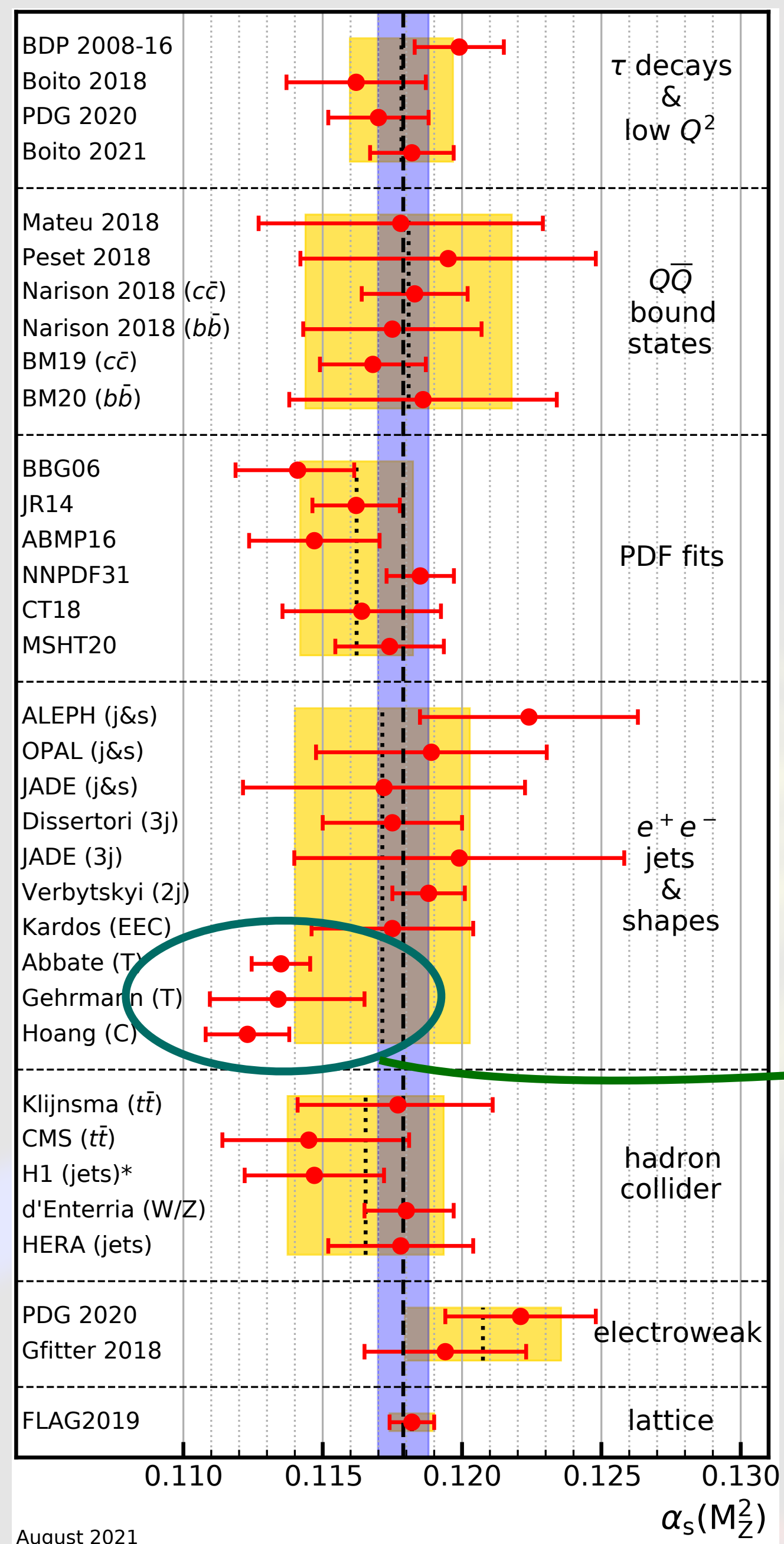
# Motivation

Uncertainties in  $\alpha_s$  propagate into almost all measurements at the LHC

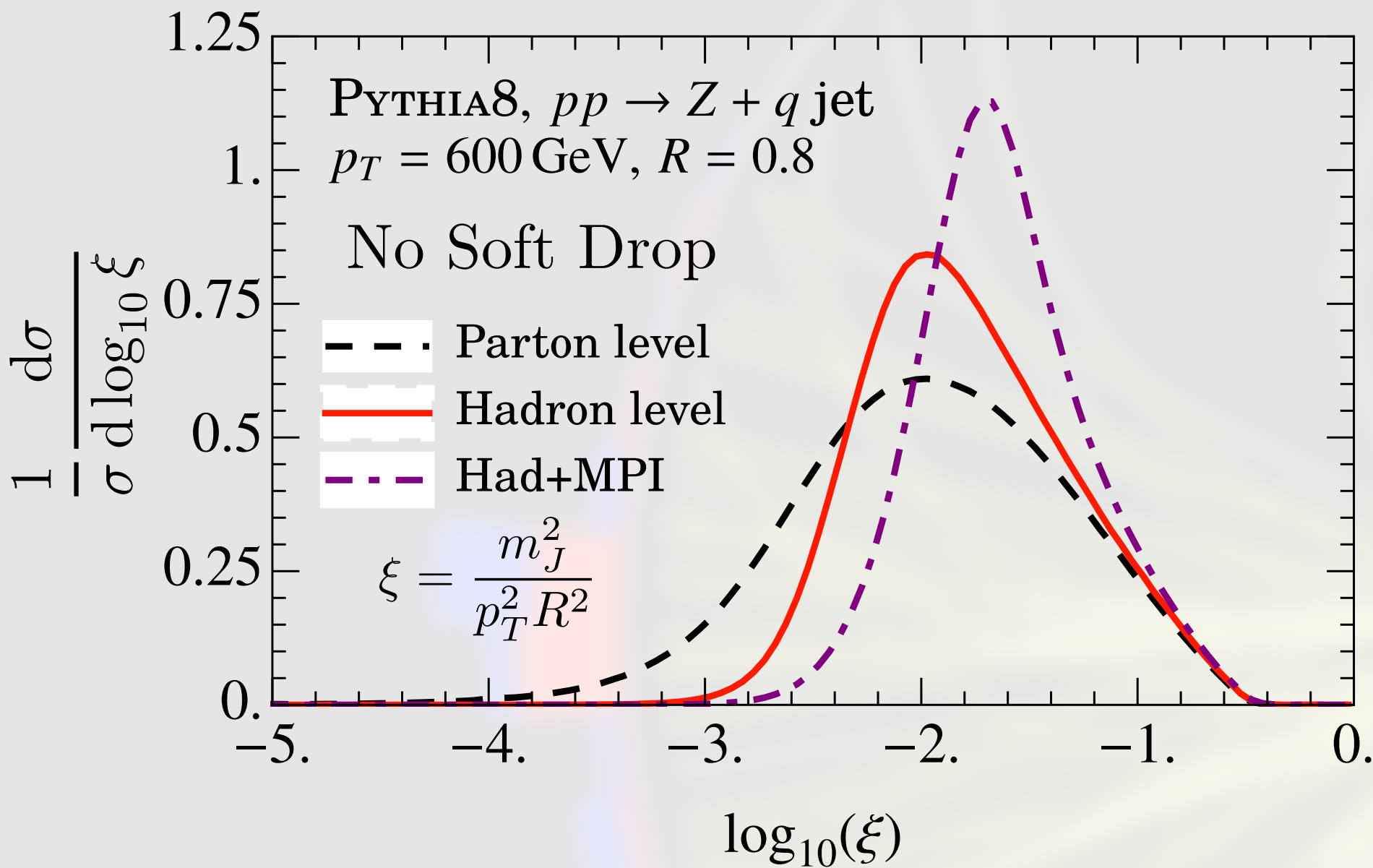
$$\alpha_s \text{ world average: } \alpha_s(m_Z) = 0.1179 \pm 0.0010$$

LEP measurements in tension with world average

*Can we get new measurements of  $\alpha_s$  at the LHC using soft drop jet mass?*

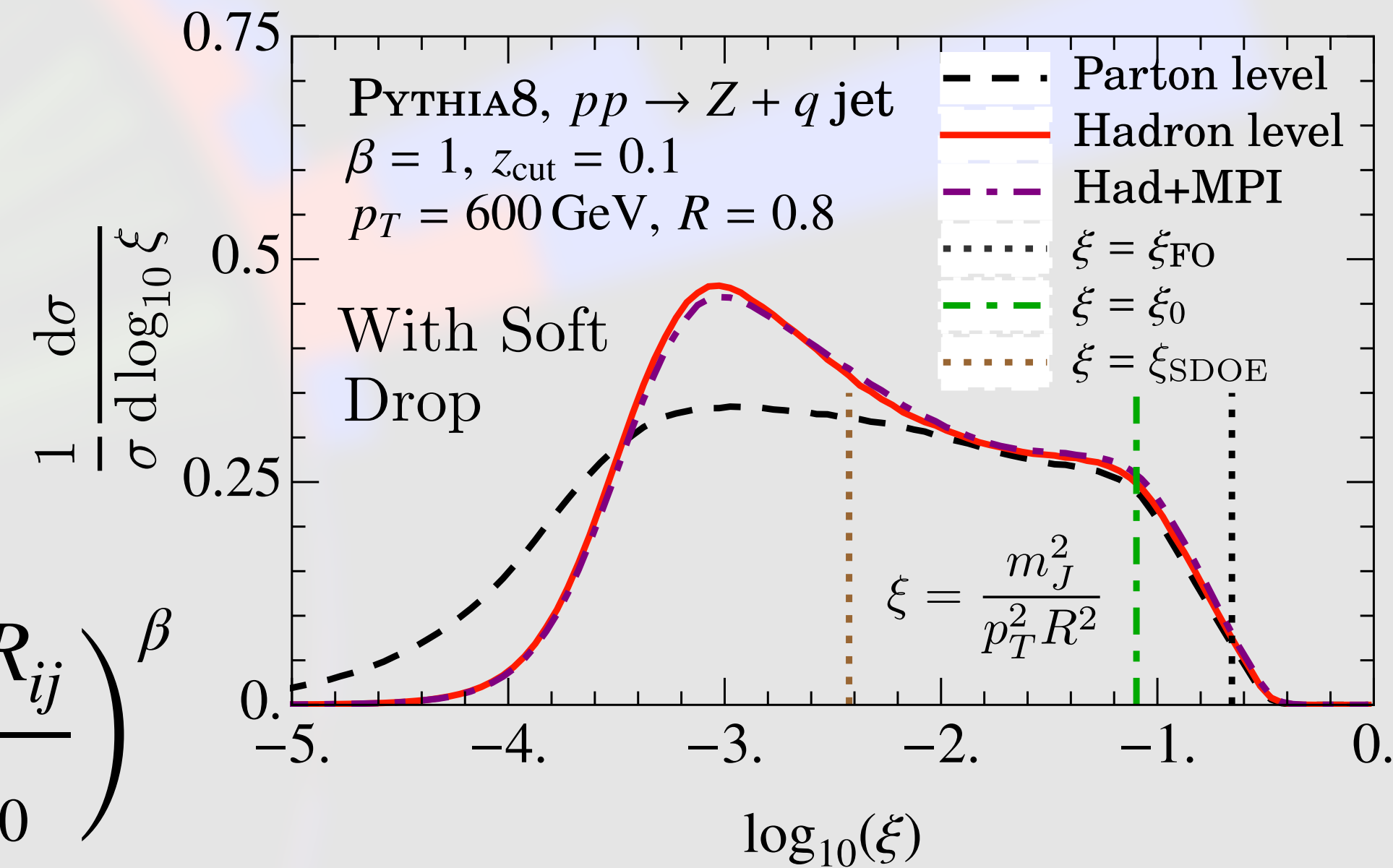


# Motivation for soft drop jet mass



Apply soft drop

$$\frac{\min\{p_{T_i}, p_{T_j}\}}{p_{T_i} + p_{T_j}} > z_{\text{cut}} \left( \frac{\Delta R_{ij}}{R_0} \right)^\beta$$



[Larkoski, Marzani, Soyez, Thaler 2014]

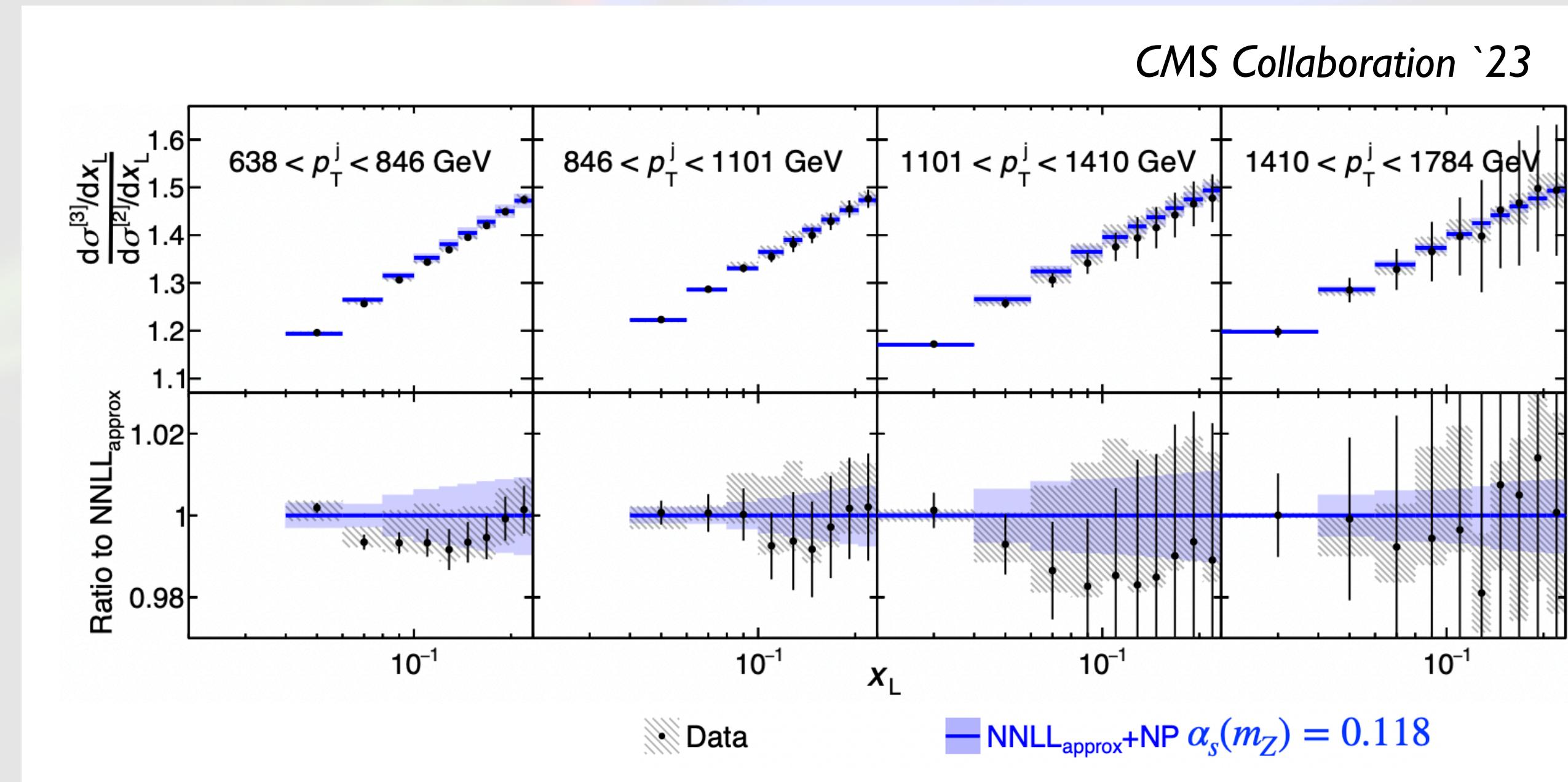
- Reduces sensitivity to the underlying event and hadronization effects
- **Measurable and calculable** observable for hadron colliders

[Les Houches 2017]

# Motivation for soft drop jet mass

*Why soft drop jet mass when we have EECs?!*

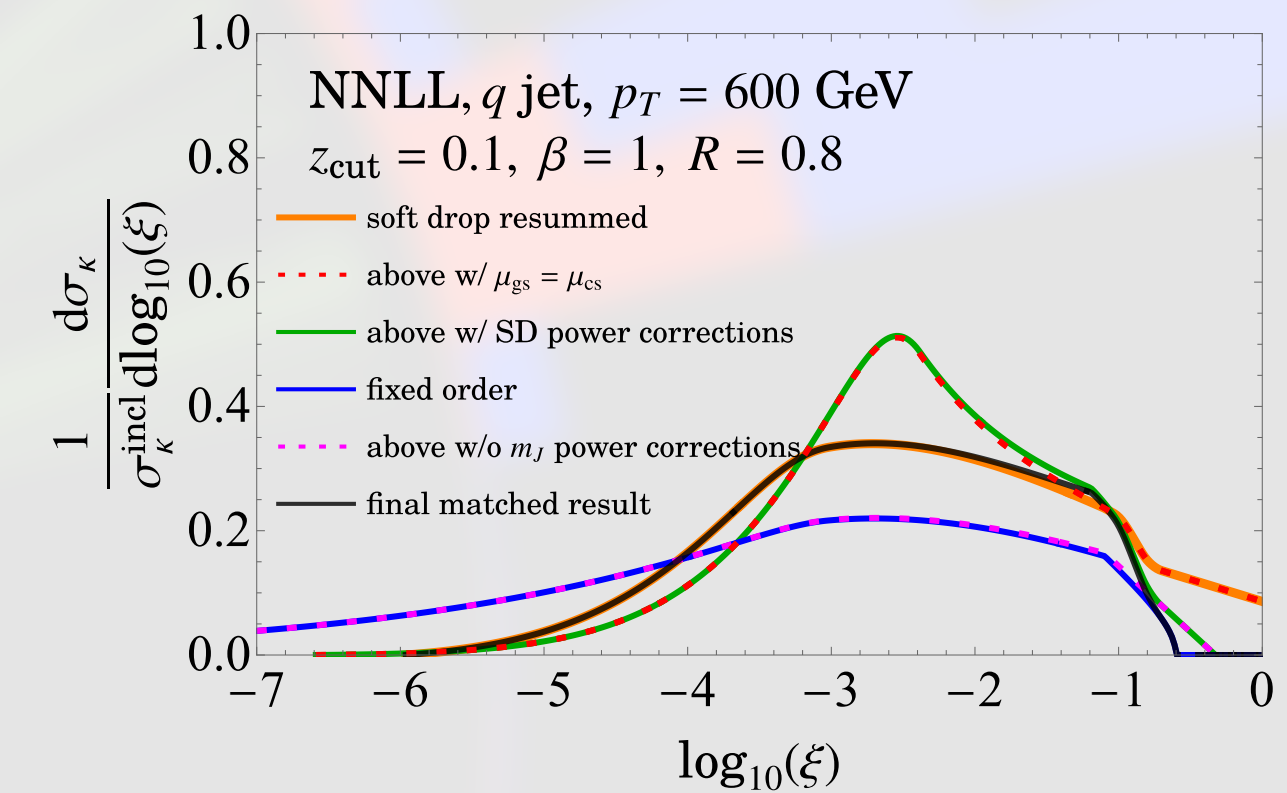
- Why not?
- Address a decade long curiosity as to whether soft drop jet mass is capable of delivering a competitive  $\alpha_s$  measurement.
- Soft drop jet mass is currently one of the **best studied and resummed jet substructure** observable.
- A precise field-theoretic understanding of hadronization in this case enables assessment of  $\alpha_s$  sensitivity **in a completely model independent way.**



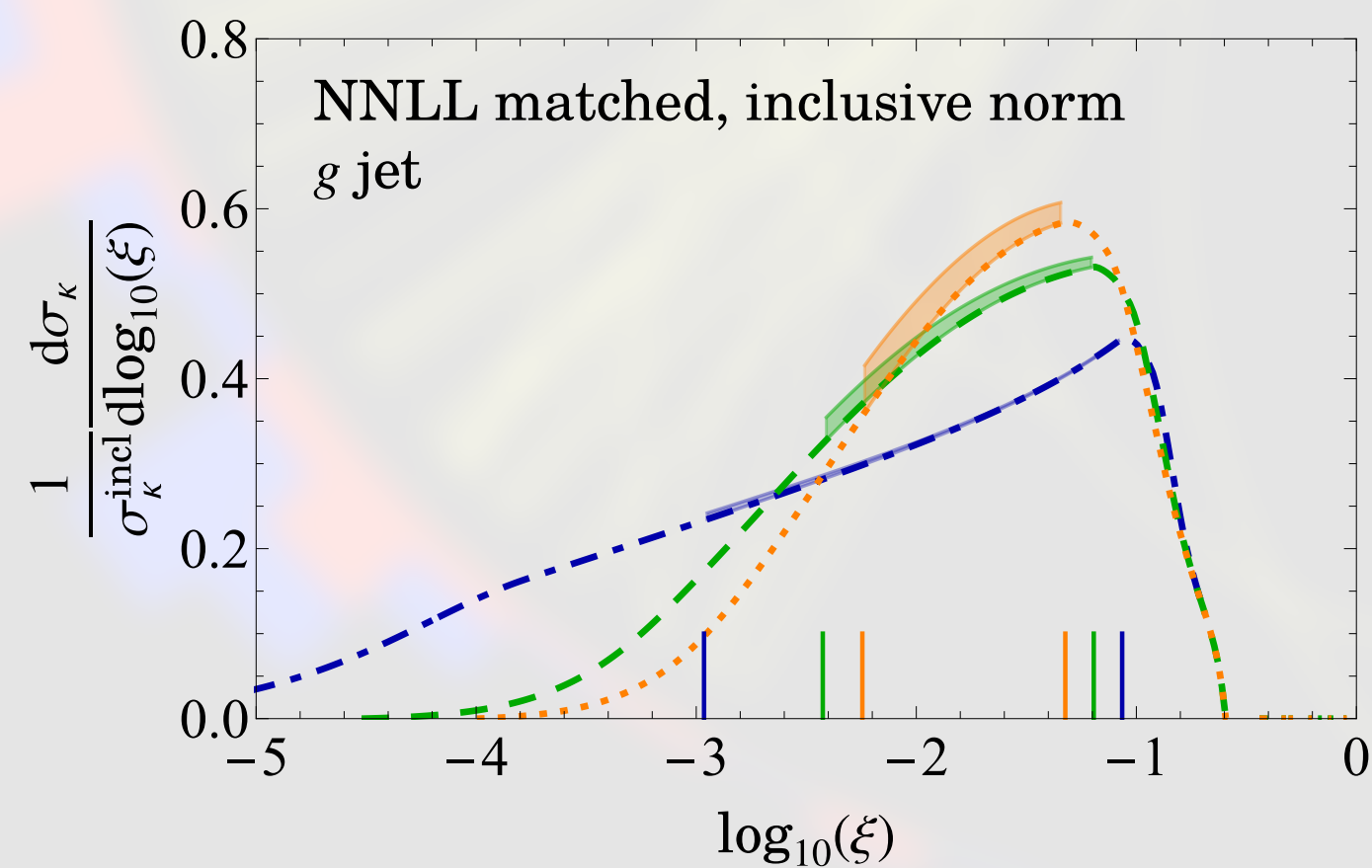
# Outline

## 1. Quark-gluon fraction and PDF dependence

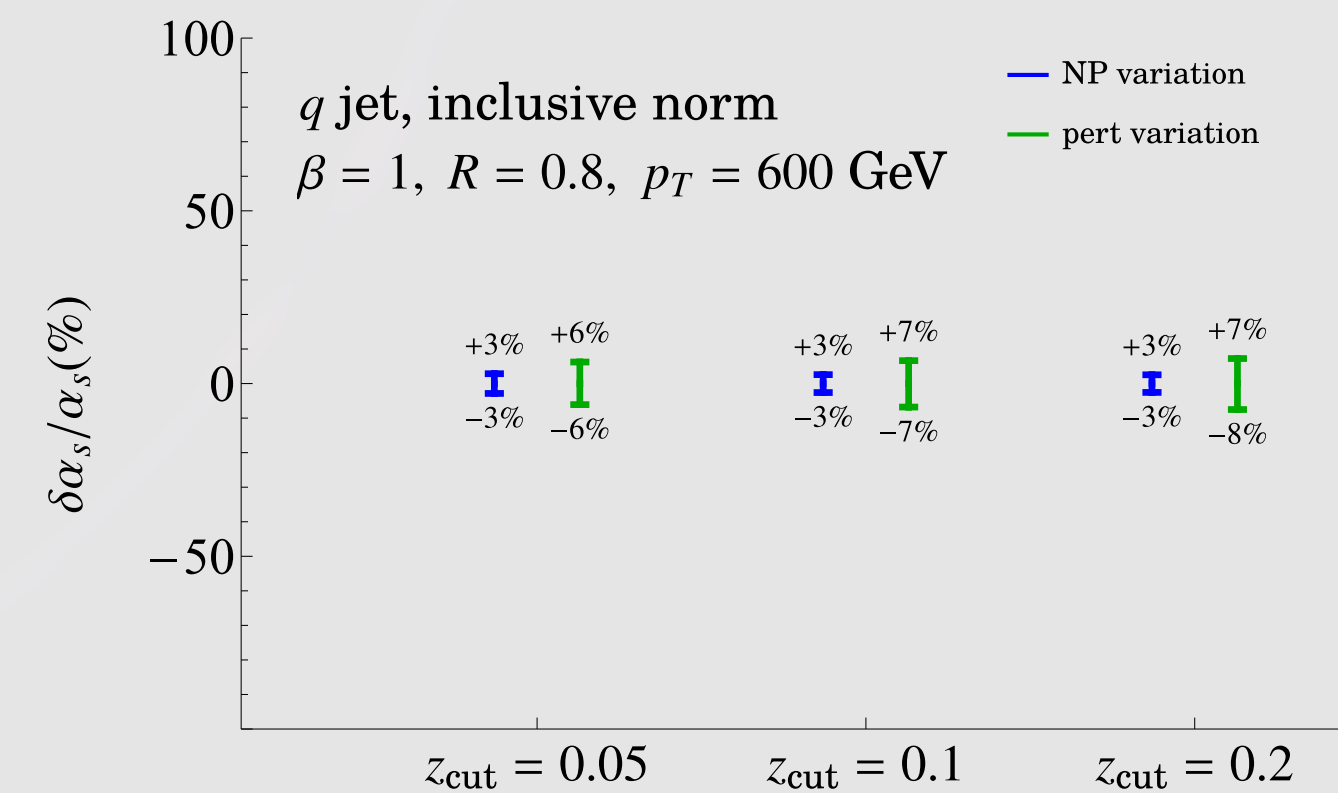
## 2. NNLL resummed cross section



## 3. Hadronization effects



## 4. Results



# Soft drop jet mass in inclusive jets

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$

Hard function for  $a + b \rightarrow c + X$

$$\frac{d^3\sigma}{dp_T d\eta d\xi} = \sum_{abc} \int \frac{dx_a dx_b dz}{x_a x_b z} f_a(x_a, \mu) f_b(x_b, \mu) H_{ab}^c \left( x_a, x_b, \eta, \frac{p_T}{z}, \mu \right) \mathcal{G}_c(z, \xi, p_T, R, \mu),$$

Parton distribution functions

Inclusive jet function

**Hard collinear factorization for  $\xi \ll 1$ :**

A single jet initiating parton  $i$

$$\mathcal{G}_c(z, \xi, p_T, R, \mu) = \sum_i \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \mathcal{J}_i(\xi, p_T, \eta, R, \mu), \quad \xi \ll 1$$

$$\mathcal{H}_{i \rightarrow j}(z, p_T R, \mu) = J_{ij}(z, p_T R, \mu) N_{\text{incl}}^j(p_T R, \mu)$$

[Kang, Lee, Liu, Ringer 2018],  
 [Cal, Lee, Ringer, Waalewijn 2020]  
 [Hannedottir, AP, Schwartz, Stewart 2022]

$$\mathcal{G}_c = \sum_{ij} J_{ij}(z, p_T R, \mu) \tilde{\mathcal{G}}_c(\xi, p_T, \eta, R, \mu)$$

Focus on this piece



# Quark-gluon fraction

q/g fraction is **internal** to our calculation  
(*not an external input, not taken from experiments*)

We can nevertheless “pull out” the q/g fractions and study their dependence on PDFs:

**Normalize to inclusive cross section:**

$$\frac{1}{\sigma_{\text{incl}}(p_T, \eta)} \frac{d^3\sigma}{dp_T d\eta d\xi} = x_q \tilde{\mathcal{G}}_q(\xi, p_T R, \mu) + x_g \tilde{\mathcal{G}}_g(\xi, p_T R, \mu)$$

$$\sigma_{\text{incl}}(p_T, \eta) \equiv \frac{d^2\sigma}{dp_T d\eta} = \sum_{a,b,c,d} f_a \otimes f_b \otimes H_{ab}^c \otimes J_{cd}, \quad x_\kappa(p_T R, \eta, \mu) \equiv \frac{\sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_{c\kappa}}{\sigma_{\text{incl}}(p_T, \eta)}$$

$$\tilde{\mathcal{G}}_\kappa(\xi, p_T R, \mu) \equiv \frac{1}{\sigma_{\text{incl}}^\kappa} \frac{d\sigma_\kappa}{d\xi}(p_T, \eta) = N_{\text{incl}}^\kappa(p_T R, \mu) \mathcal{J}_\kappa(\xi, p_T, \eta, R, \mu)$$

# Quark-gluon fraction

q/g fraction is internal to our calculation  
(*not an external input, not taken from experiments*)

We can nevertheless “pull out” the q/g fractions and study their dependence on PDFs:

PDF	$\alpha_s$ used	$x_q$	% change
NNPDF 23 LO	0.119	0.479	-6.0
NNPDF 23 NLO	0.119	0.517	1.3
NNPDF 23 NNLO	0.119	0.523	2.5
NNPDF 23 NNLO	0.120	0.514	0.84
CT18NLO_as_0119	0.119	0.514	0.87
CT18NNLO_as_0119	0.119	0.507	-0.49
MSTW2008nlo68cl	0.120	0.510	0.063
MSTW2008nlo68cl	0.117	0.514	0.87
mean	–	0.510	1.6

Using various PDFs combined with hard functions:

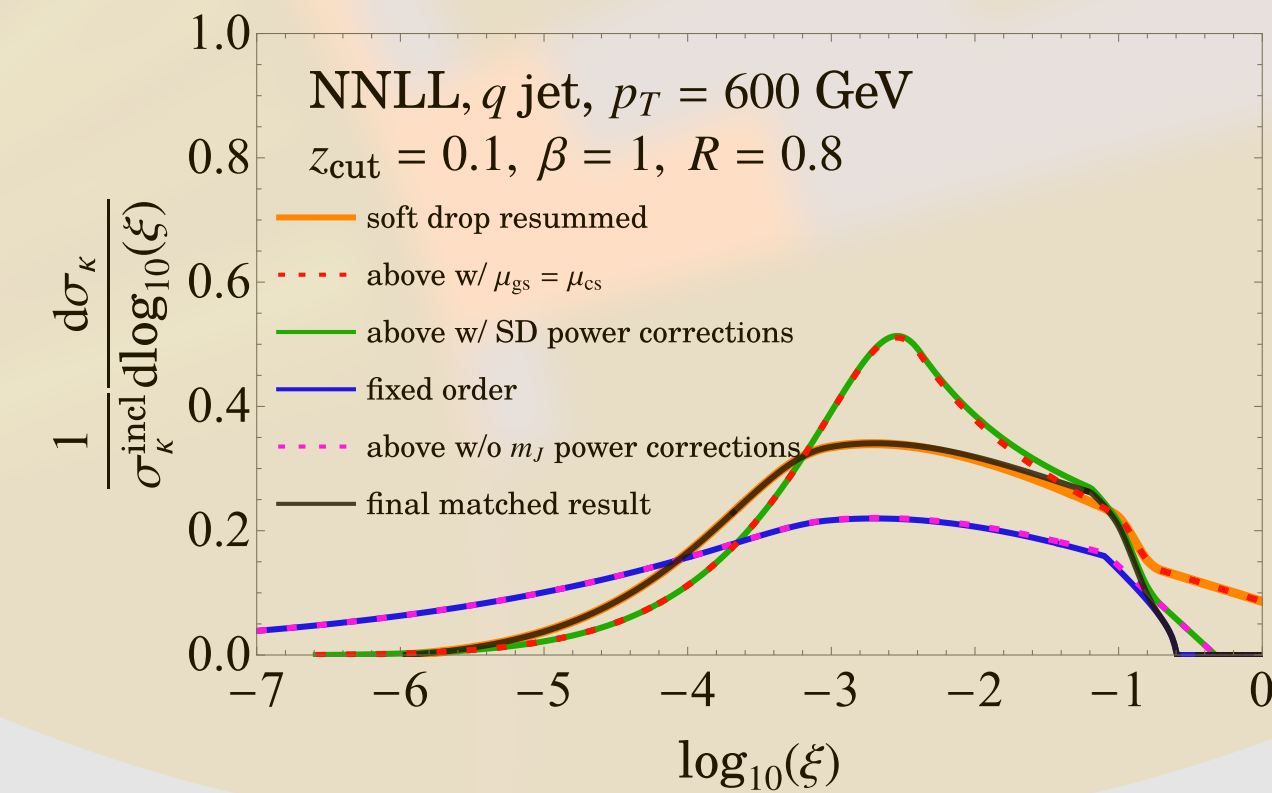
$$f_{\text{LO}} = 0.507, \quad f_{\text{NLO}} = 0.530, \quad f_{\text{NNLO}} = 0.538$$

**Subdominant effect on  $\alpha_s$  uncertainty for normalized cross section**

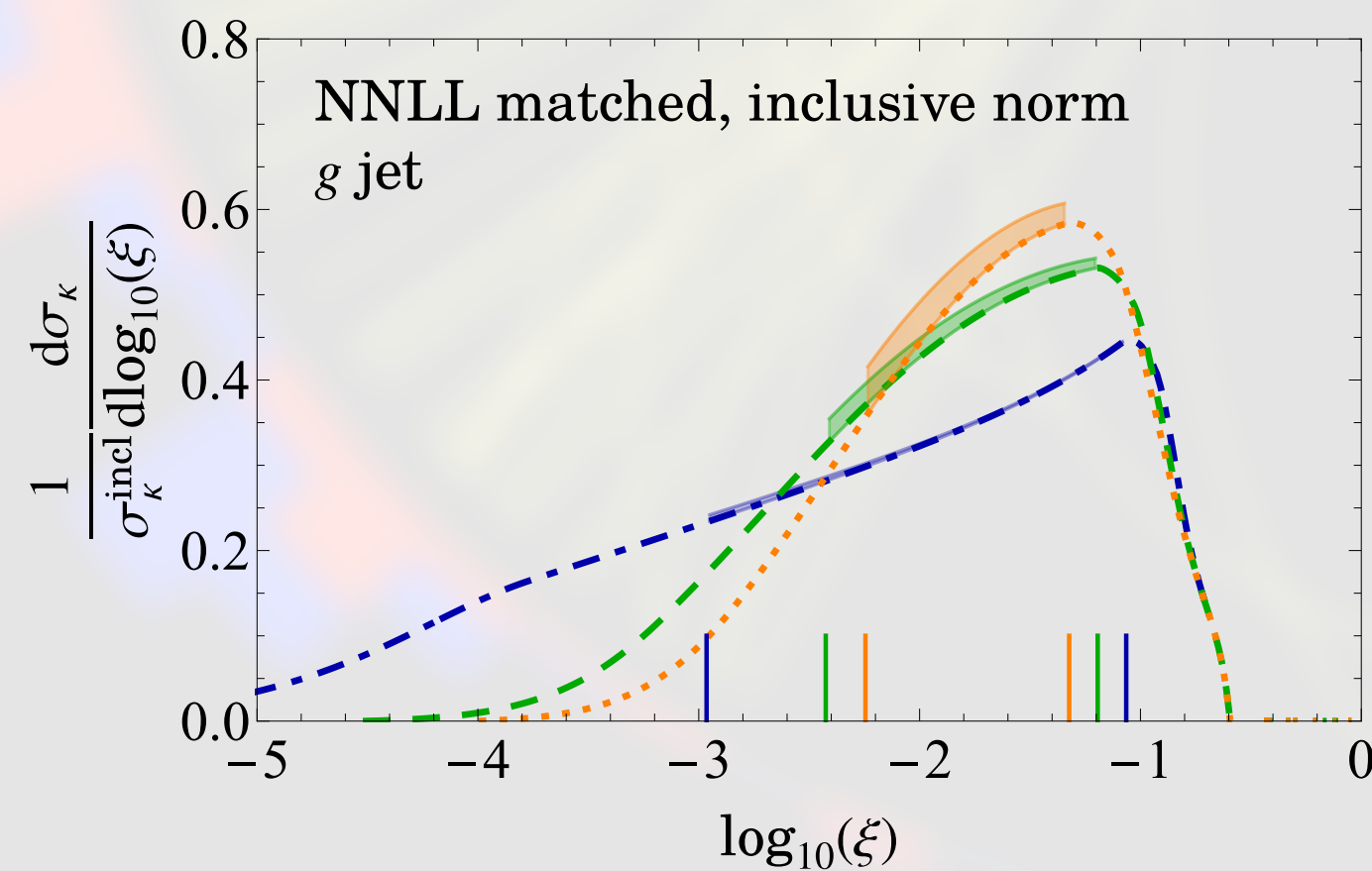
# Outline

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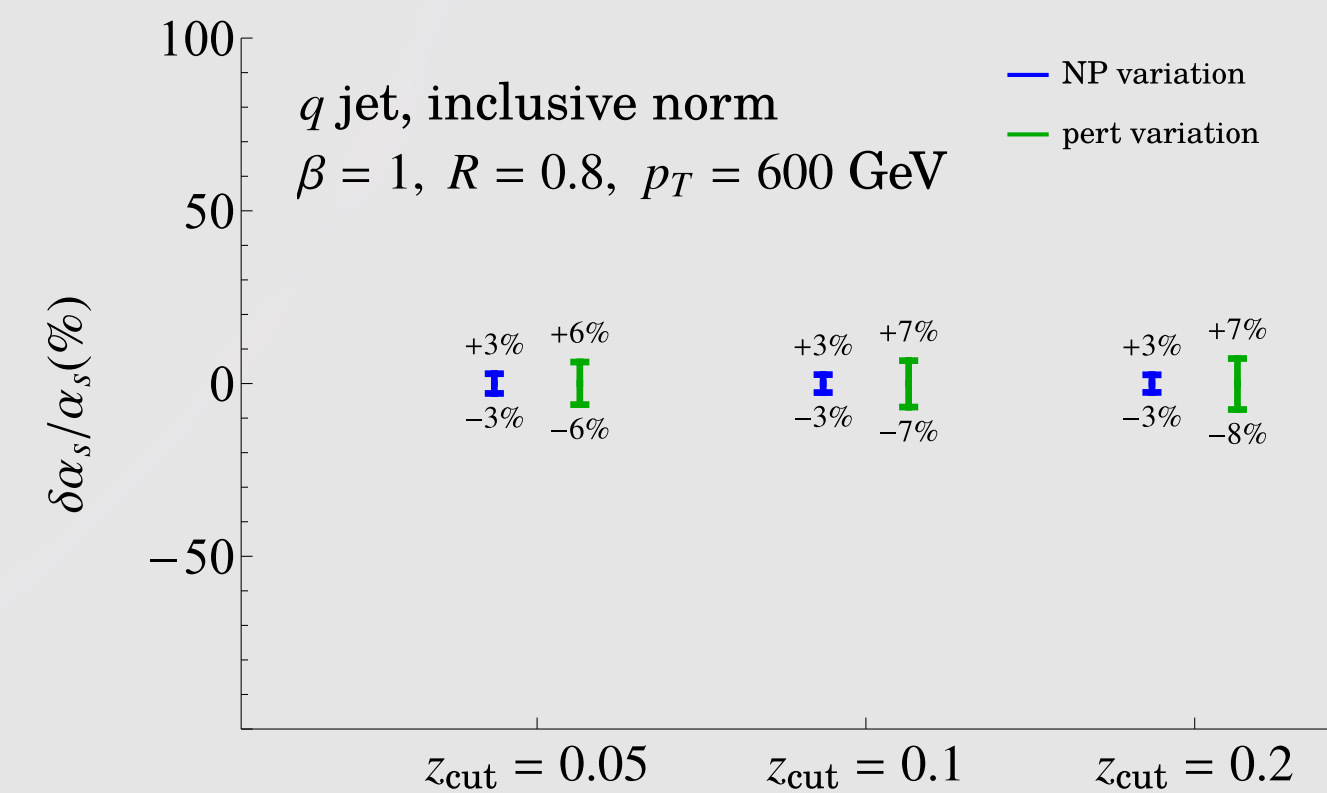
## 2. NNLL resummed cross section



## 3. Hadronization effects

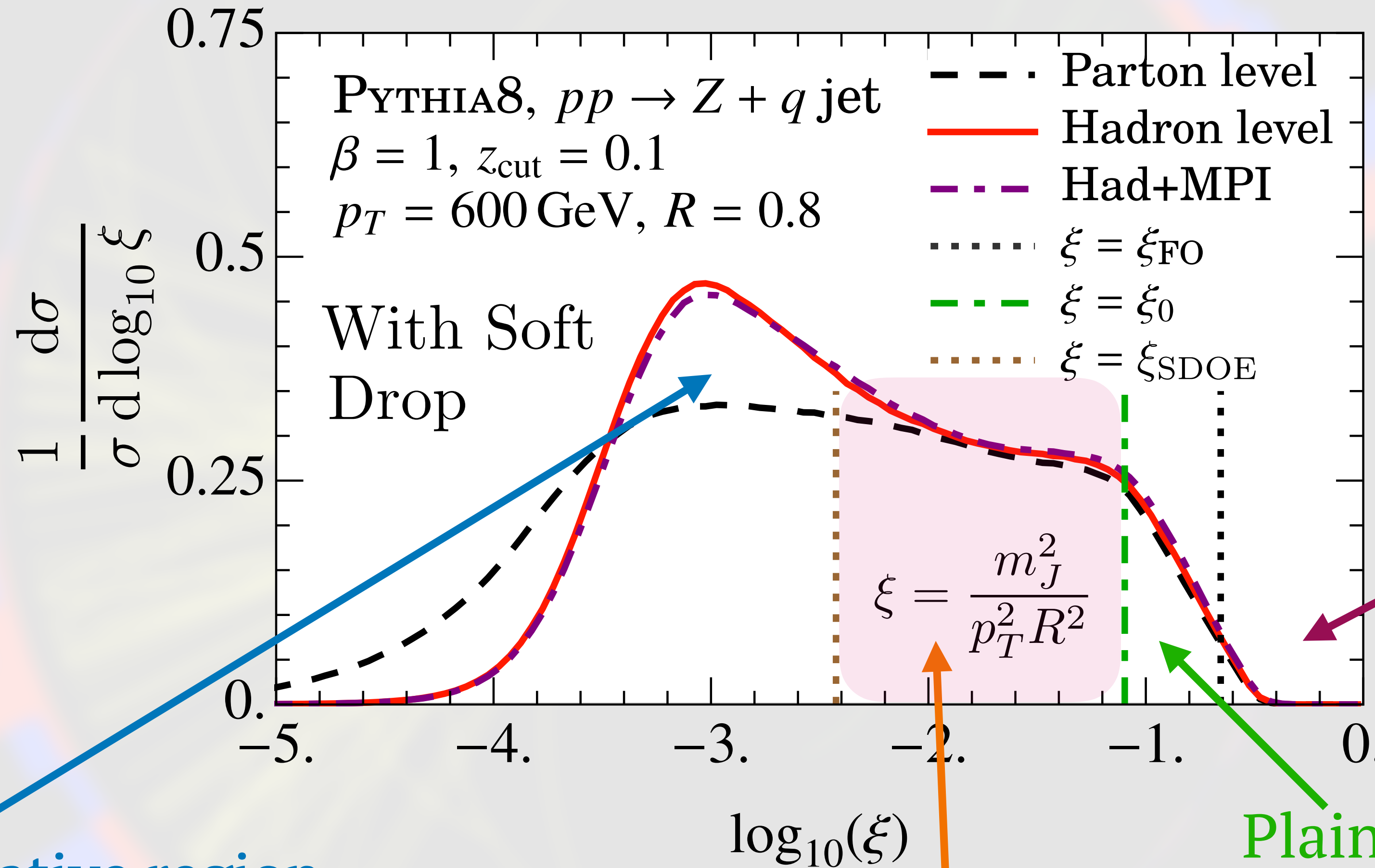


## 4. Results



# Region for fitting to $\alpha_s$

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$



Nonperturbative region  
(no theoretical control)

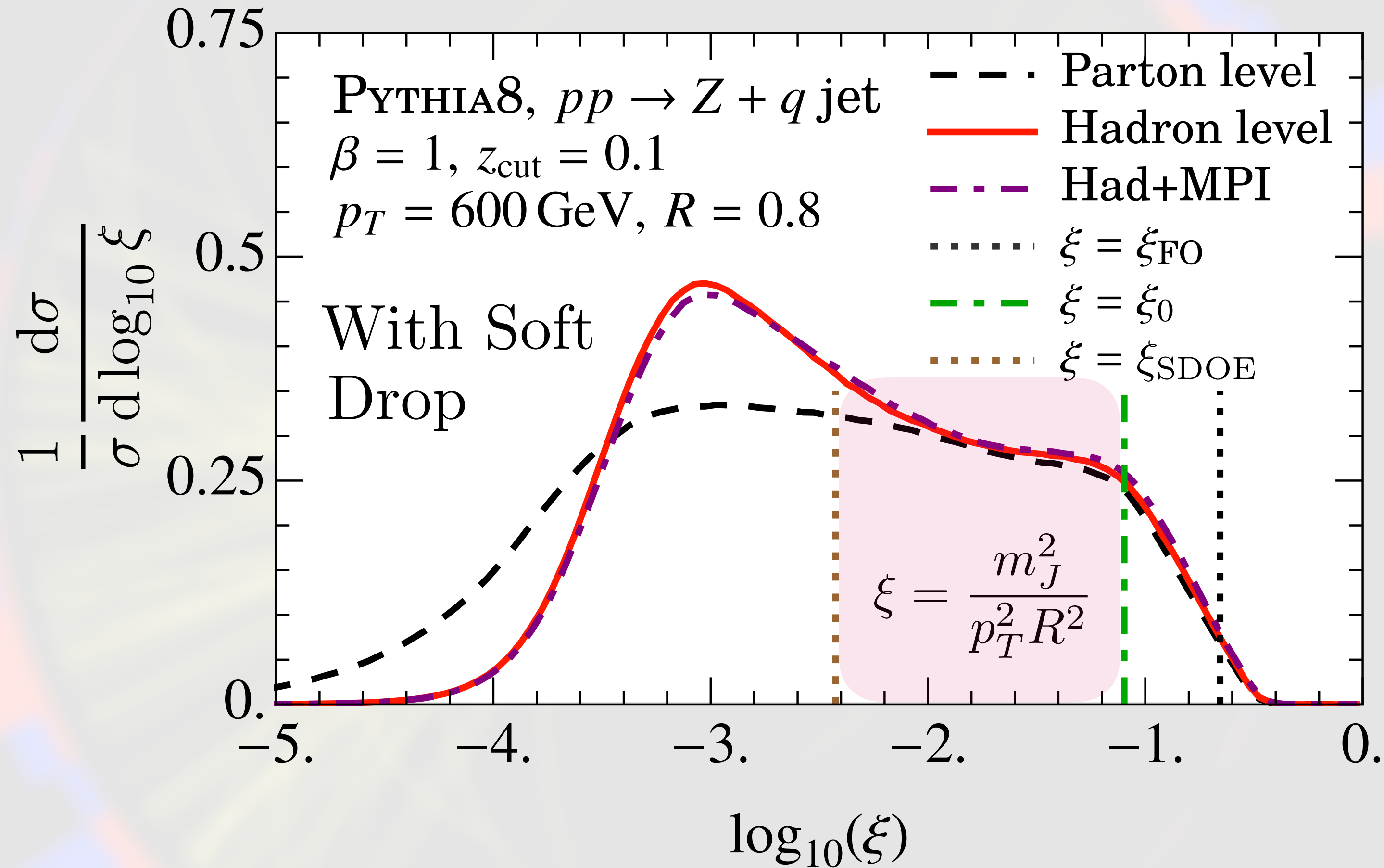
Soft drop operator expansion region (SDOE)

Plain jet mass  
resummation region

Fixed order region

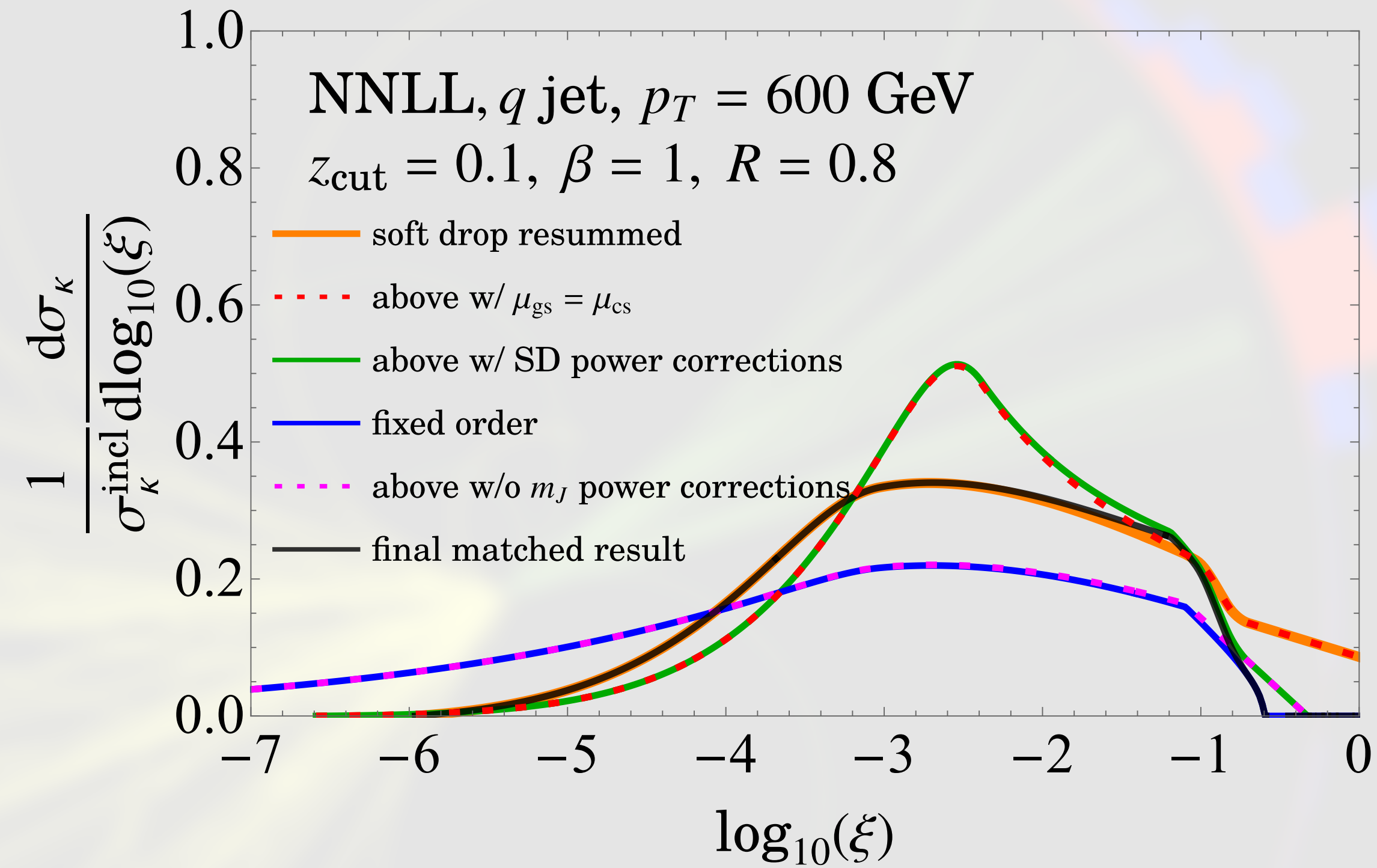
# Region for fitting to $\alpha_s$

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$



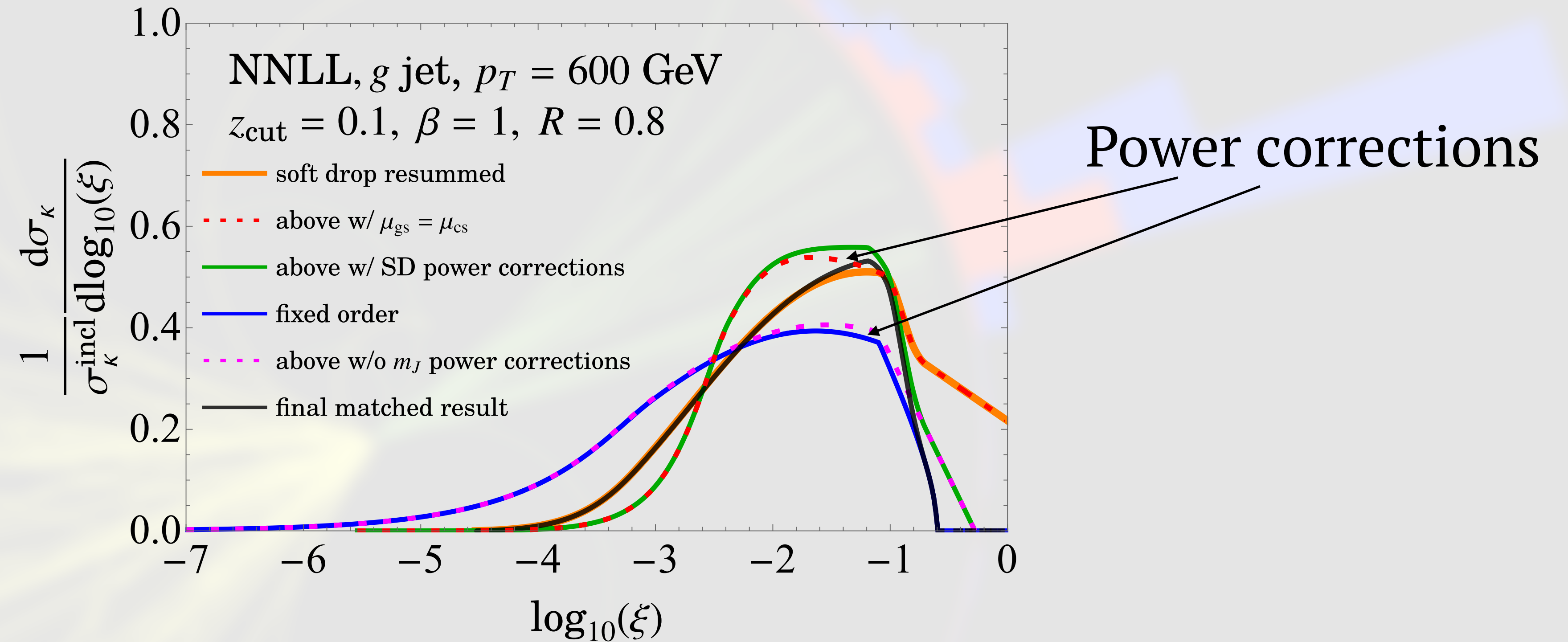
SDOE region only accessible for LHC Kinematics!

# Matched cross section (for quark jets)



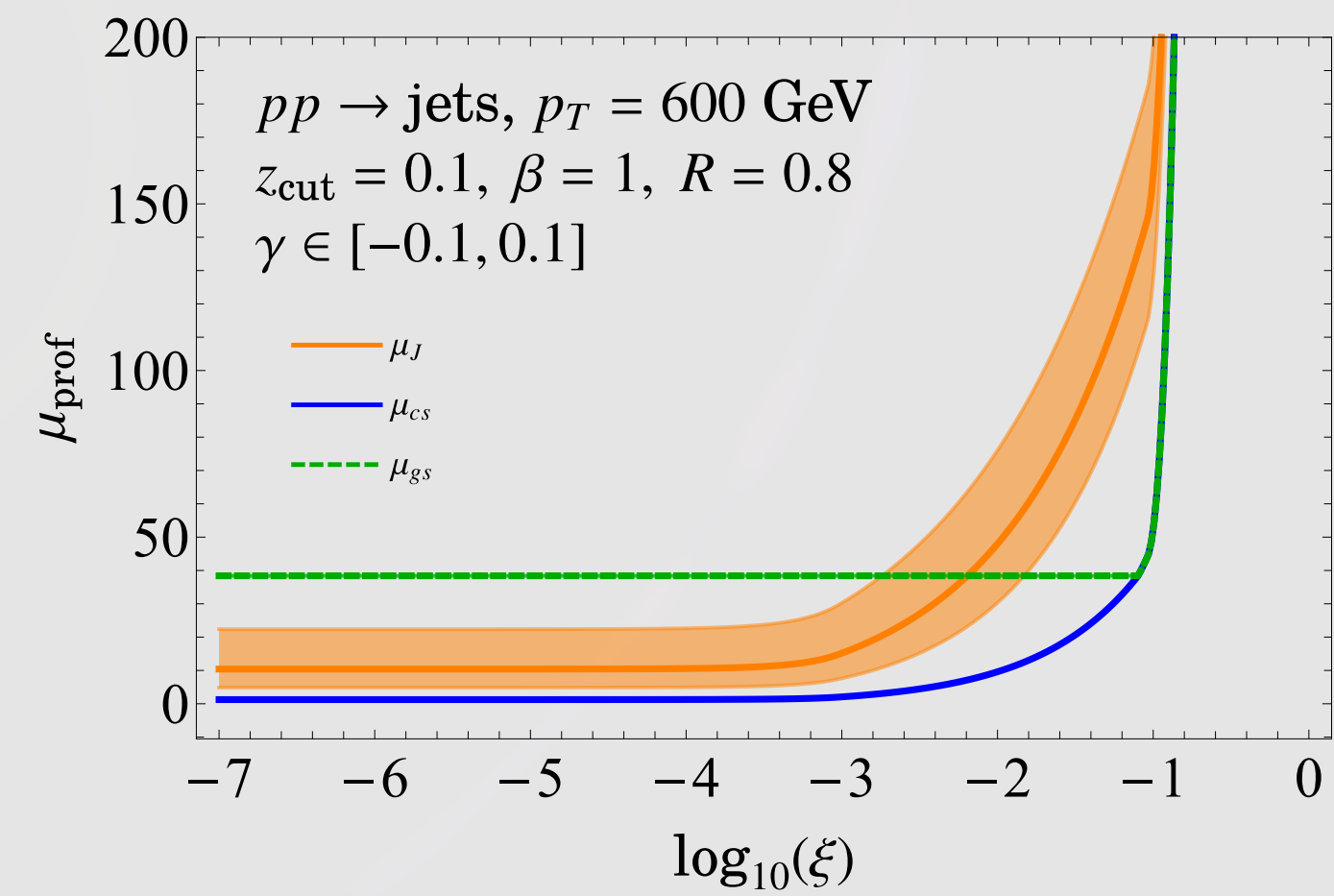
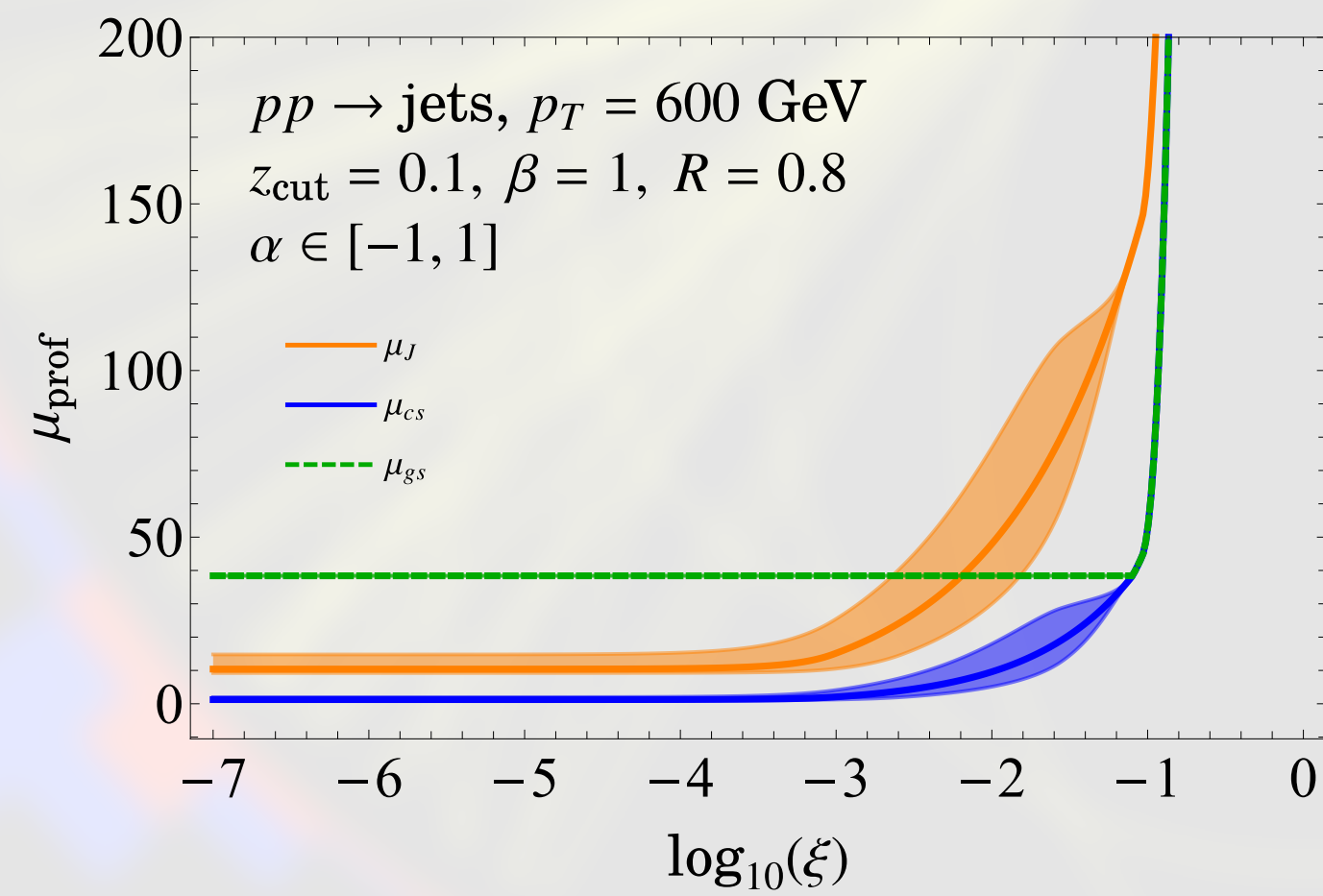
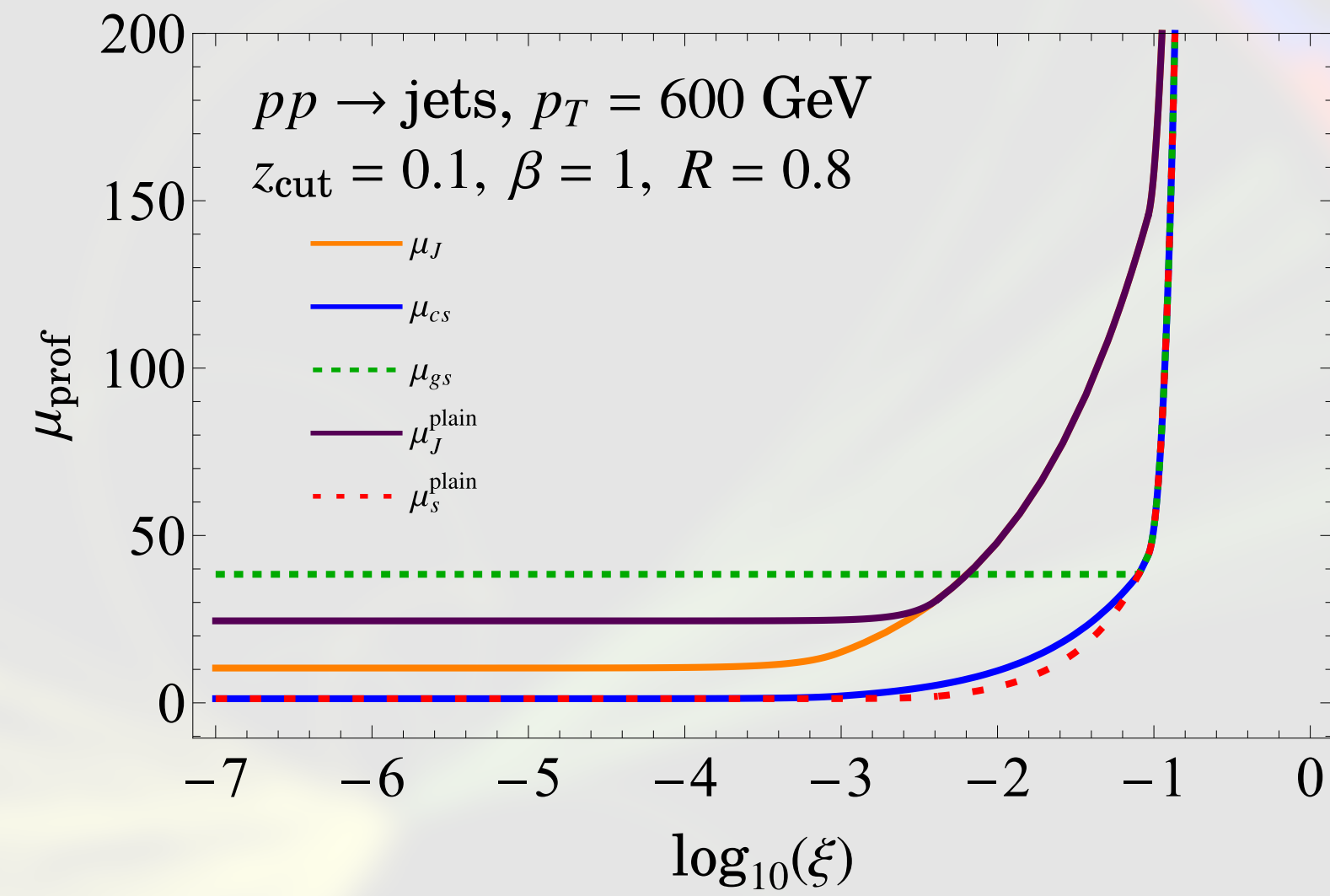
$$\begin{aligned}
 \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{matched}}(\xi) &\equiv \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{resum}}(\xi, \mu_{\text{sd} \rightarrow \text{plain}}) + \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{plain}}(\xi, \mu_{\text{plain}}) - \tilde{\mathcal{G}}_{\text{sd}}^{\text{resum}}(\xi, \mu_{\text{plain}}) \\
 &+ \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{min}}(\xi, r_g^{\text{max}}(\xi), \mu_{\text{min} \rightarrow \text{plain}}) - \tilde{\mathcal{G}}_{\text{sd}}^{\text{int}}(\xi, r_g^{\text{max}}(\xi), \mu_{\text{min} \rightarrow \text{plain}})
 \end{aligned}$$

# Matched cross section (for gluon jets)



$$\begin{aligned}
 \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{matched}}(\xi) &\equiv \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{resum}}(\xi, \mu_{\text{sd} \rightarrow \text{plain}}) + \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{plain}}(\xi, \mu_{\text{plain}}) - \tilde{\mathcal{G}}_{\text{sd}}^{\text{resum}}(\xi, \mu_{\text{plain}}) \\
 &+ \tilde{\mathcal{G}}_{\kappa, \text{sd}}^{\text{min}}(\xi, r_g^{\text{max}}(\xi), \mu_{\text{min} \rightarrow \text{plain}}) - \tilde{\mathcal{G}}_{\text{sd}}^{\text{int}}(\xi, r_g^{\text{max}}(\xi), \mu_{\text{min} \rightarrow \text{plain}})
 \end{aligned}$$

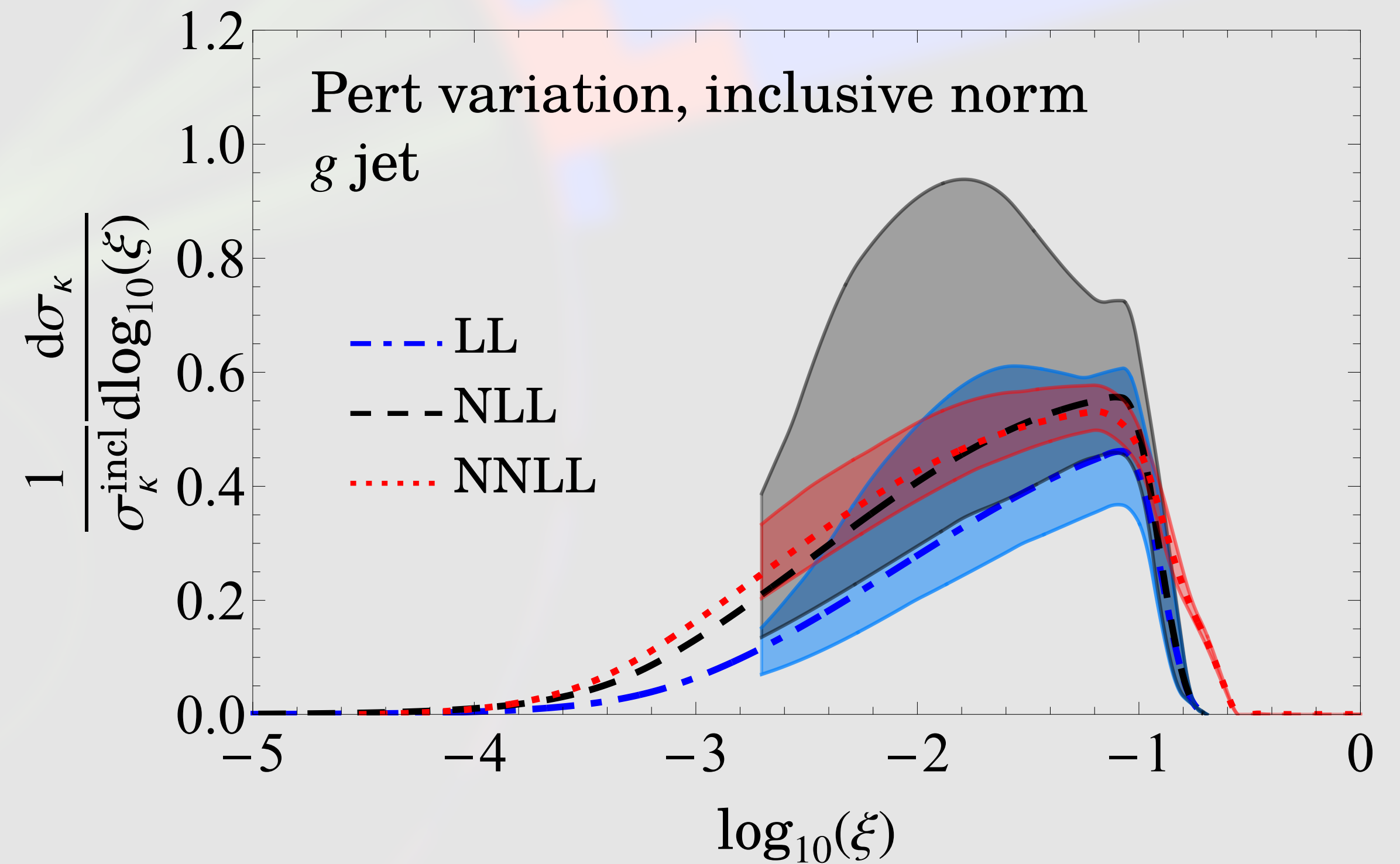
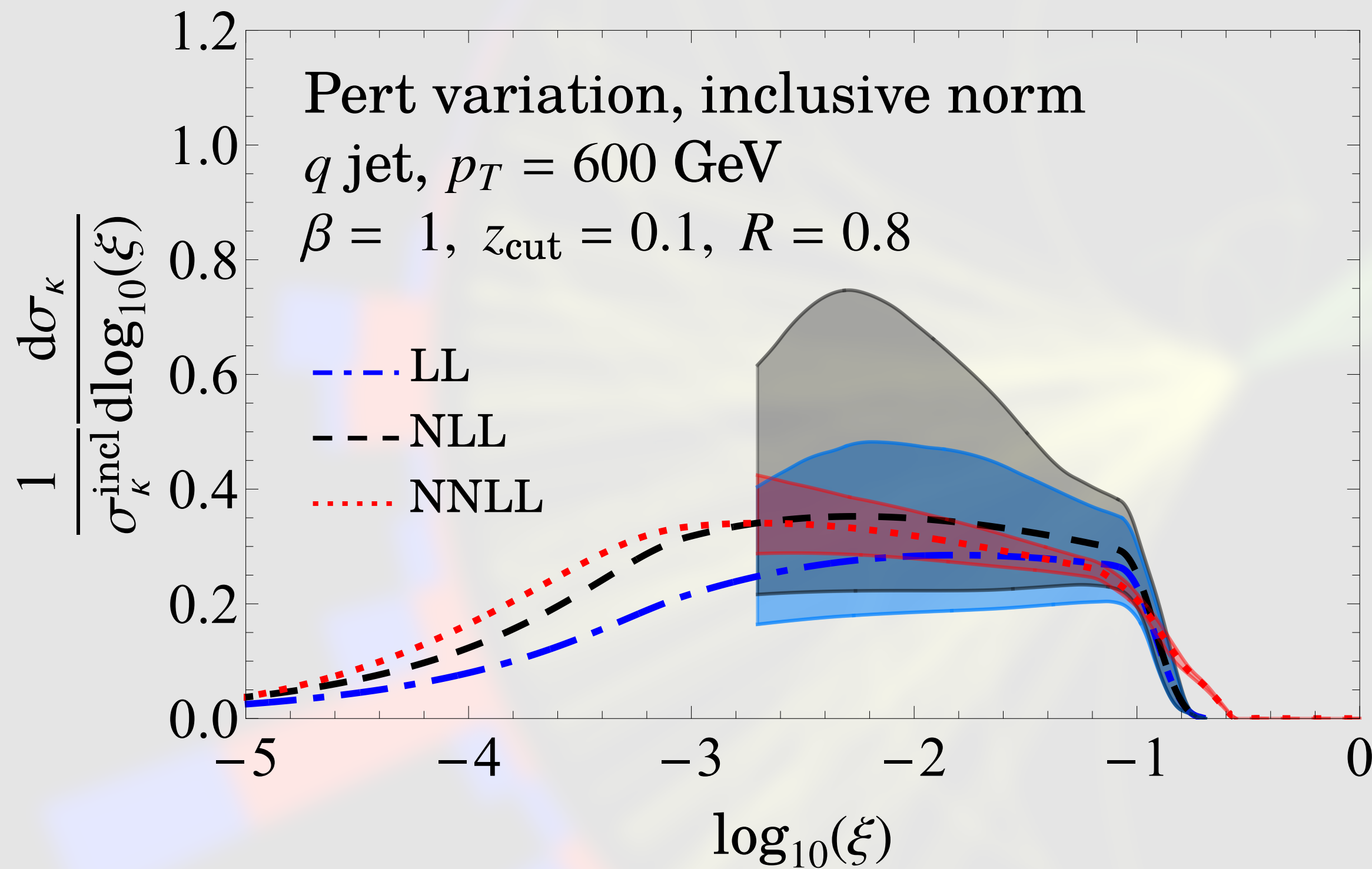
# Scale variations





# Scale variations

Convergence from NLL to NNLL:

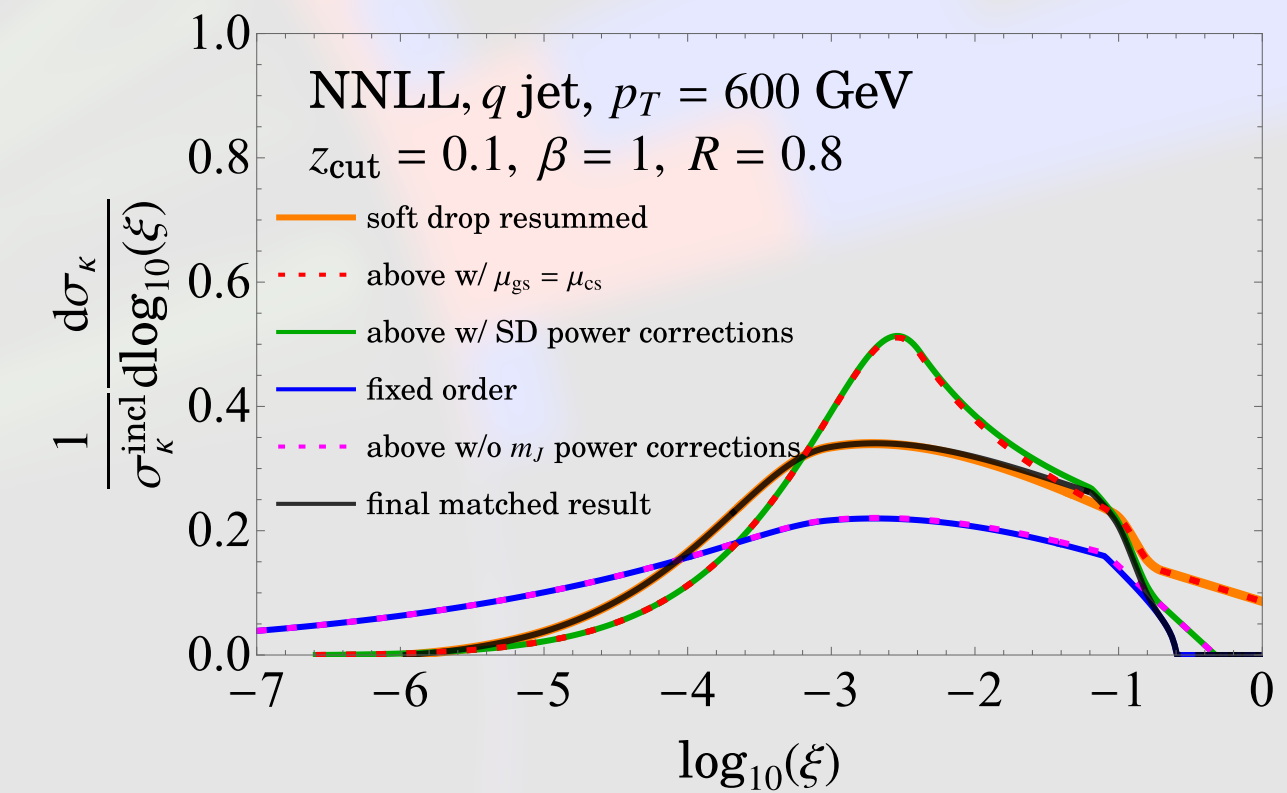


Only displaying scale variations in the SDOE region

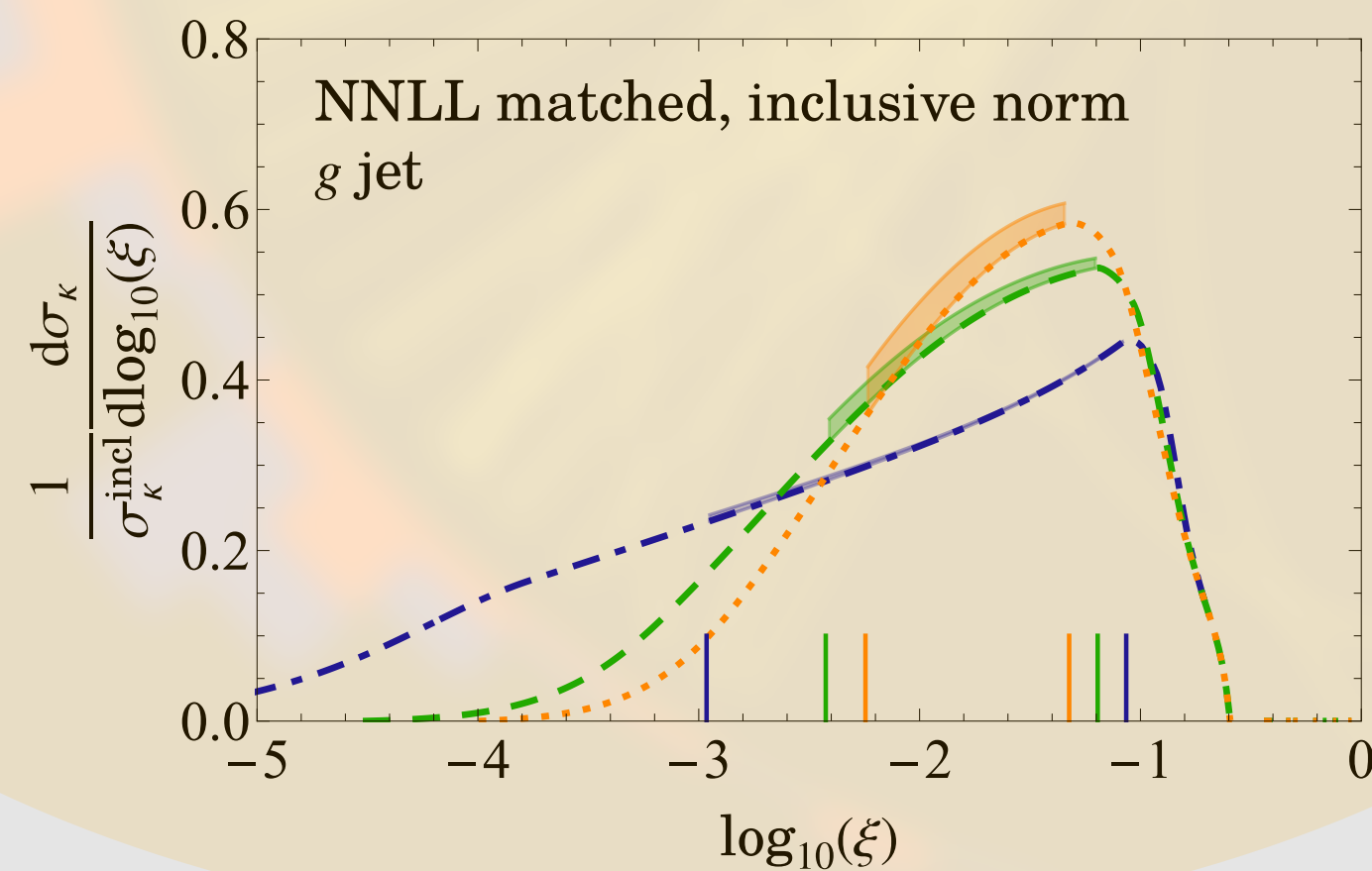
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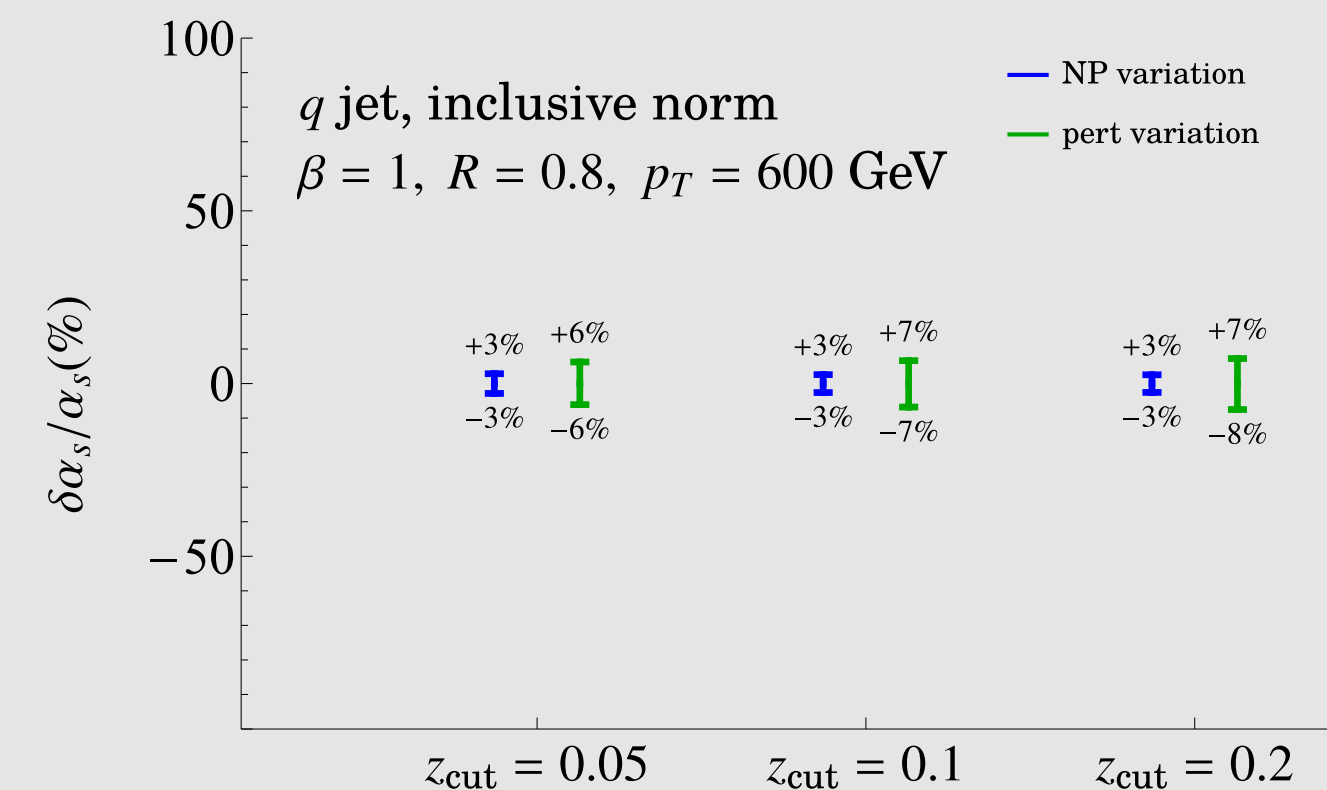
## 2. NNLL resummed cross section



## 3. Hadronization effects

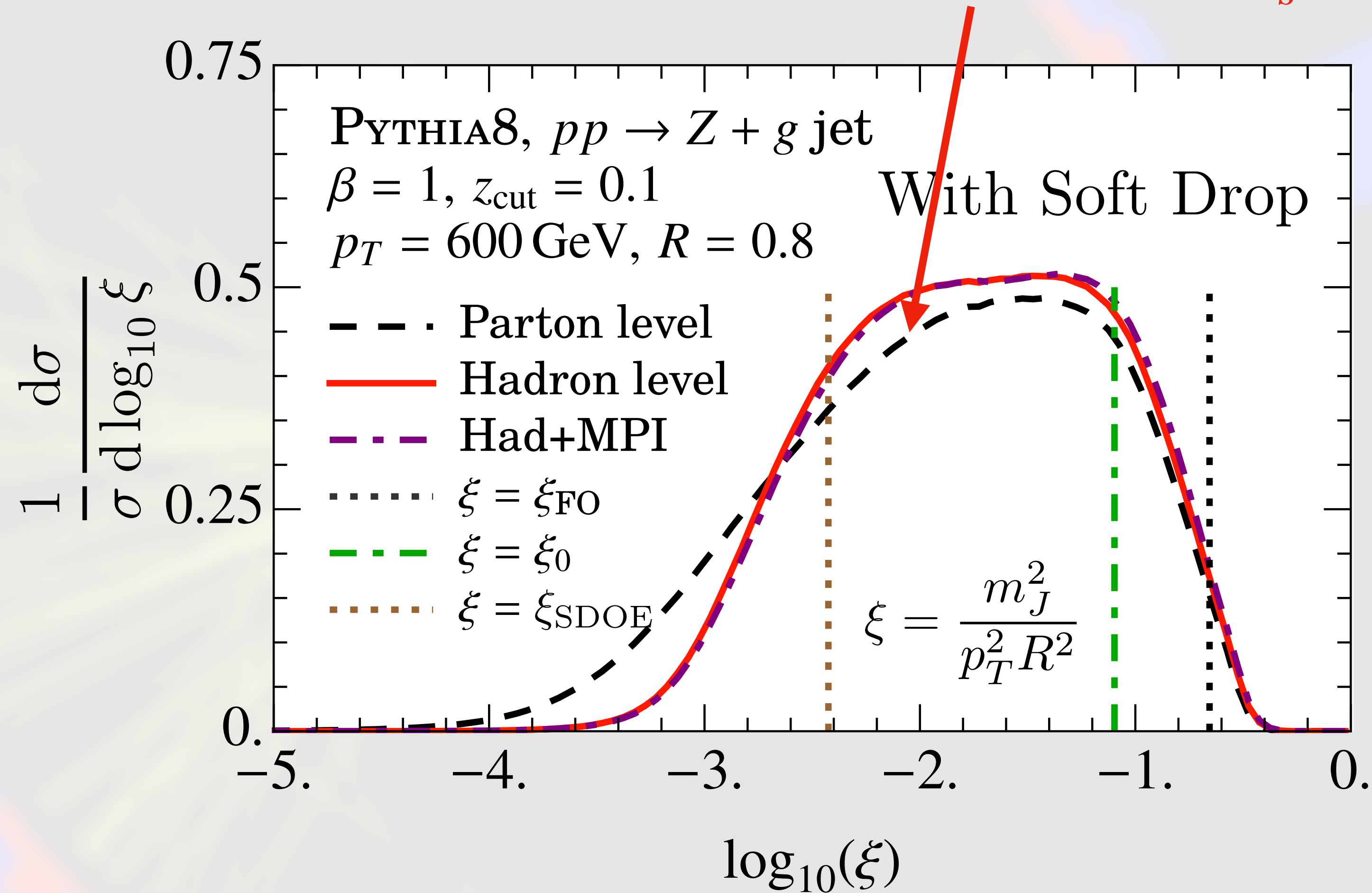


## 4. Results



# Hadronization corrections

Substantial nonperturbative effects in the  $\alpha_s$  fit region



How can we assess impact of hadronization on  $\alpha_s$   
in a **model-independent** way?

# Hadronization corrections

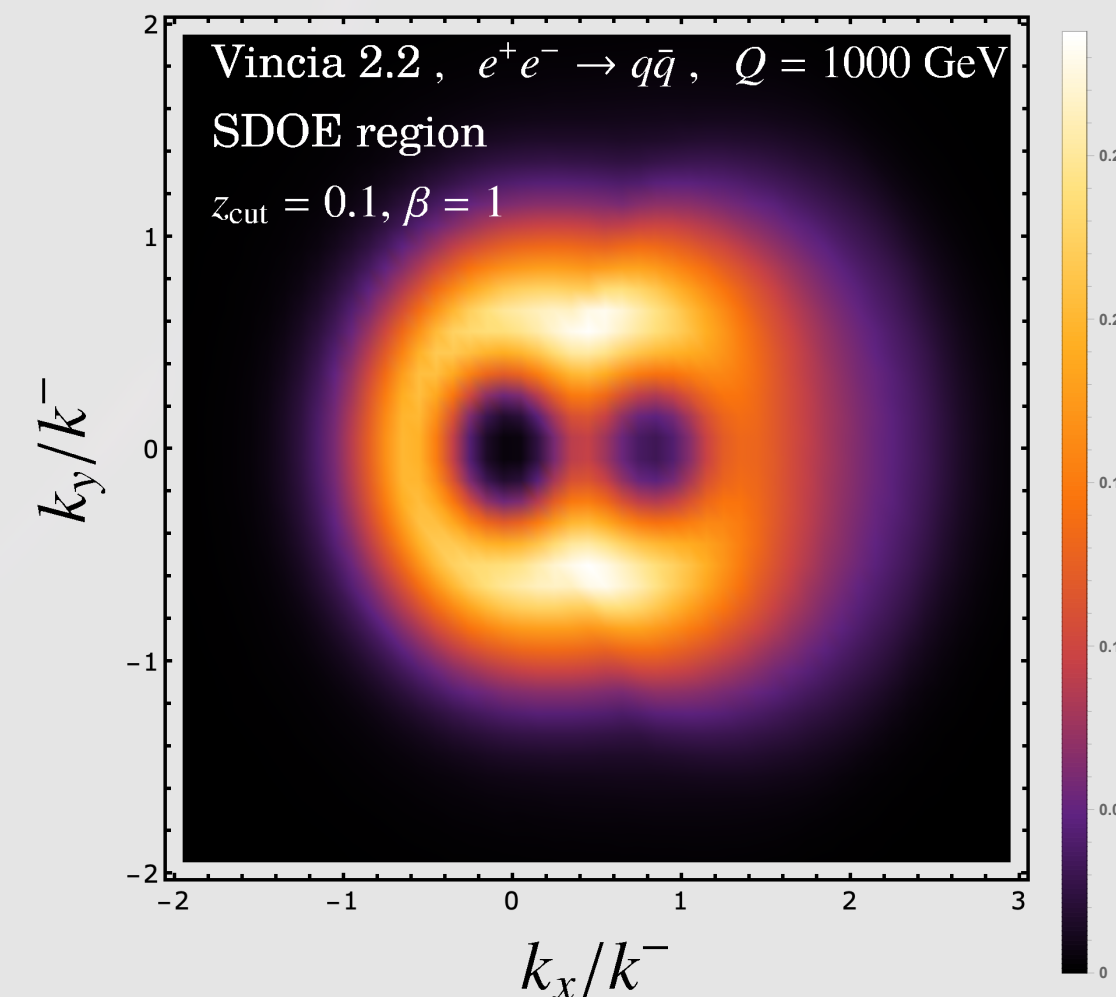
**Model-independent** statement on hadronization power corrections:

[Hoang, Mantry, AP, Stewart 2019]

$$\frac{1}{\sigma_\kappa} \frac{d\sigma_\kappa}{dm_J^2} = \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa}{dm_J^2} - Q \Omega_{1\kappa}^\circ \frac{d}{dm_J^2} \left( \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa^\circ}{dm_J^2} \right) + \frac{\Upsilon_{1,0\kappa}^\circ + \beta \Upsilon_{1,1\kappa}^\circ}{Q} \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa^\circ}{dm_J^2},$$

Perturbatively calculable Constant  $\mathcal{O}(\Lambda_{\text{QCD}})$  nonperturbative parameters

- At NLL' in [AP, Stewart, Vaidya, Zoppi 2020],
- Improved to NNLL + matching to ungroomed region in [AP JHEP 08(2023) 054]



# Hadronization corrections

**Model-independent** statement on hadronization power corrections:

[Hoang, Mantry, AP, Stewart 2019]

$$\frac{1}{\sigma_\kappa} \frac{d\sigma_\kappa}{dm_J^2} = \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa}{dm_J^2} - Q \Omega_{1\kappa}^\omega \frac{d}{dm_J^2} \left( \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa^\omega}{dm_J^2} \right) + \frac{\Upsilon_{1,0\kappa}^\odot + \beta \Upsilon_{1,1\kappa}^\odot}{Q} \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa^\odot}{dm_J^2},$$

Perturbatively calculable Constant  $\mathcal{O}(\Lambda_{\text{QCD}})$  nonperturbative parameters

$$\frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa^\omega}{dm_J^2} \equiv \int dr_g r_g \frac{1}{\hat{\sigma}_\kappa} \frac{d^2 \hat{\sigma}_\kappa}{dm_J^2 dr_g}, \quad \frac{1}{\hat{\sigma}_\kappa} \frac{d\hat{\sigma}_\kappa^\odot}{dm_J^2} \equiv \int \frac{dr_g dz_g \delta(z_g - z_{\text{cut}} r_g^\beta)}{r_g} \frac{1}{\hat{\sigma}_\kappa} \frac{d^3 \hat{\sigma}_\kappa}{dm_J^2 dr_g dz_g}.$$

Groomed jet radius

- At NLL' in [AP, Stewart, Vaidya, Zoppi 2020],
- Improved to NNLL + matching to ungroomed region in [AP JHEP 08(2023) 054]

# Aside: A diagnostic tool for hadronization models

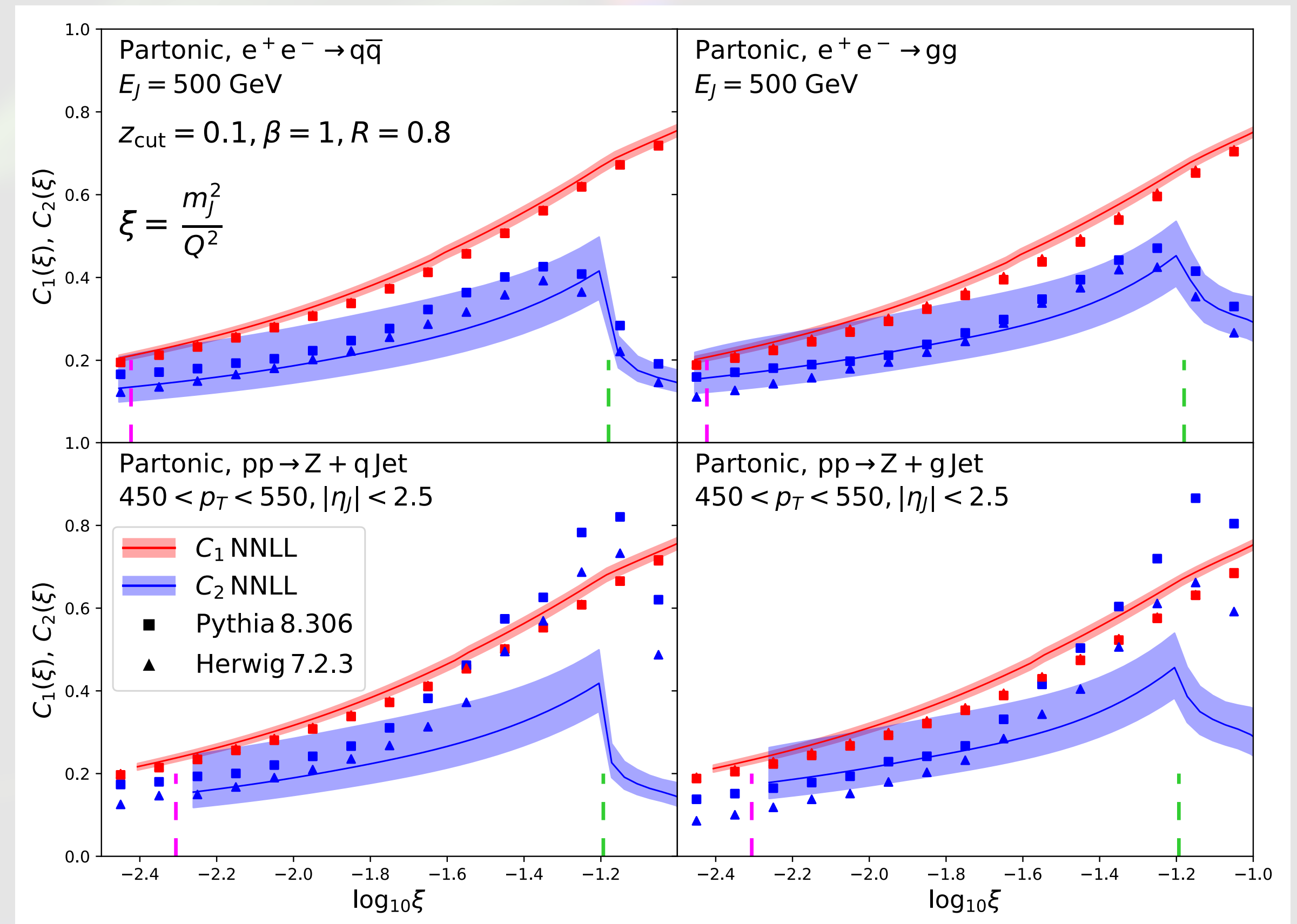
Our field-theory based analysis places **strong constraints on the universality properties of the nonperturbative power corrections** in the SDOE region

[Ferdinand, Lee, AP 2301.03605 ]

Reasonable to expect hadronization models to be consistent with these constraints.

MCs agree reasonably well with NNLL calculations of perturbative weights  $\longrightarrow$  Parton level extraction is consistent.

*(Discrepancy in  $pp$  near cusp is not a problem)*



# Aside: A diagnostic tool for hadronization models

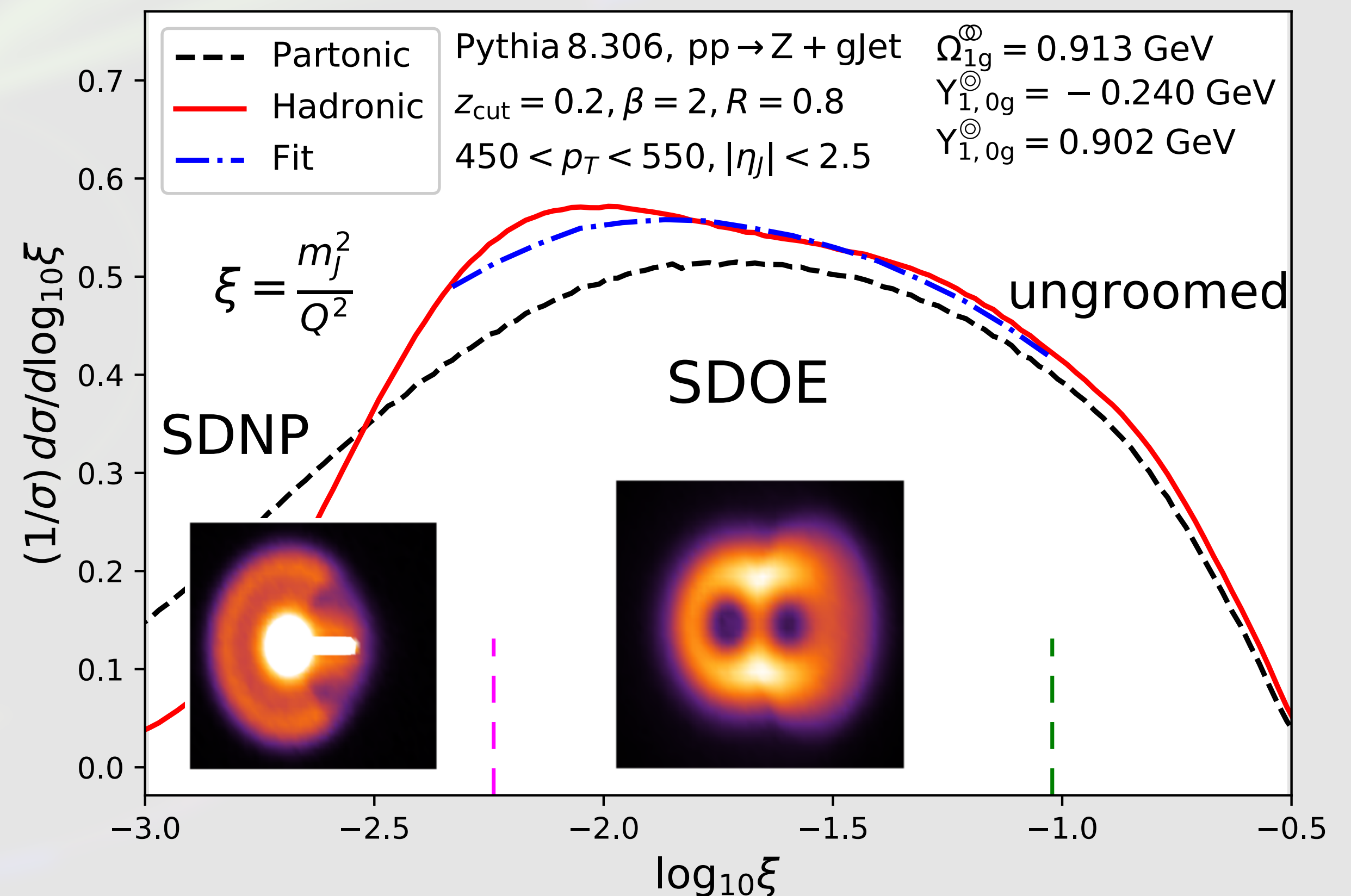
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Let us now **test the consistency of hadronization correction**

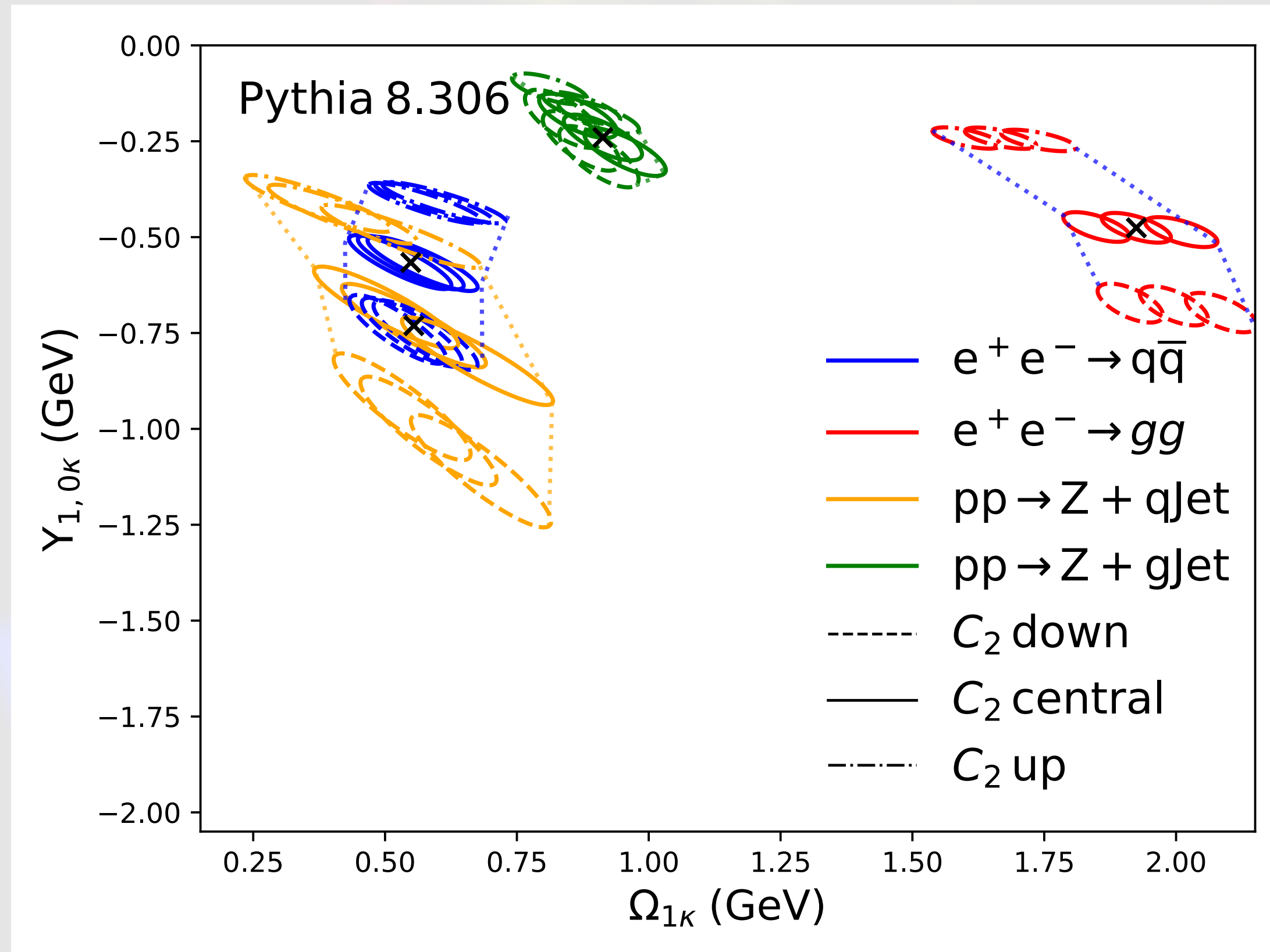


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[Ferdinand, Lee, AP 2301.03605 ]

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Quark Jets	$\Omega_{1q}^{\circledast}$ (GeV)	$\Upsilon_{1,0q}^{\circledast}$ (GeV)	$\Upsilon_{1,1q}^{\circledast}$ (GeV)	$\chi_{\min}^2/\text{dof.}$
$e^+e^- \rightarrow q\bar{q}$	$0.55^{+0.06}_{-0.03}$	$-0.57^{+0.16}_{-0.19}$	$1.06^{+0.31}_{-0.35}$	$0.77^{+0.03}_{-0.00}$
$pp \rightarrow Z + q$	$0.56^{+0.05}_{-0.14}$	$-0.73^{+0.29}_{-0.28}$	$0.89^{+0.27}_{-0.25}$	$0.65^{+0.01}_{-0.02}$
Gluon Jets	$\Omega_{1g}^{\circledast}$ (GeV)	$\Upsilon_{1,0g}^{\circledast}$ (GeV)	$\Upsilon_{1,1g}^{\circledast}$ (GeV)	$\chi_{\min}^2/\text{dof.}$
$e^+e^- \rightarrow gg$	$1.92^{+0.16}_{-0.32}$	$-0.48^{+0.23}_{-0.22}$	$0.87^{+0.25}_{-0.25}$	$3.13^{+0.05}_{-0.20}$
$pp \rightarrow Z + g$	$0.93^{+0.01}_{-0.12}$	$-0.24^{+0.11}_{-0.01}$	$0.89^{+0.20}_{-0.23}$	$1.34^{+0.05}_{-0.10}$

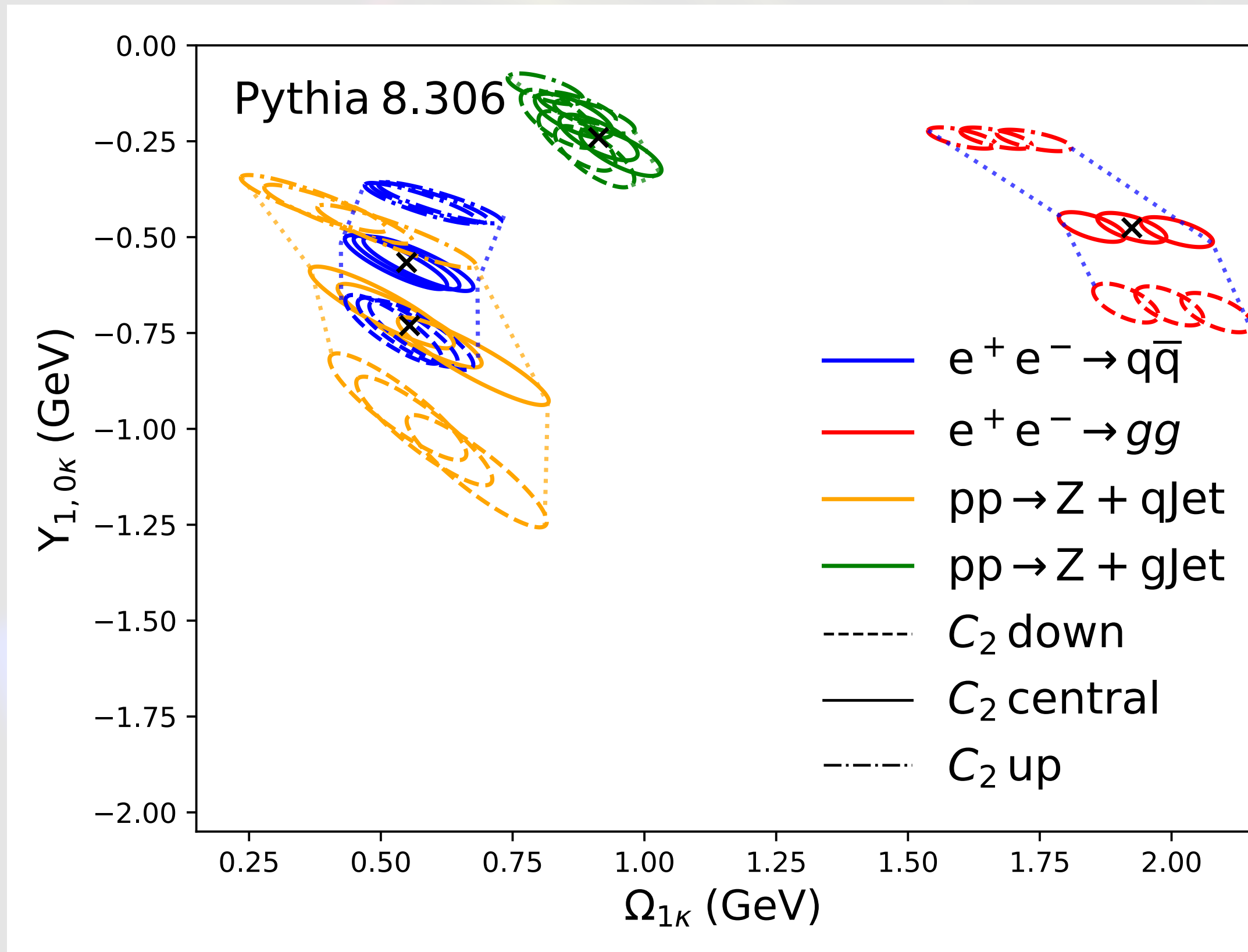


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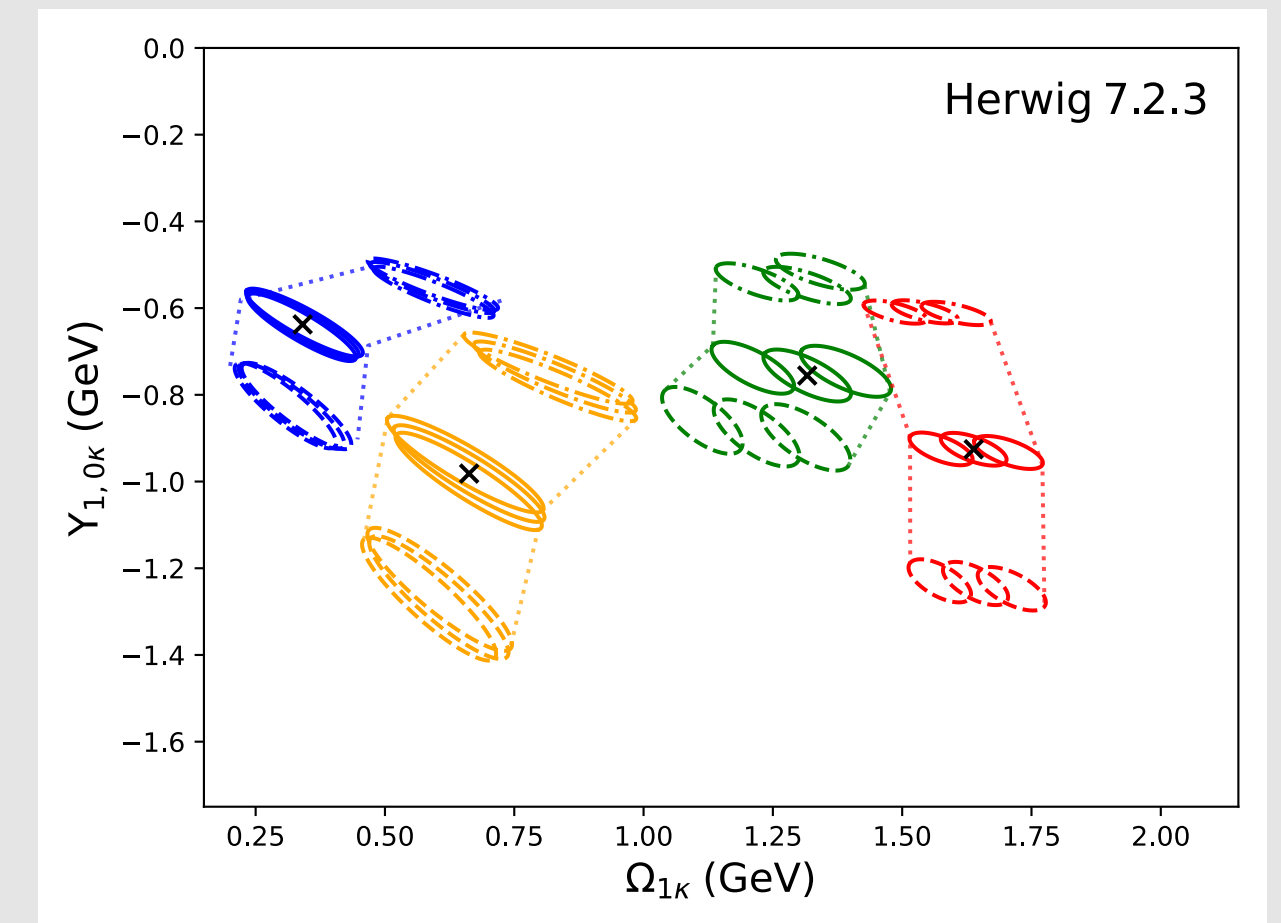
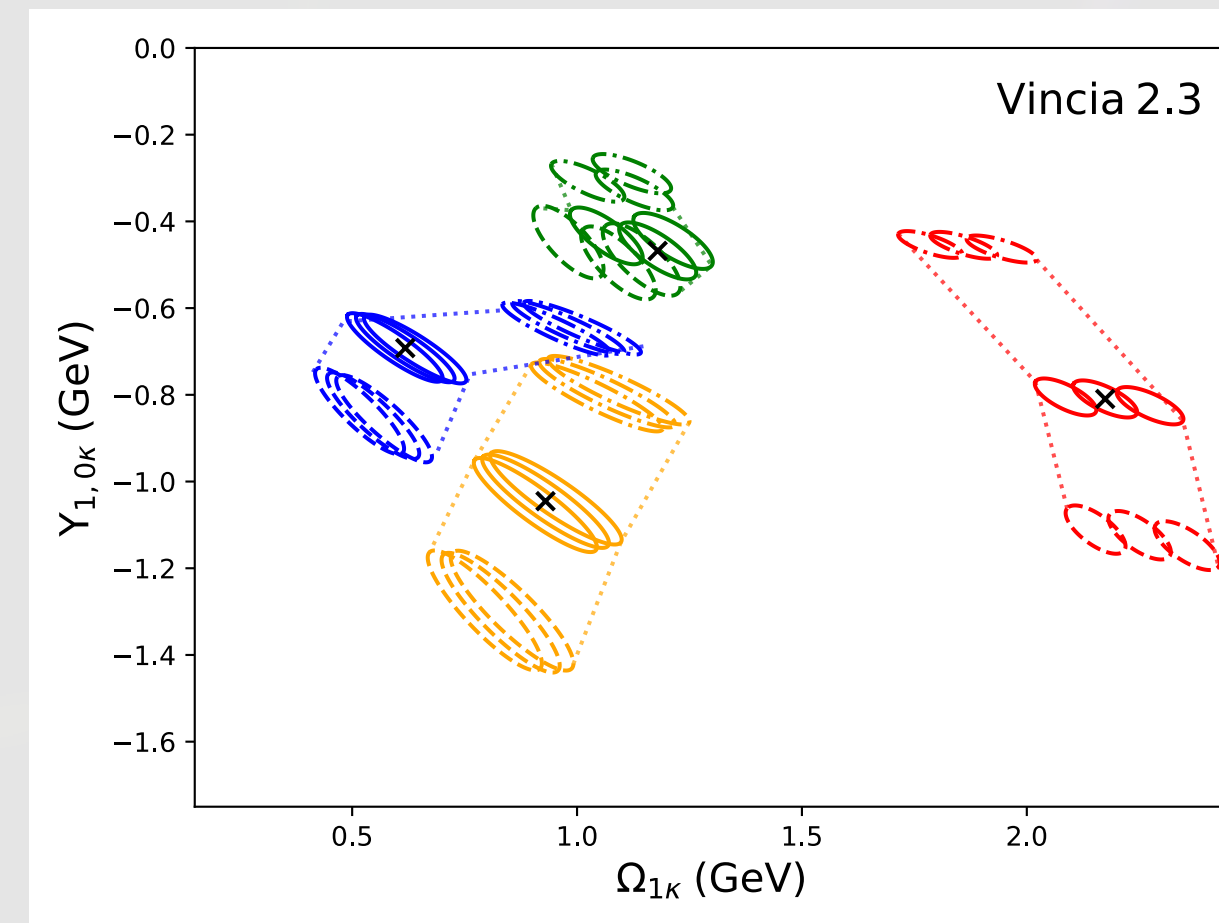
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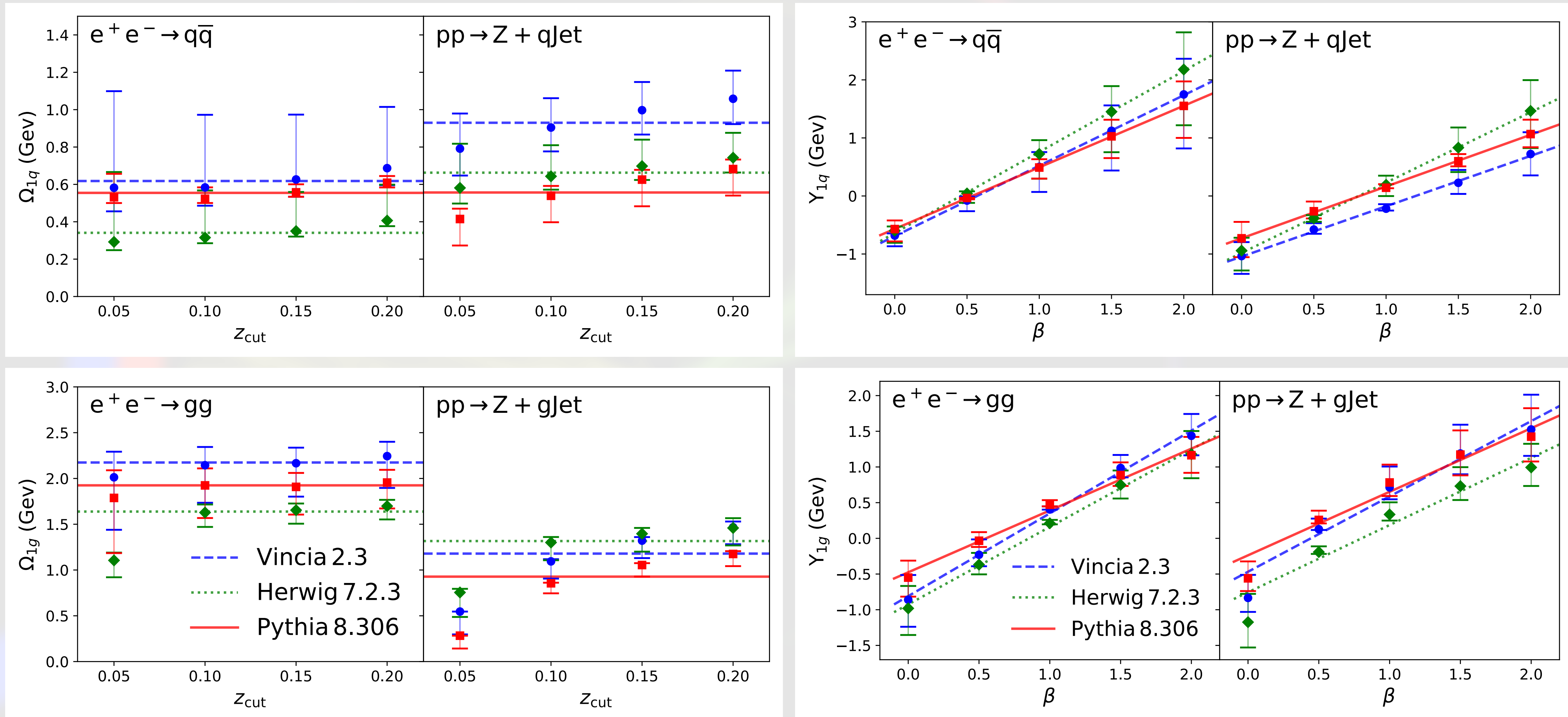
Reasonable to expect hadronization models to be consistent with these constraints.



Quark Jets	Ω <sub>1q</sub> <sup>⊙</sup> (GeV)	Υ <sub>1,0q</sub> <sup>⊙</sup> (GeV)	Υ <sub>1,1q</sub> <sup>⊙</sup> (GeV)	χ <sub>min</sub> <sup>2</sup> /dof.
e <sup>+</sup> e <sup>-</sup> → qq̄	0.55 <sup>+0.06</sup> <sub>-0.03</sub>	-0.57 <sup>+0.16</sup> <sub>-0.19</sub>	1.06 <sup>+0.31</sup> <sub>-0.35</sub>	0.77 <sup>+0.03</sup> <sub>-0.00</sub>
pp → Z + q	0.56 <sup>+0.05</sup> <sub>-0.14</sub>	-0.73 <sup>+0.29</sup> <sub>-0.28</sub>	0.89 <sup>+0.27</sup> <sub>-0.25</sub>	0.65 <sup>+0.01</sup> <sub>-0.02</sub>
Gluon Jets	Ω <sub>1g</sub> <sup>⊙</sup> (GeV)	Υ <sub>1,0g</sub> <sup>⊙</sup> (GeV)	Υ <sub>1,1g</sub> <sup>⊙</sup> (GeV)	χ <sub>min</sub> <sup>2</sup> /dof.
e <sup>+</sup> e <sup>-</sup> → gg	1.92 <sup>+0.16</sup> <sub>-0.32</sub>	-0.48 <sup>+0.23</sup> <sub>-0.22</sub>	0.87 <sup>+0.25</sup> <sub>-0.25</sub>	3.13 <sup>+0.05</sup> <sub>-0.20</sub>
pp → Z + g	0.93 <sup>+0.01</sup> <sub>-0.12</sub>	-0.24 <sup>+0.11</sup> <sub>-0.01</sub>	0.89 <sup>+0.20</sup> <sub>-0.23</sub>	1.34 <sup>+0.05</sup> <sub>-0.10</sub>



# Compare quark jets vs. gluon jets



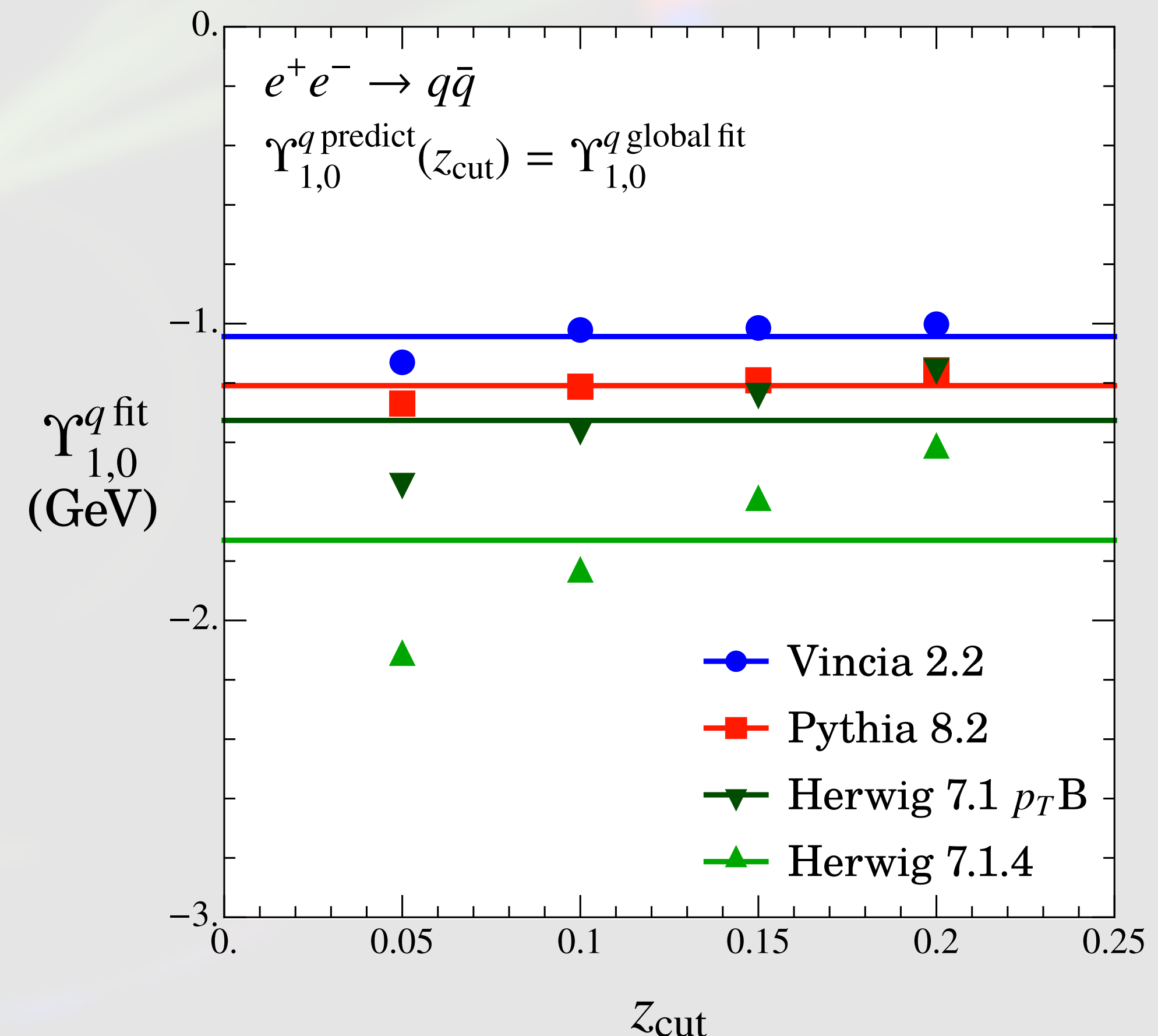
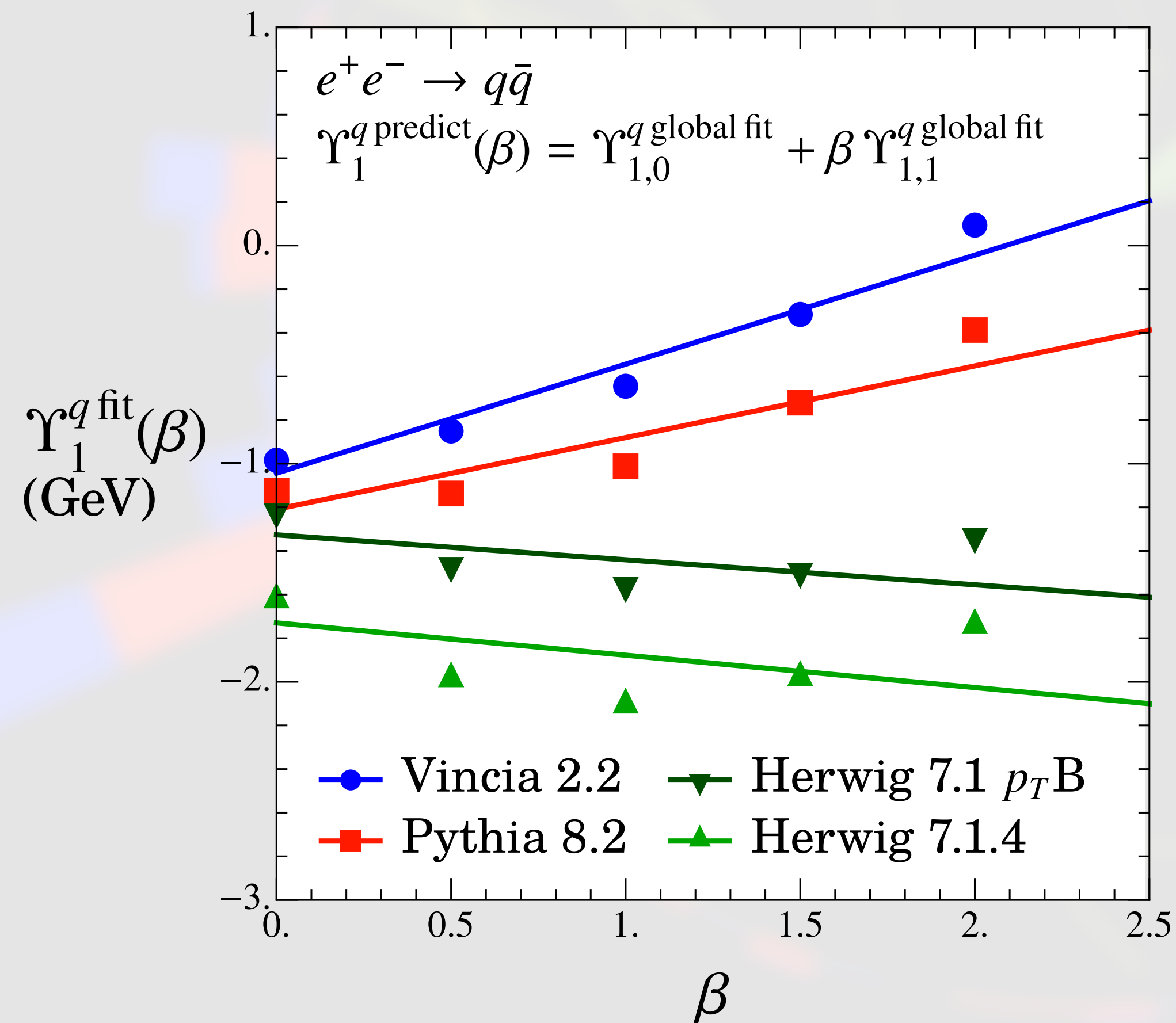
Soft drop is the unique jet substructure observable showing a rich interplay between perturbative and non-perturbative dynamics

# Universality constraints can diagnose problems in MCs!

Hoang, Mantry, AP, Stewart JHEP 12 (2019) 002

Performing fits for parameters **independently** diagnosed a problem with the default Herwig tune in 2019, later confirmed by authors, and fix incorporated in subsequent versions.

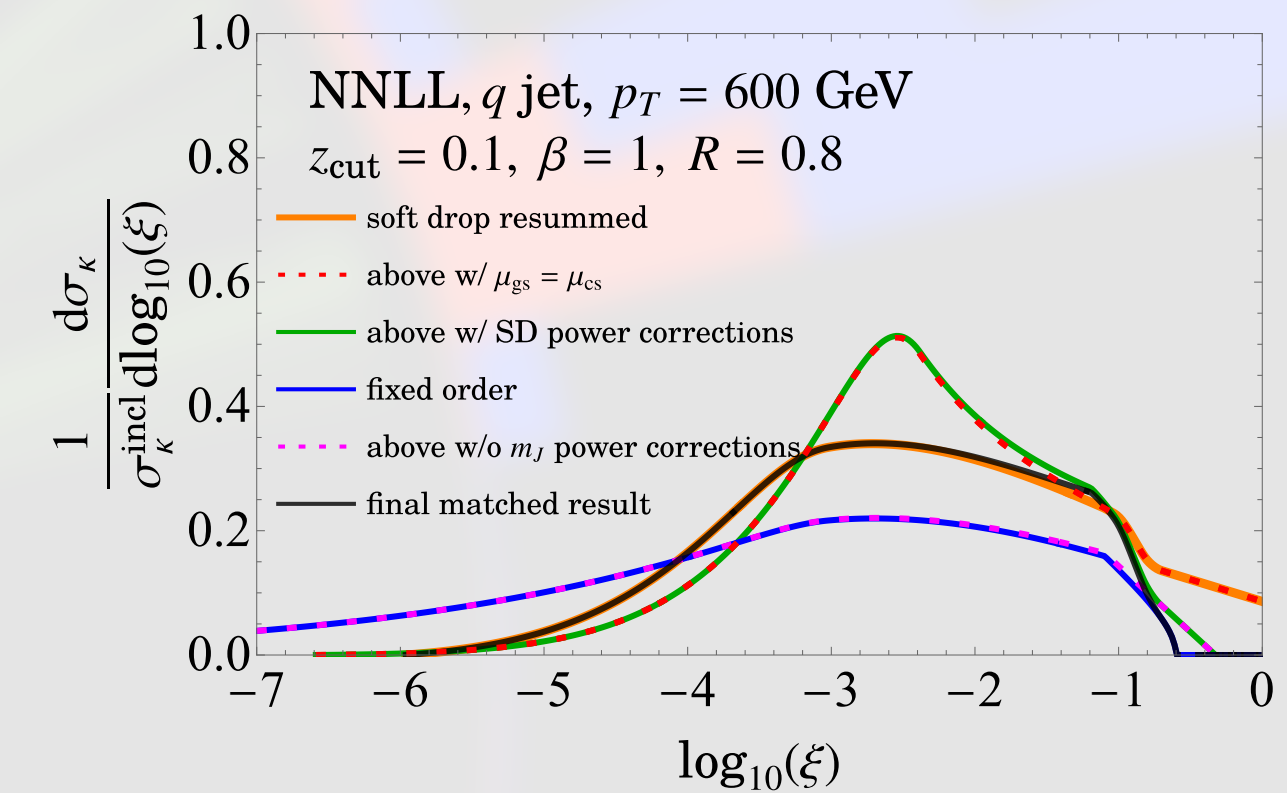
Event Generator	$\Omega_{1q}^\oplus$ (GeV)	$\Upsilon_{1,0}^q$ (GeV)	$\Upsilon_{1,1}^q$ (GeV)	$\chi_{\min}^2/\text{dof}$
PYTHIA 8.235	1.63	-1.21	0.33	0.96
VINCIA 2.2	1.22	-1.04	0.50	0.84
HERWIG 7.1.4 (default)	1.14	-1.73	-0.15	2.53
HERWIG 7.1 ( $p_T$ B)	1.14	-1.32	-0.11	0.77



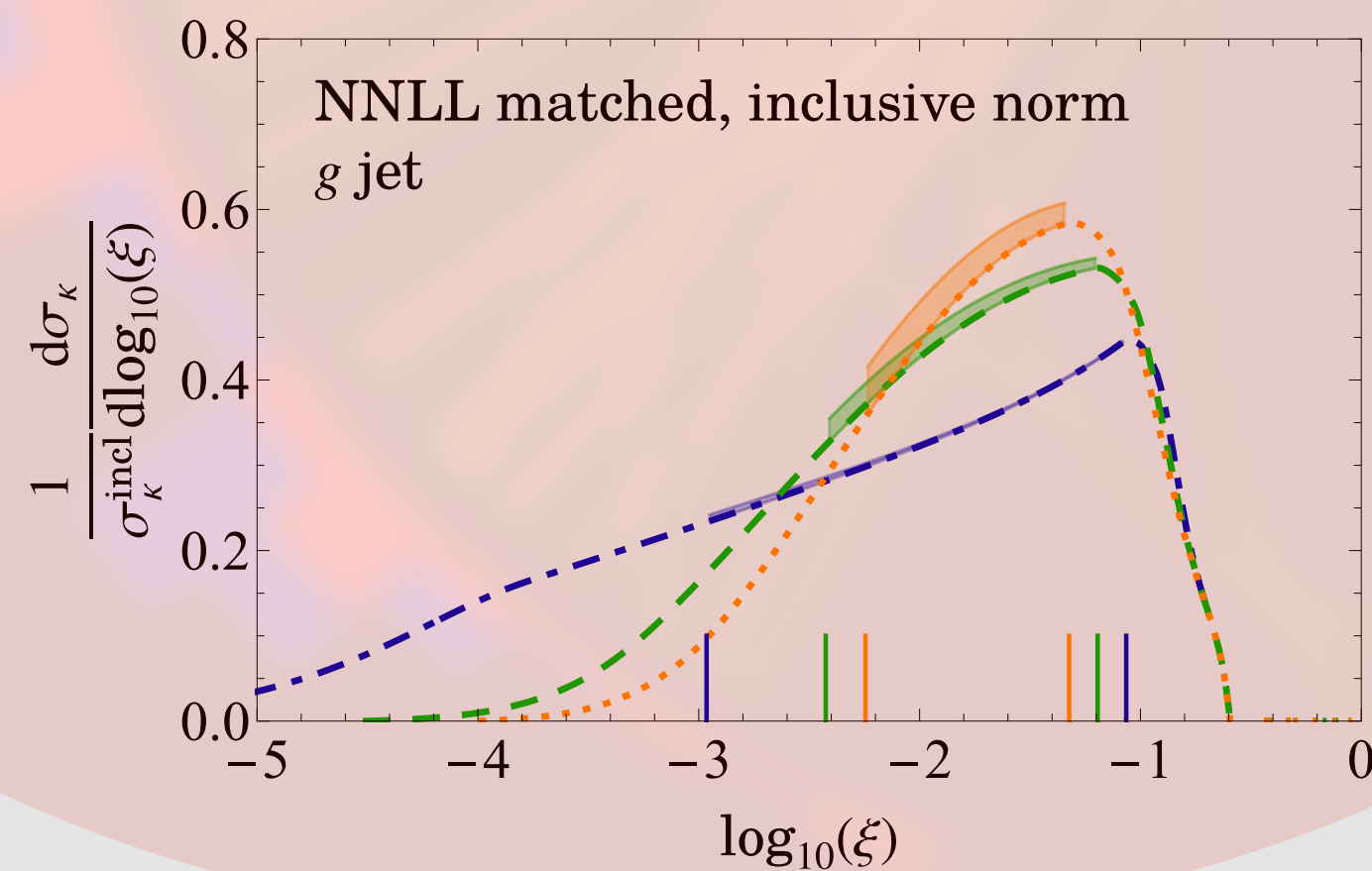
# Back to $\alpha_s$

## 1. Quark-gluon fraction and PDF dependence

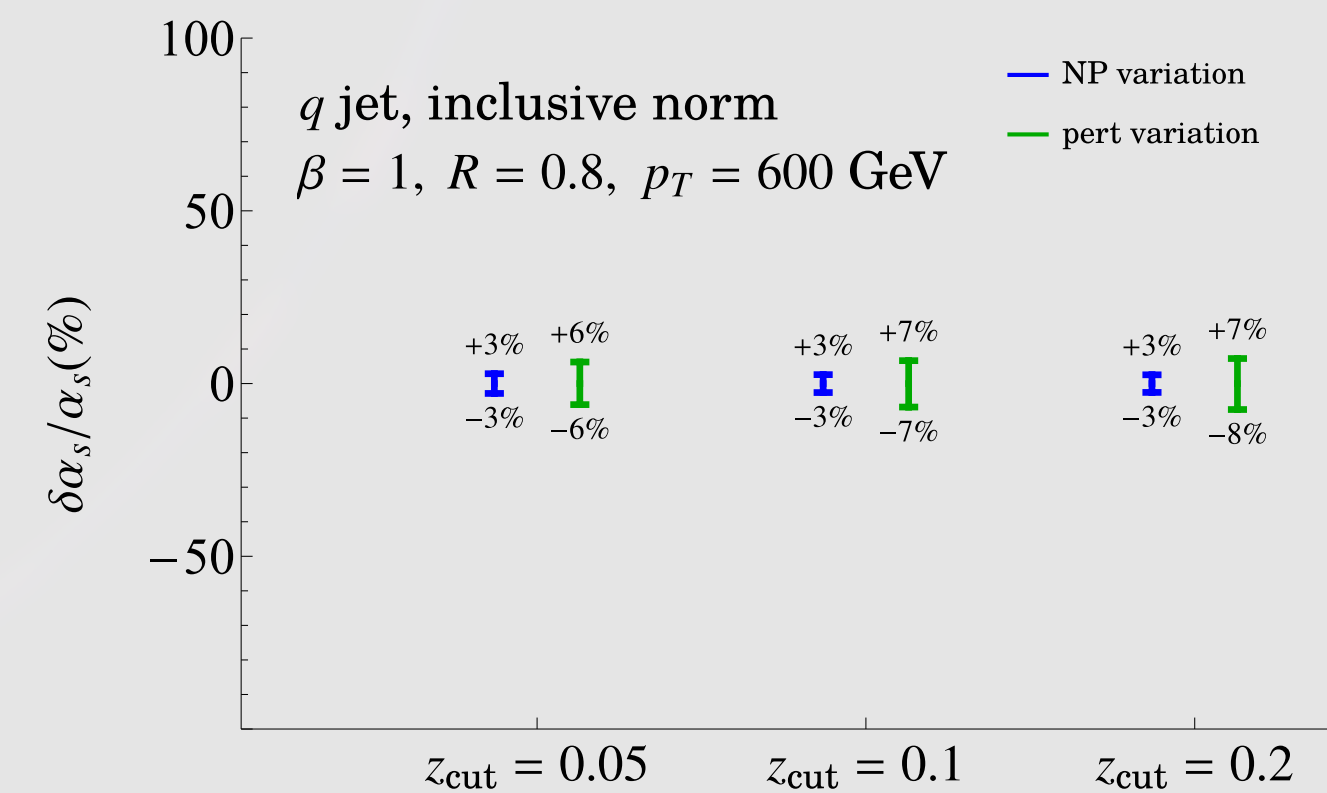
## 2. NNLL resummed cross section



## 3. Hadronization effects



## 4. Results



# What to do about hadronization?

Ideally, fit for 7 parameters to stay model-independent. Challenging?

$$(\alpha_s, \Omega_{1q}^{\otimes}, \Omega_{1g}^{\otimes}, \Upsilon_{1,0q}^{\odot}, \Upsilon_{1,0g}^{\odot}, \Upsilon_{1,1q}^{\odot}, \Upsilon_{1,1g}^{\odot})$$

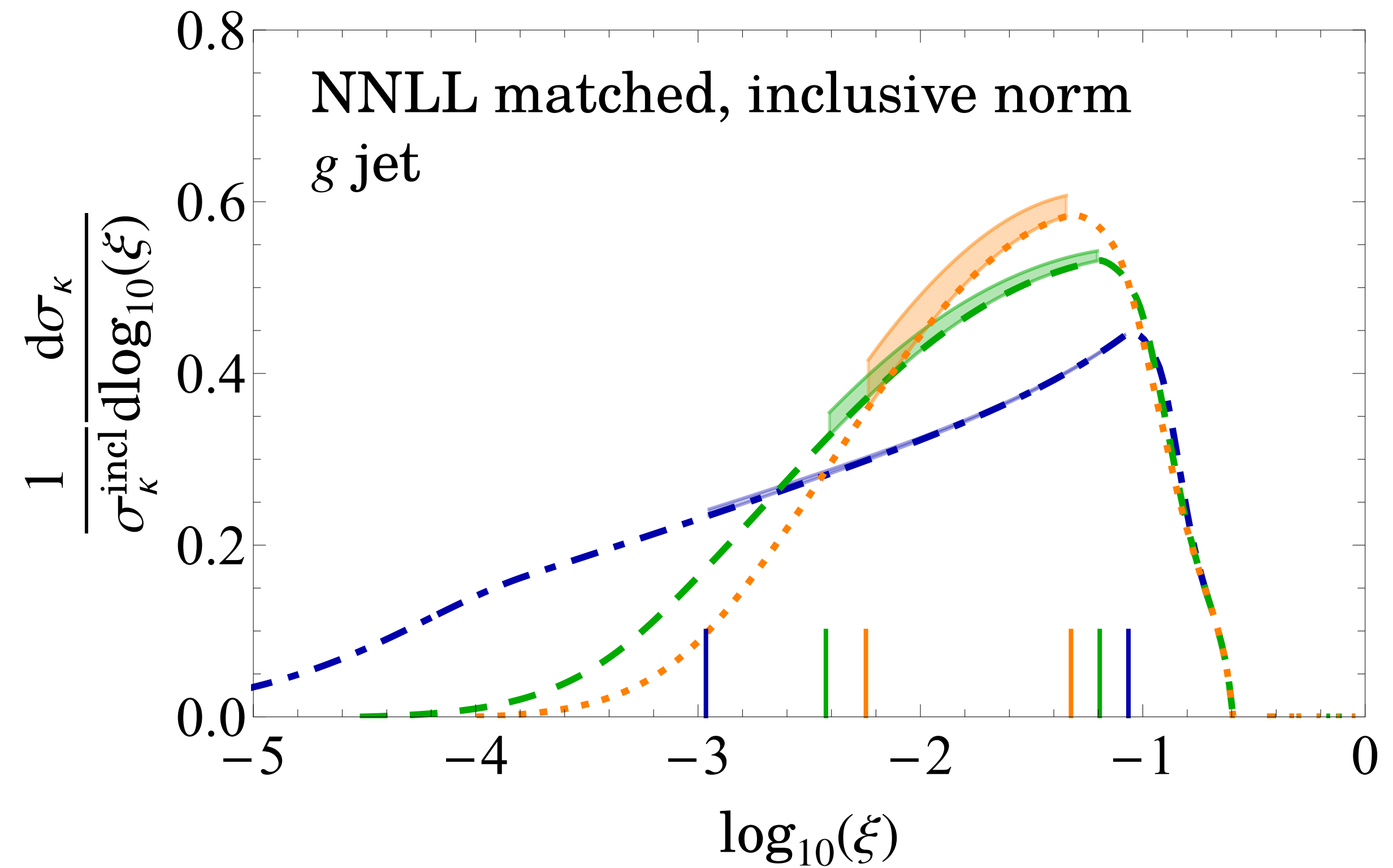
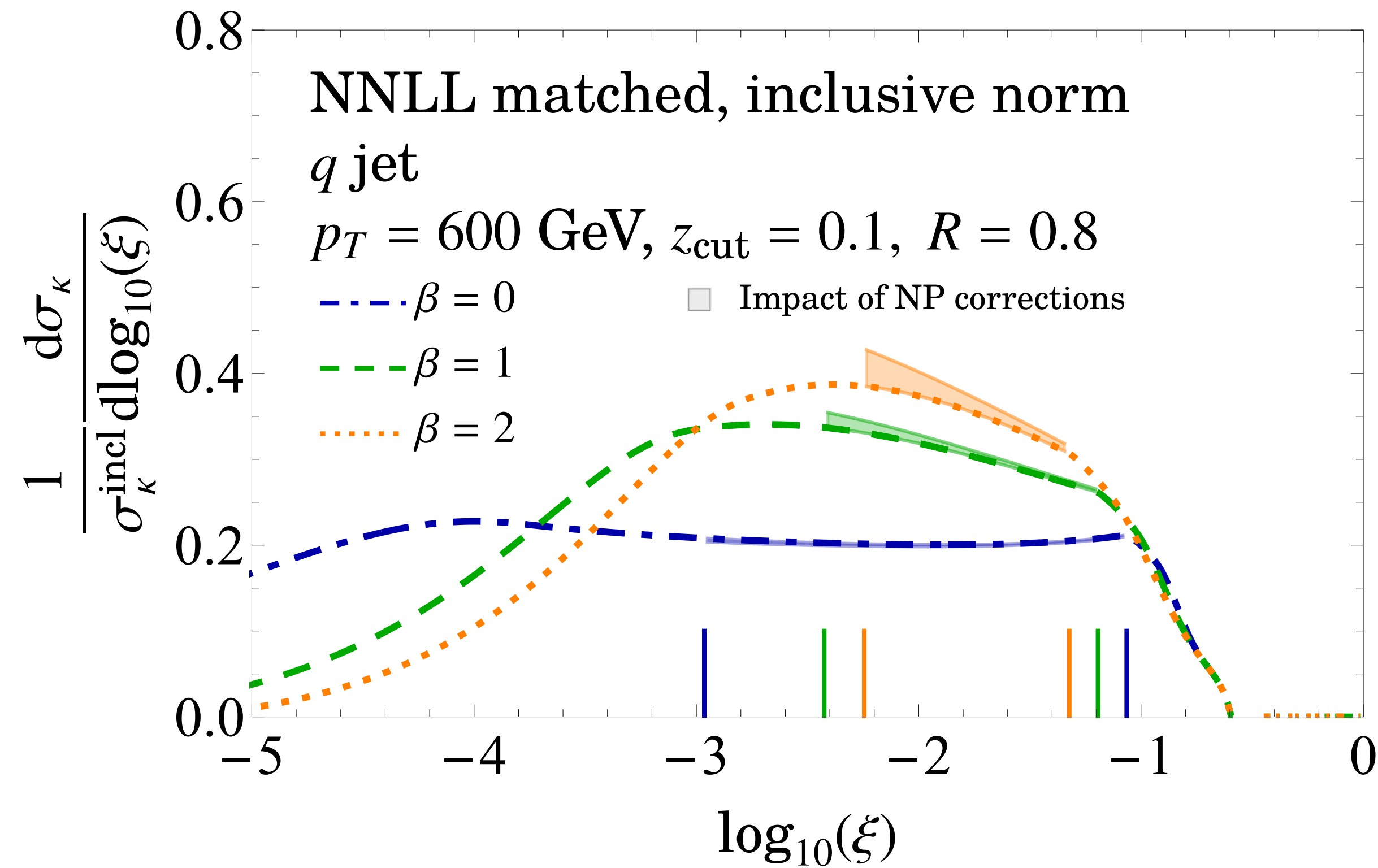
Instead, treat them as nuisance parameters

Values obtained from fit to Pythia8: [Ferdinand, Lee, AP 2301.03605 ]

$$\begin{aligned} \Omega_{1,q}^{\otimes} &= 0.55 \text{ GeV}, & \Upsilon_{1,0q}^{\odot} &= -0.73 \text{ GeV}, & \Upsilon_{1,1q}^{\odot} &= 0.90 \text{ GeV}, & \text{for quarks,} \\ \Omega_{1,g}^{\otimes} &= 0.91 \text{ GeV}, & \Upsilon_{1,0g}^{\odot} &= -0.24 \text{ GeV}, & \Upsilon_{1,1g}^{\odot} &= 0.90 \text{ GeV}, & \text{for gluons.} \end{aligned}$$

Good for uncertainty estimate, exact values not important!

# Impact of hadronization corrections







Take uncertainty as difference between parton and hadron level

# Comparison with previous work

[Frye, Larkoski, Schwartz, Yan 2016] [Marzani, Schunk, Sodes 2017]

[Anderle, Dasgupta, El-Menoufi, Helliwell, Guzzi 2020]

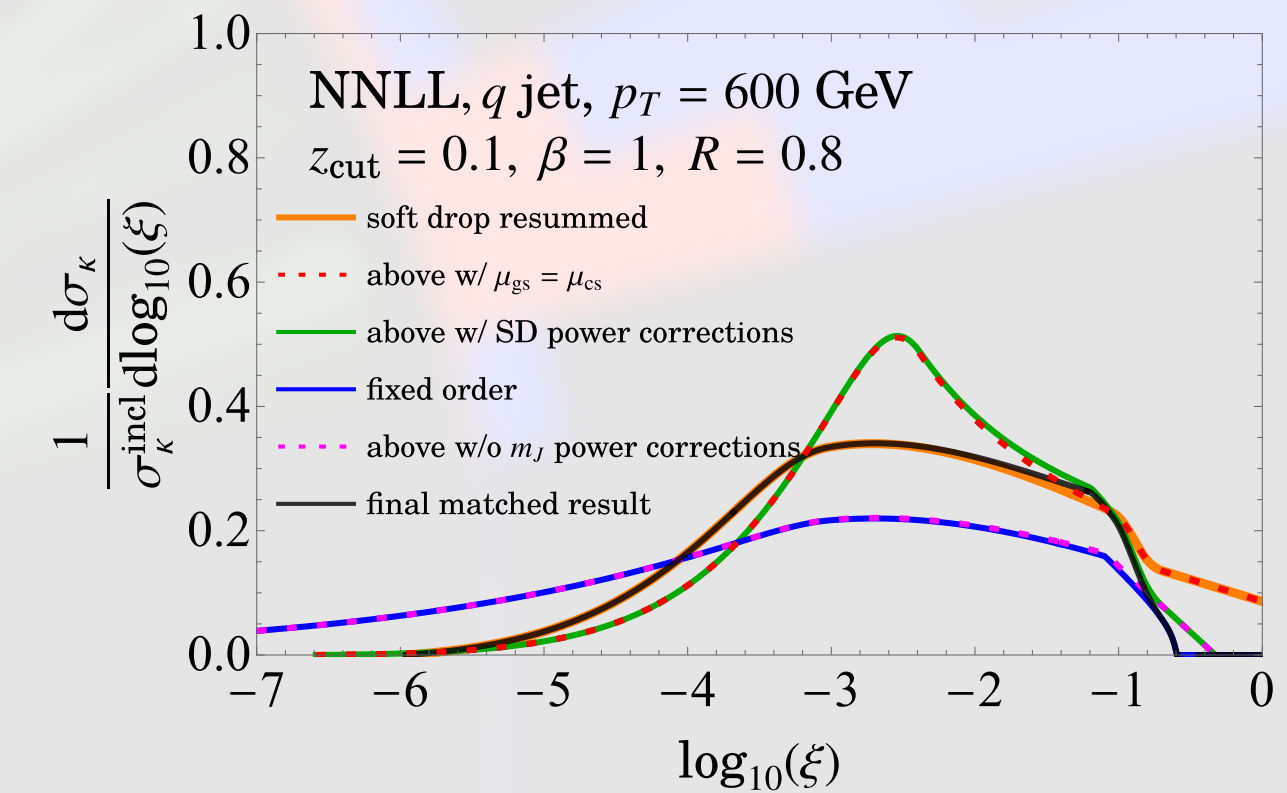
[Kang, Lee, Liu, Ringer 2018] [Larkoski, 2020] [Benkendorfer, Larkoski 2021]

- **Resummation and matching:** state-of-the-art NNLL resummation, fully analytical treatment of perturbative power corrections, included finite- $z_{\text{cut}}$  non-singular corrections (found negligible). 
- **Perturbative uncertainty:** Comprehensive variation of profile parameters that break a specific canonical-relation by randomly chosen points in a well-studied variation range. 
- **Transition into ungroomed region:** Consistent analytical matching to ungroomed region at NNLL at the soft-wide angle transition point. 
- **Nonperturbative effects:** Incorporated using model-independent field theory based formalism to assess impact on precision of  $\alpha_s$ -determination. 
- **Resummation at soft drop cusp:** 20% shift in the cusp location suggested by [Benkendorfer, Larkoski 2021] (too large?) is only a 5% modification to the range we used for our  $\alpha_s$ -sensitivity analysis.

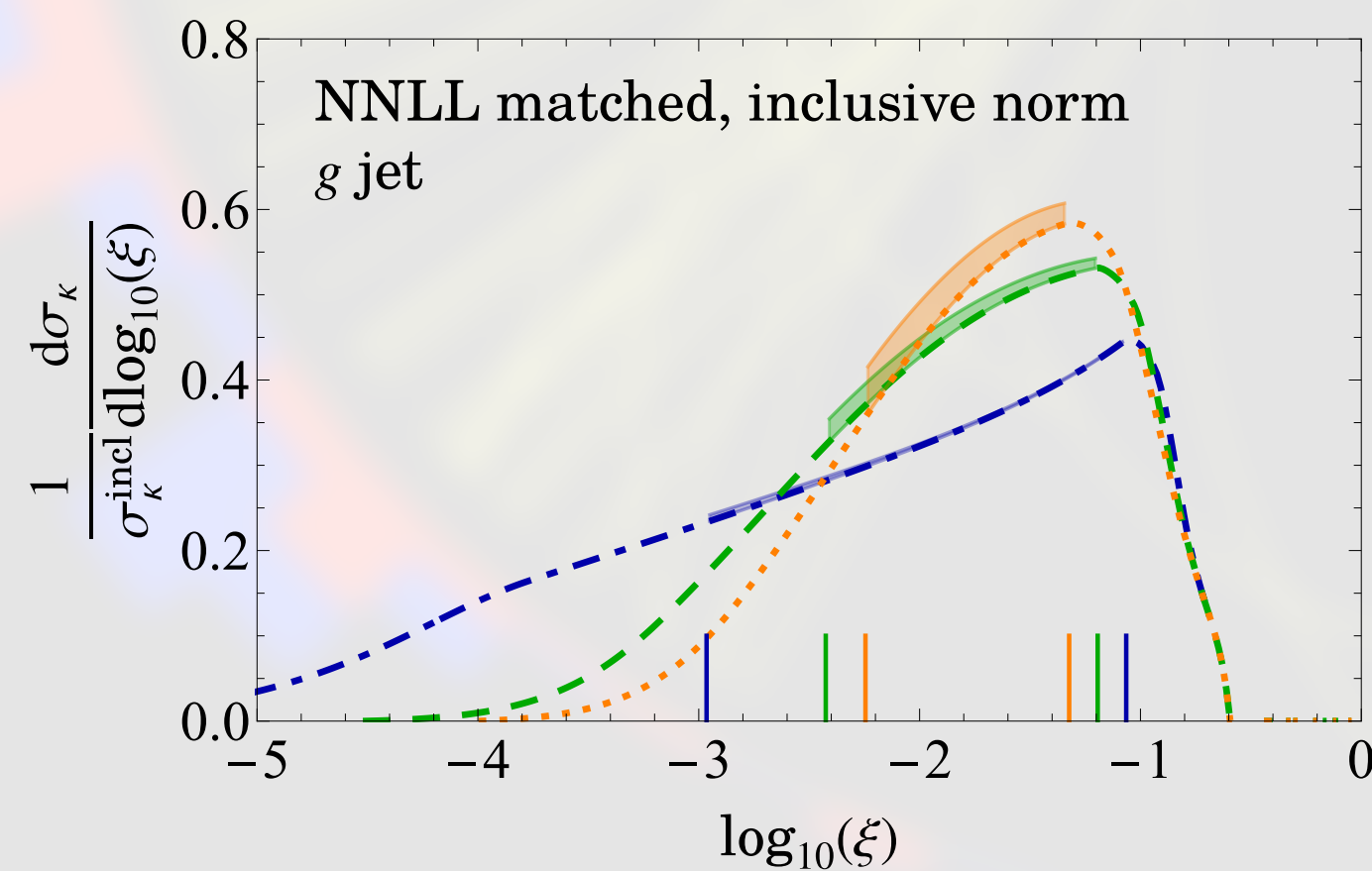
# Outline

## 1. Quark-gluon fraction and PDF dependence

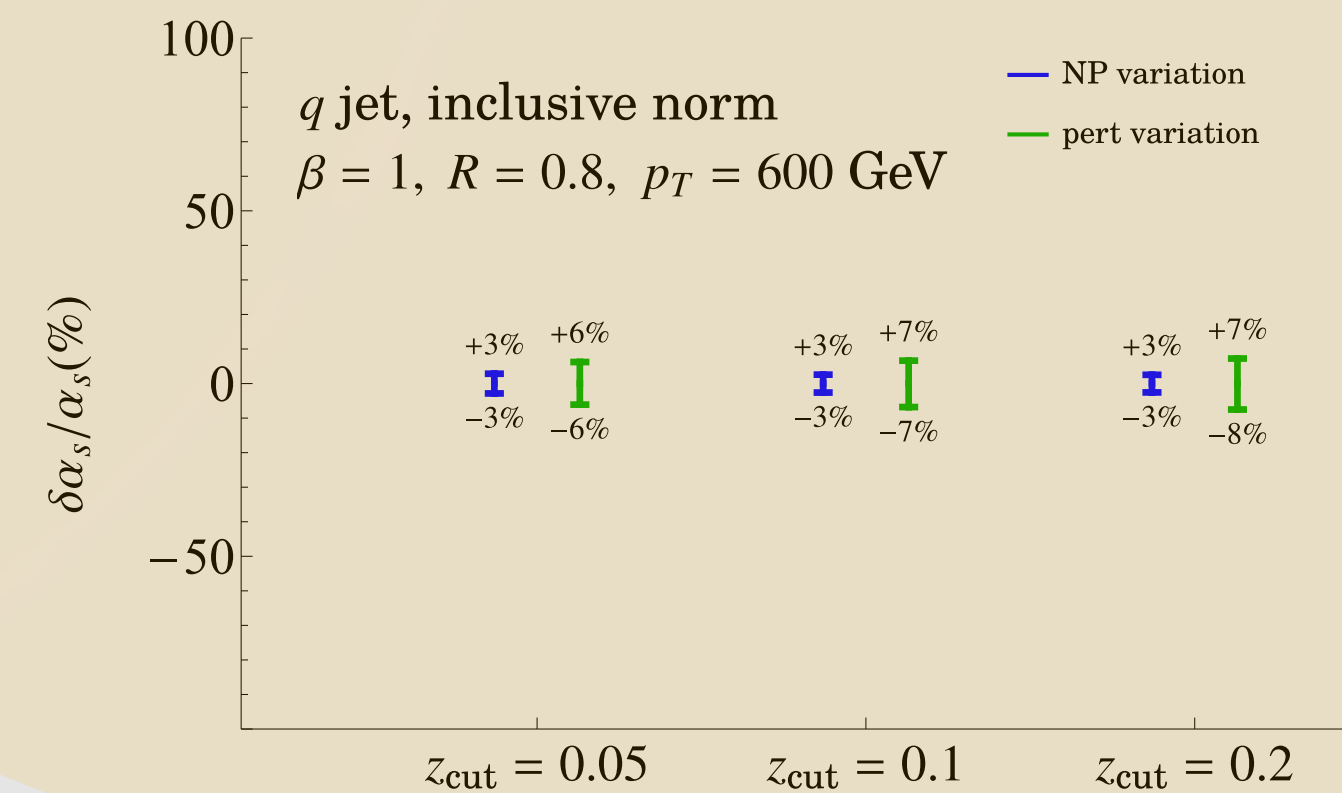
## 2. NNLL resummed cross section



## 3. Hadronization effects



## 4. Results





# Dependence on slope

Coefficient independent of  $\alpha_s, \xi$

According to **leading-logarithmic** estimate:

$$\frac{d\sigma_{\text{resum}}^k}{d \log_{10}(\xi)} \propto \exp \left[ -\alpha_s(\mu) a_k \log_{10}(\xi) \right] \approx 1 - \alpha_s(\mu) a_k \log_{10}(\xi)$$

Slope depends linearly on  $\alpha_s$

[Les Houches 2017]

# Two ways to normalize

- Normalize to inclusive cross section in the  $p_T$ - $\eta$  bin:  $\frac{1}{\sigma_{\text{incl}}} \frac{d^3\sigma}{dp_T d\eta d\xi}$ ,

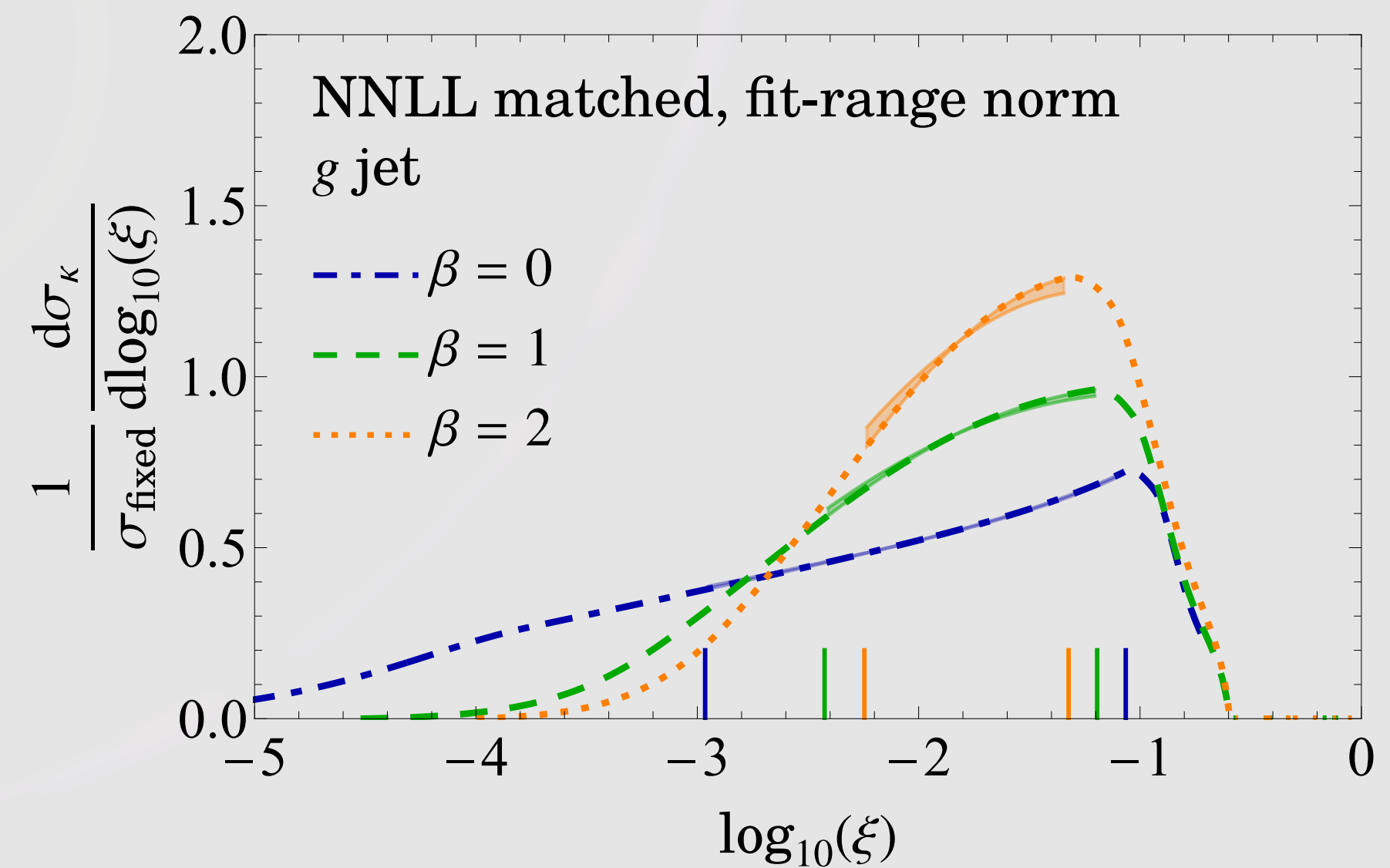
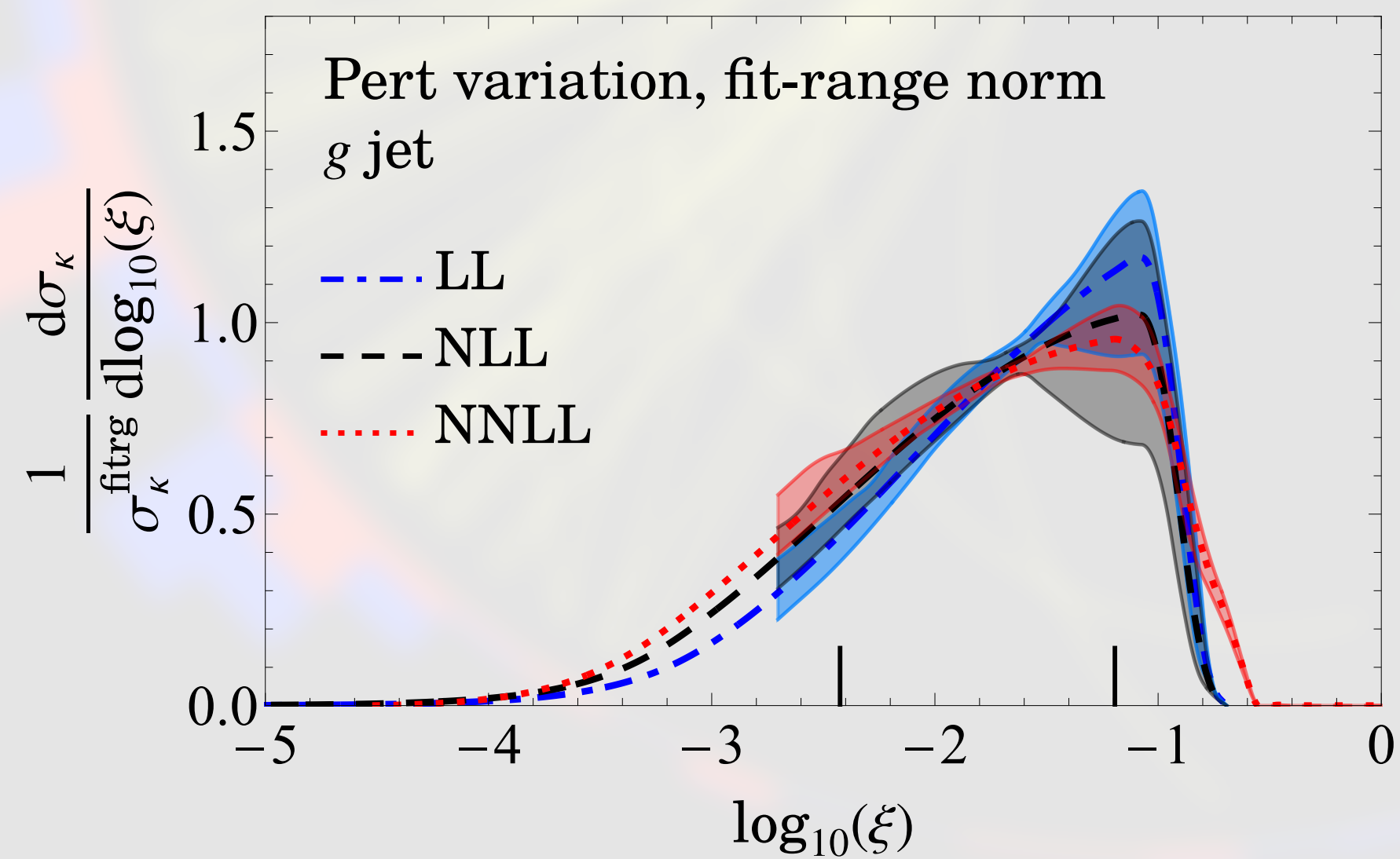
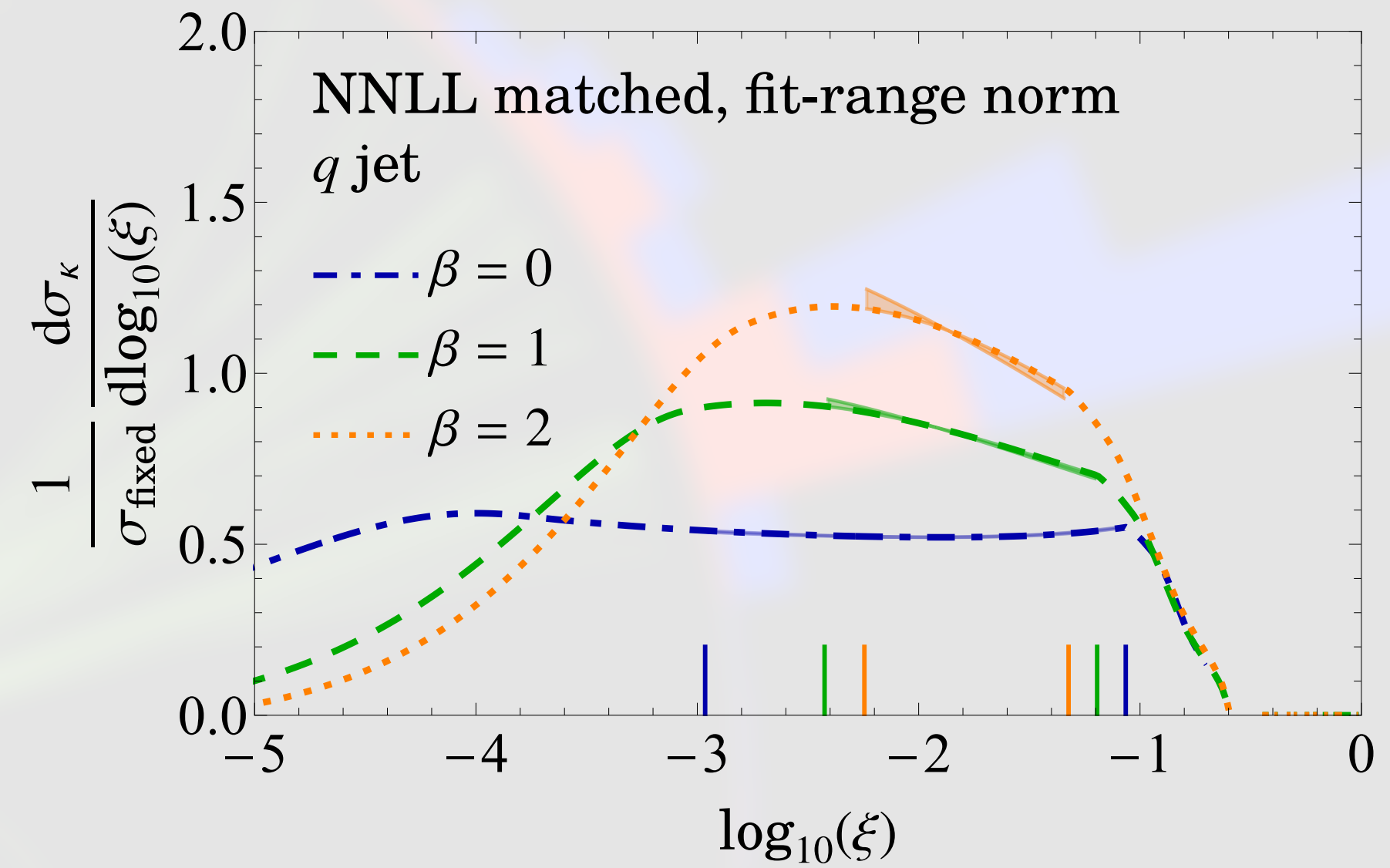
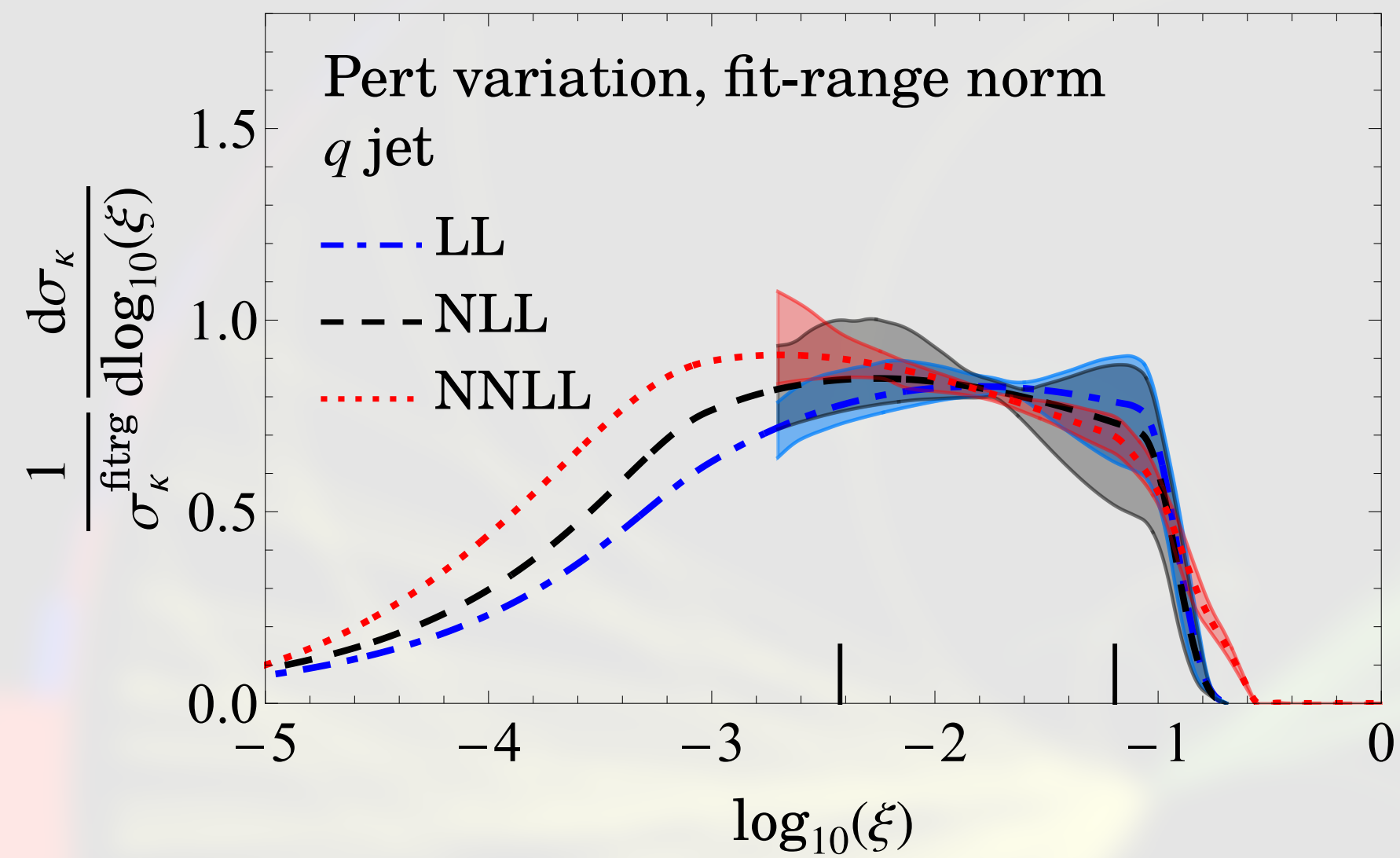
Proposed in [Kang, Lee, Liu, Ringer 2018]

- Normalize to cross section in range:  $\frac{1}{\sigma_{\text{fitrg}}} \frac{d^3\sigma}{dp_T d\eta d\xi}$

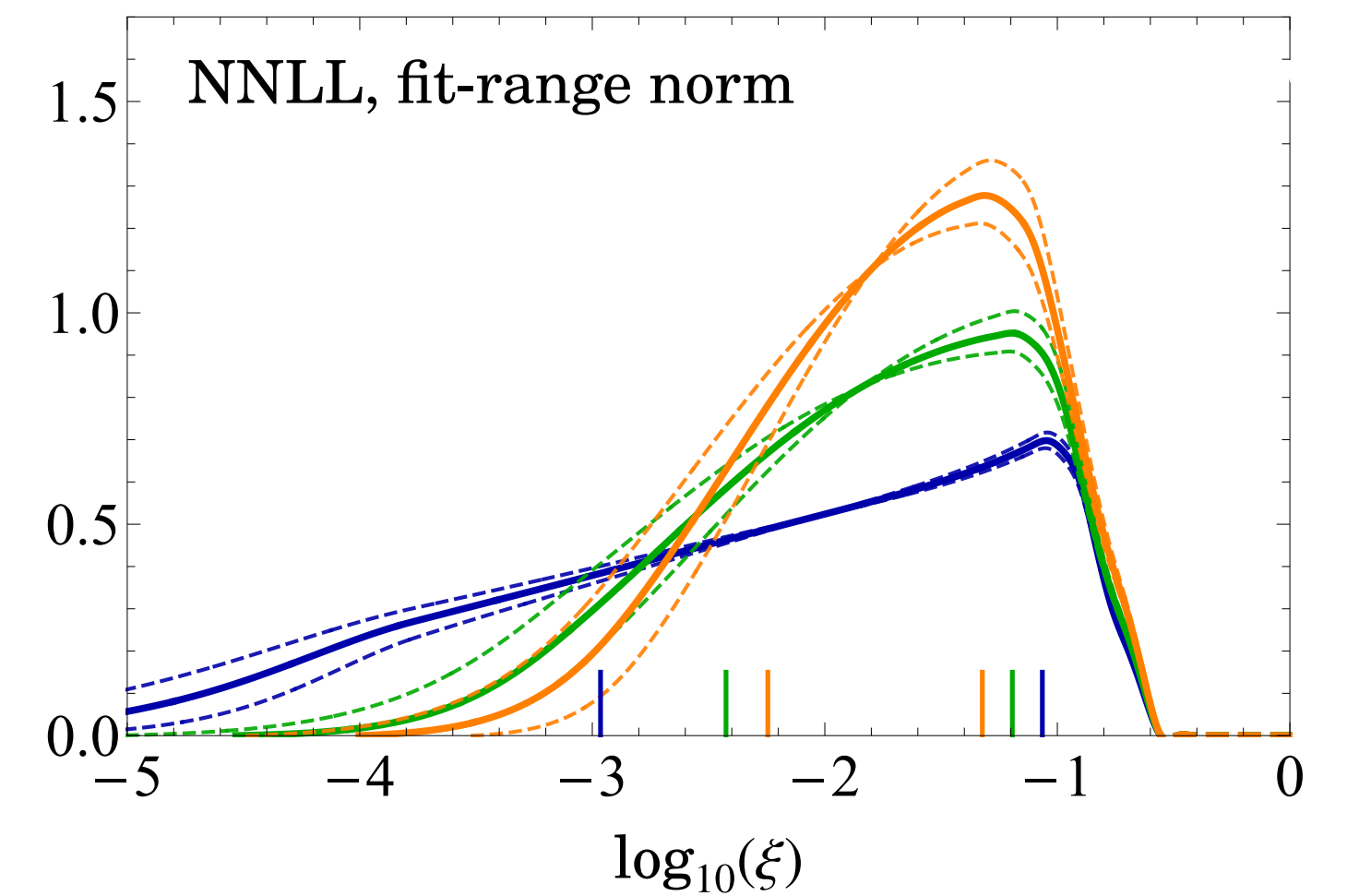
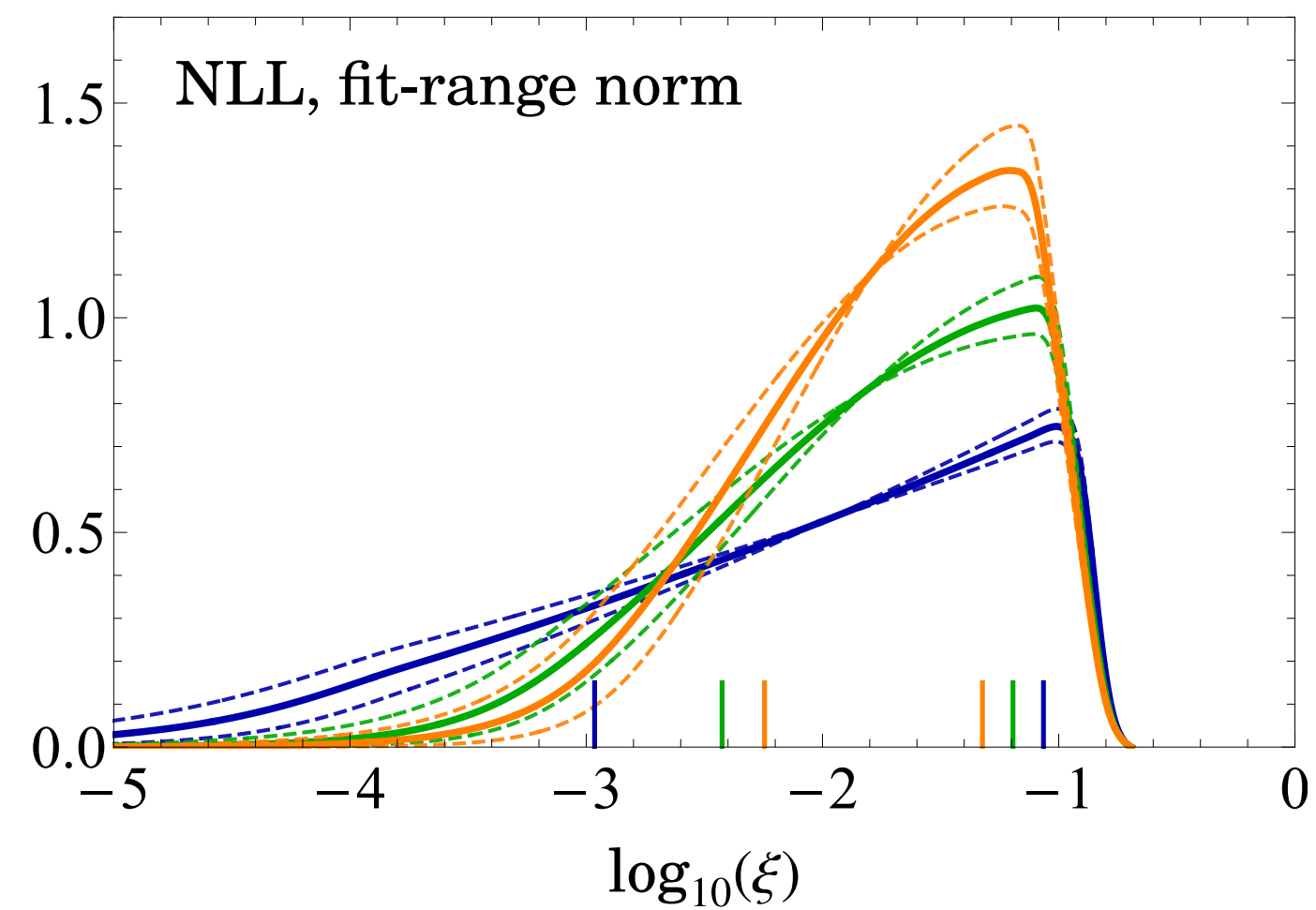
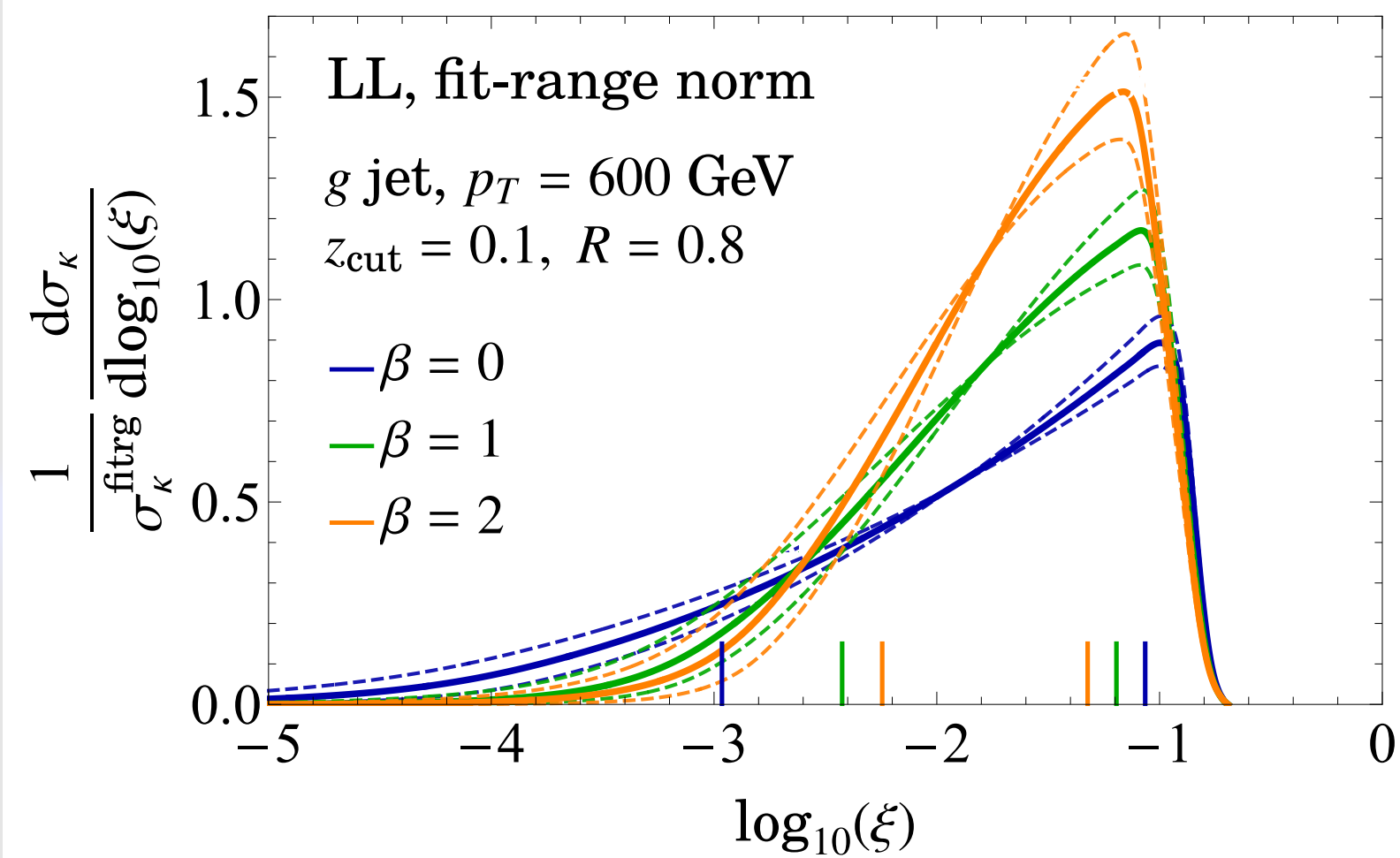
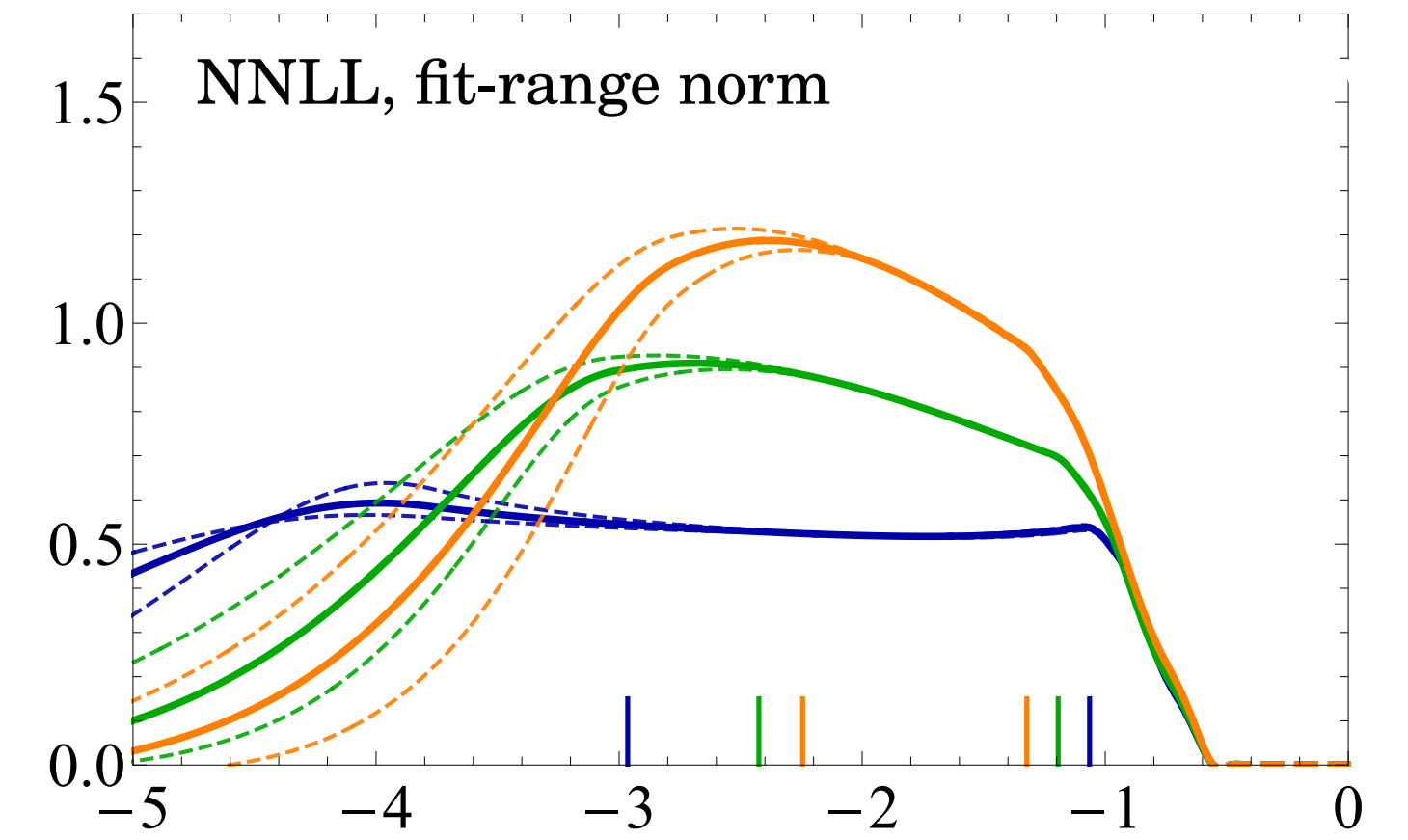
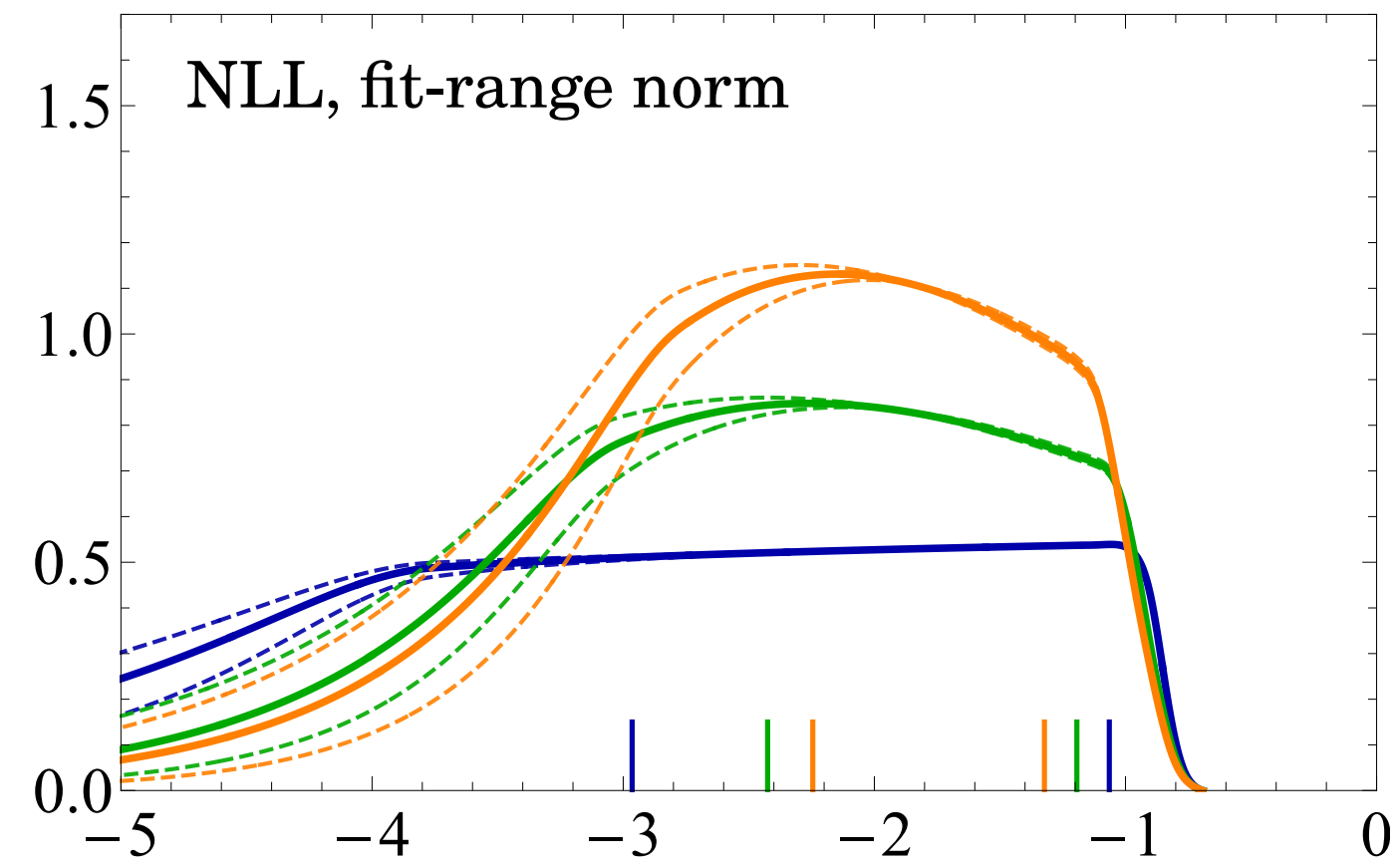
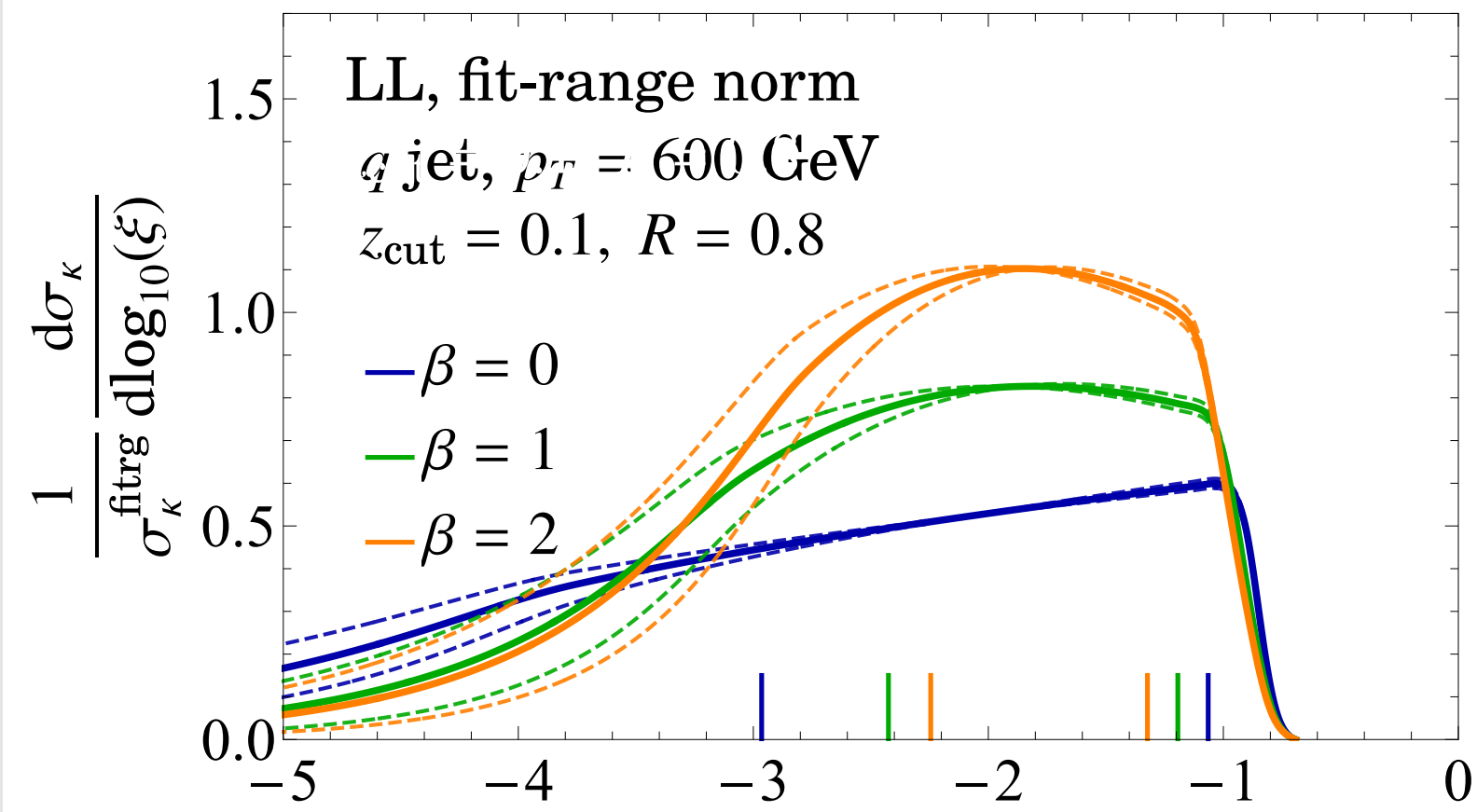
Pursued by [ATLAS 1711.08341, 1912.09837]

$$\sigma_{\text{incl}} = \frac{d^2\sigma}{dp_T d\eta}, \quad \sigma_{\text{fitrg}} = \int_{\xi_{\text{SDOE}}}^{\xi'_0} d\xi \frac{d^3\sigma}{dp_T d\eta d\xi}$$

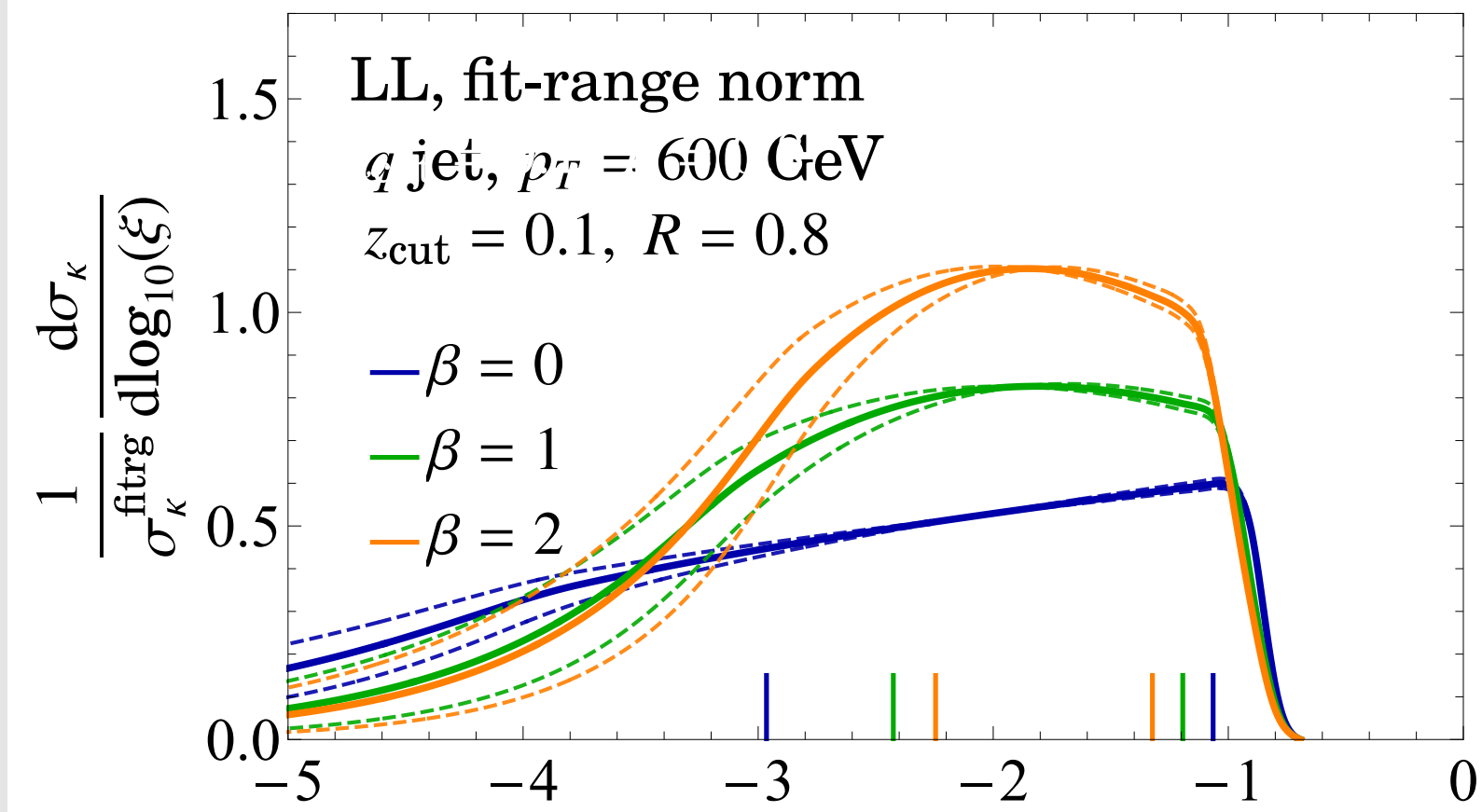
# Uncertainties in fit-range normalized spectrum



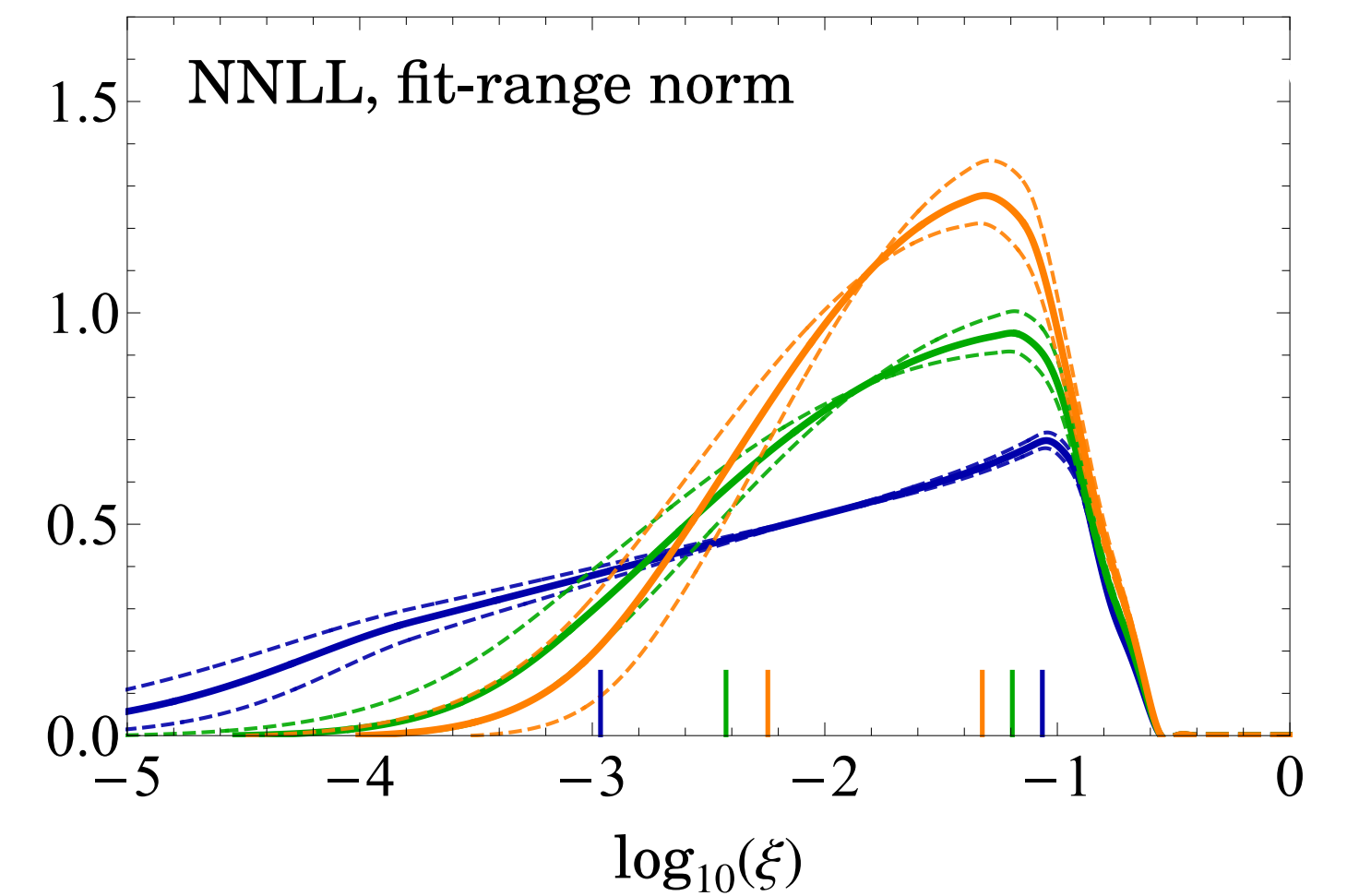
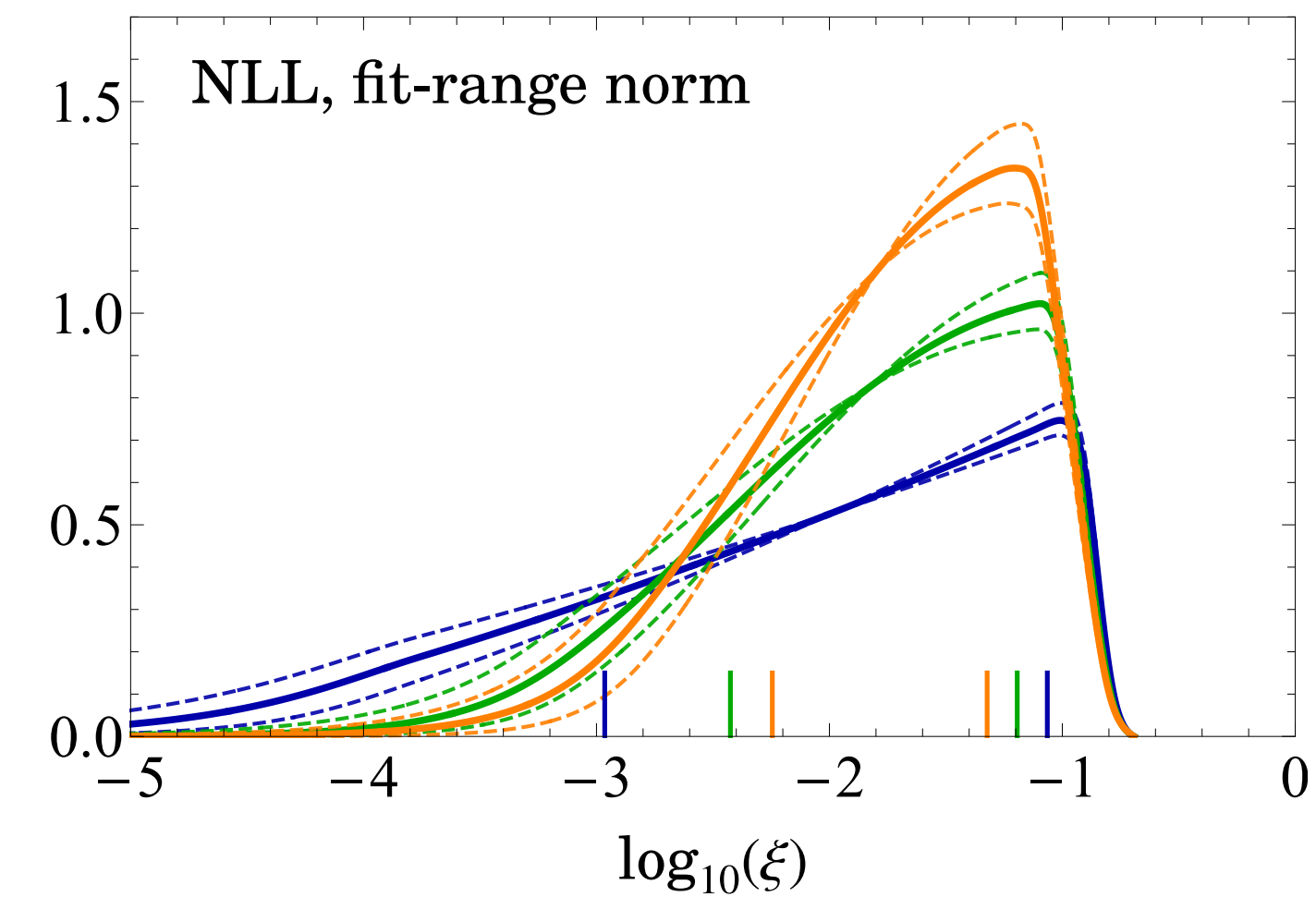
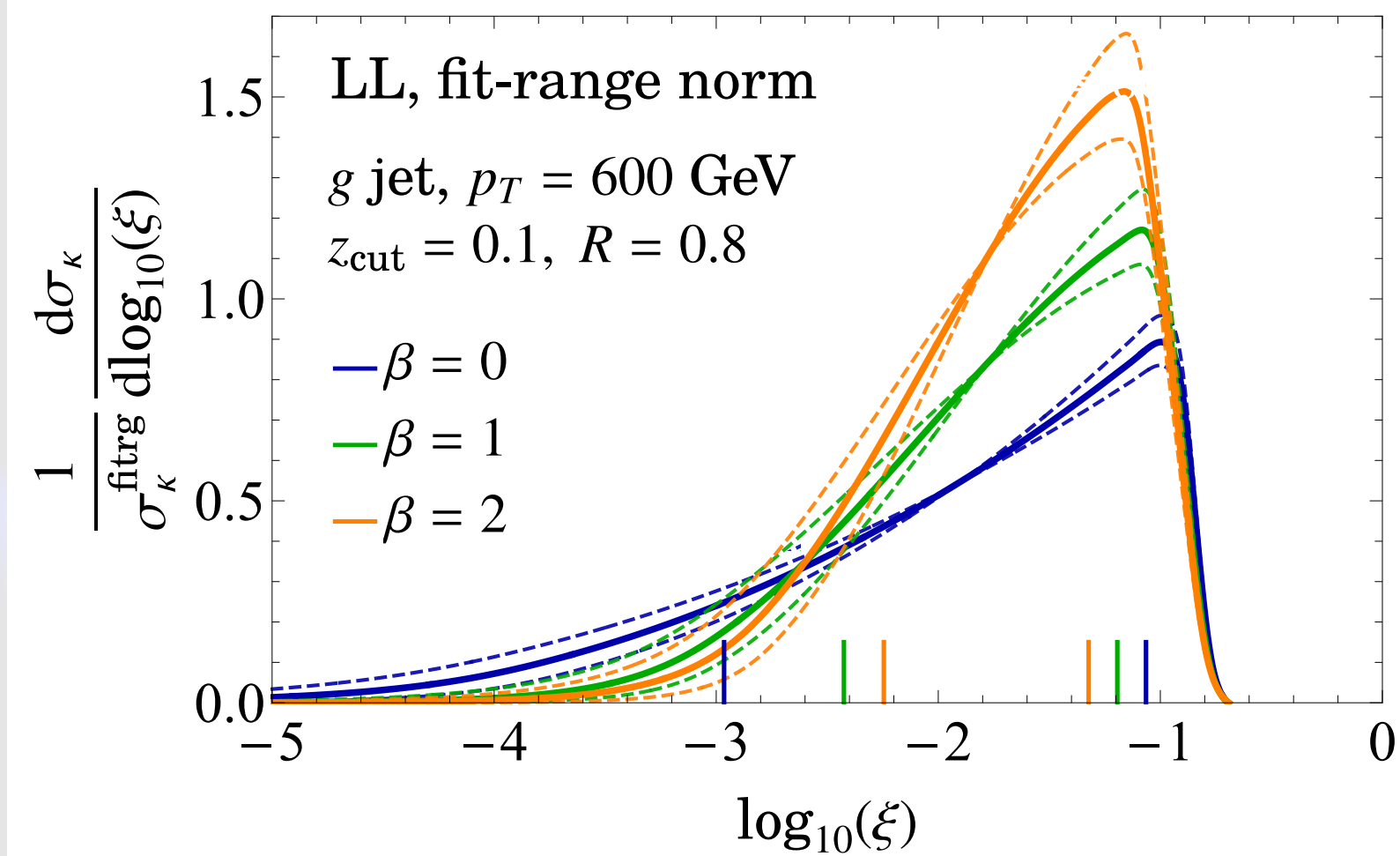
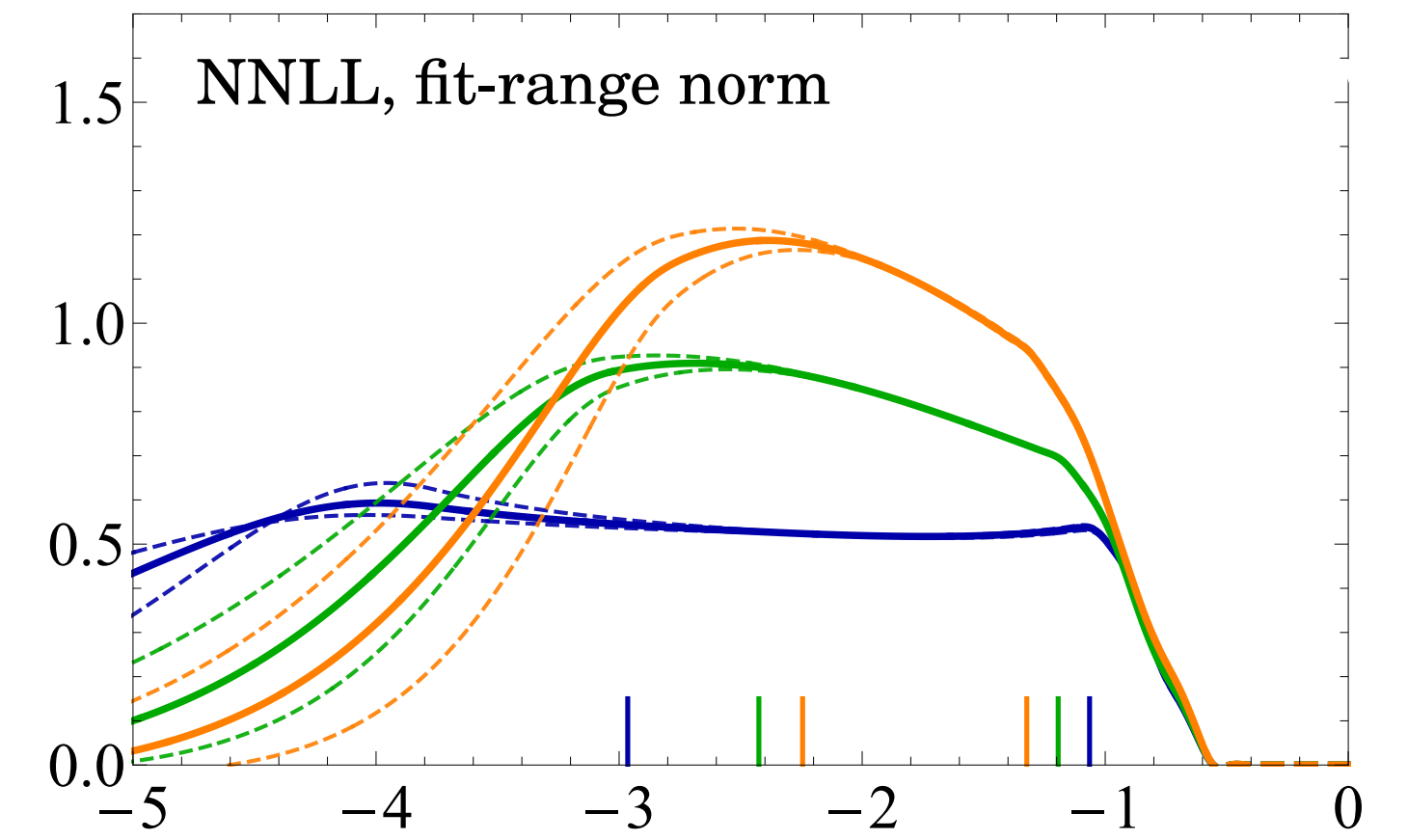
# Effects of higher order resummation on slope



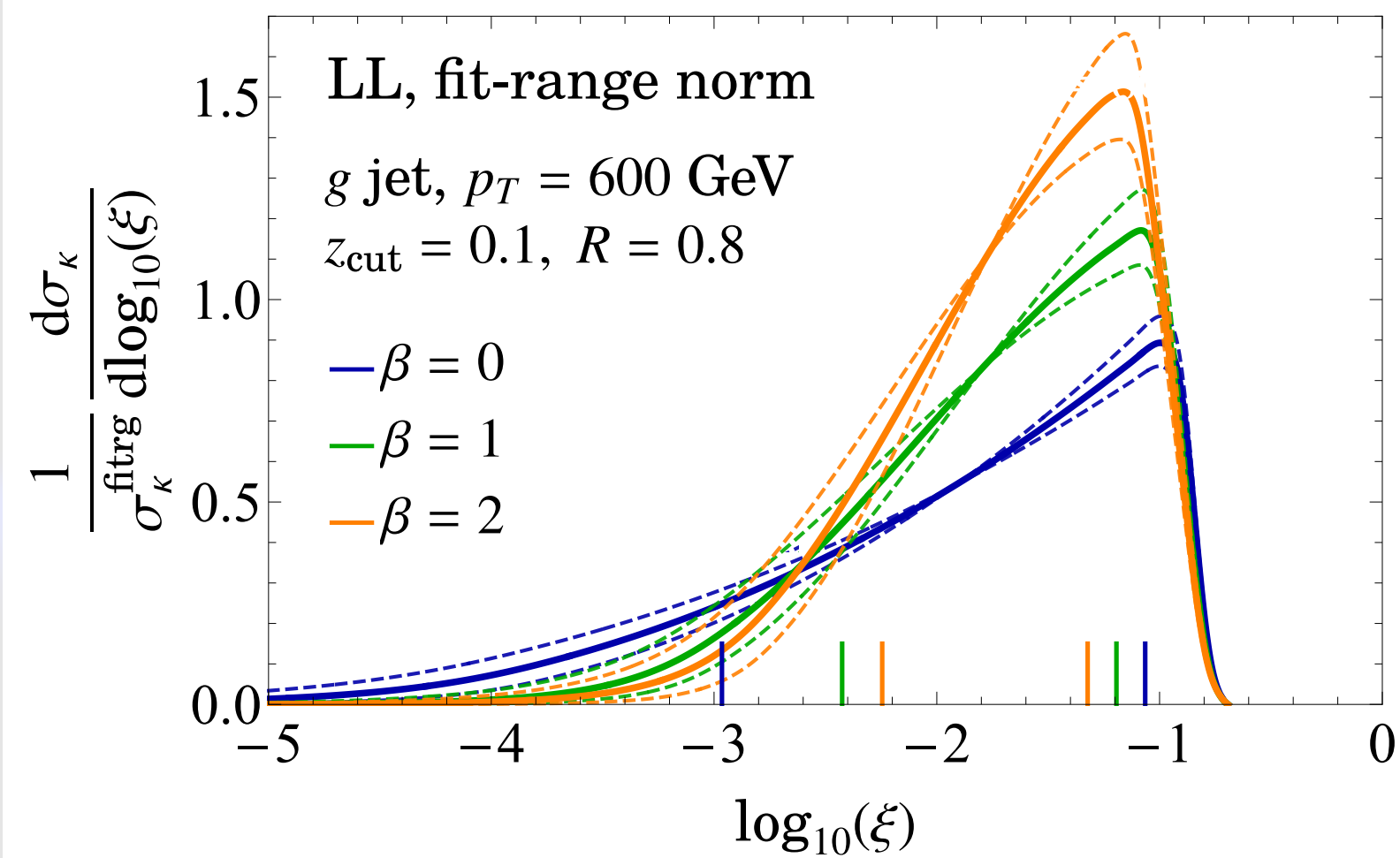
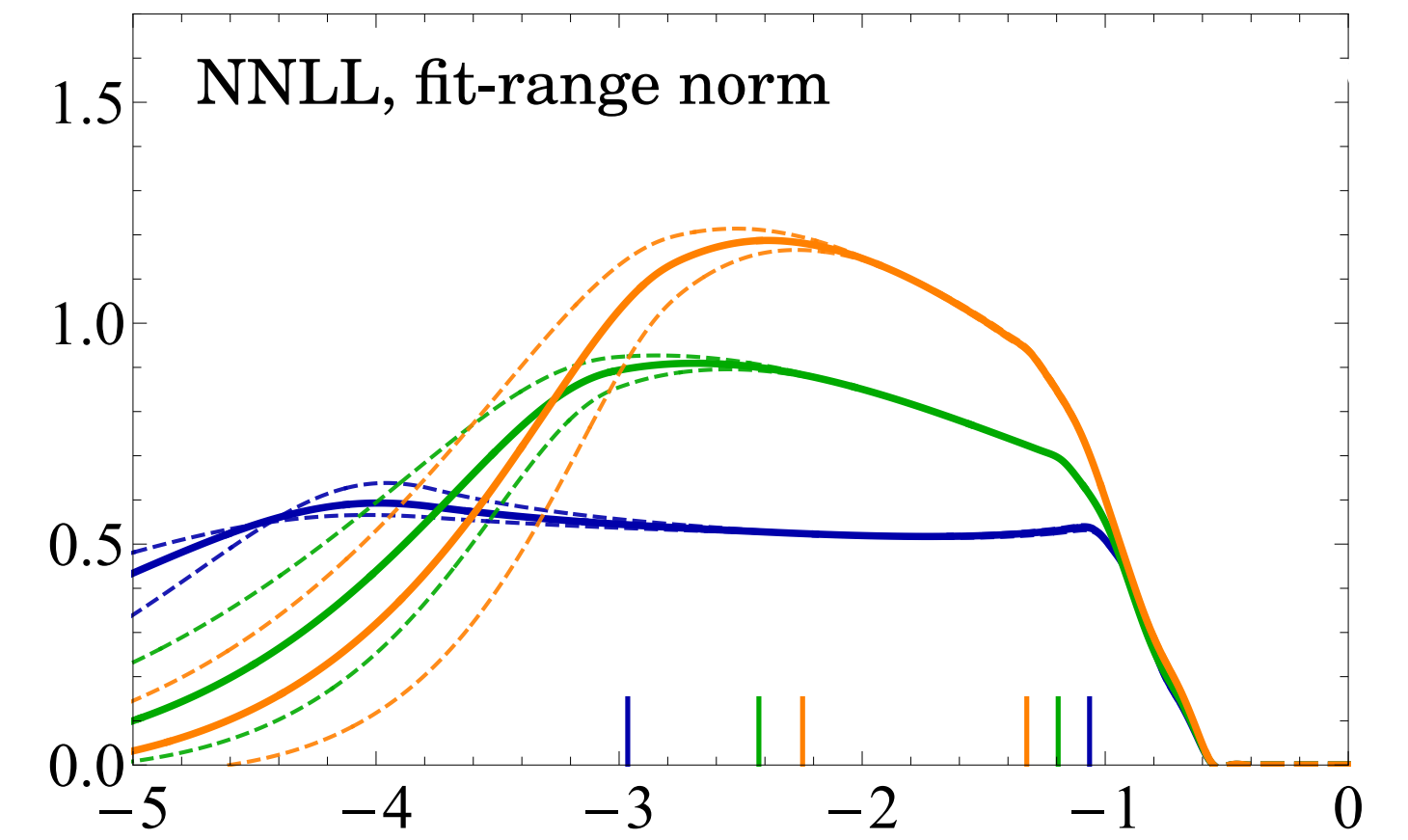
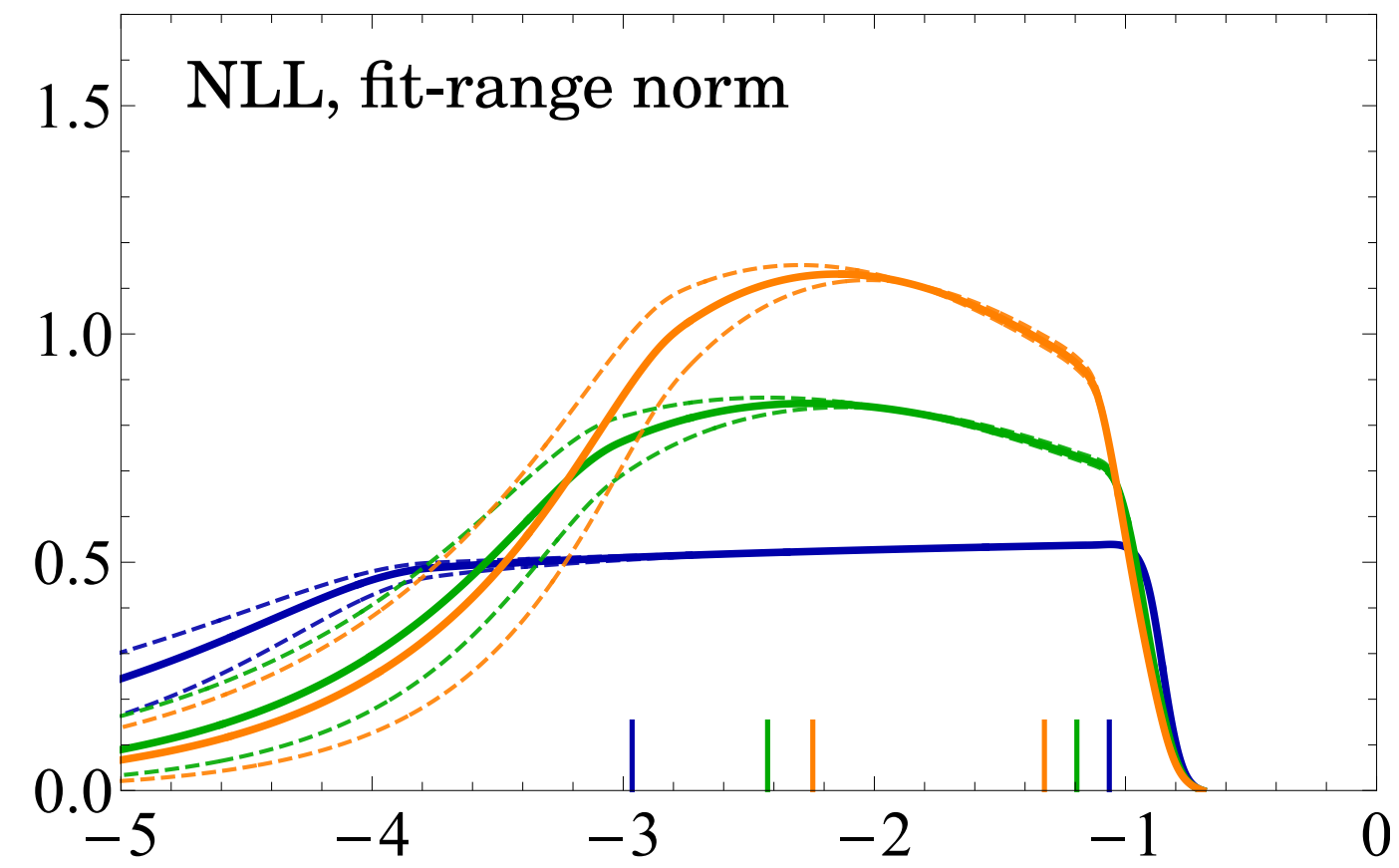
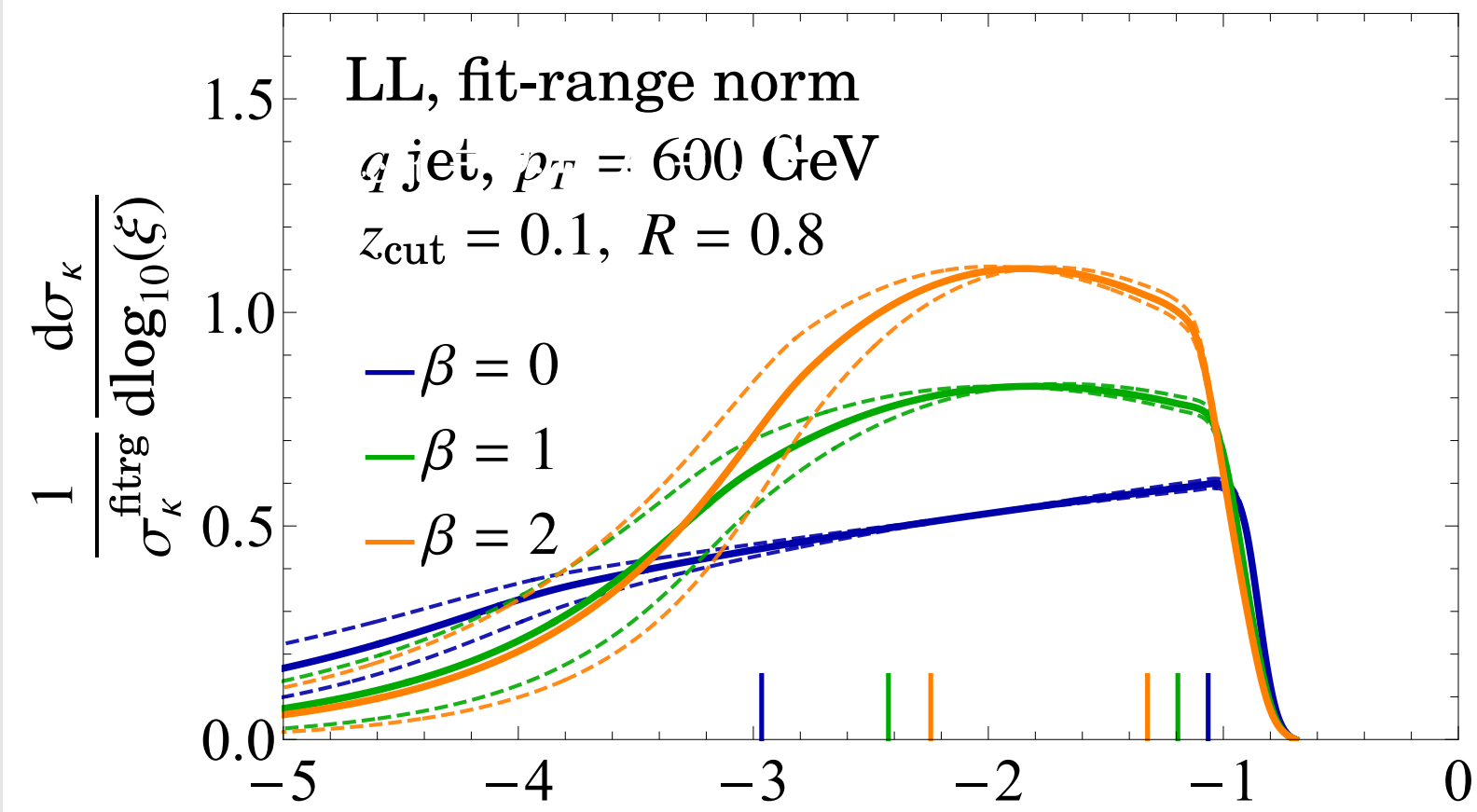
# Effects of higher order resummation on slope



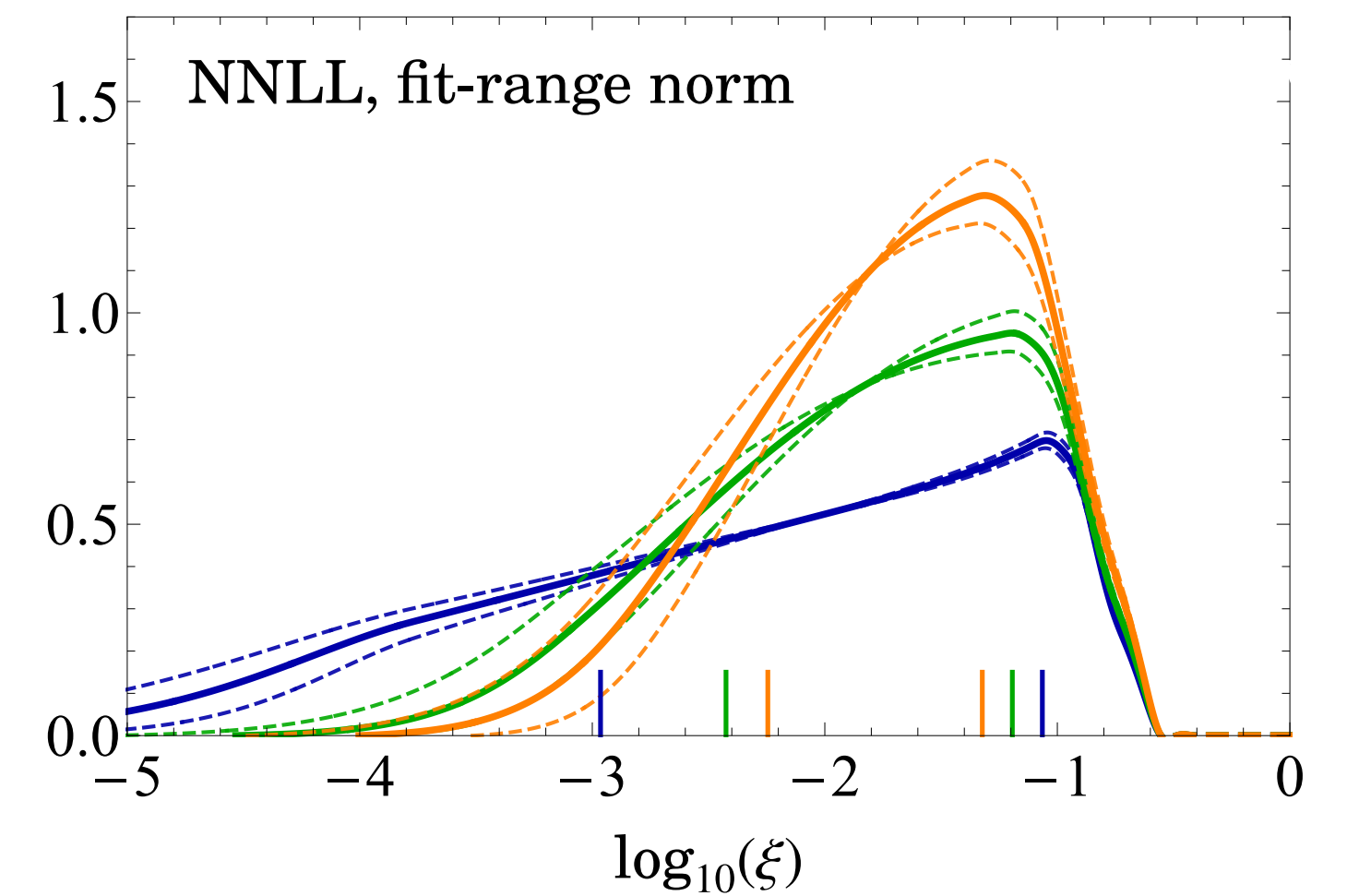
Quark jets: slope flattens



# Effects of higher order resummation on slope



Gluon jets: retain shape

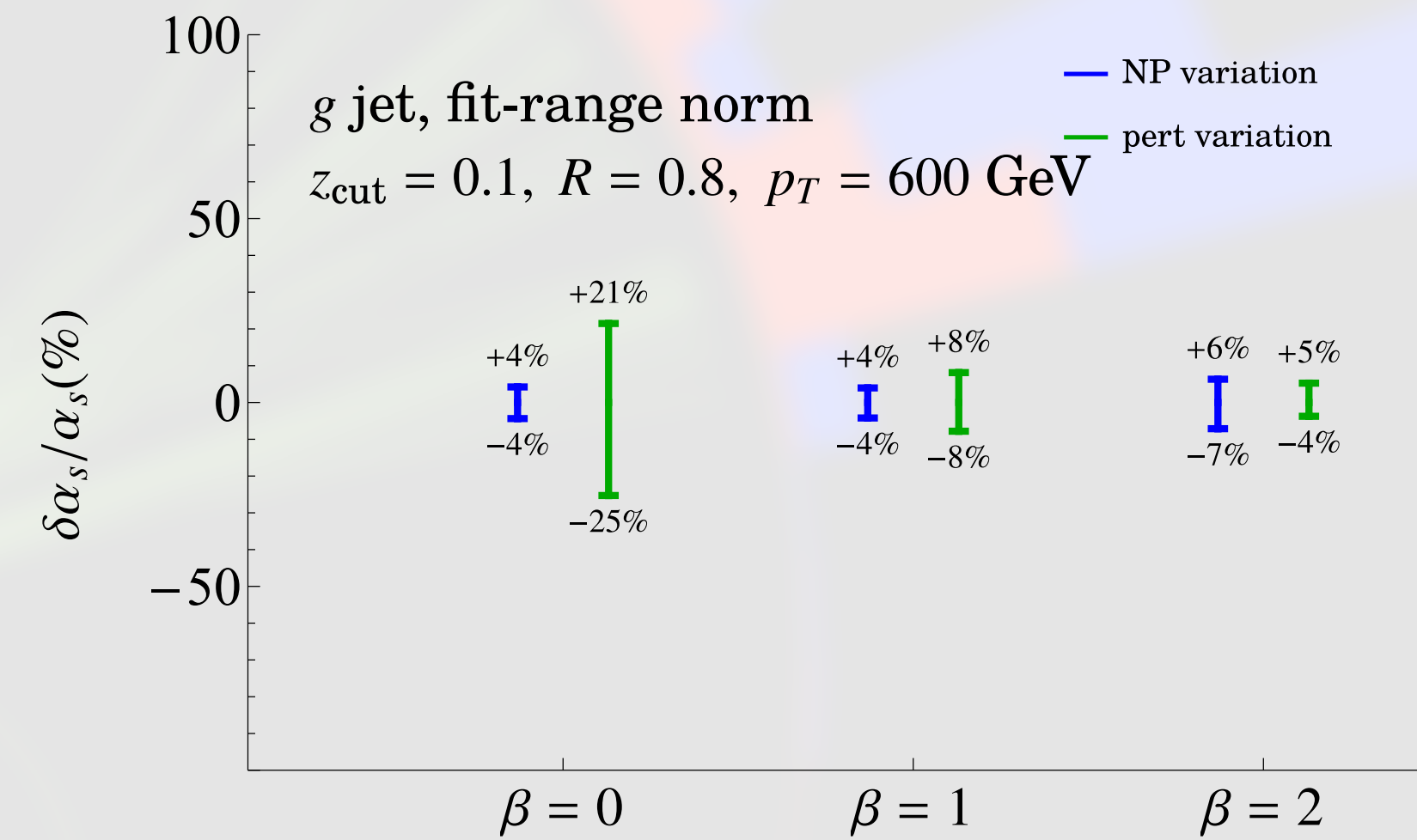
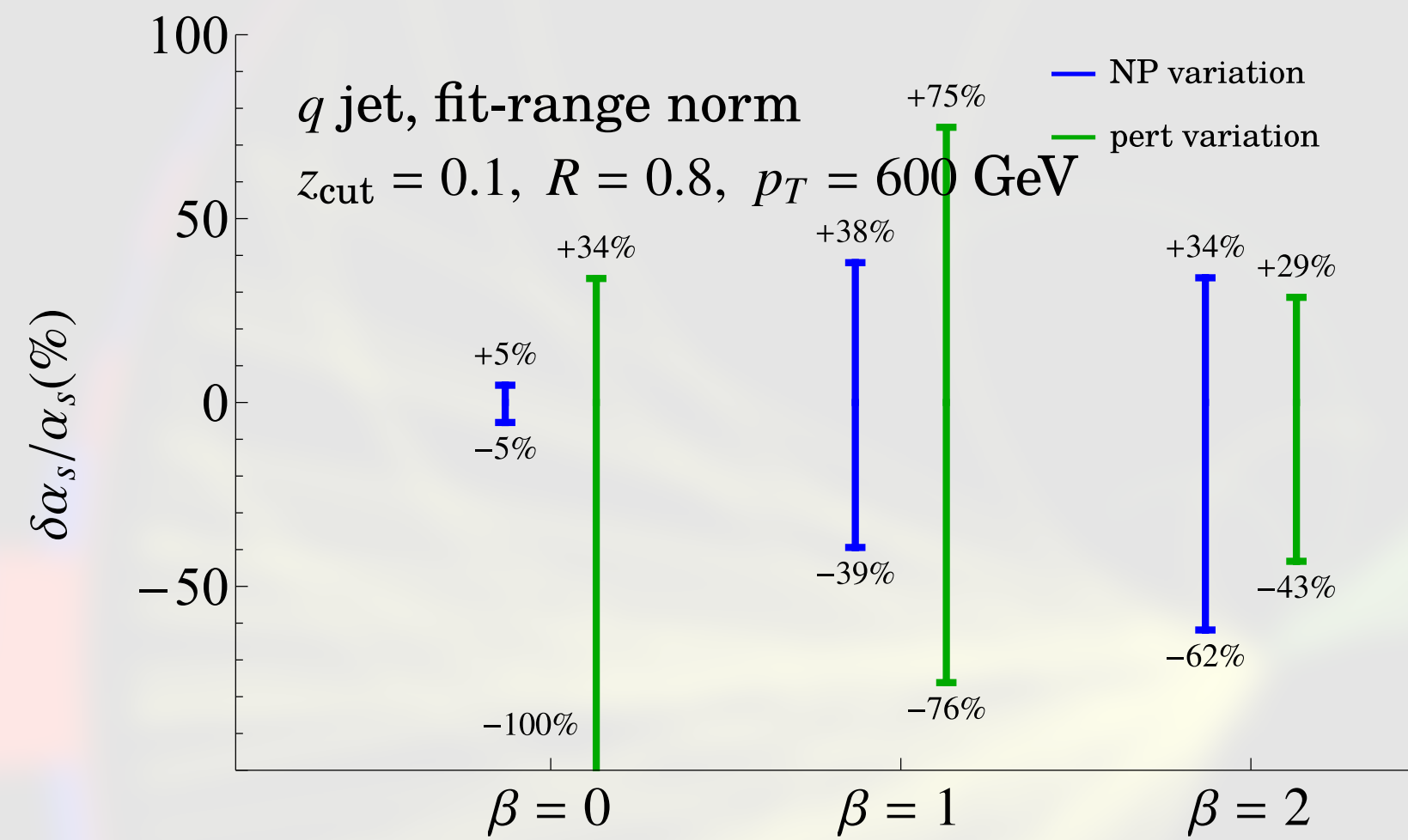


# Results for fit-range normalization

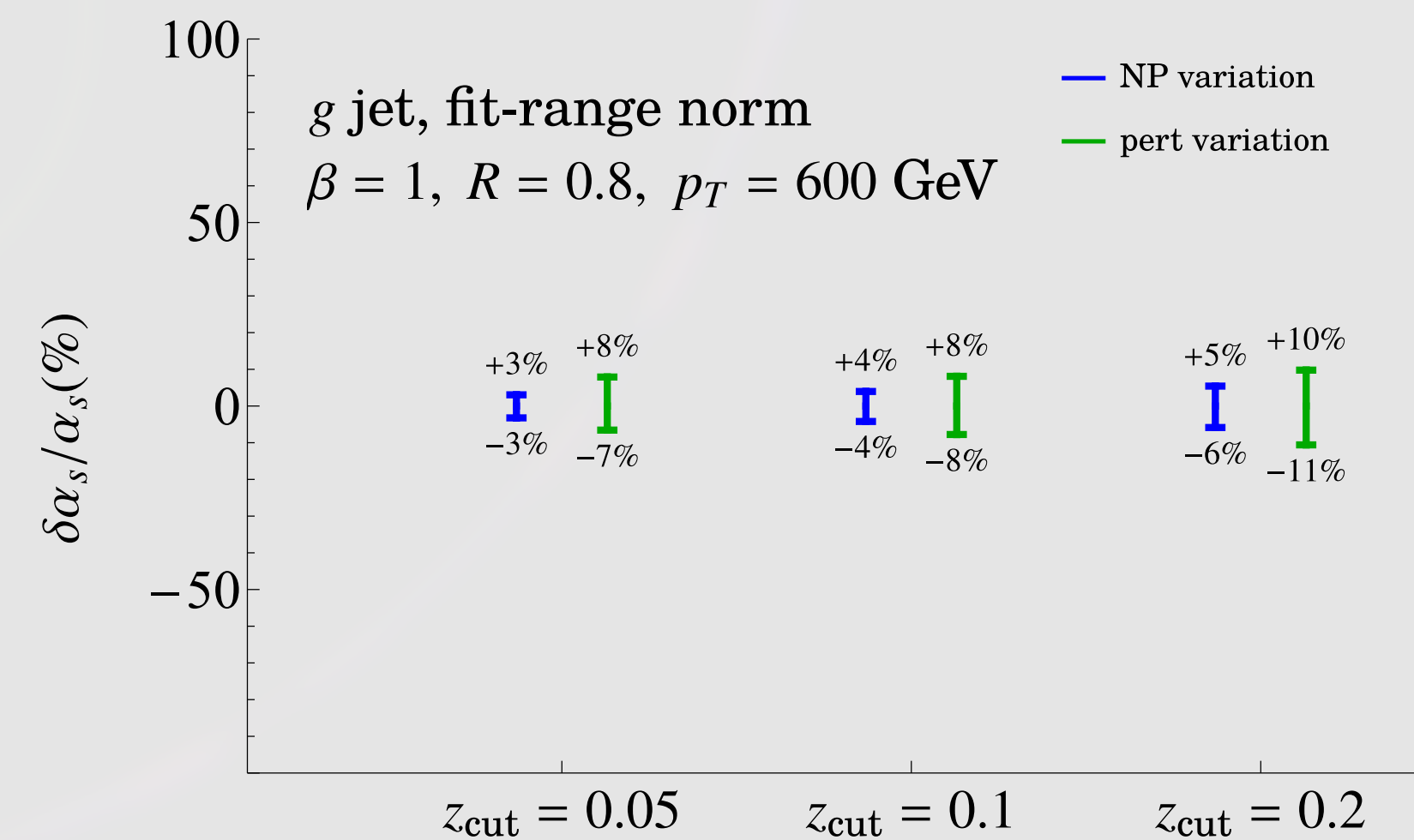
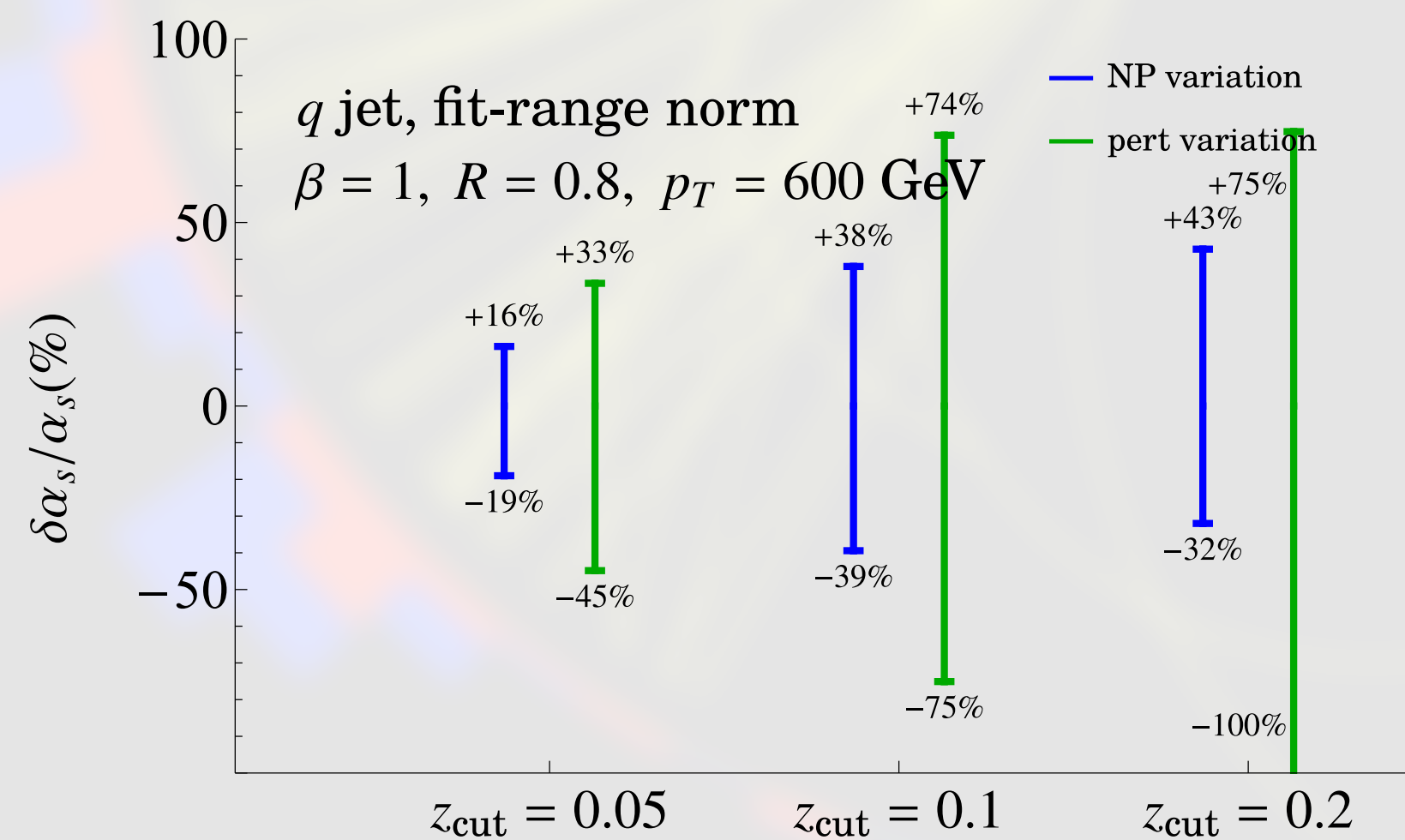
Quark jets:

Gluon jets:

Vary  $\beta$ :



Vary  $z_{\text{cut}}$ :

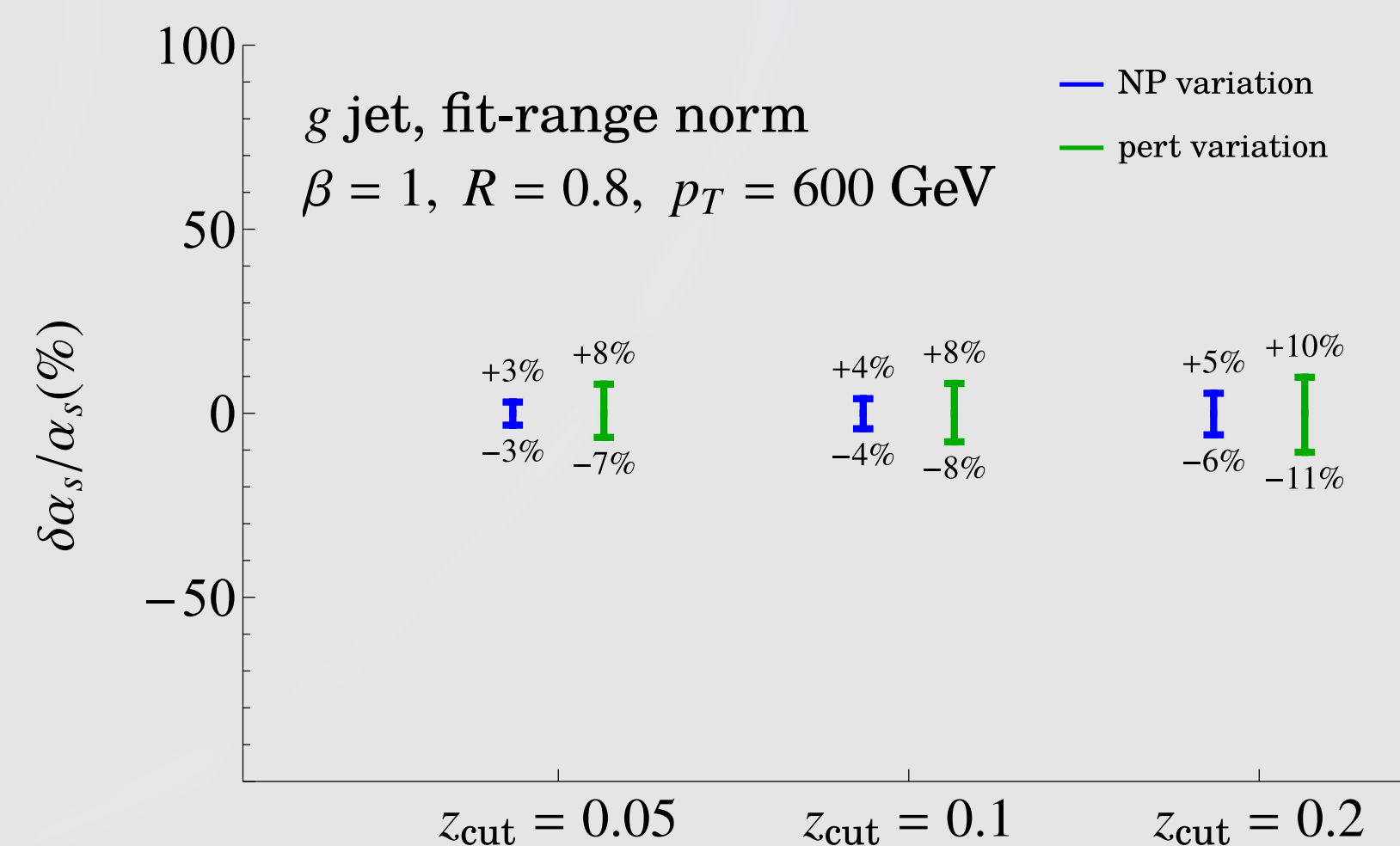
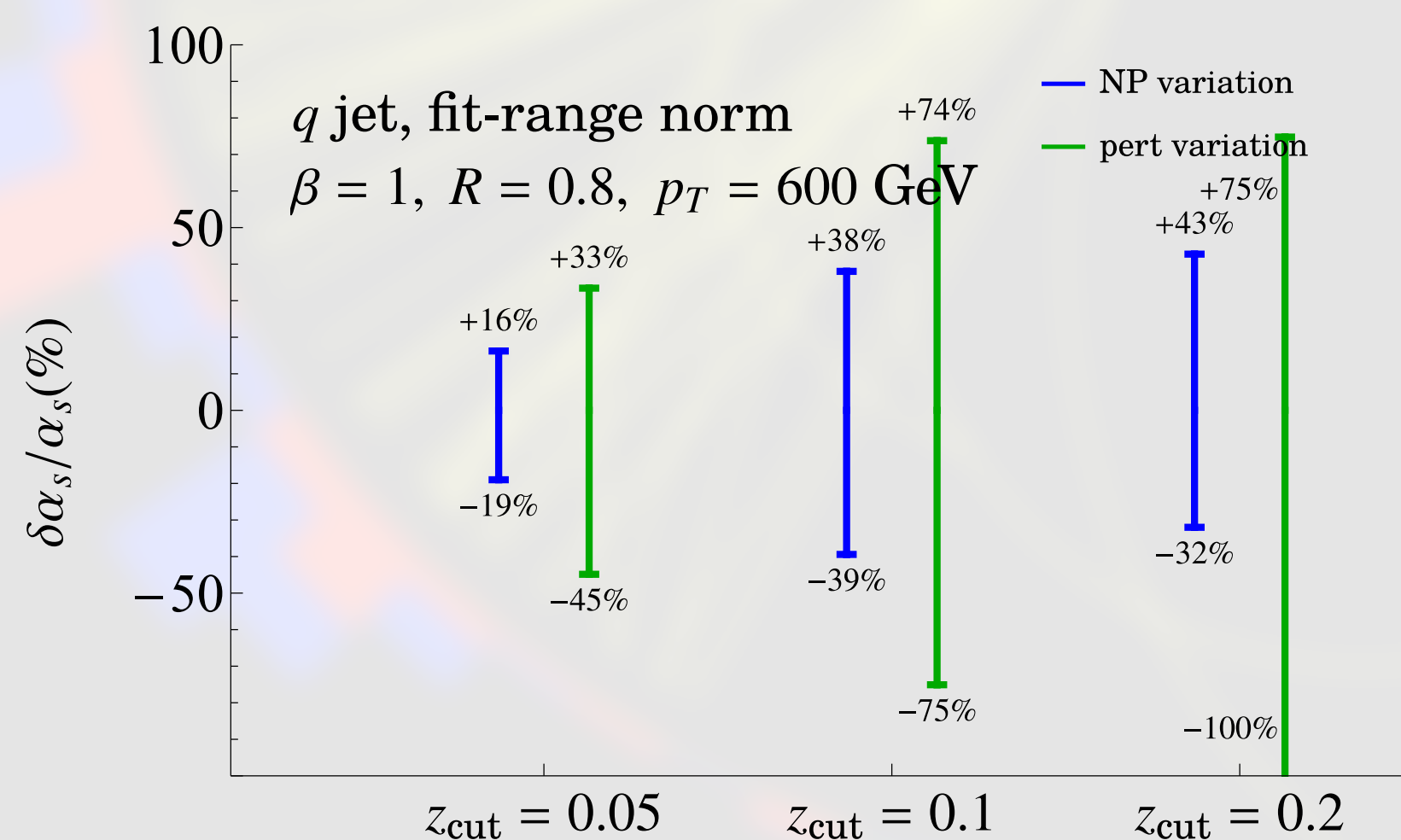
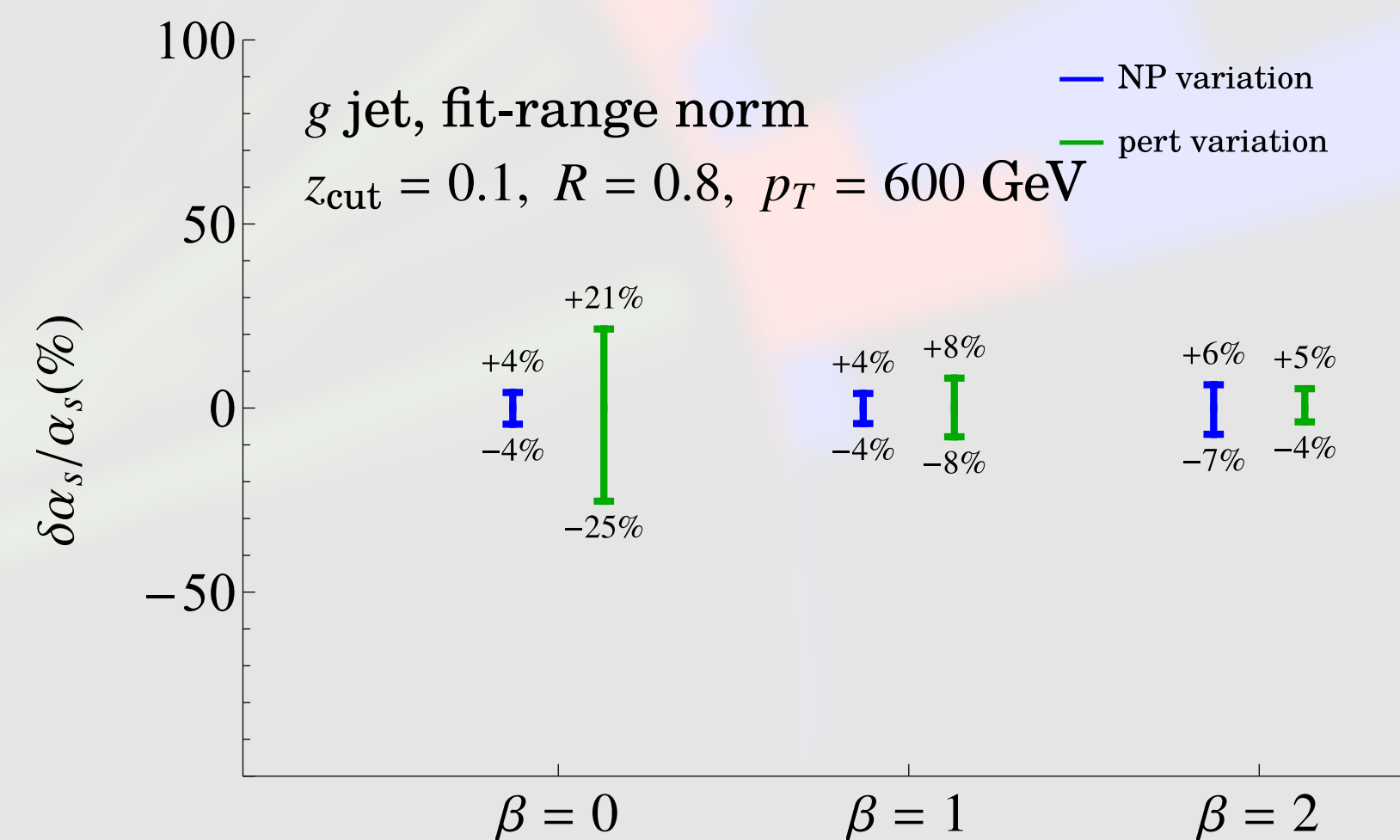
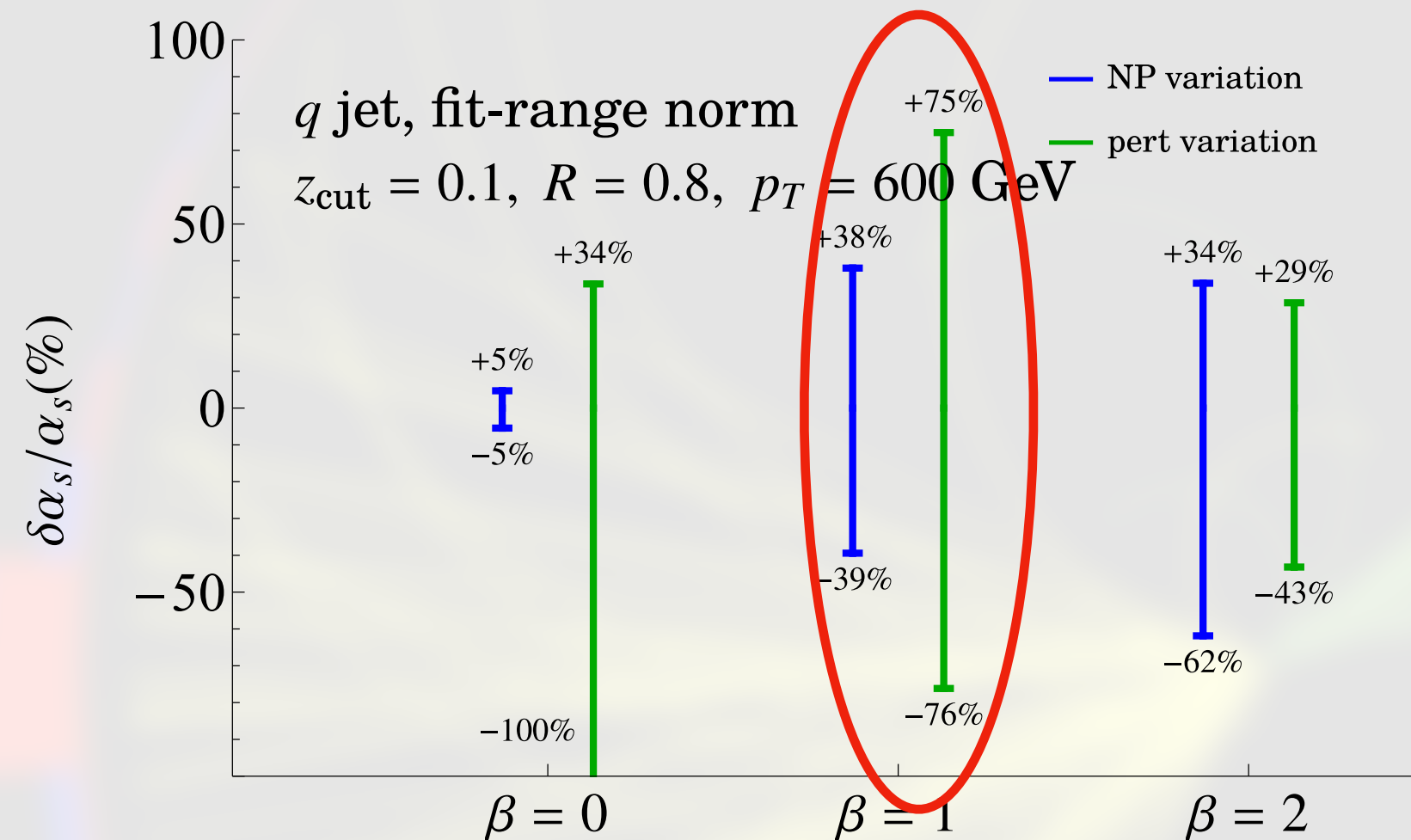


# Results for fixed-range normalization

Quark jets:

Gluon jets:

Fixed-range normalization eliminates almost all  $\alpha_s$  sensitivity of quark jets





## Results for inclusive normalization

Normalize to inclusive cross section in the  $p_T$ - $\eta$  bin:  $\frac{1}{\sigma_{\text{incl}}} \frac{d^3\sigma}{dp_T d\eta d\xi}$

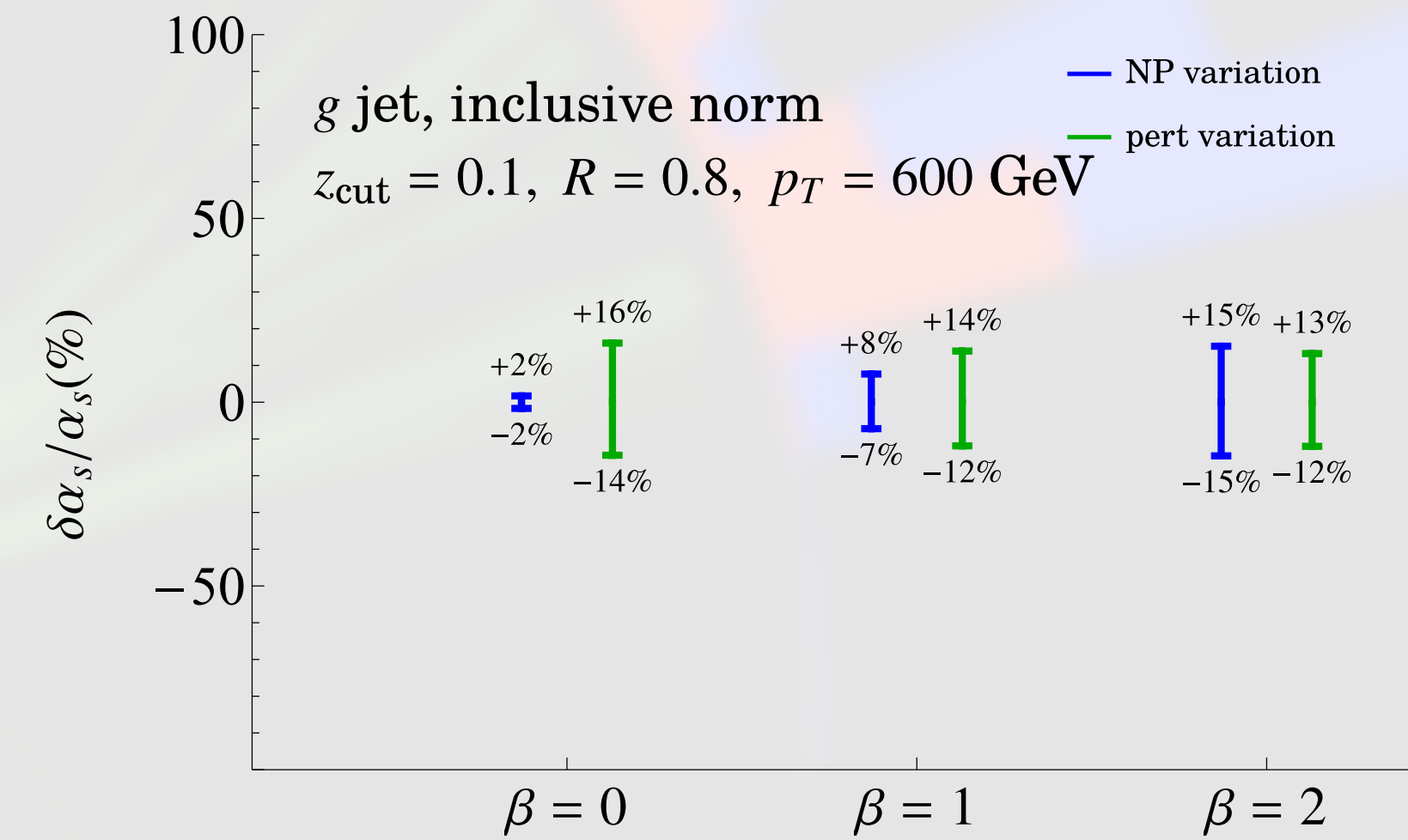
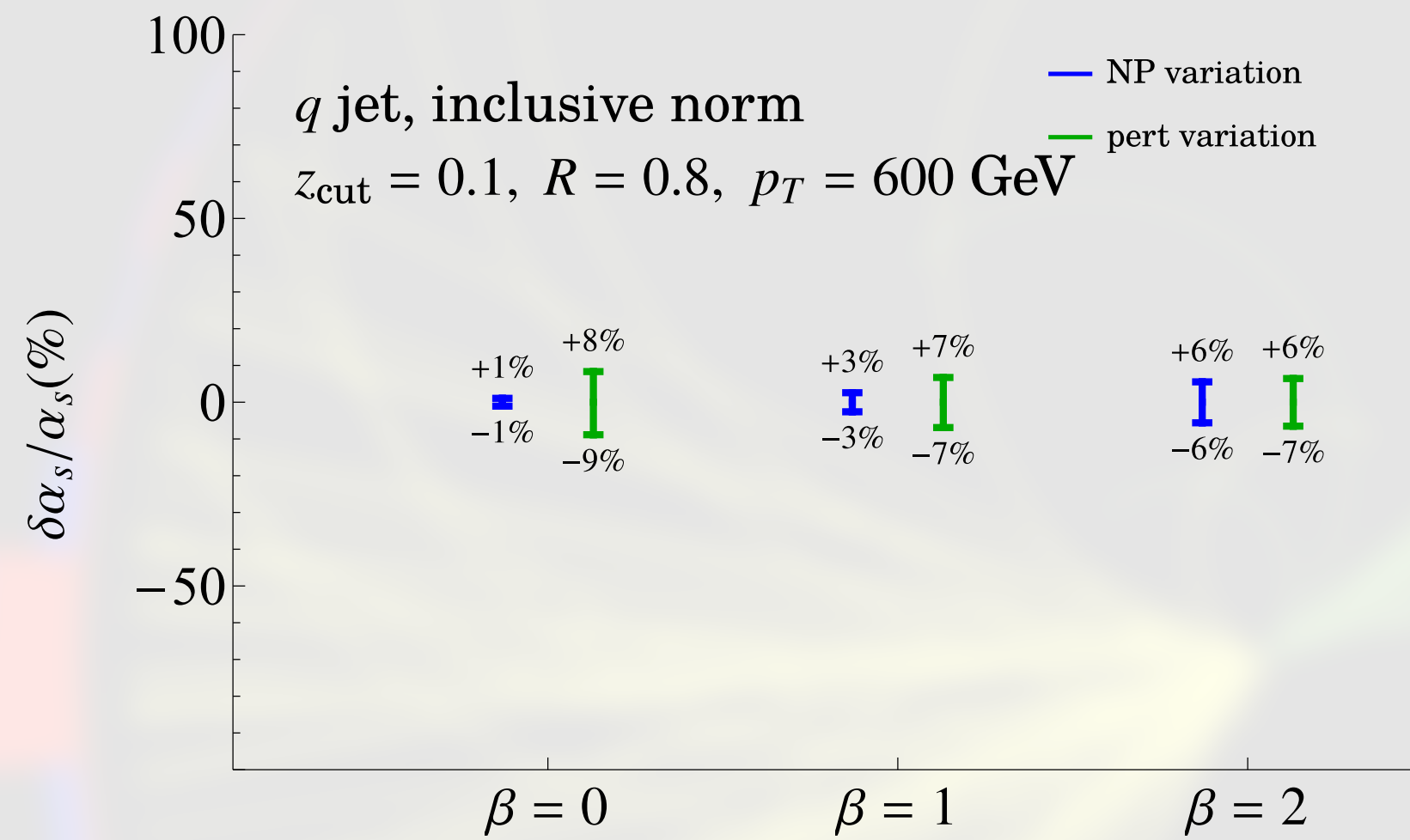
$$\sigma_{\text{incl}} = \frac{d^2\sigma}{dp_T d\eta}$$

# Results for inclusive normalization

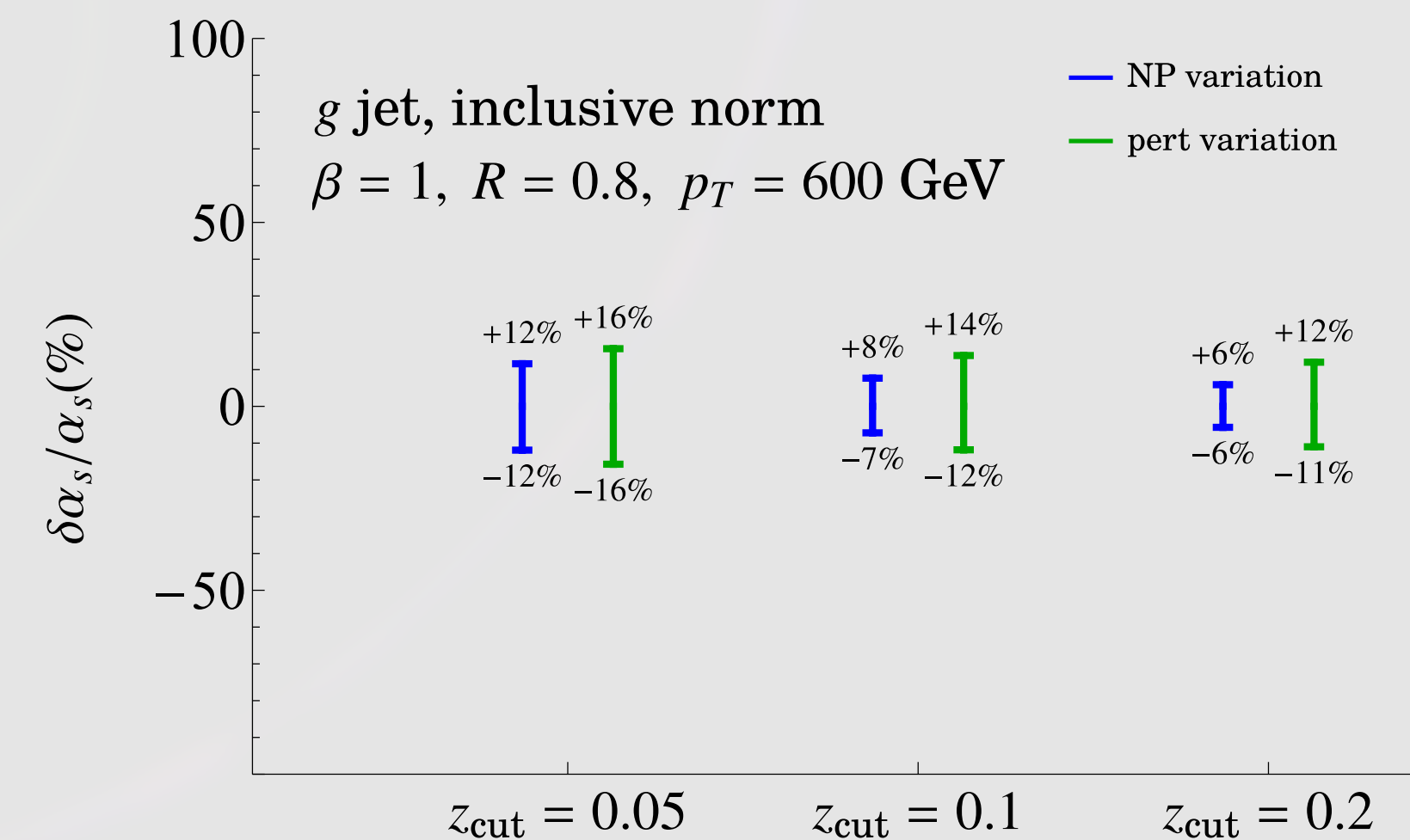
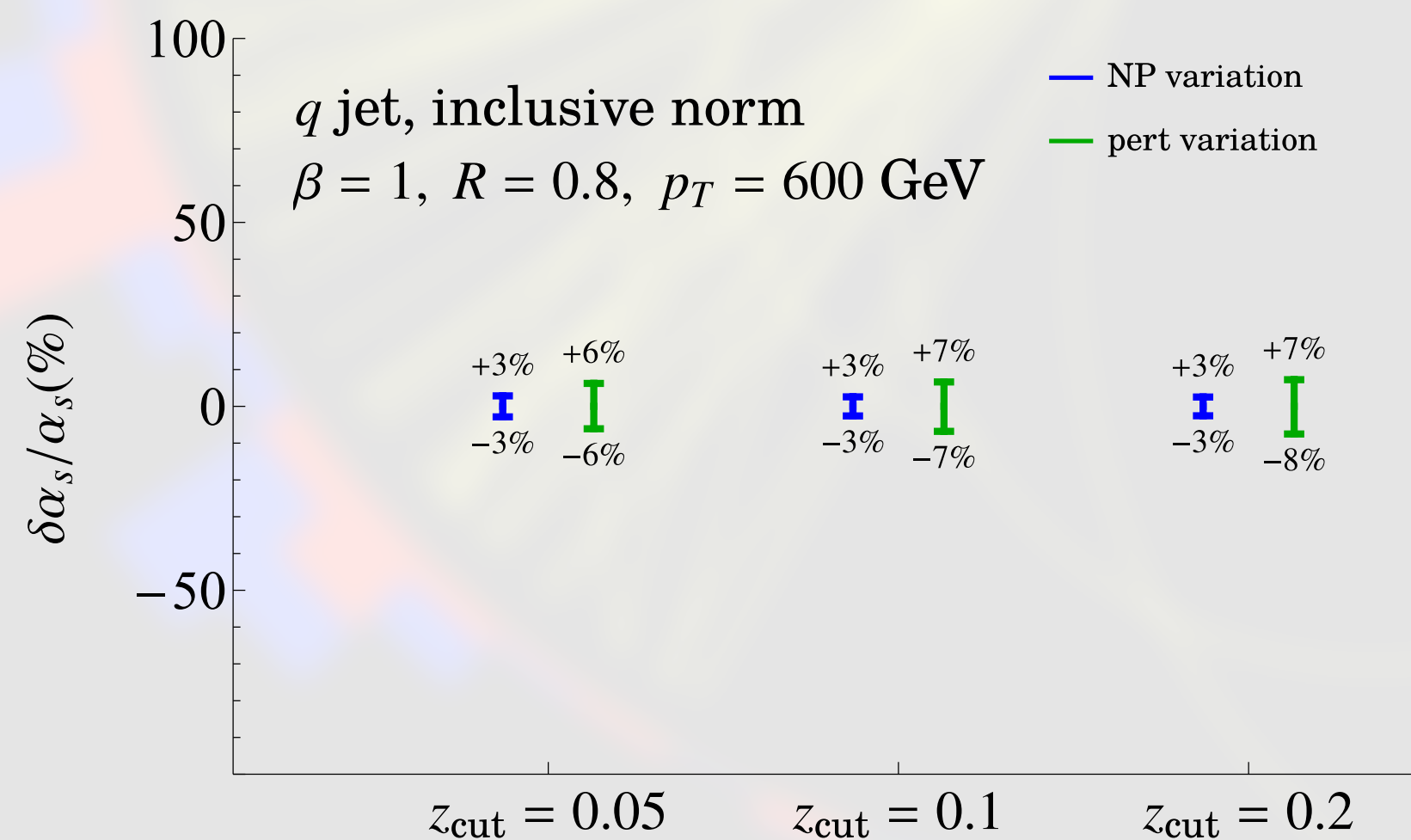
Quark jets:

Gluon jets:

Vary  $\beta$ :



Vary  $z_{\text{cut}}$ :

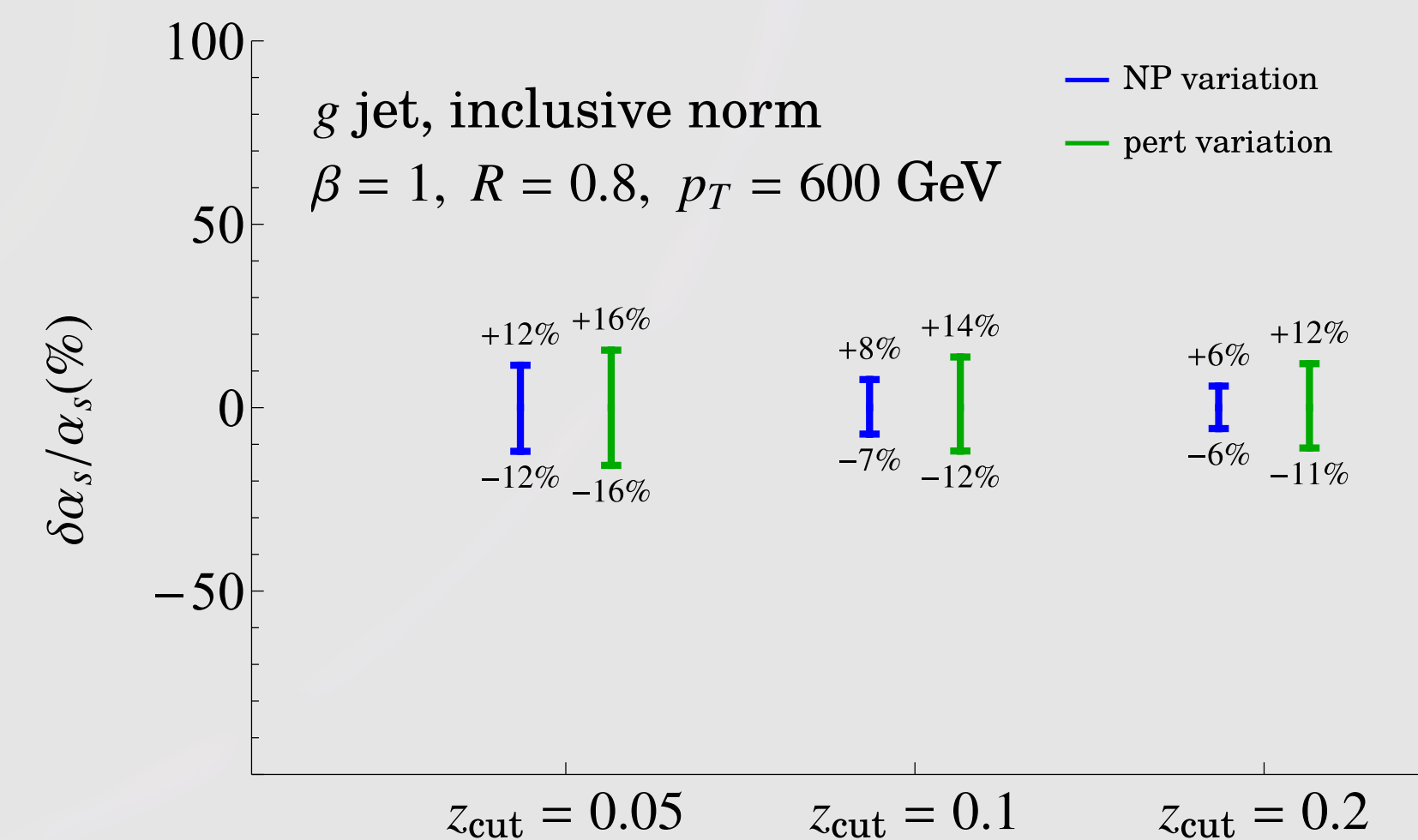
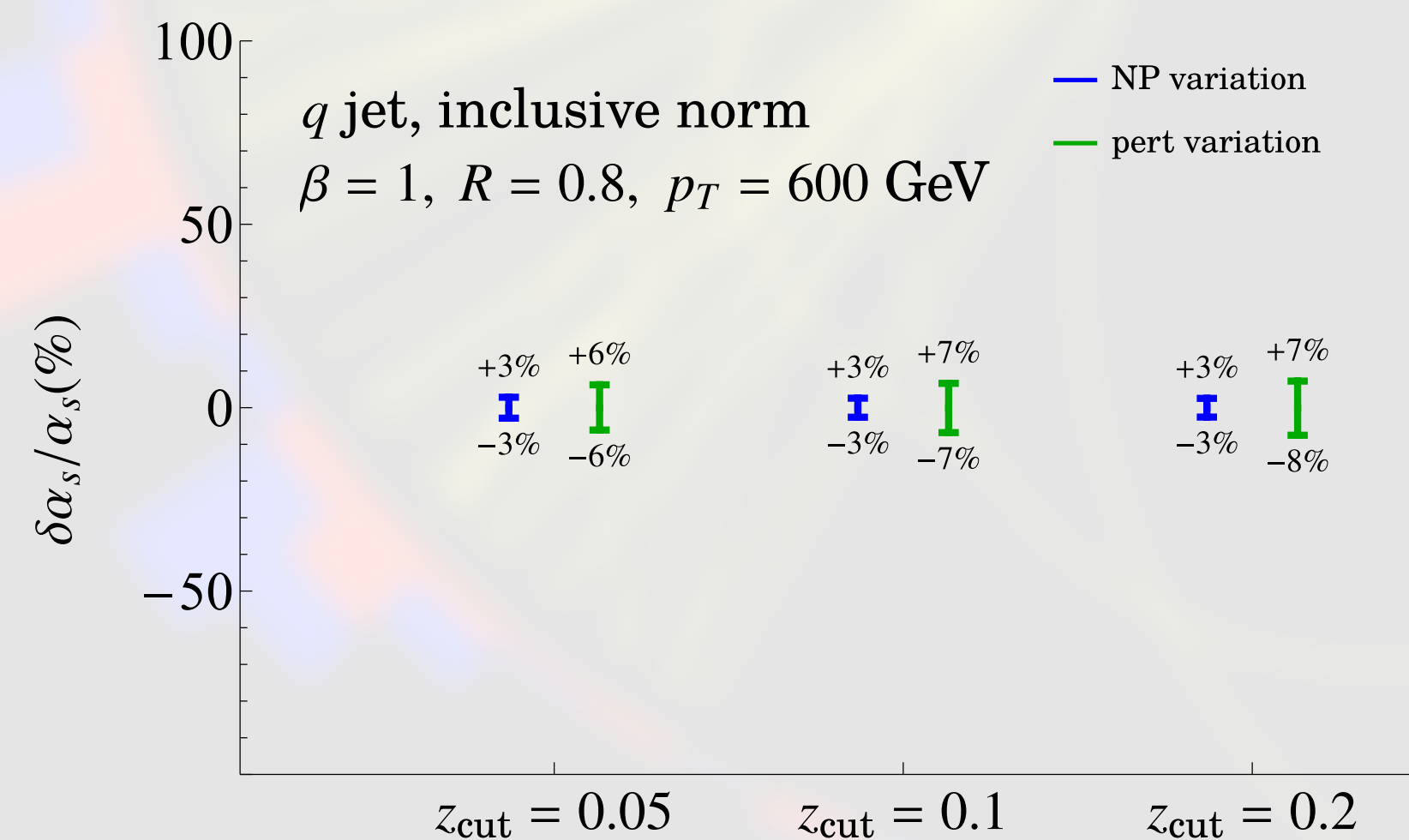
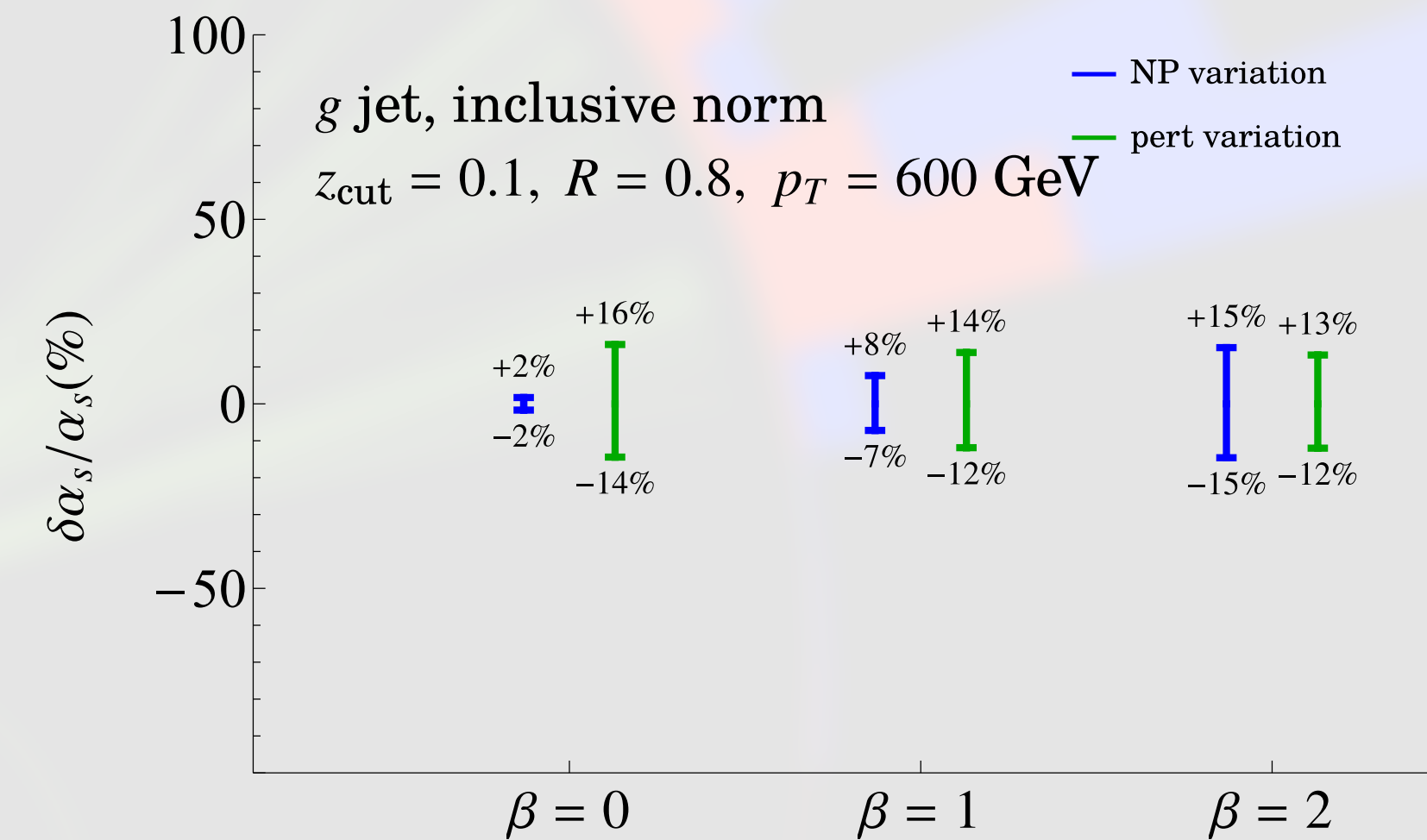
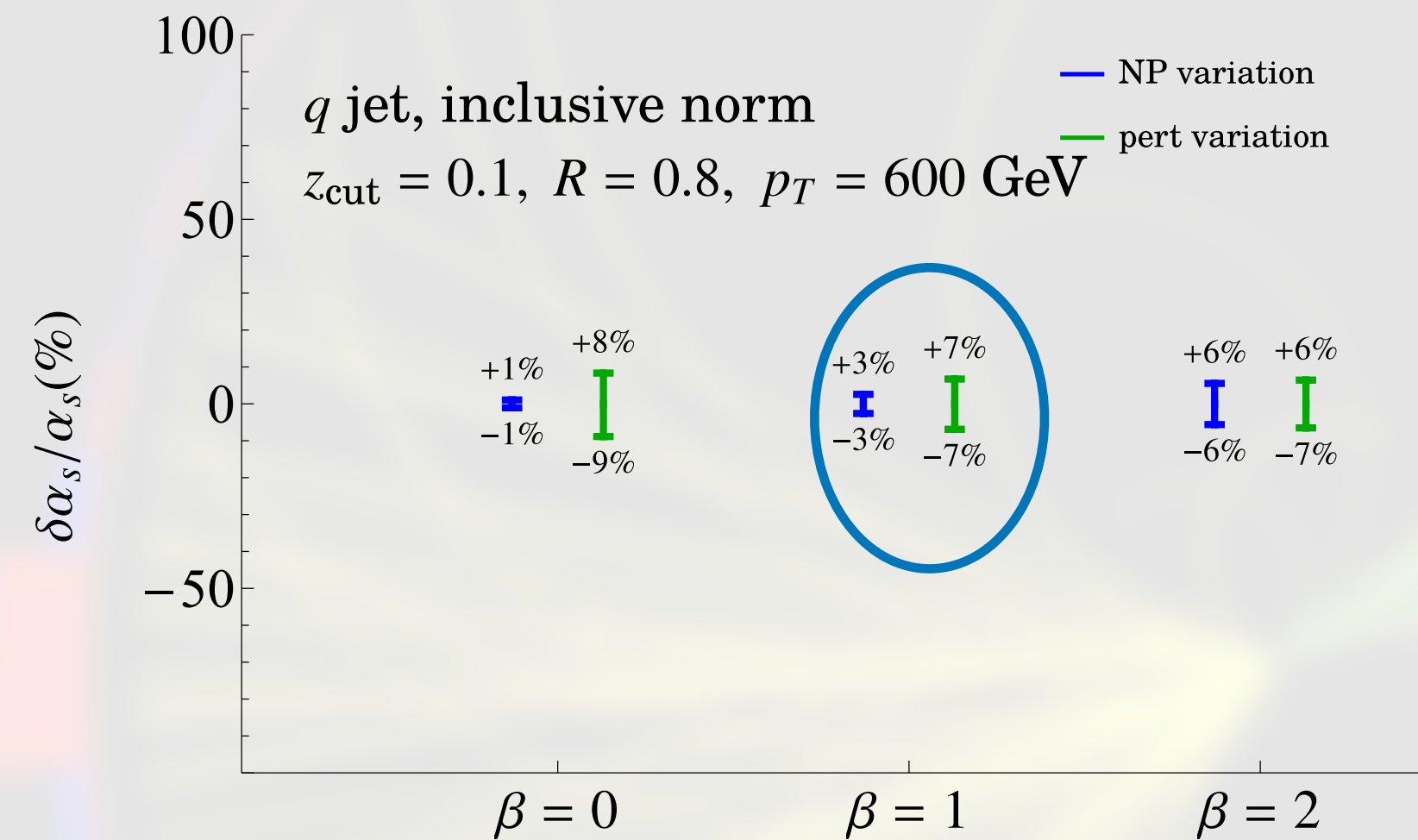


# Results for inclusive normalization

Quark jets:

Gluon jets:

Normalizing  
to the full  
spectrum  
is essential



# Prospects for improving precision

- Uncertainties can be reduced by going to **higher logarithmic orders**

- All NNLL data known

[Bell, Rahn, Talbert 2018-2020] [Frye, Larkoski, Schwartz, Yan 2016]

- Recent results at N<sup>3</sup>LL for hemisphere jets in  $e^+e^-$  collisions

[Kardos, Larkoski, Trócsányi]

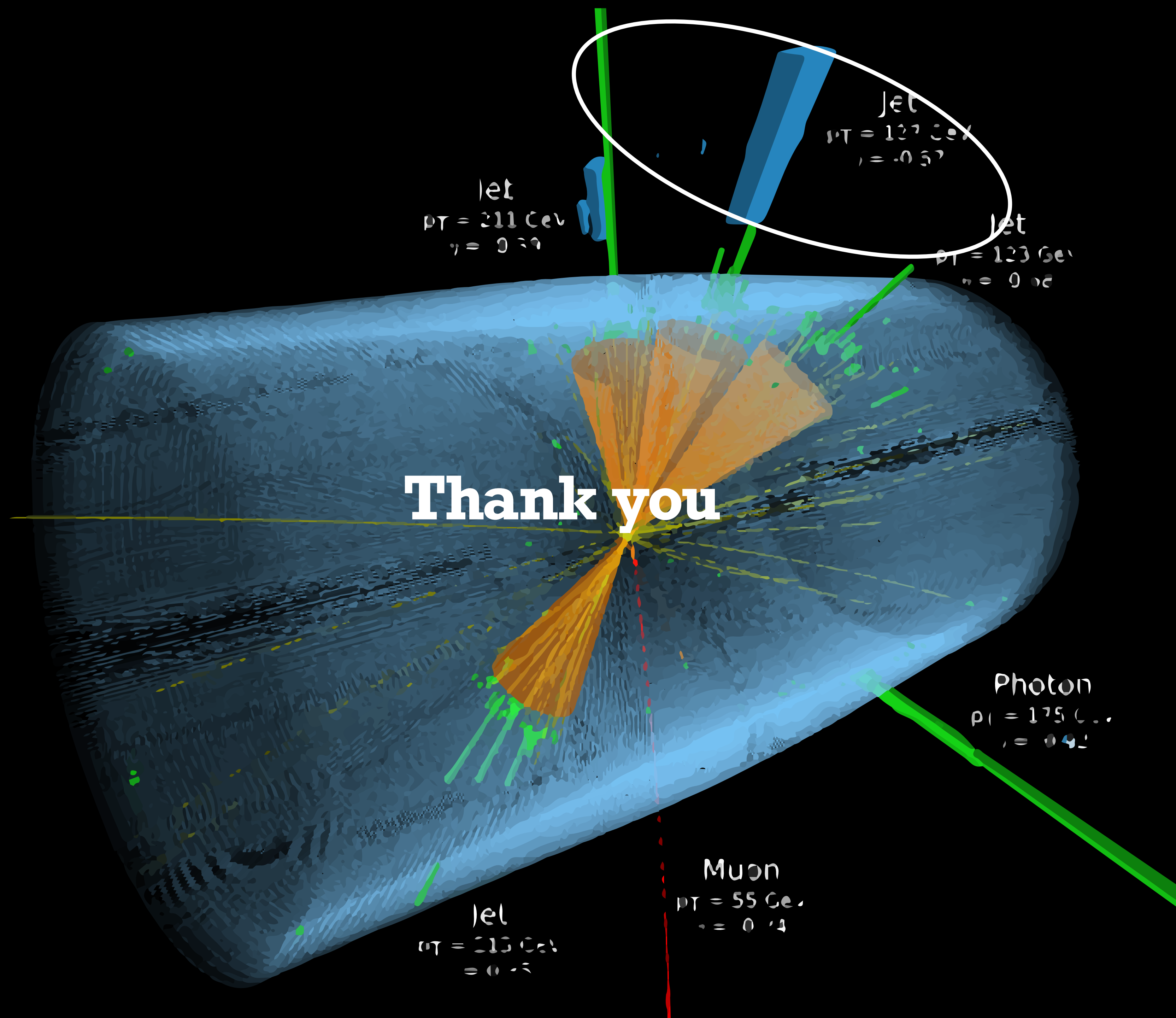
- **Need 2-loop constant pieces** for extending to N<sup>3</sup>LL

- collinear-soft (gluon jets) and

- global-soft (quark and gluon jets,  $R < \pi/2$ )

# Conclusions

- Combined **perturbative** uncertainty of around **9% for quark jets and 16% for gluon jets**.
- For  $\beta = 1$  we find **nonperturbative** uncertainty of **3% for quark jets and 8% for gluon jets**.
- q/g fraction **well defined** in theoretical calculations. PDF dependence **subdominant for normalized** cross sections.
- **Model-independent** estimate of nonperturbative power corrections
- Normalizing to **inclusive cross section** in  $p_T$ - $\eta$  bin essential to retain  $\alpha_s$ -sensitivity
- **N<sup>3</sup>LL calculations** would reduce perturbative uncertainty
- **Constrain nonperturbative parameters** using multiple  $z_{\text{cut}}$ ,  $\beta$  values.



Thank you

**Backup slides**

# Soft drop jet mass in inclusive jets

$$\xi = \frac{m_J^2}{p_T^2 R^2}$$

$$\tilde{\mathcal{G}}_\kappa(\xi, p_T R, \mu) \equiv \frac{1}{\sigma_\kappa^{\text{incl}}} \frac{d\sigma_\kappa}{d\xi}(p_T, \eta) = N_{\text{incl}}^\kappa(p_T R, \mu) \mathcal{J}_\kappa(\xi, p_T, \eta, R, \mu)$$

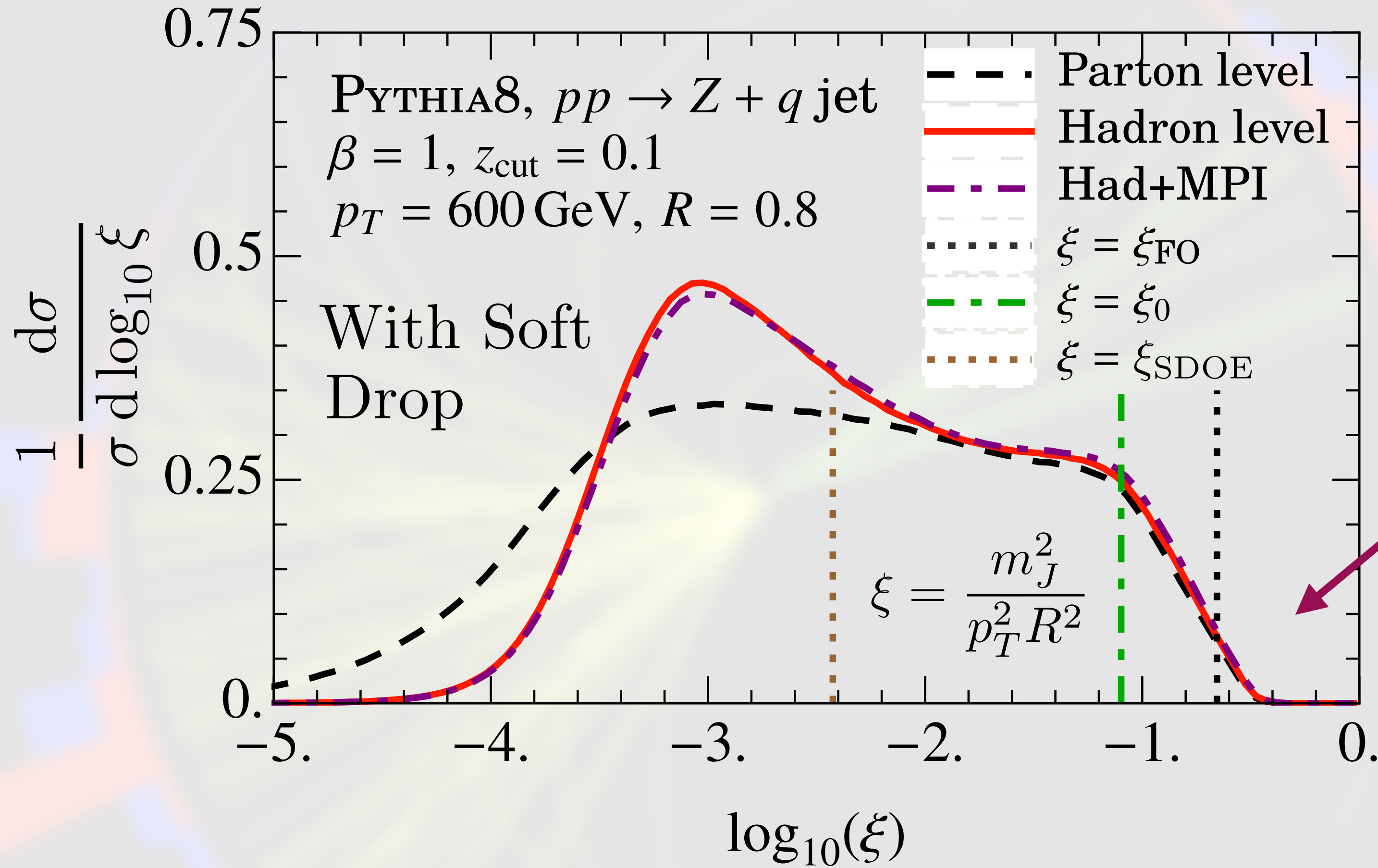
## *Energy scales:*

- Hard-collinear scale:  $Q = p_T R$
- Soft drop scale:  $Q_{\text{cut}} = z_{\text{cut}} Q \left( \frac{R}{R_0} \right)^\beta$
- Nonperturbative scale:  $\Lambda_{\text{QCD}}$



# Region for fitting to $\alpha_s$

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$

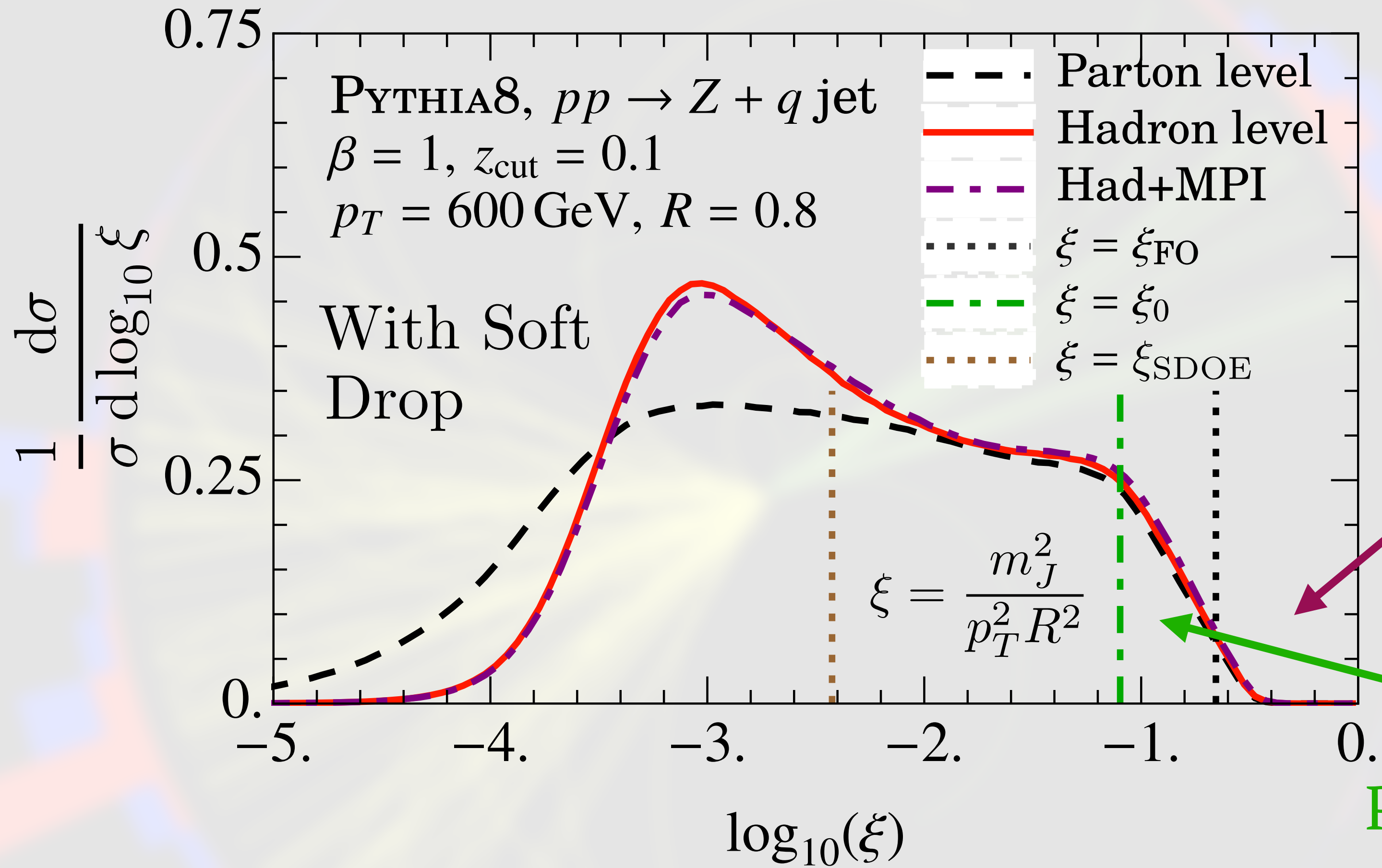


Fixed order region:  $\xi \lesssim 1$



# Region for fitting to $\alpha_s$

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



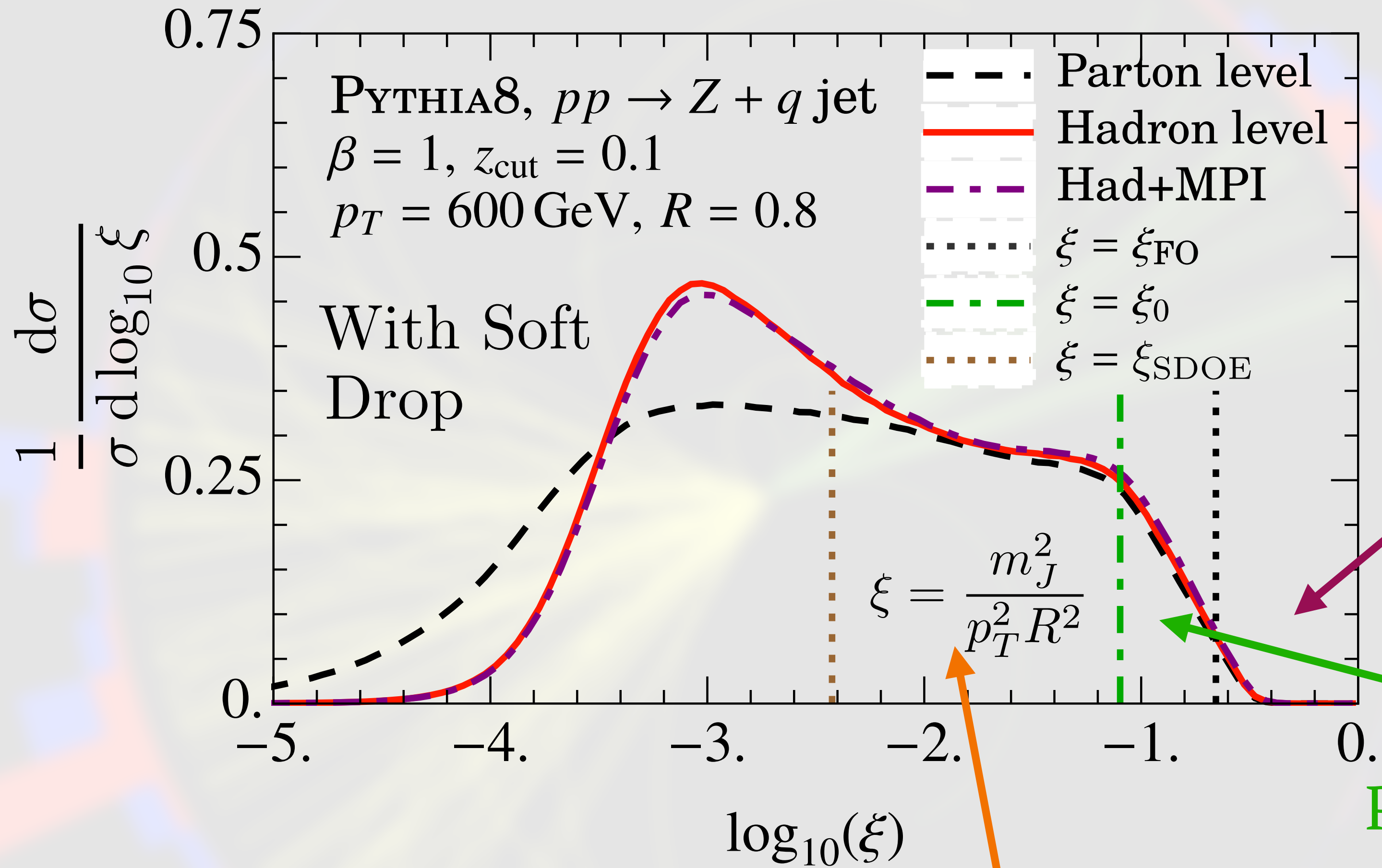
Fixed order region:  $\xi \lesssim 1$

$$\xi_0 = \frac{Q_{\text{cut}}}{Q}$$

Plain jet mass resummation region:  $\xi_0 \lesssim \xi \ll 1$

# Region for fitting to $\alpha_s$

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



Fixed order region:  $\xi \lesssim 1$

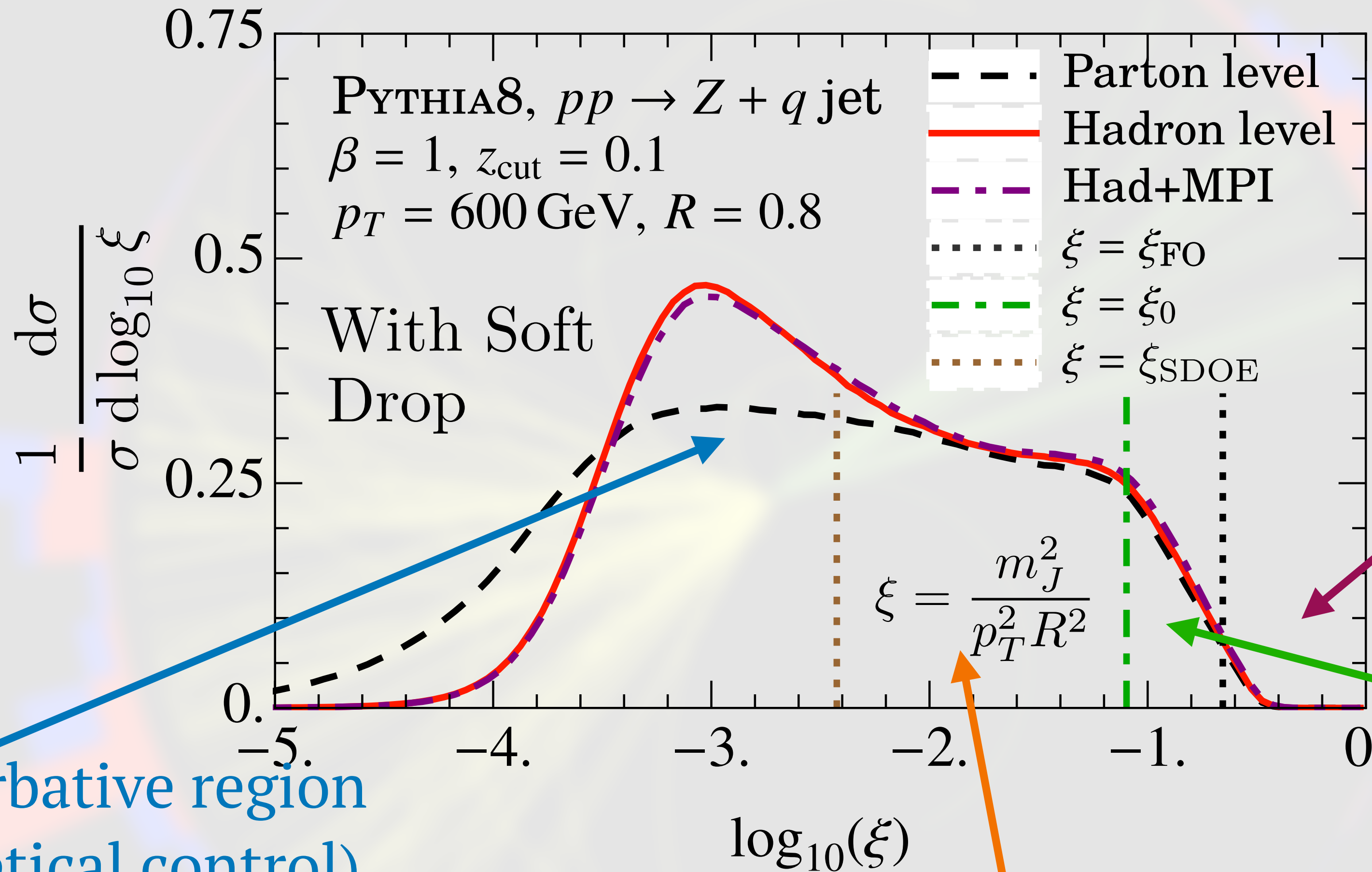
$$\xi_0 = \frac{Q_{\text{cut}}}{Q}$$

Plain jet mass resummation region:  $\xi_0 \lesssim \xi \ll 1$

Soft drop operator expansion region:  $\xi_0 \left( \frac{\Lambda_{\text{QCD}}}{Q_{\text{cut}}} \right)^{\frac{2+\beta}{1+\beta}} \ll \xi \ll \xi_0$

# Region for fitting to $\alpha_s$

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



Fixed order region:  $\xi \lesssim 1$

$$\xi_0 = \frac{Q_{\text{cut}}}{Q}$$

Plain jet mass resummation region:  $\xi_0 \lesssim \xi \ll 1$

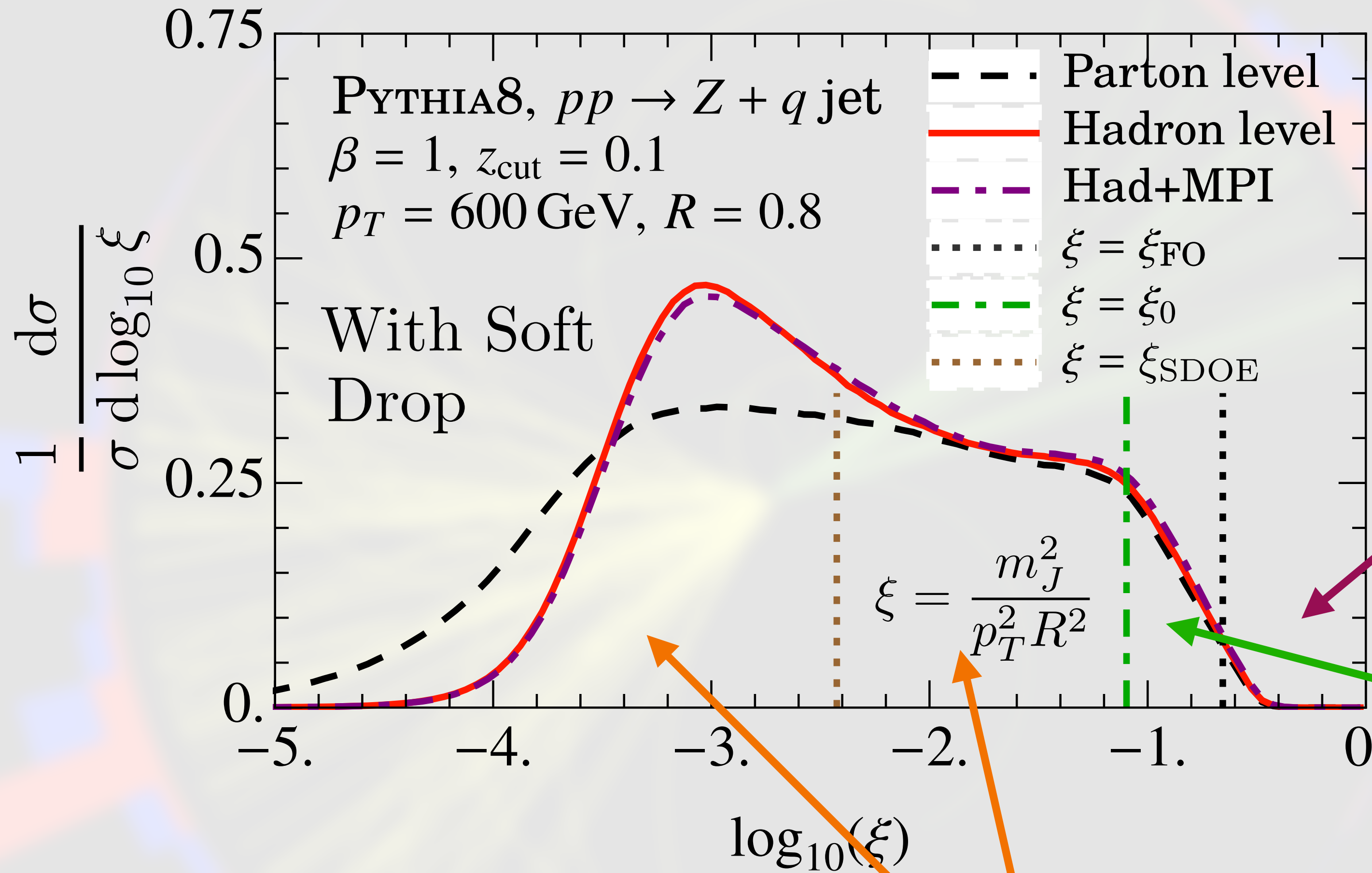
Soft drop operator expansion region:  $\xi_0 \left( \frac{\Lambda_{\text{QCD}}}{Q_{\text{cut}}} \right)^{\frac{2+\beta}{1+\beta}} \ll \xi \ll \xi_0$

Nonperturbative region  
(no theoretical control)

$$\xi \gtrsim \xi_0 \left( \frac{\Lambda_{\text{QCD}}}{Q_{\text{cut}}} \right)^{\frac{2+\beta}{1+\beta}}$$

# Large logarithms

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q^2}$$



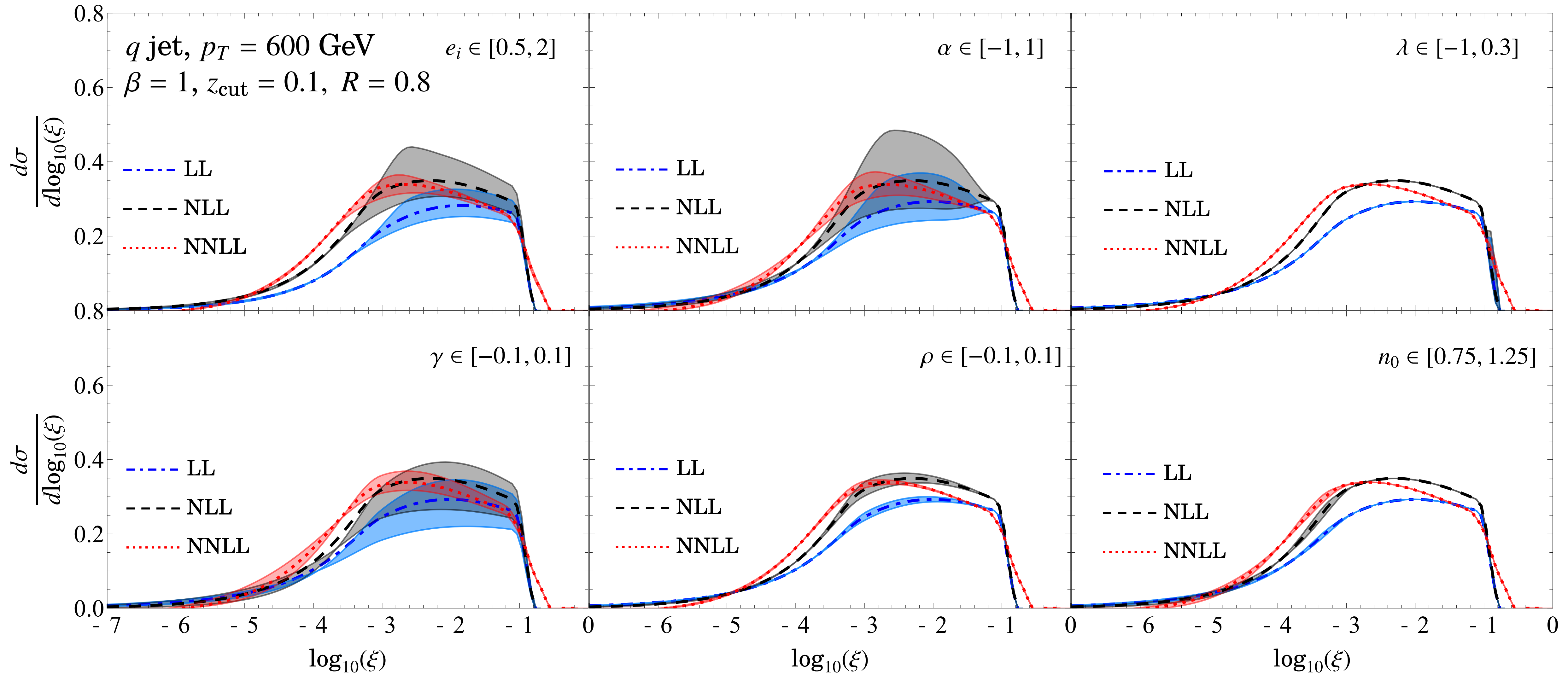
Fixed order region

$$\xi_0 = \frac{Q_{\text{cut}}}{Q}$$

Plain jet mass resummation region:  $\alpha_s^n \ln^m(\xi)$

Soft drop resummation region:  $\alpha_s^n \ln^m(\xi/\xi_0)$

# Scale variations



# Impact of scale variations: dependence on $\beta$

