Automated Calculation of Beam and Jet Functions

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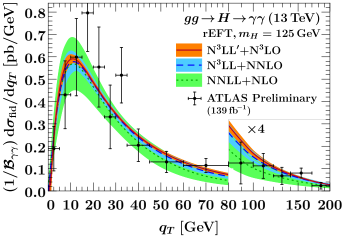
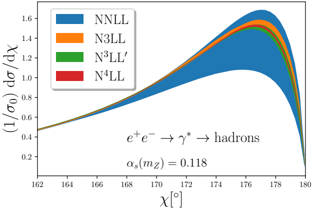
Motivation

- Resummation is required for collider observables
  - For some observables very precise $\rightarrow$ N^{3}LL and above



[Billis et al.;21]



[Duhr et al.;22]

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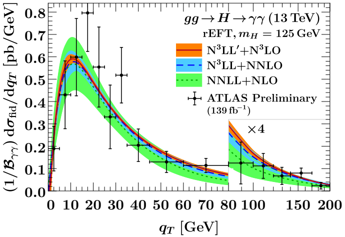
  - For generic observables only NLL accuary

    $\rightarrow$ CAESAR



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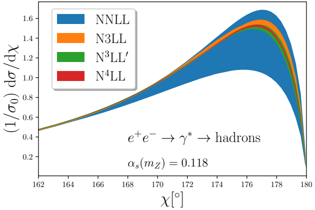
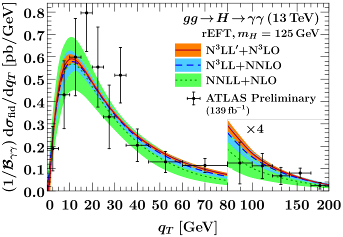


[Duhr et al.;22]

Motivation

- Resummation is required for collider observables
  - For some observables very precise $\rightarrow$ N^{3}LL and above
  - For generic observables only NLL accuary
- Our goal is to push this to N^{2}LL$'$
  - Systematic framework of SCET



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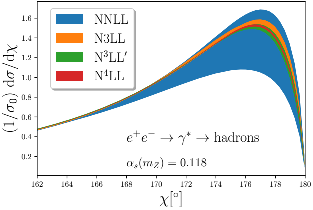
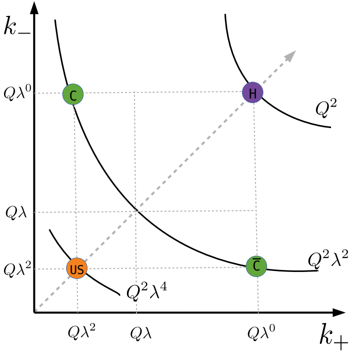


[Duhr et al.;22]

Soft-Collinear Effective Theory (SCET)

- Effective theory
  - Hard modes are integrated out
  - Soft and collinear modes
  - Leading power $\rightarrow$
    Soft and collinear modes decouple
- Typical scaling: $k^\mu \sim (k_-, k_+, k_\perp)$
  - Hard region: $k_H^\mu \sim (1, 1, 1)Q$
  - Collinear region: $k_C^\mu \sim (1, \lambda^2, \lambda)Q$
  - Ultrasoft region: $k_{US}^\mu \sim (\lambda^2, \lambda^2, \lambda^2)Q$



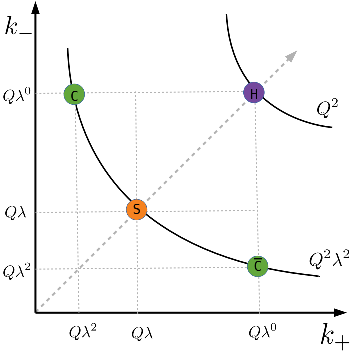
SCET-II

- Different scaling compared to SCET-I
  - Hard region: $k_H^\mu \sim (1,1,1)Q$
  - Collinear region: $k_C^\mu \sim (1,\lambda^2,\lambda)Q$
  - Soft region: $k_S^\mu \sim (\lambda,\lambda,\lambda)Q$
- Soft and collinear modes have same virtuality

$\Rightarrow$ Additional rapidity divergences

- Introduce additional regulator [Becher,Bell;12]

$$\prod_i \int \frac{\mathrm{d}^d k_i}{(2\pi)^d} \left(\frac{\nu}{k_i^- + k_i^+}\right)^\alpha \delta\left(k_i^2\right) \theta\left(k_i^{(0)}\right)$$



Factorisation

- Typical factorisation formula for LHC observables in SCET

$$\mathrm{d}\sigma \simeq H\left(\mu_F\right) \cdot B\left(\mu_F\right) \otimes \bar{B}\left(\mu_F\right) \otimes S\left(\mu_F\right)$$

- Some observables require input from final state radiation $\rightarrow$ Jet functions
  - Example: Boson-jet azimuthal decorrelation [Chien et al.;20,22]
- Resummation requires knowledge of anomalous dimensions and matching corrections

$$\underbrace{\Gamma_{\mathsf{Cusp}}, \gamma_{\mathsf{H}}, c_{\mathsf{H}}}_{\text{Observable independent}} \qquad \underbrace{\gamma_{\mathsf{B}}, \gamma_{\mathsf{J}}, \gamma_{\mathsf{S}}, c_{\mathsf{B}}, c_{\mathsf{J}}, c_{\mathsf{S}}}_{\text{Observable dependent}}$$

- SoftSERVE: [Bell,Rahn,Talbert;19,20]
  - Automated framework to calculate NNLO soft functions
- We developed a similar framework for the beam and jet functions at NNLO
  - First application: p_T-veto for quark beam function [Bell,KB,Das,Wald;22]

Automation of Beam function calculations

Beam function definitions

- Quark beam function

$$\frac{1}{2}\left[\frac{\not{n}}{2}\right]_{\beta\alpha}\mathcal{B}_{q/h}\left(x,\tau,\mu\right)=\sum_X \delta\left((1-x)P_- - \sum_i k_i^-\right)$$
$$\langle h(P)|\,\bar{\chi}_\alpha\,|X\rangle\,\langle X|\,\chi_\beta\,|h(P)\rangle\mathcal{M}(\tau;\{k_i\})$$

- Gluon beam function

$$-\mathcal{B}_{g/h}\left(x,\tau,\mu\right)=\frac{1}{xP_-}\sum_X \delta\left((1-x)P_- - \sum_i k_i^-\right)$$
$$\langle h(P)|\,\mathcal{A}^{\mu,A}_{c,\perp}\,|X\rangle\,\langle X|\,\mathcal{A}^{A}_{c,\perp,\mu}\,|h(P)\rangle\mathcal{M}(\tau;\{k_i\})$$

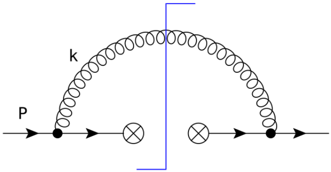
- Matching on parton distribution functions

$$\mathcal{B}_{i/h}\left(x,\tau,\mu\right)=\sum_{j\in\{q,\bar{q},g\}}\int_x^1\frac{\mathrm{d}z}{z}\mathcal{I}_{i\leftarrow j}\left(\frac{x}{z},\tau,\mu\right)f_{j/h}(z,\mu)+\mathcal{O}(\tau\Lambda_{QCD})$$

Approach at NLO

- Parametrisation:

$$k_- = (1-x)P_-,$$
$$|\vec{k}_\perp| = k_T,$$
$$\cos(\theta_k) = 1 - 2t_k$$

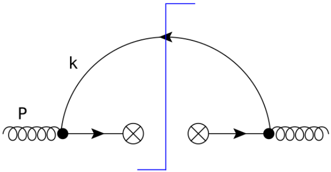


- Measurement function:

$$\mathcal{M}_1^B(\tau;k) = \exp\left[-\tau k_T \left(\frac{k_T}{(1-x)P_-}\right)^n f(t_k)\right]$$

- Master formula:

$$\mathcal{B}_{i/j}^{(1)}(x,\tau) \simeq \frac{\Gamma\left(\frac{-2\epsilon}{1+n}\right)}{1+n}(1-x)^{-1-\frac{2n\epsilon}{1+n}-\alpha} W_{i/j}^B \times \int_0^1 \mathrm{d}t_k (4t_k\bar{t_k})^{-1/2-\epsilon} f(t_k)^{\frac{2\epsilon}{1+n}}$$



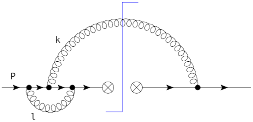
Approach at NNLO

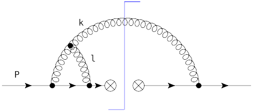
- At NNLO we need to consider two different contributions
  1) Real-Virtual contribution
     - Matrix element: NLO collinear splitting functions
     - Same parametrisation as NLO
  2) Real-Real contribution
     - Matrix element: LO triple collinear splitting functions
     - Parametrisation:

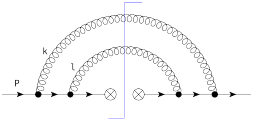
$$q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}, \quad b = \frac{k_T}{l_T}$$

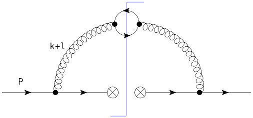
$$a = \frac{k_- l_T}{k_T l_-}, \quad t_{kl} = \frac{1 - \cos\theta_{kl}}{2}$$

$$x = \frac{k_- + l_-}{P_-}$$









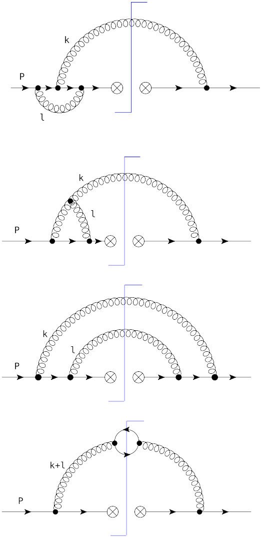
Approach at NNLO

- Measurement function at NNLO

$$\mathcal{M}_2^B(\tau;k,l) = \exp\left[-\tau q_T \left(\frac{q_T}{(1-x)P_-}\right)^n F(a,b,t_{kl},t_k,t_l)\right]$$

- Many overlapping singularities remain
  - Sector decomposition
  - Selector functions
  - Non-linear transformations

→ All singularities factorised



How to treat distributions

- In general the beam functions are distribution valued in $(1-x)^{-1}$

1)
- Direct calculation in x-space
- Expand in terms of distributions

$$(1-x)^{-1-m\epsilon} f(x) = -\frac{1}{m\epsilon} f(x)\delta(1-x) + \left[\frac{1}{1-x}\right]_+ f(x) + \dots$$
$$= -\frac{1}{m\epsilon} f(1)\delta(1-x) + \left[\frac{1}{1-x}\right]_+ f(1) + \frac{f(x)-f(1)}{1-x} + \dots$$

2)
- Resolve all distributions in Mellin space

$$\hat{\mathcal{B}}_{i/j}(N,\tau) = \int_0^1 \mathrm{d}x\, x^{N-1} \mathcal{B}_{i/j}(x,\tau)$$

SCET-I renormalisation

- The matching kernels follow the RGE

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathcal{I}_{i\leftarrow j}(x,\tau,\mu) = \left[2\frac{1+n}{n}\Gamma^i_{\text{Cusp}}L + \gamma^i_B\right]\mathcal{I}_{i\leftarrow j}(x,\tau,\mu) - 2\sum_k \mathcal{I}_{i\leftarrow k}(x,\tau,\mu)\otimes P_{k\leftarrow j}(x,\mu)$$

- One-loop solution

$$\mathcal{I}_{i\leftarrow j}(x,\tau,\mu) = \delta(1-x)\delta_{ij} + \left(\frac{\alpha_s}{4\pi}\right)\left[\left(\frac{1+n}{n}\Gamma^i_0 L^2 + \gamma^i_B L\right)\delta(1-x)\delta_{ij} - 2P^{(0)}_{i\leftarrow j}(x)L + \mathcal{I}^{(1)}_{i\leftarrow j}(x)\right] + \mathcal{O}(\alpha_s^2)$$

$\rightarrow$ Extraction of γ_B and $\mathcal{I}_{i\leftarrow j}(x)$ at NNLO

SCET-II renormalisation

- Additional scale $\nu \rightarrow$ Additional large logarithms
  - Resum them via collinear anomaly approach [Becher, Neubert; 13]

$$\left[\mathcal{I}_{i\leftarrow k}(x_1,\tau,\mu,\nu)\mathcal{I}_{j\leftarrow l}(x_2,\tau,\mu,\nu)S_{ij}(\tau,\mu,\nu)\right] \underset{\alpha=0}{\equiv} \left(q^2\bar{\tau}^2\right)^{-F_{ij}(\tau,\mu)} I_{i\leftarrow k}(x_1,\tau,\mu)I_{j\leftarrow l}(x_2,\tau,\mu)$$

- Anomaly exponent F_{ij} fulfills RGE

$$\frac{\mathrm{d}F_{ij}(\tau,\mu)}{\mathrm{d}\ln\mu} = 2\Gamma_{\mathsf{Cusp}}$$

- Slightly different RGE for the matching kernel $I_{i\leftarrow k}(x,\tau,\mu)$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} I_{i\leftarrow k}(x,\tau,\mu) = 2\left[\Gamma_{\mathsf{Cusp}}L - \gamma_H\right] I_{i\leftarrow k}(x,\tau,\mu) - 2\sum_j I_{i\leftarrow j}(x,\tau,\mu)\otimes P_{j\leftarrow k}(x,\mu)$$

$\rightarrow$ Extraction of γ_H, anomaly exponent F_{ij} and $I_{i\leftarrow k}(x)$ at NNLO

Observable status

SCET-I Observables

- Beam Thrust
- DIS Angularities

SCET-II Observables

- p_T-resummation
- Transverse Thrust
- p_T-veto

- Matching kernel can be written as

$$\begin{aligned}\mathcal{I}_{i\leftarrow j}(x) =& c^{ij}_{-1}\delta(1-x) + c^{ij}_0\left[\frac{1}{1-x}\right]_+ + c^{ij}_1\left[\frac{\log(1-x)}{1-x}\right]_+\\ &+ c^{ij}_2\left[\frac{\log^2(1-x)}{1-x}\right]_+ + c^{ij}_3\left[\frac{\log^3(1-x)}{1-x}\right]_+ + \mathcal{I}^{(2,y),\text{Grid}}_{i\leftarrow j}(x)\end{aligned}$$

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- All contributions implemented into `pySecDec` [Heinrich, et.al ;18,19,22]

p_T-veto [Preliminary]

- Measurement: $\omega_2(k,l) = \theta(\Delta - R)\max(|\vec{k}^\perp|, |\vec{l}^\perp|) + \theta(R - \Delta)|\vec{k}^\perp + \vec{l}^\perp|$
  with $\Delta = \sqrt{\frac{1}{4}\ln^2\frac{k^- l^+}{k^+ l^-} + \theta_{kl}^2}$
- Independent calculation by Dingyu Shao
  - Based on computing the difference wrt to a known reference observable
  - Asymmetric phase-space regulator
    $\rightarrow$ Method is similar to the one from
    Abreu, Gaunt, Monni, Rottoli, Szafron [2207.07037] [See talk by Gaunt]

p_T-veto [Preliminary]

- Measurement: $\omega_2(k,l) = \theta(\Delta - R)\max(|\vec{k}^{\perp}|, |\vec{l}^{\perp}|) + \theta(R-\Delta)|\vec{k}^{\perp} + \vec{l}^{\perp}|$
  with $\Delta = \sqrt{\frac{1}{4}\ln^2\frac{k^- l^+}{k^+ l^-} + \theta_{kl}^2}$
  - Focus on the gluon-to-gluon and quark-to-gluon channels

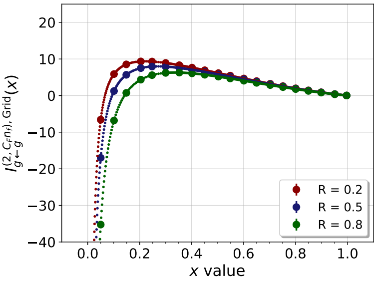
$R = 0.2$	This work	AGMRS
$c^{gg}_{-1,C_A T_F}$	$0.31(1)$	0.32
c^{gg}_{-1,C_A^2}	$-23.32(9)$	-23.32
$c^{gg}_{0,C_A T_F}$	$16.44(1)$	16.44
c^{gg}_{0,C_A^2}	$79.02(6)$	79.02

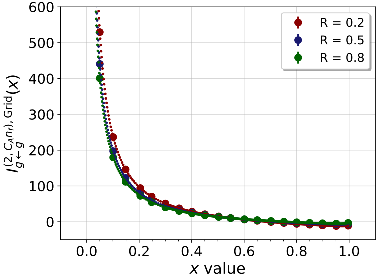
$R = 0.5$	This work	AGMRS
$c^{gg}_{-1,C_A T_F}$	$1.38(1)$	1.38
c^{gg}_{-1,C_A^2}	$-16.98(9)$	-16.98
$c^{gg}_{0,C_A T_F}$	$11.40(1)$	11.40
c^{gg}_{0,C_A^2}	$43.20(6)$	43.20

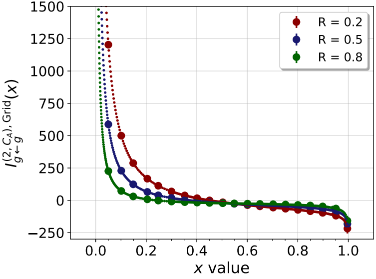
$R = 0.8$	This work	AGMRS
$c^{gg}_{-1,C_A T_F}$	$1.94(1)$	1.94
c^{gg}_{-1,C_A^2}	$-12.29(9)$	-12.29
$c^{gg}_{0,C_A T_F}$	$9.01(1)$	9.01
c^{gg}_{0,C_A^2}	$20.19(6)$	20.19

AGMRS: [Abreu et al.;22]

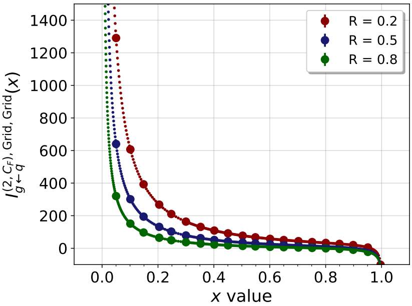
p_T-veto - $I_{g\leftarrow g}^{(2,y),\text{Grid}}(x)$ [Preliminary]

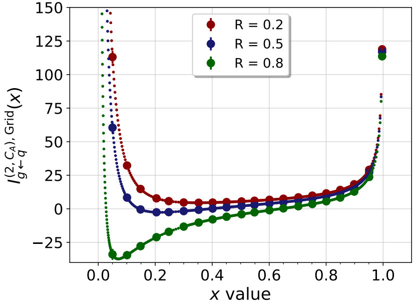






p_T-veto - $I_{g\leftarrow q}^{(2,y),\text{Grid}}(x)$ [Preliminary]





DIS Angularities [Preliminary]

- Measurement: [Zhu, Kang, Maji; 21]

$$\omega_{\mathsf{DIS}}(\{k_i\}) = \sum_i \left(k_i^+\right)^{1-\frac{A}{2}} \left(k_i^-\right)^{\frac{A}{2}}$$

  - Here we consider the cases $A = \{-1, 0, 0.5\}$
- Focus on the quark-to-quark and gluon-to-quark channels

c_{-1}^{qq}	$A=-1$	$A=0.5$
$C_F T_F$	$6.32(1)$	$-17.49(6)$
C_F^2	$-2.91(4)$	$6.41(41)$
$C_F C_A$	$-4.09(12)$	$-2.69(68)$

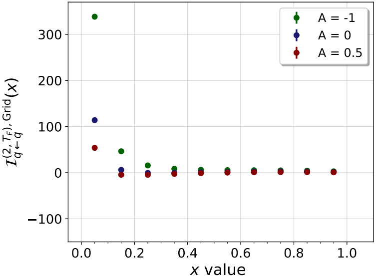
c_{0}^{qq}	$A=-1$	$A=0.5$
$C_F T_F$	$-0.15(1)$	$8.16(1)$
C_F^2	$34.19(3)$	$8.55(13)$
$C_F C_A$	$17.58(65)$	$8.64(19)$

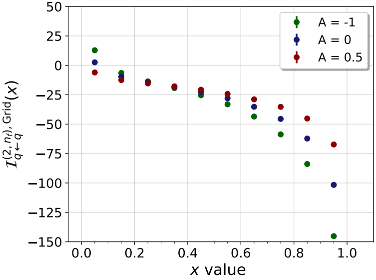
c_{1}^{qq}	$A=-1$	$A=0.5$
$C_F T_F$	$-11.85(1)$	$-5.93(1)$
C_F^2	$-54.10(3)$	$-1.46(3)$
$C_F C_A$	$22.16(3)$	$11.08(4)$

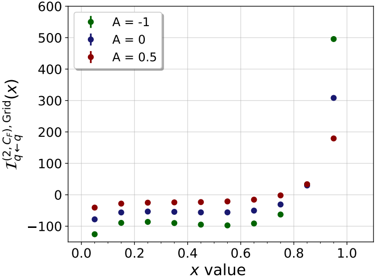
c_{2}^{qq}	$A=-1$	$A=0.5$
$C_F T_F$	$4.74(1)$	$1.18(1)$
C_F^2	0	0
$C_F C_A$	$-13.04(1)$	$-3.26(1)$

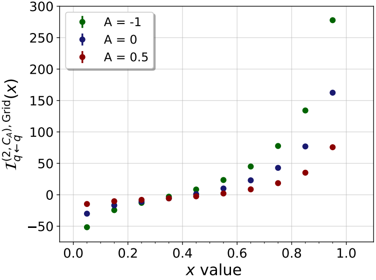
c_{3}^{qq}	$A=-1$	$A=0.5$
C_F^2	$14.22(1)$	$3.56(1)$

DIS Angularities - $\mathcal{I}_{q\leftarrow q}^{(2,y),\mathrm{Grid}}(x)$ [Preliminary]

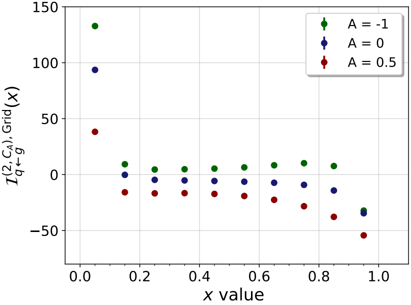


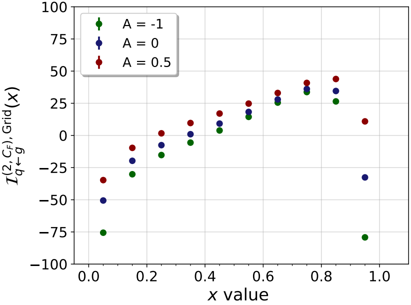






DIS Angularities - $\mathcal{I}_{q\leftarrow g}^{(2,y),\text{Grid}}(x)$ [Preliminary]





Automation of Jet function calculations

Jet function definitions

- Quark jet function

$$\frac{\not{n}}{2} J_q(Q,\tau,\mu) = \frac{1}{\pi}\sum_X (2\pi)^d \delta\left(Q - \sum_i k_i^-\right)\delta^{d-2}\left(\sum_i k_i^\perp\right)$$
$$\langle 0|\chi|X\rangle\langle X|\bar{\chi}|0\rangle \mathcal{M}(\tau;\{k_i\})$$

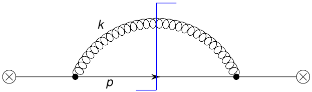
- Gluon jet function

$$-g_\perp^{\mu\nu}\delta^{ab} g_s^2 J_g(Q,\tau,\mu) = \frac{Q}{\pi}\sum_X (2\pi)^d \delta\left(Q - \sum_i k_i^-\right)\delta^{d-2}\left(\sum_i k_i^\perp\right)$$
$$\langle 0|\mathcal{A}_\perp^{\mu,a}|X\rangle\langle X|\mathcal{A}^b_{\perp,\nu}|0\rangle \mathcal{M}(\tau;\{k_i\})$$

Approach at NLO

- Parametrisation:

$$k_- = zQ, \quad p_- = \bar{z}Q,$$
$$|\vec{k}_\perp| = |\vec{p}_\perp| = k_T,$$
$$\cos(\theta_k) = 1 - 2t_k$$

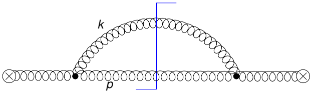


- Measurement function:

$$\mathcal{M}_1^J(\tau; k, p) = \exp\left[-\tau\,(k_T)^m \left(\frac{k_T}{Q}\right)^n (z\bar{z})^{-\frac{m+n}{1+n}n} f(z, t_k)\right]$$

- Master formula:

$$J_i^{(1)}(Q, \tau, \mu) \simeq \frac{\Gamma\left(\frac{-2\epsilon}{m+n}\right)}{m+n} \int_0^1 \mathrm{d}z (z\bar{z})^{-1-\frac{2n\epsilon}{1+n}-\alpha} W_i^J(z) \times \int_0^1 \mathrm{d}t_k (4t_k\bar{t_k})^{-1/2-\epsilon} f(z, t_k)^{\frac{2\epsilon}{m+n}}$$



Approach at NNLO

- Similar approach to the beam functions
  - Parametrisation:

$$q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}, \quad b = \frac{k_T}{l_T}$$

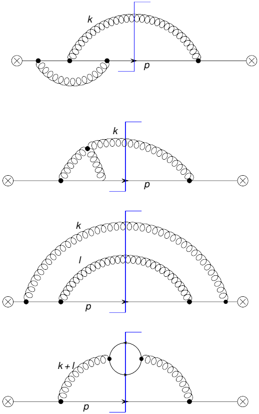
$$a = \frac{k_- l_T}{k_T l_-}, \quad t_{kl} = \frac{1 - \cos\theta_{kl}}{2}, \quad z = \frac{k_- + l_-}{Q}$$

  - Measurement function at NNLO

$$\mathcal{M}_2^J(\tau; k, l, p) = \exp\left[-\tau (q_T)^m \left(\frac{q_T}{(z\bar{z})^{\frac{m+n}{1+n}} Q} \right)^n F(z, a, b, t_{kl}, t_k, t_l) \right]$$

  - Renormalisation follows as in the beam function case
    - Slightly simpler RGE

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} J_i(Q,\tau,\mu) = 2\left[\frac{1+n}{n} \Gamma_{\mathsf{Cusp}} L + \gamma_J \right] J_i(Q,\tau,\mu)$$



Observable status

SCET-I Observables

- Thrust
- Angularities
- Transverse Thrust

SCET-II Observables

- WTA-axis Broadening
- WTA-axis p_T-resummation
- e^+e^- jet rates

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Transverse Thrust [Preliminary]

- Measurement: [Becher, Garcia i Tormo; 15]

$$\omega_{TT}(\{k_i\}) = 4 s_\theta \sum_i \left(\left| \vec{k}_i^\top \right| - \left| \vec{n}^\top \cdot \vec{k}_i^\top \right| \right)$$

  - $k_\top^\mu$ transverse to beam axis
- Quark and gluon jet anomalous dimension agree with literature [Bell,Rahn,Talbert;19,20]

γ_1^q	This work	SoftSERVE
$C_F T_F$	$-21.09(1)$	-21.09
C_F^2	$10.80(17)$	10.61
$C_F C_A$	$83.67(16)$	$83.77(3)$

γ_1^g	This work	SoftSERVE
T_F^2	0	0
$C_F T_F$	$-4.00(2)$	-4
$C_A T_F$	$-16.95(4)$	-16.99
C_A^2	$96.33(21)$	$96.33(3)$

- NNLO matching correction

c_2^q	This work
$C_F T_F$	$-5.91(3)$
C_F^2	$42.55(59)$
$C_F C_A$	$116.66(61)$

c_2^g	This work
T_F^2	$7.86(1)$
$C_F T_F$	$-47.21(12)$
$C_A T_F$	$30.69(19)$
C_A^2	$172.92(81)$

Winner-take-all recombination scheme

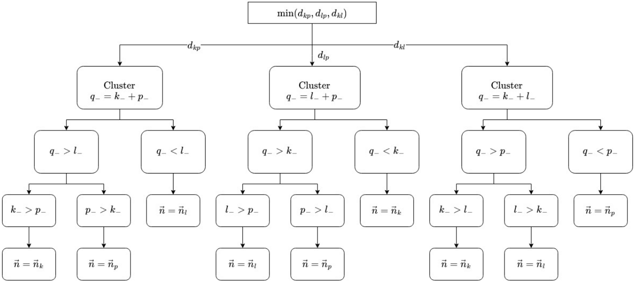
- Select the two closest particles and merge [Bertolini, Chan, Thaler; 13]

$$E_{(ij)} = E_i + E_j, \quad \vec{p}_{(ij)} = E_{(ij)} \left[\frac{\vec{p}_i}{|\vec{p}_i|} \theta(E_i - E_j) + \frac{\vec{p}_j}{|\vec{p}_j|} \theta(E_j - E_i) \right]$$

  $\rightarrow$ Massless pseudoparticle that points in the direction of the most energetic particle

  $\rightarrow$ This direction is the "winner-take-all" axis

- At leading power the soft function is the same as for the standard jet axis



WTA-axis p_T-resummation [Preliminary]

- Distance measure: k_T-algorithm, anti-k_T-algorithm and Cambridge-Aachen-algorithm
- Measurement: Vector sum of the transverse momentum projected onto the WTA axis
  $\rightarrow$ Needed for Boson-jet azimuthal decorrelation
- Quark and gluon anomaly exponent agree with literature [Bell,Rahn,Talbert;19,20]
- Quark remainder function comparison with `Event2` GSWZ:[Gutierrez-Reyes et al.;20]
  - Predictions differ by a sign, which has been confirmed in the meantime.

c_2	This work	GSWZ
$C_F T_F$	$12.72(5)$	$-13.0(3)$
C_F^2	$-13.84(37)$	$12.2(11)$
$C_F C_A$	$14.40(80)$	$-9.3(2)$
Quark:k_T-algorithm		

c_2	This work	GSWZ
$C_F T_F$	$12.27(31)$	$-12.5(3)$
C_F^2	$-25.90(59)$	$25.3(6)$
$C_F C_A$	$3.72(131)$	$-6.3(2)$
Quark:anti-k_T-algorithm		

c_2	This work	GSWZ
$C_F T_F$	$12.29(31)$	$-12.5(3)$
C_F^2	$-25.76(60)$	$24.5(6)$
$C_F C_A$	$7.41(132)$	$-6.7(2)$
Quark:Cambridge-Aachen-algorithm		

c_2	This work
T_F^2	$-5.96(1)$
$C_F T_F$	$170.48(6)$
$C_A T_F$	$9.19(5)$
C_A^2	$25.13(49)$
Gluon:k_T-algorithm	

Conclusion and Outlook

- We have reported on our automated approach for the calculation of beam and jet functions at NNLO
  - Complete setup for SCET-I and SCET-II observables

Beam functions

- DIS Angularities
- p_T-veto

Jet functions

- Transverse Thrust
- WTA-axis p_T-resummation

Outlook

- Automatic approach for polarised gluon jet and beam functions
- Development of a stand-alone `C++` code