Lattice Observables for PDFs

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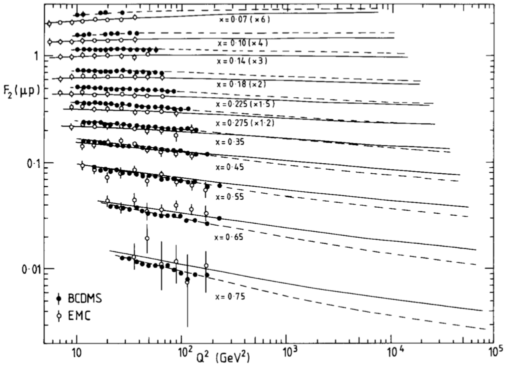
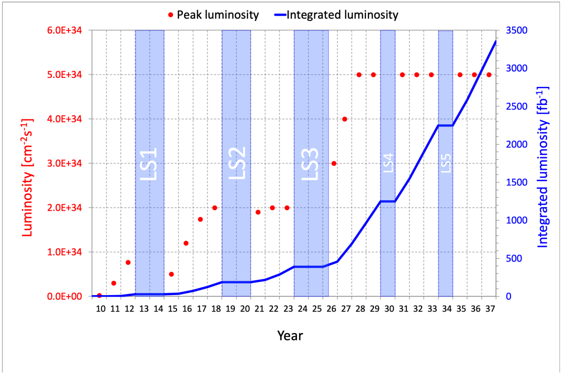


in collaboration with M Wilson, T Giani, C Monahan, K Cichy, J Karpie, K Orginos, A Radyushkin, S Zafeiropoulos

plan

1. PDFs/TMDs/GPDs in QFT
2. lattice observables
3. inverse problems

run 3/hi-lumi LHC: setting the target for theory predictions



theory predictions need to match the experimental precision
↪ increased *precision and accuracy* of *parton distribution functions*

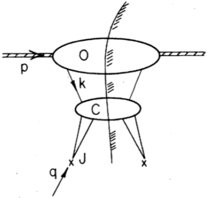
EIC and strong interactions

- hadron *tomography*
- understanding the nucleon spin
- understanding nucleons/nuclear physics from QCD
- current understanding of nucleon structure:

$$\mathcal{W}(x, k_\perp, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\Delta_\perp\cdot b_\perp}\, \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3}\, e^{i\left[(xp^+)z^- - k_\perp\cdot z_\perp\right]} \times \langle p + \frac{\Delta_\perp}{2} | \bar{\psi}(-z/2)\, \Gamma \lambda_A\, \mathcal{U}\, \psi(z/2) | p - \frac{\Delta_\perp}{2} \rangle$$

DIS hadronic tensor – factorization

$$H_{\mu\nu} = \int d^D y\, e^{iq\cdot y} \langle p | J_\mu(y) J_\nu(0) | p \rangle$$



$$H_{\mu\nu} = F_1(x,Q^2)\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) + F_2(x,Q^2)\left(p + \frac{1}{2x}q\right)_\mu \left(p + \frac{1}{2x}q\right)_\nu$$

expressed in terms of PDFs – physical, finite quantities

$$F_i(x,Q^2) = \int_x^1 \frac{d\xi}{\xi}\, C_i(\xi, Q^2, \mu^2) f_R(x/\xi, \mu^2) + \mathcal{O}(1/Q^2)$$

$$f(x) = \int \frac{dz^-}{2\pi} e^{i(xp^+)z^-} \langle p | \bar{\psi}(-z^-/2)\, \Gamma \lambda_A \mathcal{U}\, \psi(z^-/2) | p \rangle$$

lattice QCD for PDFs?

- cannot compute light-cone quantities in Euclidean field theory

- Xi. Ji (2013): compute the spatial correlator [ji 20]

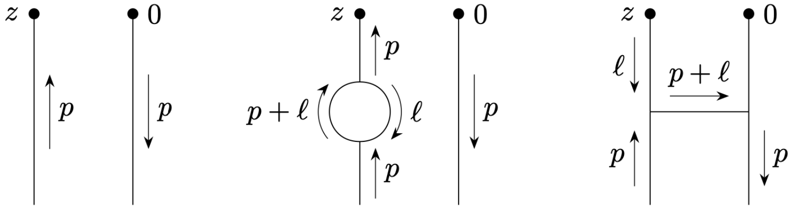
$$\langle p|\bar{\psi}(-z_3/2)\,\Gamma\lambda_A\,\mathcal{U}\,\psi(z_3/2)|p\rangle$$

- spun a lot of activity in LQCD community: **quasi-PDF**, **pseudo-PDF**, **Ioffe Time Distributions** – see recent reviews at Lattice conferences and dedicated workshops [constantinou et al 20]

- a factorization formula relates correlators in the spatial direction with the light-cone quantities (after renormalization)

toy-model computation

Ioffe time distribution $\widehat{\mathcal{M}}\left(\nu, z^2\right) = \langle p|\phi(z)\phi(0)|p\rangle$



(a) (b) (c)

at tree level

$$\widehat{\mathcal{M}}_a(\nu, z^2) = \exp[-ip \cdot z] = \exp[-i\nu] = \widehat{\mathcal{M}}^{(0)}(\nu, 0)$$

one-loop in MSbar

- light-cone separation – FT of PDFs

$$\widehat{\mathcal{M}}_R\left(\nu;\mu^2\right)=\left[1+\frac{\alpha}{6}\left(\log\frac{m^2}{\mu^2}+b\right)\right]\widehat{\mathcal{M}}^{(0)}\left(\nu,0\right) + \alpha\int_0^1 dx\,\left(1-x\right)\,\log\frac{\mu^2}{m^2\left(1-x+x^2\right)}\widehat{\mathcal{M}}^{(0)}\left(x\nu,0\right)$$

- spatial separation – FT of quasi-PDFs

$$\widehat{\mathcal{M}}_R\left(\nu,z_3^2;\mu^2\right)=\left[1+\frac{\alpha}{6}\left(\log\frac{m^2}{\mu^2}+b\right)\right]\widehat{\mathcal{M}}^{(0)}\left(\nu,0\right) + \alpha\int_0^1 dx\,\left(1-x\right)\,2K_0\left(mz_3\right)\widehat{\mathcal{M}}^{(0)}\left(x\nu,0\right)$$

factorization theorem

for $mz_3 \ll 1$ (small distances, large momenta)

$$2K_0\left(Mz_3\right) = -\log\left(m^2 z_3^2\right) + 2\log\left(2e^{-\gamma_E}\right) + \mathcal{O}\left(m^2 z_3^2\right)$$

and therefore

$$\widehat{\mathcal{M}}_R\left(\nu, -z_3^2; \mu^2\right) = \int_{-1}^{1} d\xi\, \tilde{C}\left(\xi\nu, \mu^2 z_3^2\right) \hat{f}_R\left(\xi, \mu^2\right) + O(m^2 z_3^2)$$

$$\tilde{C}\left(\xi\nu, \mu^2 z_3^2\right) = e^{i\xi\nu} - \alpha \int_0^1 dx\, (1-x) \log\left(\mu^2 z_3^2 \frac{e^{2\gamma_E}}{4}\right) e^{ix\xi\nu}$$

Wilson coefficient is IR-safe

z^2 dependence only at $\mathcal{O}(\alpha)$

QCD matrix elements

$$\boxed{M_{\Gamma,A}(z) = \bar{\psi}(z)\Gamma\lambda_A \, \mathrm{P}\exp\left(-ig\int_0^z d\eta\, A(\eta)\right)\psi(0)}$$

Ioffe time distributions

$$\mathcal{M}_{\gamma^\mu,A}(z,P) = \langle P|\mathcal{M}_{\gamma^\mu,A}(z)|P\rangle$$

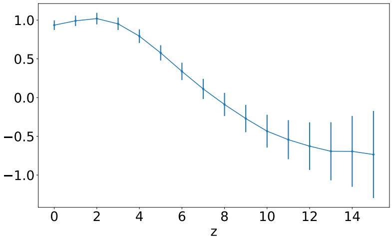
Lorentz covariance

$$\mathcal{M}_{\gamma^\mu,A}(z,P) = P^\mu h_{\gamma^\mu,A}(z\cdot P, z^2) + z^\mu h'_{\gamma^\mu,A}(z\cdot P, z^2)$$

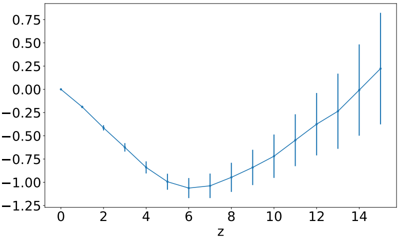
lattice observables - 1

$$\mathcal{O}^{\text{Re}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \text{Re}\left[\mathsf{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right] \qquad \mathcal{O}^{\text{Im}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \text{Im}\left[\mathsf{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right]$$

Real part



Imaginary part



[C Alexandrou et al 18]

systematic errors

- cut-off effects
- finite volume effects
- excited states contamination

- truncation effects
- higher-twist terms
- isospin breaking

$\hookrightarrow$ **renormalized lattice observables at the continuum limit**

lattice QCD yields a discrete set of datapoints to be included in PDF fits

stochastic solution of inverse problems

unknown model: $\theta \in \Theta$

Data: $Y = K\theta + \varepsilon$,   where: $\varepsilon \sim \pi_\varepsilon = \mathcal{N}(0, \Sigma)$

$$\theta \in \Theta \quad \Longrightarrow \quad \mathrm{Prob}(y|\theta) = \pi_\varepsilon(y - K\theta)$$

Bayes theorem

$$\mathrm{Prob}(\theta|y) = \frac{\mathrm{Prob}(\mathrm{y}|\theta)\mathrm{Prob}(\theta)}{\mathrm{Prob}(y)}$$

choose a parametrization, find the posterior distribution

replicas

data: $y_i, i = 1, \ldots N_{\rm dat}$ are treated as stochastic variables $\sim \mathcal{N}(Y, C)$

bootstrap: replicas simulate the fluctuations of y

$$y^{(k)} = Y + \varepsilon^{(k)}, \;\; k = 1, \ldots N_{\rm rep}$$

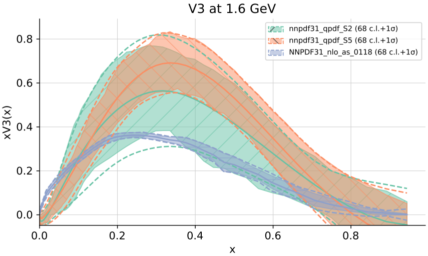
for each replica we minimize:

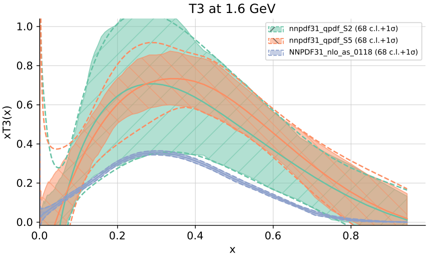
$$\chi^{2(k)}[\theta] = \frac{1}{N_{\rm dat}} \sum_{ij} \left(g[\theta]^{(k)} - y^{(k)} \right)_i C^{-1}_{ij} \left(g[\theta]^{(k)} - y^{(k)} \right)_j + \mathrm{priors}$$

$\left\{ \theta^{(k)}, k = 1, \ldots, N_{\rm rep} \right\}$ yields the probability distribution in model space

fit results

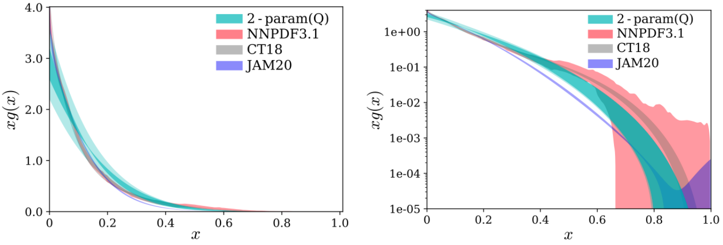
lattice data can be included in NNPDF fits, just like any other data!





[ldd et al 19, using ETMC data]

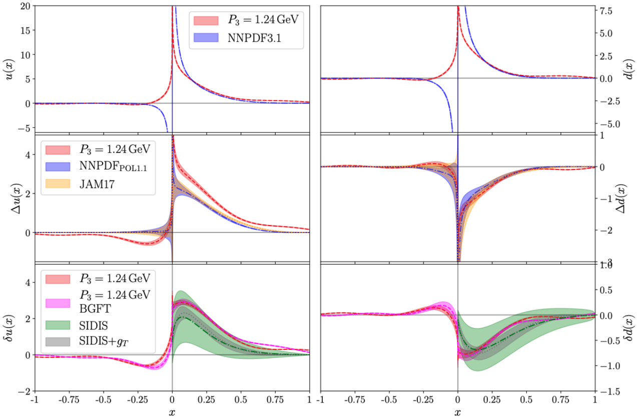
HadStruc



$$m_\pi = 358 \text{ MeV}$$

[Khan et al 21]

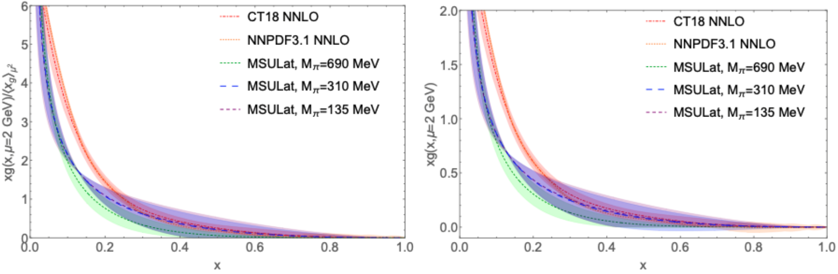
ETMC



$$m_\pi = 260 \text{ MeV}$$

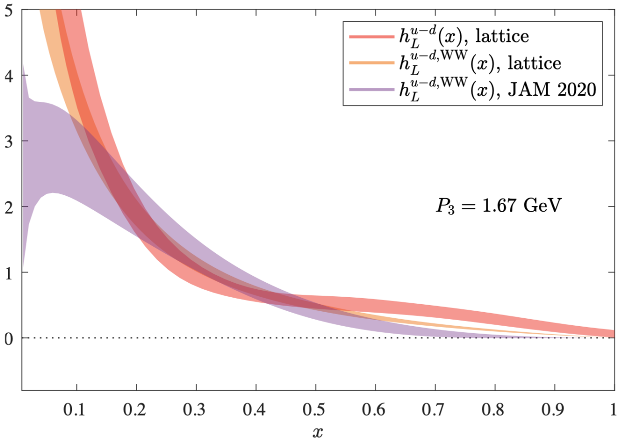
[Alexandrou et al 21]

MSULat



[Fan et al 21]

twist-3 and Wandzura-Wilczek



$m_\pi = 260$ MeV

[Bhattacharya et al 21]

outlook

- light-cone PDFs + factorization describe the structure of the proton
- current extraction from data is very precise + improving
- lattice quantities needs to be properly renormalised and extrapolated to the continuum limit
  $\hookrightarrow$ are on the same footing as experimental data
- lattice data provide complementary information, can be included in global fits like any other data (inverse problem)
- identify the areas where a significant phenomenological impact from lattice QCD is possible
- GPDs for EIC