

# Top-pair events with $B$ -hadrons at the LHC

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Based on arXiv:2102.08267 and arXiv:2210.06078

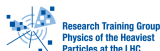
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Collaborative Research Center TRR 257



Particle Physics Phenomenology after the Higgs Discovery



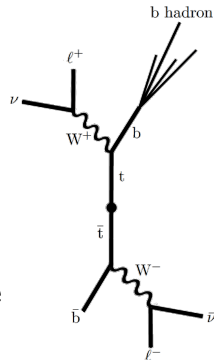
# Top-pairs with $B$ -hadrons

- Process considered:

$$p p \rightarrow t(\rightarrow B W^+ + X) \bar{t}(\rightarrow \bar{b} W^-)$$

$$\quad \quad \quad \hookrightarrow \ell^+ \nu_\ell \quad \quad \quad \hookrightarrow \ell^- \bar{\nu}_\ell$$

- Measurements involving  $b$ -jets suffer from large jet energy scale uncertainties
- Measurements of  $B$ -hadron momenta very precise  $\Rightarrow$  high-precision top-mass determination
- Production of hadrons is a non-perturbative effect

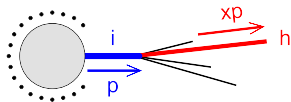
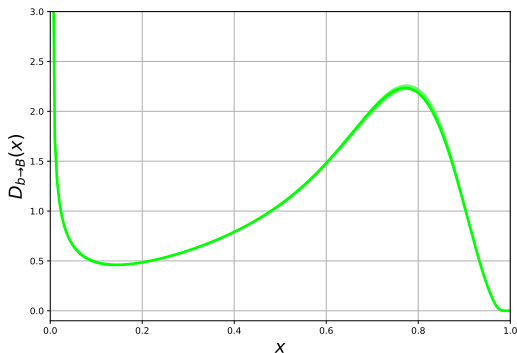


# Introduction to fragmentation

- Idea: describe production of hadrons using two steps
  - ① The production of partons using perturbation theory
  - ② The (non-perturbative) fragmentation of these partons into the observed hadrons
- Transition parton  $\rightarrow$  hadron in the final state
- Mathematically similar to transition hadron  $\rightarrow$  parton in the initial state
- Factorisation accurate up to  $\mathcal{O}\left(\frac{m_h}{Q}\right)$

# Fragmentation functions

- ‘Probability distribution’ to find a hadron  $h$  with a fraction  $x$  of the parton  $i$ 's momentum:  $D_{i \rightarrow h}(x)$
- Only considers longitudinal kinematics;  $i, h$  massless
- Non-perturbative: fitted to data
- Scale dependent
- Analogous to PDFs
- No parton showers used



# The software

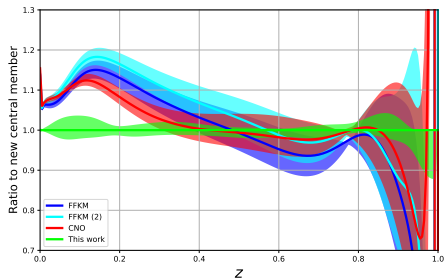
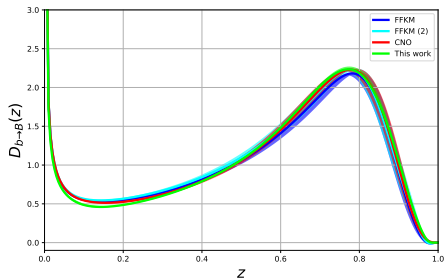
- Calculations were performed using C++ library STRIPPER
- Many NNLO firsts over the years. Recently:
  - Isolated photon + dijet production at the LHC  
*Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia (2023)*
  - Event shapes at the LHC  
*Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet (2023)*
  - $Wb\bar{b}$  production at the LHC *Hartanto, Poncelet, Popescu, Zoia (2022)*
  - Three-jet production at the LHC *Czakon, Mitov, Poncelet (2021)*
  - Diphoton + jet at the LHC *Chawdhry, Czakon, Mitov, Poncelet (2021)*
  - Exact top-mass effects in Higgs production at the LHC  
*Czakon, Harlander, Klappert, Niggetiedt (2021)*
  - ...
- First implementation of fragmentation in a general code for NNLO cross sections
- Fully general implementation; not limited to cases presented in this talk

# New fragmentation function fits

- No fits based on PFF approach available at NNLO
- Required for fully consistent results
- First paper: FF sets based on three different compromises
- Two based on NNLO calculation within SCET/HQET  
*Fickinger, Fleming, Kim, Mereghetti (2016)*
- One based on NLO calculation within PFF approach  
*Cacciari, Nason, Oleari (2006)*
- Different compromises consistent within uncertainties
- Nonetheless better to use a consistent fit

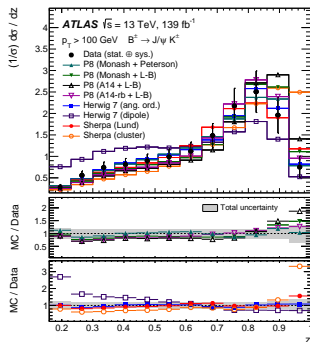
# First NNLO fit within the PFF approach

- Based on data from ALEPH, DELPHI, OPAL and SLD.
- **Blue/cyan**: based on Fickinger, Fleming, Kim, Mereghetti (2016)
- **Red**: based on Cacciari, Nason, Oleari (2006)
- **Green**: Czakon, TG, Mitov, Poncelet (2022)
- First NNLO PFF with NNLL soft-gluon resummation



# Incorporating $B$ -hadron decays

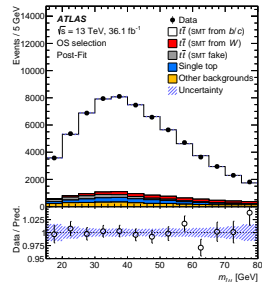
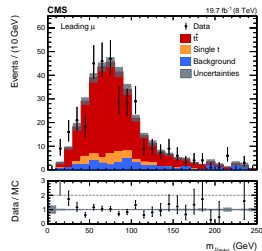
- First paper: predictions for fully reconstructed  $B$ -hadrons only
- Full reconstruction of  $B$ -hadrons difficult in practice
- Not enough  $t\bar{t}$  events for distributions  
 $\Rightarrow$  Cannot compare first results to experiment at present
- Could compare to data if process changed to  $p p \rightarrow B + X$
- Fully reconstructed  $B$ -hadrons in arXiv:2108.11650 (ATLAS)
- Not a problem for the software, but process lacks information on  $m_t$





# Incorporating $B$ -hadron decays

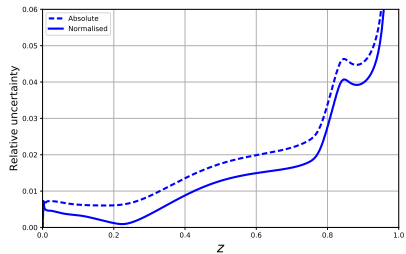
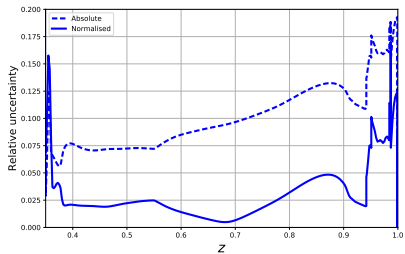
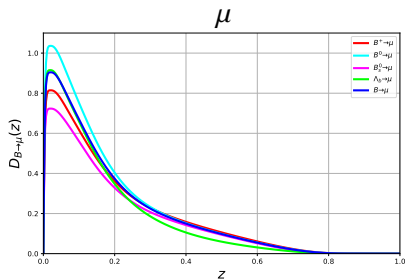
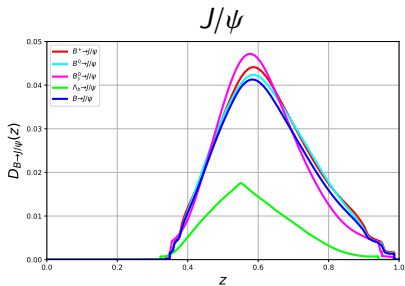
- Solution: incorporate  $B$ -hadron decays
- Only reconstruct some decay products  
 $\Rightarrow$  Significantly boost statistics
- Examples:  
 CMS: arXiv:1608.03560 ( $B \rightarrow J/\psi + X$ )  
 ATLAS: arXiv:2209.00583 ( $B \rightarrow \mu + X$ )
- Still considering top-pair production,  
 but comparison with experiment now possible



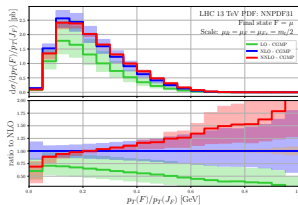
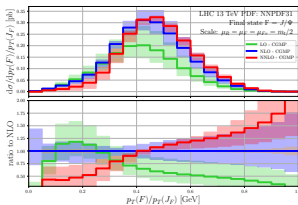
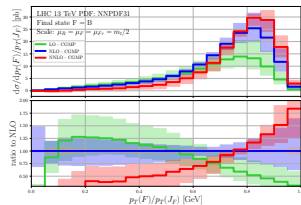
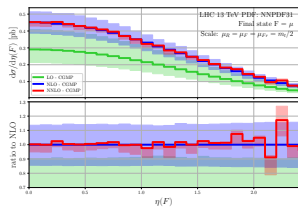
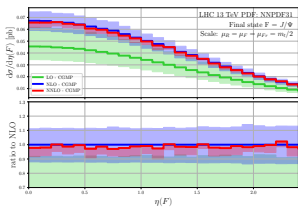
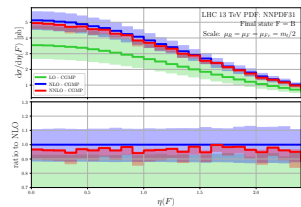
# Including $B$ -hadron decays in theory predictions

- $B$ -hadron treated as massless  $\Rightarrow$  cannot decay
- Most obvious solution:
  - 1 Map massless  $B$ -hadron momentum to massive one
  - 2 Decay massive  $B$ -hadron using external package
- Not ideal:
  - Momentum remapping ambiguous
  - Need to interface to external package (e.g. EvtGen)
- Easier and more consistent solution:
  - 1 Modify fragmentation function to incorporate the decay
  - 2 Run the program as usual, no modifications required
- Need the 'fragmentation function'  $D_{B \rightarrow d}$  for the decay  $B \rightarrow d$
- Can be obtained from the differential decay rate using EvtGen

# $B$ -hadron decay fragmentation functions

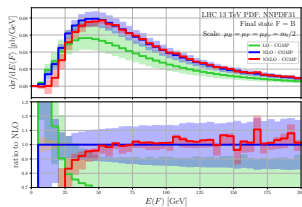


# Results: pseudorapidity and jet-ratio spectra

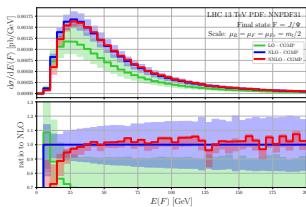
**B**
**J/ψ**
**μ**


# Results: energy and transverse momentum spectra

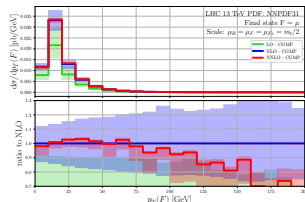
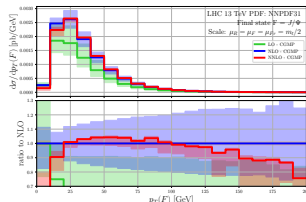
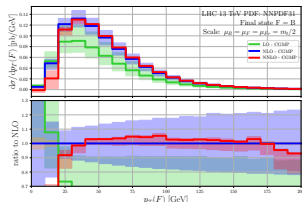
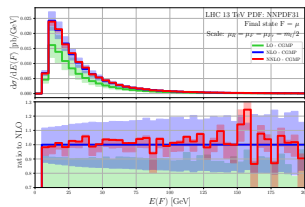
$B$



$J/\psi$



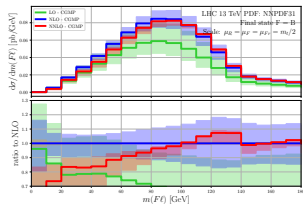
$\mu$



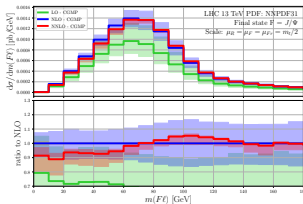
# Results: invariant mass spectra

Two versions:  $m(F\ell) \equiv m(F\ell^+)$  and  $m(F\ell)_{\min} \equiv \min(m(F\ell^+), m(F\ell^-))$

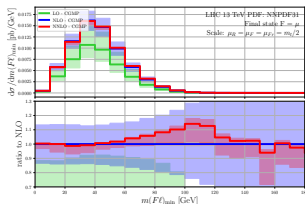
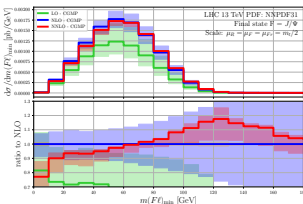
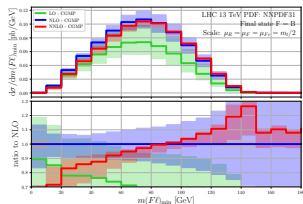
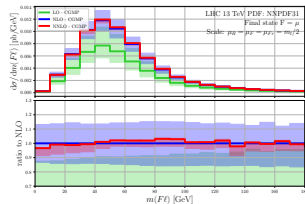
$B$



$J/\psi$

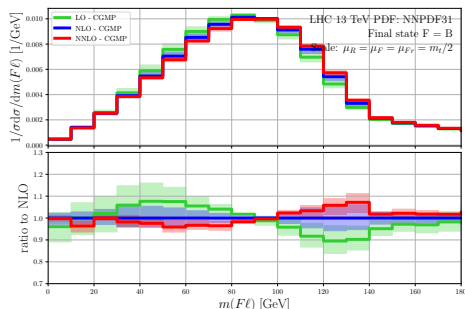


$\mu$

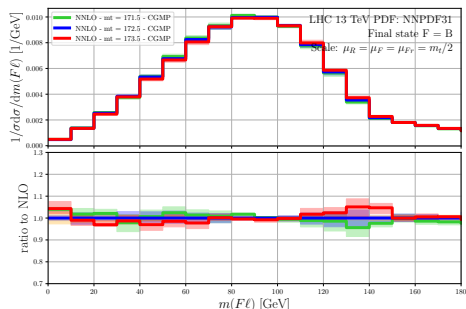


# Measuring $m_t$ using the $m(B\ell)$ spectrum

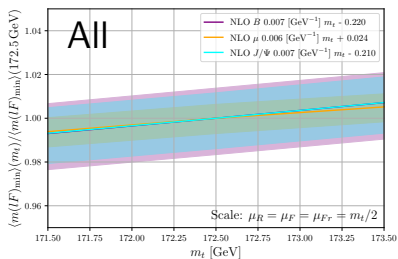
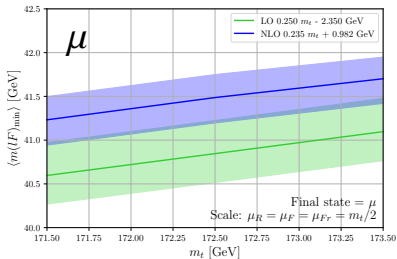
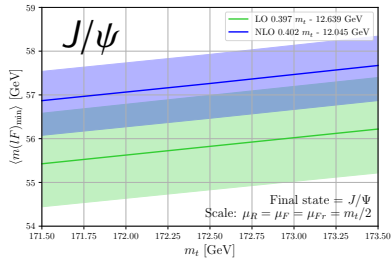
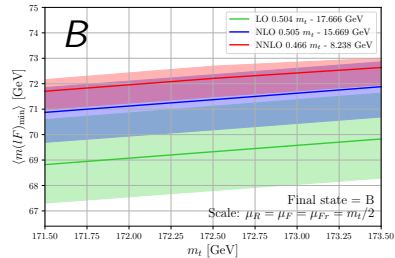
## LO, NLO, NNLO for fixed $m_t$



## NNLO, varying $m_t$ by 1 GeV



- Note: NNLO effects almost identical to a  $\sim 1$  GeV shift in  $m_t$

Measuring  $m_t$  using  $\langle m(F\ell)_{\min} \rangle$ 



# Conclusion & outlook

- First NNLO+NNLL  $B$ -hadron FF fit within PFF approach
- Introduced novel method of integrating decays into FFs
- First application: top-quark pairs at the LHC
- Much smaller uncertainties at NNLO than at NLO  
⇒ Reduction of theory uncertainties on extracted  $m_t$
- Framework completely general: any process, any hadron, any decay

We are very interested in comparing to data in dedicated studies!

# Backup slides

# Phase-space cuts

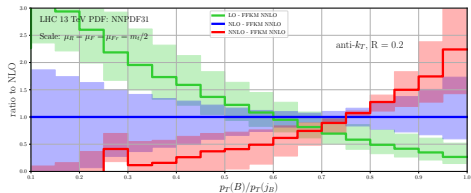
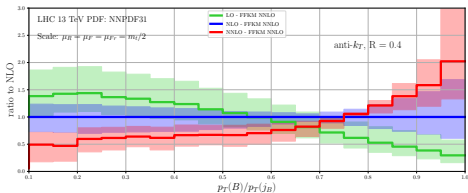
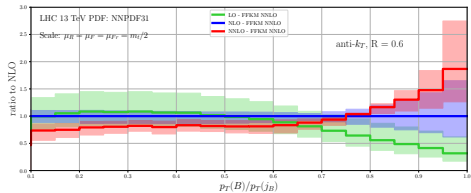
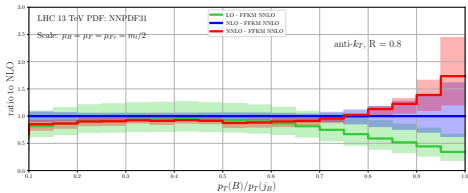
Differential distributions:

- $p_T(\ell) > 25 \text{ GeV}$ ,  $|\eta(\ell)| < 2.5$
- at least 2 anti- $k_T$  jets ( $R = 0.4$ ) with  $p_T(j) > 25 \text{ GeV}$  and  $|\eta(j)| < 2.5$
- $\Delta R(\ell, j) > 0.4$
- $p_T(F) > 8 \text{ GeV}$  and  $|\eta(F)| < 2.5$ ,  $F$  must be part of one jet

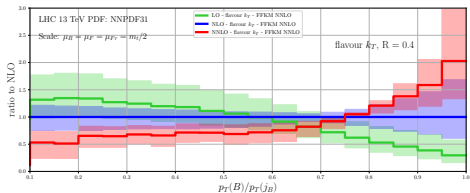
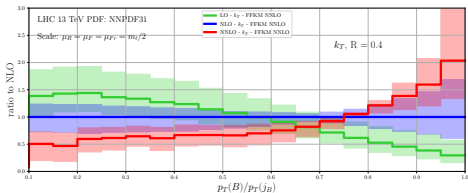
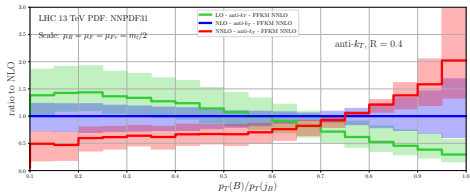
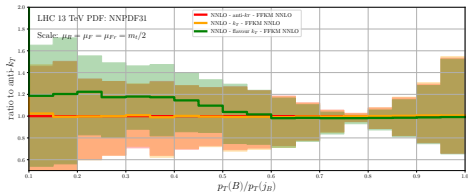
Top-quark-mass extraction:

- $p_T(\ell) > 25 \text{ GeV}$ ,  $|\eta(\ell)| < 2.5$
- $p_T(F) > 8 \text{ GeV}$  and  $|\eta(F)| < 2.5$

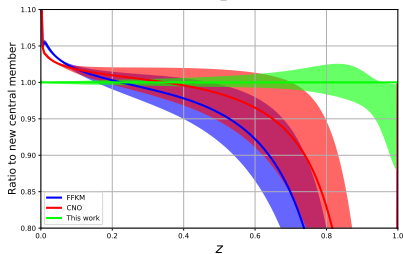
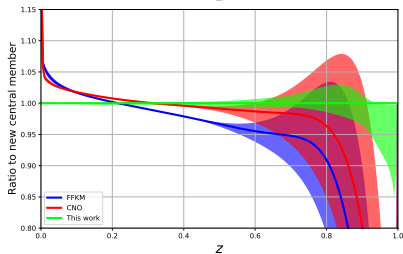
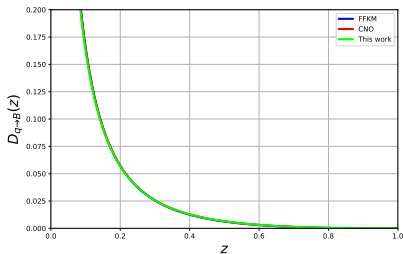
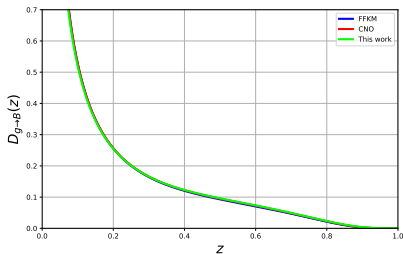
# Jet ratio: $R$ -dependence



# Jet ratio: jet-algorithm-dependence



# First NNLO fit within the PFF approach



# Including B-hadron decays in theory predictions

- Assume isotropic decay:  $d\Gamma(B \rightarrow \mu + X) = f(E_\mu) dE_\mu d\cos\theta_\mu d\phi_\mu$
- Valid for spin-0 particles (e.g. weakly-decaying B-mesons)
- Normalize  $E_\mu$  using  $m_B \Rightarrow f(E_\mu) dE_\mu \rightarrow f(y) dy$
- Boost from B-hadron rest frame to  $E_B \gg m_B$  and integrate over the angles and  $y$ , fixing  $z = E_\mu/E_B$

$$\Rightarrow \frac{d\Gamma(B \rightarrow \mu + X)}{dy} \rightarrow D_{B \rightarrow \mu}(z)$$

- $D_{B \rightarrow \mu}$  is the 'fragmentation function' for transition  $B \rightarrow \mu$

# Including B-hadron decays in theory predictions

- Can calculate  $D_{B \rightarrow \mu}$  once and for all
- $D_{B \rightarrow \mu}$  combines with known  $D_{i \rightarrow B}$  via convolution
- Only requirement: must know  $f(E_\mu)$
- Can be obtained using e.g. EvtGen
- Works for any descendant, not just muons
- Vast amount of data from B-factories  
 $\Rightarrow f(E_\mu)$  expected to be more precise than  $D_{i \rightarrow B}$



## Decay fragmentation function derivation

$$d\Gamma(B \rightarrow d + X) = \frac{1}{4\pi} f(E_d^{\text{rest}}) dE_d^{\text{rest}} d\cos(\theta) d\phi \stackrel{y = \frac{E_d^{\text{rest}}}{m_B}}{=} \frac{m_B}{4\pi} f(y m_B) dy d\cos(\theta) d\phi$$

Boost to  $E_B \gg m_B$  along the z-axis and fix  $z = \frac{E_d}{E_B}$  using

$$\begin{aligned} \delta\left(z - \frac{E_d}{E_B}\right) &= \delta\left(z - \gamma_B \frac{E_d^{\text{rest}} + \beta_B \cos(\theta) \sqrt{(E_d^{\text{rest}})^2 - m_d^2}}{E_B}\right) \\ &\approx \delta\left(z - \frac{E_d^{\text{rest}} + \cos(\theta) \sqrt{(E_d^{\text{rest}})^2 - m_d^2}}{m_B}\right) \\ &= \delta\left(z - y - \cos(\theta) \sqrt{y^2 - \frac{m_d^2}{m_B^2}}\right) \end{aligned}$$

## Decay fragmentation function derivation

Integrating over the angles and  $y$  yields

$$\begin{aligned}
 & \frac{d\Gamma(B \rightarrow d + X)}{dz} \\
 &= \int_0^{2\pi} \int_{-1}^1 \int_0^1 \frac{m_B}{4\pi} f(y m_B) \delta\left(z - y - \cos(\theta) \sqrt{y^2 - \frac{m_d^2}{m_B^2}}\right) dy d\cos(\theta) d\phi \\
 &= \int_0^1 \frac{m_B}{2\sqrt{y^2 - \frac{m_d^2}{m_B^2}}} f(y m_B) \theta\left(1 - \frac{(z-y)^2}{y^2 - \frac{m_d^2}{m_B^2}}\right) dy \\
 &= \int_{\frac{z}{2} + \frac{m_d^2}{2zm_B^2}}^1 \frac{m_B}{2\sqrt{y^2 - \frac{m_d^2}{m_B^2}}} f(y m_B) dy \equiv \Gamma_B D_{B \rightarrow d}(z)
 \end{aligned}$$