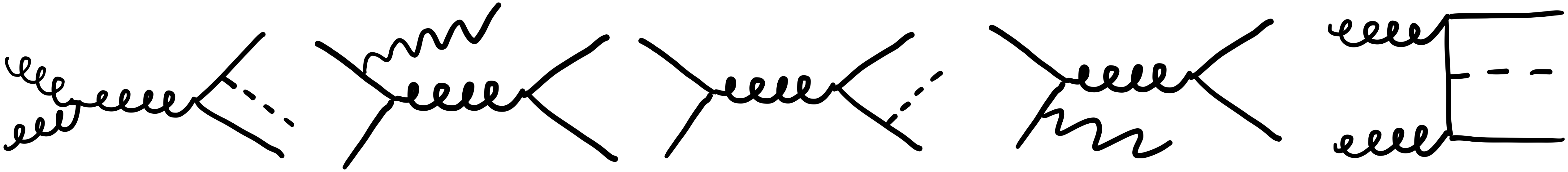


PRECISION QCD FOR ASSOCIATED TOP PRODUCTION

QCD@LHC2023

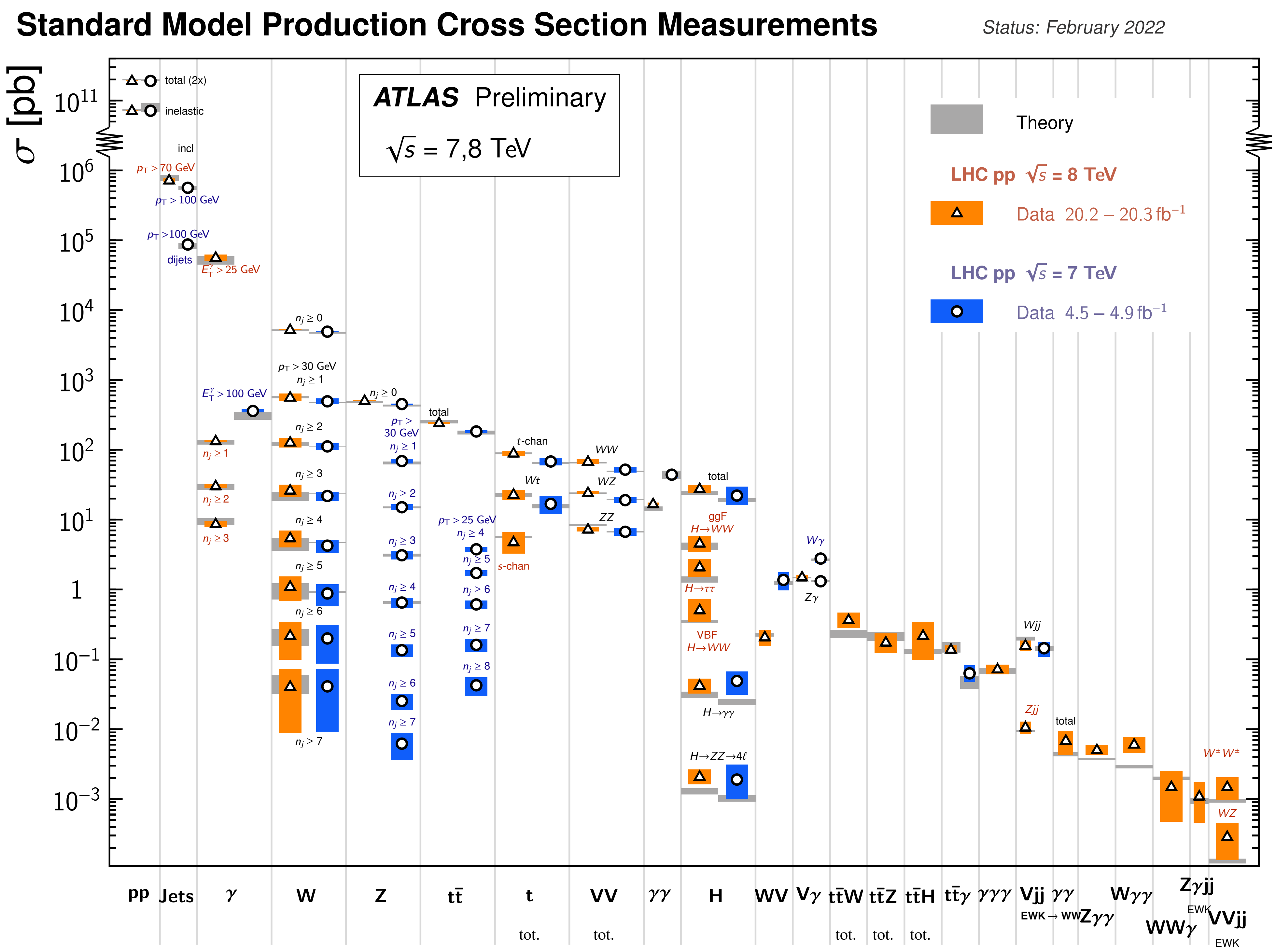
Durham , 08/09/2023

UNIVERSITÀ
DEGLI STUDI
DI MILANO

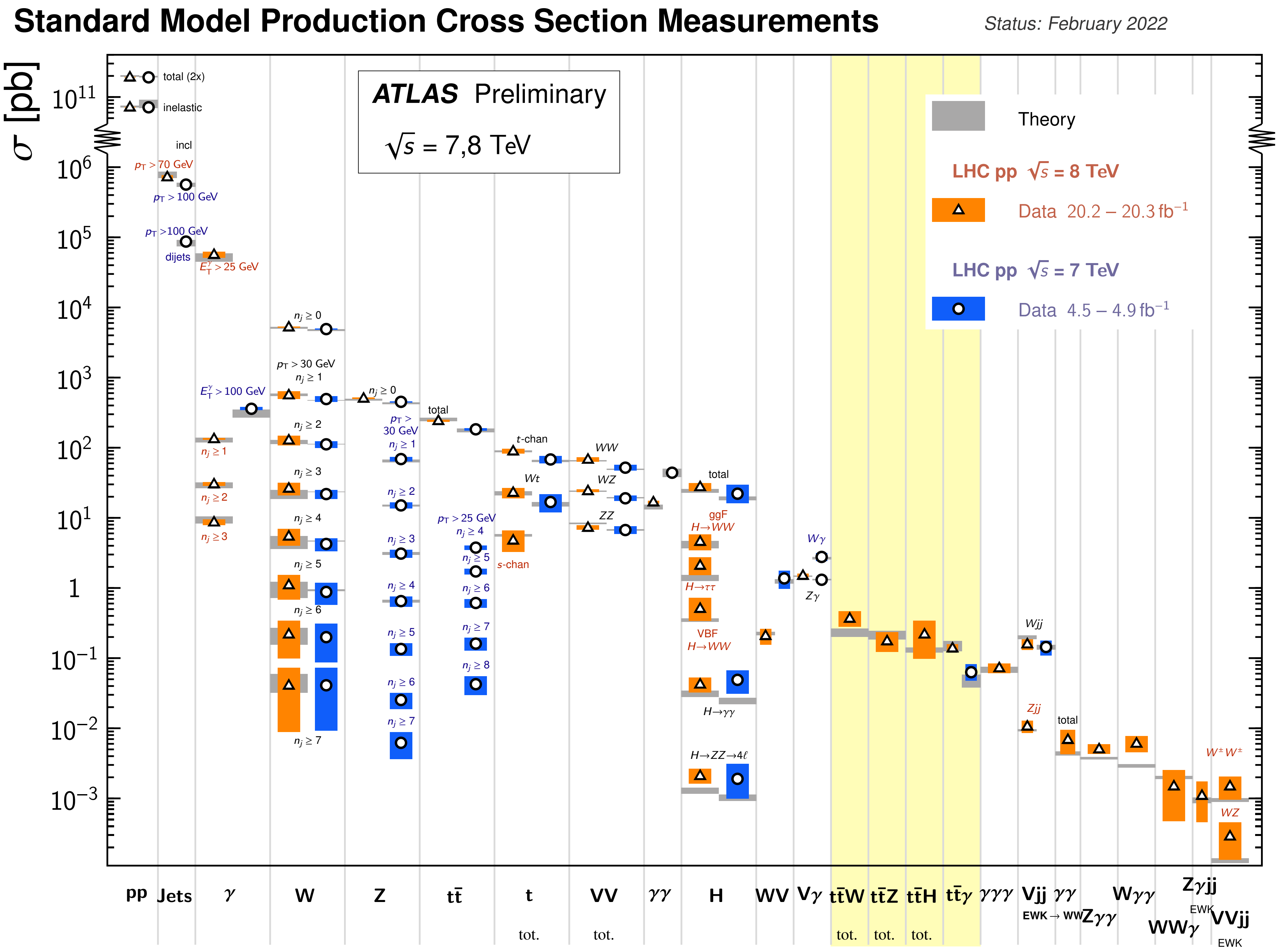
Simone Devoto

In collaboration with: L. Buonocore, S. Catani, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini

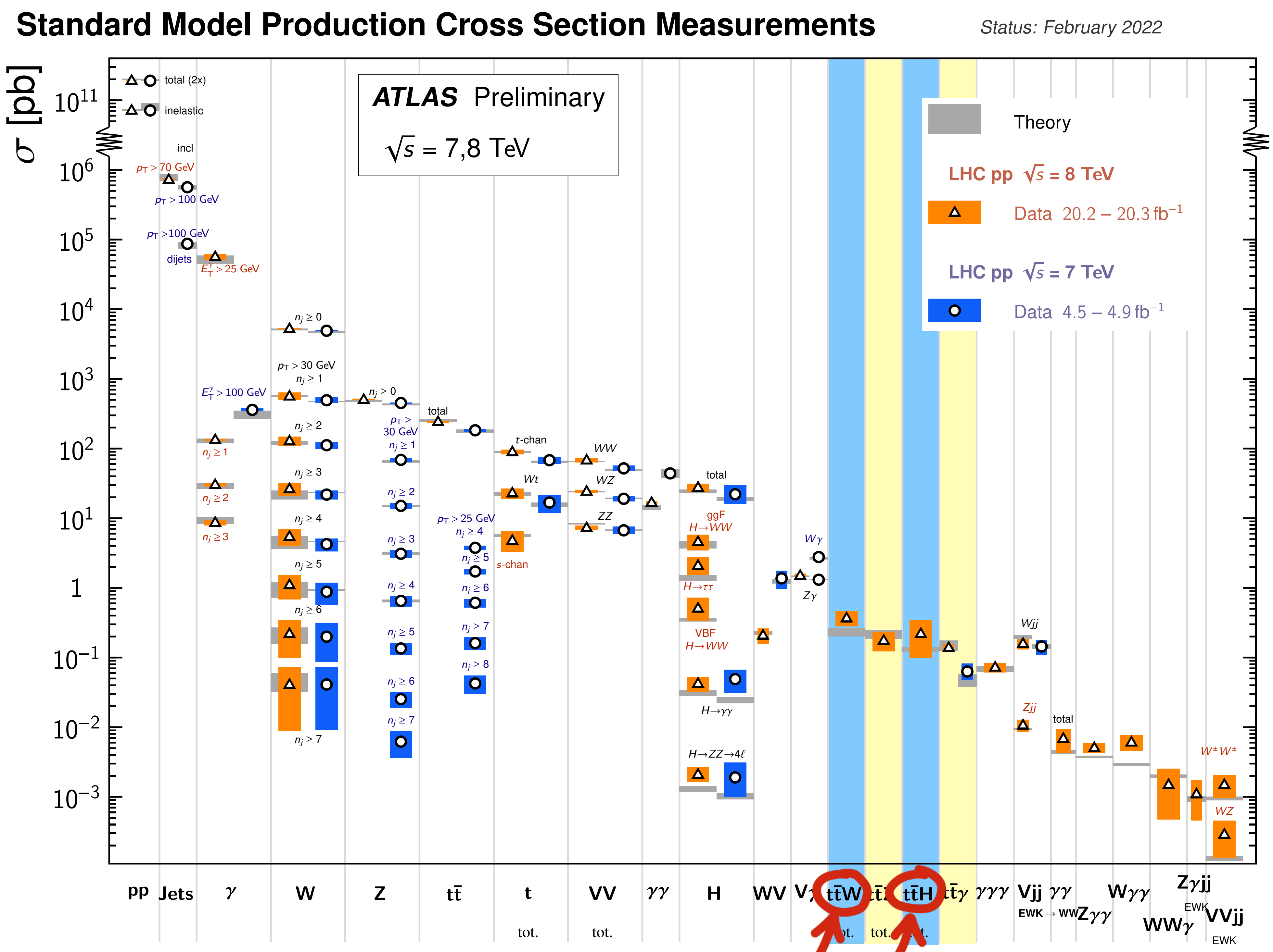
INTRODUCTION



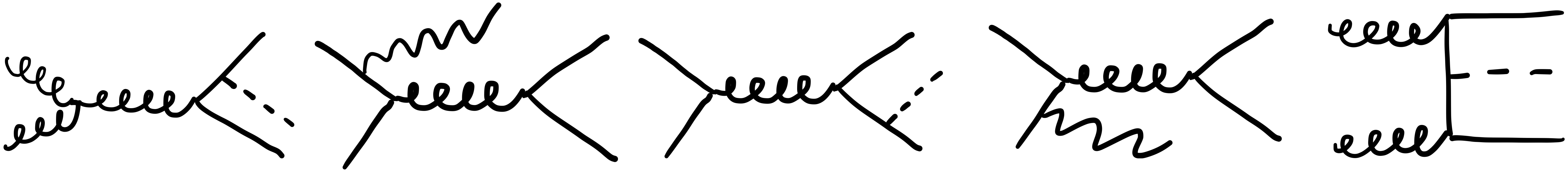
INTRODUCTION



INTRODUCTION

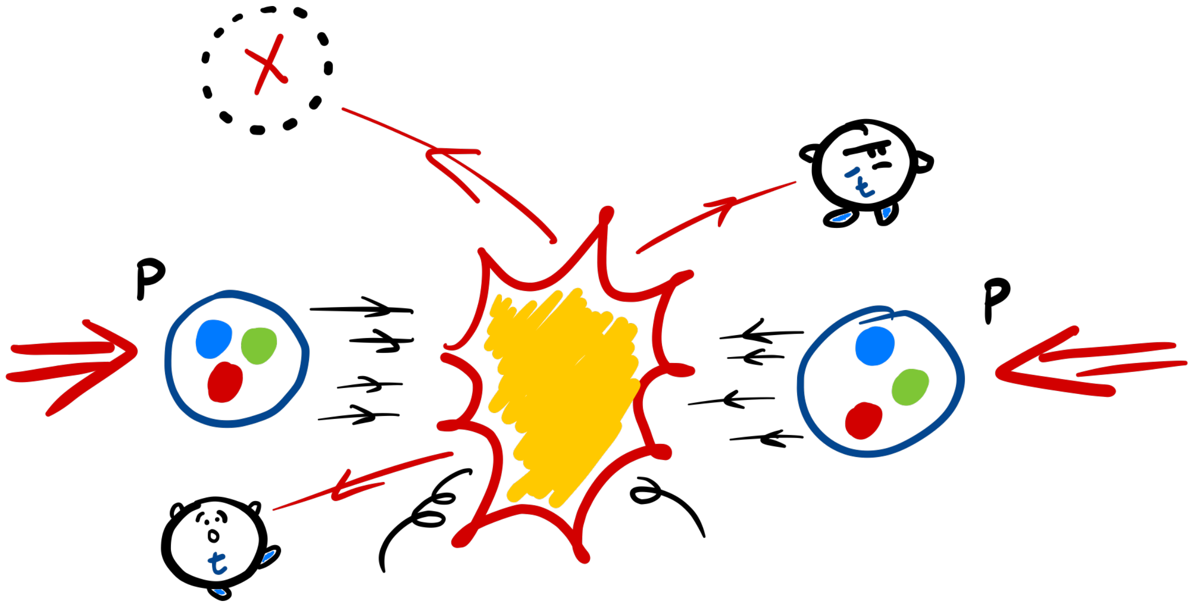


This talk!



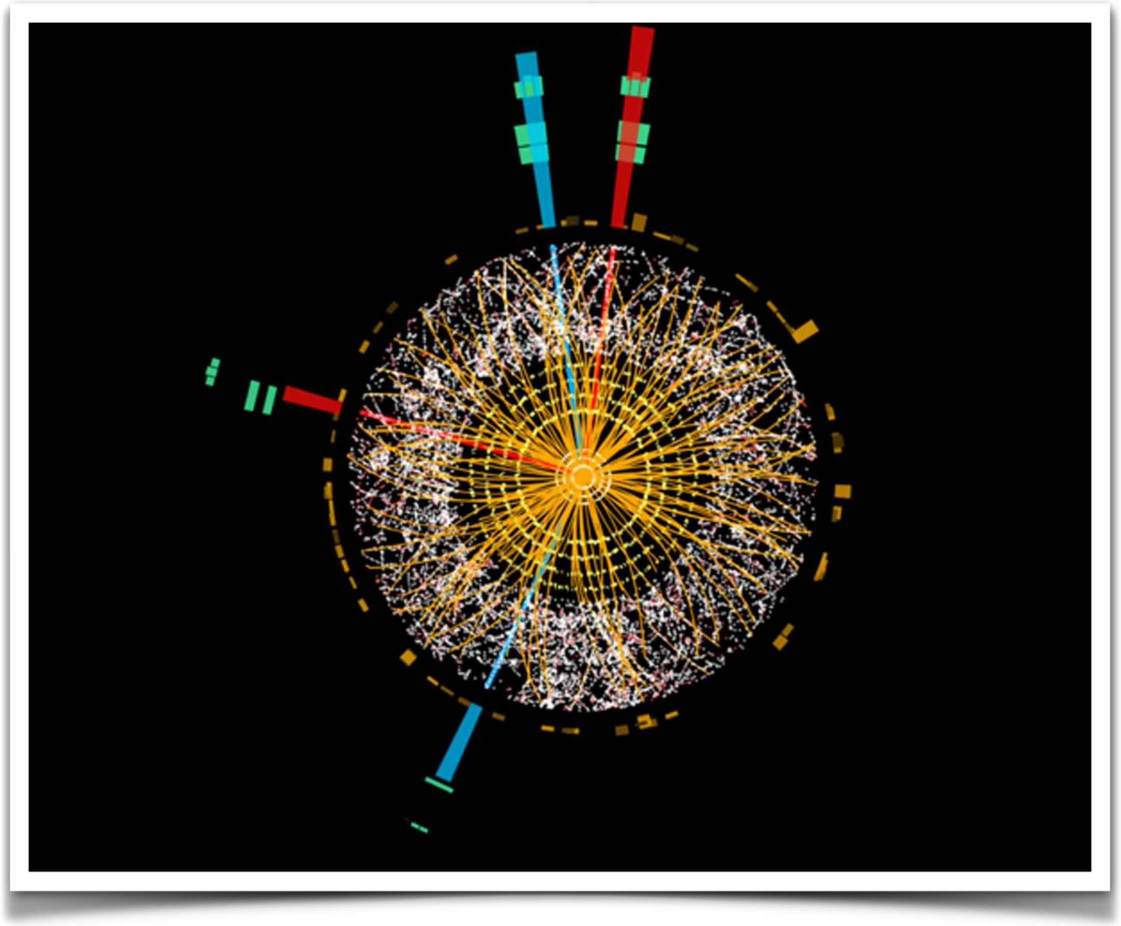
CONTENTS

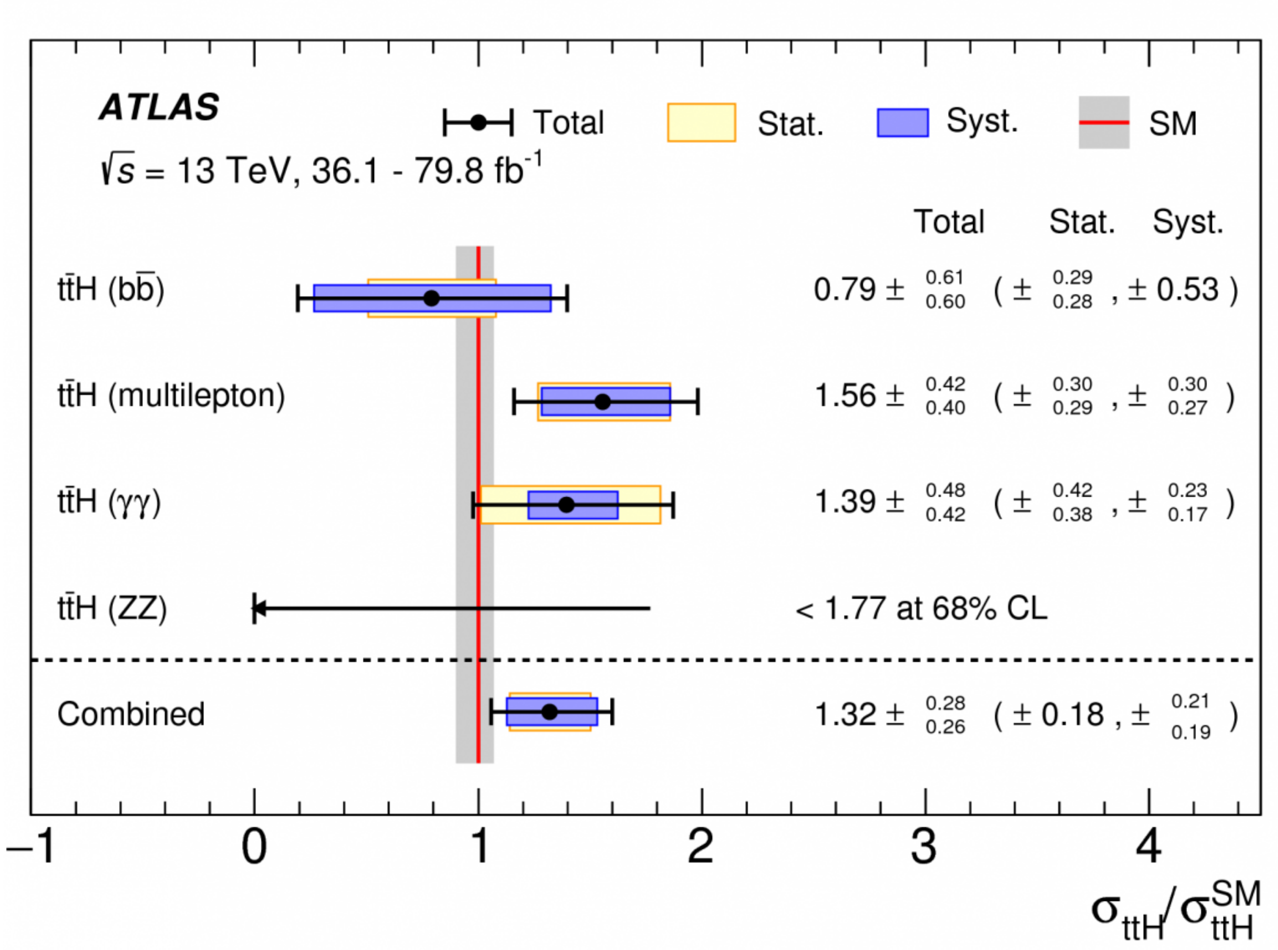
- **Motivations**;
- Theory bottlenecks:
  - subtraction;
  - two-loop amplitudes;
- **$t\bar{t}H$ @ NNLO;**
- **$t\bar{t}W$ @ NNLO**;
- **Conclusions**.



INTRODUCTION $(t\bar{t}H)$

- The **discovery of the Higgs boson** in 2012 confirmed one of the most glaring predictions of the Standard Model.
- The **study of the Higgs boson** proprieties is one of the priorities of LHC.
- Special role played by the **top quark**: strong coupling because of the top mass!
- $t\bar{t}H$ production allows direct measurement of the **top-quark Yukawa coupling**! (possible window on new physics scenarios…)





[M.Cepeda et al.: arXiv 1902.00134]

STATUS OVERVIEW ($t\bar{t}H$)

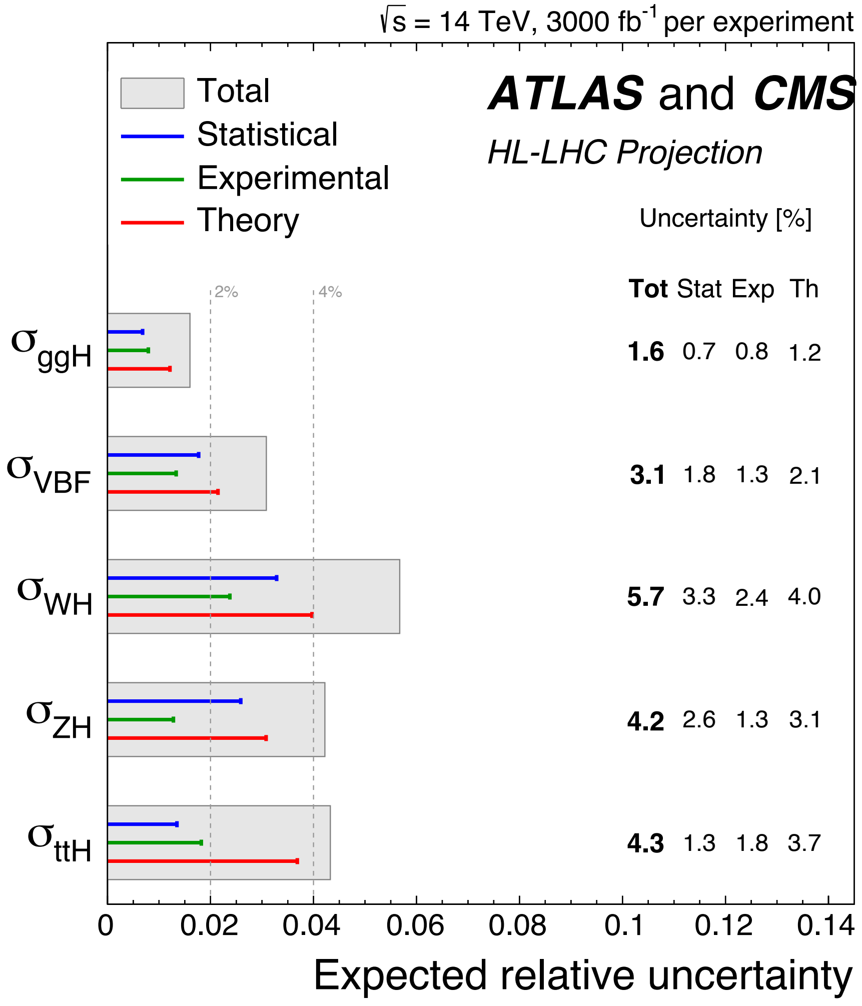
THEORY

- **NLO QCD**: *[W. Beenakker, S. Dittmaier, M. Krämer, B. Plumper, M. Spira, and P. Zerwas; 0107081, 0211352], [L. Reina and S. Dawson; 0107101], [L. Reina, S. Dawson, and D. Wackeroth; 0109066], [S. Dawson, L. Orr, L. Reina, and D. Wackeroth; 0211438], [S. Dawson, C. Jackson, L. Orr, L. Reina, and D. Wackeroth; 0305087], [A. Denner and R. Feger, 1506.07448];*
- **NLO EW**: *[S. Frixione, V. Hirschi, D. Pagani, H. Shao, and M. Zaro; 1407.0823, 1504.03446], [Y. Zhang, W.-G. Ma, R.-Y. Zhang, C. Chen, and L. Guo; 1407.1110];*
- **NLO QCD + EW**: *[A. Denner, JN. Lang, M. Pellen, and S. Uccirati; 1612.07138];*
- **Resummation of soft gluons**: *[A. Kulesza, L. Motyka, T. Stebel, and V. Theeuwes; 1509.02780, 1704.03363], [A. Broggio, A. Ferroglia, B. D. Pecjak, A. Signer, and L. L. Yang; 1510.01914], [A. Broggio, A. Ferroglia, B. D. Pecjak, and L. L. Yang; 1611.00049], [A. Broggio, A. Ferroglia, R. Frederix, D. Pagani, B. D. Pecjak, and I. Tsinikos; 1907.04343], [W.-L. Ju and L. L. Yang; 1904.08744], [A. Kulesza, L. Motyka, D. Schwartländer, T. Stebel, and V. Theeuwes; 2001.03031]*
- **First steps to NNLO**: *off-diagonal channels [S. Catani, I. Fabre, M. Grazzini, S. Kallweit; 2102.03256]*

- Current theoretical uncertainties $\mathcal{O}(10\%)$

EXPERIMENTS

- **ATLAS collaboration**: *[1806.00425];*
- **CMS collaboration**: *[1804.02610].*

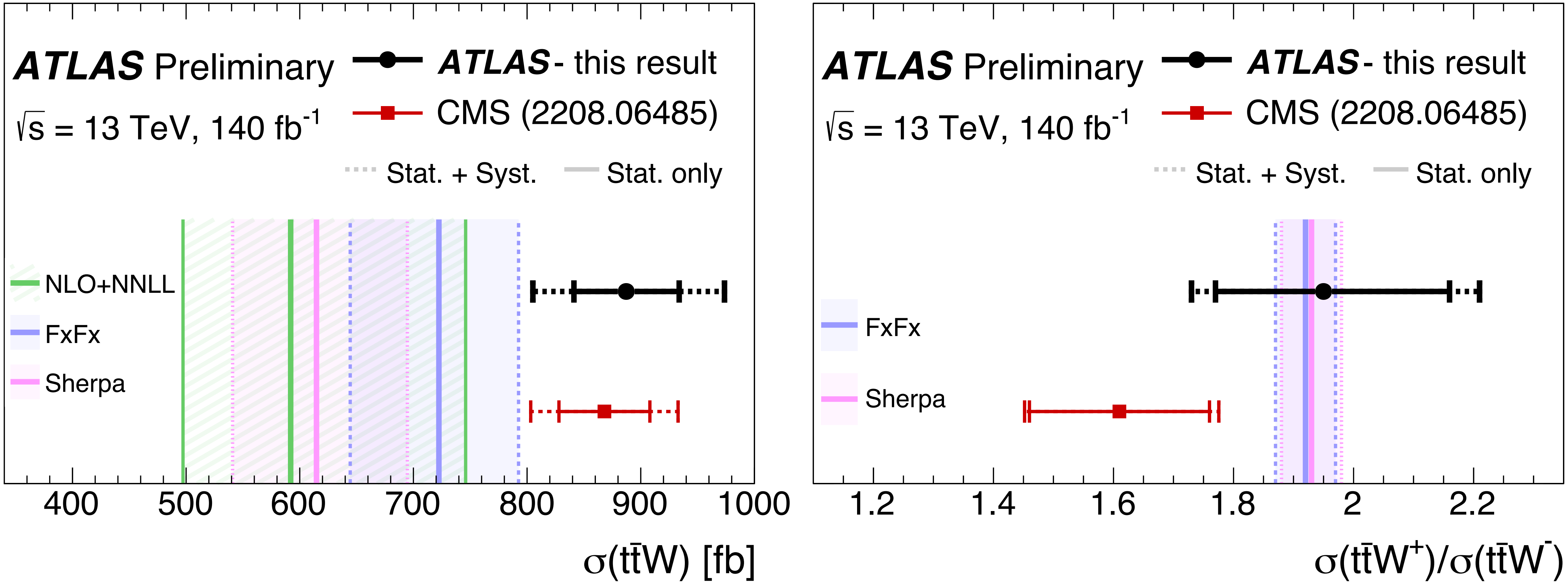


[M.Cepeda et al.: arXiv 1902.00134]

- Current experimental uncertainties $\mathcal{O}(20\%)$
- Expected at the end of HL-LHC $\mathcal{O}(2\%)$

INTRODUCTION ($t\bar{t}W$)

- Together with $t\bar{t}H$ production, one of the **most massive** Standard Model (SM) signatures accessible at the LHC;
- Relevant as a $t\bar{t}H$ **background**;
- Measurements carried out by the ATLAS and CMS collaborations lead to rates consistently **higher than the SM predictions**;
- Most recent measurements confirm excess at the 2σ **level**.



[ATLAS-CONF-2023-019]

STATUS OVERVIEW $(t\bar{t}W)$

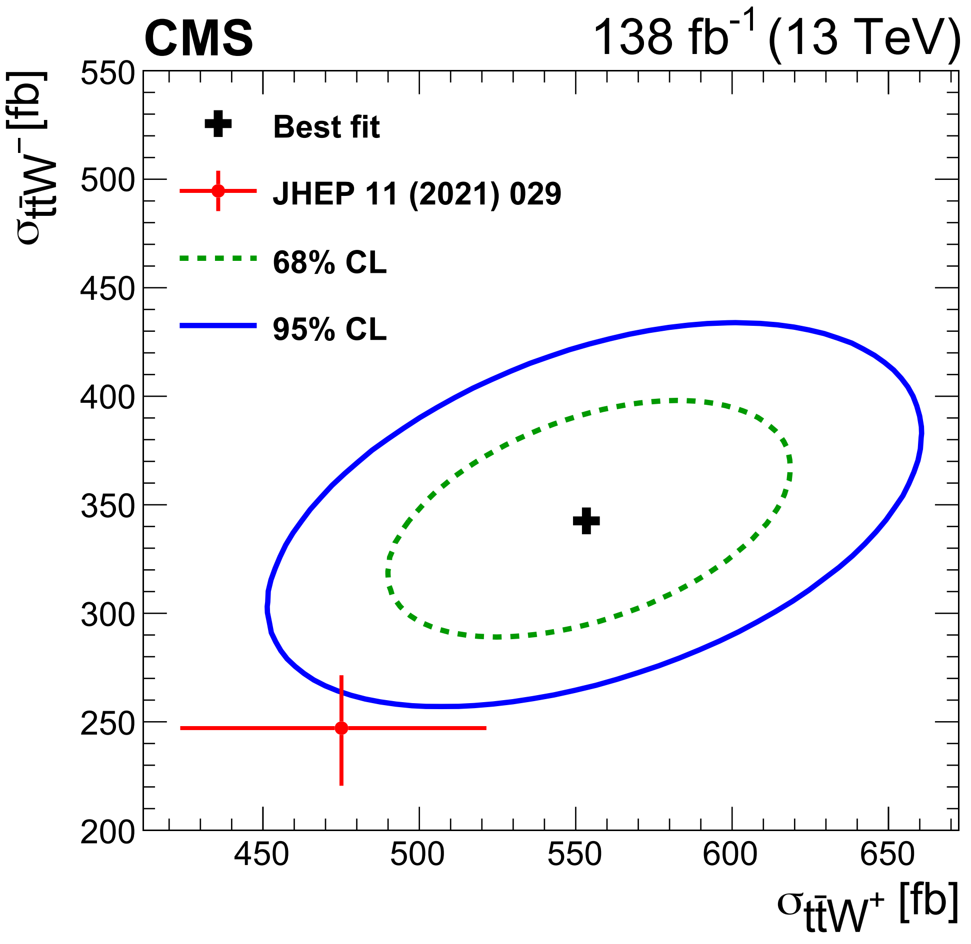
THEORY

- NLO QCD:
  [S. Badger, J. M. Campbell, R. K. Ellis, 1011.6647], [J. M. Campbell, R. K. Ellis, 1204.5678], [A. Denner, G. Pelliccioli, 2102.03264];
- NLO QCD with light jet : *[G. Bevilacqua, H. Y. Bi, F. Febres Cordero, H. B. Hartanto, M. Kraus, J. Nasufi, L. Reina, and M. Worek , 2109.1581, 2305.03802]*
- NLO QCD + EW:
  [S. Frixione, V. Hirschi, D. Pagani, H. S. Shao, M. Zaro, 1504.03446], [R. Frederix, D. Pagani, M. Zaro, 1711.02116], [Denner, Pelliccioli, 2020]
- Resummation of soft gluons:
  [H. T. Li, C. S. Li, S. A. Li, 1409.1460] [A. Broggio, G. Ferroglia, G. Ossola, B. D. Pecjak, 1607.05303], [A. Kulesza, L. Motyka, D. Schwartlaender, T. Stebel, V. Theeuwes, 1812.08662]
- NLO QCD + EW (on-shell) predictions supplemented with multi-jet merging as la FxFx: *[R. Frederix, S. Frixione, 1209.6215] [R. Frederix, I. Tsinikos, 2108.07862]*

- Current theoretical uncertainties $\mathcal{O}(10\%)$

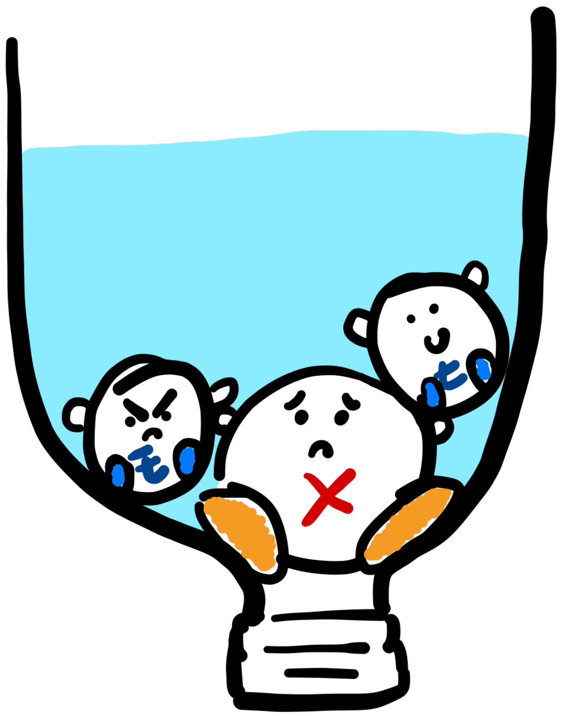
EXPERIMENTS

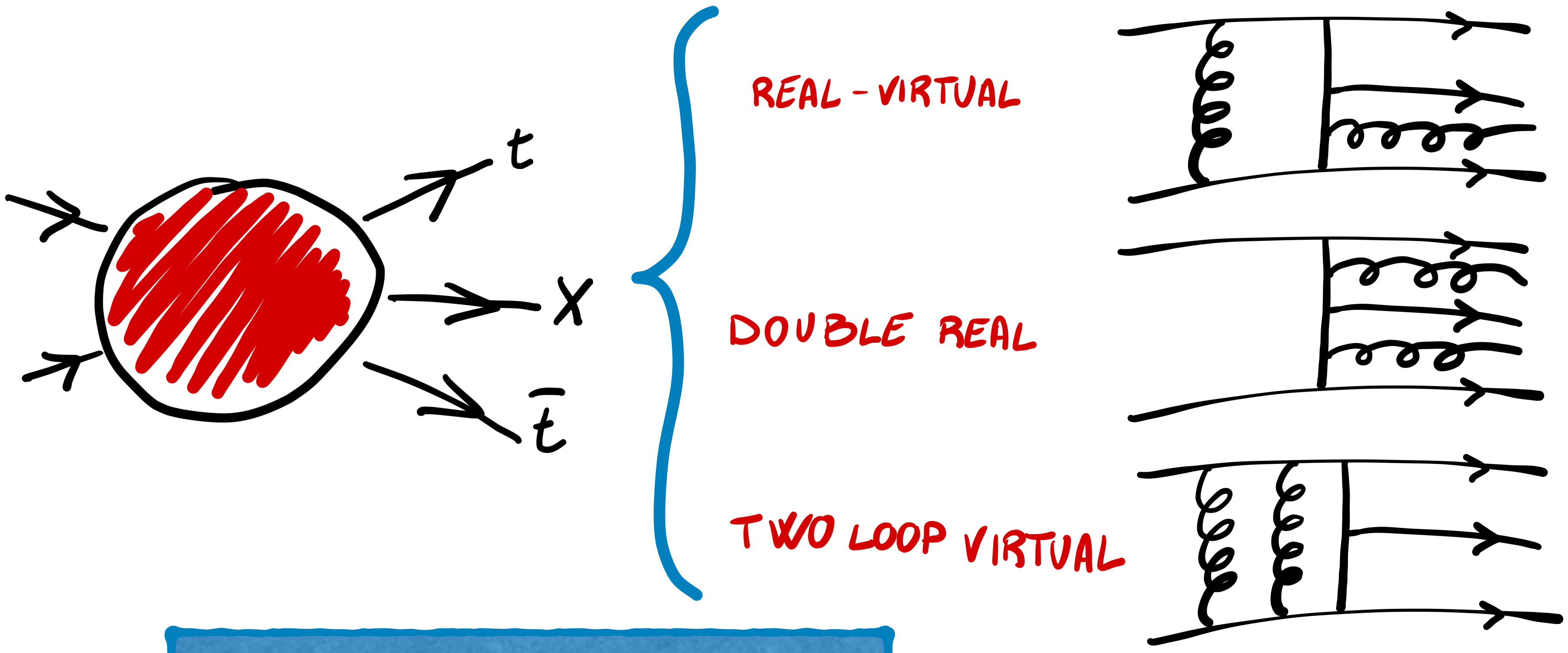
- ATLAS collaboration: *[ATLAS-CONF-2023-019];*
- CMS collaboration: *[2208.06485].*



- Theory-experiment tension at 2σ level;
- Explained by higher order corrections?

THEORY BOTTLENECKS





What makes NNLO challenging?

Subtraction procedure

- We use **q_T-subtraction**;
- We **generalised** the method to this class of processes.

Two loop amplitudes

- Not known: current frontier!
- We developed **approximations**.

q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini Phys.Rev.Lett. 98 (2007)]

$$d\sigma^F_{NNLO} = d\sigma^F_{NNLO}\Big|_{q_T=0} + d\sigma^F_{NNLO}\Big|_{q_T\neq 0}$$

$$d\sigma^{F+jets}_{NLO}$$

$$\boxed{d\sigma^F_{NNLO} = \mathscr{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{NLO} - d\sigma^{CT}_{NLO}\right]}$$

HARD COLLINEAR COEFFICIENT

Contains information on virtual corrections to the process.

$$\mathscr{H}^F_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathscr{H}^{(2)}$$

Contains the genuine **2-loop contribution**:

$$H^{(2)} = \frac{2\,Re\left(\mathscr{M}^{(2)}(\mu_{IR},\mu_R)\mathscr{M}^{(0)}\right)}{|\mathscr{M}^{(0)}|^2}$$

- APPROXIMATED -

Includes:

- one-loop squared contribution;
- **soft parton contribution**.

- EXACT -

SOFT PARTON CONTRIBUTION

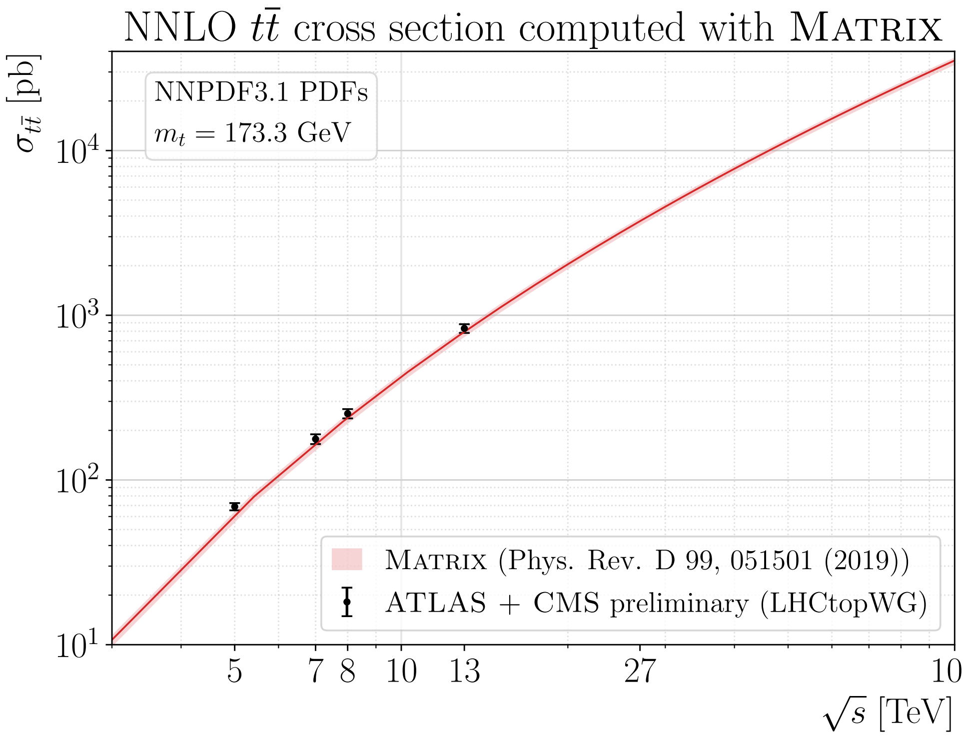
[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786
SD, J. Mazzitelli, In preparation]

The soft contribution from a massive final state was a key ingredient to extend q_T subtraction to a massive coloured final state.

Soft contributions to heavy-quark (Q) production

[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786]

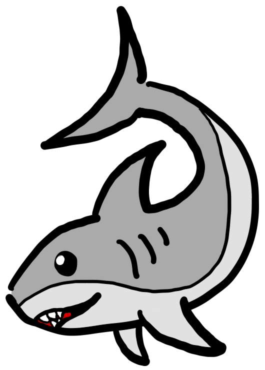
- Applied to top pair and bottom pair production: *[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019, 2020]*;
- Mostly **analytic** expressions;
- Assumption of $Q\bar{Q}$ **back-to-back** at LO.



NEW: generalisation to $Q\bar{Q}F$ kinematics

[SD, J. Mazzitelli, IN PREPARATION]

- removed the back-to-back assumption;
- Extra contribution computed **numerically**;
- On-the-fly numerical integration implemented in a **library**: **SHARK**
  Soft function for **H**eavy quark production in **AR**bitrary **K**inematics



2-LOOP CONTRIBUTION

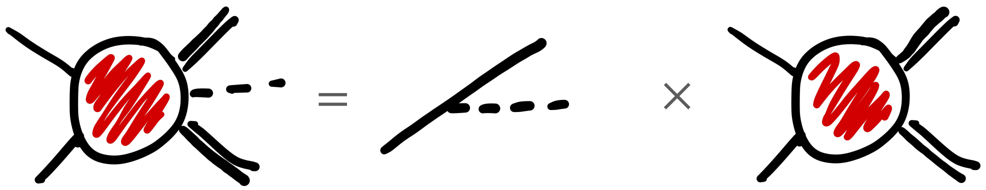
$$H^{(2)}_{t\bar{t}X} = \frac{2\,Re\left(\mathscr{M}^{(2)}_{t\bar{t}X}(\mu_{IR},\mu_R)\mathscr{M}^{(0)}_{t\bar{t}X}\right)_{appr.}}{|\mathscr{M}^{(0)}_{t\bar{t}X}|^2_{appr.}}$$

subtraction scale μ_{IR}
(we use $\mu_{IR} = Q_{t\bar{t}H}$)

- We need to find an approximation of the virtual amplitude;
- We apply the approximation both on the numerator and denominator of $H^{(2)}_{t\bar{t}X}$: effectively a **reweighting**.

Two independent approximations

Soft approximation



- Captures the leading behaviour when the energy and **mass of the associated boson** are smaller than the other relevant scales

Massification procedure

- Captures the leading behaviour when the **mass of the top pair** are smaller than the other relevant scales

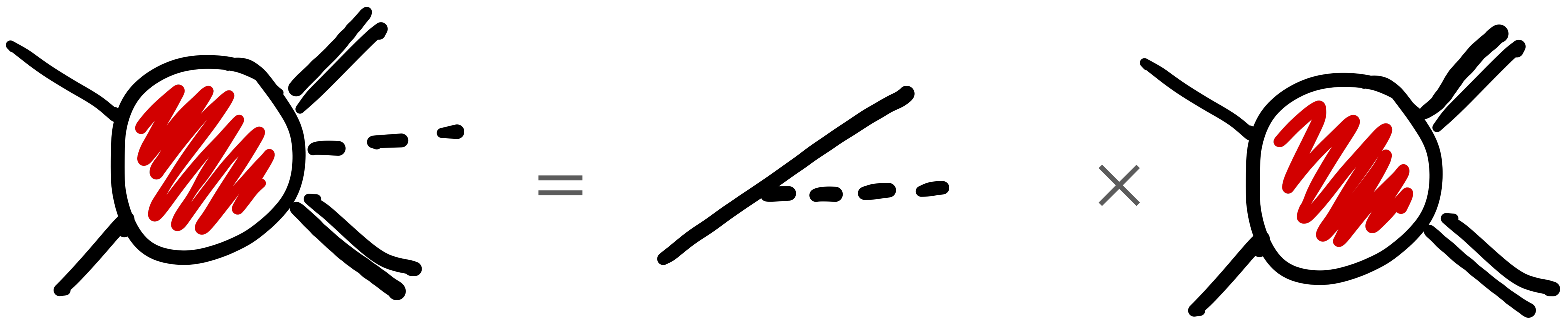
SOFT APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + X(k)$

Soft approximation:

$k \to 0, \quad m_X \ll m_t$



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}H}\big(\{p_i\},k\big) \simeq F\big(\alpha_S(\mu_R); m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\,\mathcal{M}_{q\bar{q}'\to t\bar{t}}\big(\{p_i\}\big)$$

$$\mathcal{M}_{q\bar{q}'\to t\bar{t}W}\big(\{p_i\},k\big) \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k} - \frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}\big(\{p_i\}\big)$$

- The formula captures the leading behaviour in the **soft limit** $k \to 0$: the emission from highly **off-shell top propagators** is **not captured**.
- The formula can be obtained both from the **eikonal approximation** and the **low energy theorems**;
- To use the approximation, we need a **recoil prescription** to map the $t\bar{t}X$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}X} \to Q_{t\bar{t}}$);

MASSIFICATION PROCEDURE

[A. A. Penin: 0508127]
[A. Mitov, S. Moch: 0612149]
[T. Becher and K. Melnikov: 0704:3582]

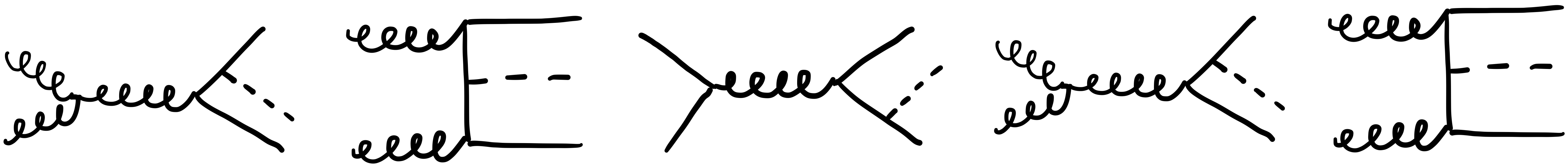
Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + X(k)$

Massification procedure:

$m_t \ll Q$

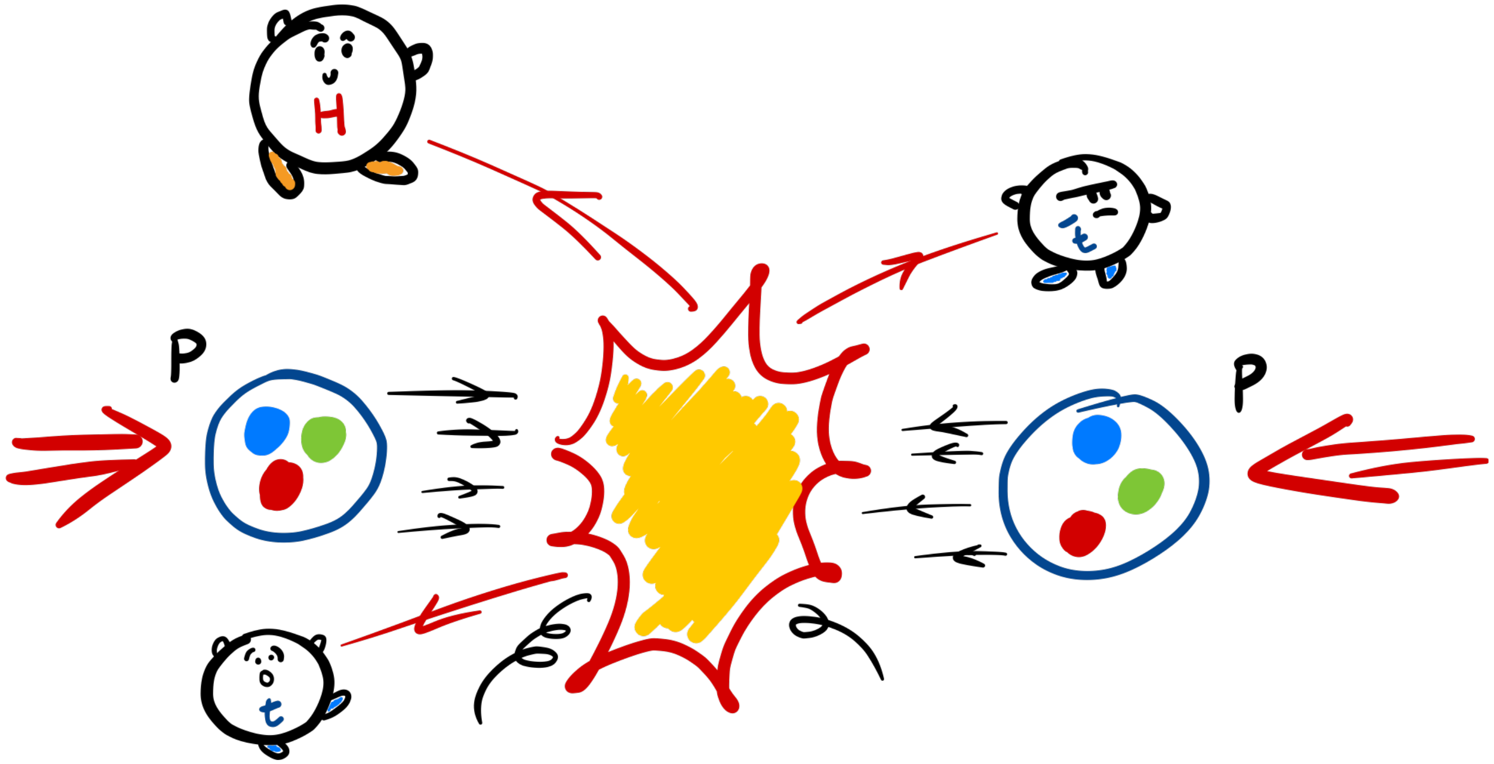
$$\mathcal{M}(\{p_i\}, k; \mu_R, \epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_S(\mu_R), \frac{m_t}{\mu_R}, \epsilon\right) \mathcal{M}^{m_t=0}(\{p_i\}, k; \mu_R, \epsilon)$$

- The perturbative factor $Z_{[q]}^{(m_t|0)}$ was computed in *[A. Mitov, S. Moch: 0612149]*;
- The procedure retrieves the correct **mass logarithms**;
- The contribution from **massive top loops** is **not captured**;
- Successfully employed to derive and cross check results for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ amplitudes *[M. Czakon, A. Mitov, S. Moch: 0705.1975]*;
- Successfully applied to $b\bar{b}W$ production *[L. Buonocore, SD, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2212.04954]*.



$t\bar{t}H$ PRODUCTION

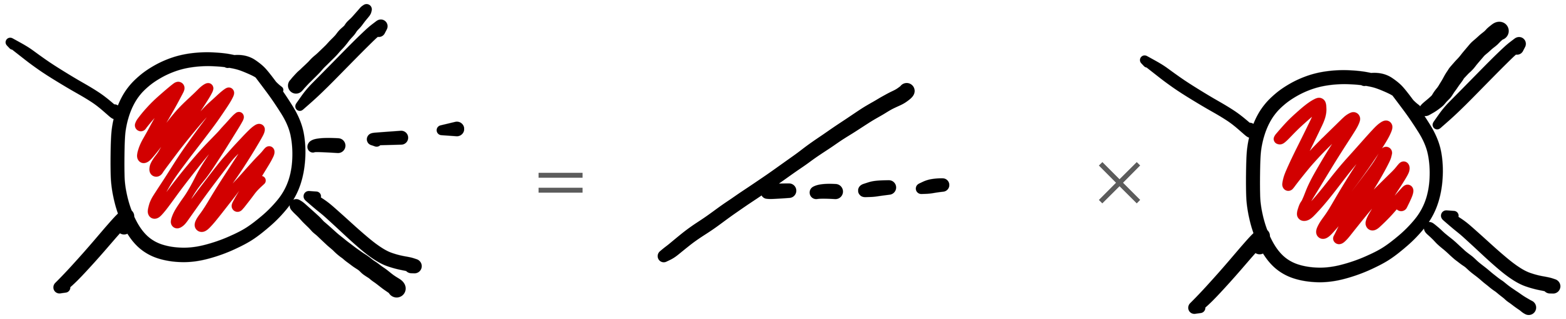
[ArXiv:2210.07846]



CHOICE OF THE APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

- Amplitudes for the process $c\bar{c} \to t\bar{t}$ available *[Czakon (2008); Barnreuther et al.(2013)]*: **we can use the soft approximation**.



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}H}\big(\{p_i\},k\big) \simeq F\big(\alpha_S(\mu_R);m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathcal{M}_{q\bar{q}'\to t\bar{t}}\big(\{p_i\}\big)$$

- The perturbative function $F\big(\alpha_S(\mu_R);m_t/\mu_R\big)$ is an **effective coupling** which also takes into account the **renormalisation** of the mass and of the wave function;
- To map the $t\bar{t}H$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}H} \to Q_{t\bar{t}}$), we use the **$\mathrm{q_T}$ recoil prescription**:
  - We reabsorb the Higgs momentum equally in the initial-state parton momenta;
  - We leave unchanged the top and anti-top momenta.

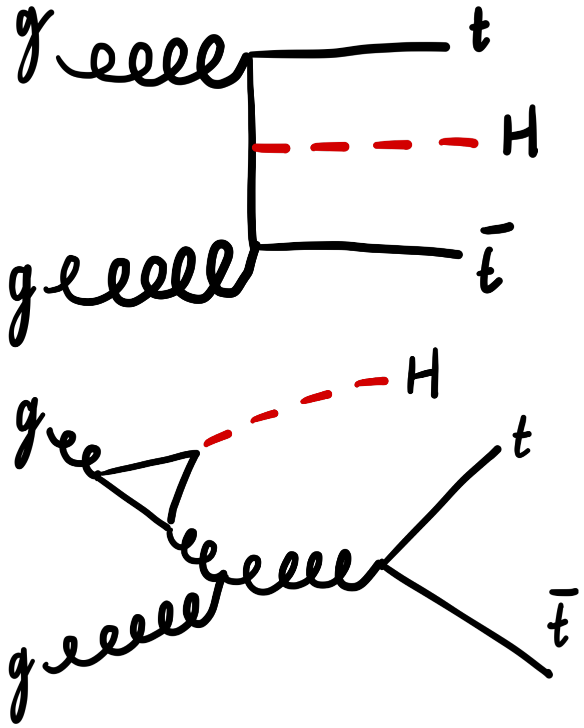
TESTING THE APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

To **validate** our procedure: test the approximation at NLO!

$\Delta\sigma_{\mathrm{NLO,H}}$[fb]	13 TeV		100 TeV	
	gg	$q\bar{q}$	gg	$q\bar{q}$
Exact	88.62	7.826	8205	217.0
Soft Approximation	61.92	7.413	5612	206.0
Difference	**30.1%**	**5.27%**	**31.6%**	**5.06 %**

- Deviation w.r.t. exact computation is about **30%** for the gg **channel** and **5%** for the $q\bar{q}$ **channel**;
- Deviation **independent** of kinematic variables;
- **Better agreement** for $q\bar{q}$ channel can be explained by the presence, both at LO and NLO, of diagrams where a **Higgs boson is radiated from a virtual top** only present in the gg channel.



UNCERTAINTIES ESTIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

How to estimate the NNLO uncertainties?

- We use the **deviation from the exact results at NLO** as a **lower bound** on the NNLO uncertainty;
- We multiply by a **tolerance factor** of **3**;
- We combined **linearly** the uncertainty for the gg and $q\bar{q}$ channel;

How to test the NNLO uncertainties?

- Check the effect of using **different recoil prescription**;
- Check the effect of using a **different subtraction scales** $\mu_{IR} \rightarrow 2\,\mu_{IR}$, $\mu_{IR} \rightarrow 1/2\,\mu_{IR}$.

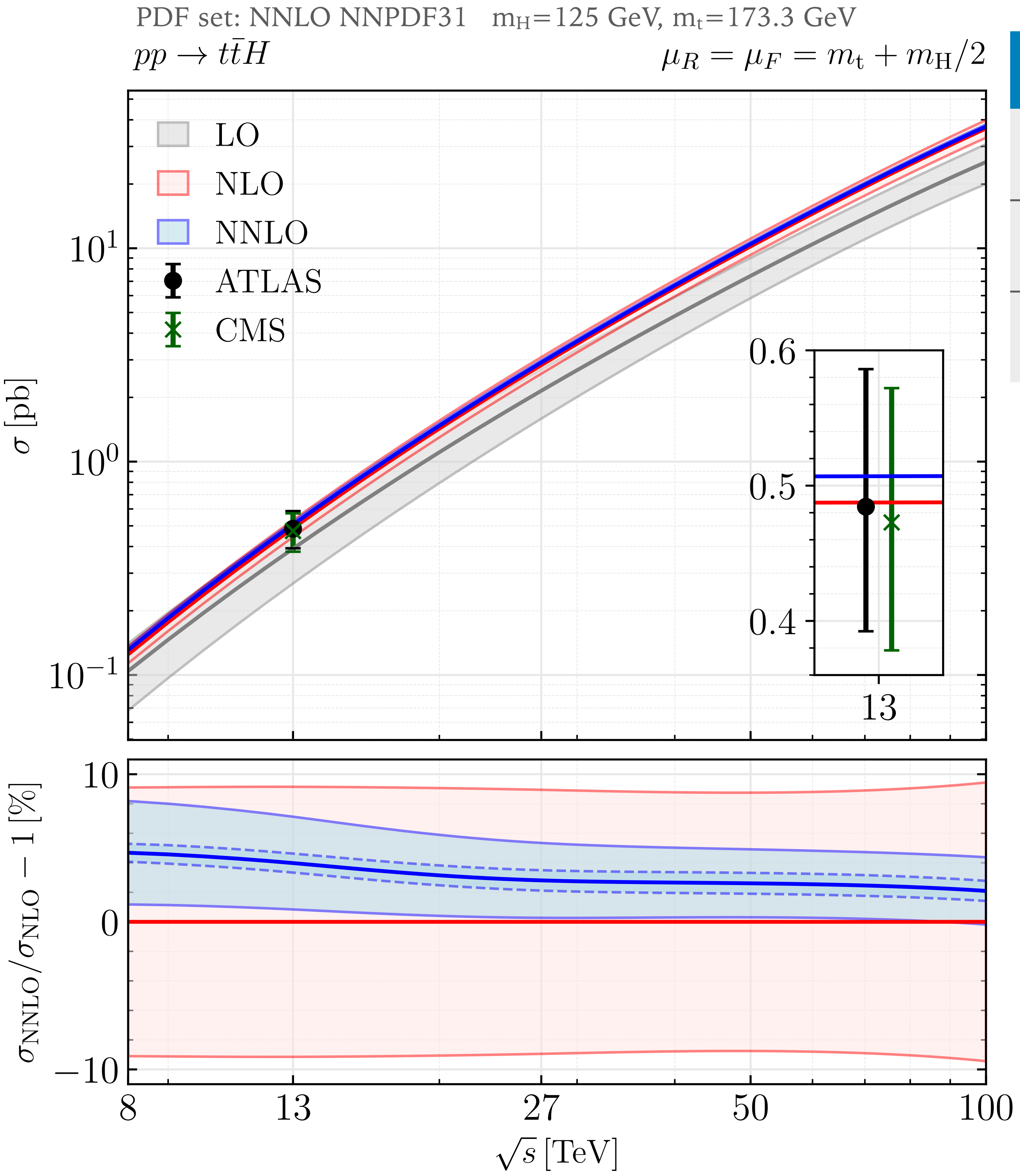
Final uncertainty:

- $\pm 15\,\%$ **on** $\Delta\sigma_{\mathrm{NNLO}}$
- $\pm 0.6\,\%$ **on** σ_{NNLO}

Effect on the total cross section modulated by the (small) contribution of the hard factor: about ***1%*** *of the LO cross section in the gg and* ***2-3%*** *in the $q\bar{q}$ channel.*

RESULTS

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]



σ [pb]	13 TeV	100 TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

Numerical + soft Higgs uncertainties

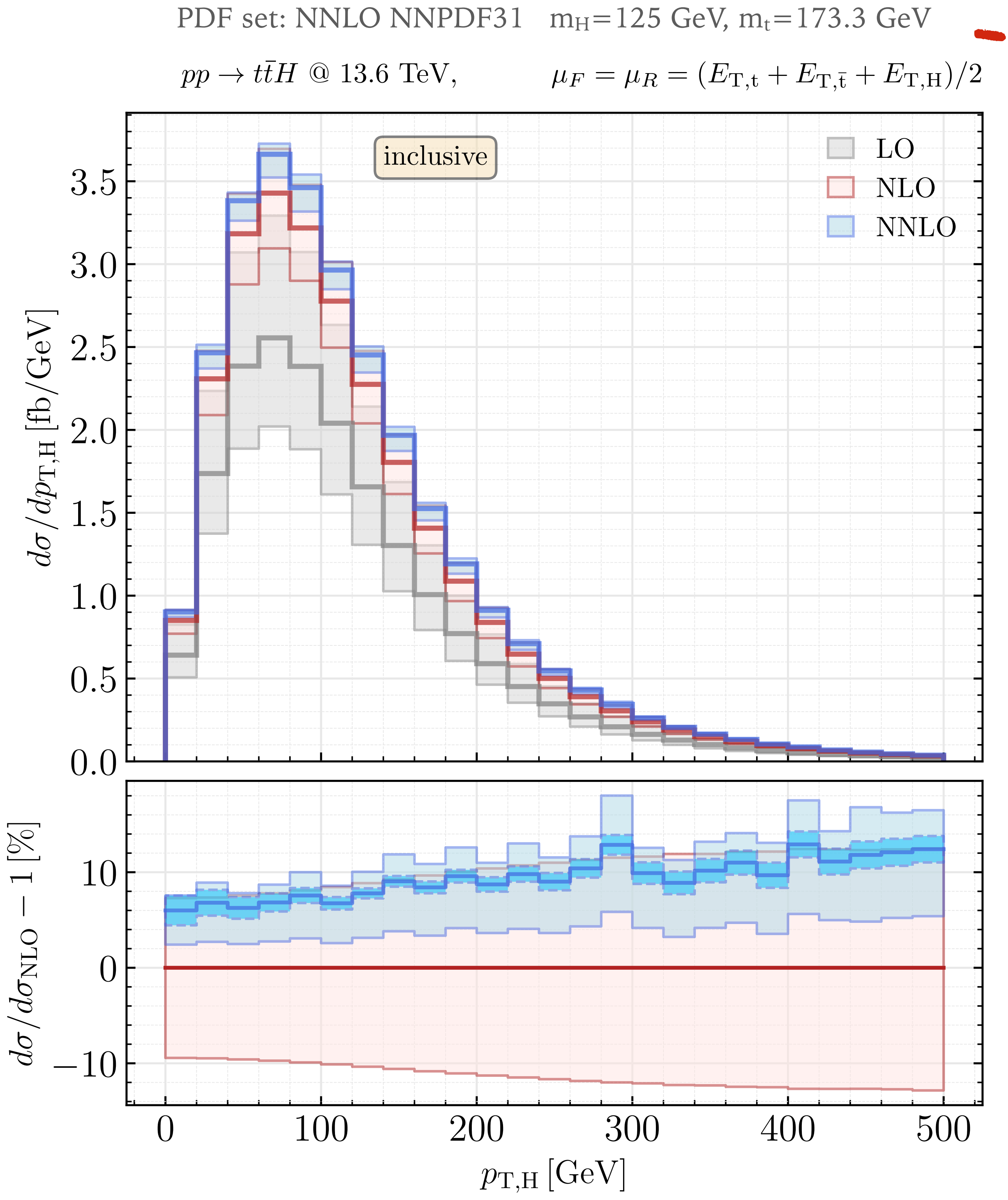
Scale uncertainties

- **NNLO corrections**: **+4%** (13 TeV), **+2%** (100 TeV);
- Reduction of **scale uncertainties**;
- Soft approximation uncertainty significantly **smaller** than remaining perturbative uncertainties.

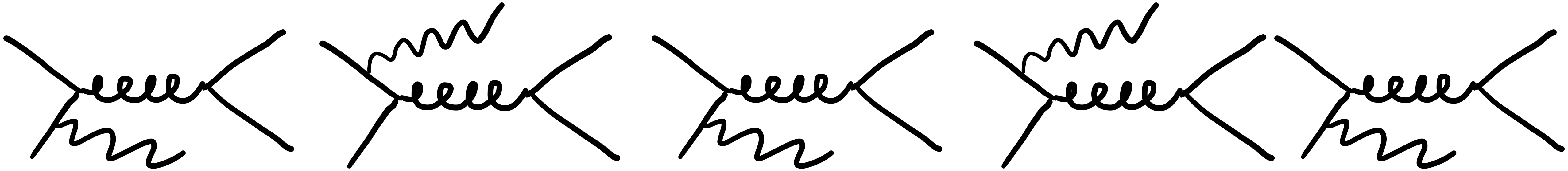
RESULTS

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: IN PREPARATION]

-PRELIMINARY RESULTS-

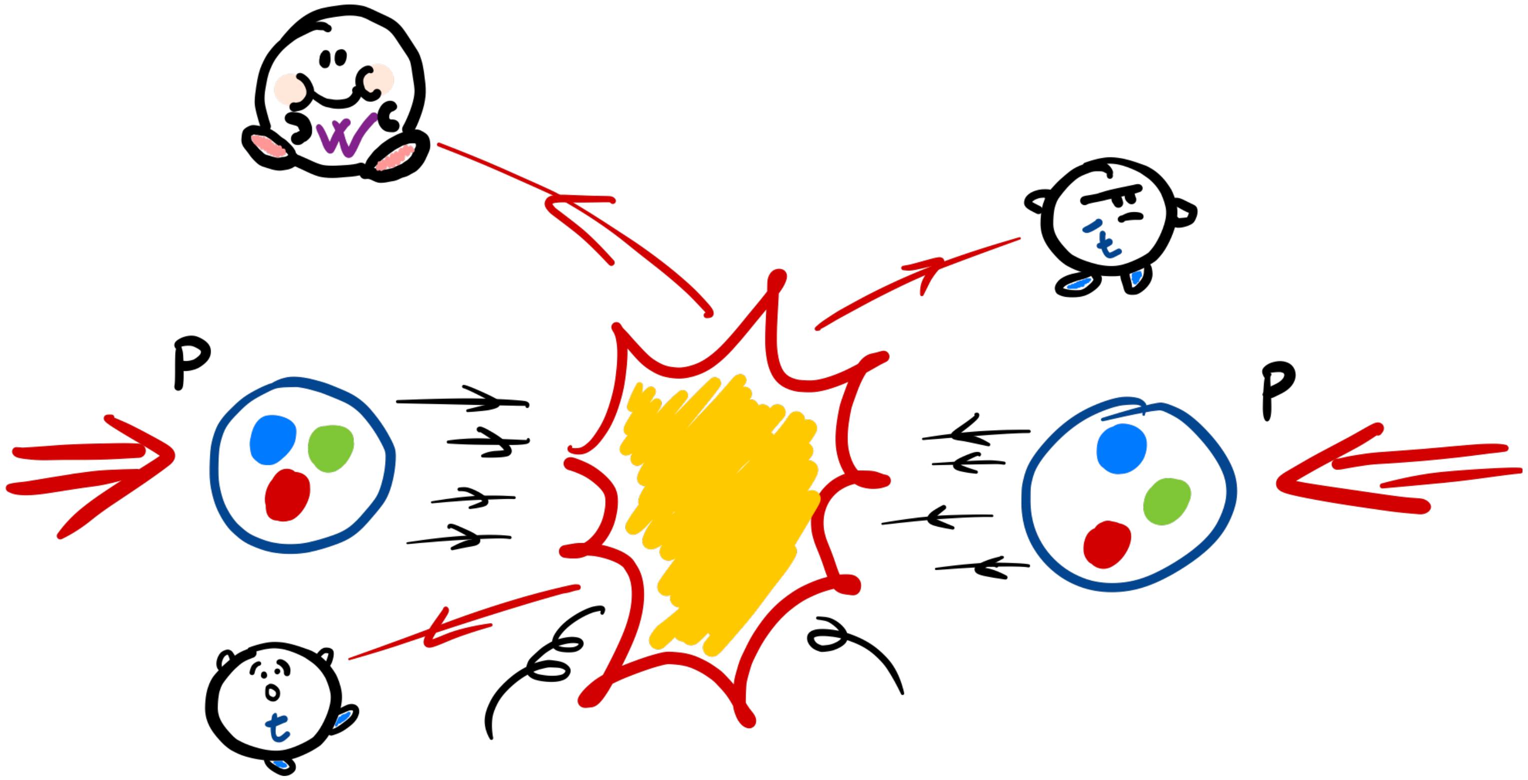


- First results for **differential distributions**;
- Soft approximation uncertainty computed on a **bin-by-bin basis**;
- NLO and NNLO uncertainty bands **overlap**;
- Soft approximation uncertainty **of the same order** over all the spectrum.



$t\bar{t}W$ PRODUCTION

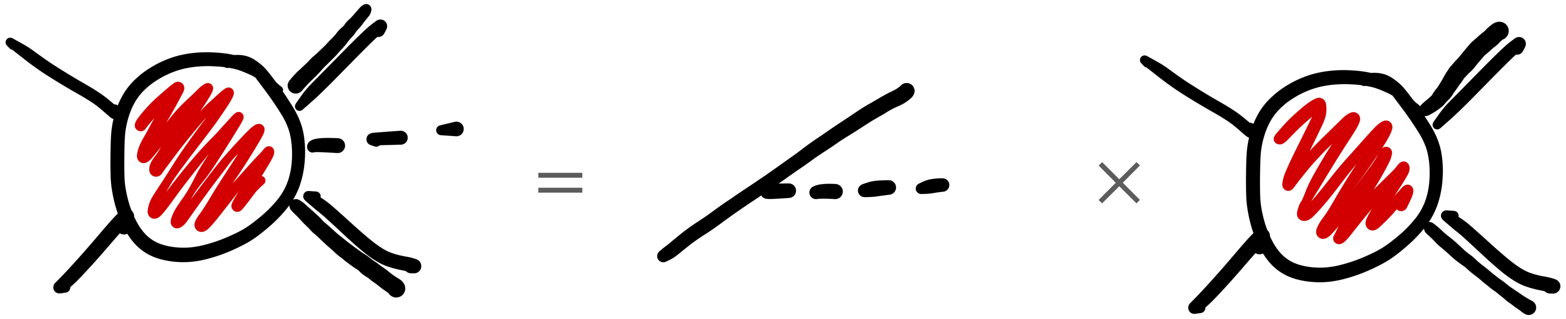
[ArXiv:2306.16311]



CHOICE OF THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

- Amplitudes for the process $c\bar{c} \to t\bar{t}$ available *[P. Bärnreuther, M. Czakon, P. Fiedler: 1312.6279]*: **we can use the soft approximation**.



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}W}\big(\{p_i\},k\big) \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k} - \frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}\big(\{p_i\}\big)$$

- The soft emission of a W selects the **helicity configuration** $\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}$;
- In contrast with the $t\bar{t}H$ case, the soft W is emitted by the **initial-state partons**;
- To map the $t\bar{t}W$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}W} \to Q_{t\bar{t}}$), we use use a **prescription symmetrised** with respect to the one employed for $t\bar{t}H$ case:
  - We reabsorb the W momentum equally in the top-quark momenta;
  - We leave unchanged the initial-state parton momenta.

CHOICE OF THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

- Amplitudes for the massless process $c\bar{c} \to q\bar{q}W$ available *[S. Abreu, F. Febres Cordero, H. Ita, M. Klinkert, B. Page, V. Sotnikov: 2110.07541]*: **we can use the massification procedure**;

$$\mathcal{M}(\{p_i\},k;\mu_R,\epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_S(\mu_R),\frac{m_t}{\mu_R},\epsilon\right)\mathcal{M}^{m_t=0}(\{p_i\},k;\mu_R,\epsilon)$$

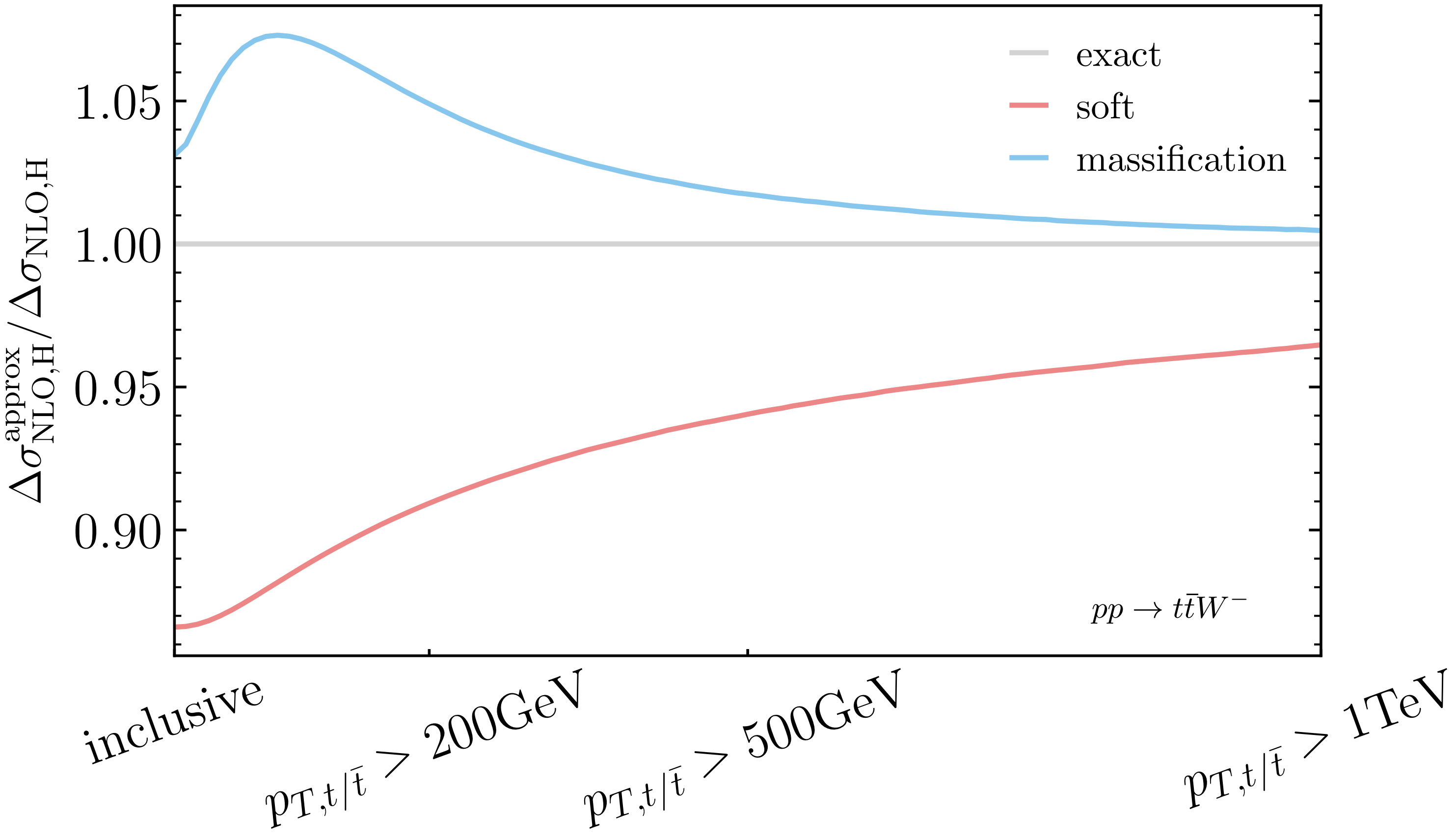
- Massification of the amplitudes implemented in a **C++ library**, **WQQAmp** *[L. Buonocore, L. Rottoli, C. Savoini, https://gitlab.com/lrottoli/WQQAmp]*;
- We need to map the massless kinematics into a massive one: we do it by preserving the momentum of the $t\bar{t}$ pair.

TESTING THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

To validate our procedure: test the approximations at NLO!

- Both approximations provide a **good estimation** also at the inclusive level;
- We observe a **pattern**: **soft approximation undershoots** the exact result, while the **massification procedure overshoots**;
- As expected, both approximations get closer to the exact result when a **harder cut** is imposed

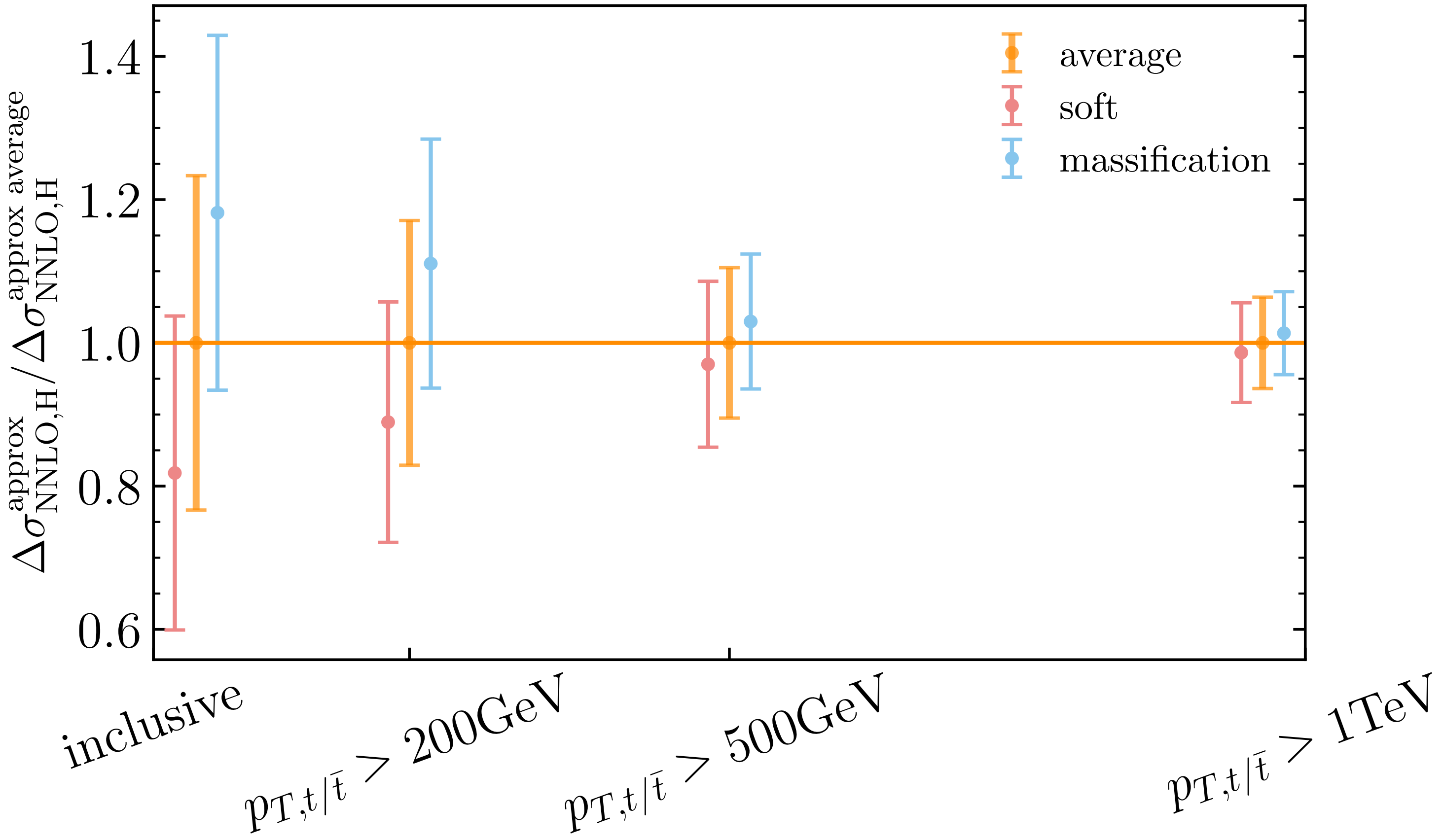


UNCERTAINTIES ESTIMATION

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

How to estimate the NNLO uncertainties of each approximation?

- ➤ **Method 1**: we take the difference between exact and approximated result at NLO and we multiply by a **tolerance factor** of **2**;
- ➤ **Method 2**: we consider the effect of using a **different subtraction scales** $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$;
- ➤ The uncertainty is defined as the **maximum between these two estimates**.



- ➤ The two approximations are **fully consistent**;
- ➤ Our best prediction is obtained by taking their **average** and **linearly combing** the uncertainties.

Final uncertainty:

- **±25 % on $\Delta\sigma_{\text{NNLO,H}}$**
- **±2 % on σ_{NNLO}**

RESULTS

*[L. Buonocore, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]*

LHC@13TeV	$\sigma_{t\bar{t}W^+}\ [fb]$	$\sigma_{t\bar{t}W^-}\ [fb]$	$\sigma_{t\bar{t}W}\ [fb]$	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.0^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\,\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\,\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\,\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\,\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\,\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\,\%$	$2.007^{+2.1\%}_{-2.1\%}$

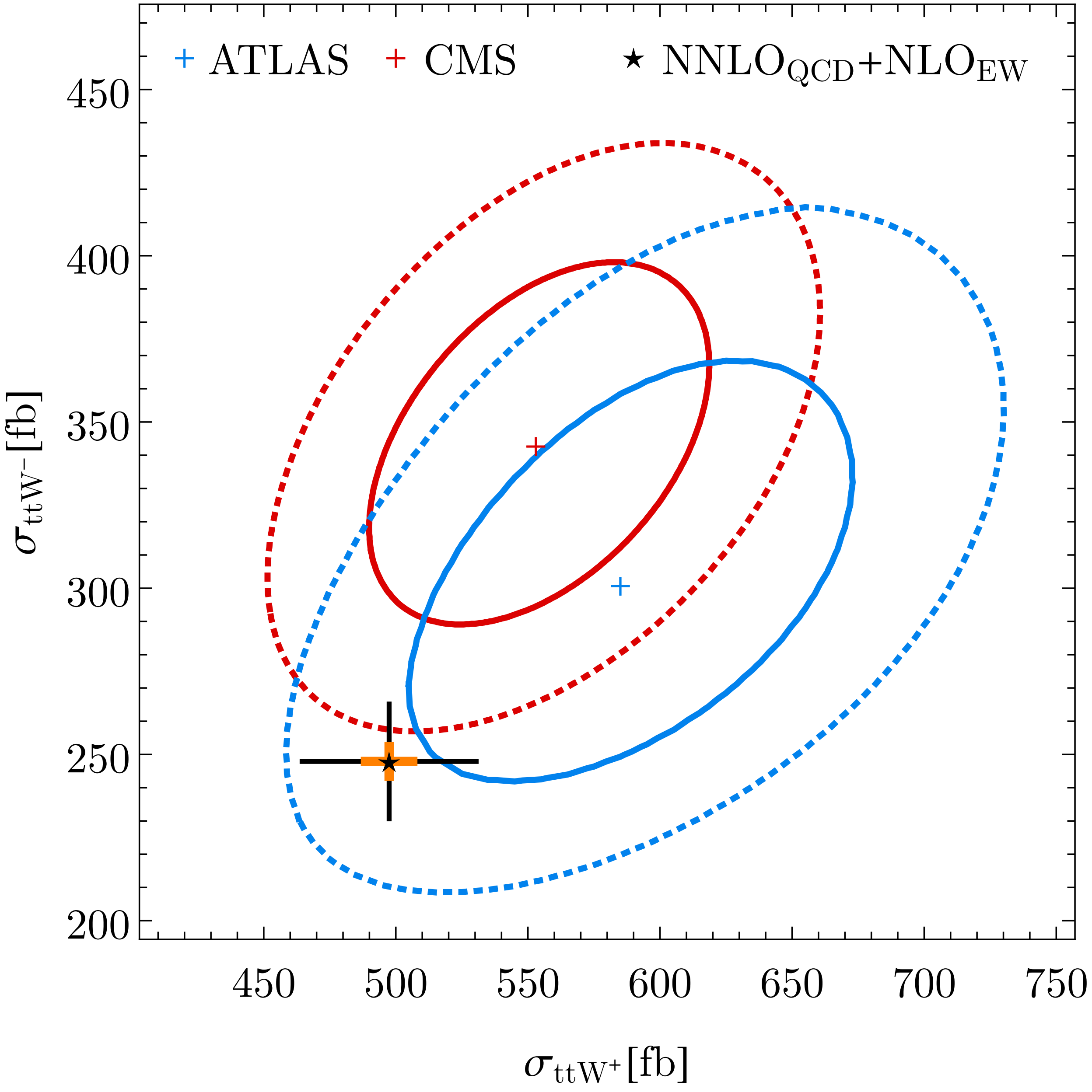
Scale uncertainties

Uncertainties from 2 loop amplitudes

- We choose $\mu_0 = M/2$;
- NNLO predictions show first sign of **perturbative convergence**;
- ratio $\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ have a **very stable** perturbative behaviour;
- **PDF uncertainties** $\pm 1.8\,\%$ (computed with MATRIX + PINEAPPL interface *[SD, T. Ježo, S. Kallweit, C. Schwan, in preparation]*)
- α_S **uncertainties** $\pm 1.8\,\%$;
- by combining with EW corrections, we get our **best prediction**;
- to be conservative, scale uncertainties for $\mathrm{NNLO_{QCD}+NLO_{EW}}$ are **symmetrised**.

RESULTS

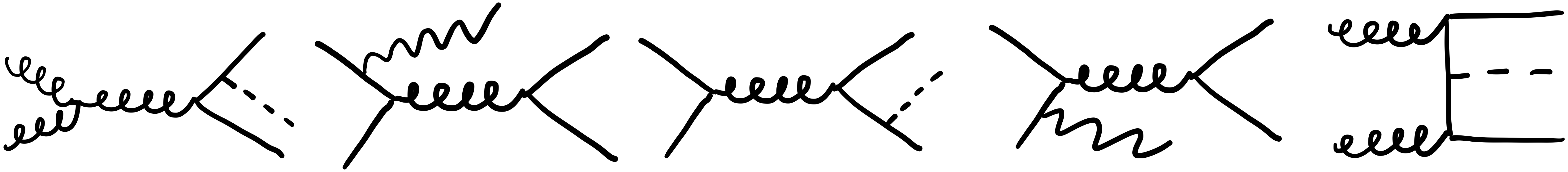
[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]



- We compare our best prediction to **ATLAS and CMS measurements**;
- With respect to the **FxFx prediction**, the current theory reference, higher rate and smaller uncertainties;

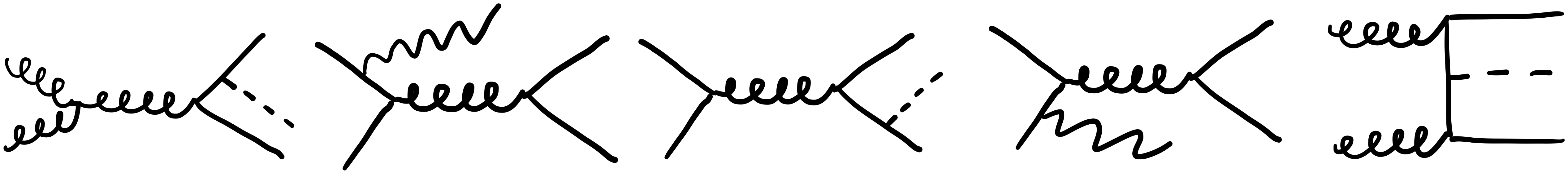
$$\sigma_{t\bar{t}W}^{NNLO_{QCD}+NLO_{EW}} = 745.3^{+6.7\%}_{-6.7\%}$$
$$\sigma_{t\bar{t}W}^{FxFx} = 722.3^{+9.7\%}_{-10.8\%}$$

- Tension remains at the $1\sigma - 2\sigma$ **level**.



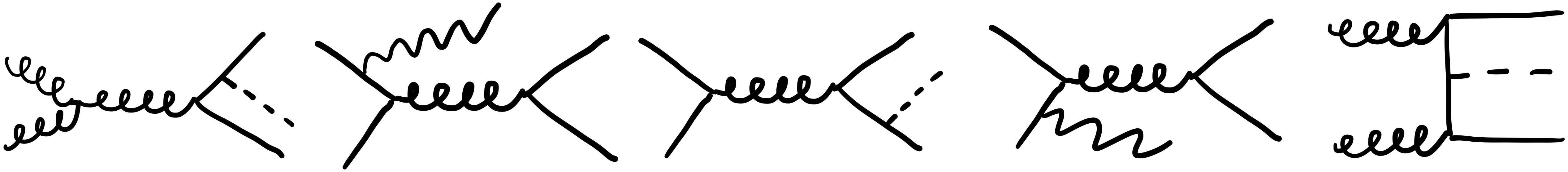
SUMMARY & OUTLOOK

- We computed within q_T subtraction formalism the **NNLO QCD corrections** to $t\bar{t}H$ production and $t\bar{t}W$ production;
- The **missing ingredients** we needed for the computation are:
  - **NNLO soft contribution** in arbitrary kinematics;
  - **two-loop amplitudes** (**massification** and/or **soft approximation**);
- **First** (almost) exact computations at NNLO QCD for a **$2 \rightarrow 3$ process** with massive coloured particles.



SUMMARY & OUTLOOK

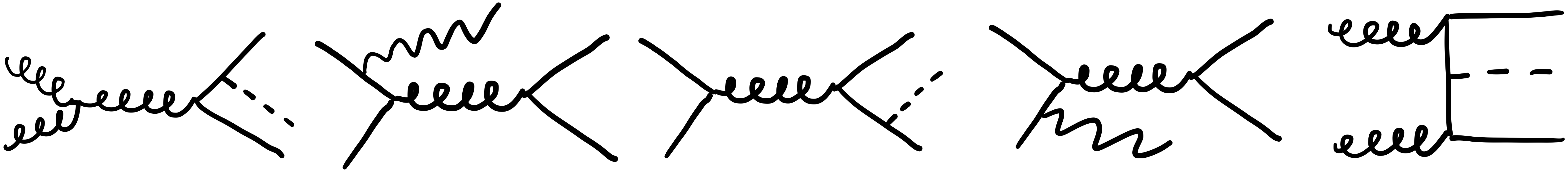
- Differential distributions;
- Further phenomenological studies.



SUMMARY & OUTLOOK

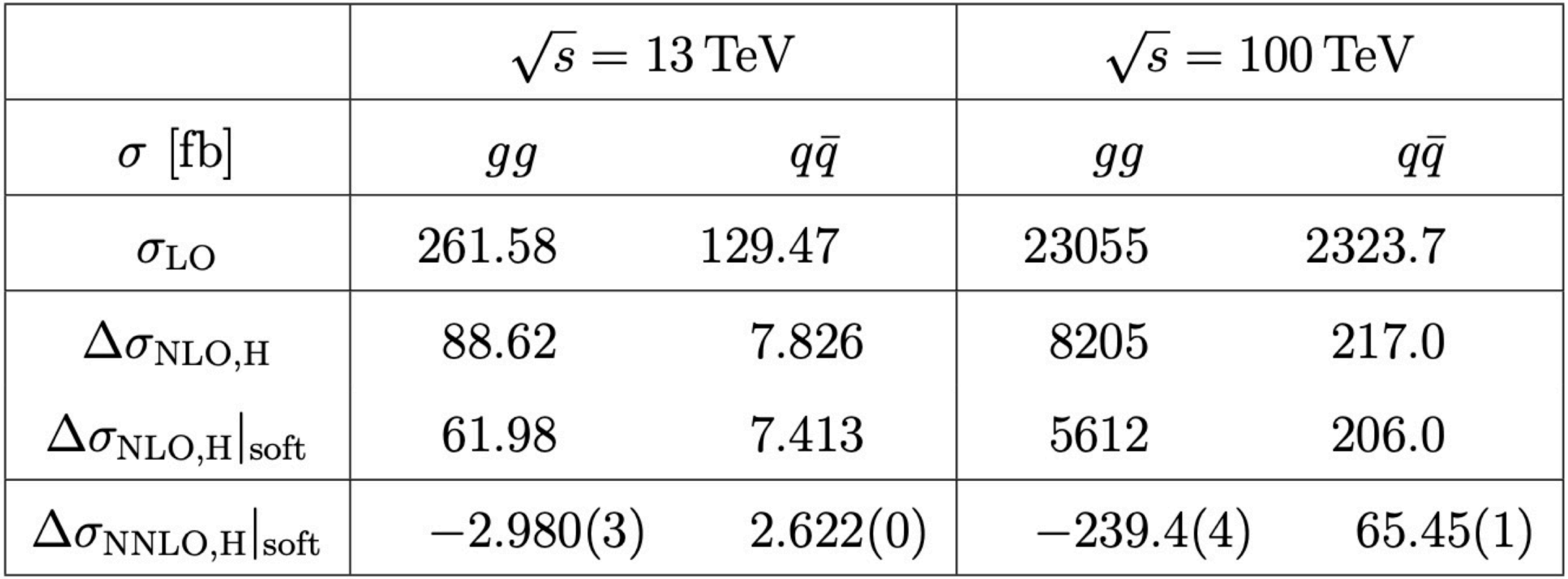
- **Differential distributions**;
- Further phenomenological studies.

THANKS!



BACKUP SLIDES

TOTAL CROSS SECTION



	$\sqrt{s} = 13\,\text{TeV}$		$\sqrt{s} = 100\,\text{TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\vert_{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO,H}}\vert_{\text{soft}}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

- Soft Higgs approximation at LO:
  - gg channel: factor 2.3 ($\sqrt{s} = 13$ TeV)/factor 2.0 ($\sqrt{s} = 100$ TeV)
  - $q\bar{q}$ channel: factor 1.11 ($\sqrt{s} = 13$ TeV)/factor 1.06 ($\sqrt{s} = 100$ TeV)
- At LO there is no reweighting!

CHANGING THE SUBTRACTION SCALE

$$H^{(2)}_{t\bar{t}H} = \frac{2\,Re\Big(\mathcal{M}^{(2)}_{t\bar{t}H}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)}_{t\bar{t}H}\Big)_{soft}}{|\,\mathcal{M}^{(0)}_{t\bar{t}H}|^2_{soft}}$$

- The subtraction scale μ_{IR} is the scale at which the IR poles are subtracted (equivalently, at which the soft approximation is applied);
- Effect of using a different subtraction scales $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$.
  - gg channel +164%/-25% ($\sqrt{s} = 13$ TeV)
    +142%/-20% ($\sqrt{s} = 100$ TeV)
  - $q\bar{q}$ channel +4%/-0% ($\sqrt{s} = 13$ TeV)
    +3%/-0% ($\sqrt{s} = 100$ TeV)

SOFT HIGGS APPROXIMATION

Eikonal approximation

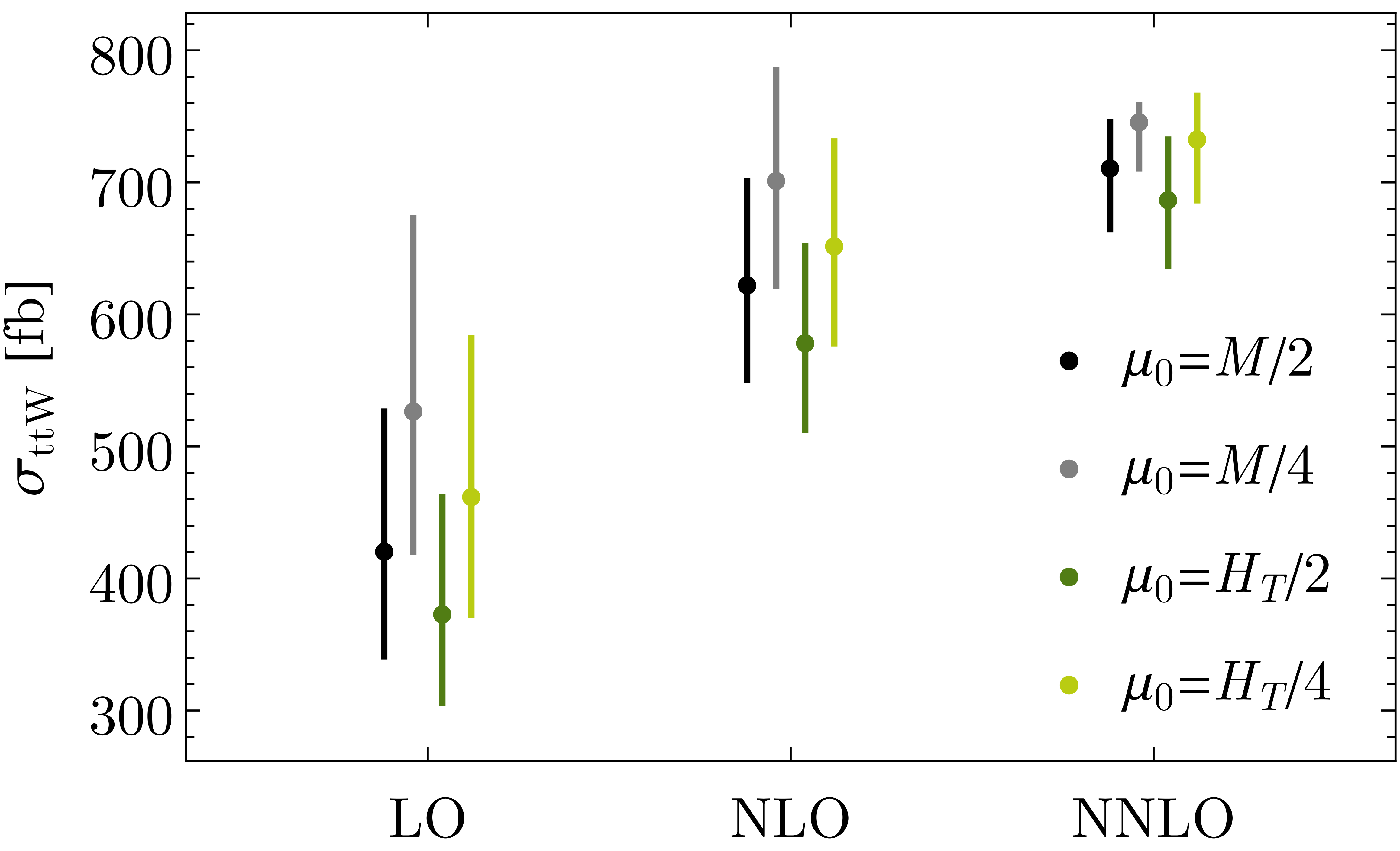
$$\lim_{k\to 0}\mathscr{M}_{t\bar{t}H}\big(\{p_i\},k\big) = F\big(\alpha_S(\mu_R);m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathscr{M}_{t\bar{t}}\big(\{p_i\}\big)$$

Low Energy Theorem

$$\lim_{q\to 0}\mathscr{M}^{\mathrm{bare}}(p\to p+q) = \frac{1}{v}m_0\frac{\partial}{\partial m_0}\mathscr{M}^{\mathrm{bare}}(p\to p)\Big|_{p^2=m^2}$$

$$F(\alpha_S(\mu_R);m_t/\mu_R) = \;1+\frac{\alpha_S(\mu_R)}{2\pi}\big(-3\,C_F\big)$$
$$+\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2\left(\frac{33}{4}C_F^2-\frac{185}{12}C_FC_A+\frac{13}{6}C_F(n_L+1)-6\,C_F\beta_0\ln\frac{\mu_R^2}{m_t^2}\right)+\mathcal{O}(\alpha_S^3)$$

$t\bar{t}W$: DIFFERENT SCALE CHOICES



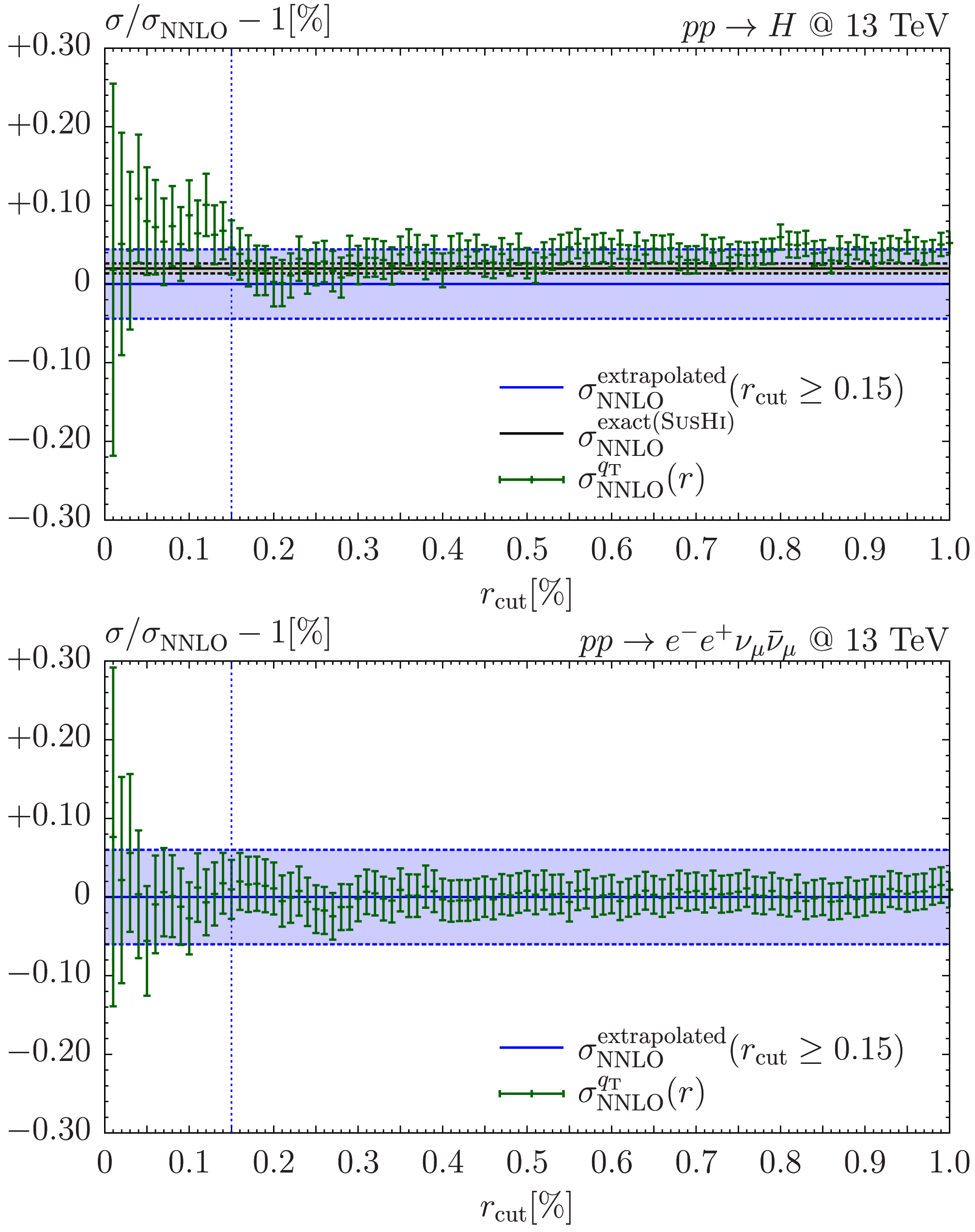
THE SLICING

$$\boxed{d\sigma^{F}_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]}$$

$d\sigma^{F+jets}_{(N)LO}$ and $d\sigma^{CT}_{(N)LO}$ are separately divergent. In practice, q$_T$ subtraction is implemented as a slicing method:

- ➤ introducing a cutoff $r_{cut} = Q/M$;
- ➤ performing the limit $r_{cut} \to 0$.

Quality of the $q_T \to 0$ extrapolation can be understood looking at the r_{cut} dependence



[M.Grazzini, S. Kallweit, M. Wiesemann: arXiv 1711.06631]

r_{CUT} DEPENDENCE