

Photon splitting corrections to soft-photon resummation

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QCD@LHC

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Outline

Motivation

Algorithm

Lepton definitions

Results

Academic case: on-shell $Z \rightarrow e^+e^-$

Realistic case: $pp \rightarrow e^+e^-$

Conclusions

Introduction

- ▶ QED corrections to leptonic final states are needed
- ▶ Either: QED parton shower in analogy to QCD
- ▶ Or: soft-photon resummation (YFS) [Yennie, Frautschi, Suura '61](#)
- ▶ Implemented in SHERPA with hard corrections up to NLO EW + NNLO QED [Krauss, Schönherr '08](#)
- ▶ Initial-state YFS also implemented in SHERPA [Krauss, Price, Schönherr '22](#)

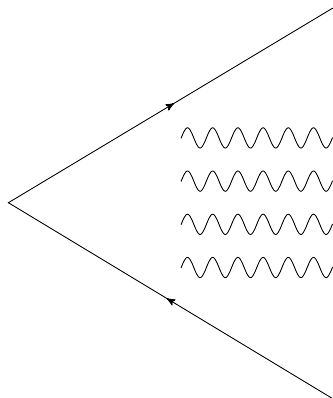
The YFS soft-photon resummation

$$d\Gamma^{\text{YFS}} = d\Gamma_0 e^{\alpha Y(\omega_{\text{cut}})} \sum_{n_\gamma=0}^{\infty} \frac{1}{n_\gamma!} \left[\prod_{i=1}^{n_\gamma} d\Phi_{k_i} \alpha \tilde{S}(k_i) \Theta(k_i^0 - \omega_{\text{cut}}) \mathcal{C} \right]$$

- ▶ $Y(\omega_{\text{cut}})$ is the YFS form factor containing the real and virtual soft-photon divergences, $E_\gamma < \omega_{\text{cut}}$
- ▶ \tilde{S} is the eikonal (soft emission effects) for $E_\gamma > \omega_{\text{cut}}$
- ▶ \mathcal{C} corrects the eikonal to the full hard emission up to a given order (here NNLO)
- ▶ YFS contains no description of charged particle production, $\gamma \rightarrow f\bar{f}$ and the associated $\ln m_f$

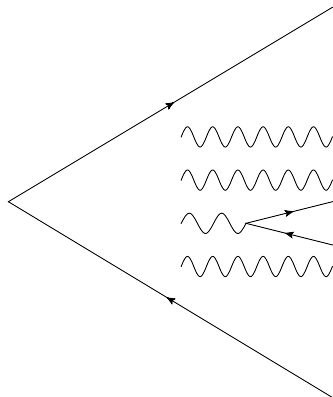
YFS and photon splittings

- ▶ All charged particles are massive within the YFS framework, which regulates collinear divergences
- ▶ Hence $\gamma \rightarrow f\bar{f}$ is IR finite but logarithmically enhanced for light flavours
- ▶ $\gamma \rightarrow e^+e^-$ will induce the largest corrections



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Photon splittings cont'd

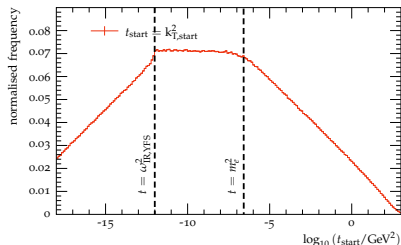
- ▶ We implemented a photon splitting algorithm which allows $\gamma \rightarrow f\bar{f}$ to occur, where $f = e, \mu, \tau, \pi, K$
- ▶ The splittings into light charged hadrons (pions and kaons) use scalar QED (without a form factor as a first approximation)
- ▶ Note that we treat hadrons as the DoF instead of quarks since $E_\gamma \lesssim$ hadronisation scale

Photon splitting algorithm

- ▶ One-step dipole parton shower
- ▶ Input: primary charged particles and coherently emitted soft photons - works for any setup¹ given there is something to emit a photon and something to absorb recoil
- ▶ We reconstruct the scale t_{start} (GeV^2) from the input
- ▶ Evolution continues until the IR cutoff $t_0 = 4m_e^2$
- ▶ This algorithm applies a factor $\alpha_0 \log(t_{\text{start}}/4m_f^2)$ for each pair produced

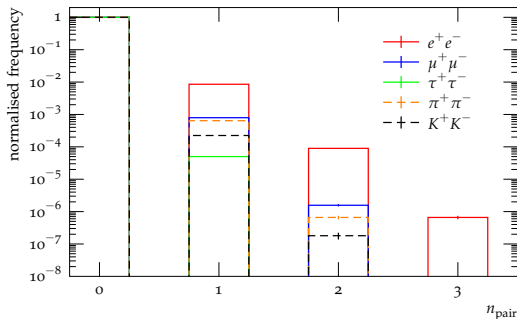
¹Currently decays via Photons++, in future e^+e^- via YFS++ 

Scale choice



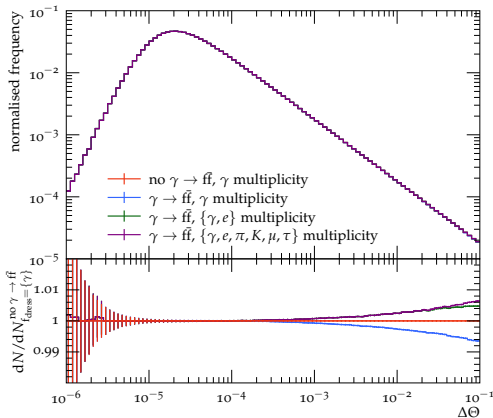
- ▶ What should we use as the ordering variable?
- ▶ $t = k_T^2$ for reconstructing starting scale ($f \rightarrow f\gamma$)
- ▶ $t = q^2$ for photon splittings ($\gamma \rightarrow f\bar{f}$)
- ▶ This is the most physical choice [Brodsky, Lepage, Mackenzie '83](#)

Secondary flavour distribution for $Z \rightarrow e^+e^-$



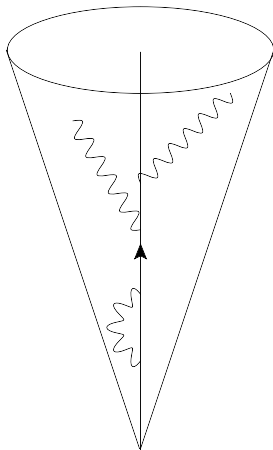
- ▶ Subsequent pair production decreases
 $\mathcal{P}(2) \sim \mathcal{P}(1)^2$
- ▶ Flavour suppression
 $\sim \log(m_f^2)$

Angular structure of photon splitting corrections



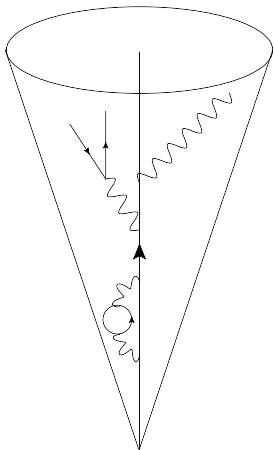
- ▶ On-shell $Z \rightarrow e^+ e^-$
- ▶ For IR safety, $E_\gamma > 0.1\text{MeV}$
- ▶ Hard or wide-angle photons are more likely to split than soft or collinear ones
- ▶ At small $\Delta\Theta$, no difference in multiplicity
- ▶ At larger $\Delta\Theta$, we observe particles other than photons
- ▶ The majority of these are electrons

Rethinking object definitions



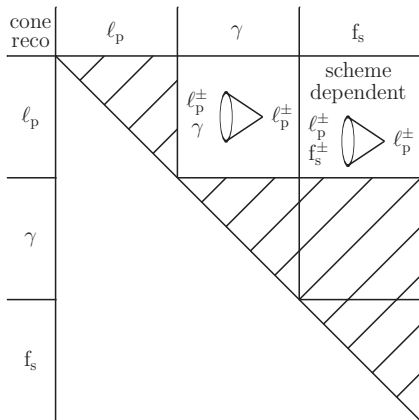
- ▶ For massless leptons, cone dressing with only photons is problematic
- ▶ Because we exclude real $\ell^+\ell^-$, there is nothing to cancel the virtual collinear singularity
- ▶ For massive leptons, there are contributions $\sim \log(m_\ell)$

Rethinking object definitions



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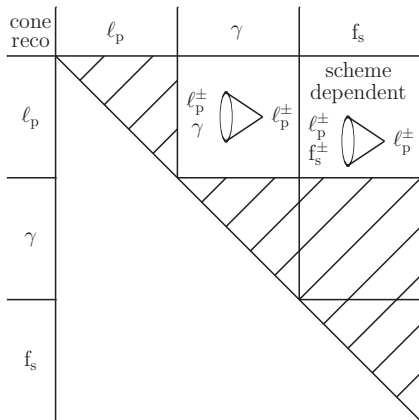
Flavour-aware lepton dressing



We consider the following schemes:

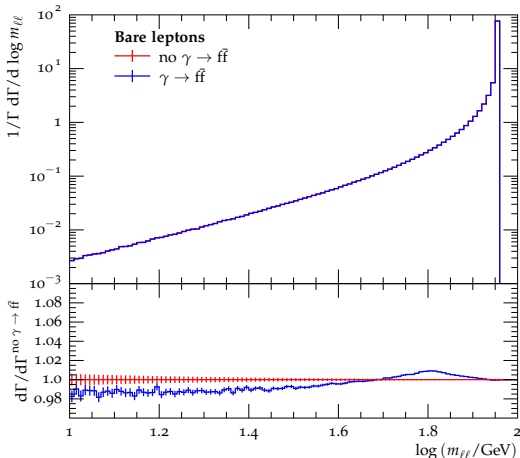
- ▶ $f_{\text{dress}} = \{\gamma\}$
- ▶ $f_{\text{dress}} = \{\gamma, e\}$
- ▶ $f_{\text{dress}} = \{\gamma, e, \pi, K\}$
- ▶ $f_{\text{dress}} = \{\gamma, e, \pi, K, \mu, \tau\}$

Flavour-aware lepton dressing

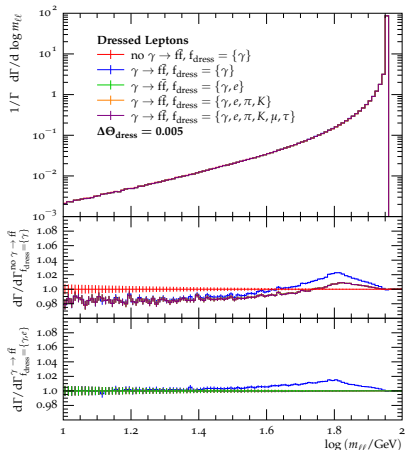


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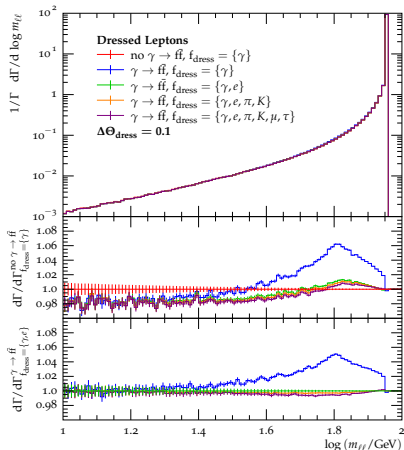
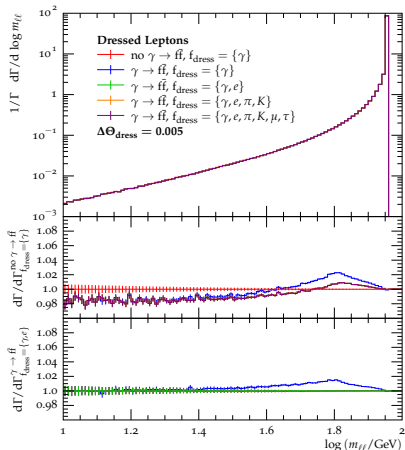
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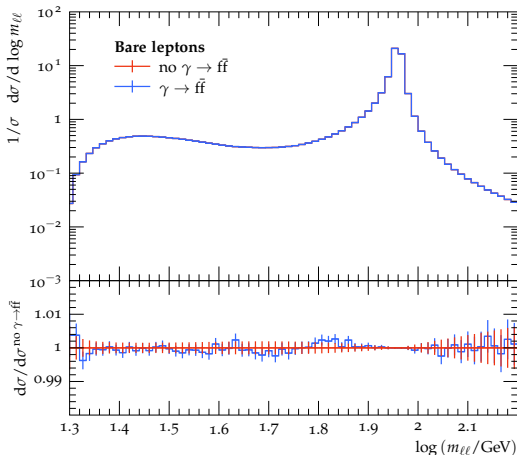
Academic case: on-shell $Z \rightarrow e^+e^-$ Dilepton invariant mass for on-shell $Z \rightarrow e^+e^-$ 

- ▶ Primary electrons identified using energy
- ▶ Small recoil effect on bare primary leptons visible below Z mass
- ▶ Reference is YFS (photon emission corrections only)

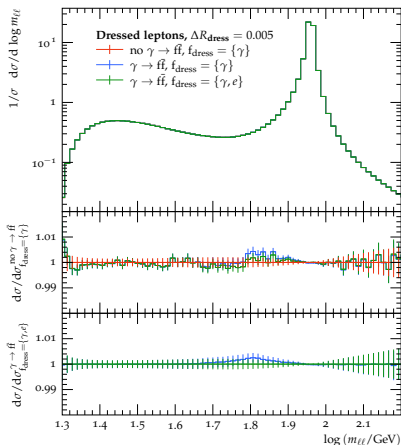
Academic case: on-shell $Z \rightarrow e^+e^-$ Dilepton invariant mass for on-shell $Z \rightarrow e^+e^-$ 

- ▶ Distance measure is $\Delta\Theta_{\text{dress}} = ((\Delta\theta)^2 + (\Delta\phi)^2)^{1/2}$
- ▶ Left: small dressing cone
- ▶ Upper ratio plot wrt. YFS (photon emission corrections only)
- ▶ Lower ratio plot wrt. YFS + photon splittings dressed with photons and electrons

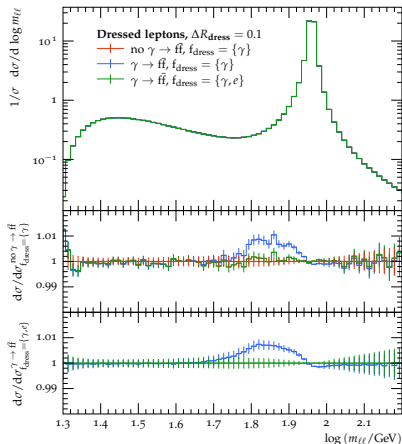
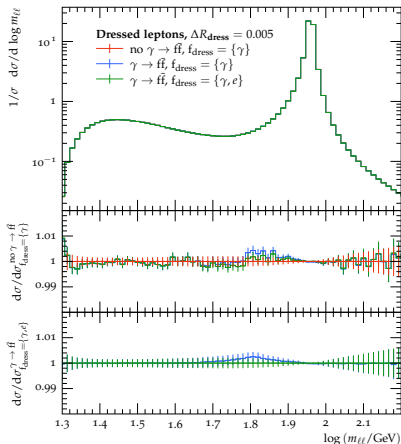
Academic case: on-shell $Z \rightarrow e^+e^-$ Dilepton invariant mass for on-shell $Z \rightarrow e^+e^-$ 

Realistic case: $pp \rightarrow e^+e^-$ Dilepton invariant mass for $pp \rightarrow e^+e^-$ (preliminary)

- ▶ Reference is YFS (photon emission corrections only)
- ▶ No significant correction without improved statistics

Realistic case: $pp \rightarrow e^+e^-$ Dilepton invariant mass for $pp \rightarrow e^+e^-$ (preliminary)

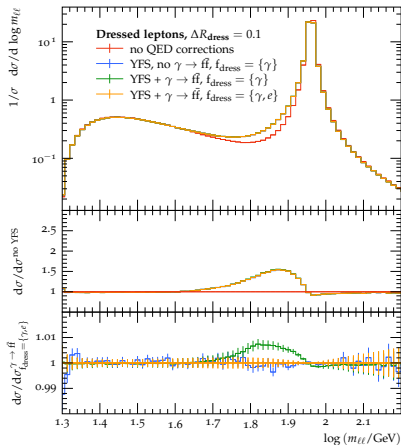
- ▶ Distance measure is $\Delta R_{\text{dress}} = ((\Delta\eta)^2 + (\Delta\phi)^2)^{1/2}$
- ▶ Left: small dressing cone
- ▶ Correction from $\gamma \rightarrow f\bar{f}$ now statistically significant due to recombination of momenta
- ▶ Upper ratio plot wrt. YFS
- ▶ Lower ratio plot wrt. YFS + photon splittings dressed with photons and electrons

Realistic case: $pp \rightarrow e^+e^-$ Dilepton invariant mass for $pp \rightarrow e^+e^-$ (preliminary)

Conclusions

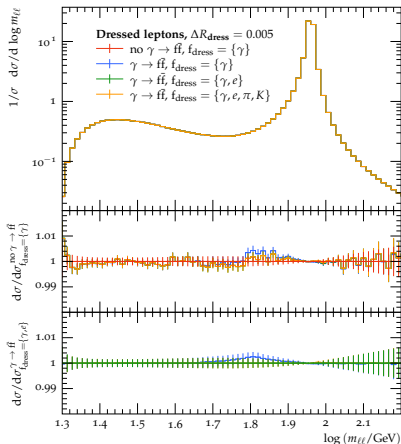
- ▶ We introduced an automated method for including photon splitting corrections to the YFS implementation in SHERPA
- ▶ $\gamma \rightarrow f\bar{f}$ introduces corrections at the per mille level for bare leptons and at the percent level for photon-dressed leptons
- ▶ By introducing novel flavour-aware dressing strategies, we limit these corrections and reduce dependence on the cone size
- ▶ Both the photon splitting method and the dressing strategies are general and applicable to a wide range of setups
- ▶ Included in 3.0beta release - see Eno's talk

Backup: Dilepton invariant mass for $pp \rightarrow e^+e^-$



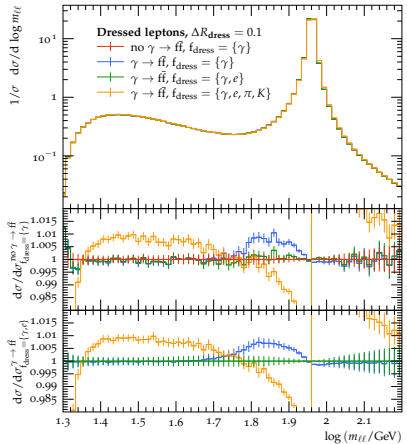
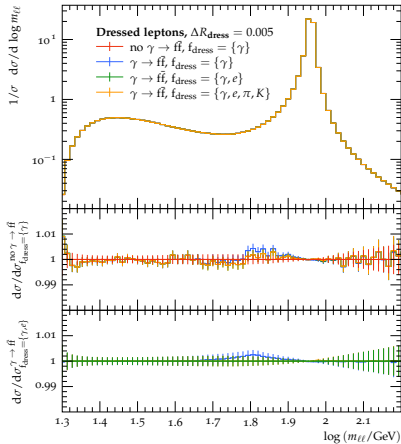
- ▶ QED corrections are $\sim 50\%$
- ▶ Photon splittings give a 1% correction to photon-dressed leptons
- ▶ This correction disappears when electrons are included in the dressing

Dilepton invariant mass for $pp \rightarrow e^+e^-$ (preliminary)

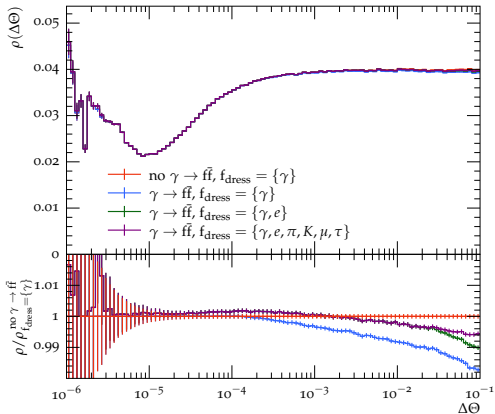


- ▶ Include light charged hadrons in dressing
- ▶ Does this work in a hadron collider environment?
- ▶ Left: small dressing cone - no further benefit
- ▶ Right: larger dressing cone - detrimental

Dilepton invariant mass for $pp \rightarrow e^+e^-$ (preliminary)



Backup: Energy density of a dressed lepton



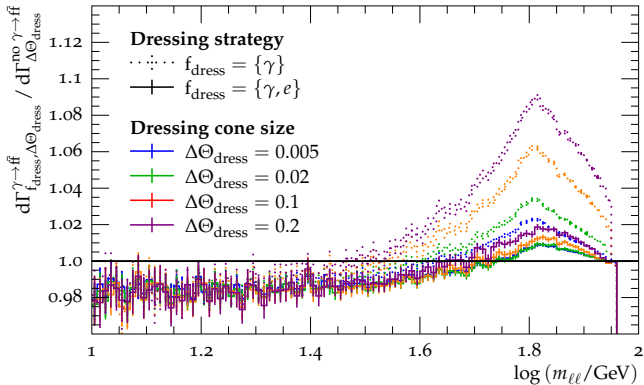
- ▶ Distance measure is $\Delta\Theta_{\text{dress}} = ((\Delta\theta)^2 + (\Delta\phi)^2)^{1/2} = 0.1$
- ▶ Photon splittings cause loss in energy of dressed lepton
- ▶ Including electrons in dressing recoups most of lost energy

Splitting functions

$$\begin{aligned}
 S_{s_{\tilde{ij}}(\tilde{k}) \rightarrow s_i \gamma_j(k)} &= -Q_{\tilde{ij}\tilde{k}}^2 \alpha(0) \left[\frac{2}{1-z+zy} - \frac{\tilde{V}_{\tilde{ij},\tilde{k}}}{v_{ij,k}} \left(2 + \frac{m_i^2}{p_i p_j} \right) \right] \\
 S_{f_{\tilde{ij}}(\tilde{k}) \rightarrow f_i \gamma_j(k)} &= -Q_{\tilde{ij}\tilde{k}}^2 \alpha(0) \left[\frac{2}{1-z+zy} - \frac{\tilde{V}_{\tilde{ij},\tilde{k}}}{v_{ij,k}} \left(1 + z + \frac{m_i^2}{p_i p_j} \right) \right] \\
 S_{\gamma_{\tilde{ij}}(\tilde{k}) \rightarrow s_i \bar{s}_j(k)} &= S_{\gamma_{\tilde{ij}}(\tilde{k}) \rightarrow f_i \bar{f}_j(k)} = -Q_{\tilde{ij}\tilde{k}}^2 \alpha(0) \left[1 - 2z(1-z) - z_+ z_- \right]
 \end{aligned}$$

Catani et al. '02, Dittmaier et al. '08, Schumann, Krauss '08

Backup: Dressing cone size dependence

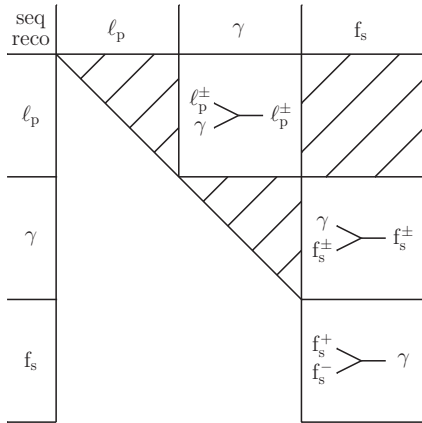


Backup: Leptonic W decay

- ▶ The W is charged and the neutrino is not, so instead of an FF dipole we have an FI dipole $W - \ell$
- ▶ Large W mass suppresses photon emissions, so neglect it as an emitter
- ▶ Modify kinematic variables and splitting functions – we keep the W eikonal term

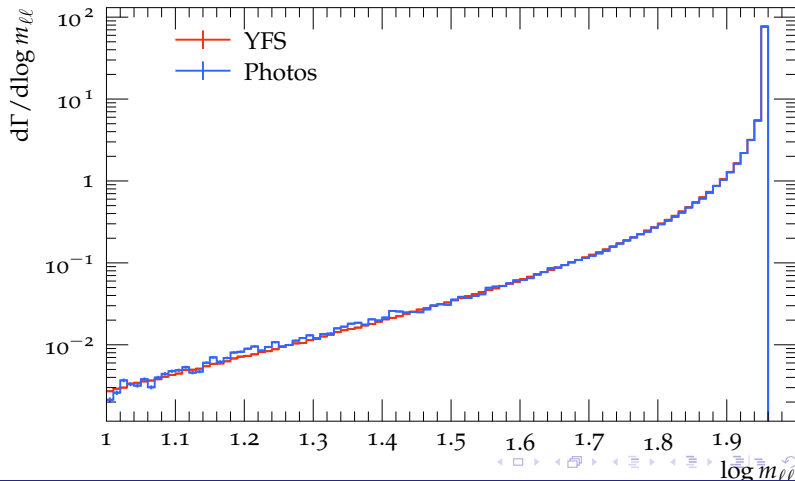
Basso et al. '16

Backup: Sequential recombination dressing

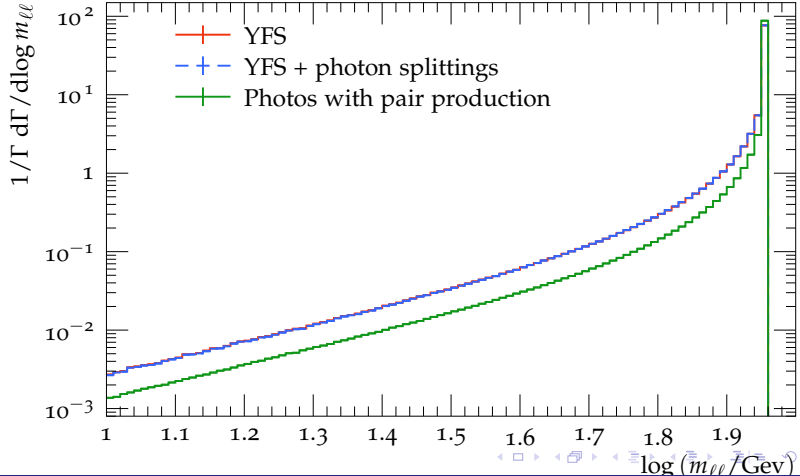


- ▶ Similarities with QCD jets & ability to distinguish flavour
- ▶ Flavour- k_\perp algorithm?
- ▶ Future work only - this would not be backwards-compatible

Backup: Comparison with Photos



Backup: Comparison with Photos



Backup: Comparison with Photos

