A Logarithmically Accurate Resummation In C++

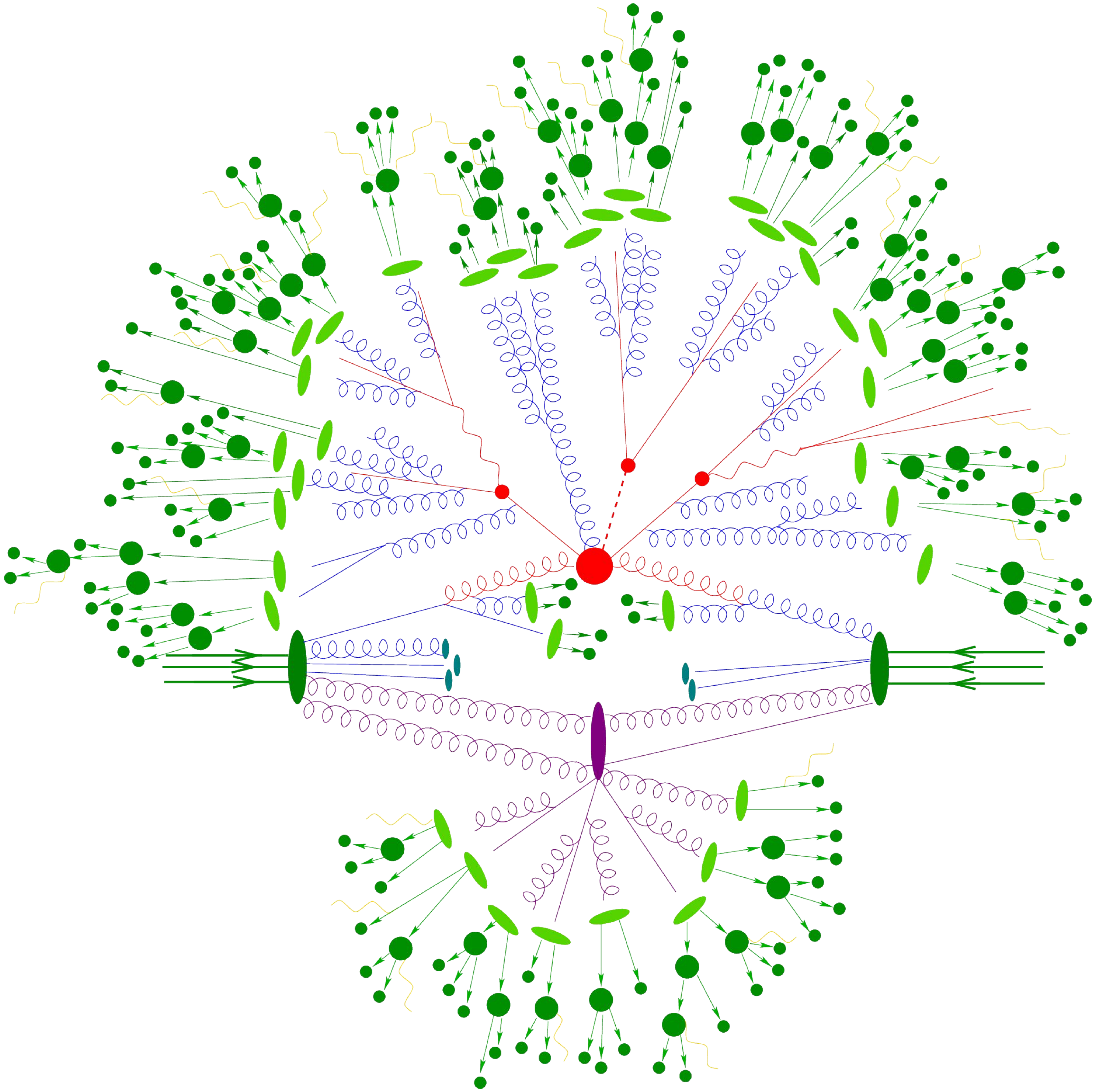
QCD@LHC 2023, 4 September 2023

[arXiv:2208.06057]

Daniel Reichelt, work in collaboration with Florian Herren, Stefan Höche, Frank Krauss and Marek Schönherr

colliders for theorists

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays



A Logarithmically Accurate Resummation In C++

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays

See Talk by Gavin Salam this morning

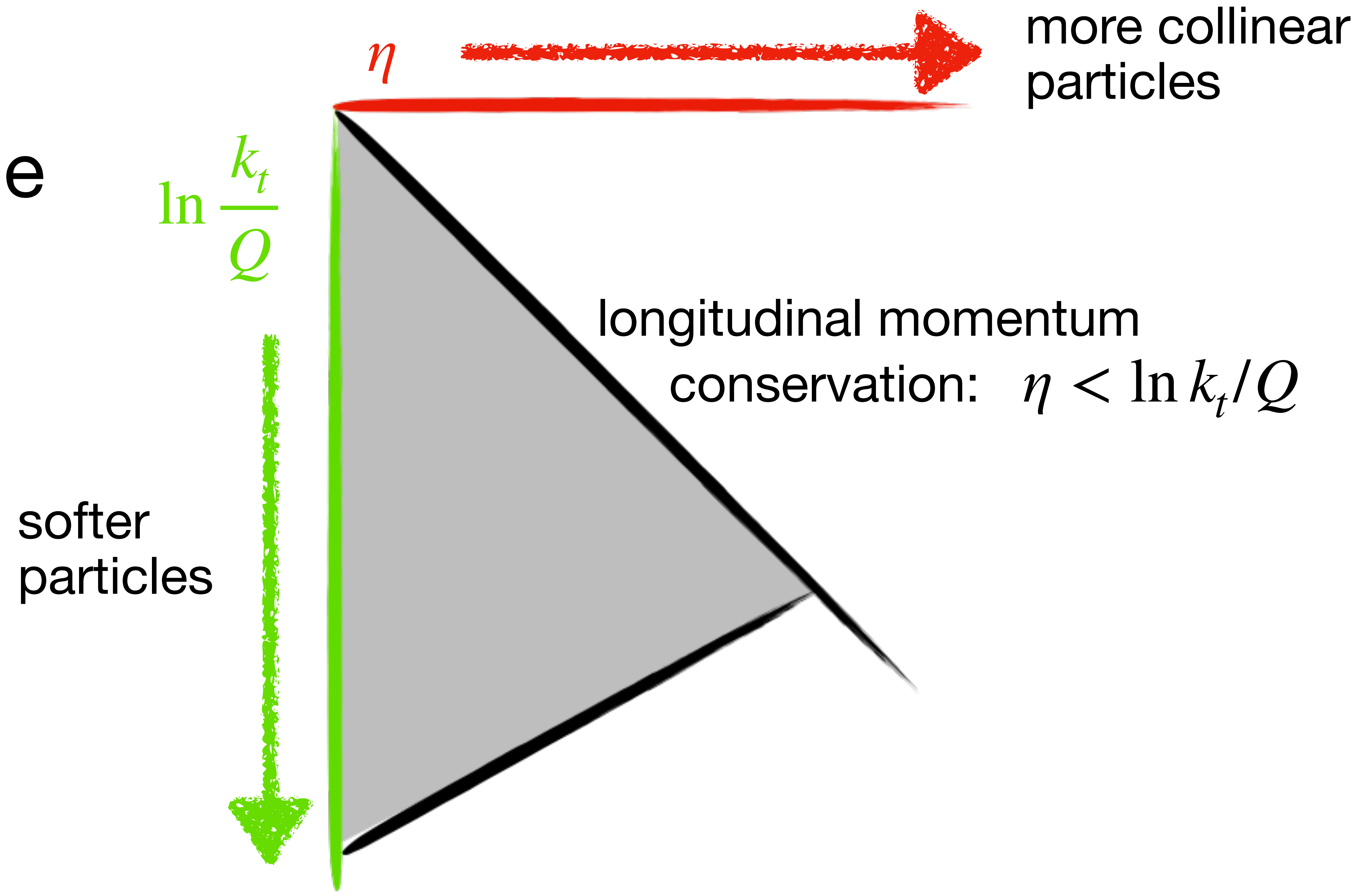
This Talk:

Why?

- parton showers resum large logs ~ NLL, but open questions on actual accuracy
- starting work towards NNLL/NLO evolution → probably better resolve this first
- recent formal discussion → current dipole showers need reworking [Dasgupta,Dreyer,Hamilton,Monni,Salam ’18]

parton showers - Cliff notes version

- no-emission probability (sudakov factor) $\sim \exp\left[-\int_{t_0}^{t_1} \frac{dk_t}{k_t} dz \frac{\alpha_S}{2\pi} P(z)\right]$
- splitting kernels $P(z)$ captures soft and collinear limits of matrix elements
- fill phase space ordered in evolution variable $(k_t, \theta, q^2, \ldots) \Rightarrow$ here k_t ordered shower
- generate new final state after emission according to recoil scheme



splitting of Eikonal

Starting point: eikonal

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} \equiv \frac{W_{ik,j}}{E_j^2}$$

naive implementation leads to soft double counting need to split into ij and kj collinear terms [Marchesini, Webber ‘88]

Option 1:

$$W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}\,, \qquad \text{where} \qquad \tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

- e.g. Angular ordered shower, downside: problems with NGLs

Option 2: follow [Catani, Seymour ’97]

$$W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}\,, \qquad \text{where} \qquad \bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{jk})}$$

- full phase space coverage, splitting functions remain positive definite

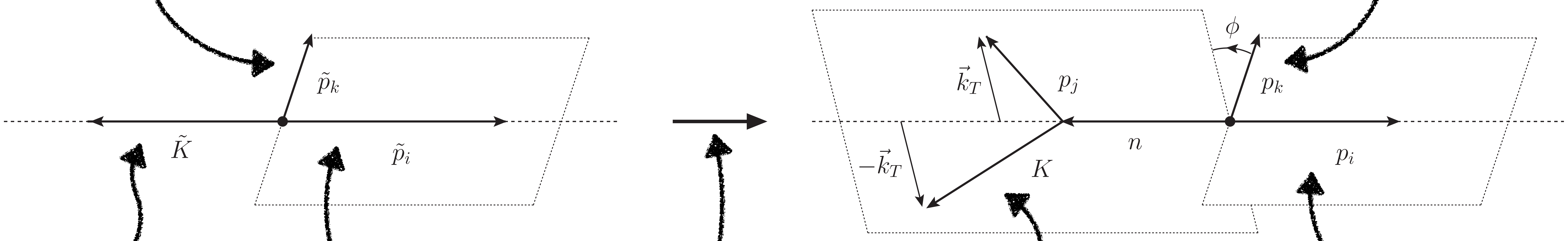
Note related ideas in [Forshaw, Holguin, Plätzer ’20]

kinematics - global recoil scheme

- Before splitting:
- After splitting:

colour spectator

$p_k = \tilde{p}_k$



$\sum$ other momenta k_i

splitter

$p_i = z\tilde{p}_i$

$K^2 = \tilde{K}^2$

$\tilde{K} + \tilde{p}_i = K + p_i + p_j$

$k_i^\mu \to \Lambda^\mu{}_\nu k_i^\nu$

[Catani, Seymour ’97]

$$\Lambda^\mu{}_\nu = g^\mu{}_\nu - \frac{(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{K\cdot\tilde{K} + \tilde{K}^2} + 2\frac{K^\mu \tilde{K}_\nu}{\tilde{K}^2} \quad \to \quad \Lambda^\mu{}_\nu \tilde{K}^\nu = K^\mu$$

effect of recoil on accuracy - multiple emissions

- QCD coherence → factorised emissions
- observables dependece correlated → how to extract NLL without additional information?
- method from [Banfi, Salam, Zanderighi '05]: need explicit soft-collinear limit*:

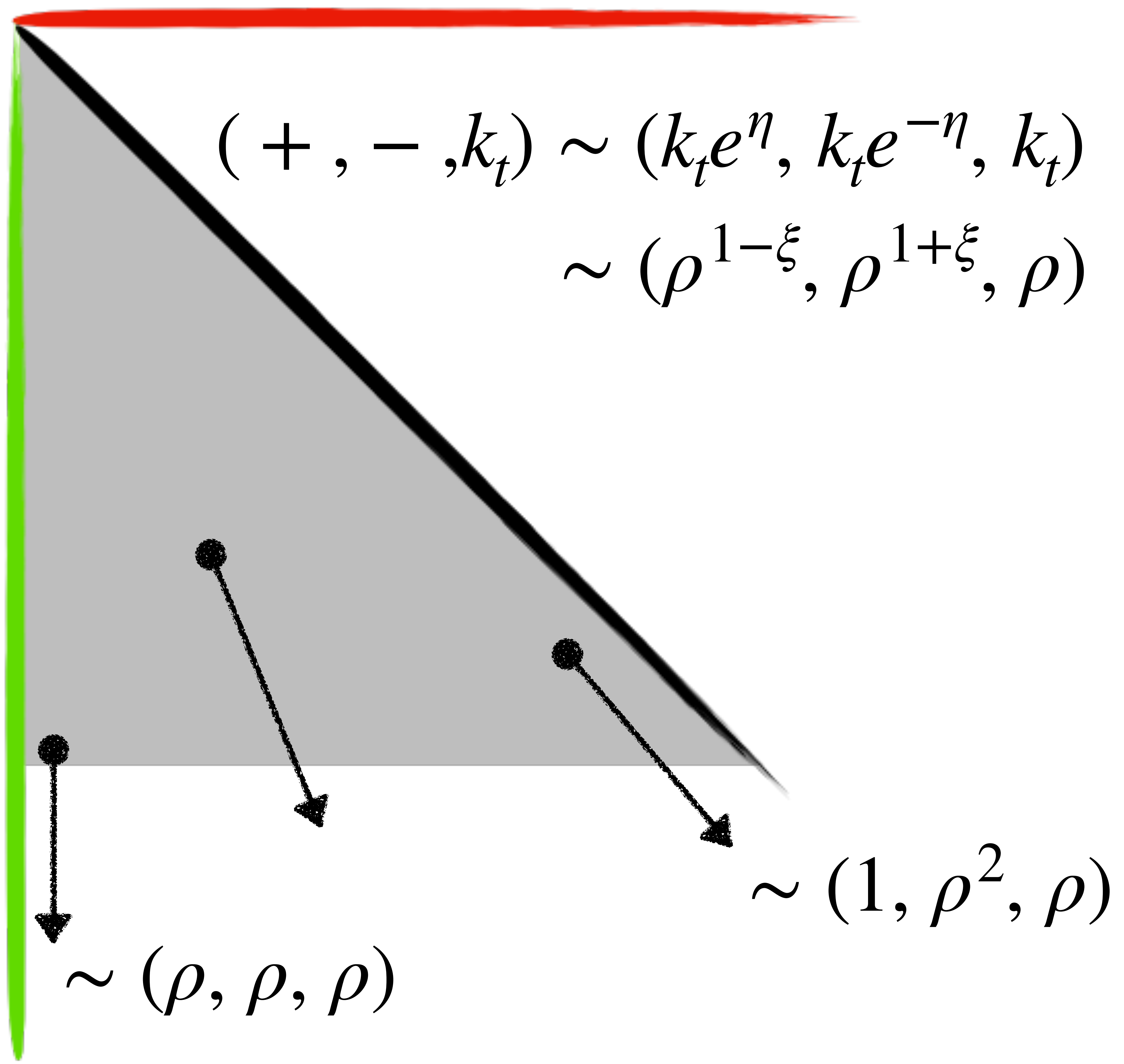
$$k_t^\rho = k_t \rho \qquad \xi = \frac{\eta}{\eta_{\max}}$$

$$\eta^\rho = \eta - \xi \ln \rho$$

and assume

$$V(k_i^\rho) = \rho V(k_i)$$

→ numerically evaluate integrals in this limit



* again assume $V(k_t, \eta) \sim k_t/Q$ for brevity

effect of recoil on accuracy

- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \to 0$)?*
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]

See Talk by Gavin Salam this morning

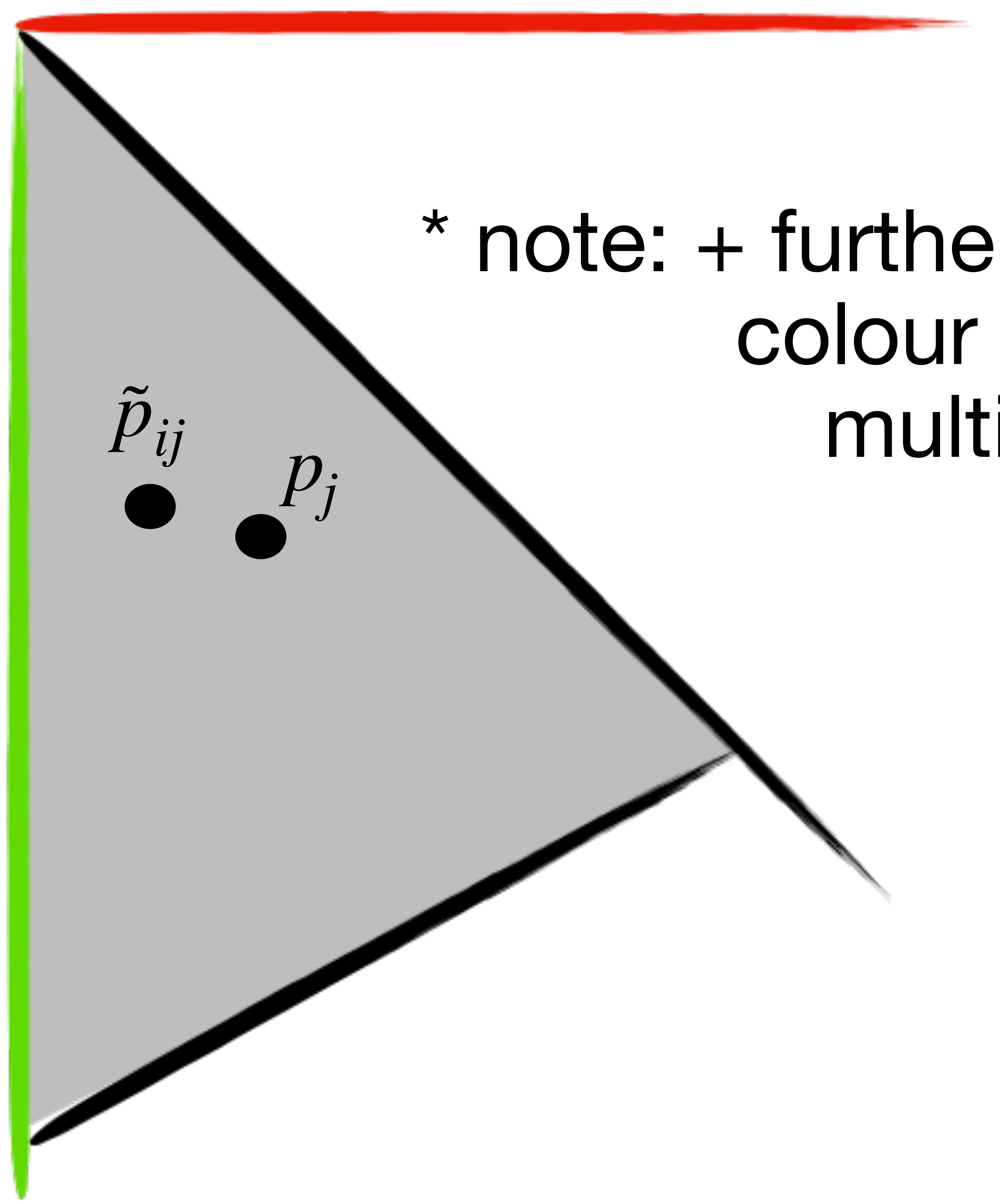
$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$

- consider situation where we first emit $\tilde{p}_{ij}$ from p_a, p_b, then emit p_j, $\tilde{p}_{ij} \to p_i, p_j$

- transverse momentum of p_i will be

$$\sim k_t^{ij} + k_t^j$$

$$\Rightarrow \frac{\Delta k_t^{ij}}{k_t^{ij}} \to \frac{\rho k_t^j}{\rho k_t^{ij}} = \mathcal{O}(1)$$



* note: + further problems for colour assignment in multiple emissions

analytic proof of accuracy

$$\Lambda^{\mu}{}_{\nu}(K,\tilde{K}) = g^{\mu}_{\nu} + \tilde{K}^{\mu}A_{\nu} + X^{\mu}B_{\nu}$$

vanishes in soft limit

work out $\rho \to 0$ limit: $A^{\nu} \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^{\nu}}{\tilde{K}^2} - \frac{X^{\nu}}{\tilde{K}^2}\,,$ and $B^{\nu} \xrightarrow{\rho\to 0} \frac{\tilde{K}^{\nu}}{\tilde{K}^2}$

apply to soft momentum p_l:

$$\frac{\Delta p_l^{0,3}}{p_l^{0,3}} \sim \rho^{\,1-\max(\xi_i,\xi_j)}$$

$$\frac{\Delta p_l^{1,2}}{p_l^{1,2}} \sim \rho^{\,(1-\xi_l)(\max(\xi_i,\xi_j)-\xi_l)}$$

compare to $\frac{\Delta k_t}{k_t} \sim \mathcal{O}(1)$ from local dipole scheme

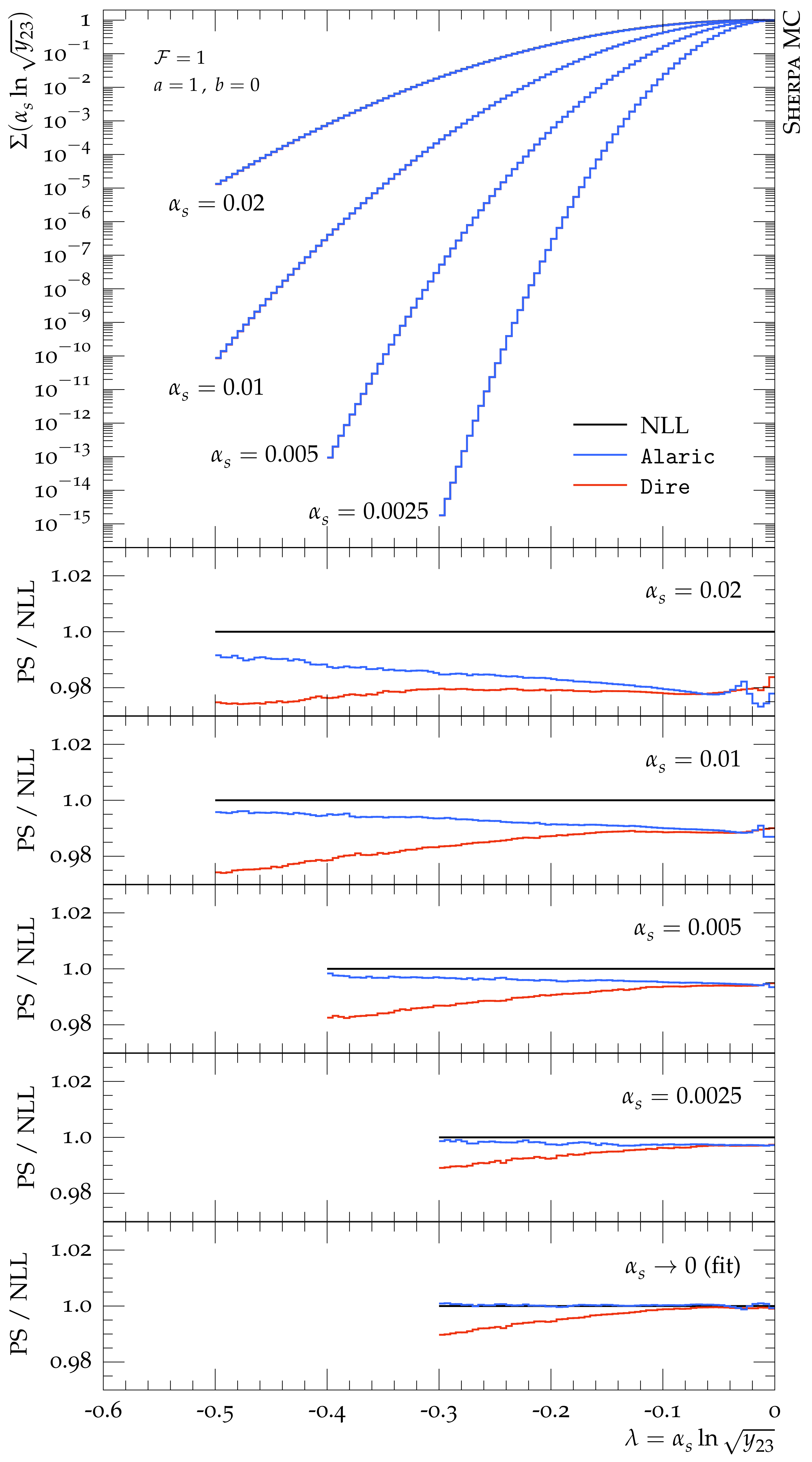
numerical validation I

- Limit $\alpha_s \to 0$ with $\lambda = \alpha_s L =$ const. of

$$\frac{\Sigma^{\text{Shower}}}{\Sigma^{\text{NLL}}} \sim \exp\left(f^{LL}_{\text{Shower}} - Lg_1(\alpha_s^n L^n)\right)$$
$$\times \exp\left(f^{NLL}_{\text{Shower}} - g_2(\alpha_s^n L^n)\right)$$
$$\times \exp\left(\mathcal{O}(\alpha_s^{n+1} L^n)\right)$$

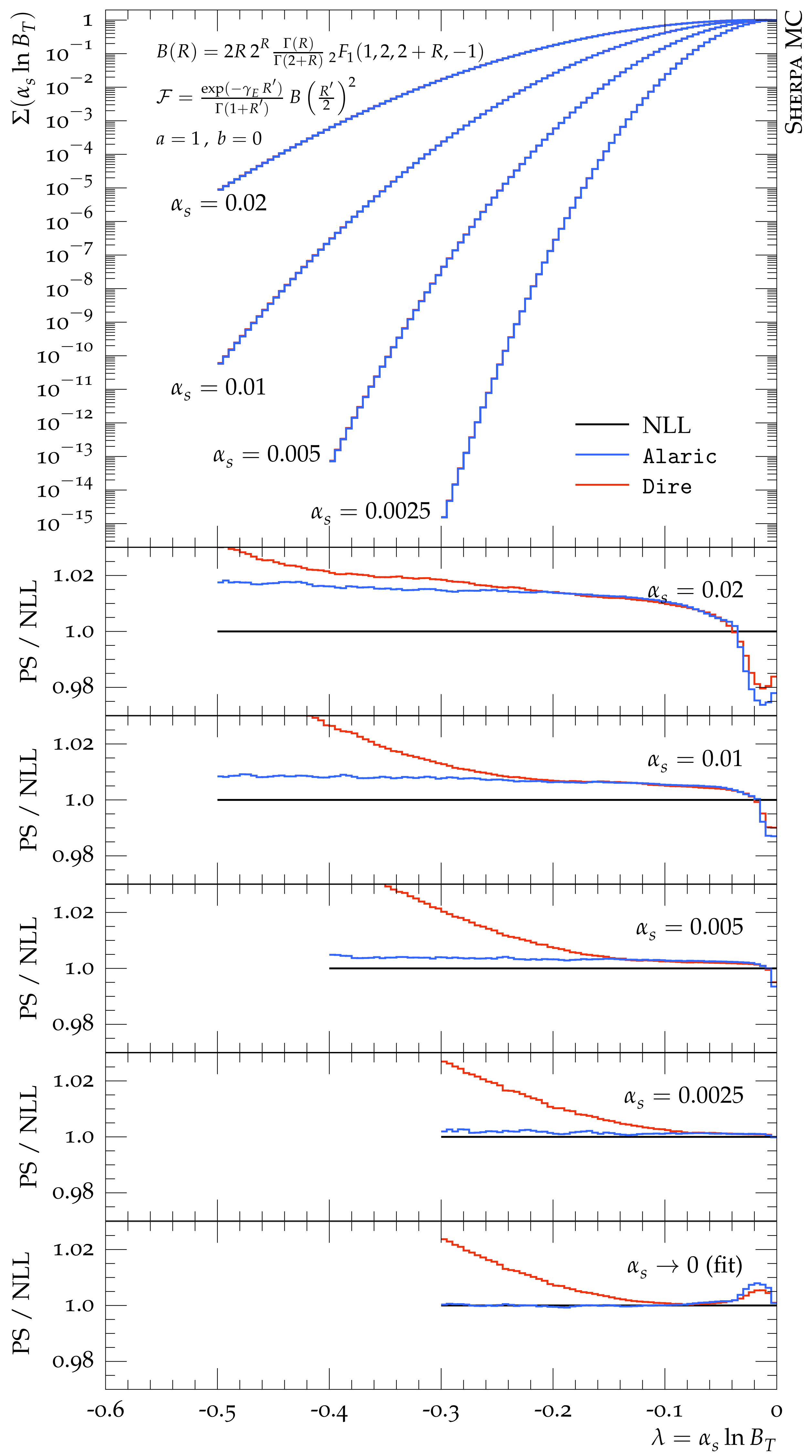
$\to 1$ if shower reproduces LL, NLL logs

- Observable: jet resolution y_{23} in Cambridge jet measure, $\mathcal{F} = 1 \to$ only largest emission matters, check that additional shower emissions vanish

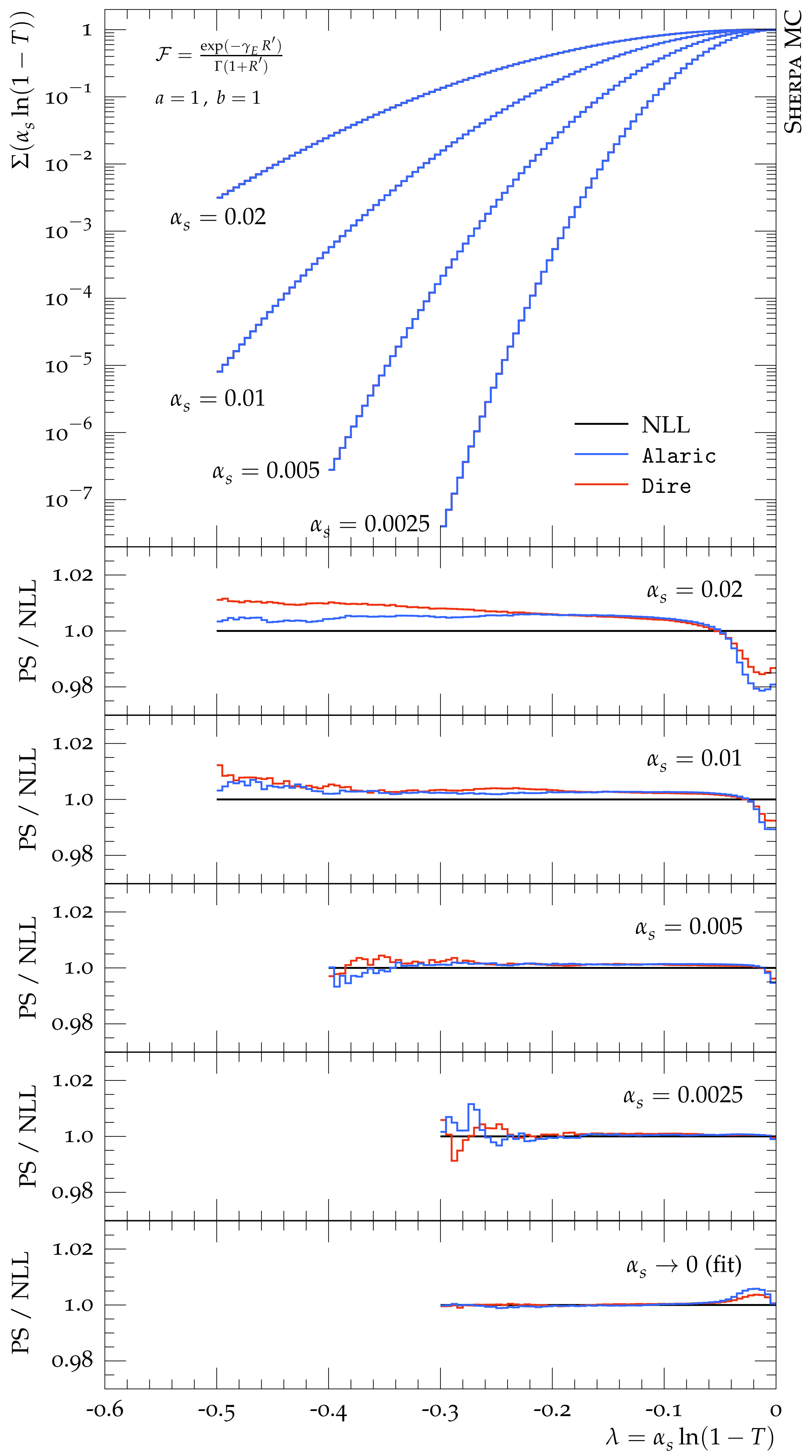


numerical validation II

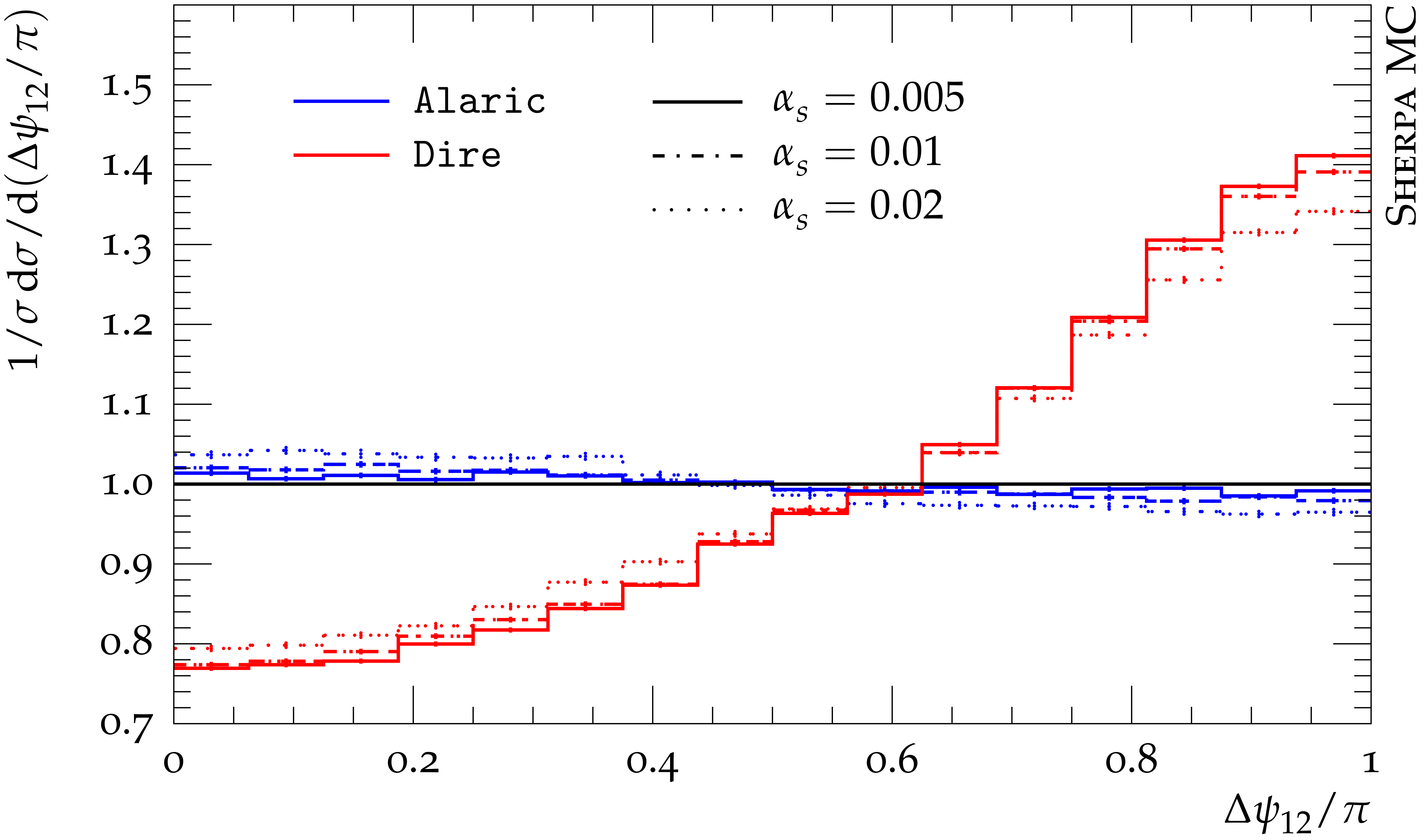
- total broadening $B_T = B_L + B_R$
- scaling k_t like, similar to y_{23}
- but non-trivial $\mathscr{F}$ function



- thrust $\tau = 1 - t$
- scaling like virtuality $k_t e^{-\eta}$
- standard function $\mathscr{F} = \dfrac{\exp(-\gamma_E R)}{\Gamma(1+R')}$
- no evidence for NLL violation even for standard showers

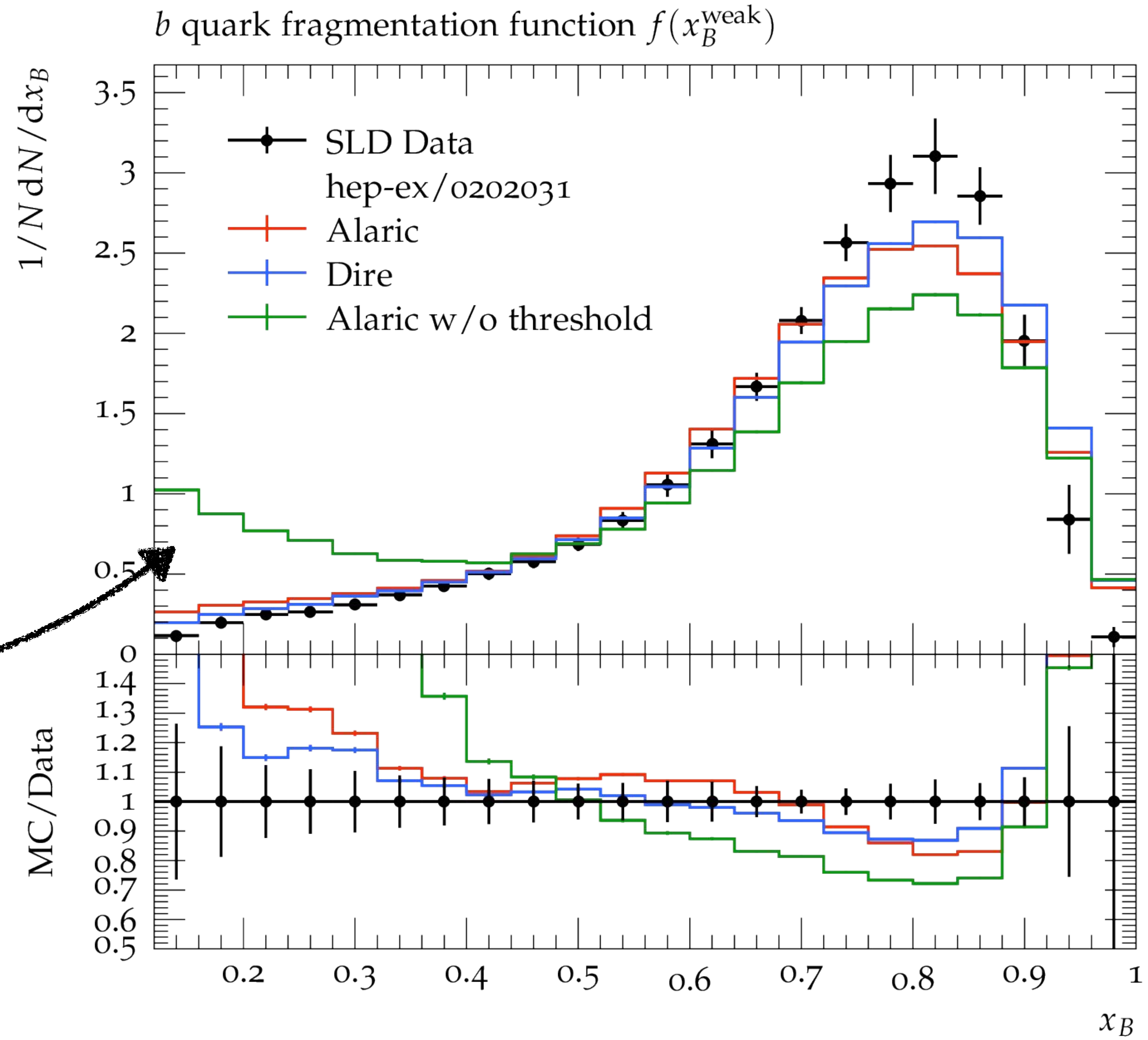


numerical validation III



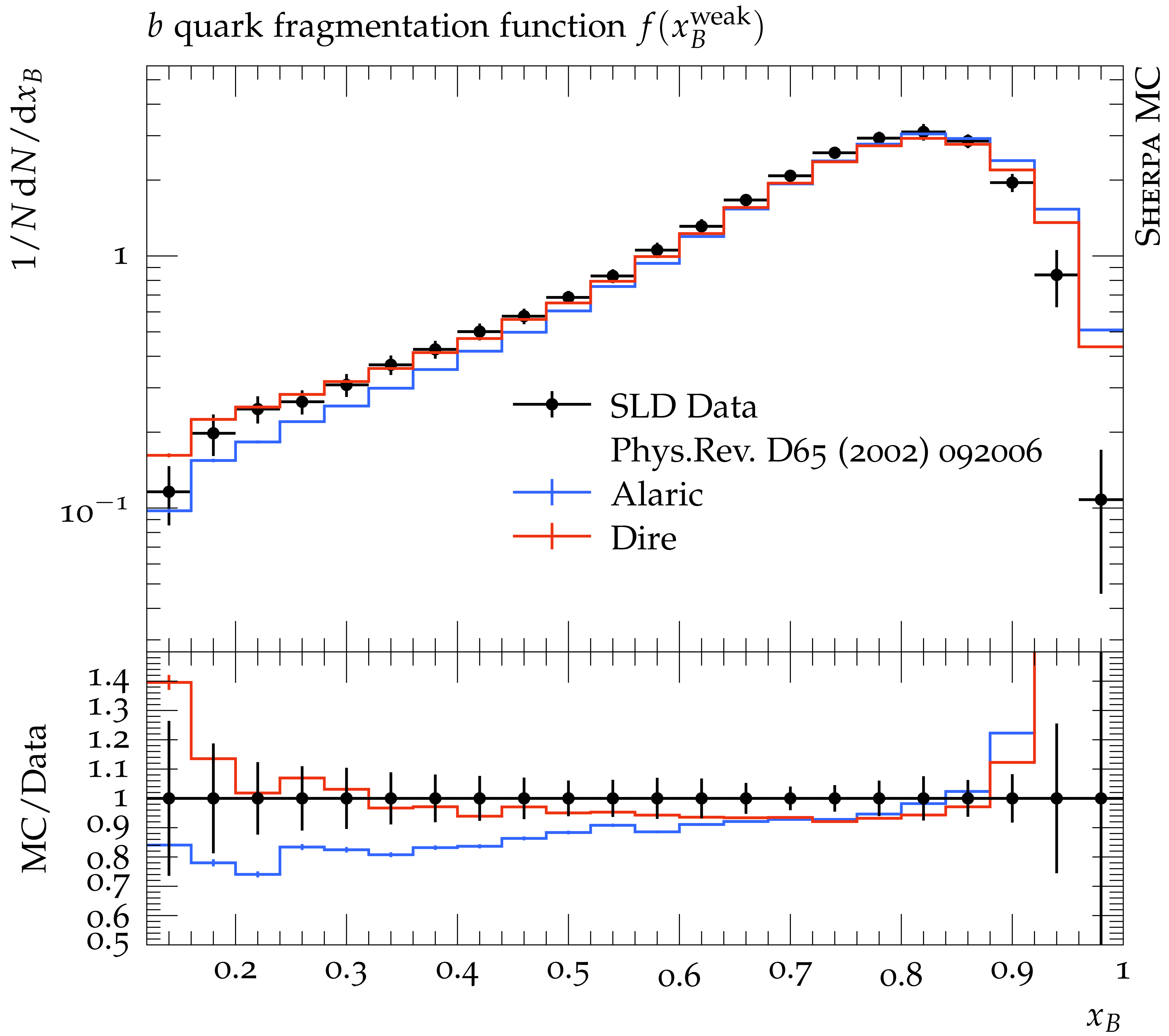
pheno, details and b fragmentation

- first caveat: no quark masses implemented yet
- problem for cluster hadronisation → use Lund model via Pythia
- + need flavour threshold for $g \to b\bar{b} / g \to c\bar{c}$ splittings
- Dire parton shower as implemented in Sherpa as reference, Lund model tuned for Alaric $\sigma = 0.3$ GeV, $a = 0.4$, $b = 0.36$ GeV^{-2} and for Dire $\sigma = 0.3$ GeV, $a = 0.4$, $b = 0.46$ GeV^{-2}



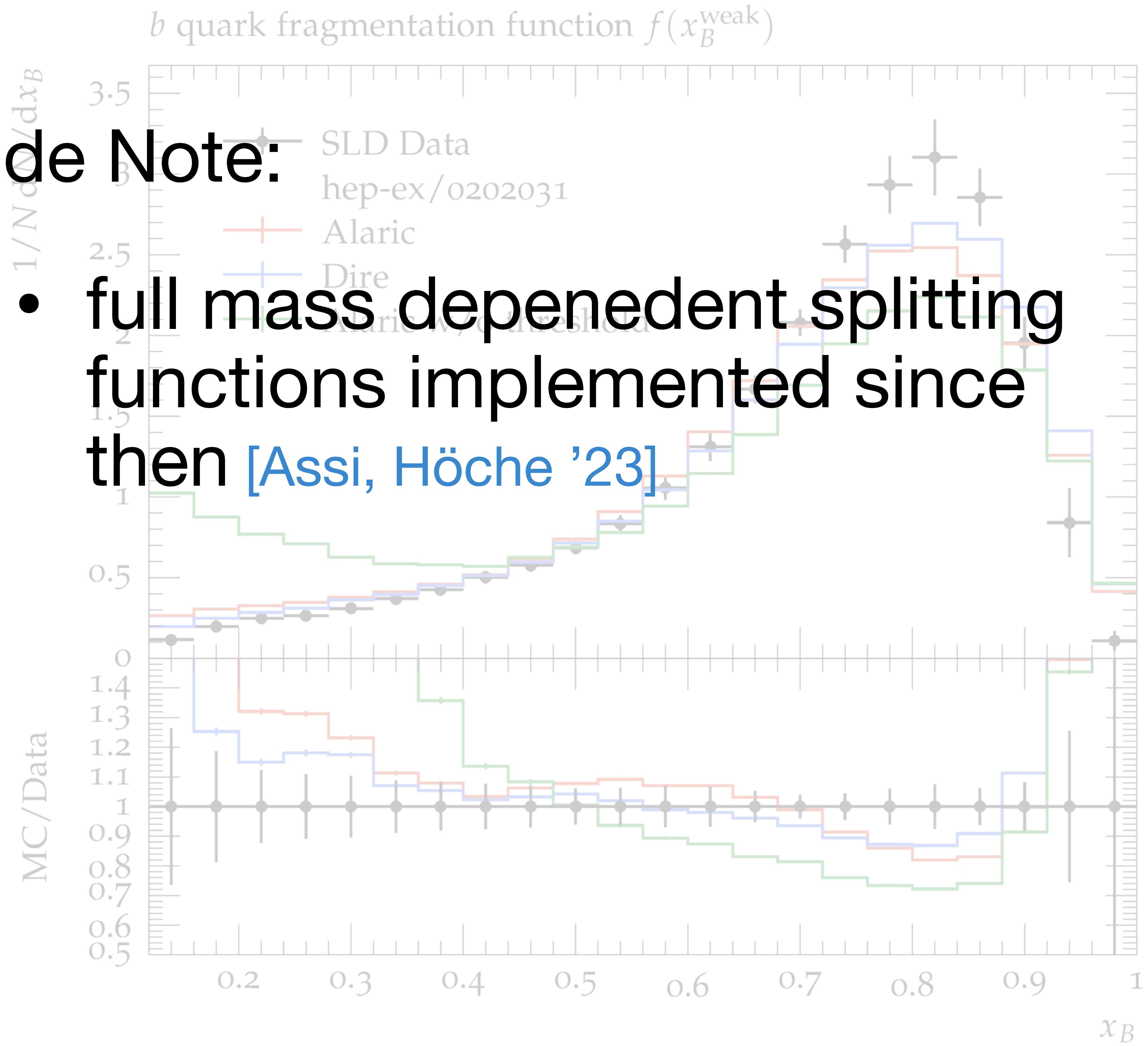
$$x_B \sim \frac{E_{B\text{-Hadron}}}{E_{\text{tot.}}/2}$$

pheno, details and b fragmentation



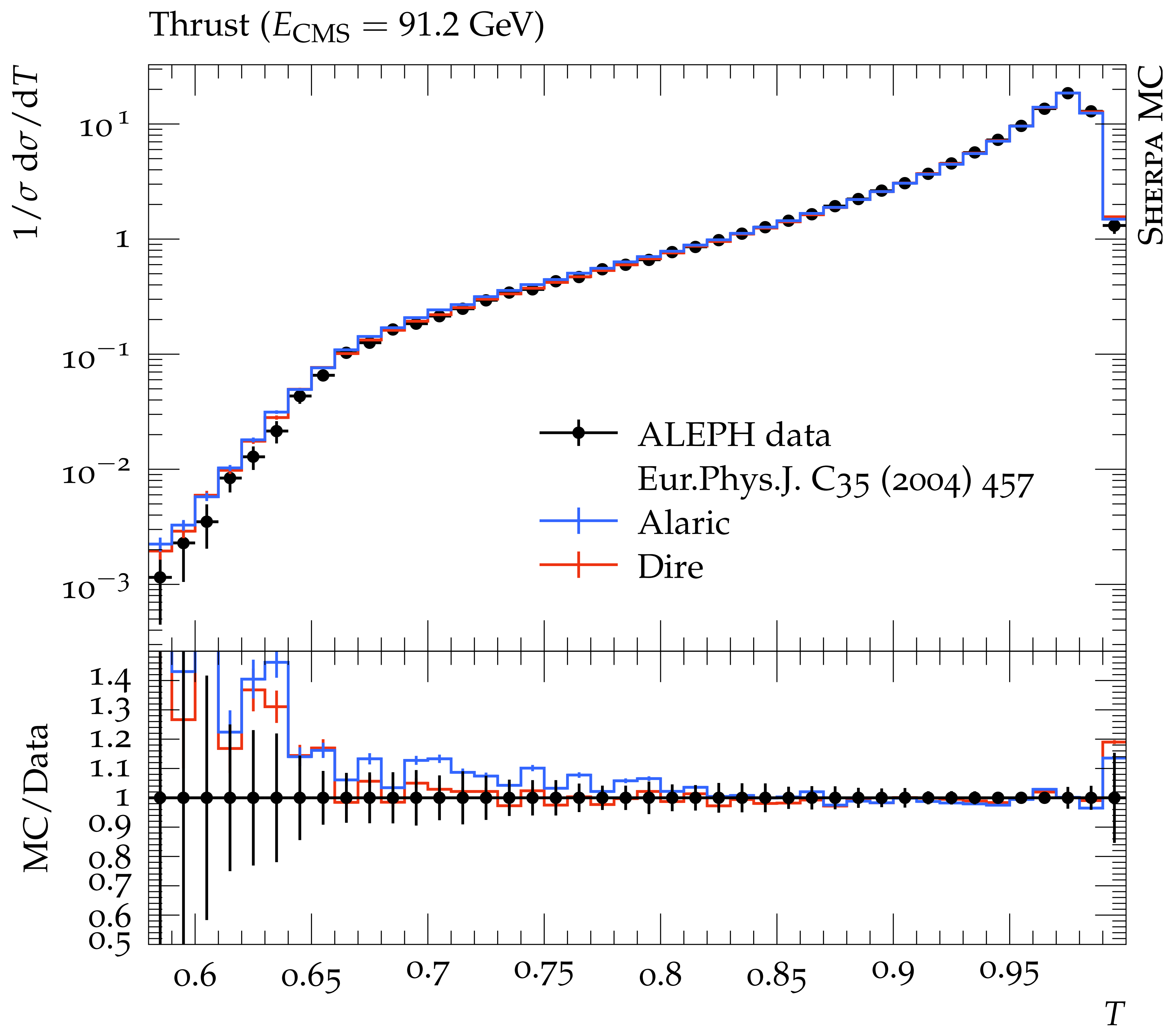
Side Note:

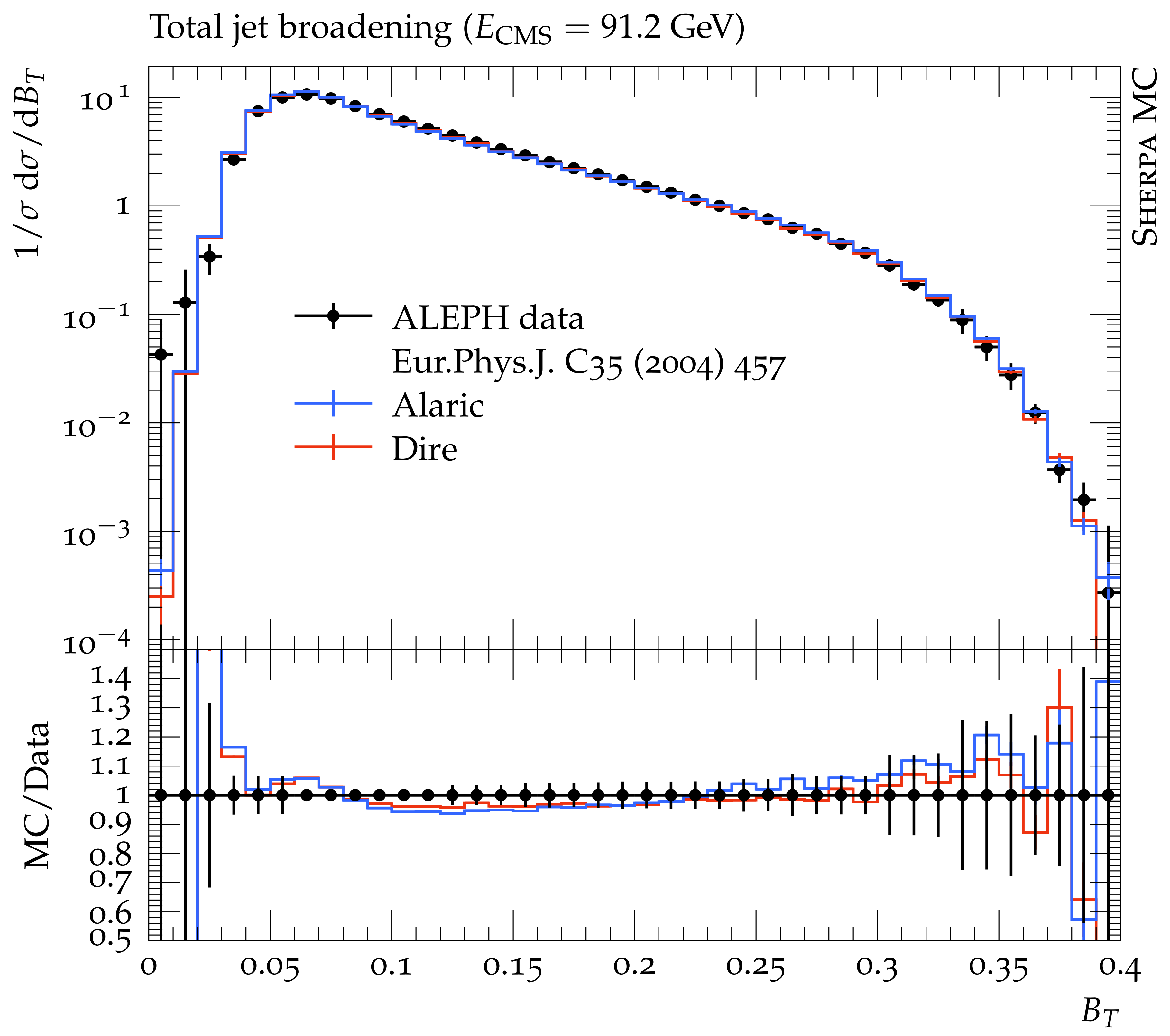
- full mass depenedent splitting functions implemented since then [Assi, Höche '23]



$$x_B \sim \frac{E_{B-\text{Hadron}}}{E_{\text{tot.}}/2}$$

pheno, LEP observables





Thrust:

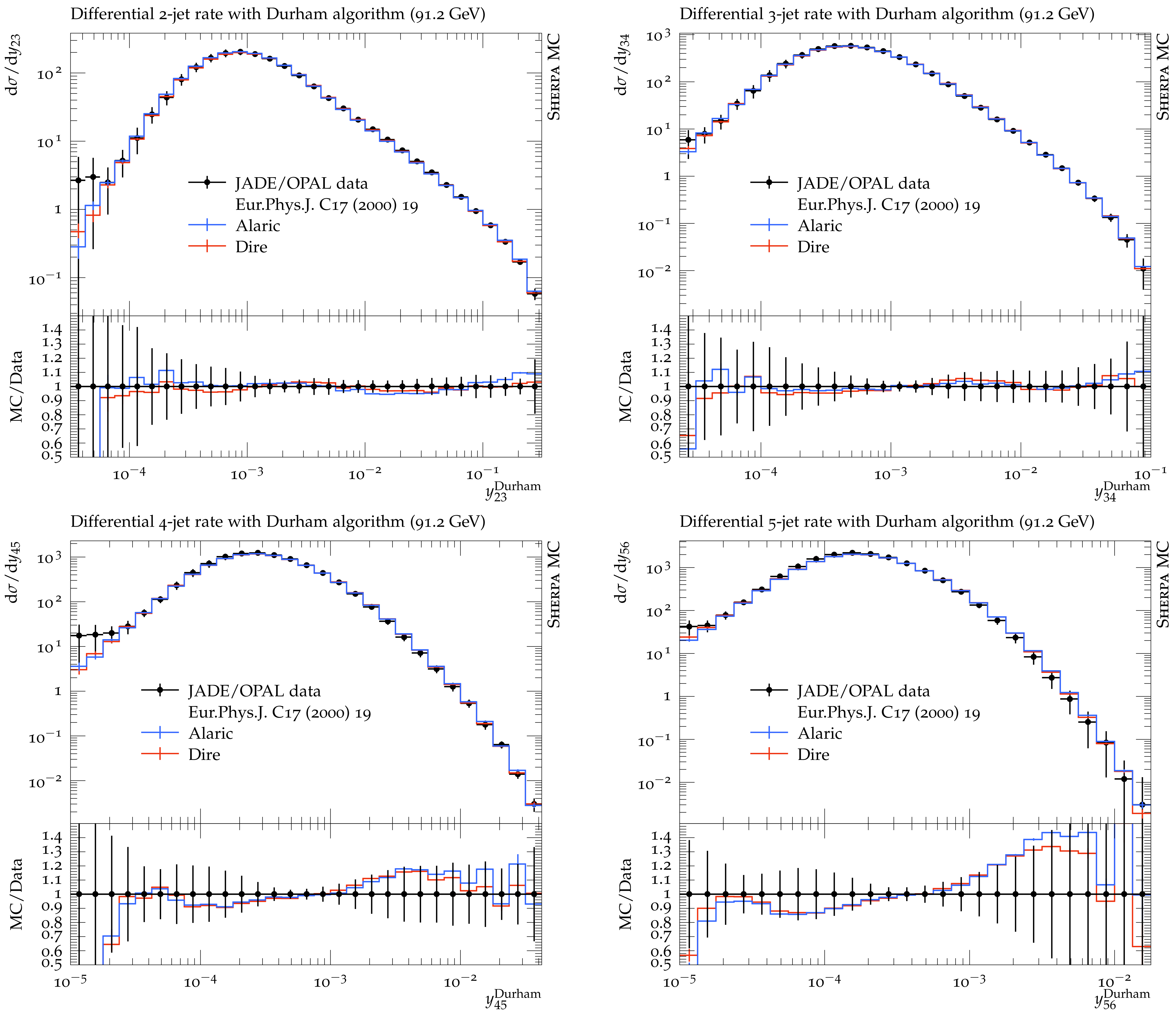
- Note this is T, not 1-T: soft physics is to the right
- Note there is no matching, relevant for small T

Total Broadening:

- soft physics is left hand side
- some deviations from data, but similar to Dire

pheno, LEP observables

- Durham resolution scales $y_{n,n+1} \sim k_t^2/Q^2$
- higher Born multiplicities → sensitivity to multiple emissions increased
- again, note no matching/merging involved



Alaric initial state shower

- Formalism presented in [Herren, Höche, Krauss, DR, Schönherr '22] general and applicable to initial state evolution
- practical considerations:
    - precise definition of evolution variable
    - PDFs, clear in principle, but more choices to make
    - distribution of recoil (i.e. definition of $\tilde{K}$)

Alaric initial state shower

- Formalism presented in [Herren, Höche, Krauss, DR, Schönherr '22] general and applicable to initial state evolution
- practical considerations:
  - precise definition of evolution variable/scales
  - PDFs, clear in principle, but more choices to make
  - distribution of recoil (i.e. definition of $\tilde{K}$)

in principle stay k_t ordered:

$$t_{FS} = vz(1-z)2\tilde{K}\tilde{p}_i$$

$$t_{IS} = v\frac{(1-z)}{z}2\tilde{K}\tilde{p}_i$$

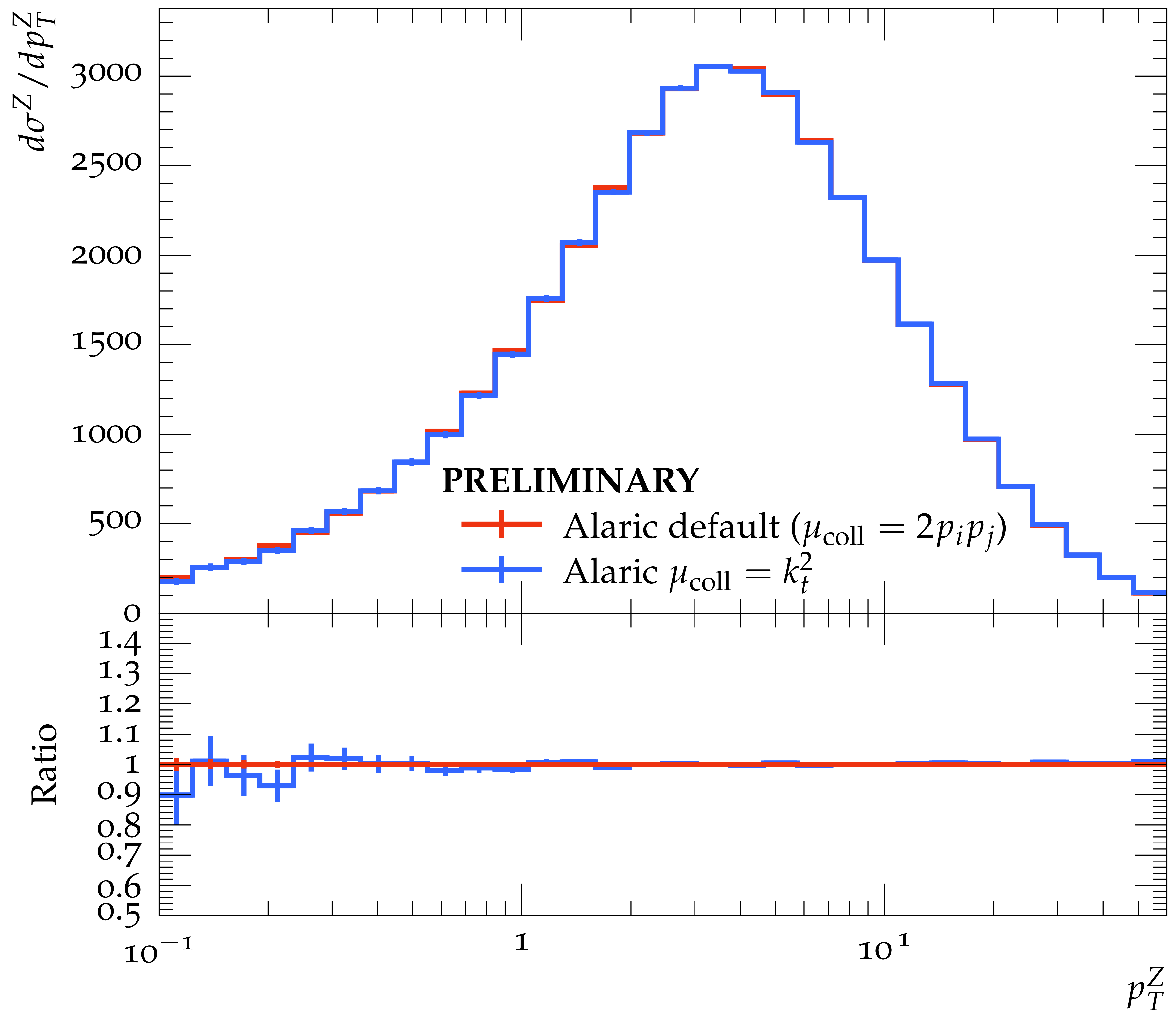
scale in collinear (SL) evolution:

- choice between k_t^2 vs. virtuality

choice between k_t^2 vs. virtuality

scale in collinear (SL) evolution:

- choice between k_t^2 vs. virtuality
- little to no effect in p_T^Z spectrum



Alaric initial state shower

- Formalism presented in [Herren, Höche, Krauss, DR, Schönherr '22] general and applicable to initial state evolution
- practical considerations:
  - precise definition of evolution variable/scales
  - PDFs, clear in principle, but more choices to make
  - distribution of recoil (i.e. definition of $\tilde{K}$)

PDF choice:

- initial studies made using CT14nnlo
- use virtuality as shower scale

Alaric initial state shower

- Formalism presented in [Herren, Höche, Krauss, DR, Schönherr ‘22] general and applicable to initial state evolution
- practical considerations:
    - precise definition of evolution variable/scales
    - PDFs, clear in principle, but more choices to make
    - distribution of recoil (i.e. definition of $\tilde{K}$)

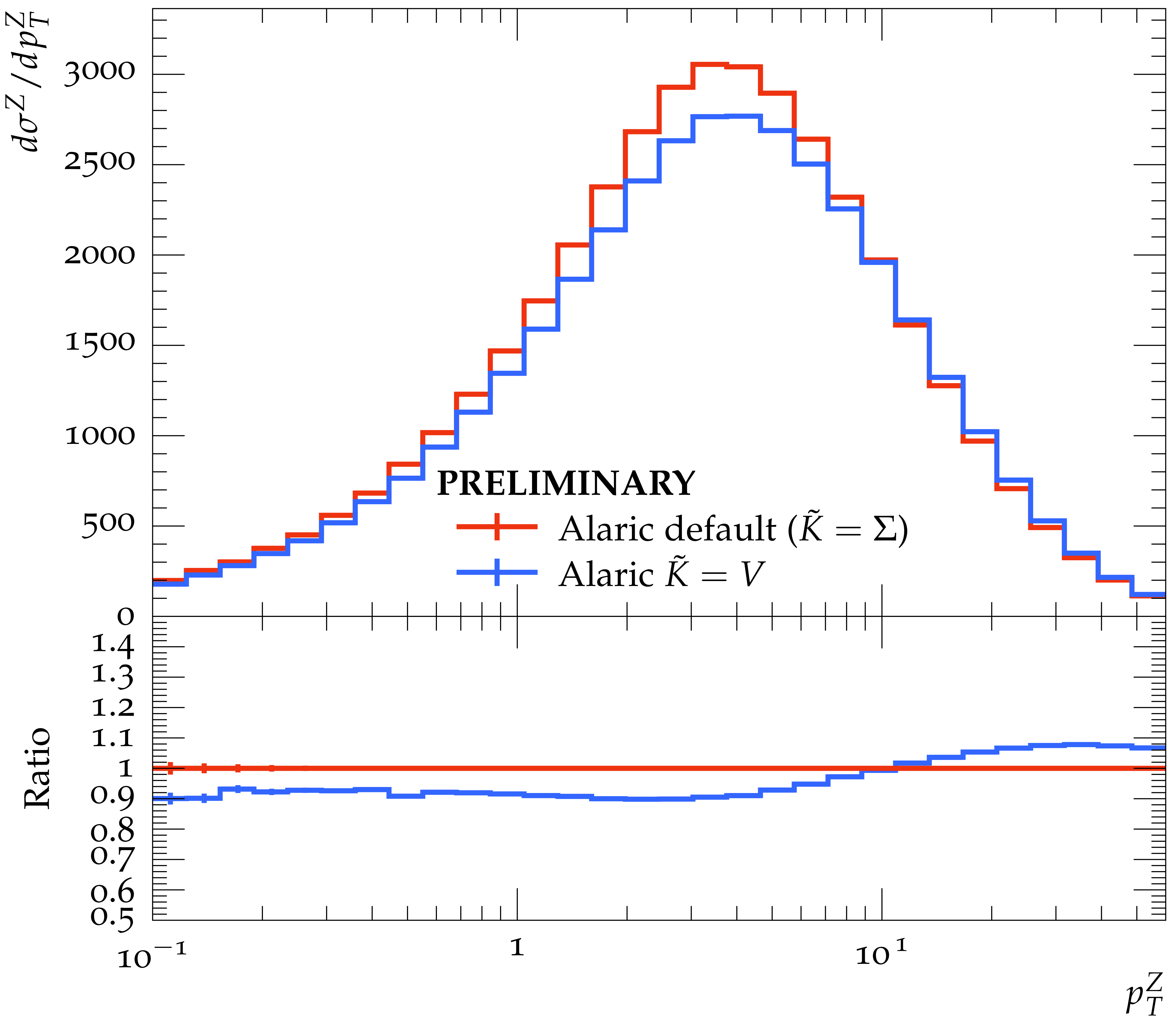
Alaric IS - choice of $\tilde{K}$

- effect of recoil (i.e. Lorentz transformation) vanishes for soft particles → in limit, should not matter if

$$\tilde{K} = \Sigma = \sum_{\text{FS}} p_i \qquad \text{or}$$

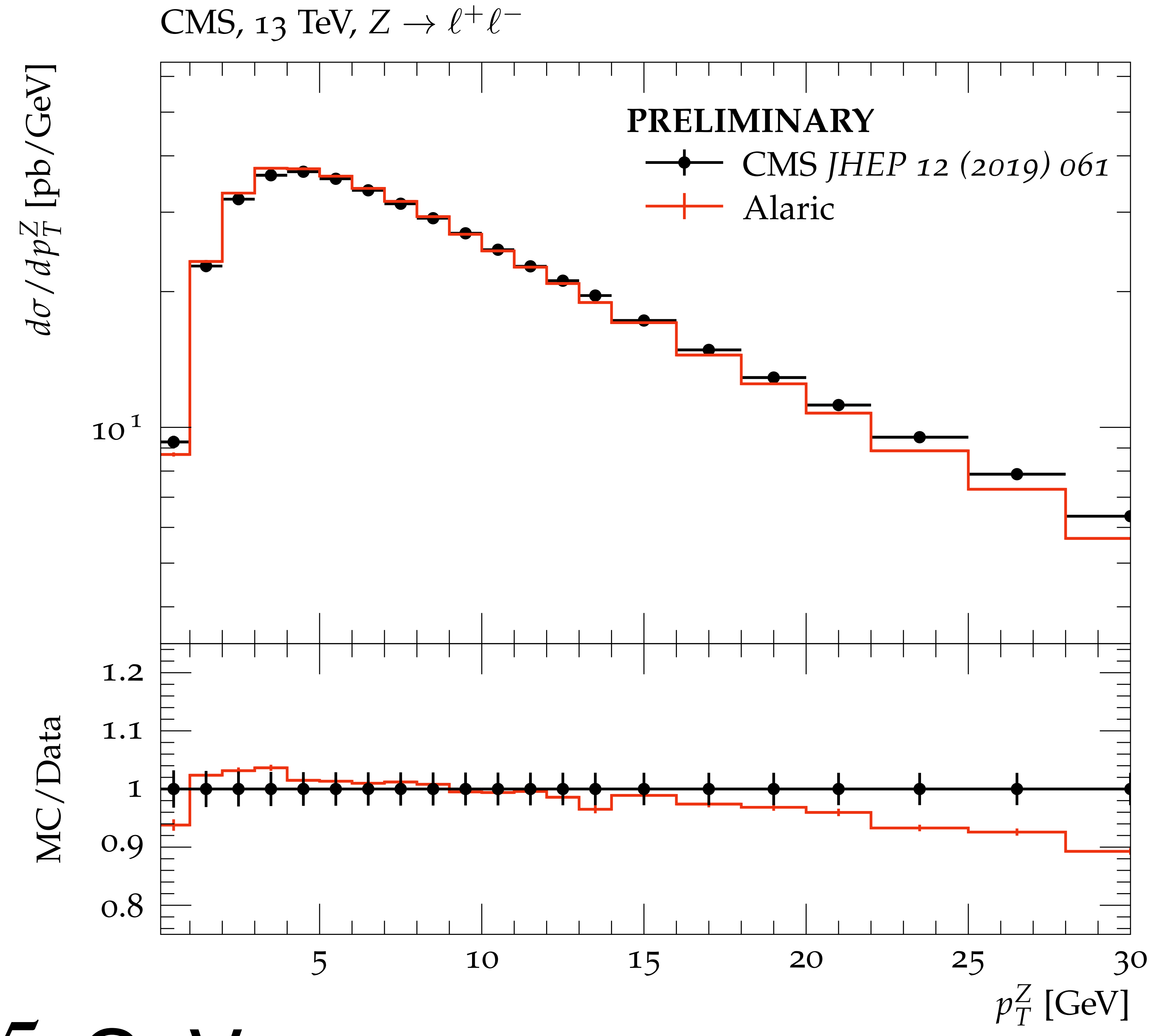
$$\tilde{K} = p_V = p_Z$$

- in practice shift ~ 10%

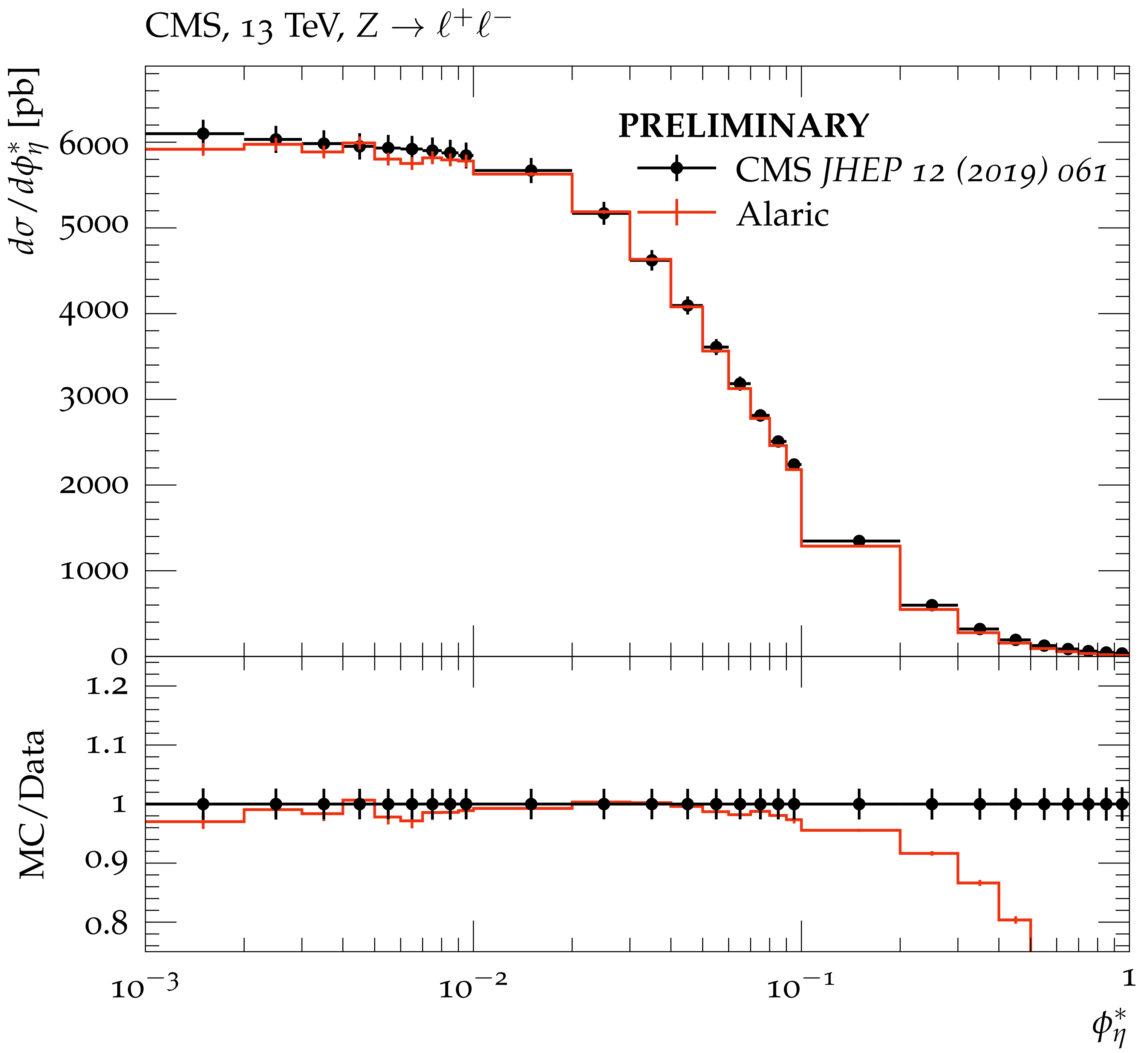


Alaric IS pheno

- first results:
  - DY transverse momentum spectrum
  - intrinsic transverse momentum model relevant at $p_t^Z \sim 1 - 5$ GeV
  - flat wrt. data in $5 \text{ GeV} < p_t^Z < 20 \text{ GeV}$
  - missing HO corrections above that

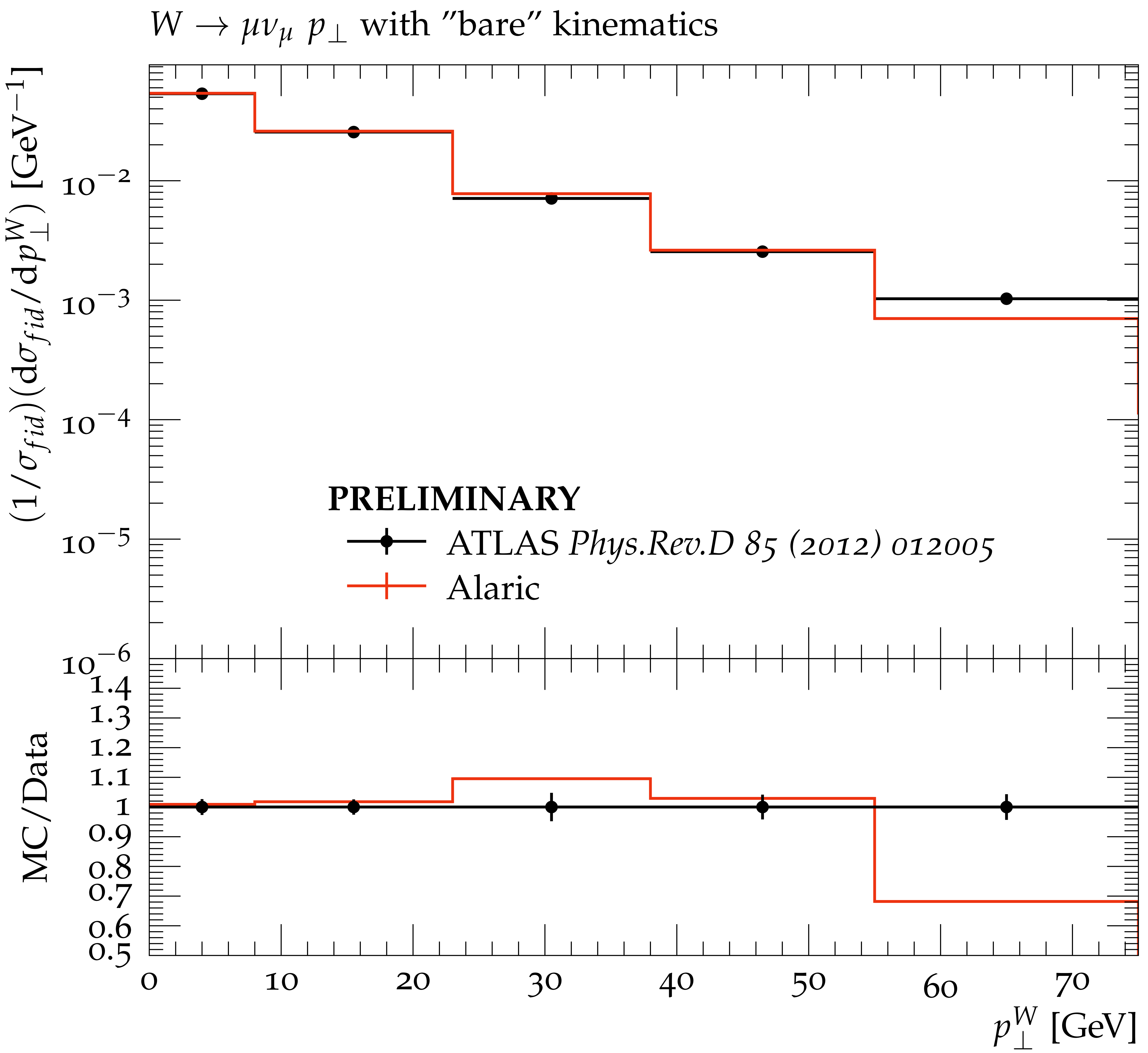


- similar picture from ϕ^*:



Alaric IS pheno

- transverse momentum of W bosons
- compare to ATLAS data @ $\sqrt{s} = 7$ TeV
- note much lower resolution than previous plot



A Logarithmically **A**ccurate **R**esummation **I**n **C++**

- NLL resummation in CAESAR formalism as definition and validation of parton shower accuracy
- New parton shower Alaric
  - partial fractioning of eikonal → positive definite splitting function with full phase space coverage
  - global kinematics scheme enables analytic proof of NLL accuracy + numerical validation
  - included in Sherpa framework and first pheno results @ LHC