THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
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NLO matching with KrkNLO

theory and progress

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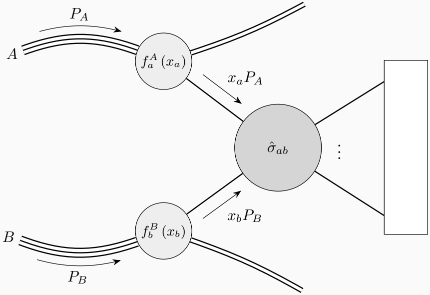
QCD@LHC 2023

Matching at NLO

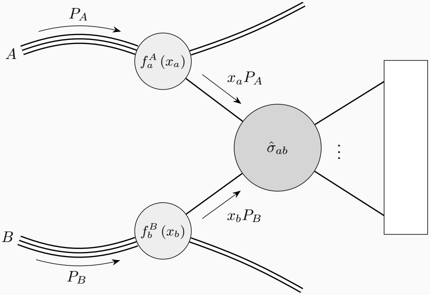
NLO matching



Interested in some[1] function $\mathcal{O}$ of phase-space:

$$\mathsf{d}\sigma_{AB}[\mathcal{O}](\mu_\mathsf{F}, \mu_\mathsf{R}) = \sum_{a,b} f_a^A(\xi_1; \mu_\mathrm{F}) \otimes_{\xi_1} \mathrm{d}\hat{\sigma}_{ab}[\mathcal{O}](\xi_1, \xi_2; \mu_\mathrm{F}, \mu_\mathrm{R}) \otimes_{\xi_2} f_b^B(\xi_2; \mu_\mathrm{F}).$$

NLO matching



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[1] infrared and collinear safe

Building a cross-section

For fixed-order calculations: expand perturbatively (and subtract)

$$\mathrm{d}\sigma_{ab}^{\mathrm{NLO}}\left[\mathcal{O}\right](\xi_1,\xi_2) = \left(\frac{\alpha_s}{2\pi}\right)^k \left\{\mathrm{d}\Phi_m(\xi_1 P_1, \xi_2 P_2)\left[\mathrm{B}(\Phi_m)\right]\mathcal{O}(\Phi_m)\right\}$$
$$+ \left(\frac{\alpha_s}{2\pi}\right)^{k+1} \left\{\mathrm{d}\Phi_m(\xi_1 P_1, \xi_2 P_2)\left[\mathrm{V}(\Phi_m)\right]\mathcal{O}(\Phi_m)\right.$$
$$\left. + \,\mathrm{d}\Phi_{m+1}(\xi_1 P_1, \xi_2 P_2)\left[\mathrm{R}(\Phi_{m+1})\right]\mathcal{O}(\Phi_{m+1})\right\}$$

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&+ \left(\frac{\alpha_s}{2\pi}\right)^{k+1} \left\{\mathrm{d}\Phi_m \left[\mathrm{V}(\Phi_m) + \mathrm{I}(\Phi_m)\right] \mathcal{O}(\Phi_m)\right. \\
&\qquad + \mathrm{d}x\, \mathrm{d}\Phi_m(x\xi_1 P_1, \xi_2 P_2) \left[(\mathrm{P}+\mathrm{K})_{ac}(x) \cdot \mathrm{B}_{cb}(\Phi_m)\right] \mathcal{O}(\Phi_m) \\
&\qquad + \mathrm{d}x\, \mathrm{d}\Phi_m(\xi_1 P_1, x\xi_2 P_2) \left[(\mathrm{P}+\mathrm{K})_{bc}(x) \cdot \mathrm{B}_{ac}(\Phi_m)\right] \mathcal{O}(\Phi_m) \\
&\qquad \left. + \mathrm{d}\Phi_{m+1} \left[\mathrm{R}(\Phi_{m+1}) - \mathrm{S}(\Phi_{+1}) \cdot \mathrm{B}(\tilde{\Phi}_m)\right] \mathcal{O}(\Phi_{m+1})\right\}
\end{aligned}$$

New legs from old[2]

What is a parton shower?

At its heart:

$$\mathrm{PS}\left[\mathcal{O}(\Phi_m)\right] = \Delta_{\mu_s}^{Q(\Phi_m)}\,\mathcal{O}(\Phi_m)$$

[2] Based on ongoing work with Andrzej Siódmok and Simon Plätzer.

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where the Sudakov form factor is

$$\Delta_{\mu_s}^{Q(\Phi_m)} = \exp\left[-\sum_\alpha \int \mathrm{d}\Phi_{+1}\, \Theta[\mu_s < \mu(\Phi_{+1}) < Q(\Phi_m)]\, P_m^{(\alpha)}(\Phi_{+1})\, \Theta_{\mathrm{PS}}^{(\alpha)}\right]$$

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The matched NLO cross-section shouldn't spoil the fixed-order result:

$$\hat{\sigma}^{\mathsf{NLO+PS}}[\mathcal{O}] = \hat{\sigma}^{\mathsf{NLO}}[\mathcal{O}]$$

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$$\hat{\sigma}^{\mathrm{NLO+PS}}[\mathcal{O}] = \hat{\sigma}^{\mathrm{NLO}}[\mathcal{O}] + \mathcal{O}\left(\alpha_s^2\right) + \mathcal{O}\left(\frac{\mu_s}{Q}\right)$$

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factorisation scheme (α_s^{k+1})	$\Delta f_a \otimes_{\xi_1} \mathrm{B} + \mathrm{B} \otimes_{\xi_2} \Delta f_b$	

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$\mathcal{O}(\Phi_m)$: restore the cancellation required by the matching condition by modifying the PDF factorisation scheme

- collinear convolution terms can only go into the PDF
- where to put end-point contributions $\propto \delta(1-x)$?

Familiar ingredients

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For dipoles, we already know the answer from dipole subtraction:[3]

$$-\sum_{\alpha} \int \mathrm{d}\Phi_{+1}\; \Theta[\mu_s < \mu(\Phi_{+1}) < Q(\Phi_m)]\; P_m^{(\alpha)}(\Phi_{+1})\; \Theta_{\mathrm{PS}}^{(\alpha)} = \sum_{(\alpha)} \mathrm{I}^{(\alpha)} + \mathrm{d}x \left(\mathrm{P}^{(\alpha)} + \mathrm{K}^{(\alpha)} \right)$$

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This provides the recipe for the PDF transformation (more later).

[3] Again, subject to the caveat of full phase-space coverage

Final recipe[4]

$$
\begin{aligned}
\mathrm{d}\Phi_m\, \Theta_{\mathrm{cut}}\,[\Phi_m] \quad & \Bigg[\Big\{ \mathrm{B}(\Phi_m) + \mathrm{V}(\Phi_m) + I(\Phi_m) + \Delta_0^{\mathrm{FS}} \Big\} \Delta_{\mu_s}^{Q(\Phi_m)}\, \mathcal{O}(\Phi_m) \\
& + \sum_\alpha \mathrm{d}\Phi_{+1}^{(\alpha)} \left\{ \frac{\mathrm{R}^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}{\mathrm{PS}^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}\, \Theta_{\mathrm{PS}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}\right] \mathrm{PS}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}\right] \Theta_{\mu_s}^{(\alpha)}\, \mathcal{O}(\Phi_{m+1}^{(\alpha)}) \right\} \Bigg]
\end{aligned}
$$

1. generate a Born phase-space point, ME and shower:
   - if an emission is generated, reweight to R
   - if not, reweight to B + V
2. matching complete; allow the shower to proceed!

[4] S. Jadach et al. “Matching NLO QCD with parton shower in Monte Carlo scheme — the KrkNLO method”. arXiv: 1503.06849 [hep-ph], Stanislaw Jadach et al. “New simpler methods of matching NLO corrections with parton shower Monte Carlo”. arXiv: 1607.00919 [hep-ph].

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This is NLO accurate, but differs from other methods at higher orders.

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Krk PDF scheme

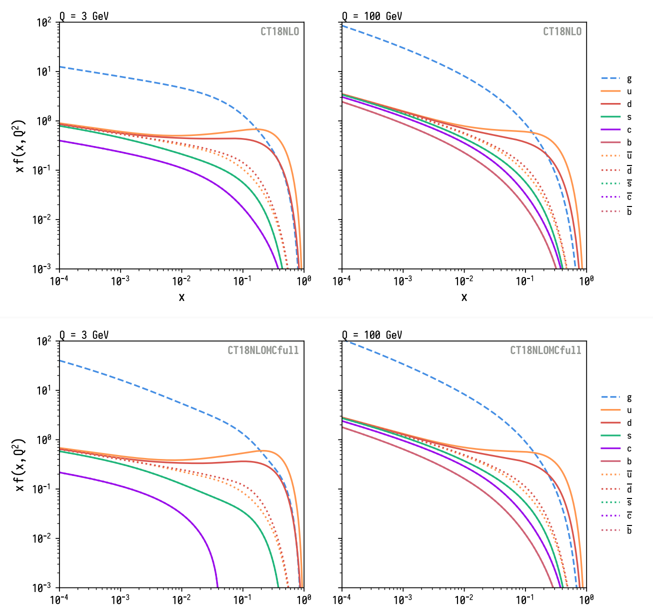
Krk (/MC/CS) factorisation scheme[5]

From the dipole operators, we can write down the convolution terms:

$$
\begin{aligned}
f_q^{\mathrm{Krk}}(x,\mu_{\mathrm{F}}) = f_q^{\overline{\mathrm{MS}}}(x,\mu_{\mathrm{F}})
&- \frac{\alpha_s(\mu_{\mathrm{F}})}{2\pi}\,\frac{3}{2}C_{\mathrm{F}}\;f_q^{\overline{\mathrm{MS}}}(x,\mu_{\mathrm{F}}) \\
&+ \frac{\alpha_s(\mu_{\mathrm{F}})}{2\pi}\;C_{\mathrm{F}}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_q^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\quad\left[\frac{1+z^2}{1-z}\log\frac{(1-z)^2}{z}+1-z\right]_+\right] \\
&+ \frac{\alpha_s(\mu_{\mathrm{F}})}{2\pi}\;C_{\mathrm{A}}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left[z^2+(1-z)^2\right]\log\frac{(1-z)^2}{z}+2z(1-z)\right] \\
f_g^{\mathrm{Krk}}(x,\mu_{\mathrm{F}}) = f_g^{\overline{\mathrm{MS}}}(x,\mu_{\mathrm{F}})
&- \frac{\alpha_s(\mu_{\mathrm{F}})}{2\pi}\;C_{\mathrm{A}}\left[\frac{\pi^2}{3}+\frac{341}{72}-\frac{59}{36}\frac{N_f T_{\mathrm{R}}}{C_{\mathrm{A}}}\right]\;f_g^{\overline{\mathrm{MS}}}(x,\mu_{\mathrm{F}}) \\
&+ \frac{\alpha_s(\mu_{\mathrm{F}})}{2\pi}\;C_{\mathrm{A}}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left[4\left[\frac{\log(1-z)}{1-z}\right]_+ - 2\frac{\log z}{1-z}\right.\right. \\
&\qquad\left.\left. +2\left(\frac{1}{z}-2+z(1-z)\right)\ln\frac{(1-z)^2}{z}\right]\right] \\
&+ \frac{\alpha_s(\mu_{\mathrm{F}})}{2\pi}\;C_{\mathrm{F}}\sum_{q_f,\bar{q}_f}\left[\int_x^1 \frac{\mathrm{d}z}{z} f_q^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left[\frac{1+(1-z)^2}{z}\log\frac{(1-z)^2}{z}+z\right]\right]
\end{aligned}
$$

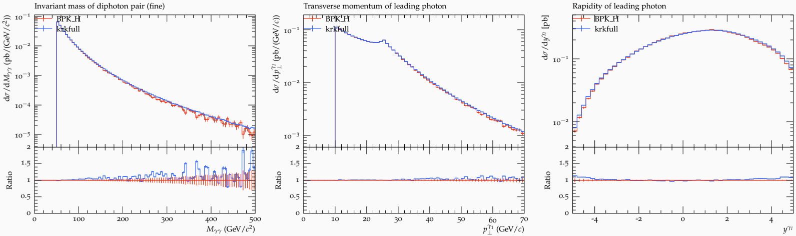
5 S. Jadach et al. "Parton distribution functions in Monte Carlo factorisation scheme". arXiv: 1606.00355 [hep-ph].

Applied to LHAPDF6 grids:



Validation

Do we reproduce the Herwig (Matchbox) automated P and K operators?



Validation

Seeing Sudakovs

To verify the real weight, we must *unweight* the Sudakov:

- numerical integration of dipole kernels considered in shower algorithm;
- over the same splitting phase-space/kinematic region used in the shower algorithm;
- with the same scales, PDF arguments, α_s etc

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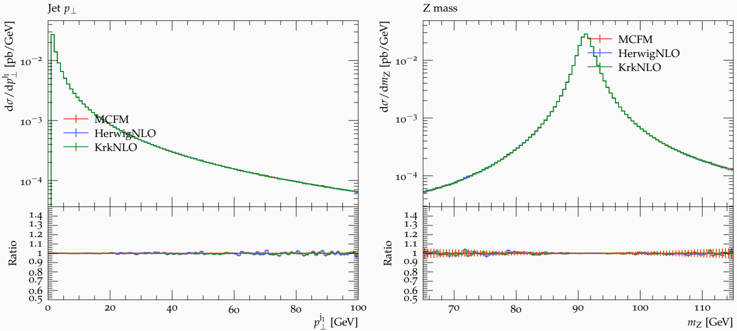
$$\Delta_{\mu_s}^{Q(\phi_m)} = \exp\left[-\sum_{\alpha}\int \mathrm{d}q(\phi_m)\;\Theta[\mu_s < \mu(q) < Q(\phi_m)]\;P_m^{(\alpha)}(q)\;\Theta_{\mathrm{PS}}^{(\alpha)}\right]$$

This is non-trivial!

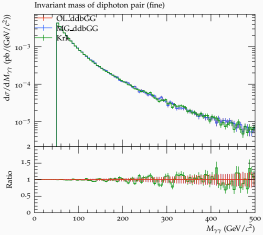
Does it work?

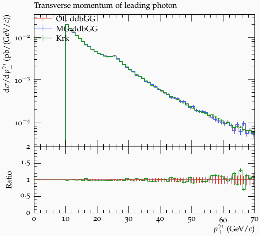
Drell-Yan:

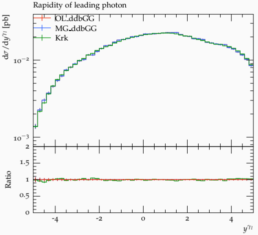


What about the virtuals?

Diphoton:







Roadmap

- new processes
- PDF factorisation scheme[6]
- automation!

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…and physics results (very soon)

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Thank you!