Recent Progress on Track Functions

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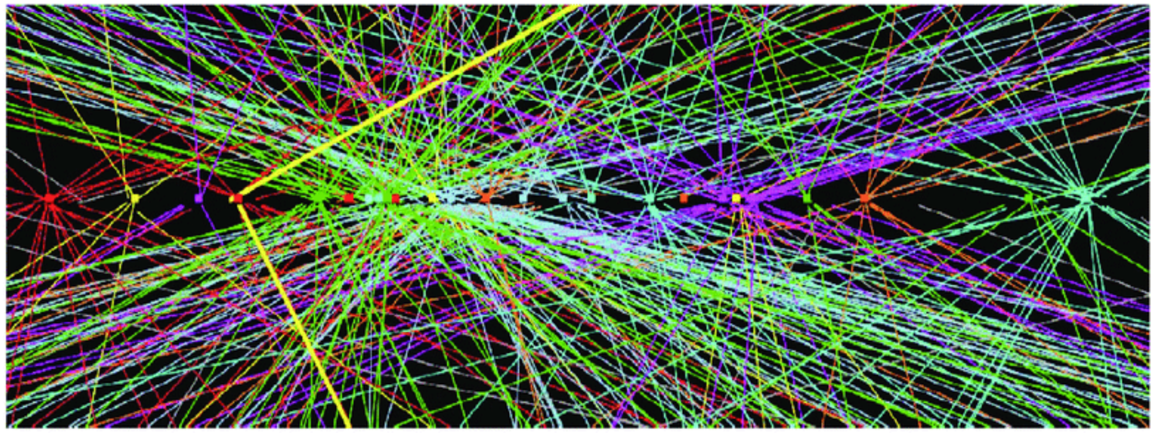
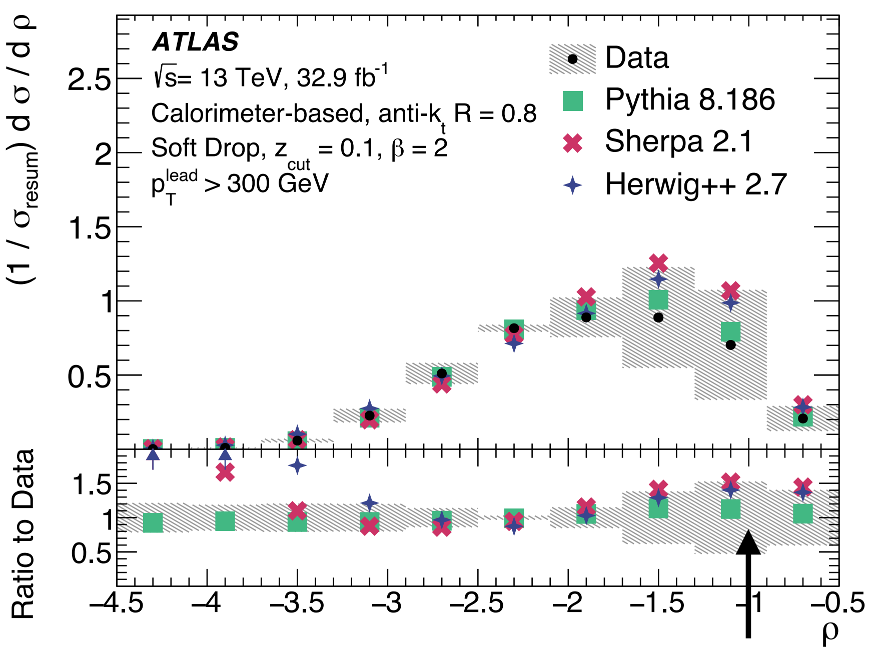
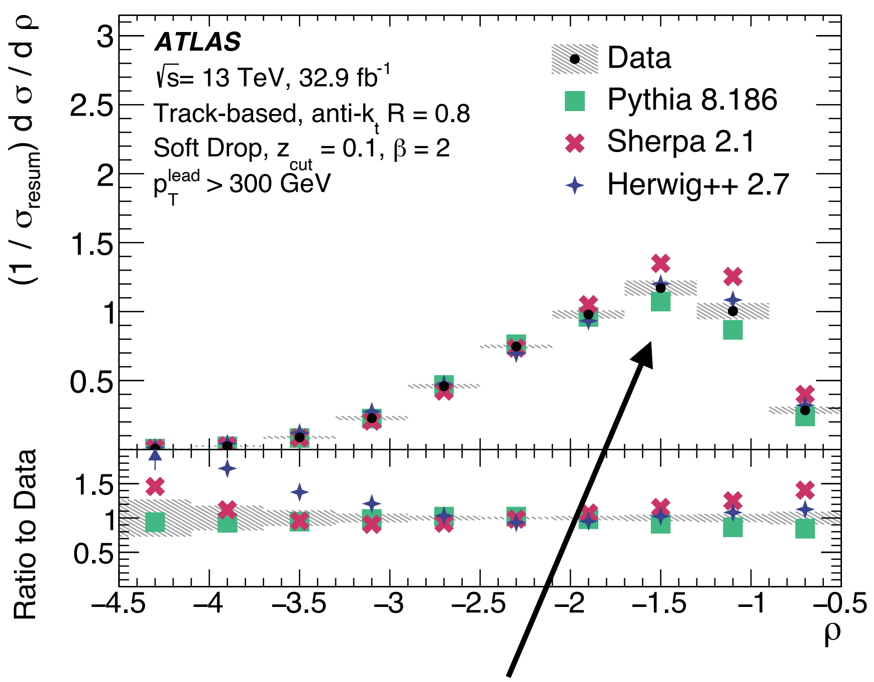
Motivation for track-based measurements

- Superior angular resolution
  → good for jet substructure
- Pile-up removal



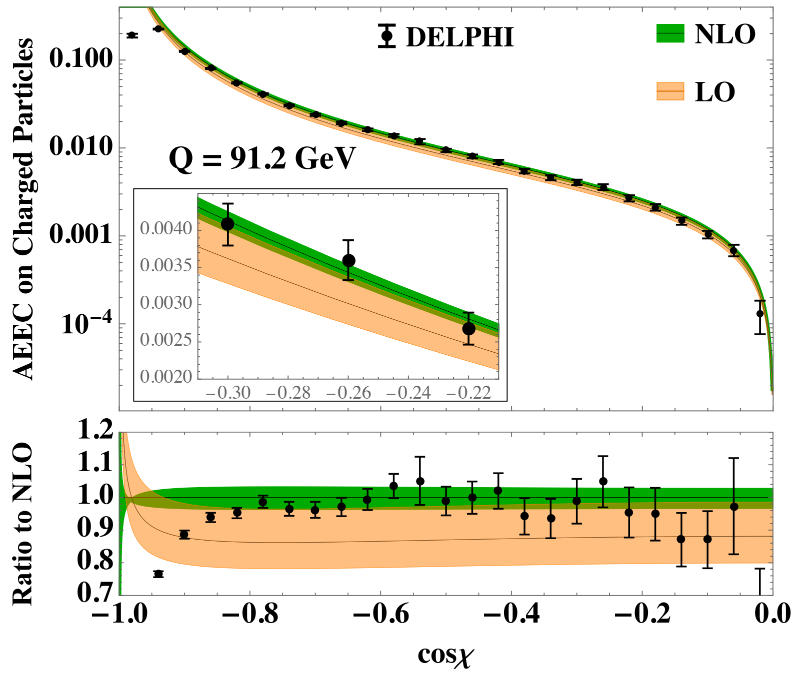
- E.g. for groomed $\rho = \ln(m^2/p_T^2)$





Introduction

- Track-based measurements sensitive to hadronization
  → can be modeled in parton showers.
- Track functions offer **systematically** improvable framework.
- Recently extended to $\mathcal{O}(\alpha_s^2)$ → **high precision possible**!
- Track functions are being extracted by ATLAS.
- Particularly easy to use for energy correlators [see also Kyle's talk]



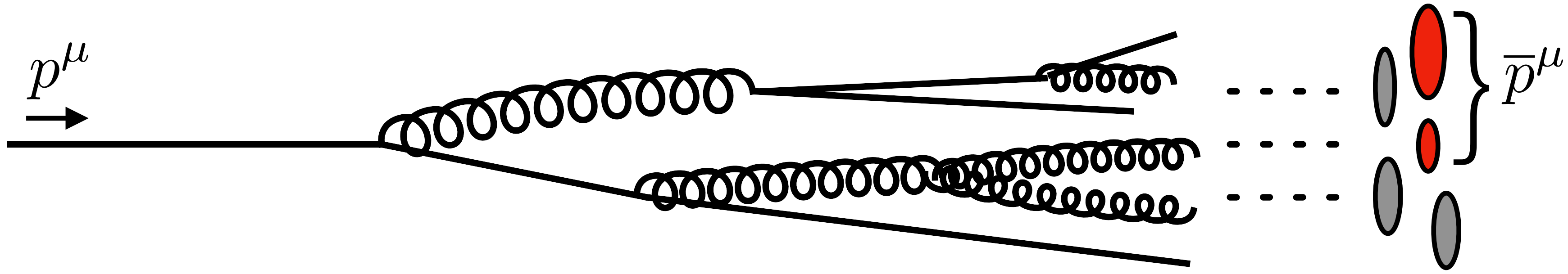
Outline

1. Track function basics
2. Track functions at α_s^2
3. Phenomenology with track functions

Bonus: studying factorization violation with vector angularities.

1. Track functions basics

1. Track function



- $T_i(x, \mu)$ describes **total** momentum fraction x of initial parton i converted to tracks, i.e.

$$\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

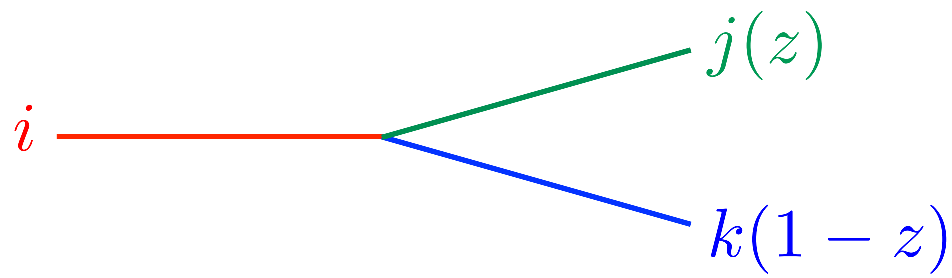
- Nonperturbative, process-independent function.

- Conservation of probability:

$$\int_0^1 \mathrm{d}x \, T_i(x) = 1$$

[Chang, Procura, Thaler, WW]

1. Track function at order α_s



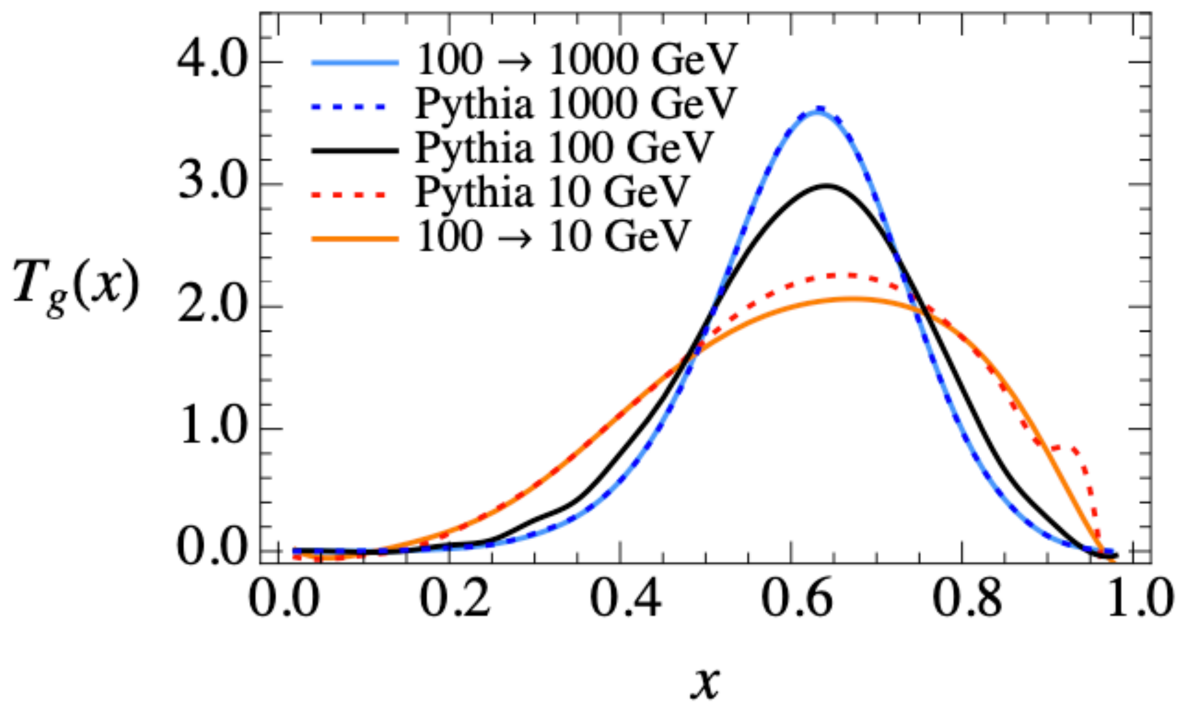
splitting function

$$T^{(1)}_{i,\mathrm{bare}}(x) = \sum_j \int \mathrm{d}z \left[\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right) P_{ji}(z)\right] \int \mathrm{d}x_1\, T^{(0)}_j(x_1,\mu)$$
$$\times \int \mathrm{d}x_2\, T^{(0)}_k(x_2,\mu)\,\delta\big[x - z x_1 - (1-z) x_2\big]$$

summing contribution of branches

- $1/\epsilon_{\mathrm{IR}}$ cancels against IR pole in partonic cross section.
- $1/\epsilon_{\mathrm{UV}}$ is renormalized, leads to evolution of track function

1. Evolution at order α_s



$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T_i(x,\mu) = \sum_{j,k} \int \mathrm{d}z\, \frac{\alpha_s}{4\pi} P_{ji}(z) \int \mathrm{d}x_1\, T_j(x_1,\mu) \int \mathrm{d}x_2\, T_k(x_2,\mu)$$

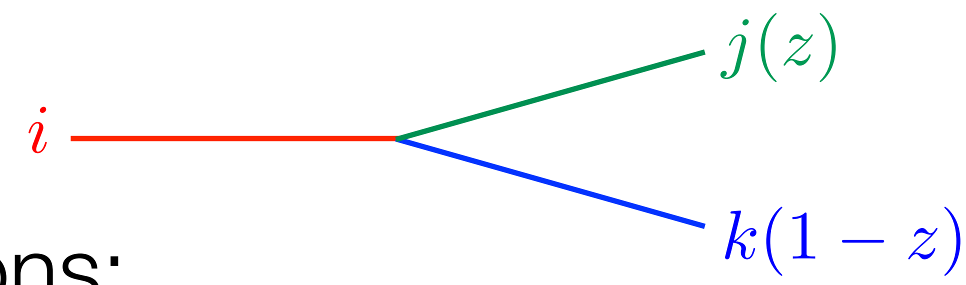
$$\times\, \delta[x - z x_1 - (1-z) x_2]$$

[Chang, Procura, Thaler, WW]

- Consistent with extraction from Pythia at different energies.
- Simplifies for integer moments: $[x - z x_1 - (1-z) x_2]^N$

binomial expansion

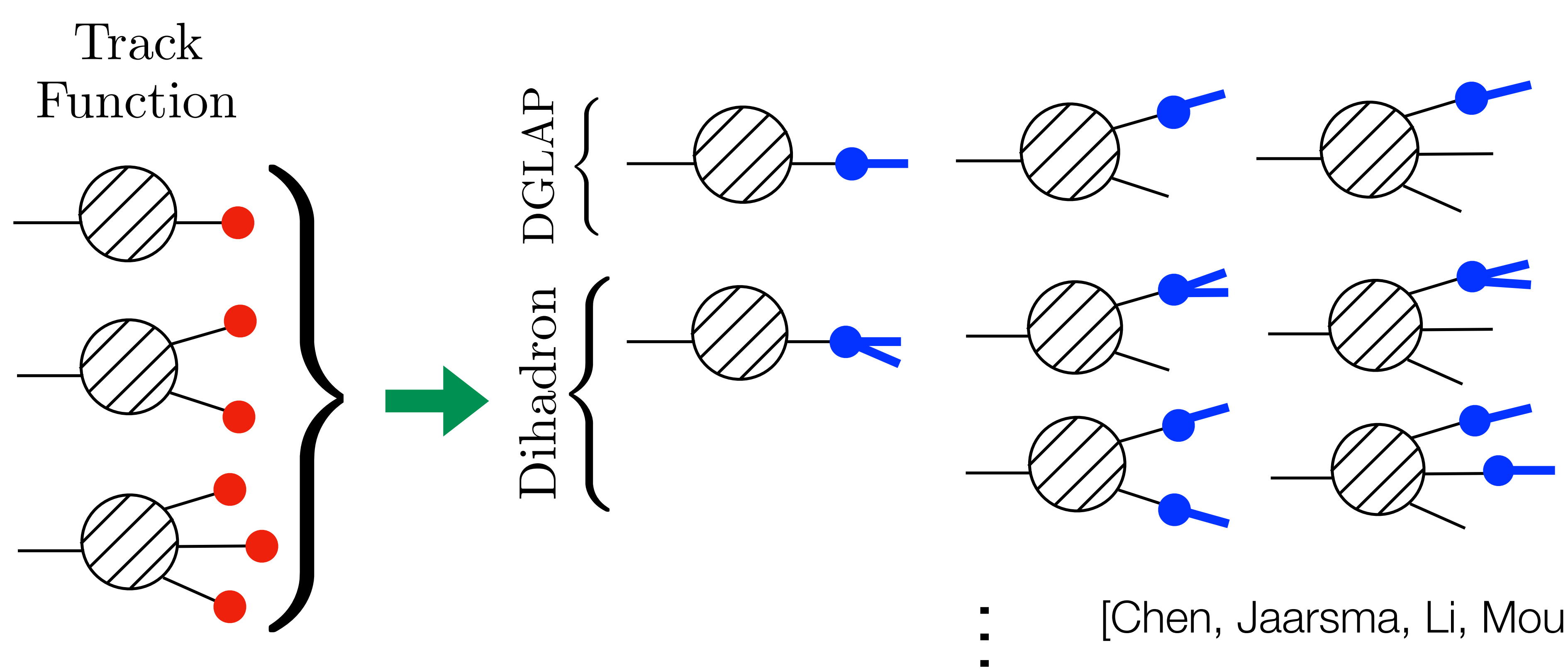
1. Relation to (multi-)hadron fragmentation



- Track vs. fragmentation functions:

$$\delta[x - zx_1 - (1-z)x_2] \quad \rightarrow \quad \delta[x - zx_1] + \delta[x - (1-z)x_2]$$

- Evolution for (multi-)hadron fragmentation follows from tracks:



[Chen, Jaarsma, Li, Moult, WW, Zhu]

1. Using track functions

- Consider a cross section differential in observable e

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \sum_N \int \mathrm{d}\Pi_N \, \frac{\mathrm{d}\sigma_N}{\mathrm{d}\Pi_N} \, \delta[e - \hat{e}(\{p_i^\mu\})]$$

- At leading order, for track-based measurement $\bar{e}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\bar{e}} = \sum_N \int \mathrm{d}\Pi_N \, \frac{\mathrm{d}\sigma_N}{\mathrm{d}\Pi_N} \, \underbrace{\int \prod_{i=1}^{N} \mathrm{d}x_i \, T_i(x_i)}_{\text{hadronization}} \, \delta[e - \hat{e}(\{x_i p_i^\mu\})]$$

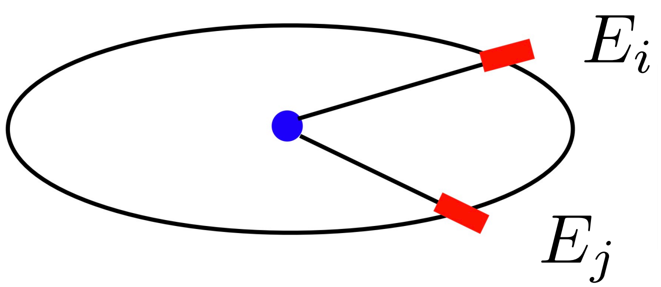
- Beyond leading order, there is a cancellation of IR divergences, $\mathrm{d}\sigma_N \to \mathrm{d}\bar{\sigma}_N$, similar to fragmentation functions/PDFs.

[Chang, Procura, Thaler, WW]

1. Track-based energy correlators

- Weighted cross section in e^+e^- collisions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\chi} = \sum_{i,j} \int \mathrm{d}\sigma\, \frac{E_i E_j}{Q^2}\, \delta(\cos\chi - \cos\theta_{ij})$$



[Basham, Brown, Ellis, Love]

- Tracks are essential for small χ

- Conversion to tracks is simple:

$$E_i \to \int \mathrm{d}x_i\, T_i(x_i)\, x_i E_i = T_i(1) E_i$$

[Chen, Moult, Zhang, Zhu]

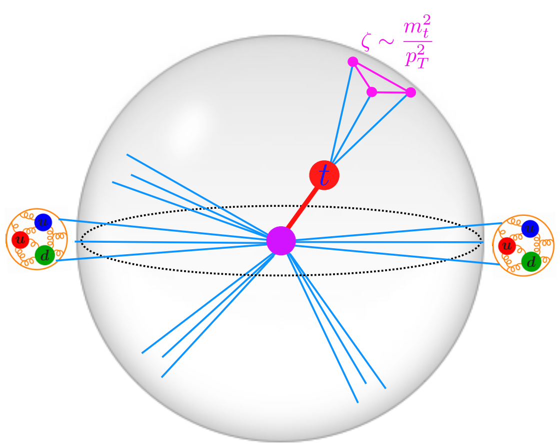
- Contact term $\chi = 0$ involves $T_i(2)$.

1. Track-based weighted energy correlators

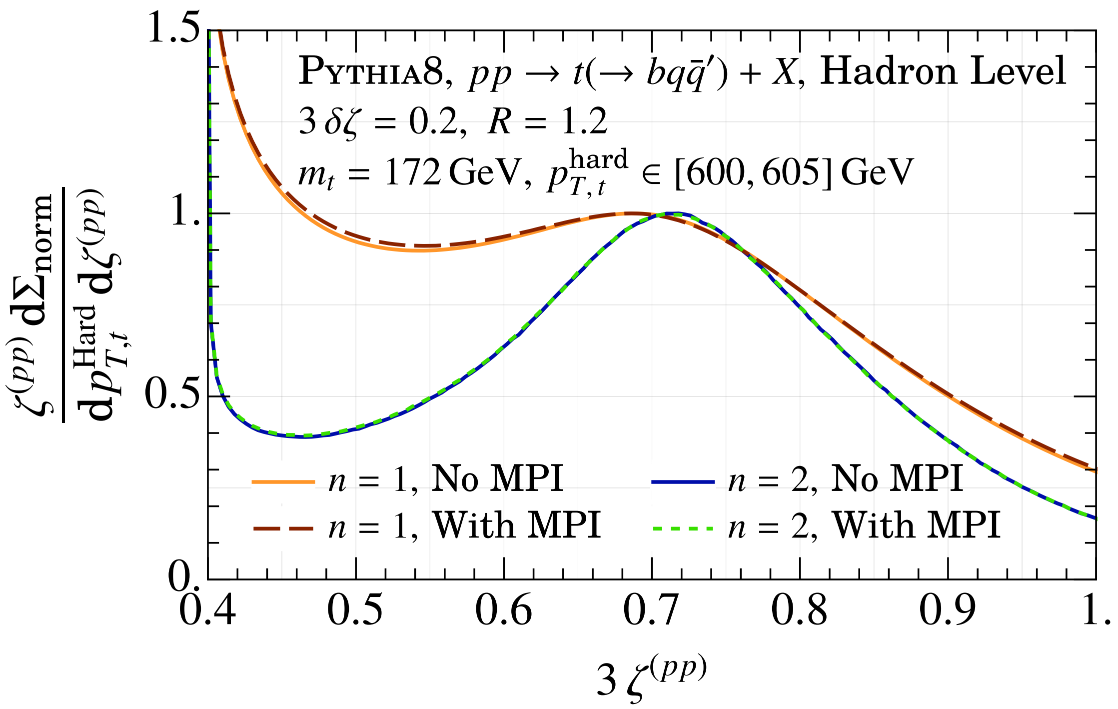
- N-point energy correlators involve at most the Nth moment
- Modifying weights to suppress soft radiation is easy for **tracks**:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\chi} = \sum_{i,j} \int \mathrm{d}\sigma \, \frac{E_i^n E_j^n}{Q^{2n}} \, \delta(\cos\chi - \cos\theta_{ij})$$

- Advantageous for top quark mass determination:



[Holguin, Moult, Pathak, Procura]



2. Track functions at α_s^2

2. Calculation in a nut shell

- $T_i(x) \propto 1/\epsilon_{\rm UV} - 1/\epsilon_{\rm IR} = 0$ → need a scale.
- Extract it from $J_i(s,x)$, with s invariant mass of **all** particles.
- Consistency of factorization implies same UV poles as $J_i(s)$
  [Becher, Neubert; Becher, Bell]

2. Calculation in a nut shell

- $T_i(x) \propto 1/\epsilon_{\rm UV} - 1/\epsilon_{\rm IR} = 0$ → need a scale.
- Extract it from $J_i(s,x)$, with s invariant mass of **all** particles.
- Consistency of factorization implies same UV poles as $J_i(s)$
  [Becher, Neubert; Becher, Bell]
- Remaining $1/\epsilon$ poles are infrared and cancel in

want to extract

$$J(s,x,\mu) = \delta(s)T(x,\mu) + a_s\,\delta(s)\left[\mathcal{J}^{(1)}_{1\to 1}T(x,\mu) + \mathcal{J}^{(1)}_{1\to 2}\otimes TT(x,\mu)\right]$$
$$+ a_s^2\,\delta(s)\left[\mathcal{J}^{(2)}_{1\to 1}T(x,\mu) + \mathcal{J}^{(2)}_{1\to 2}\otimes TT(x,\mu) + \mathcal{J}^{(2)}_{1\to 3}\otimes TTT(x,\mu)\right] + \dots$$

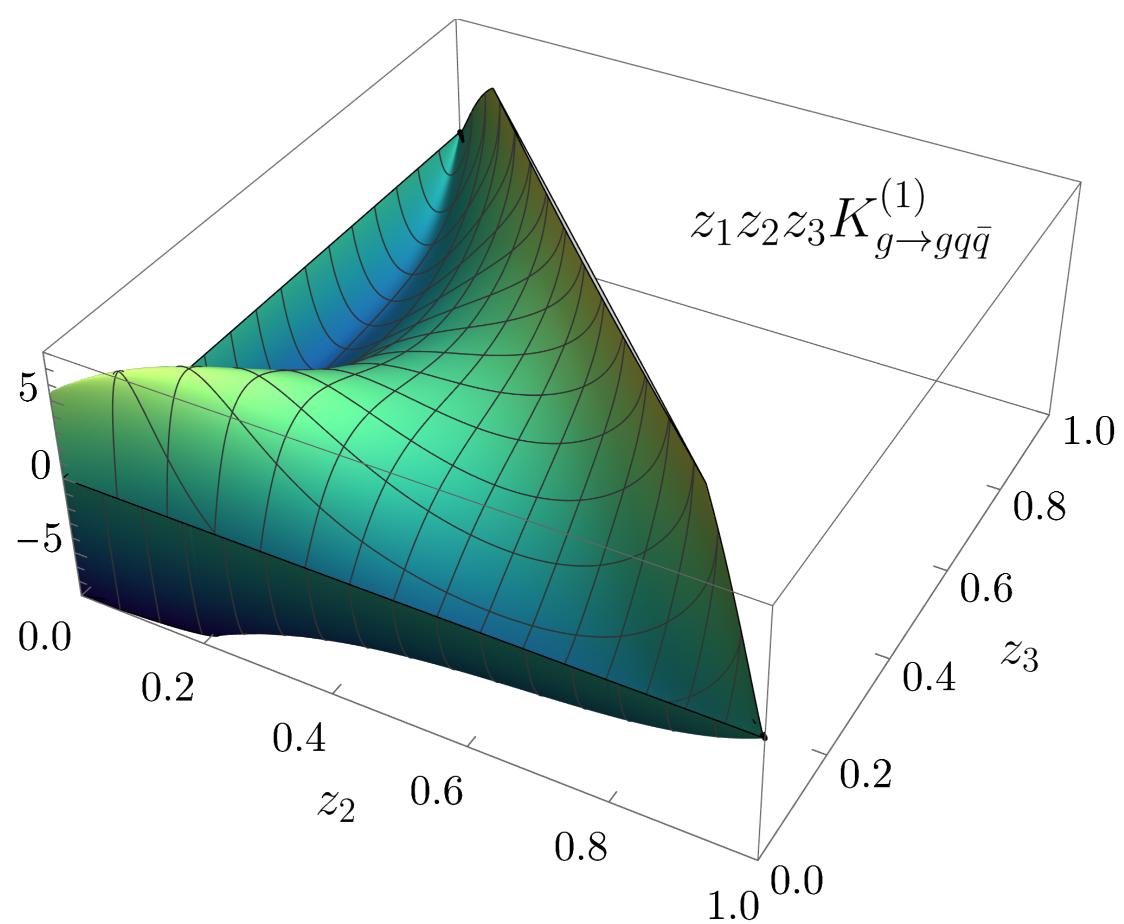
- Only need $\delta(s)$ term and IR poles (i.e. drop second line).
- To isolate poles in $J_i(s,x,\mu)$, use sector decomposition.

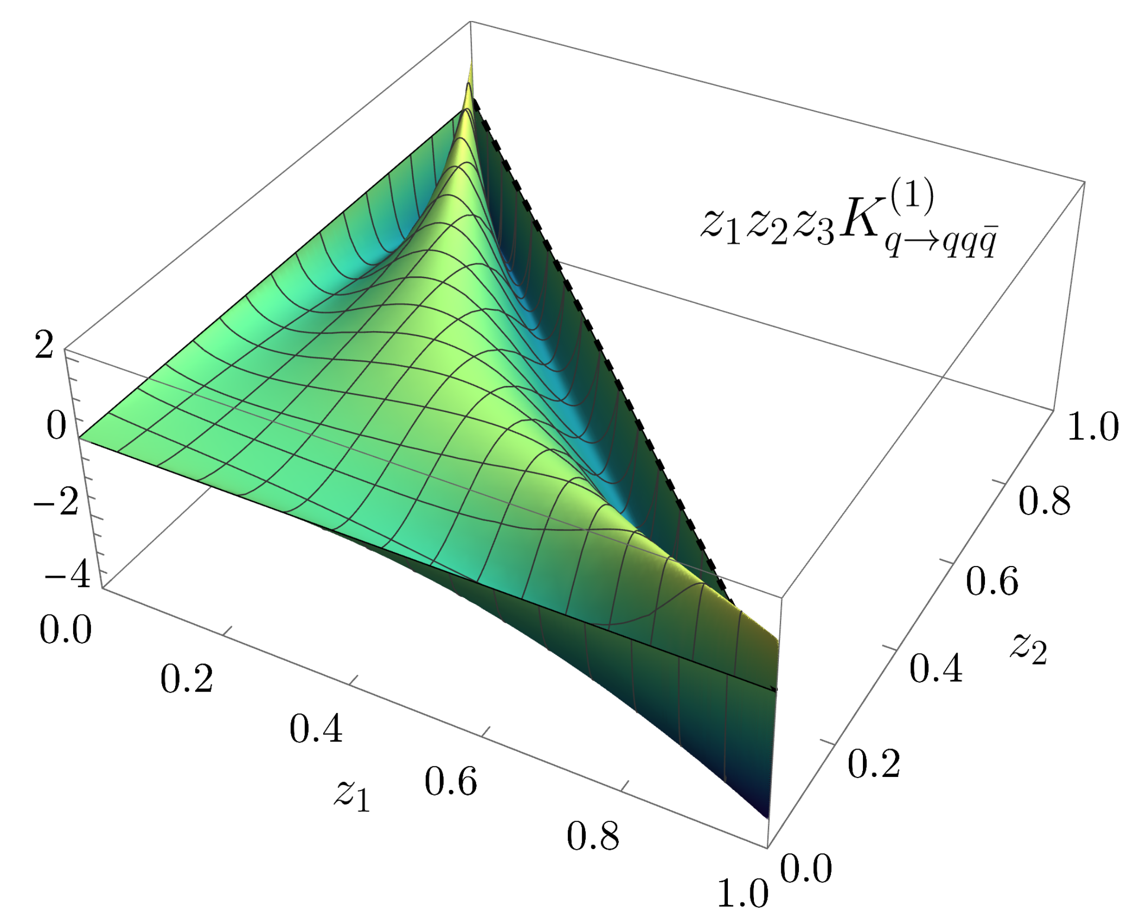
[Li, Moult, Schrijnder van Velzen, WW, Zhu; Jaarsma, Li, Moult, WW, Zhu; Chen, Jaarsma, Li, Moult, WW, Zhu]

2. Track function evolution at NLO

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T(x,\mu) = a_s\Big[K^{(0)}_{1\to 1}\otimes T(x,\mu) + K^{(0)}_{1\to 2}\otimes TT(x,\mu)\Big] + a_s^2\Big[K^{(1)}_{1\to 1}\otimes T(x,\mu) + K^{(1)}_{1\to 2}\otimes TT(x,\mu) + K^{(1)}_{1\to 3}\otimes TTT(x,\mu)\Big]$$

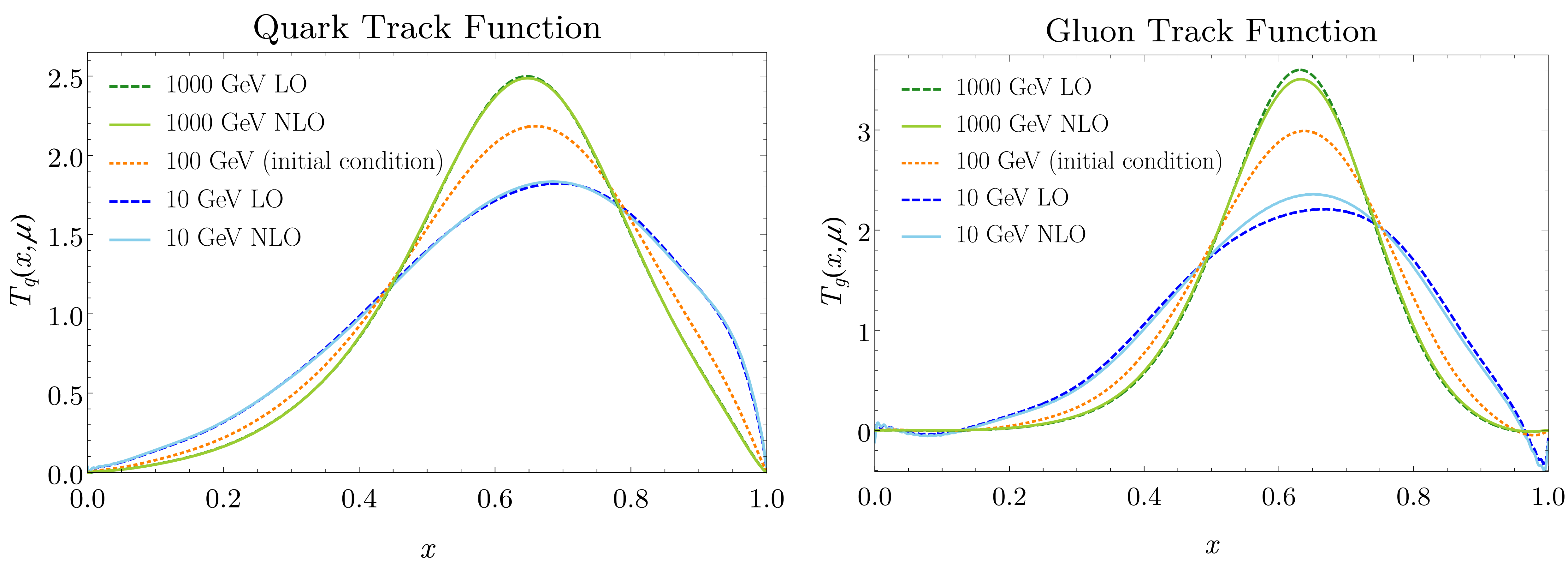
- Kernels are 10 pages and available electronically.





[Chen, Jaarsma, Li, Moult, WW, Zhu]

2. Results for track function evolution



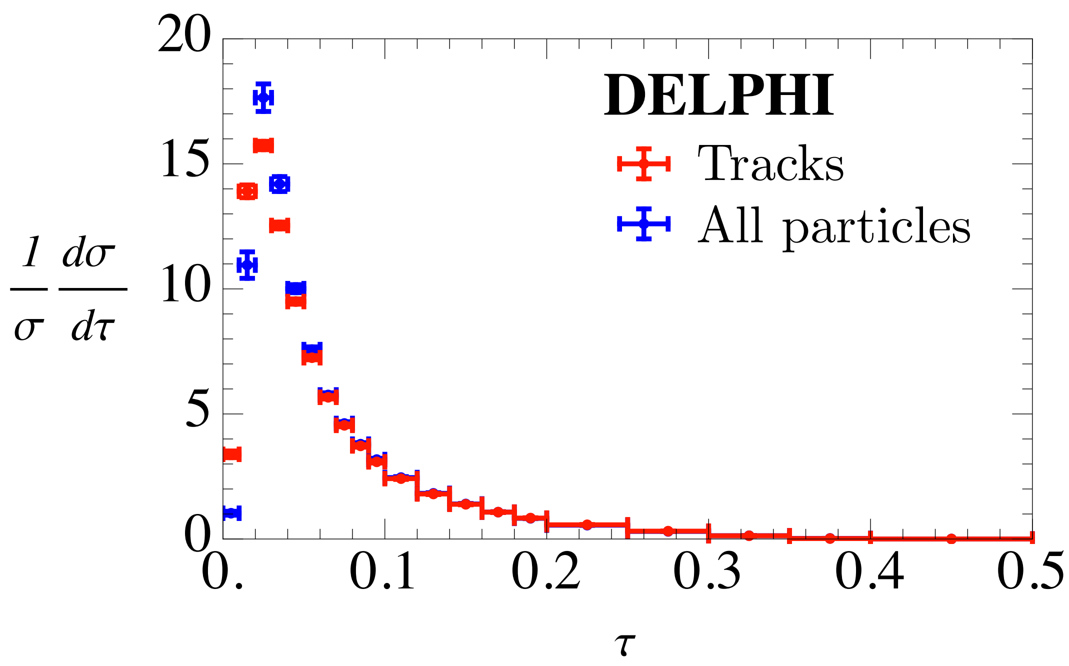
- Evolution is milder than for fragmentation functions or PDFs.
- NLO evolution is a small correction, especially for quarks.
- Evolution code is available (based on moments, Fourier or Legendre wavelets).

[Chen, Jaarsma, Li, Moult, WW, Zhu]

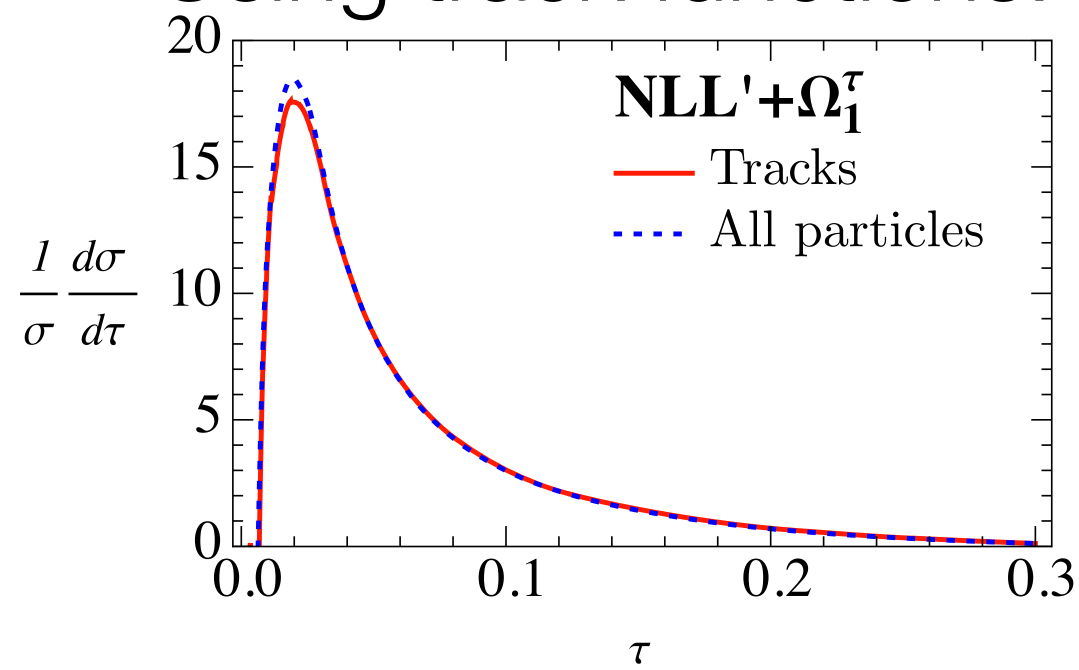
3. Phenomenology with track functions

3. Track thrust

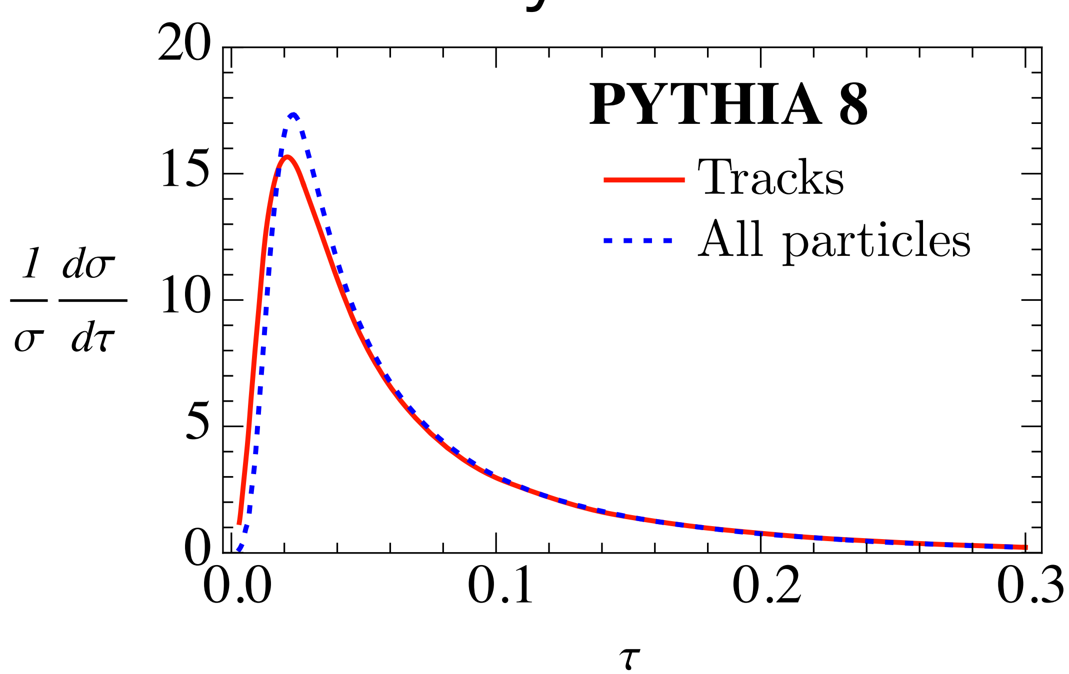
DELPHI:



Using track functions:



Pythia:

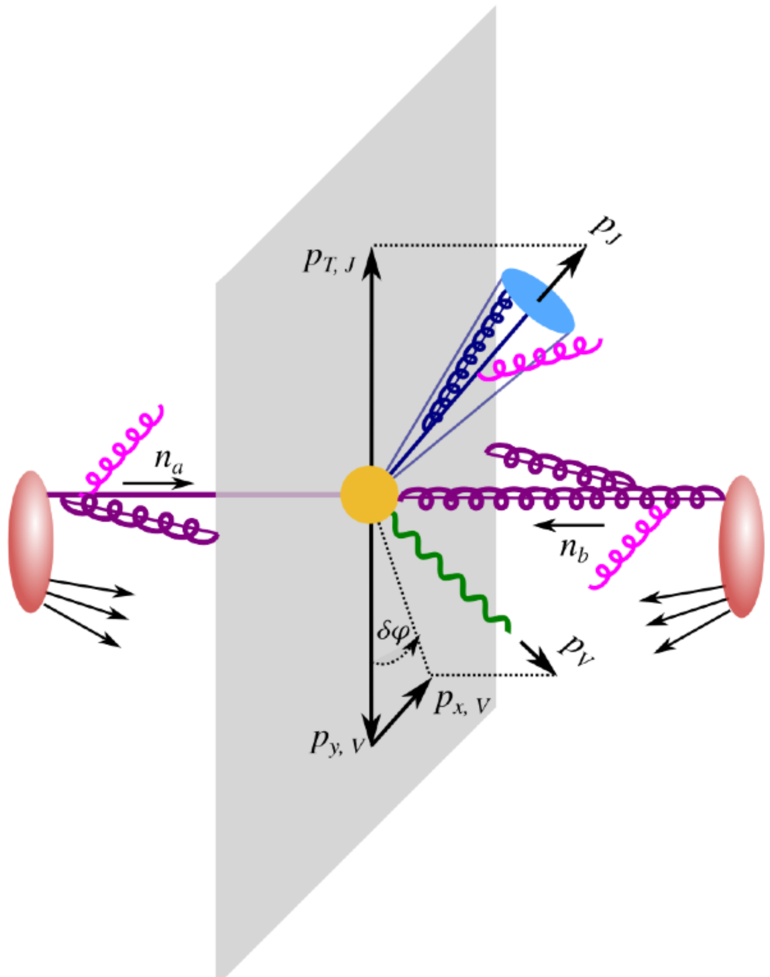


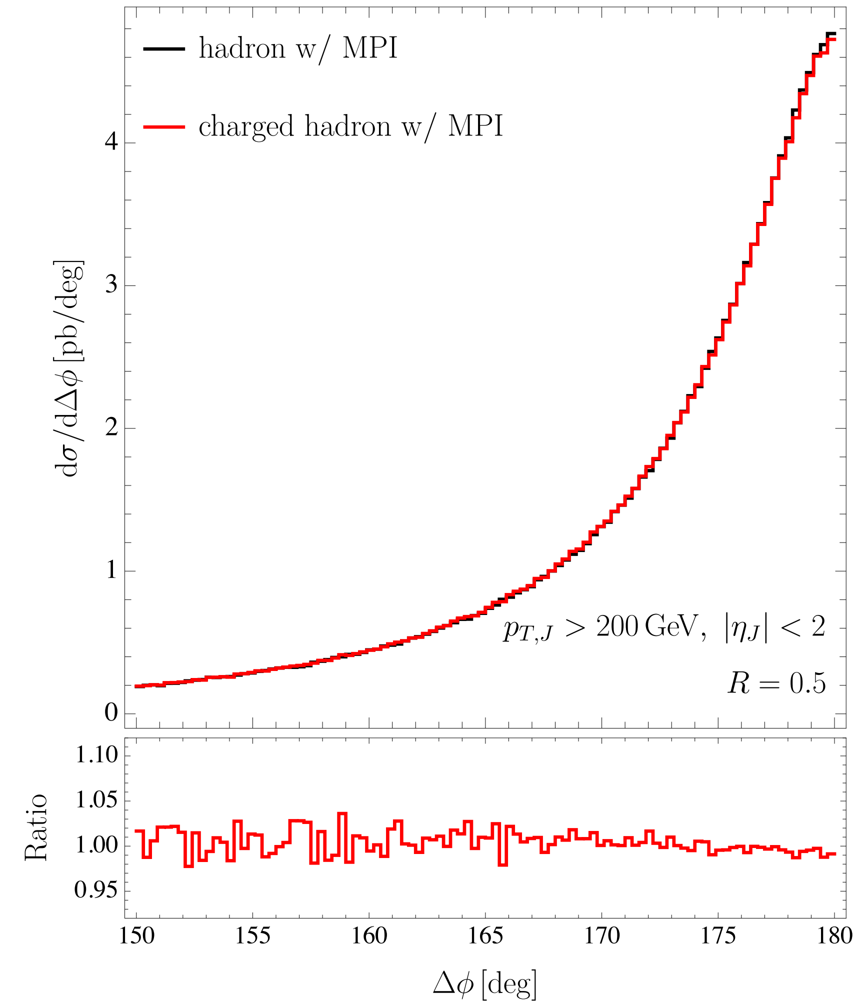
Good agreement, but…

- Not easy calculation + effect of tracks accidentally small.
- Differences in peak from nonperturbative effects.

[Chang, Procura, Thaler, WW]

3. Azimuthal decorrelation in V+jet on tracks





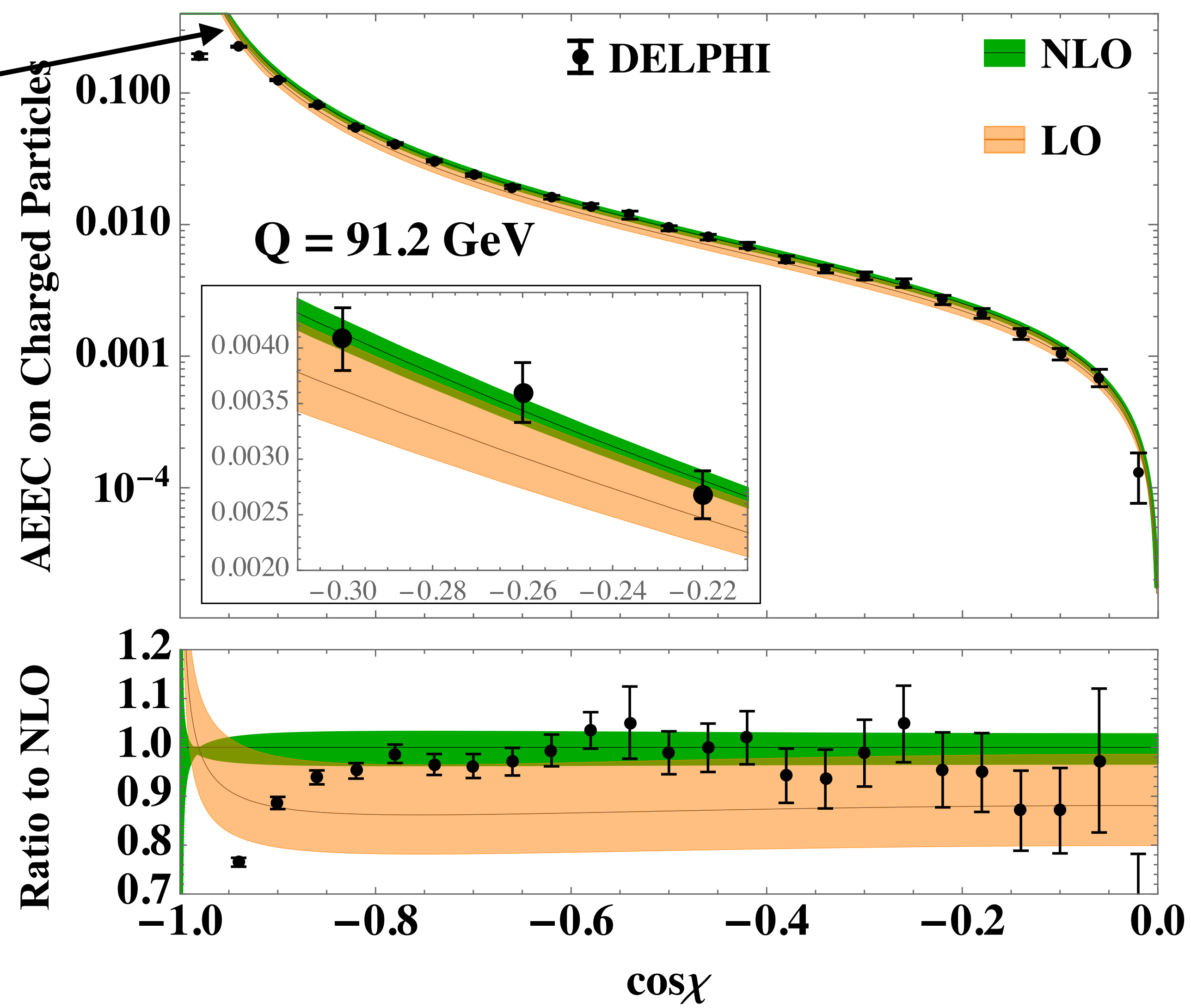
- Azimuthal angle $\Delta\phi$ between vector boson and jet requires resummation in back-to-back limit.
- Using **recoil-free** recombination scheme, tracks only affect collinear (not soft) radiation → beyond NLL, so very small.

[Chien, Rahn, Schrijnder van Velzen, Shao, WW, Wu]

3. Track-based EEC

- First $\mathcal{O}(\alpha_s^2)$ result for track-based measurement:

resummation needed

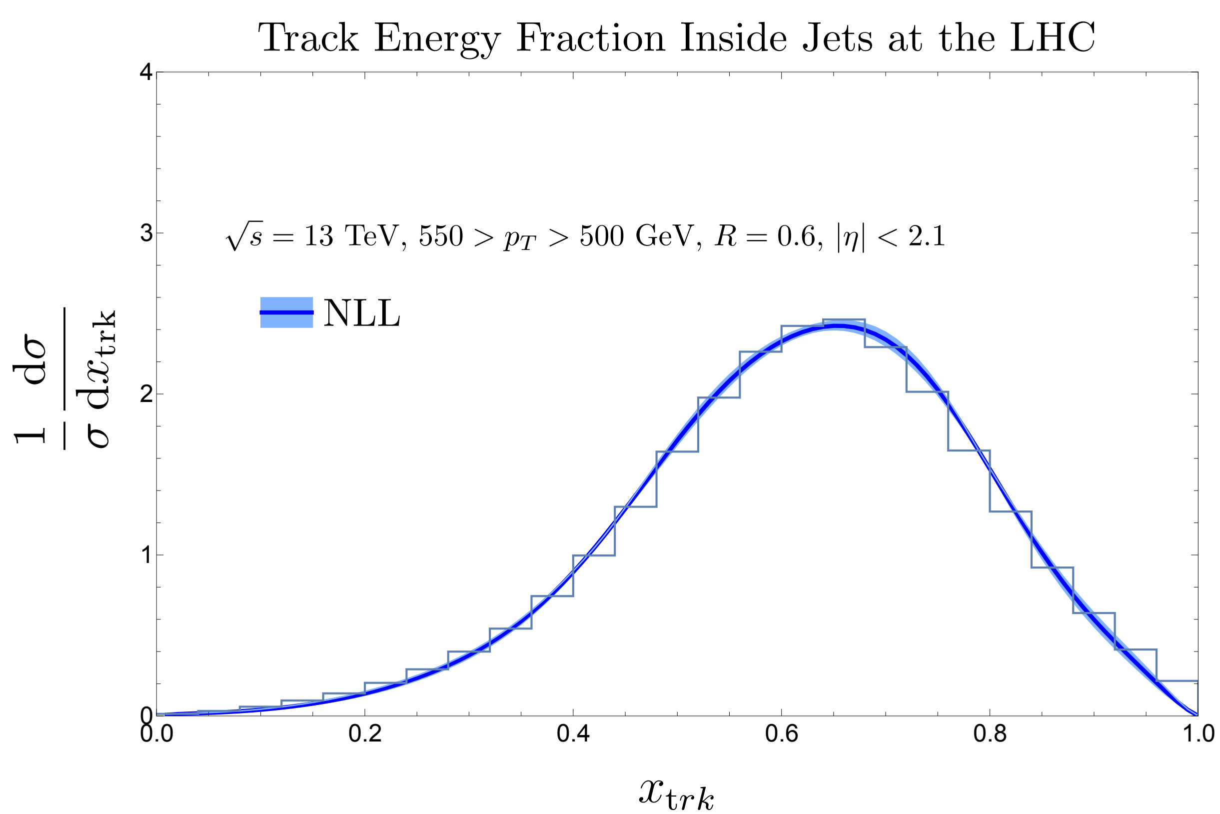


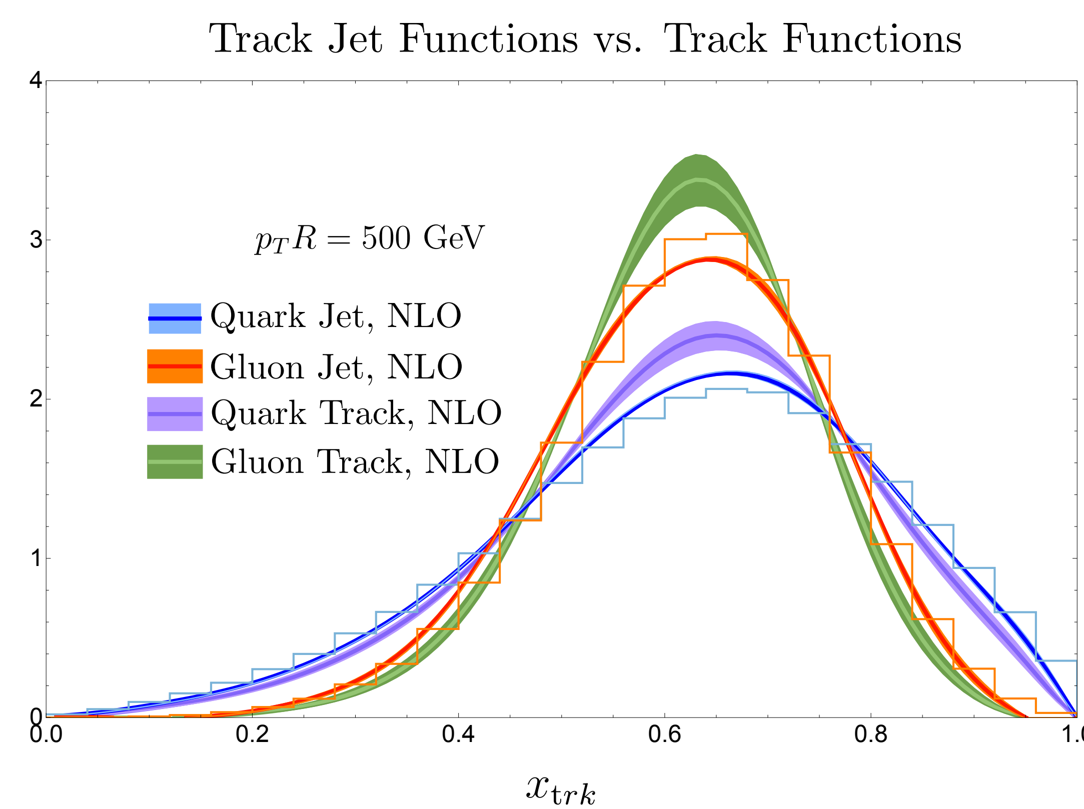
$$\mathrm{AEEC}(\cos\chi) = \mathrm{EEC}(\cos\chi) - \mathrm{EEC}(-\cos\chi)$$

- Uncertainty reduced at NLO, tantalizing agreement with data.

[Li, Moult, Schrijnder van Velzen, WW, Zhu]

3. Charged energy fraction in jets at LHC

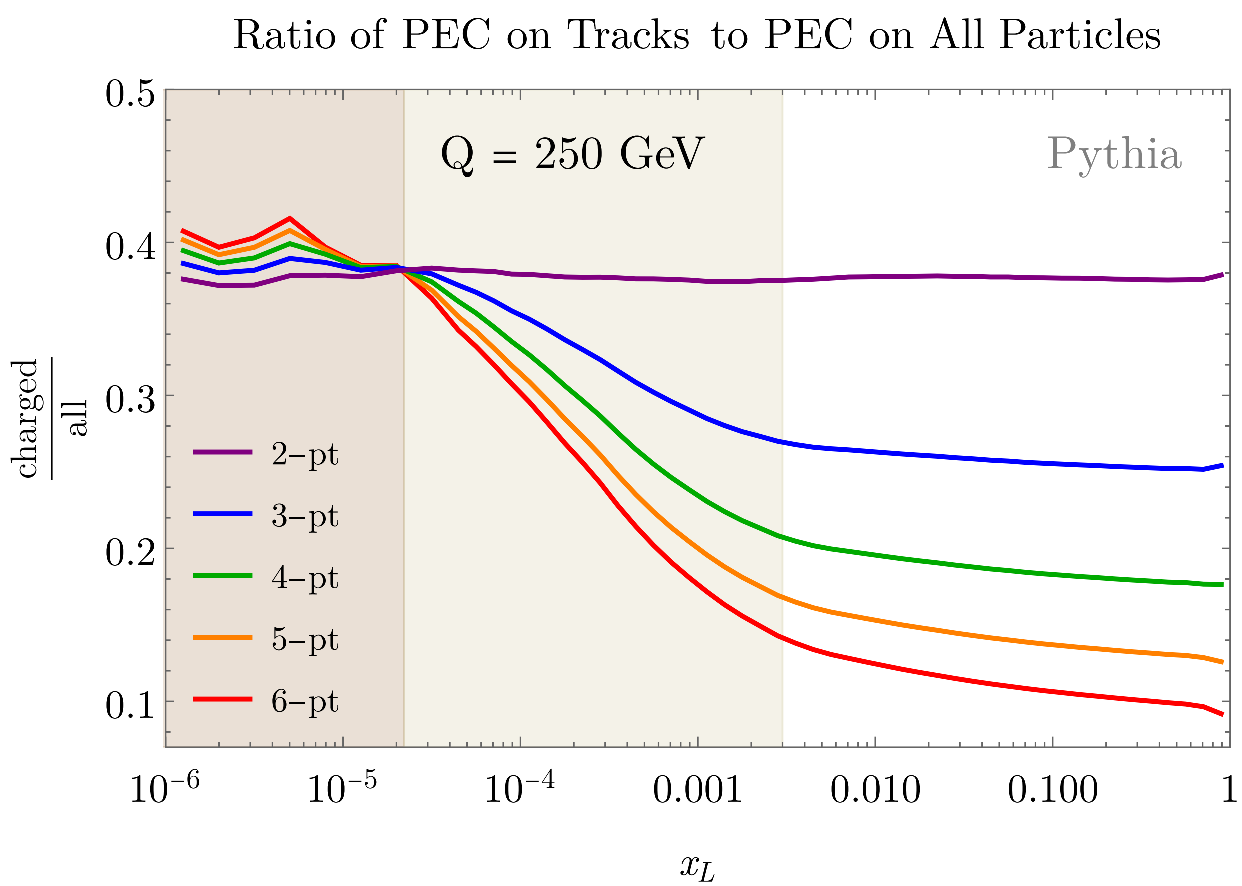


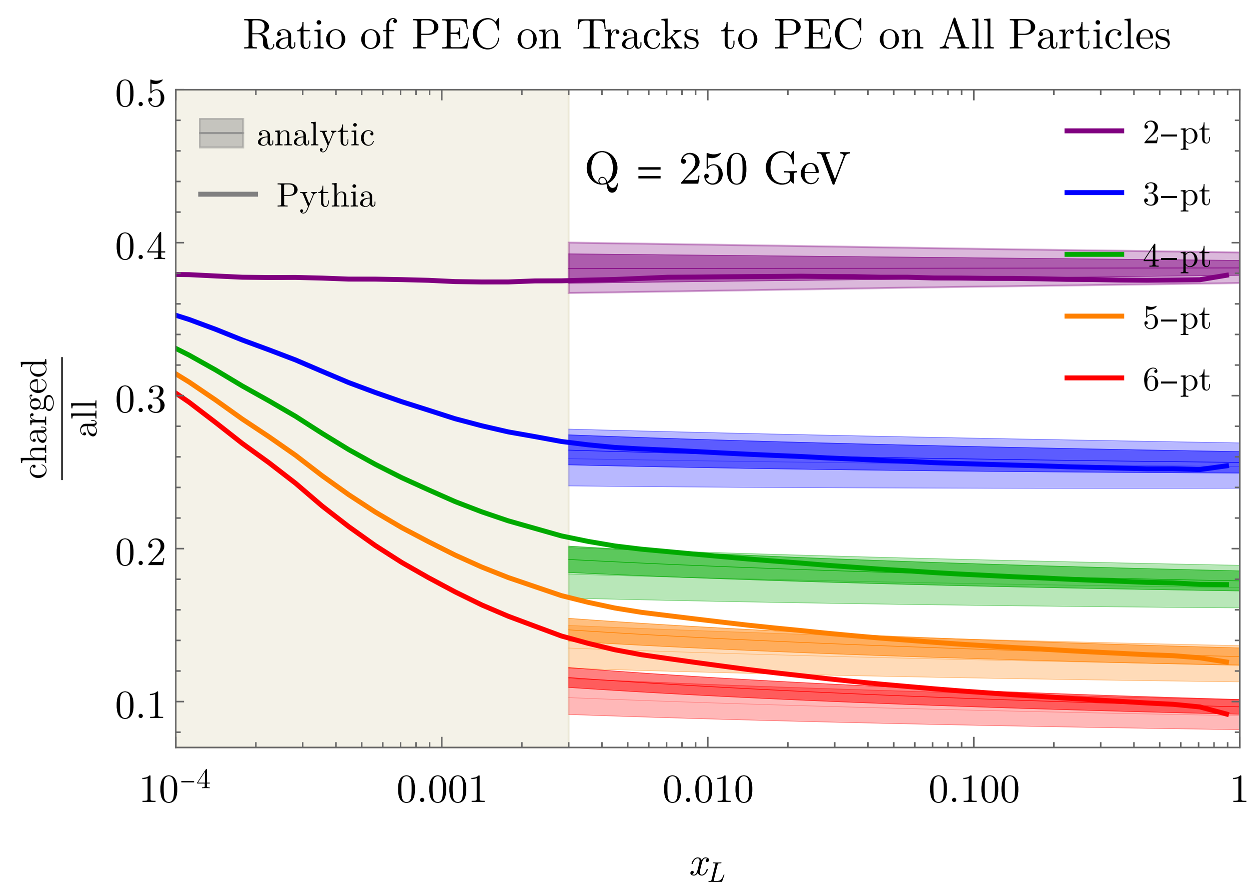


- At LO this is the track function → use this to extract it!
- We also include effects of hard scattering (quark vs. gluon) and jet formation (nontrivial, see right panel).

[Lee, Moult, Ringer, WW]

3. Projected *N*-point energy correlator

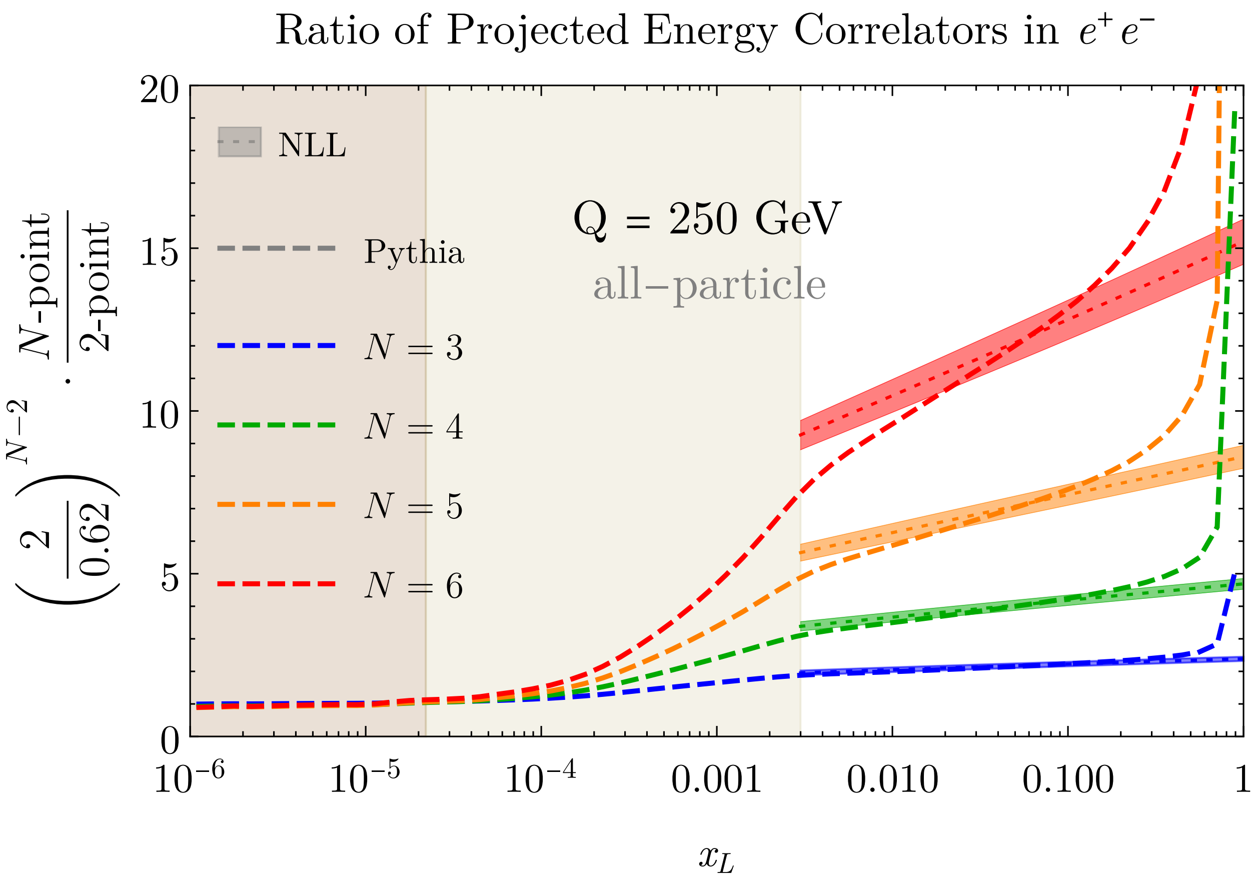


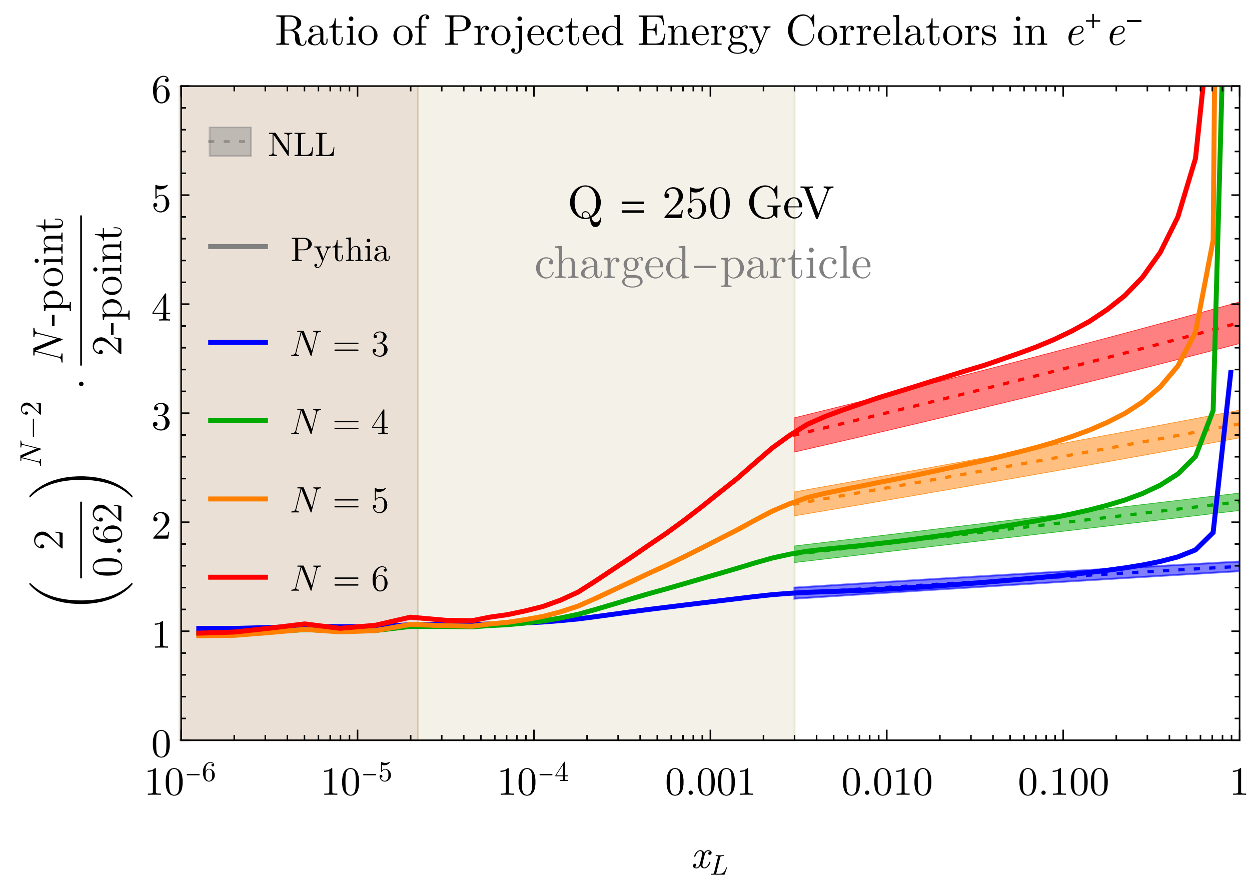


- Projected means: fix largest angle, $x_L = (1 - \cos\theta)/2$
- Ratio of charged to all particles is constant for 2-point, but **not higher point.**

[Jaarsma, Li, Moult, WW, Zhu]

3. Projected *N*-point energy correlator

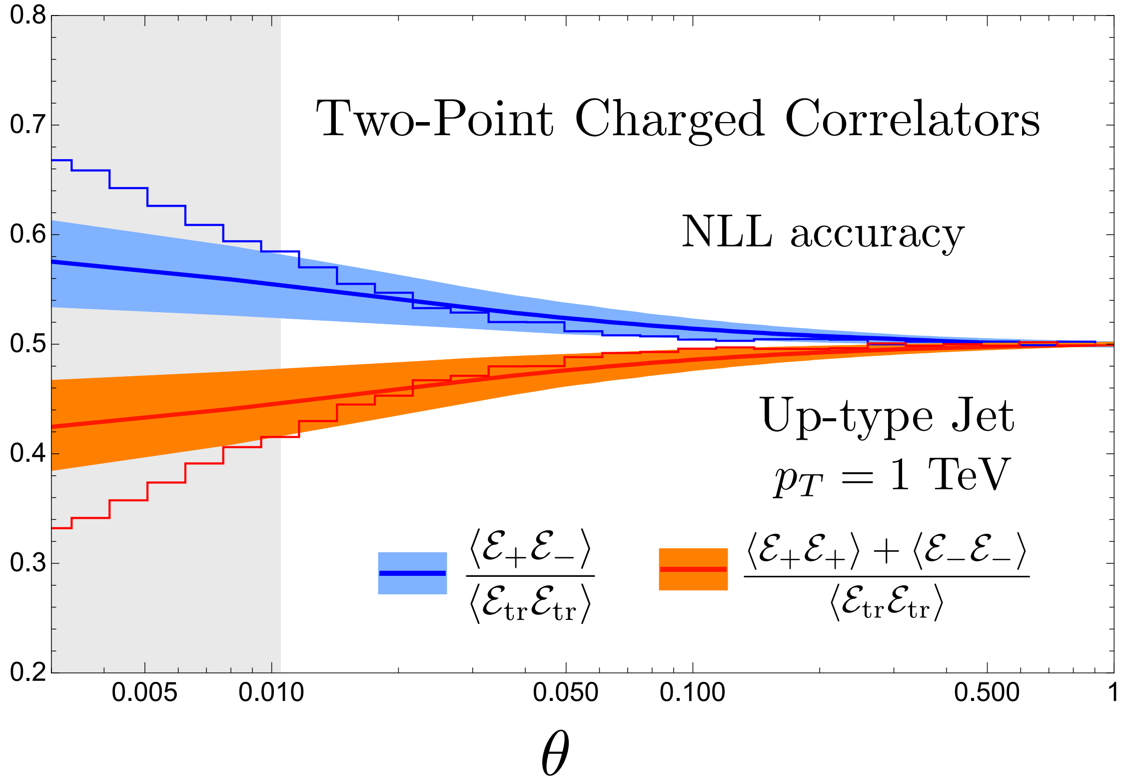


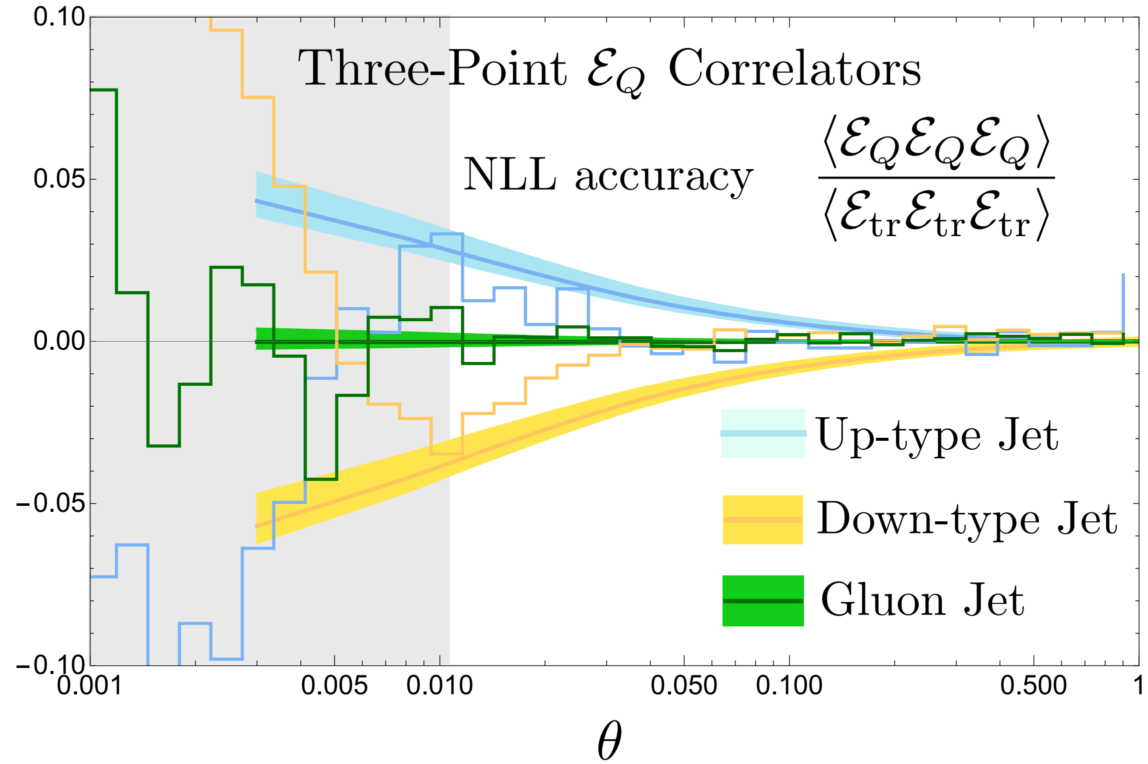


- All vs. charged particles are qualitatively similar, e.g. slope increases with *N*
- Quantitative difference is **calculable** with track functions!

[Jaarsma, Li, Moult, WW, Zhu]

3. Energy correlators taking charge





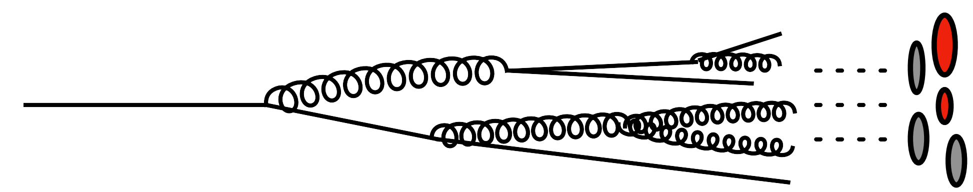
Can also consider correlators on:

- $\mathcal{E}_\pm$: energy of positively/negatively charged particles
- $\mathcal{E}_\mathcal{Q}$: charged-weighted energy

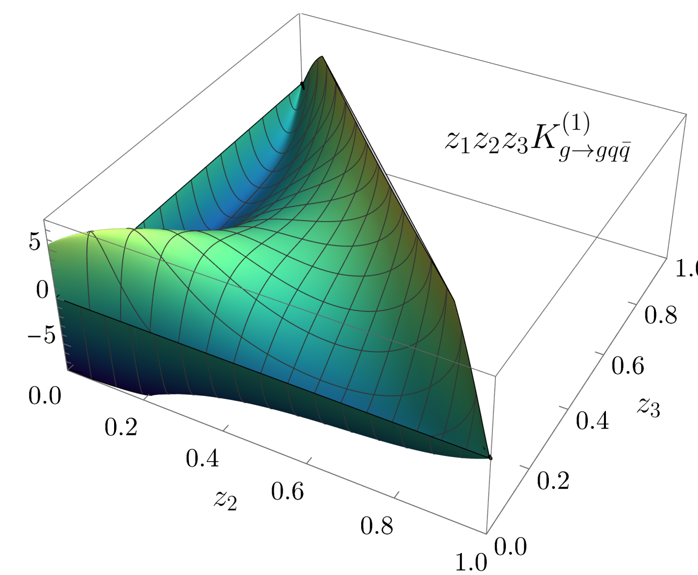
[Lee, Moult]

Conclusions and outlook

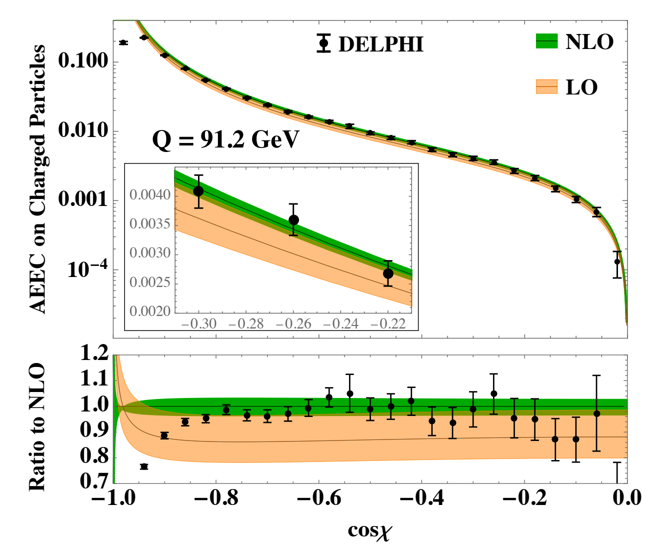
- Tracks offer superior angular resolution and reduce pile-up.



- Track functions now at order α_s^2



- It's evolution contains that of multi-hadron fragmentation functions
- Applications: thrust, azimuthal decorrelation. energy correlators, …



- Energy correlators are particular simple as they only use moments.

Bonus: studying factorization violation with vector angularities

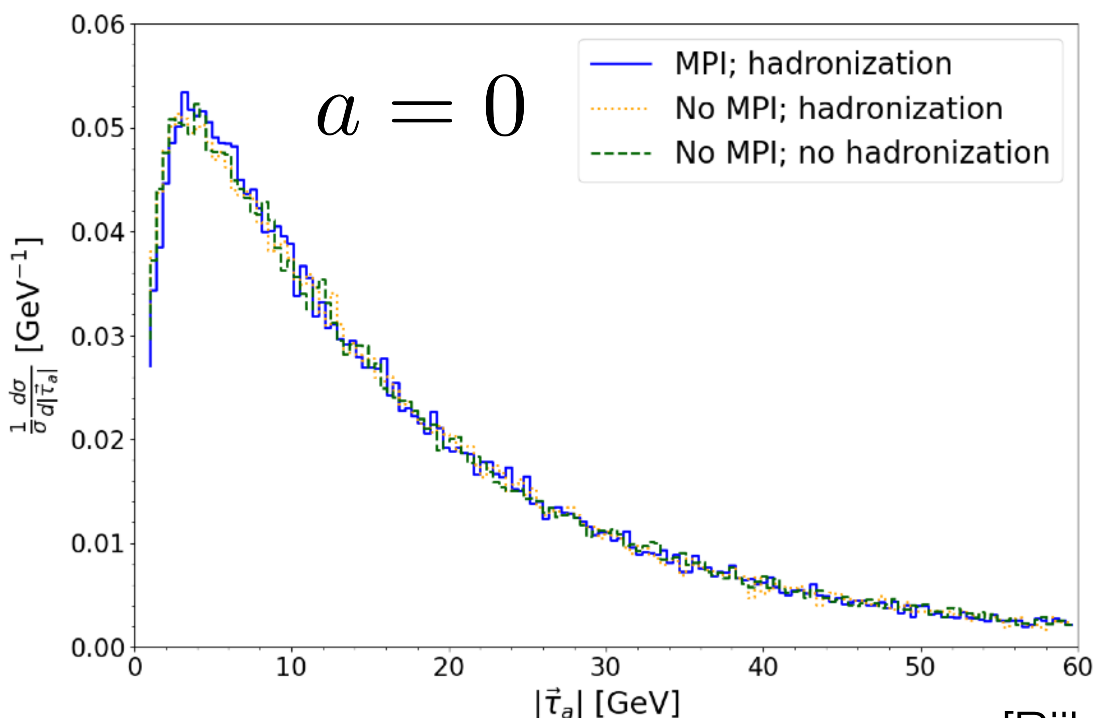
Vector angularities and factorization violation

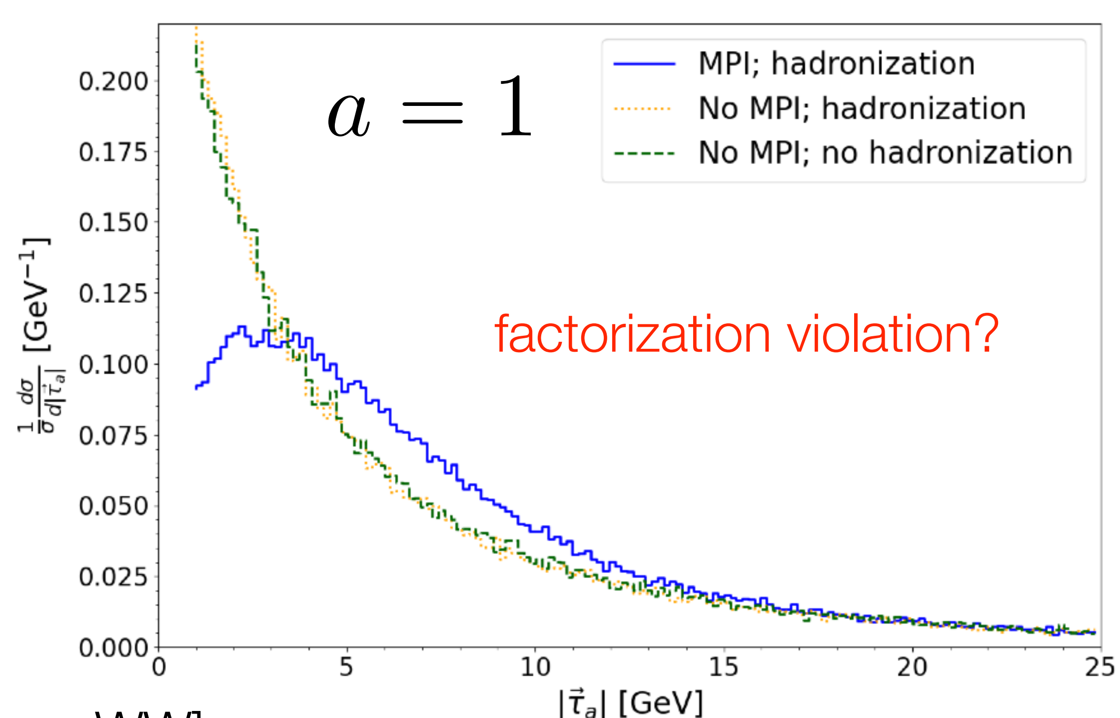
- Factorization proven for Drell-Yan transverse momentum
  [Bodwin; Collins, Soper, Sterman]

- **Vector** angularities generalize transverse momentum:

$$\vec{\tau}_a = \sum_i \vec{k}_{\perp,i} e^{-a|y_i|},$$

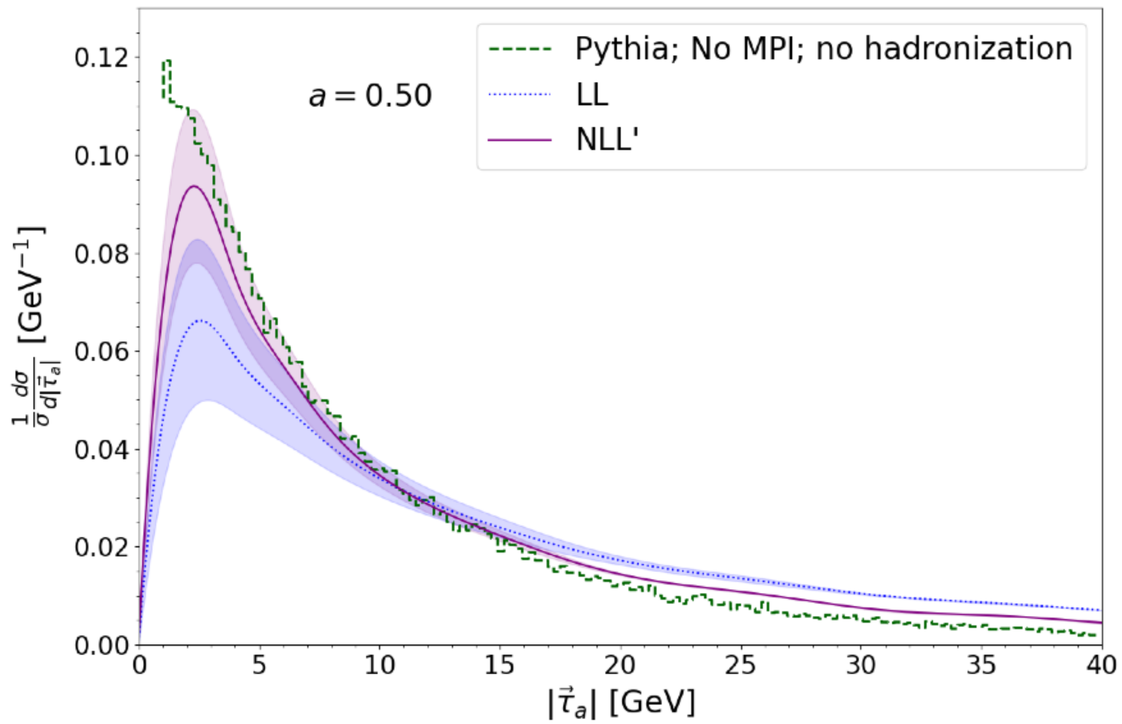
- Use Pythia's MPI as proxy for factorization violation:

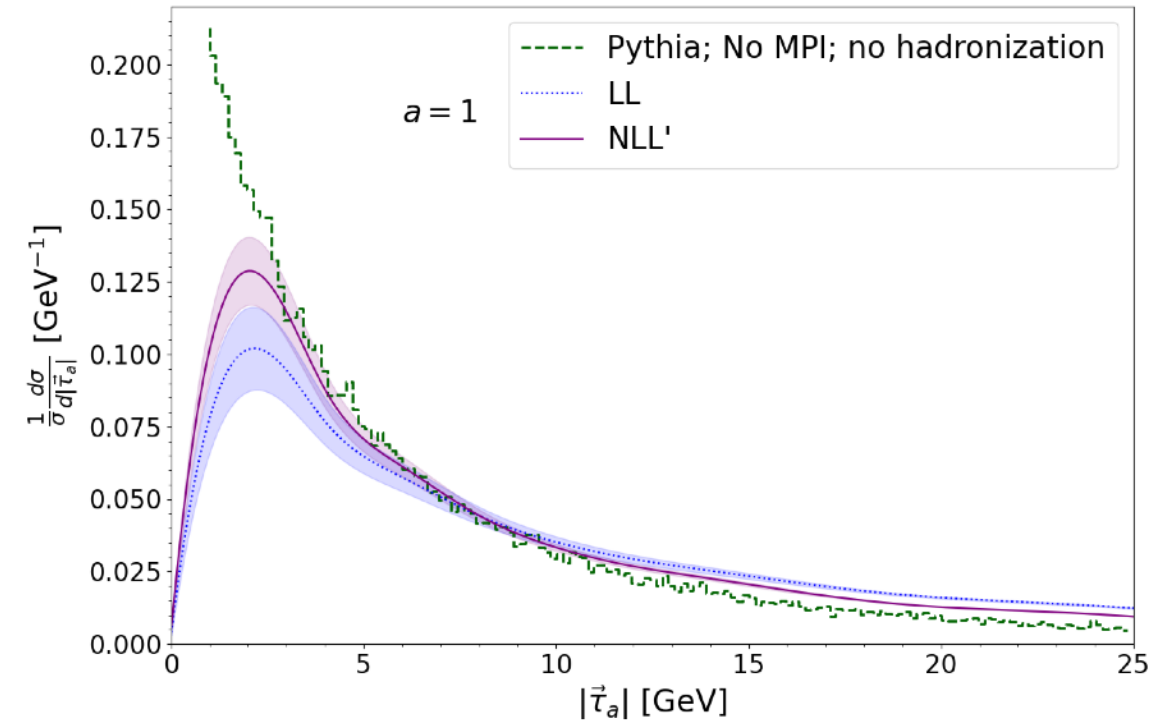




[Bijl, Niedenzu, WW]

Resummation for vector angularities





- Factorization:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}^2\vec{\tau}_a} = \sum_q \sigma_{0,q} H(Q^2,\mu) \int \frac{\mathrm{d}^2\vec{b}_\perp}{(2\pi)^2} e^{-\mathrm{i}\vec{\tau}_a\cdot\vec{b}_\perp} \tilde{B}_q\Big(\frac{\vec{b}_\perp}{(Qe^Y)^a}, x_1, \mu\Big) \tilde{B}_{\bar{q}}\Big(\frac{\vec{b}_\perp}{(Qe^{-Y})^a}, x_2, \mu\Big)\, \tilde{S}(\vec{b}_\perp, \mu)$$

- Compared to transverse momentum resummation: also in impact parameter space, but no rapidity divergences.
- Precise predictions could provide baseline for studying factorization violation