Double Parton Scattering theory.

recent progress

September 6, 2023

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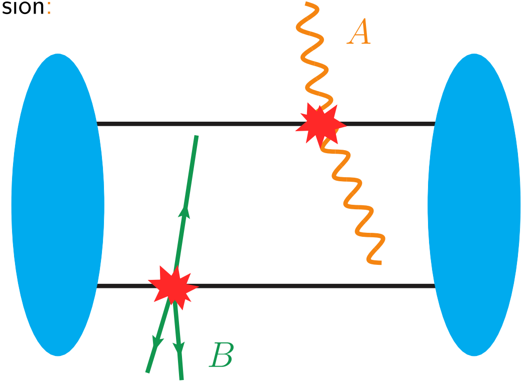
Part I

A brief introduction to Double Parton Scattering.

What is double parton scattering?

Double parton scattering (DPS) describes two individual hard interactions in a single hadron-hadron collision:



- Already observed at previous colliders at CERN and at the Tevatron.
- More data available from the LHC and more to come from HL-LHC.

DPS is naturally associated with the situation where the final state can be separated into two subsets A and B with individual hard scales Q_A and Q_B.

When is DPS relevant and why is it interesting?

DPS is power suppressed compared to single parton scattering (SPS) for integrated cross sections, but leading power for cross sections differential in the transverse momenta of final states ($\boldsymbol{q}_A$ and $\boldsymbol{q}_B$):

SPS vs. DPS cross sections:

- integrated cross sections: $\dfrac{\sigma_{\text{DPS}}}{\sigma_{\text{SPS}}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
- differential cross sections: $\dfrac{\mathrm{d}\sigma_{\text{DPS}}}{\mathrm{d}\boldsymbol{q}_A \mathrm{d}\boldsymbol{q}_B} \sim \dfrac{\mathrm{d}\sigma_{\text{SPS}}}{\mathrm{d}\boldsymbol{q}_A \mathrm{d}\boldsymbol{q}_B} \sim \mathcal{O}\left(\frac{1}{\Lambda^2 Q^4}\right)$

Relative importance of DPS increases with collision energy as smaller momentum fractions are probed:

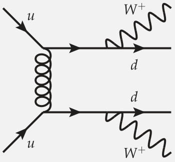
Small-x behaviour:

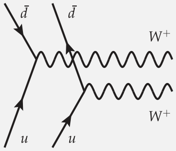
$\sigma_{\text{DPS}} \sim f^4$ vs. $\sigma_{\text{SPS}} \sim f^2$ with PDFs f.

When is DPS relevant and why is it interesting?

For coupling suppressed final states in SPS DPS may give leading contributions:

Like-sign W production:





SPS suppressed by $\mathcal{O}(\alpha_s^2)$ and characterized by additional jets!

DPS gives access to information about hadron structure not accessible in other processes:

Parton correlations accessible in DPS:

- spatial distribution of partons inside the hadron.
- spin correlations between partons (even for unpolarized hadrons).
- colour correlations (even for colourless final states).

Factorization for DPS.

In close analogy to the SPS case a factorization theorem for the DPS cross section can be derived:

DPS factorization theorem:

$$\sigma^{AB}_{\mathrm{DPS}} = \frac{1}{1+\delta_{AB}} \sum_{a_1,a_2,b_1,b_2} \hat{\sigma}_{a_1 b_1 \to A} \otimes \hat{\sigma}_{a_2 b_2 \to B} \otimes \underbrace{\int \mathrm{d}^2\boldsymbol{y}\, F_{a_1 a_2}(\boldsymbol{y}; Q_A, Q_B)\, F_{b_1 b_2}(\boldsymbol{y}; Q_A, Q_B)}_{\mathcal{L}_{a_1 a_2 b_1 b_2}(Q_A, Q_B)}$$

where:

- $\hat{\sigma}$ are partonic cross sections
- $F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2)$ are position space double parton distributions (DPDs)

For the production of colourless final states factorization has been established with the same level of rigor as in the SPS case (also in the TMD case)!

[Paver, Treleani, 1982, 1984; Mehkfi, 1985] (for tree level formula)

[Diehl, Ostermeier, and Schäfer, 2012; Diehl, Gaunt, PP, and Schäfer, 2015; Diehl and Nagar, 2019] (for full factorization proof)

The DPS “pocket formula”.

Assuming no parton-parton correlations whatsoever, DPDs are often approximated as:

$$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) = f_{a_1}(x_1; \mu_1) f_{a_2}(x_2; \mu_2) G(\boldsymbol{y})$$

Plugging this into the DPS factorization formula yields:

DPS pocket formula:

$$\sigma_{\text{DPS}}^{AB} = \frac{1}{1+\delta_{AB}} \sigma_{\text{SPS}}^{A}\, \sigma_{\text{SPS}}^{B} \int \mathrm{d}^2\boldsymbol{y}\, G^2(\boldsymbol{y}) = \frac{1}{1+\delta_{AB}} \frac{\sigma_{\text{SPS}}^{A}\, \sigma_{\text{SPS}}^{B}}{\sigma_{\text{eff}}}$$

where σ_{eff} should be a process independent quantity encapsulating the relative importance of DPS.

$\longrightarrow$ Can be extracted from experimental data!

The DPS "pocket formula".

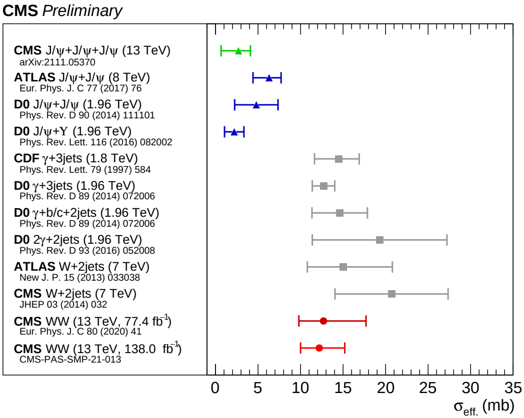


Figure: Overview of the effective cross section σ_{eff} measured in different processes [CMS-PHO-EVENTS-2022-016]. Also measured in other processes not shown here.

Small distance limit of DPDs.

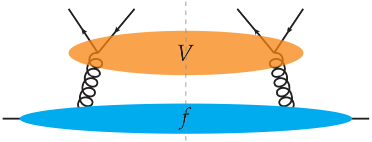
Operator product expansion of DPDs for $\boldsymbol{y} \to 0$:

$$F_{a_1 a_2}(\boldsymbol{y};\mu,\mu) \overset{\boldsymbol{y}\to 0}{=} F^{\text{int}}_{a_1 a_2}(\boldsymbol{y};\mu,\mu) + F^{\text{split}}_{a_1 a_2}(\boldsymbol{y};\mu,\mu)$$

where $F^{\text{int}}_{a_1 a_2}$ and $F^{\text{split}}_{a_1 a_2}$ can be expressed in terms of twist-4 distributions and PDFs, respectively.

F^{split} is enhanced with respect to F^{int} by a factor of $\boldsymbol{y}^{-2}$, making it the leading contribution at small $\boldsymbol{y}$:

$$F_{a_1 a_2}(\boldsymbol{y};\mu,\mu) \overset{\boldsymbol{y}\to 0}{\approx} F^{\text{split}}_{a_1 a_2}(\boldsymbol{y};\mu,\mu) = \frac{1}{\pi \boldsymbol{y}^2} V_{a_1 a_2, a_0}(\boldsymbol{y},\mu) \underset{12}{\otimes} f_{a_0}(\mu)$$



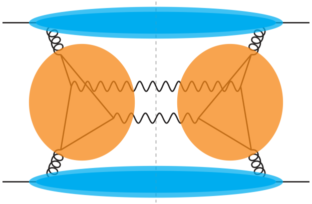
Issues with the DPS cross section?

$$\int \mathrm{d}^2\boldsymbol{y} F_{a_1 a_2}(\boldsymbol{y}) F_{b_1 b_2}(\boldsymbol{y}) \sim \int \frac{\mathrm{d}^2\boldsymbol{y}}{y^4}$$

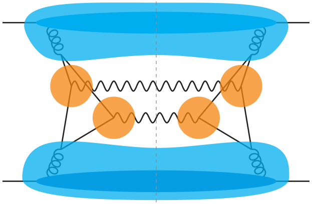
UV divergent cross section?

Disentangling SPS and DPS.

SPS-DPS ambiguity for contributions of the following form:



SPS?



DPS?

Diehl-Gaunt-Schönwald subtraction formalism:

Double counting between SPS and DPS requires a subtraction term:

$$\sigma = \sigma_{\text{SPS}} + \sigma_{\text{DPS}} - \sigma_{\text{sub}}\,, \qquad \sigma_{\text{sub}} = \sigma_{\text{DPS}} \text{ with } F_{ij} \to F_{ij}^{\text{split}}$$ [Diehl, Gaunt, and Schönwald, 2017]

The UV divergence of the DPS cross section is regulated with a lower cut-off ($\boldsymbol{y} \gtrsim 1/\min(Q_A, Q_B)$).

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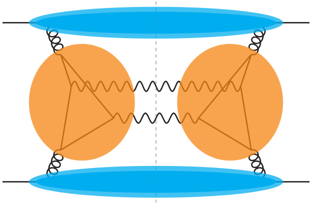
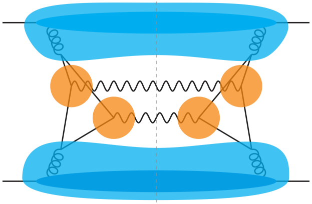


SPS for small y



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What is the state of the art of DPS theory?

Recent years have brought remarkable progress in DPS theory:

- Full QCD description of DPS. [Blok et al., 2011; Diehl et al., 2011; Manohar and Waalewijn, 2012; Ryskin and Snigirev, 2012]
- Factorization proof for DPS. [Diehl, Gaunt, Ostermeier, PP, Schäfer, 2015; Diehl and Nagar, 2019]
- Consistent combination of SPS and DPS. [Gaunt and Stirling, 2011; Diehl, Gaunt and Schönwald, 2017]

Basic theory framework has been largely worked out!

Open issues in DPS theory:

- DPDs are not constrained by experimental data.
- Missing higher order perturbative results.
- Publicly available tools for DPS computations are still missing.

Many of these issues are actively being worked on!

Part II

Recent progress in DPS theory: DPDs.

Recent theory results for DPDs.

Recent results for colour non-singlet DPDs:

- Proof of the violation of positivity bounds. [Diehl, Gaunt, Pichini, PP, 2021]
- Computation of NLO evolution kernels. [Diehl, Fabry, Vladimirov, 2022]
- Study of perturbative colour correlations in DPS cross sections. [Blok, Mehl, 2022]

Recent results for small-y DPDs:

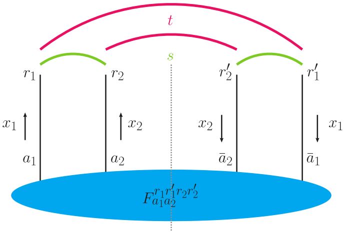
- Computation of the $1 \to 2$ splitting kernels at NLO. [Diehl, Gaunt, PP, Schäfer, 2019; Diehl, Gaunt, PP, 2021]
- Consistent treatment of mass effects in small-y DPDs. [Diehl, Nagar, PP, 2023]

Recent results for DPDs from the lattice:

- Computation of the first Mellin moment of DPDs. [Bali et al., 2021; Zimmermann, Reitinger, 2022]
- Extension of the quasi-PDF approach to DPDs. [Jaarsma, Rahn, Waalewijn, 2023]

Colour structure of DPDs.

The four colour indices of a DPDs can be coupled to an overall colour singlet in a variety of ways:



- The fields are coupled pairwise to irreducible representations R_i of $SU(N)$ such that $R_1 R_2$ (RR') is a colour singlet.
- The full colour structure is decomposed in terms of these combinations (t-channel):

$$F_{a_1 a_2}^{r_1 r_1' \, r_2 r_2'}(x_1, x_2, \boldsymbol{y}) \sim \sum_{R_1, R_2} P_{R_1 R_2}^{r_1 r_1' \, r_2 r_2' \;\; R_1 R_2} F_{a_1 a_2}(x_1, x_2, \boldsymbol{y})$$

Colour representations for DPDs (t-channel):

- $R_1 R_2 = 1\,1,\ 8\,8$ for $a_1 a_2 = qq'$.
- $R_1 R_2 = 1\,1,\ 8\,A$, and $8\,S$ for $a_1 a_2 = qg$.
- $R_1 R_2 = 1\,1,\ A\,A,\ S\,S,\ A\,S,\ S\,A,\ 10\,\overline{10},\ \overline{10}\,10$ and $27\,27$ for $a_1 a_2 = gg$.

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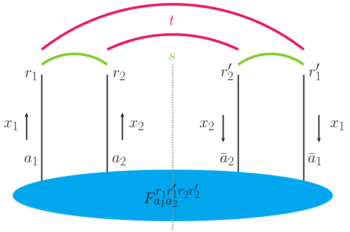

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$$F^{r_1 r_1' \, r_2 r_2'}_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) \sim \sum_{R,R'} P^{r_1 r_2 \, r_1' r_2'}_{RR'} \, F^{RR'}_{a_1 a_2}(x_1, x_2, \boldsymbol{y})$$

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Violation of positivity bounds for colour non-singlet DPDs. [Diehl, Gaunt, Pichini, PP, 2021]

s-channel distributions allow for a density interpretation, implying positivity of the distributions.

Positivity bound for s-channel DPDs:

$F^{RR'}_{a_1 a_2} \geq 0$ for all RR' [Kasemets and Mulders, 2014]

It can be shown that this bound can be violated:

- LO DGLAP evolution to higher scales is not guaranteed to preserve positivity.
- For small y the perturbative DPDs are negative in certain kinematics at NLO.

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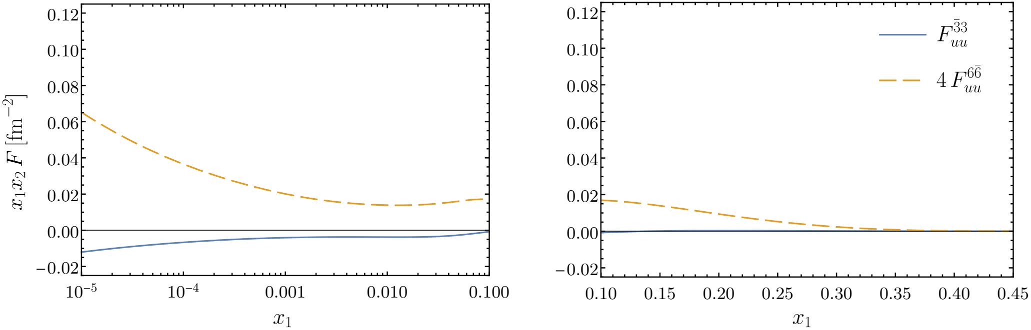

Figure: The s-channel distributions $F_{uu}^{\bar{3}3}$ and $F_{uu}^{6\bar{6}}$ computed with the two-loop splitting formula for $x_1 = x_2$ as a function of x_1.

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- LO DGLAP evolution to higher scales is not guaranteed to preserve positivity.
- For small $\boldsymbol{y}$ the perturbative DPDs are negative in certain kinematics at NLO.

This leaves only the small-$\boldsymbol{y}$ behaviour as reliable constraint for colour non-singlet DPDs!

Colour non-singlet DGLAP kernels at NLO.

[Diehl, Fabry, Vladimirov, 2022]

Evolution of colour non-singlet DPDs:

$$\frac{\mathrm{d}\,{}^{R_1R_2}F_{a_1a_2}(x_1,x_2,\boldsymbol{y};\mu_1,\mu_2,\zeta_p)}{\mathrm{d}\log\mu_1^2} = \sum_{b_1,R_1'} {}^{R_1R_1'}P_{a_1b_1}(x_1';\mu_1,x_1^2\zeta_p) \underset{x_1}{\otimes} {}^{R_1'R_2}F_{b_1a_2}(x_1',x_2,\boldsymbol{y};\mu_1,\mu_2,\zeta_p)$$

Due to the rapidity dependence the ${}^{RR'}P_{ab}$ are not trivially related to the regular DGLAP kernels!

Recent determination of the NLO ${}^{RR'}P_{ab}$ kernels:

- Rapidity independent terms could be obtained from published results for the NLO DGLAP kernels.
- Full results including rapidity dependence were obtained from TMD small distance matching kernel calculations.

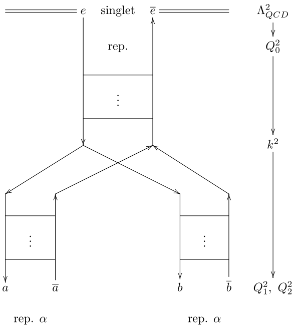
Studies of the numerical impact of these NLO kernels are work in progress and will be published soon!

Colour correlations in DPS cross sections.

[Blok, Mehl, 2022]

It has long been understood that contributions from colour non-singlet DPDs are strongly Sudakov suppressed.

A recent study suggests that there are cases without such suppression:



- Two partons originating from a perturbative splitting at a scale k close to the scales of the two hard scatterings Q_1, Q_2.
- Single DGLAP evolution from $\Lambda_{\rm QCD}$ to k without Sudakov suppression.
- Colour non-singlet double DGLAP evolution from k to Q_1, Q_2.
- For k close enough to Q_1, Q_2 Sudakov suppression expected not to be too strong.

The “splitting scale”.

At which scale μ_{split} should the splitting be evaluated?

The natural scale of the splitting is set by the interparton distance y of the observed partons:

$$\mu_{\text{split}}(y) \sim \frac{1}{y}$$

How to avoid evaluation of the perturbative splitting at non-perturbative scales for large y?

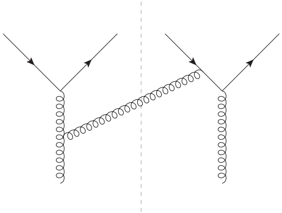
Regularized splitting scale:

$$\mu_{\text{split}}(y) = \frac{b_0}{y^*(y)}\,, \qquad \text{e.g.} \quad y^*(y) = \frac{y}{\sqrt[4]{1 + y^4/y_{\max}^4}}\,, \qquad y_{\max} = \frac{b_0}{\mu_{\min}}$$

Splitting DPDs at NLO.

[Diehl, Gaunt, PP, Schäfer, 2019; Diehl, Gaunt, PP, 2021]

LO splitting DPDs exhibit a huge dependence on μ_{split}, hinting at the importance of higher orders!



Computation of the NLO $1 \to 2$ splitting kernels ${}^{R_1R_2}V^{(2)}_{a_1a_2,a_0}$:

- Bare kernels from Feynman diagrams for partonic DPDs a_1a_2 in parton a_0.
- Consistent regularization of rapidity divergences.
- Renormalized kernels obtained through RGE analysis.

Structure of NLO kernels:

$$
\begin{aligned}
{}^{R_1R_2}V^{(2)}_{a_1a_2,a_0}(z,u,\boldsymbol{y};\mu,\zeta) &= {}^{R_1R_2}V^{[2,0]}_{a_1a_2,a_0}(z,u) + L\; {}^{R_1R_2}V^{[2,1]}_{a_1a_2,a_0}(z,u) \\
&\quad + \left(L \log\frac{\mu^2}{\zeta} - \frac{L^2}{2} + c_{\overline{\text{MS}}}\right) \frac{{}^{R_1}\gamma_J^{(0)}}{2}\; {}^{R_1R_2}V^{(1)}_{a_1a_2,a_0}(z,u)
\end{aligned}
$$

where $L = \log\frac{y^2\mu^2}{b_0^2}$ and ${}^{R}\gamma$ is the anomalous dimension of the CS-kernel for DPDs.

Splitting DPDs at NLO.

[Diehl, Gaunt, PP, Schäfer, 2019; Diehl, Gaunt, PP, 2021]

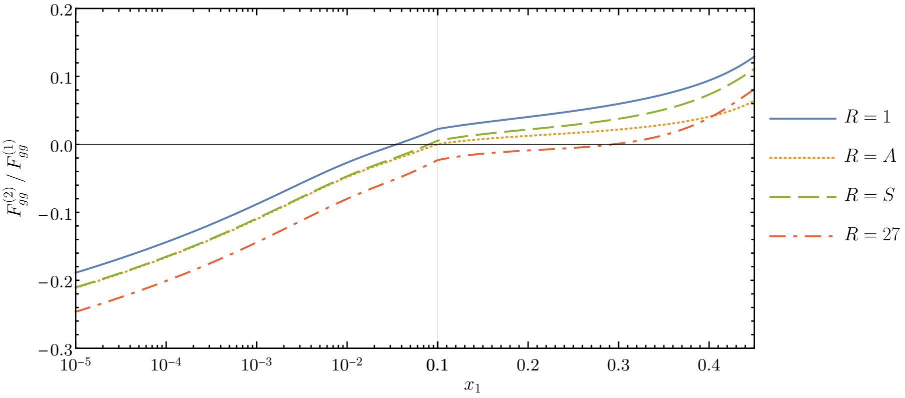


Figure: The ratio ${}^{RR}F_{gg}^{(2)} / {}^{RR}F_{gg}^{(1)}$ for $x_1 = x_2$ as a function of x_1.

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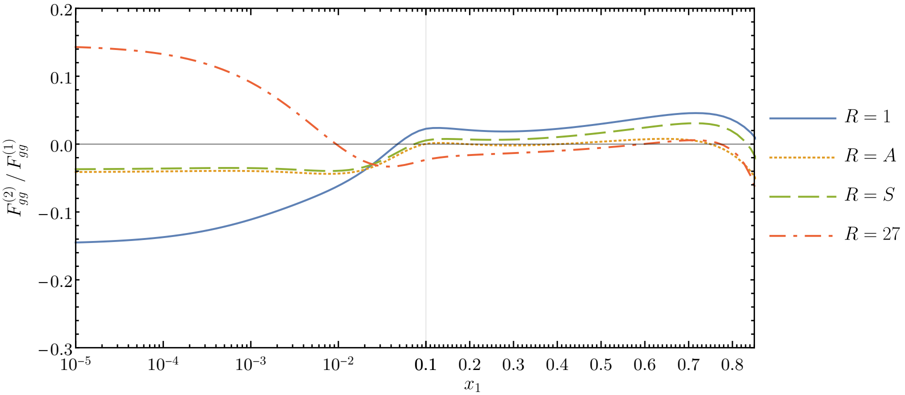

Figure: The ratio ${}^{RR}F_{gg}^{(2)} / {}^{RR}F_{gg}^{(1)}$ for $x_2 = 0.1$ as a function of x_1.

Splitting DPDs at NLO.

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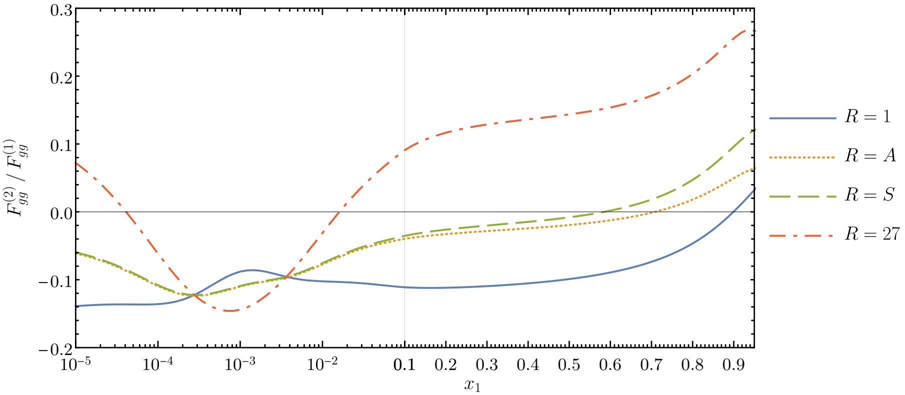

Figure: The ratio ${}^{RR}F_{gg}^{(2)} / {}^{RR}F_{gg}^{(1)}$ for $x_2 = 10^{-3}$ as a function of x_1.

Splitting DPDs at NLO.

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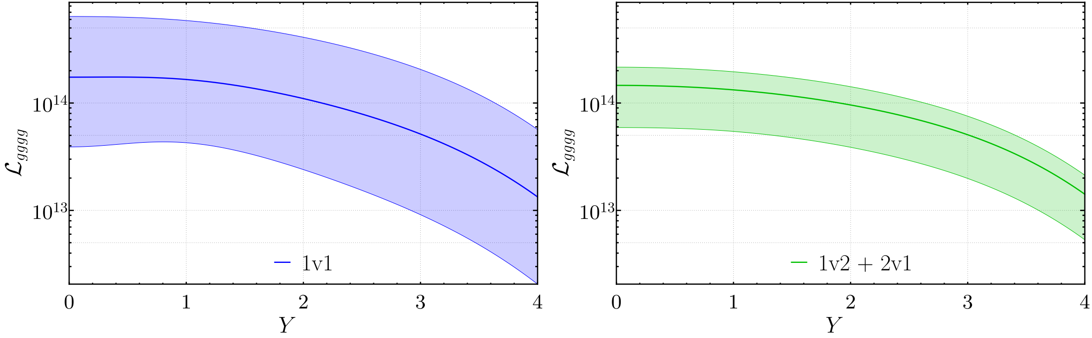

(a) 1v1 @ LO

(b) 1v2 + 2v1 @ LO

Figure: Splitting scale variation of the $gggg$ luminosity for double dijet production $(\mu_1 = \mu_2 = 25\,\mathrm{GeV})$ as a function of the rapidity Y of the produced dijets.The bands correspond to a variation of the splitting scale by a factor of two around its central value. 1v1 denotes the contribution where the partons in both DPDs originate from splitting, whereas 1v2 and 2v1 correspond to the contributions where only in one DPD the partons originate from splitting.

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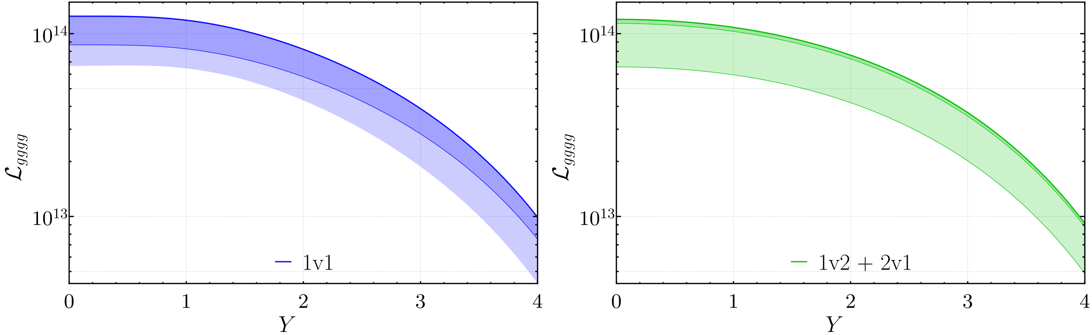

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Mass effects in splitting DPDs.

[Diehl, Nagar, PP, 2023]

How to treat heavy quarks Q in the small-y DPDs?

Neglecting mass effects:

- Q decouples for $\mu_{\text{split}} < \gamma m_Q \sim m_Q$.
- Q massless for $\mu_{\text{split}} > \gamma m_Q \sim m_Q$.

Including mass effects:

- Q decouples for $\mu_{\text{split}} < \alpha m_Q \ll m_Q$.
- Q massive for $\alpha m_Q < \mu_{\text{split}} < \beta m_Q$.
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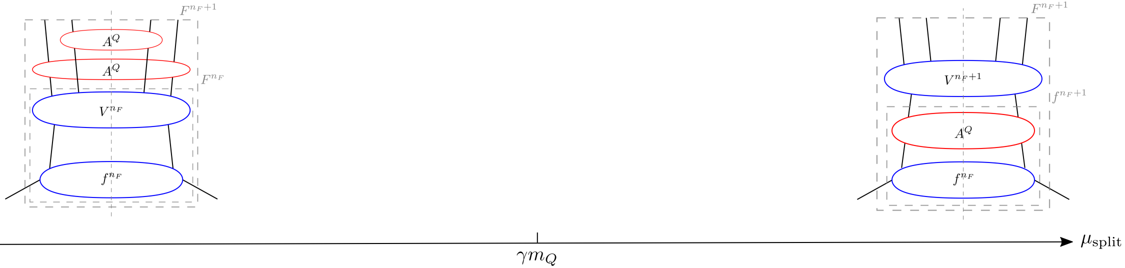

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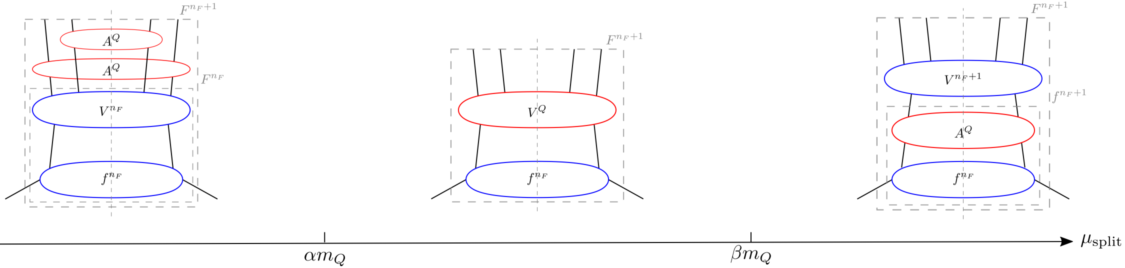

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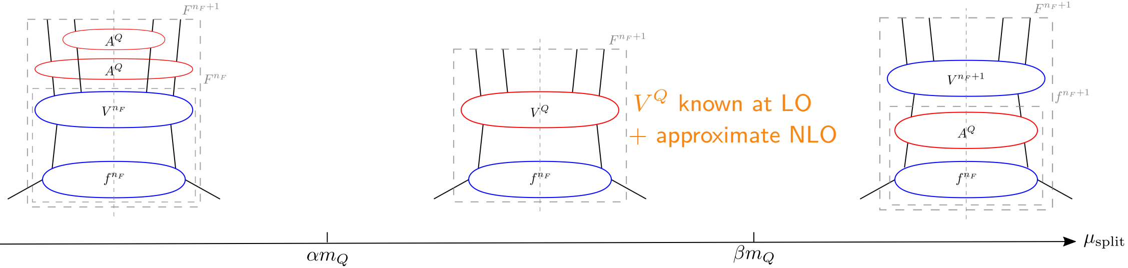


Mass effects in splitting DPDs.

[Diehl, Nagar, PP, 2023]

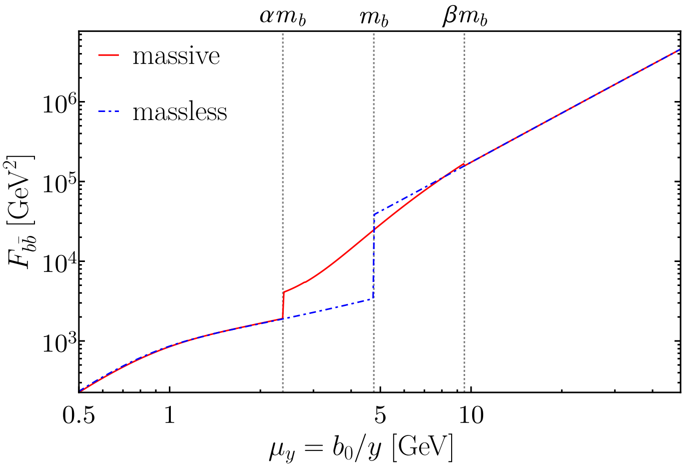


Figure: The $b\bar{b}$ DPD from LO splitting with and without mass effects at $x_1 = x_2 = 1.8 \times 10^{-3}$ evolved to $\mu_1 = \mu_2 = 25\,\text{GeV}$ with LO evolution as a function of $\mu_y = b_0/y$ for $\alpha = 0.5$ and $\beta = 2$.

Mellin moments of DPDs.

[Bali, Diehl, Gläßle, Schäfer, Zimmermann, 2021]

DPDs are defined as matrix element of two non-local twist-2 operators. How to put this on the lattice?

Mellin moments of DPDs:

$$I^{m\,n}_{a_1 a_2}(\boldsymbol{y}) = \int \mathrm{d}x_1\, x_1^{m-1} \int \mathrm{d}x_2\, x_2^{n-1} F_{a_1,a_2}(x_1, x_2, \boldsymbol{y})$$

Mellin moments contain only local operators $\longrightarrow$ accessible on the lattice!

Determination of $I^{1\,1}_{qq'}$ on the lattice: Computation of two current proton matrix elements.

Results:

- Different $\boldsymbol{y}$-dependence for different flavour combinations.
- Small spin-spin correlations, larger spin-orbit correlations.
- Mellin moments of DPDs factorize only approximately.

Mellin moments of DPDs.

[Bali, Diehl, Gläßle, Schäfer, Zimmermann, 2021]

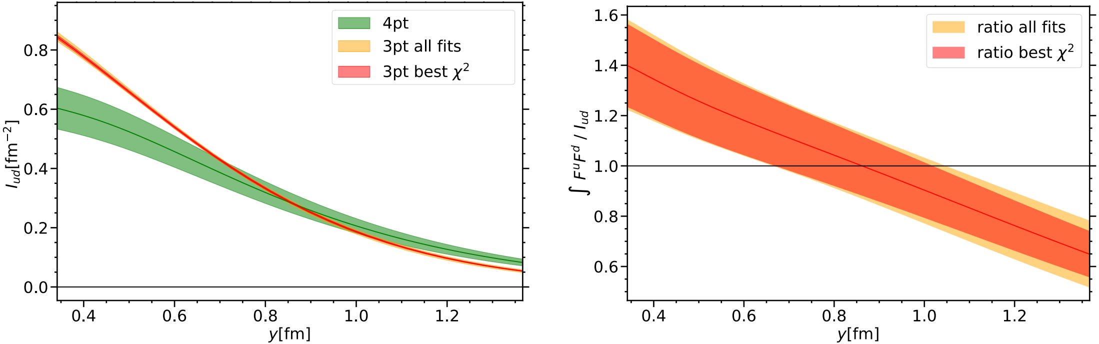


Figure: Comparison of the Mellin Moment $I_{ud}^{1\,1}$ (4pt) with its factorized approximation (3pt).

DPDs from LaMET.

[Jaarsma, Rahn, Waalewijn, 2023; Zhang, 2023]

Extend the successful quasi-PDF approach to DPDs:

Matching DPDs onto quasi-DPDs:

$$
{}^{R_1R_2}\tilde{F}_{a_1a_2}(x_1, x_2, \boldsymbol{y}; \mu, \tilde{\zeta}_p, \tilde{P}^z) = \sum_{\substack{R_1', R_2' \\ a_1', a_2'}} \int_0^1 \frac{\mathrm{d}x_1'}{x_1'} \frac{\mathrm{d}x_2'}{x_2'} \, {}^{R_1R_1'}C_{a_1a_1'}\left(\frac{x_1}{x_1'}, x_1'\tilde{P}^z, \mu\right) {}^{R_2R_2'}C_{a_2a_2'}\left(\frac{x_2}{x_2'}, x_2'\tilde{P}^z, \mu\right)
$$
$$
\times \exp\left[\frac{1}{2} {}^{R_1'}J(y;\mu) \ln\left(\frac{\tilde{\zeta}_p}{\zeta_p}\right)\right] {}^{R_1'R_2'}F_{a_1'a_2'}(x_1', x_2', \boldsymbol{y}; \mu, \zeta_p) \,,
$$

with quasi-DPDs ${}^{R_1R_2}\tilde{F}_{a_1a_2}$ and matching kernels ${}^{RR'}C_{aa'}$.

$\longrightarrow$ Access to the full x_1- and x_2-dependence of DPDs from the lattice!

Open questions:

- Computation of the DPS soft factor on the lattice?
- Renormalization of quasi-DPDs?

Part III

Recent progress in DPS theory: Tools.

A parton shower for DPS.

[Cabouat, Gaunt, 2020; Cabouat, Gaunt, Ostrolenk, 2019]

Existing parton showers include DPS, but mostly based on the assumptions of the pocket formula.

`dShower` goes beyond this and implements an ISR parton shower for DPS built on unfactorized DPDs!

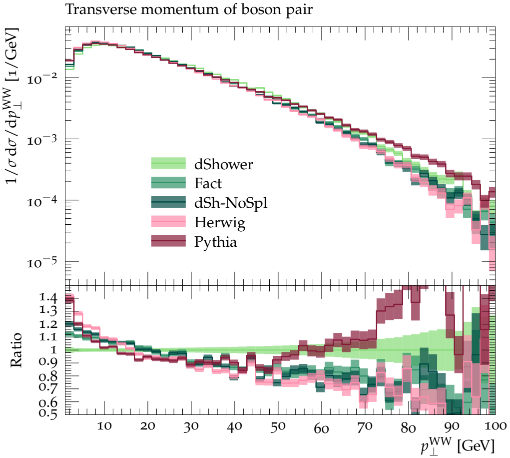
Features:

- Two partons $a_1 a_2$ initiating two hard scatterings with momentum fractions x_1 and x_2 separated by $\boldsymbol{y}$ are evolved backwards from the hard scale guided by the double DGLAP equation and full DPDs.
- Possibility of parton merging at scales $\mu \sim 1/y$:
  if a merging happens the further backward evolution is performed with a regular single parton shower, if no merging happens the splitting contribution to the DPDs is removed in further backward evolution.
- Consistent combination of SPS and DPS plus parton showers:
  Implementation of the DGS subtraction formalism, extended to the fully-differential case.

`dShower` provides an important step towards a full implementation of the DPS theory!

A parton shower for DPS.

[Cabouat, Gaunt, 2020; Cabouat, Gaunt, Ostrolenk, 2019]



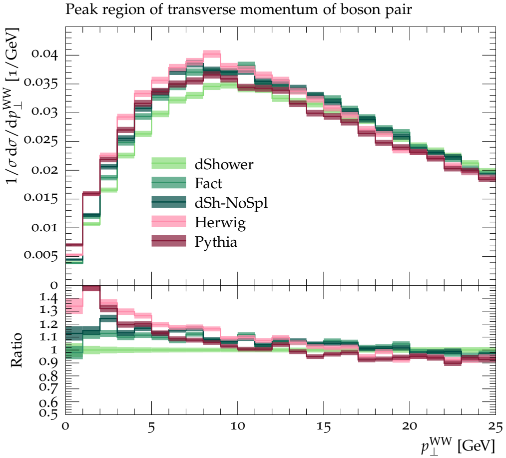


Figure: Comparison of dShower with other parton showers for the $p_\perp$ spectrum in DPS WW production.

Fast and precise evolution for DPDs.

[Diehl, Nagar, PP, Tackmann, 2022]

`ChiliPDF` is a C++ library for the evolution and interpolation of PDFs and DPDs! (will be public)

Design:

- DPDs are discretized in x_1, x_2, and y on Chebyshev grids, allowing for high interpolation accuracy with fewer points than e.g. splines.
- No gridding in μ_1 and μ_2 – evolution is performed on the fly using higher-order Runge-Kutta algorithms.

Features:

- Evolution and flavour matching for DPDs (unpolarized and polarized, colour singlet and non-singlet) at the highest available order.
- Small-$\boldsymbol{y}$ splitting DPDs at NLO.
- Evaluation of sum rules for unpolarized colour singlet DPDs.
- Computation of DPS luminosities.

Tools: ChiliPDF.

Fast and precise evolution for DPDs.

[Diehl, Nagar, PP, Tackmann, 2022]

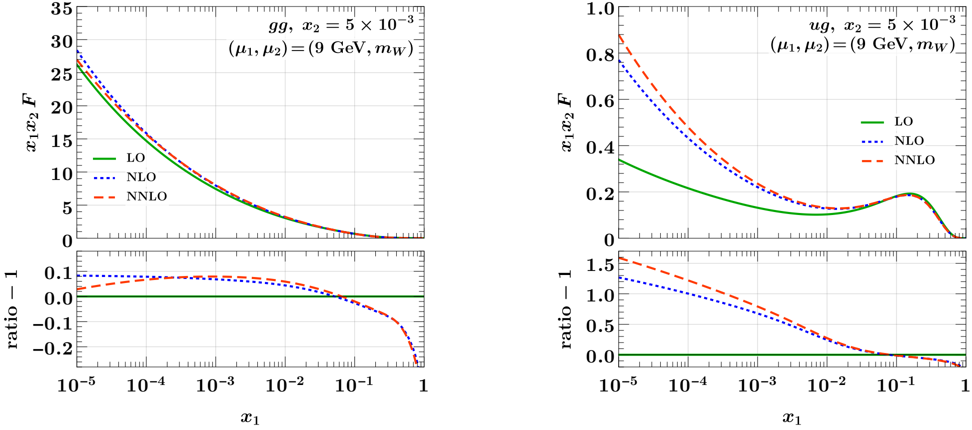


Figure: Unpolarized DPDs at $x_2 = 5 \times 10^{-3}$, evolved from $(\mu_1, \mu_2) = (2\,\text{GeV}, 2\,\text{GeV})$ to $(9\,\text{GeV}, m_W)$ at different perturbative orders. The ratio shown in the small panels is taken with respect to the LO result.

Part IV

But wait, there's more!

Advances in DPS theory and phenomenology.

Besides the work covered in detail there's a lot of activity in the field.

Recent work on DPS:

- Triple parton scattering (TPS):
  Triple J/Ψ production as a golden channel for TPS. [Shao, Zhang, 2019; Blok, Mehl, 2023]
  Sum rules and models for triple parton distributions (TPDs) [Fedkevych, Gaunt, 2022]
- DPS in photon-proton interactions: DPS at the EIC? [Rinaldi, 2022]
- Study of DPS contributions to four jet production at the LHC. [Fedkevych, Kuleza, 2020]
- New multiparton interaction (MPI) observables. [Andersen, Monni, Rottoli, Salam, Soto-Ontoso, 2023]
- Factorization of DPDs for small momentum fractions. [Golec-Biernat, Staśto, 2022]
- ...

Hopefully we'll see even more progress in the near future!