Multi-loop amplitudes for V+jet production
at hadron colliders

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based on 2301.10849, 2306.10170 and 2307.15405
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Overview

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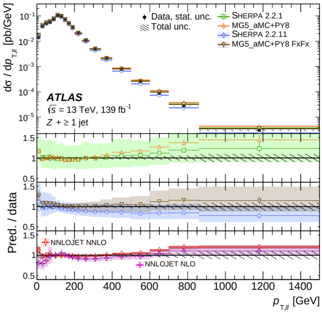

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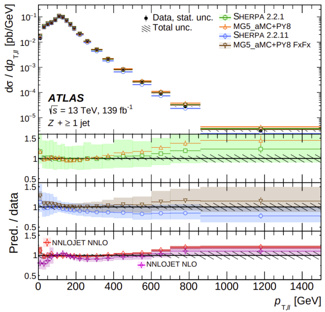

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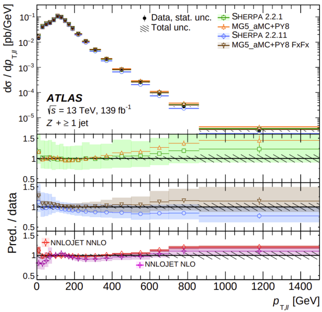

- Background for Higgs properties studies and new physics searches (via missing E_T) [Lindert, Pozzorini et al. 2017]
  - extrapolation of background to signal region (especially at high p_T)

Theory state of the art

$(Z, W^{\pm})$+jet completed using antenna subtraction [Gehrmann-de Ridder et al. 2016] and N-jettiness slicing [Boughezal et al. 2016] at

$$\boxed{\mathrm{NNLO}_{QCD}} + \mathrm{NLO}_{\mathrm{EW}}$$

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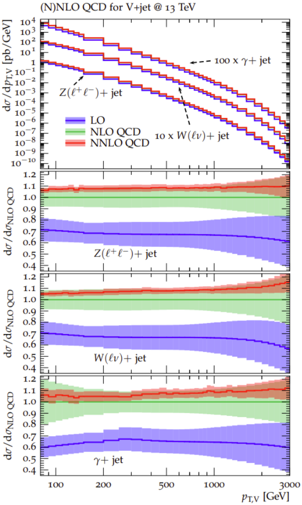

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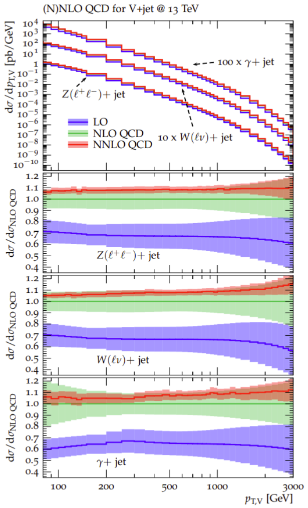
Hard scatter is the first step to 1%-level:

$$d\sigma_{h_1 h_2 \to X}(p_1, p_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_i, \mu) f_j(x_2, \mu)$$

$$\times \boxed{d\sigma_{ij \to X}(x_1 p_1, x_2 p_2, \mu)} + O(\Lambda_{QCD}/Q)$$

$$\downarrow$$

$$d\hat{\sigma}_0(1 + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} \boxed{+\alpha_s^3 \sigma^{(3)}} + \ldots)$$



Amplitudes state of the art

Required matrix elements for N^{3}LO

- Tree-level: $2 \to 5$ ✓
- 1 loop: $2 \to 4$ ✓
- 2 loops: $2 \to 3$ (all integrals [Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia, 2023], and leading colour amplitude [Abreu, Cordero, Ita, Klinkert, Page, 2021])
- 3 loops: $2 \to 2$ **!**

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Very timely: golden era of NNLO predictions, dawn of N^3LO amplitudes (first all massless $2 \to 2$ integrals [Henn, Mistlberger, Smirnov, Wasser 2020] and QCD amplitude [Caola, von Manteuffel, Tancredi 2021, + Bargiela, Chakraborty, Gambuti 21-22])

$2 \to 2$ amplitudes with an external mass

- 1 loop amplitude [Giele, Glover 1992]
- 2 loop amplitude [Garland, Gehrmann, Glover, Koukoutsakis, Remiddi, 2002]: first demonstration of method of differential equations, introduction of 2d HPLs [Gehrmann, Remiddi, 2001], symmetric alphabet which closes under crossings

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- IBP reduction automation, finite field methods etc.

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New results!

1. 2 loop amplitude with axial vector coupling of Z ($\to$ completion at 2L)
   - confirm non-singlet agreement between V and A-V
   - singlet will contribute to diff. observables (eg. angular correlations)
2. 3 loop amplitude at leading colour → *extend 2L to* ϵ^2

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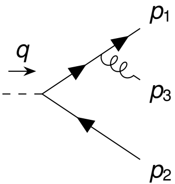
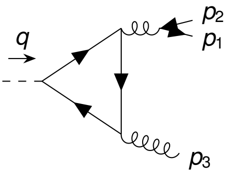
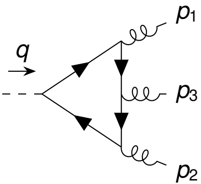


$q\bar{q}$ channel non-singlet



$q\bar{q}$ channel singlet



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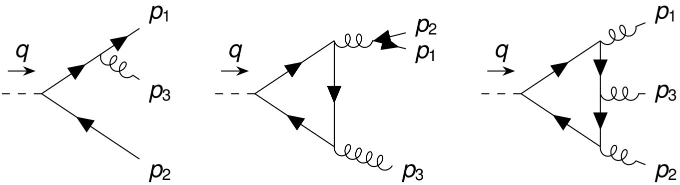

$q\bar{q}$ channel non-singlet | $q\bar{q}$ channel singlet | gluonic channel

- SM coupling for Z has a vector (γ^μ) and axial-vector ($\gamma^\mu\gamma^5$) part
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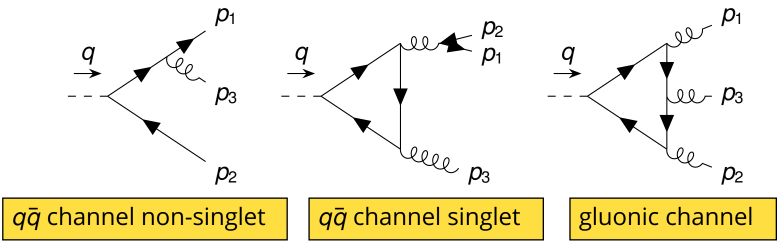

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  - $q\bar{q}$ non-singlet has to agree between V and AV (check)
  - 2L singlet cancels in the sum over degenerate quark pairs (new result for massless b and massive t in the loop)
- 3 loops: As $N \sim N_f$, dominant colour factors are N^3, $N^2 N_f$, NN_f^2 and N_f^3 (only planar diagrams)

Tensor basis

Traditional approach in CDR

List all combinations of p_i^μ and $g^{\mu\nu}$

↓ apply symmetries, fix gauge

Compute scalar form factors wrt this basis:
$A^\mu = \sum_i F_i T_i^\mu$

↓ express tensor basis in spin. hel. formalism

Construct helicity amplitudes as combinations of form factors $\times$ spinor structures (fix $d = 4$)

Use this a priori?

Realization:
- combinations of FFs do not contribute ($N_{FFs} > N_{helamps}$)
- evanescent terms from d-dimensional Dirac algebra do not appear in hel. amps

Tensor basis

in t' Hooft-Veltman scheme [Peraro, Tancredi 2019]

1. Consider basis $q_i = \{p_1, p_2, p_3, v_A\}$ with $v_A^\mu = \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma}$

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Levi-Civita symbol

- v_A and γ_5 in Larin scheme: $\gamma^\mu \gamma^5 \to \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} \gamma_\nu \gamma_\rho \gamma_\sigma$ contain the Levi-Civita symbol $\epsilon^{\mu\nu\rho\sigma}$
- products of more than two ϵ ambiguous
- Trade $g^{\mu\nu} = \sum_{i,j=1}^{3} b_{ij} \cancel{p_i^\mu p_j^\nu} + b v_A^\mu v_A^\nu$

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Result

Minimal basis with at most one ϵ, no mixing between odd and even structures

- $6 + 6$ even+odd structures for $Vgq\bar{q}$: $2_g \times 2_{q\bar{q}} \times 3_V$
- $12 + 12$ even+odd structures for $Vggg$: $2_g^3 \times 3_V$

Example tensor basis

Vggg

$$\begin{aligned}
A^{\text{vector}}_{\mu_1\mu_2\mu_3\mu} = &+F_1\, p_3^{\mu_1} p_1^{\mu_2} p_2^{\mu_3} p_1^{\mu} + F_2\, p_3^{\mu_1} p_1^{\mu_2} p_2^{\mu_3} p_2^{\mu} + F_3\, p_3^{\mu_1} p_1^{\mu_2} g^{\mu_3\mu} \\
&+ F_4\, p_3^{\mu_1} p_2^{\mu_3} g^{\mu_2\mu} + F_5\, p_3^{\mu_1} p_1^{\mu} g^{\mu_2\mu_3} + F_6\, p_3^{\mu_1} p_2^{\mu} g^{\mu_2\mu_3} \\
&+ F_7\, p_1^{\mu_2} p_2^{\mu_3} g^{\mu_1\mu} + F_8\, p_1^{\mu_2} p_1^{\mu} g^{\mu_1\mu_3} + F_9\, p_1^{\mu_2} p_2^{\mu} g^{\mu_1\mu_3} \\
&+ F_{10}\, p_2^{\mu_3} p_1^{\mu} g^{\mu_1\mu_2} + F_{11}\, p_2^{\mu_3} p_2^{\mu} g^{\mu_1\mu_2} + F_{12}\, \mathcal{T}_E^{\mu_1\mu_2\mu_3\mu} \\
A^{\text{axial}}_{\mu_1\mu_2\mu_3\mu} = &+G_1\, p_3^{\mu_1} p_1^{\mu_2} p_2^{\mu_3} v_A^{\mu} + G_2\, p_3^{\mu_1} p_1^{\mu_2} v_A^{\mu_3} p_1^{\mu} + G_3\, p_3^{\mu_1} p_1^{\mu_2} v_A^{\mu_3} p_2^{\mu} \\
&+ G_4\, p_3^{\mu_1} v_A^{\mu_2} p_2^{\mu_3} p_1^{\mu} + G_5\, p_3^{\mu_1} v_A^{\mu_2} p_2^{\mu_3} p_2^{\mu} + G_6\, p_3^{\mu_1} \mathcal{T}_O^{\mu_2\mu_3\mu} \\
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\end{aligned}$$

with

$$\begin{aligned}
\mathcal{T}_O^{\mu_1\mu_2\mu_3} &\equiv g^{\mu_1\mu_2} v_A^{\mu_3} + g^{\mu_1\mu_3} v_A^{\mu_2} + g^{\mu_2\mu_3} v_A^{\mu_1} \\
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cf. [Gehrmann, Tancredi, Weihs 2013] where 14 even structures were used

IBP reduction

- Obtain integrand with QGRAF [Nogueira 1993] and FORM [Vermaseren 2000]
- Contribution of each diagram to each helicity amp. is a sum of scalar integrals

	loops	props (+ISPs)	IR top sectors	coupling	integrals	rank	masters
gg	2	7(+2)	1PL + 2NPL	vector	11.7k	4	24
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	3 (LC)	10(+5)	3PL	vector	97k	5	291
	3 (full)		3PL+26NPL		$\sim 1M$	6	???

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- Diagram matching/shifts with Reduze 2 [von Manteuffel,Studerus 2012] and reduction with KIRA 2 [Klappert, Lange, Maierhöfer, Usovitsch 2021]

Master integrals

extension of 2L integrals from ϵ^0 [Gehrmann, Remiddi 2001] to ϵ^2:

- new canonical basis [Henn 2013] using candidates from DLogBasis [Henn, Mistlberger, Smirnov, Wasser 2020] and heuristic study of leading singularities:

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$\longrightarrow$ solution possible to all orders in terms of GHPLs with alphabet $\{y, z, 1-y, 1-z, 1-y-z, y+z\}$

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Boundary conditions

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3. basis is UV finite $\rightarrow$ even at thresholds allow only IR-type singularity

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ongoing work at 3L:

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- completion of planar topologies [Canko, Syrrakos 2021]

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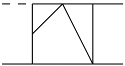
Main findings

1. 2 new letters $\{1-2y+y^2-z, y-y^2-z\}$
2. adjacency conjecture of letters $1-y$, $1-z$ and $y+z$ violated [Dixon, McLeod, Wilhelm 2020]



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- Ultimate renormalization check: Verify Ward identities
  Example for $q\bar{q}$ channel pure-singlet:

$$S_\epsilon^{-2}\bar{G}_i^{(2),p} \boxed{- \left(\frac{3\beta_0}{2\epsilon}\right) S_\epsilon^{-1}} \bar{G}_i^{(1),p} \boxed{+ C_F T_R \left(\frac{3}{2\epsilon} + \frac{3}{4}\right)} \bar{G}_i^{(0),n} \boxed{-Z^a} \bar{G}_i^{(1),p}$$

α_s

n/p mixing in Larin

fix to recover correct anomaly

Ward identities

Finite Z_a determined by requiring that the renormalized AV current satisfies the axial anomaly relation

one order lower

$$\partial_\mu \langle J_5^\mu(x) \mathcal{O}(x_1, \ldots, x_n) \rangle = \frac{\alpha_s}{8\pi} \langle G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}(x) \mathcal{O}(x_1, \ldots, x_n) \rangle$$

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Example: (iff $Z^a = 2C_F$)

$$q^\mu A^{a,ext,(1L)}_\mu = + \frac{4(y-z)}{q^4 yz(1-y-z)} \bar{u}(p_2) p\!\!\!/_3 u(p_1) v_A^\nu - \frac{4(y+z)}{q^4 y(1-y-z)^2} \bar{u}(p_2) v\!\!\!/_A u(p_1) p_1^\nu = A^{\text{tree}}_{\text{pseudo}}$$

Results and checks

Results at *xjetamps.hepforge.org*

- **Ready to use:**
  - finite remainders IR subtracted in SCET
  - analytically continued to all production channels: $q\bar{q} \to Vg$, $qg \to Vq$, $\bar{q}g \to V\bar{q}$, and $gg \to Vg$
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All results checked at 1L against OpenLoops [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 2019] and 2L amplitudes up to ϵ^0 with the literature

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