Determining where PBHs form

New horizons in primordial black holes physics

Sam Young, University of Leiden, 21.06.2023

What does AI think a PBH looks like?

Talk overview

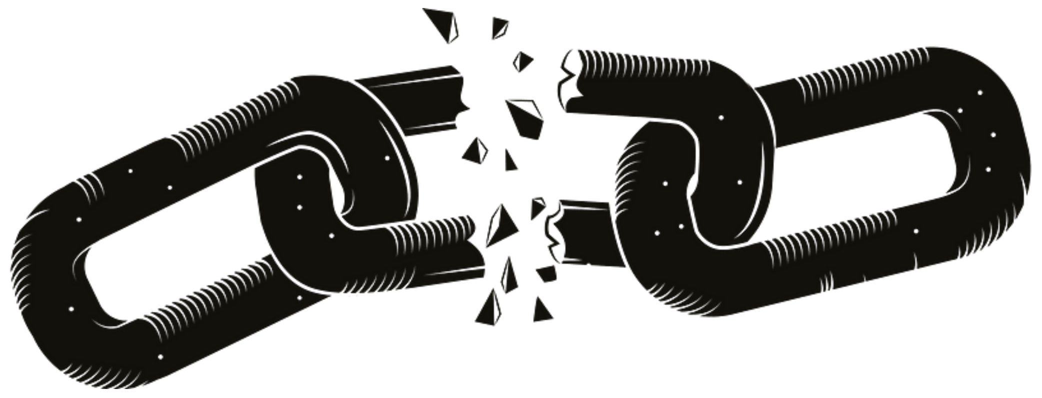
- Sorry not to be there in person
- Short summary of several years research, including ongoing work

SY, Musco, Byrnes [1904.00984]
SY [1905.01230]
Gow, Byrnes, Cole, SY [2008.03289]
SY [2201.13345]

Predicting observables for upcoming surveys →

Generation of initial conditions (e.g. inflation) | Formation of PBHs (From collapse of density perturbations) | Observations/ constraints

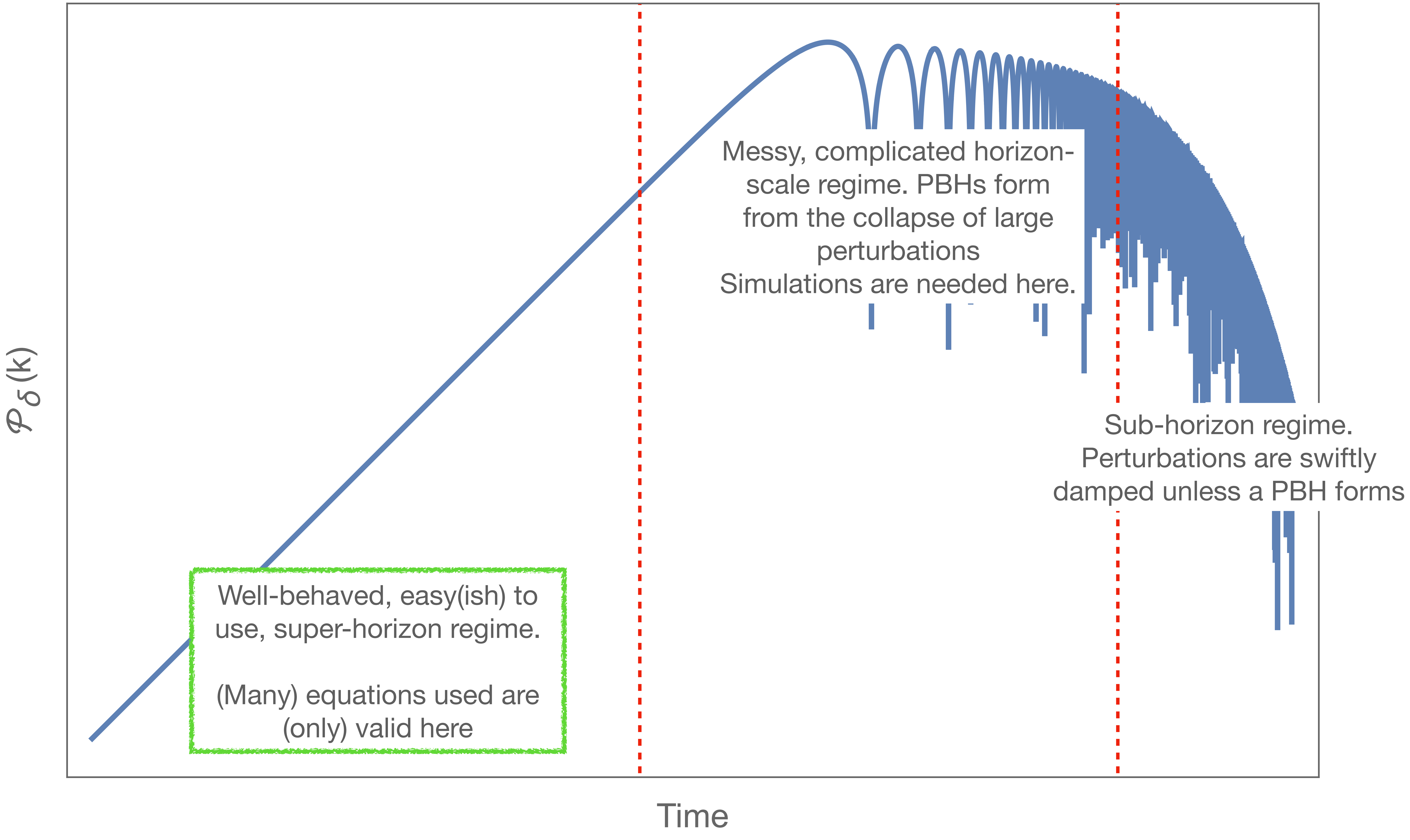
← Using existing observations to constrain cosmological (inflation) models



What if we get the formation criteria wrong?

1. What variables can/should we use to describe perturbations for PBH calculations?
2. What is the correct smoothing function, and how can we calculate it from simulations?

Working in the super-horizon regime



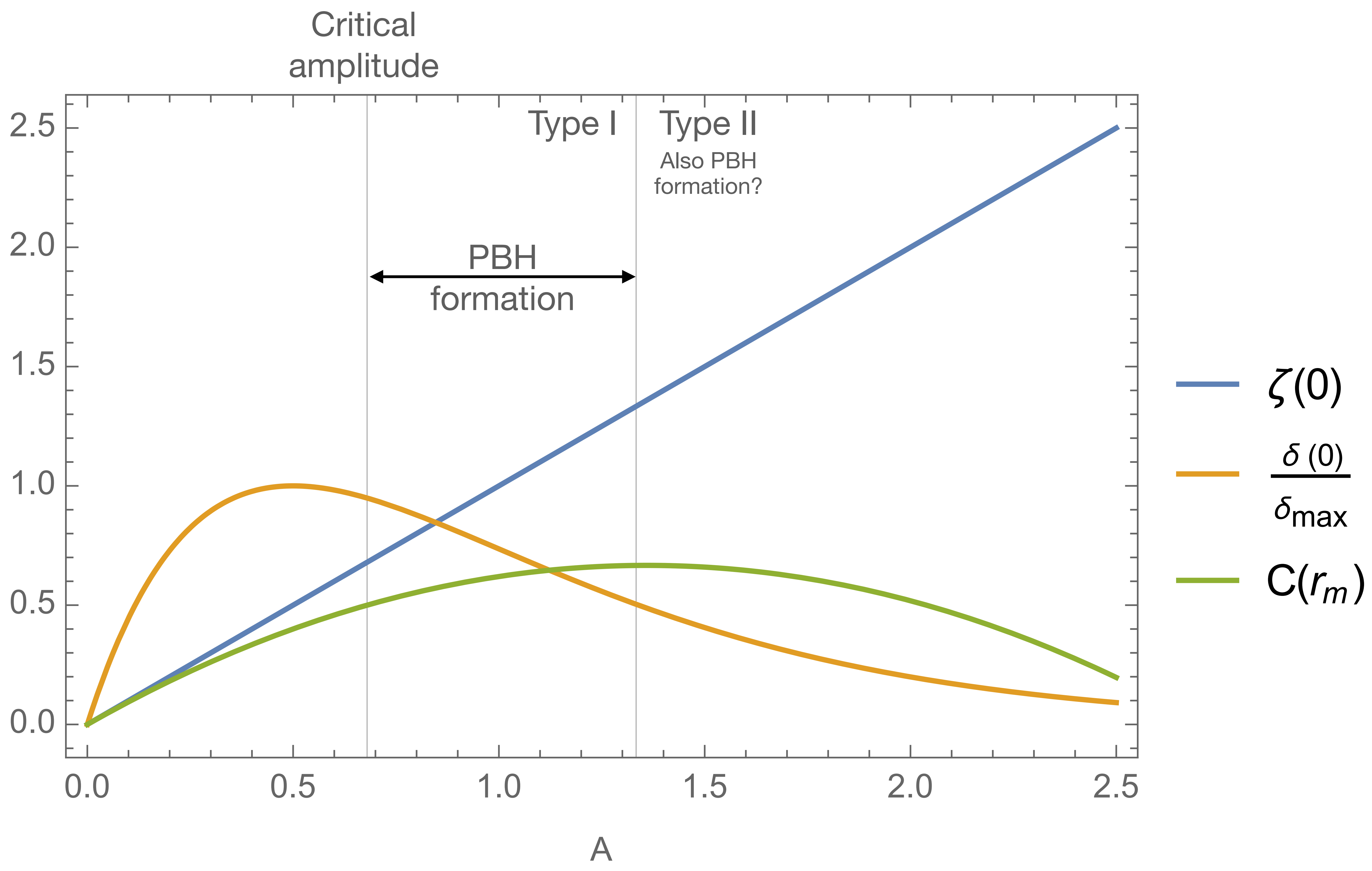
Why does the variable matter?

Lets consider an individual perturbation

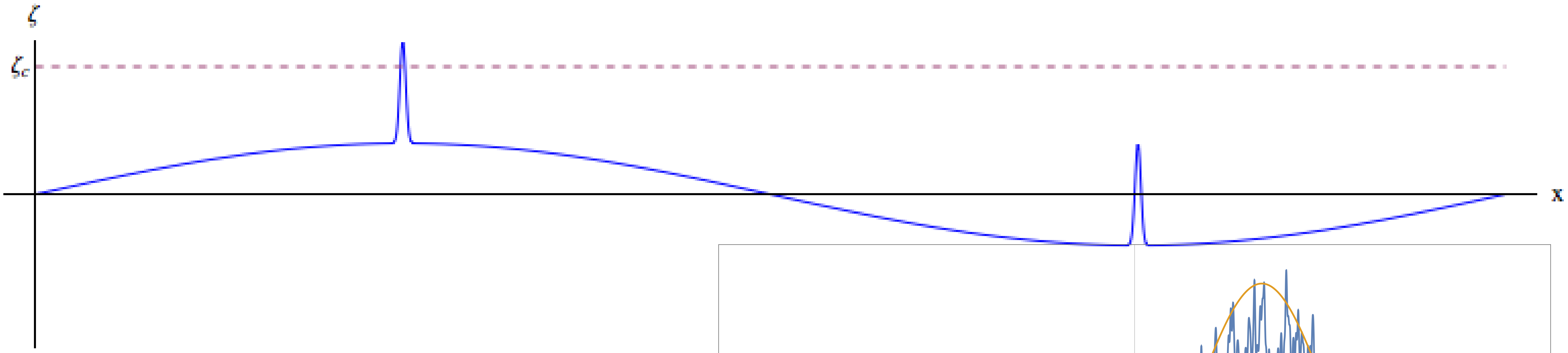
$$\zeta = A \exp\left(-\frac{r^2}{r_m^2}\right)$$

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} = -\frac{2(1+w)}{5+3w} \exp\left(-2\zeta(r)\right) \left(\zeta''(r) + \frac{2}{r}\zeta'(r) + \frac{1}{2}\zeta'(r)^2\right)$$

$$C = -\frac{3(1+w)}{5+3w} r\zeta'(r) + \left(2 + r\zeta'(r)\right)$$

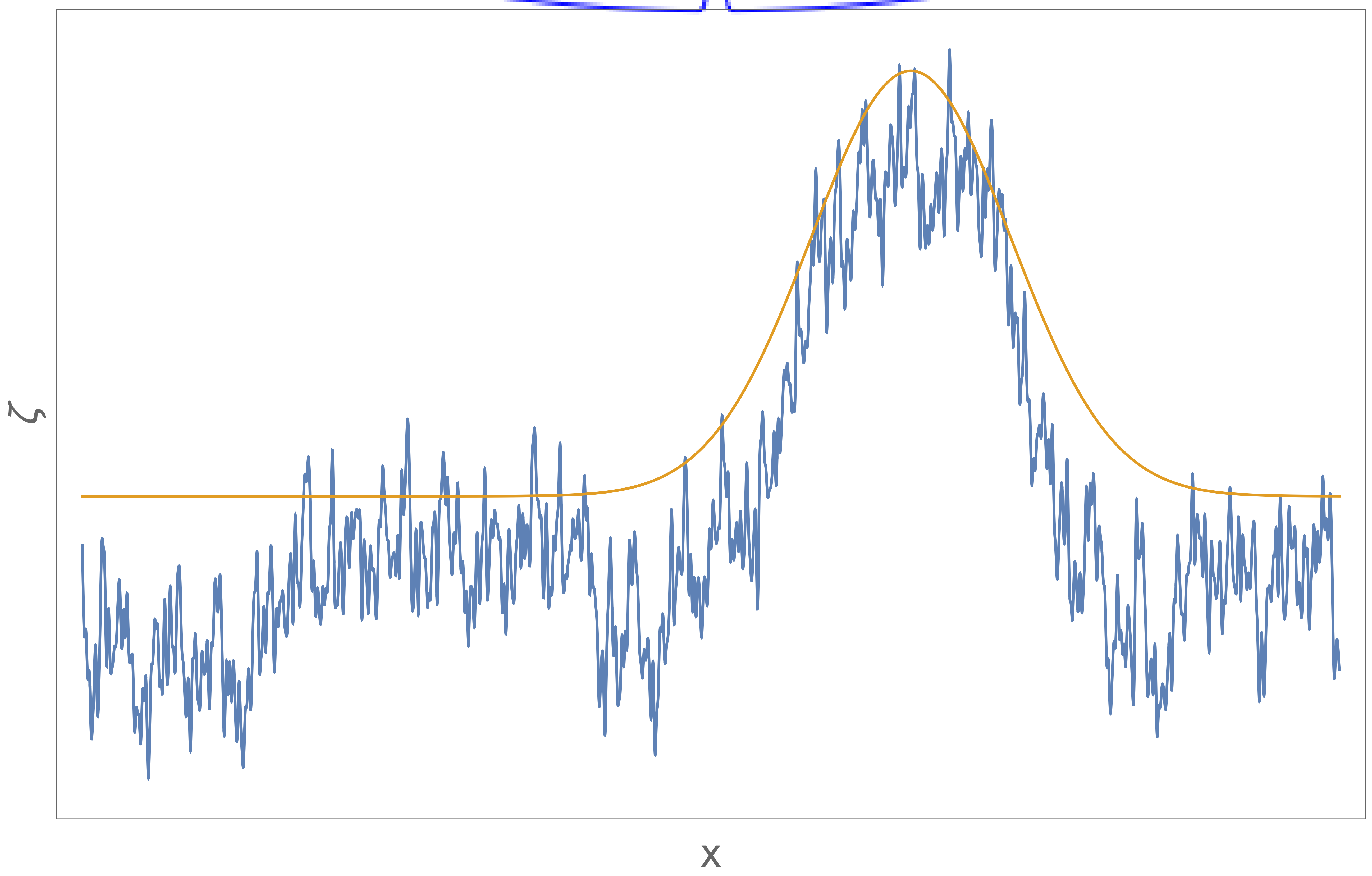


The curvature perturbation ζ



$$\mathrm{d}S^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{e}^{2\zeta}\mathrm{d}\mathbf{X}^2$$

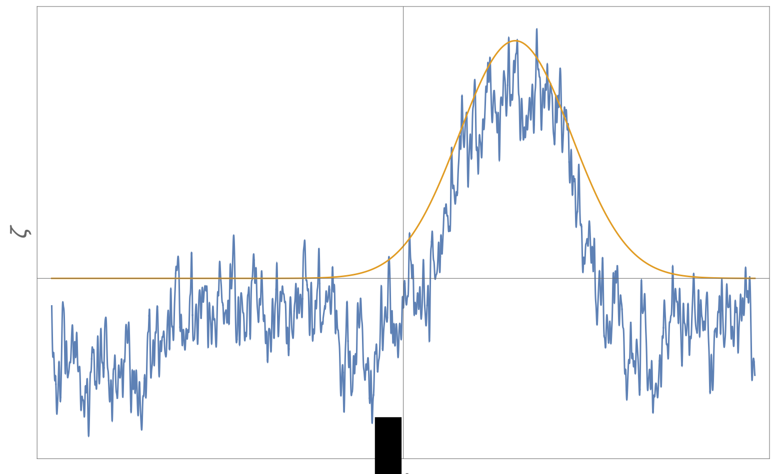
- Can be useful for narrow power spectra
- Simple to use
  - Inflationary models typically predict $\mathscr{P}_\zeta(k)$ and its statistics
- Suffers from background contamination/environmental effects



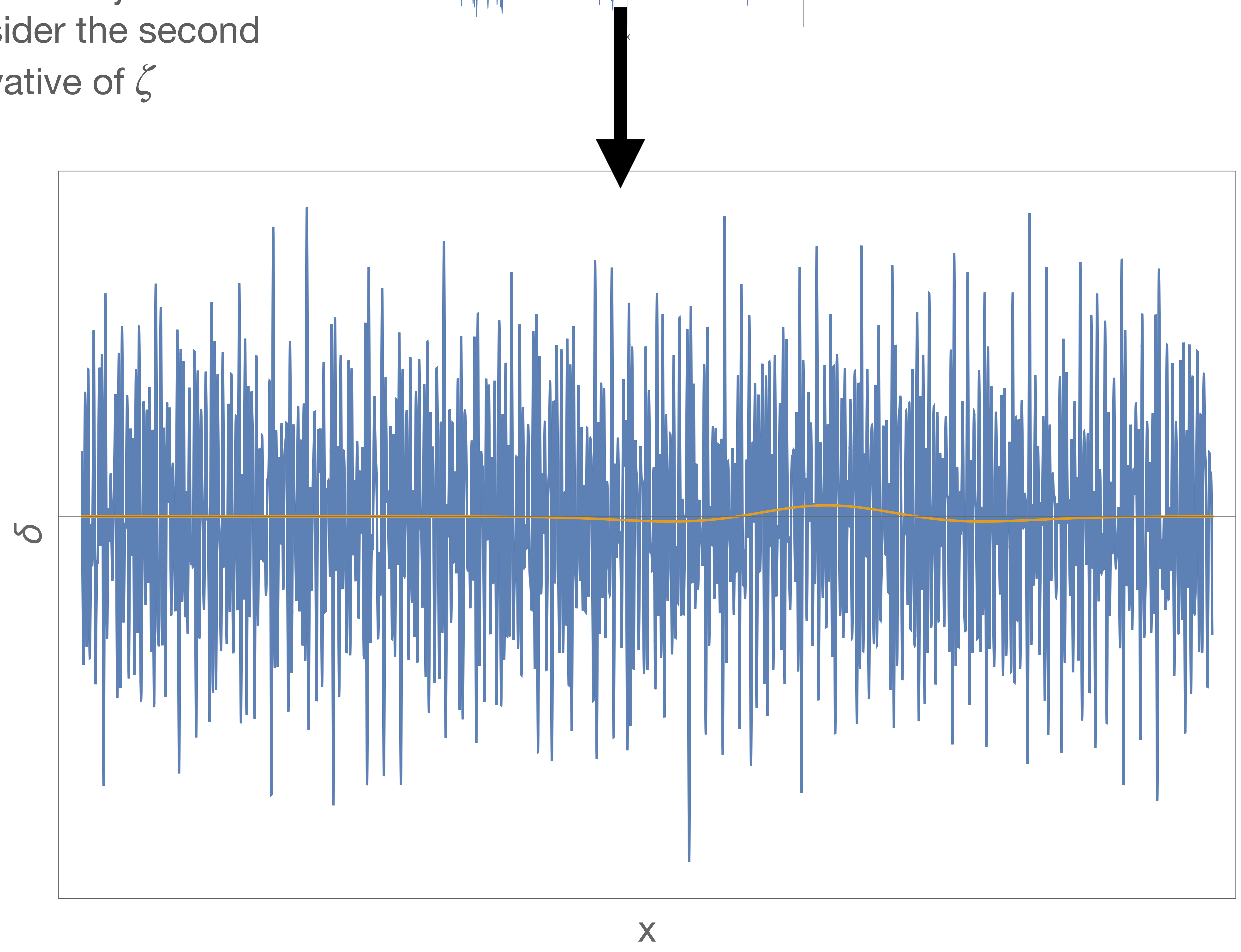
The density contrast δ

Linear order

Or you can just consider the second derivative of ζ



$$\delta = -\frac{2(1+\omega)}{5+3\omega}\frac{1}{(aH)^2}\nabla^2\zeta$$



- Dominated by small-scale modes
- Time-dependent

  →Typically determined at horizon entry with linear transfer function

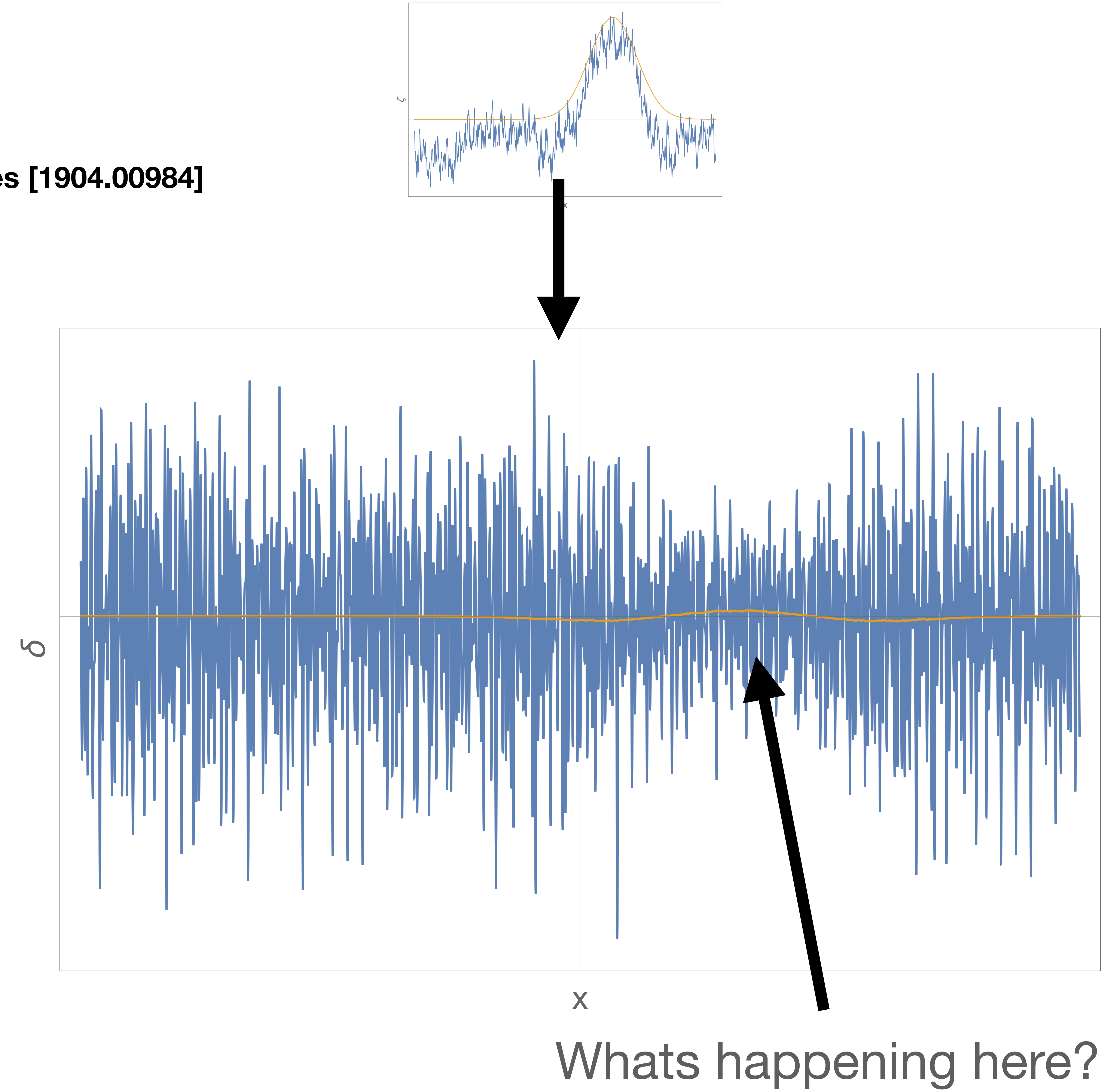
- Again, not very useful except for narrow power spectra
- Non-linear corrections?

The density contrast

Non-linear corrections, SY, Musco, Byrnes [1904.00984]

$$\delta = -\frac{2(1+\omega)}{5+3\omega}\frac{1}{(aH)^2}e^{-2\zeta}\left(\nabla^2\zeta + \frac{1}{2}\left(\bar{\nabla}\zeta\right)^2\right)$$

- Still dominated by small-scale modes
- Contamination by background perturbations
- Doesn't increase monotonically with ζ
- Complicated statistics (which are often misleading anyway)



The compaction function

Smooth the problems away?

The time-dependance of these cancels out

This can be thought of (almost) as the rescaled average density of a region

$$C = 2\frac{\delta M}{R} = R^2H^2\int \mathrm{d}^3\mathbf{y}\,\delta(\mathbf{x}-\mathbf{y})W_{\mathrm{TH}}(\mathbf{y})$$

- Assuming spherical symmetry, this can be calculated from ζ:

$$C = -\frac{2}{a\mathrm{e}^{\zeta(R)}R}\frac{3H^2}{8\pi}\int_0^R \mathrm{d}\left(a\mathrm{e}^{\zeta(r)}r\right)\left[4\pi\left(a\mathrm{e}^{\zeta(r)}r\right)^2\right]\times\frac{4}{9}\left(\frac{1}{aH}\right)^2\mathrm{e}^{-2\zeta(r)}\left(\zeta''(r)+\frac{2}{r}\zeta'(r)+\frac{1}{2}(\zeta'(\mathbf{r}))^2\right)$$

- This has a simple solution

$$C = -\frac{2}{3}r\zeta'(r)\big(2+r\zeta'(r)\big) = C_1 - \frac{3}{8}C_1^2$$

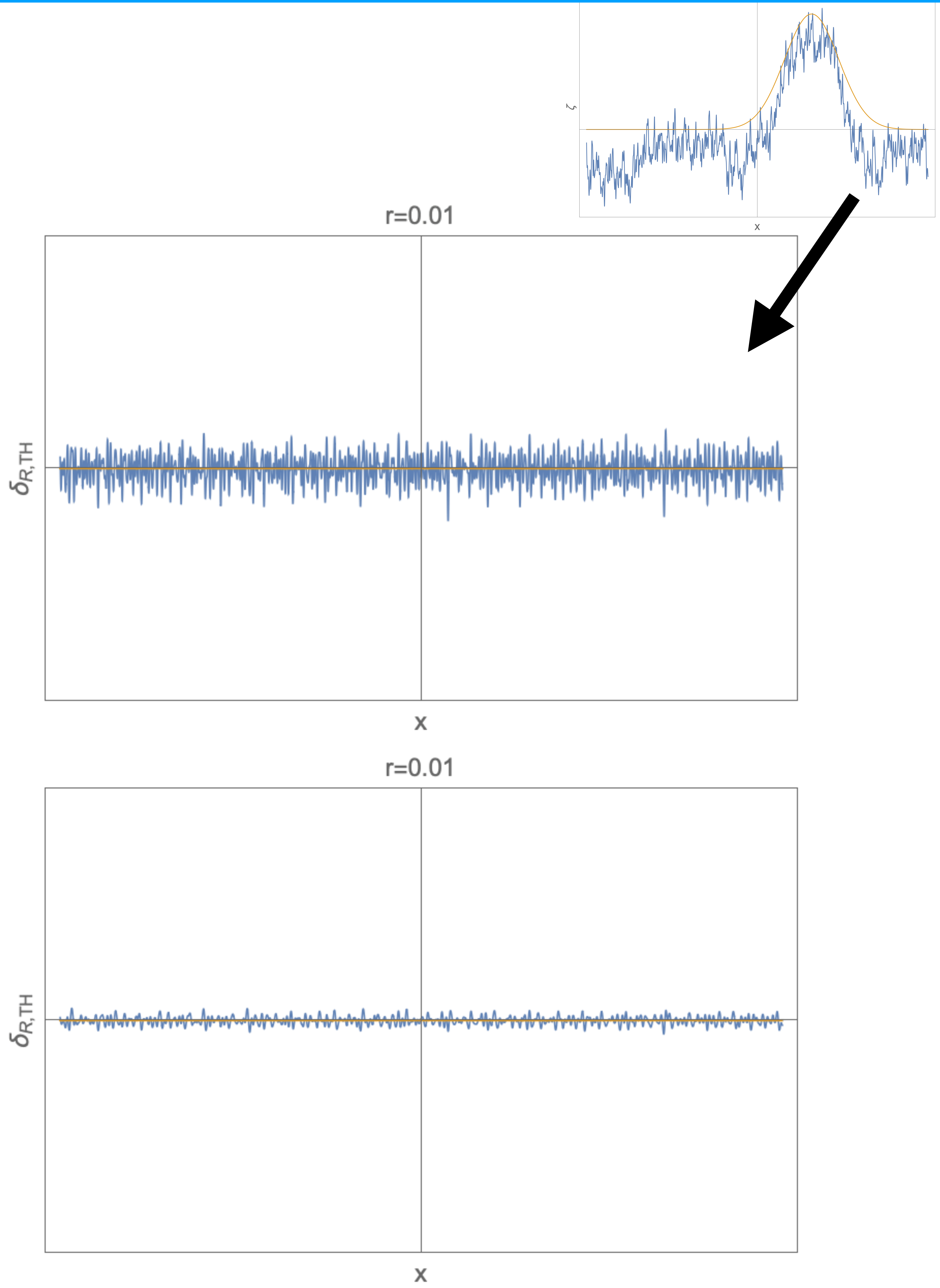
This is very nice if we know the (non-Gaussian) statistics of ζ.
SY [2201.13345]
Gow et al [2211.08438]
Ferrante et al [2211.01728]

- But we don’t have to use a top-hat window function

The compaction C

Different smoothing functions

- Top-hat: $W_{TH} = \frac{1}{V}\theta_H(\mathbf{x} - R)$
    - Analytic formula: $C = -\frac{3(1+w)}{5+3w} r\zeta' + \left(2 + r\zeta'\right)$
    - (But this may not be valid for broad power spectra)
        - SY [2201.13345]
    - Smoothing is not very efficient (we'll see why later)
    - Problems for (close-to) scale invariant or power-law spectra, $\sigma_0^2 \to \infty$
- Gaussian: $W_G = \frac{1}{V}\exp\left(-\frac{x^2}{2R^2}\right)$
    - Very efficient smoothing
    - Suitable for broad power spectra
    - No analytic relation



Context: Options for calculating the abundance

(Assuming Gaussian statistics)

- $\nu = C/\sigma_0$
- Press-Schechter approach/threshold statistics
  - $n \sim \delta_{\mathrm{D}}(\nu - \bar{\nu})$
  - $\bar{n} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\nu^2}{2}\right)$

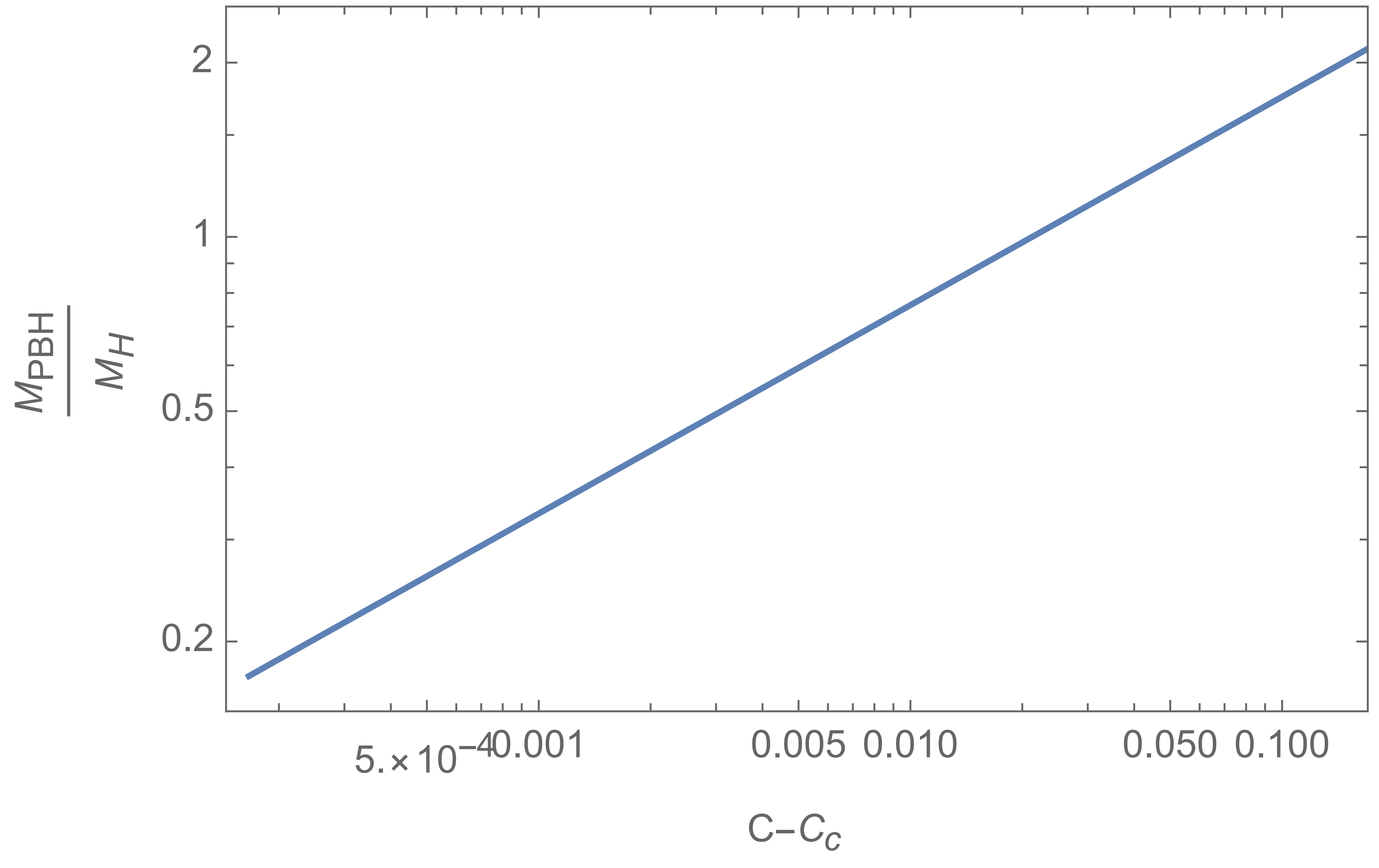
Fraction of the universe where the density is in the range $\bar{\nu} \to \bar{\nu} + \mathrm{d}\bar{\nu}$

- BBKS peaks theory
  - $n \sim \delta_{\mathrm{D}}^{(3)}(\eta_i)\theta_{\mathrm{H}}(\lambda_3)\delta_{\mathrm{D}}(\nu - \bar{\nu})$
  - $\bar{n} = \frac{1}{3^{3/2}(2\pi)^2}\frac{\sigma_1^3}{\sigma_0^3}\nu^3 \exp\left(-\frac{\nu^2}{2}\right)$

Number density of peaks with height in the range $\bar{\nu} \to \bar{\nu} + \mathrm{d}\bar{\nu}$ (high-peak limit)

- YM peaks theory (SY & Musso, 2001.06469, see also Germani & Sheth, 1912.07072)
  - $n \sim \delta_{\mathrm{D}}(\eta_0)\theta_{\mathrm{H}}(\zeta_{00})\delta_{\mathrm{D}}^{(3)}(\eta_i)\theta_{\mathrm{H}}(\lambda_3)\delta_{\mathrm{D}}(\nu - \bar{\nu})$
  - $\bar{n} = \frac{16\sqrt{2}}{3^{3/2}\pi^{5/2}}\frac{\sigma_{RR}\sigma_0^3}{\sigma_2\sigma_1^3 R^7\sqrt{1-\gamma_{0,2}^2}}\alpha\nu^4 \exp\left(-\frac{1+\frac{16\sigma_0^2}{R^4\sigma_2^2}-\frac{8\sigma_0\gamma_{0,2}}{R^2\sigma_2}}{1-\gamma_{0,2}^2}\frac{\nu^2}{2}\right)$

Number density of peaks with height in the range $\bar{\nu} \to \bar{\nu} + \mathrm{d}\bar{\nu}$ and scale $R \to R + \mathrm{d}R$ (high-peak limit)



$$M_{\mathrm{PBH}} = K M_{\mathrm{H}} \left(C - C_c\right)^{\gamma}$$

PBH mass depends on both the amplitude and scale of a perturbation

Exponential dependance on σ_0^2

Where does the smoothing function appear in the calculation?

- The abundance is exponentially sensitive to C_c and σ_0^2

$$\sigma_0^2 \sim \int_0^\infty \frac{\mathrm{d}k}{k} \mathscr{P}_\delta(k) \tilde{W}^2(k, R_s) T^2(k, R_H)$$

$\propto k^4 \mathscr{P}_\zeta$

~ 1 on superhorizon scales

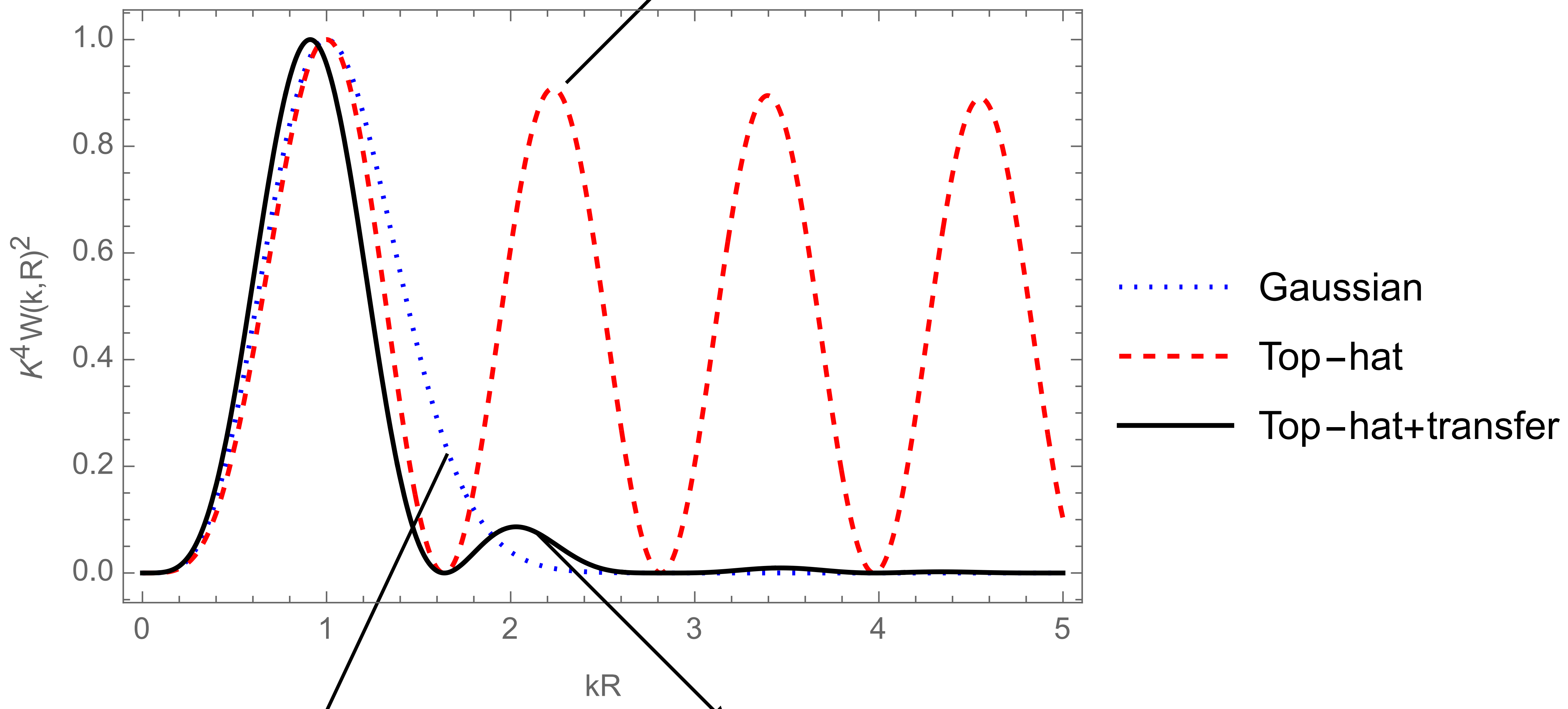
Fourier transform of smoothing function

- Linear transfer function:

$$T\left(k, R\right) = 3\frac{\sin(kR/\sqrt{3}) - (kR/\sqrt{3})\cos(kR/\sqrt{3})}{(kR/\sqrt{3})^3}$$

- Different $\tilde{W}(k, R_s)$ can change PBH abundance by orders of magnitude
    - (Actually, it can change it from $0 \to \infty$)
- Which one should we use?

Highly oscillatory, which matches the (linear) transfer function on sub-horizon scales



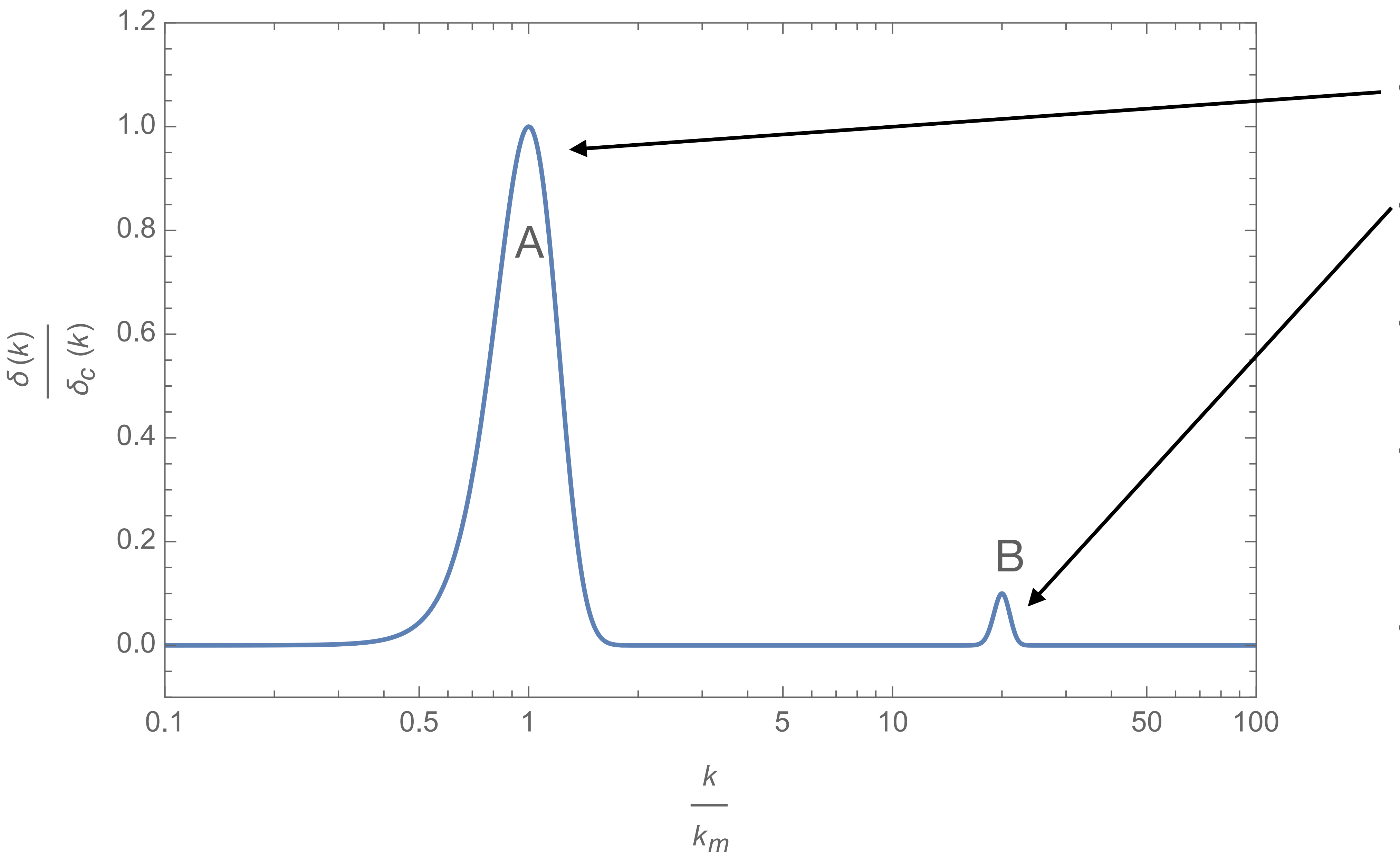
Efficient smoothing

Authors often set $R_s = R_H$ so that the transfer function provides additional smoothing. How valid is this?

But…which is the correct smoothing function?

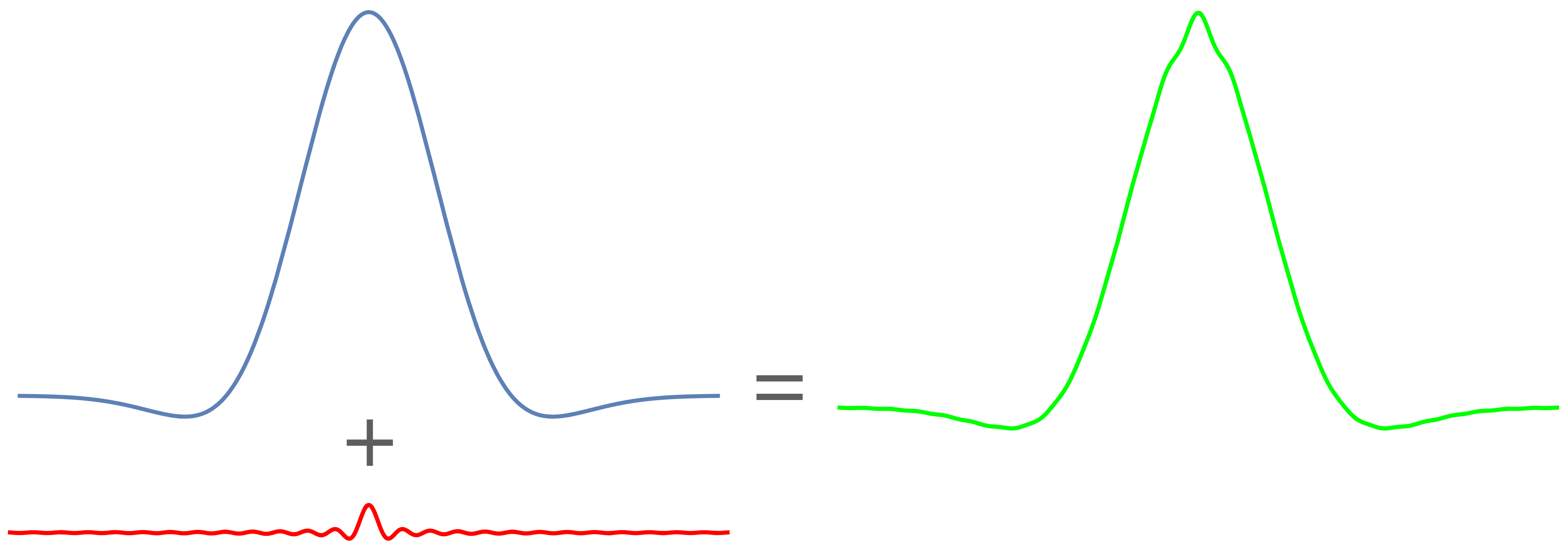
Lots of problems but no solutions (so far)

- Limit ourselves (first) to the smoothed density contract/compaction
- What does a smoothing function do? Which one should we use?
  - Smoothes out smaller (larger k) modes
  - This helps us isolate specific scales
  - But what does a smoothing function actually tell us?



- Lets consider a PBH forming at this scale
- How important is e.g. this scale?
- How important is e.g the height of A compared to the height of B?
- e.g. If $B = 0.1$, can A be 0.1 smaller and still form a PBH? Or can only be 0.01 smaller?
- The smoothing function assigns a weight to each Fourier mode

Simulation set-up



- Background profile: $\delta_{BG}(r) = A\left(1 - \frac{2}{3}\frac{r^2}{r_m^2}\right)\exp\left(-\frac{r^2}{r_m^2}\right)$
- Perturbation to the profile: $\delta_p(r) = Bk^2\frac{\sin(kr)}{kr}$
- Run simulations to find the critical value A_c for given $\{B, k\}$
  - Or $\bar{A}_c$ for $B = 0$
- Assume that there is a “correct” window function $W(x, R)$ (or $\tilde{W}(k, R)$), which makes the critical value constant:

$$\overbrace{\bar{C}_{w,c}}^{\propto \bar{A}} = \overbrace{C_{BG,w}}^{\propto A} + \overbrace{C_{p,w}}^{\propto Bk^2} = \text{constant}$$

$$\downarrow$$

$$c_1\bar{A}_c = c_1A_c + c_2Bk^2\tilde{W}(k, R)$$

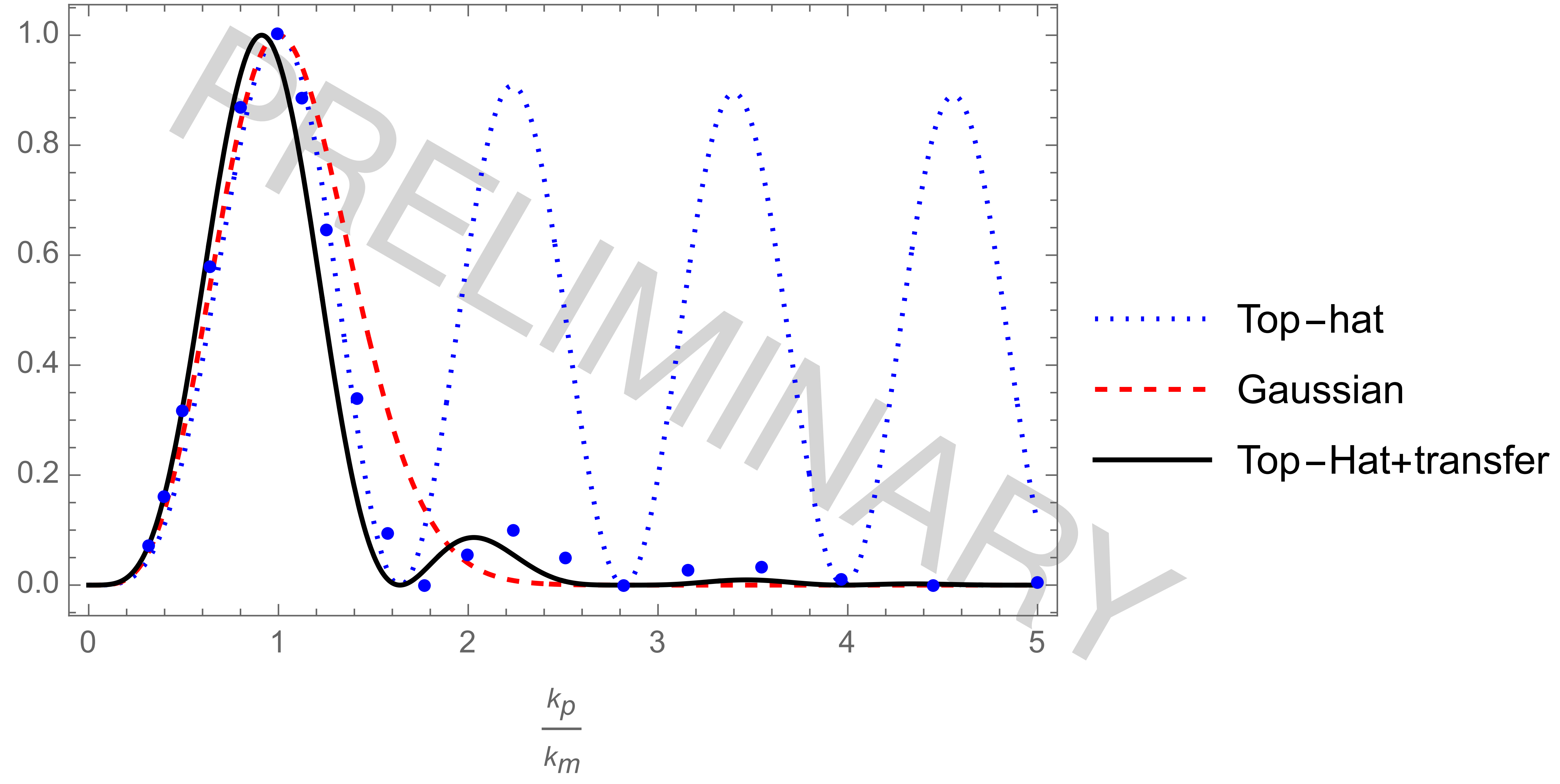
(Preliminary) results from simulations

Using Albert Escriva's code: [1907.13065]

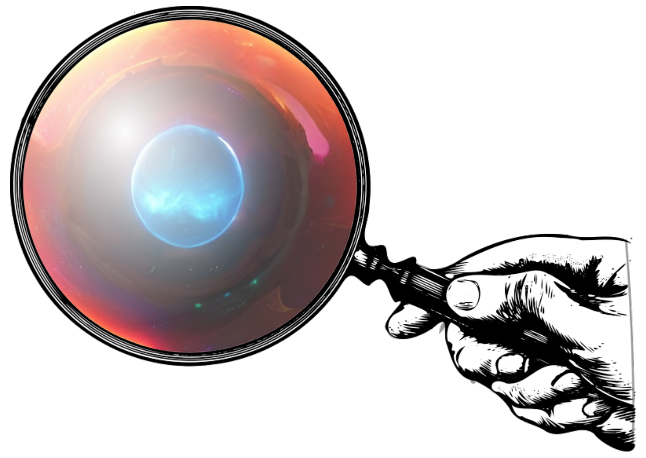
- Expected: damped oscillatory behaviour
- **Need more data and checks, but results coming soon**
- Moderately well fit by

$$k^2\tilde{W}(k,R) \propto k^2\tilde{W}_{TH}(k,R)T(k,R)$$

- Linear transfer function seems to (surprisingly?) be a good fit

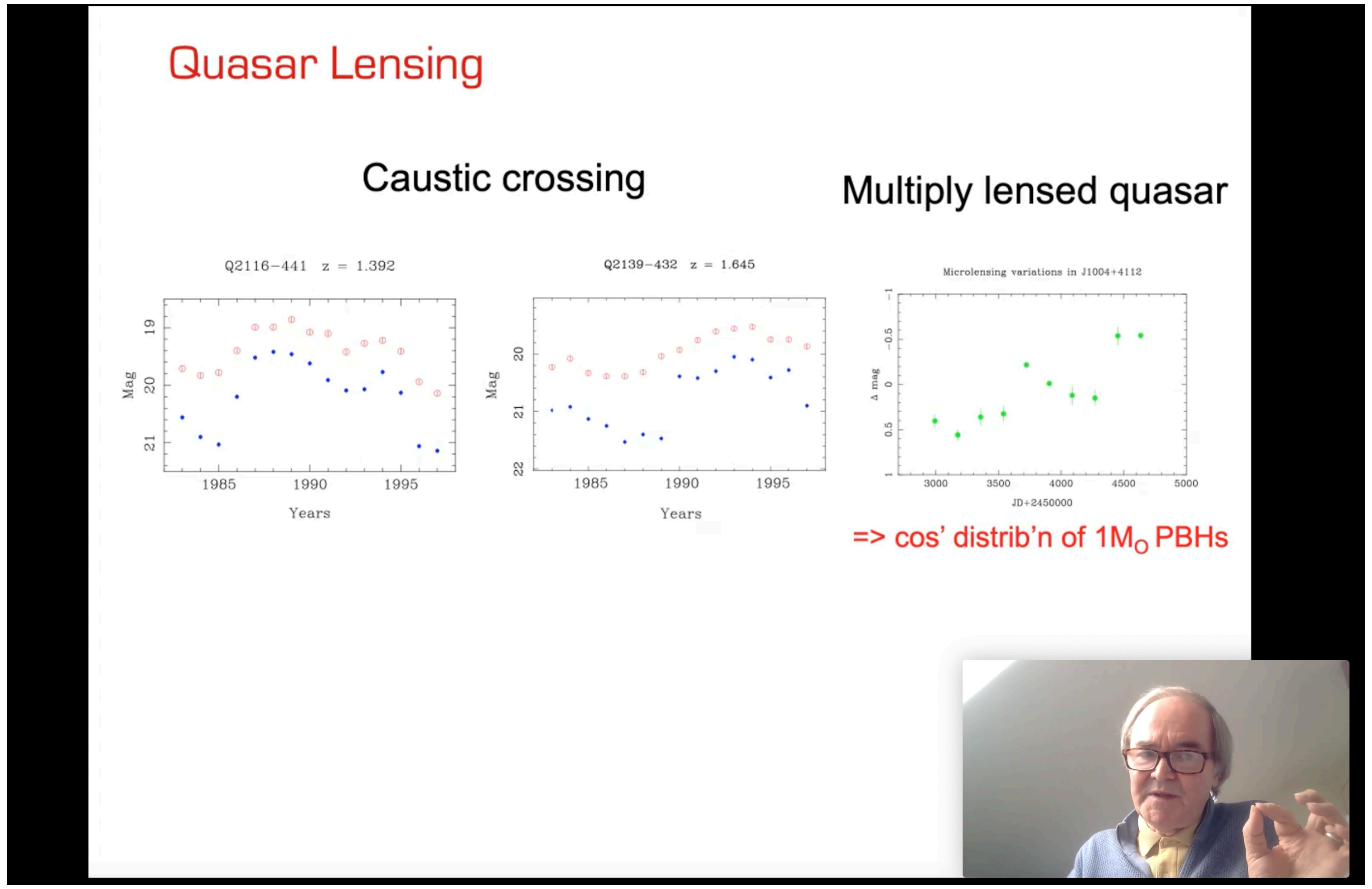


Zooming in on primordial black holes



Monthly online seminar series/journal club

- The meeting is held monthly on the first Monday of each month
- Next meeting in October
- Aims to nurture discussion and foster collaborations
- Please email me if you want to be added to the mailing list
    - Let me know if you would like to speak, or would like to nominate someone else
    - young@lorentz.leidenuniv.nl



Prof Bernard Carr, Inaugural meeting October 2022

Summary

- The smoothed density contrast/compaction should be used for PBH calculation
- Ongoing work to determine the appropriate window function
  - Also need to determine how this will affect the calculations, which is non-trivial
- Don't forget to email me about the monthly meeting!
  young@lorentz.leidenuniv.nl