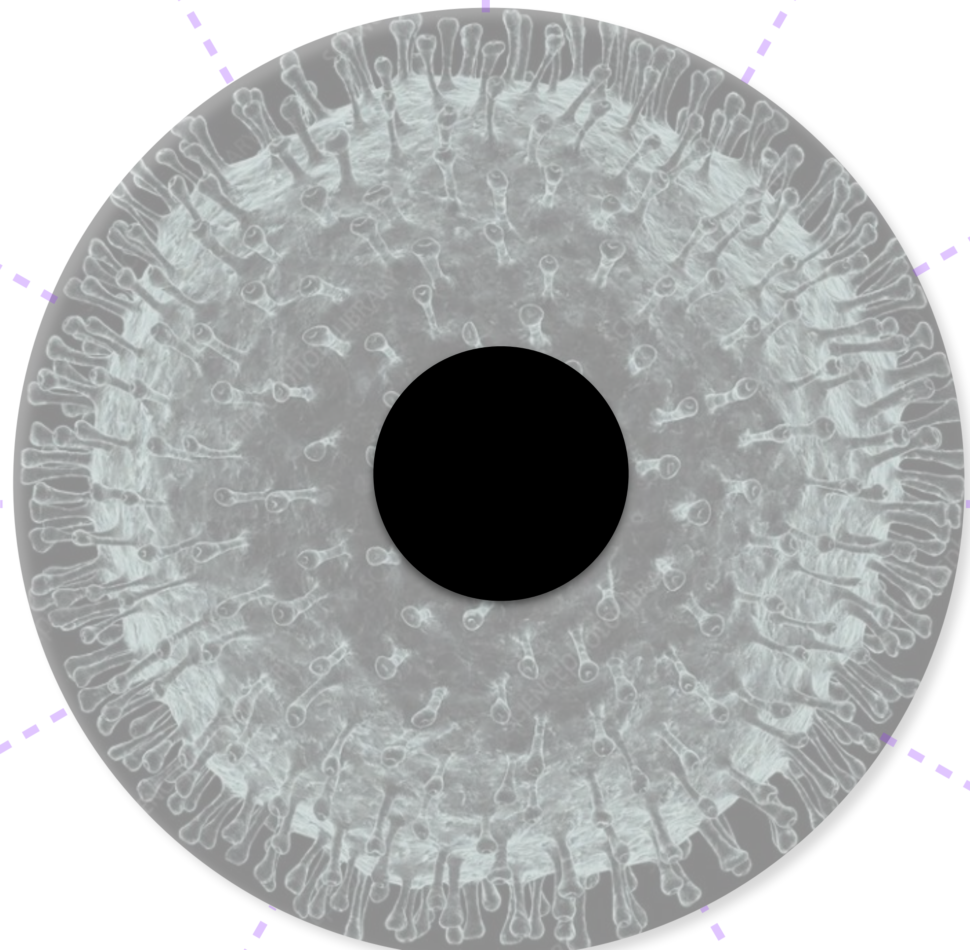
Primordial Extremal Black Holes



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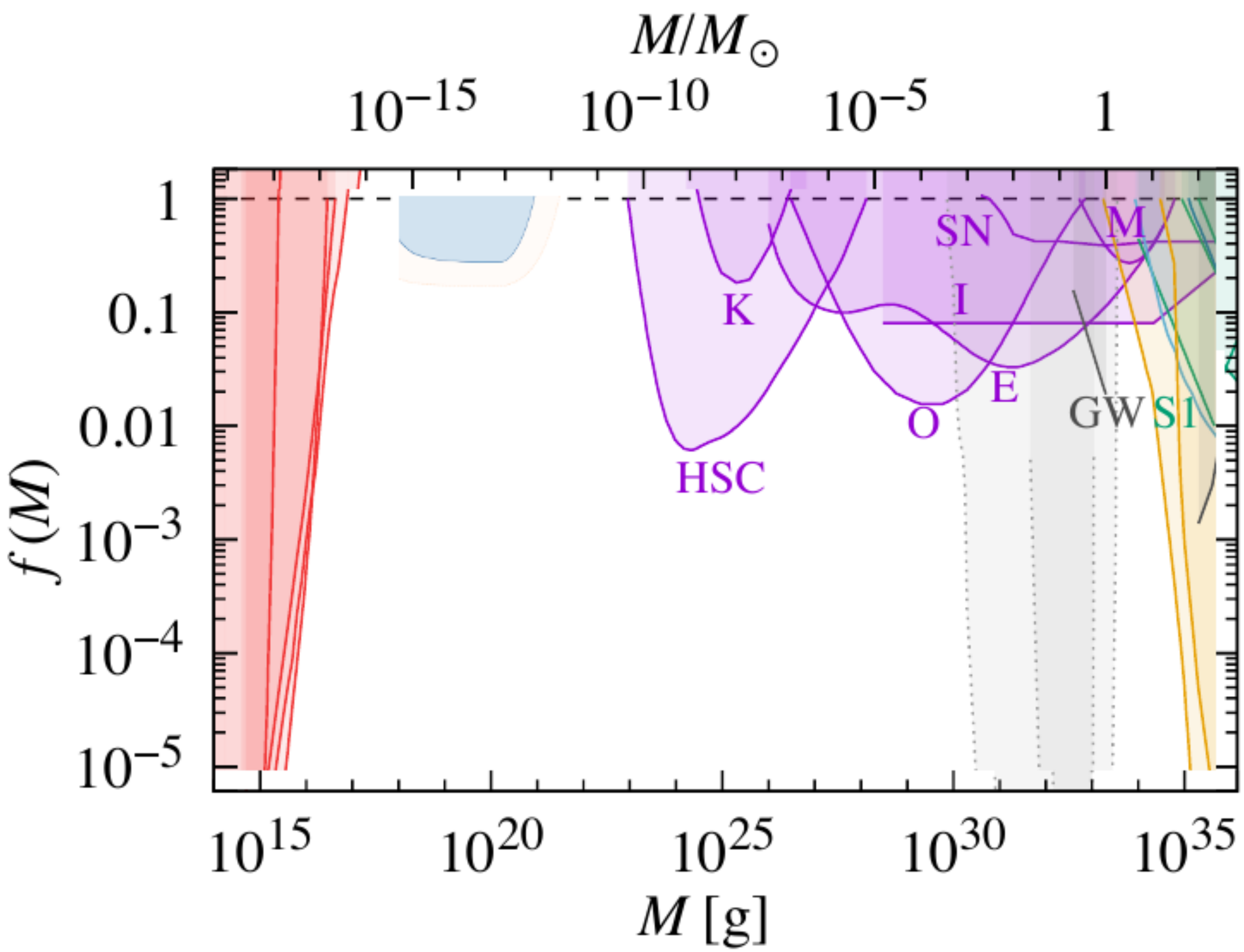
New Horizons in Primordial Black Hole Physics

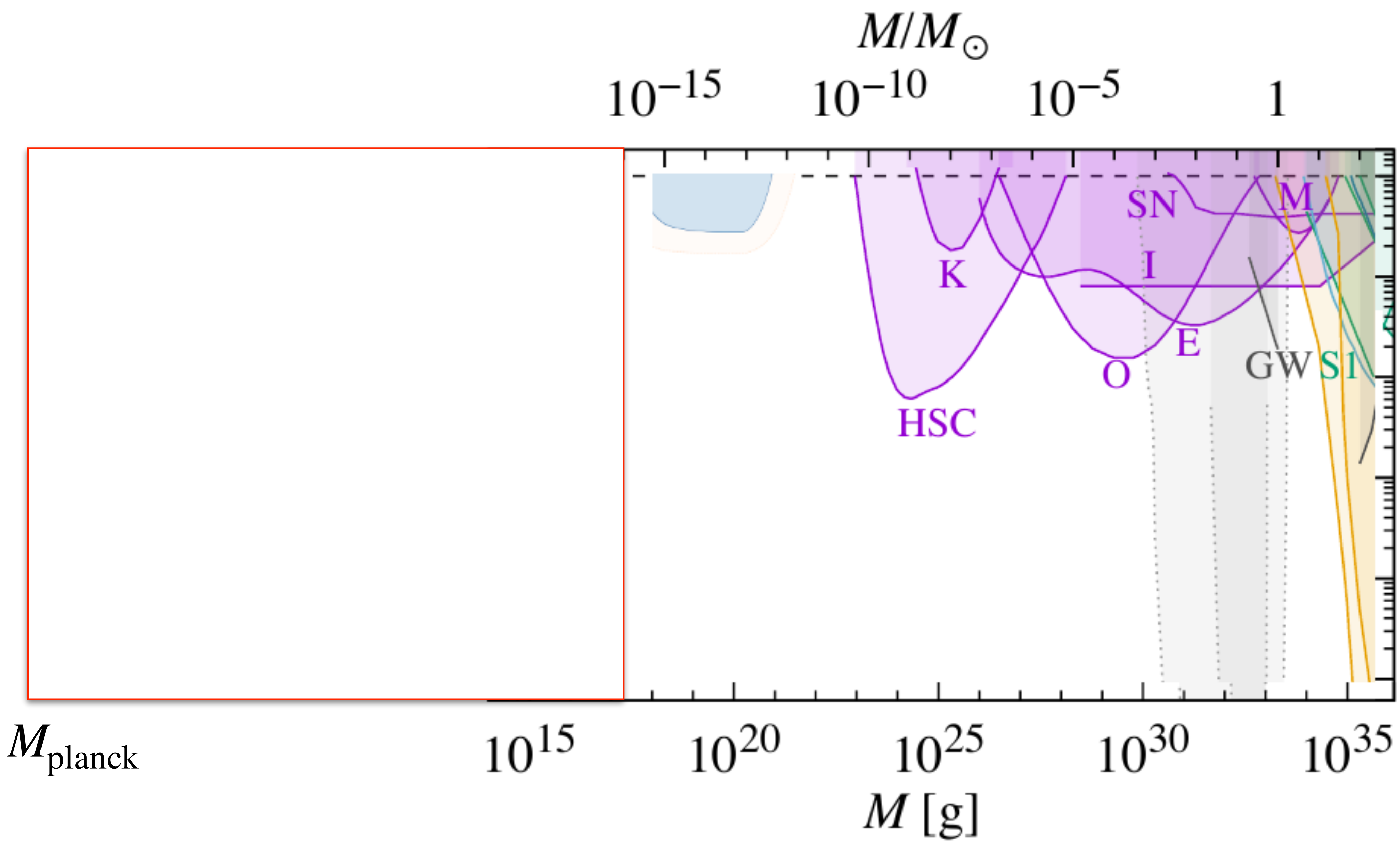
June 21, 2023

Motivation



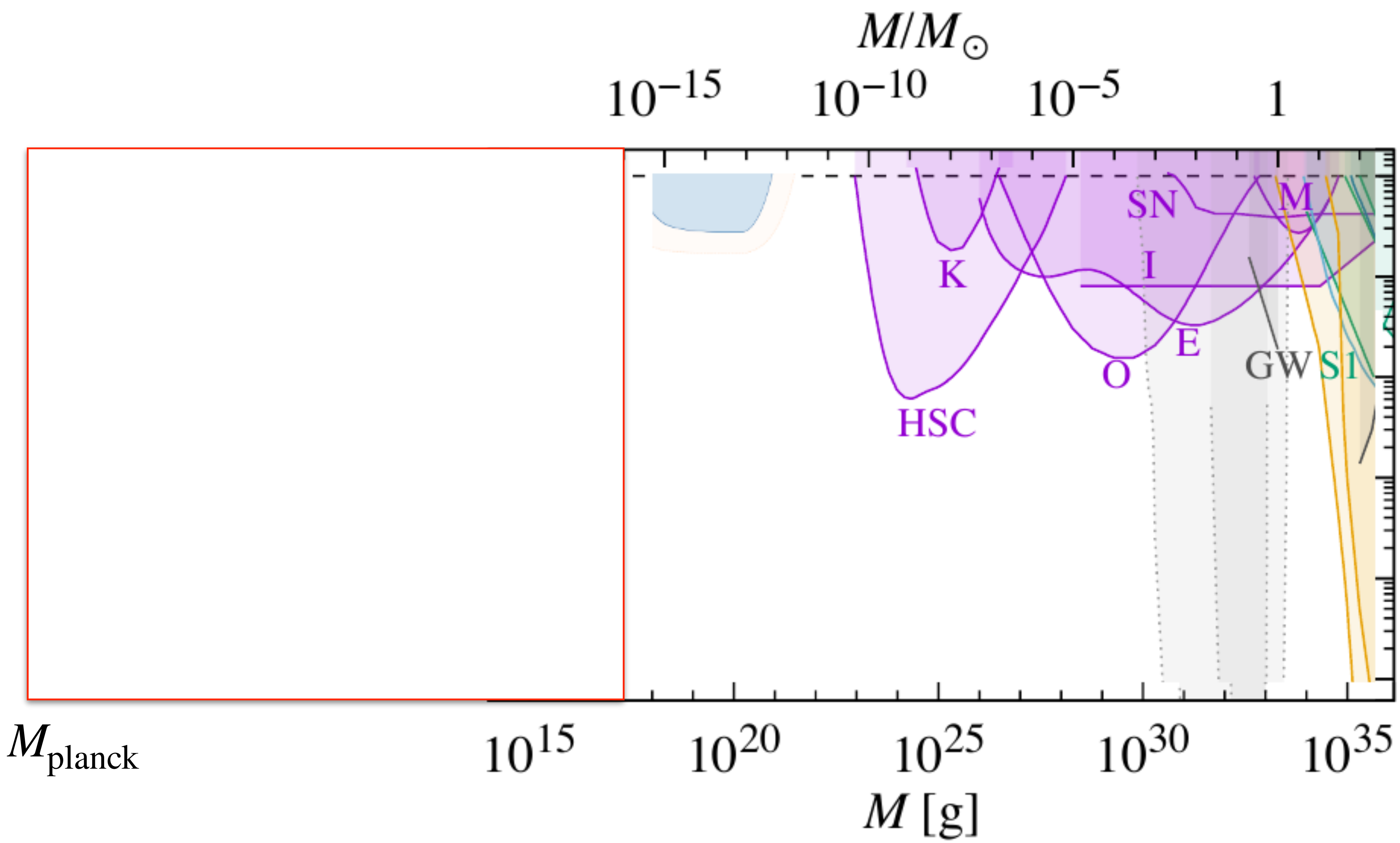
from slides of Peter Tinyakov here

Motivation



$M/M_\odot$
10^{-15}
10^{-10}
10^{-5}
1
SN
M
K
I
E
O
GW
S1
HSC
M_{planck}
10^{15}
10^{20}
10^{25}
10^{30}
10^{35}
M [g]

Motivation



Go beyond the Schwarzschild PBH

Reissner-Nordstrom Black Hole

- **Charged or Reissner-Nordstrom (RN) black hole**

$$ds^2 = -B_{\rm RN}(r)dt^2 + B_{\rm RN}(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$B_{\rm RN}(r) = 1 - \frac{2\,G\,M}{r} + \frac{G\,(Q_{\rm E}^2\,e^2 + Q_{\rm M}^2\,h^2)}{4\pi r^2}$$

- **The outer horizon radius is**

$$r_+ = \frac{\left(M_{\rm BH} + \sqrt{M_{\rm BH}^2 - M_{\rm eBH}^2}\right)}{M_{\rm pl}^2} \qquad M_{\rm eBH} = \frac{\sqrt{Q_{\rm E}^2 e^2 + Q_{\rm M}^2 h^2}}{\sqrt{4\pi}} M_{\rm pl}$$

- **Temperature is suppressed when close to the eBH state**

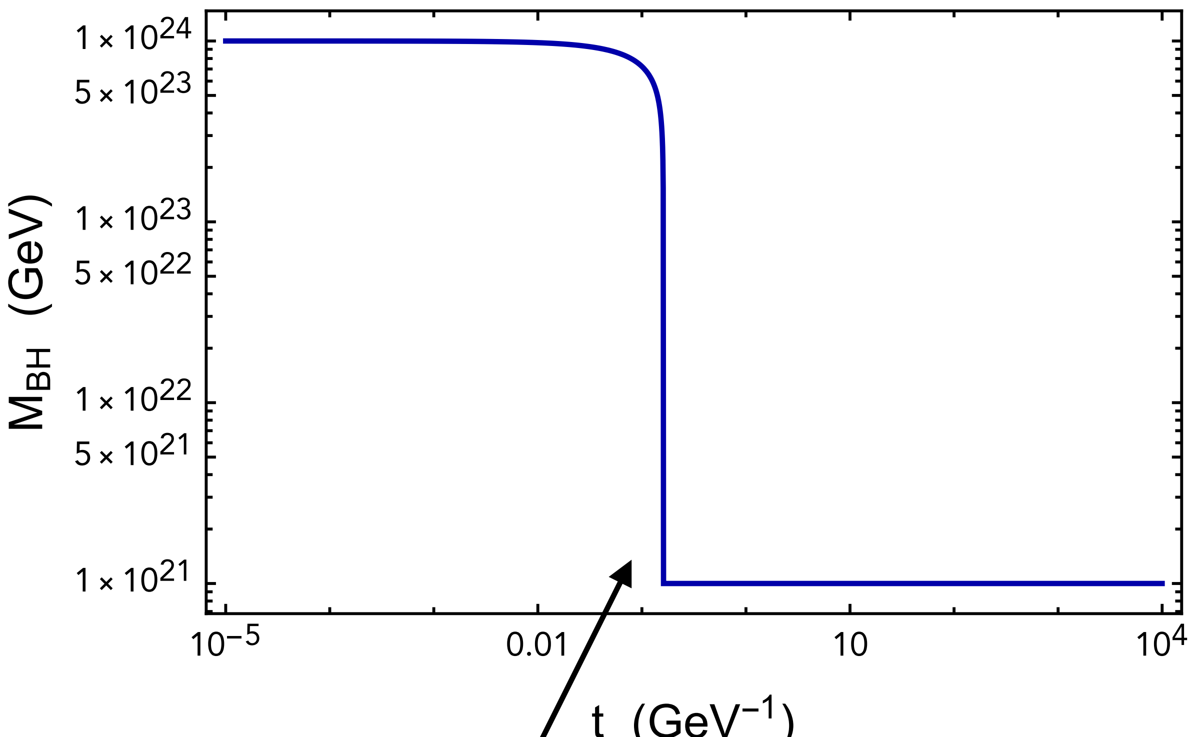
$$T(M_{\rm BH}, M_{\rm eBH}) = \frac{M_{\rm pl}^2}{2\pi} \frac{\sqrt{M_{\rm BH}^2 - M_{\rm eBH}^2}}{\left(M_{\rm BH} + \sqrt{M_{\rm BH}^2 - M_{\rm eBH}^2}\right)^2}$$

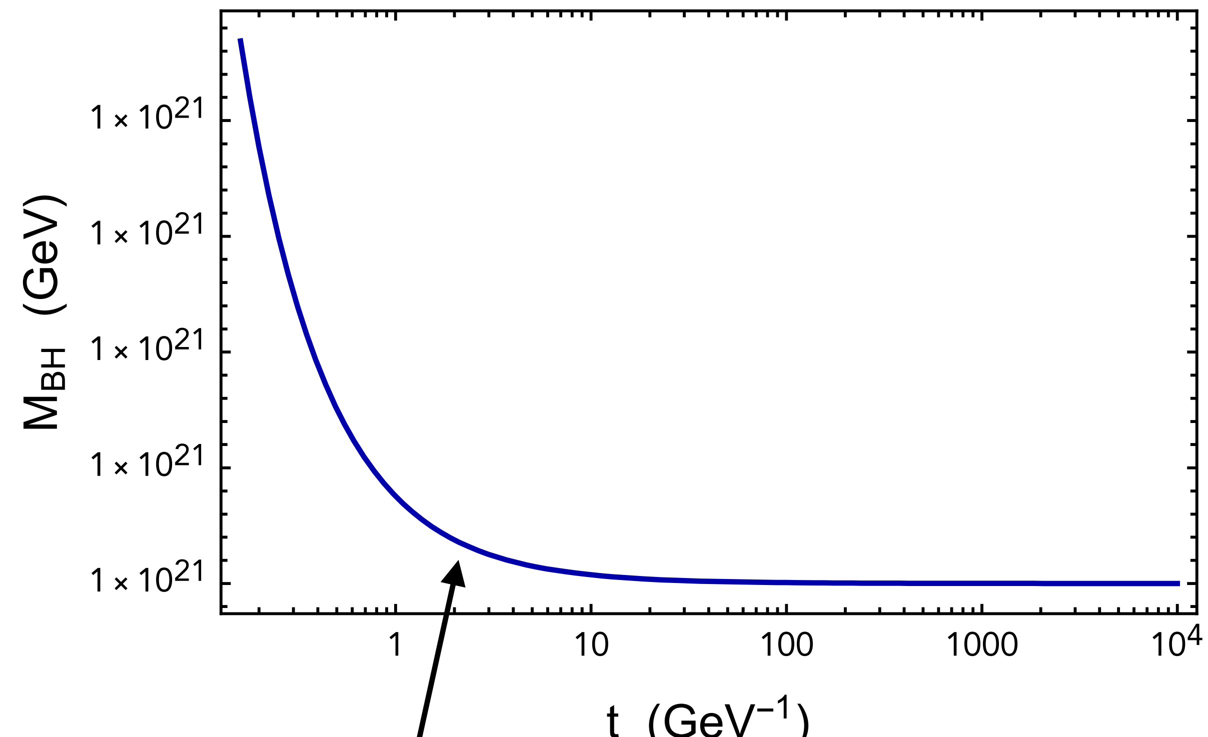
Evolution of a RN Black Hole

- **A PBH with a charge Q will evolve towards a near extremal one, which has suppressed T**

$$\frac{dM_{\rm BH}}{dt} \approx -\frac{\pi^2}{120}\, g_*\, 4\pi r_+^2 \left[T(M_{\rm BH}, M_{\rm eBH})\right]^4$$

semi-classical w/o greybody factor





$$\tau \approx \frac{5120\pi}{g_*}\,\frac{M_{\rm BH}^3}{M_{\rm pl}^4}$$

$$M_{\rm BH}(t) = M_{\rm eBH} + \frac{120\pi\, M_{\rm eBH}^4}{g_*\, M_{\rm pl}^4\, t}$$

$$T_{\rm eBH} = \sqrt{\frac{60\, M_{\rm eBH}}{\pi\, g_*\, t}}$$

Breakdown of Semi-classical

$$M_{\rm BH}(t) = M_{\rm eBH} + \frac{120\pi\, M_{\rm eBH}^4}{g_*\, M_{\rm pl}^4\, t} \qquad T = \sqrt{\frac{60\, M_{\rm eBH}}{\pi\, g_*\, t}}$$

$$M_{\rm BH} = M_{\rm eBH} + \frac{2\pi^2\, M_{\rm eBH}^3\, T^2}{M_{\rm pl}^4} \qquad E \equiv M_{\rm BH} - M_{\rm eBH} = \frac{2\pi^2\, M_{\rm eBH}^3\, T^2}{M_{\rm pl}^4}$$

- **This semi-classical description breaks down when**

$$E \sim T$$

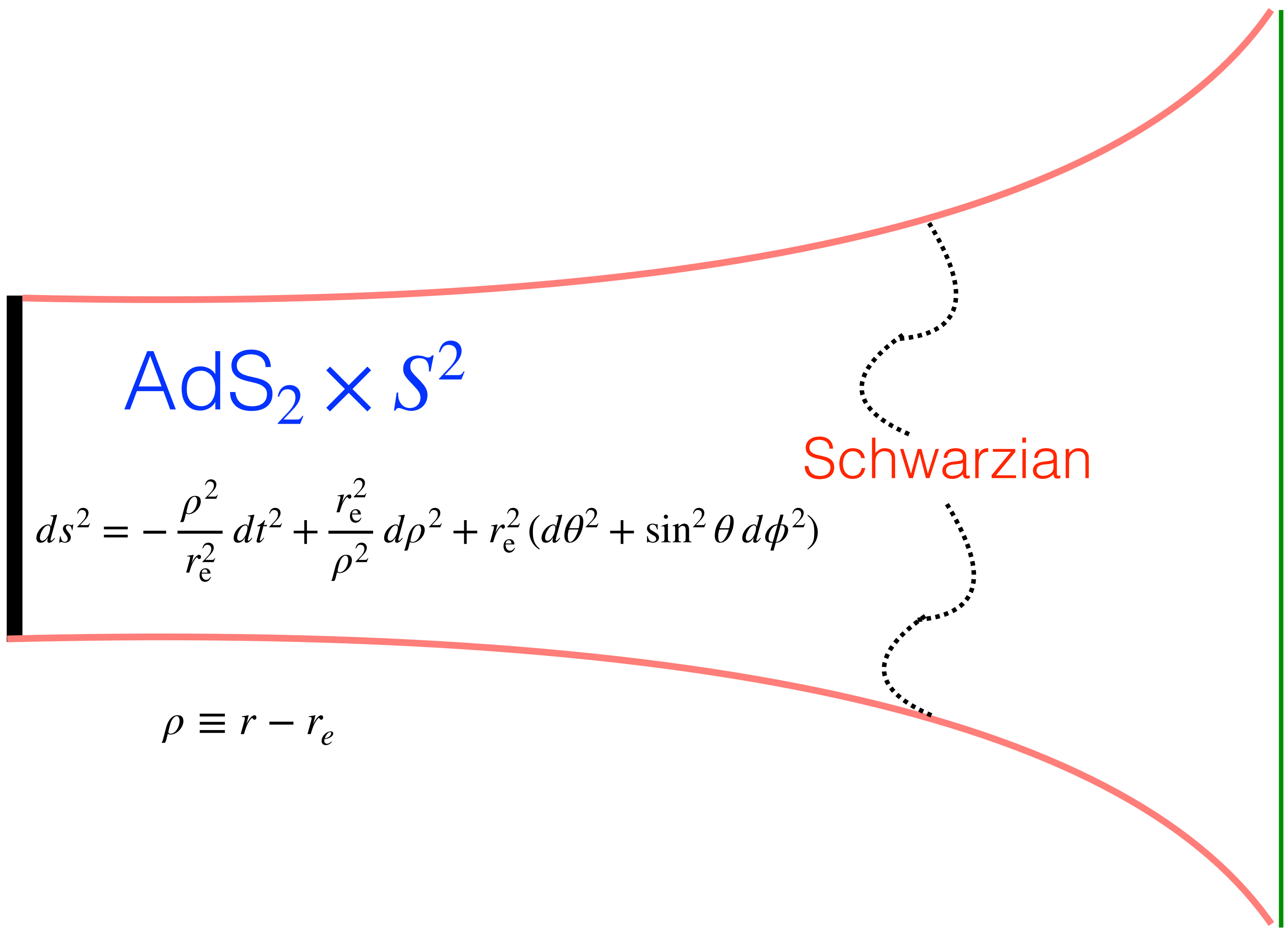
- **Or when the T is below a "gap scale"**

$$\Lambda_{\rm gap} \equiv \frac{M_{\rm pl}^4}{M_{\rm eBH}^3}$$

Preskill, Schwarz, Shapere, Trivedi, Wilczek, '1991

Maldacena, Michelson, Strominger, '1998

Near-extremal Black Hole



$$Z_{\rm sch} \propto \int \frac{\mathscr{D}f(\tau)}{\mathrm{SL}(2,R)} \exp\left[\frac{N\gamma}{4\pi^2}\int_0^\beta d\tau\,\left\{\tan\big(\pi T f(\tau)\big), \tau\right\}\right]$$

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2$$

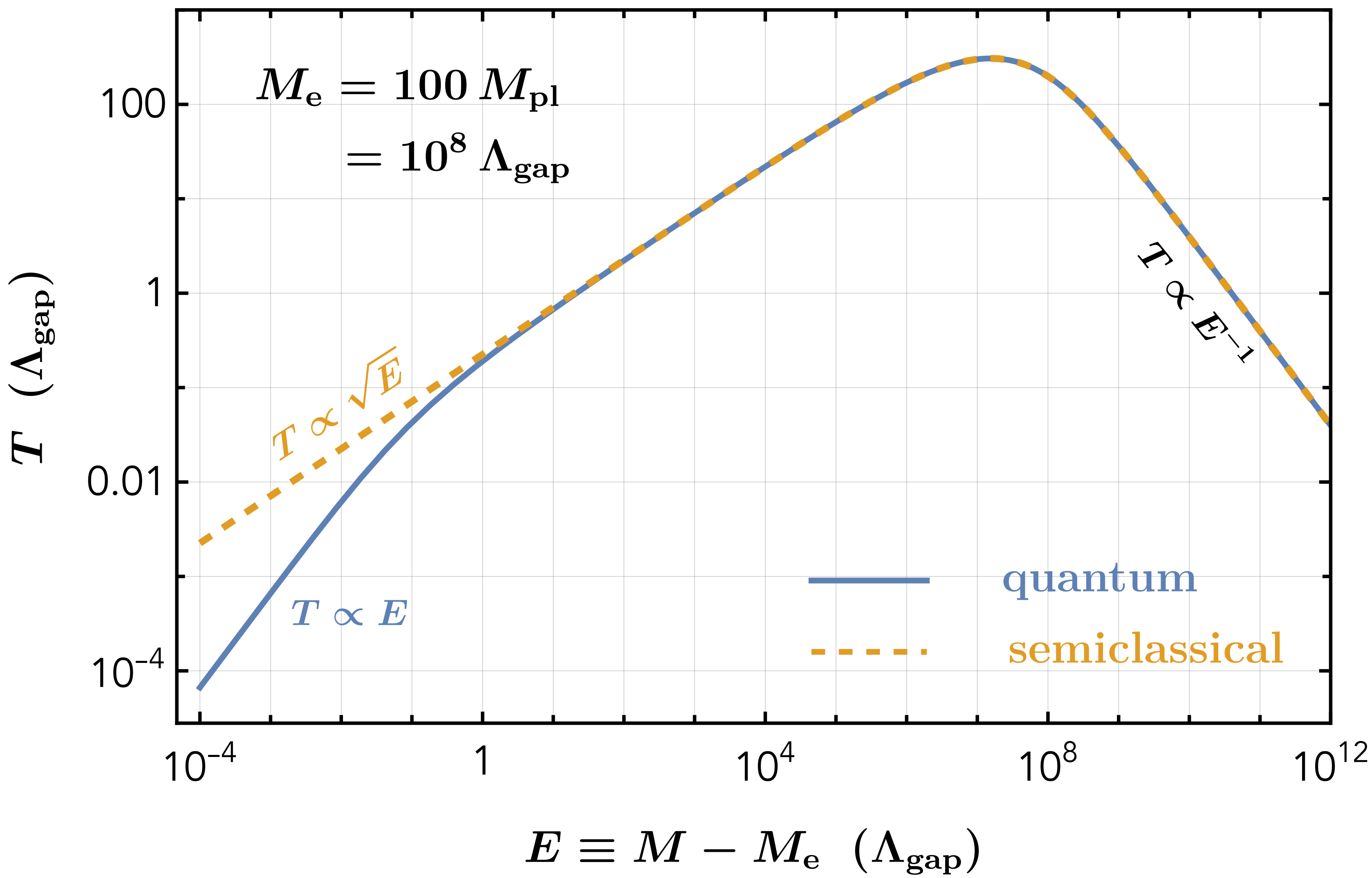
Almheiri, Polchinski, 1402.6334

Maldacena, Stanford, Yang, 1606.01857

Stanford, Witten, 1703.04612

Iliesiu, Turiaci, 2003.02860

Including Quantum Effect



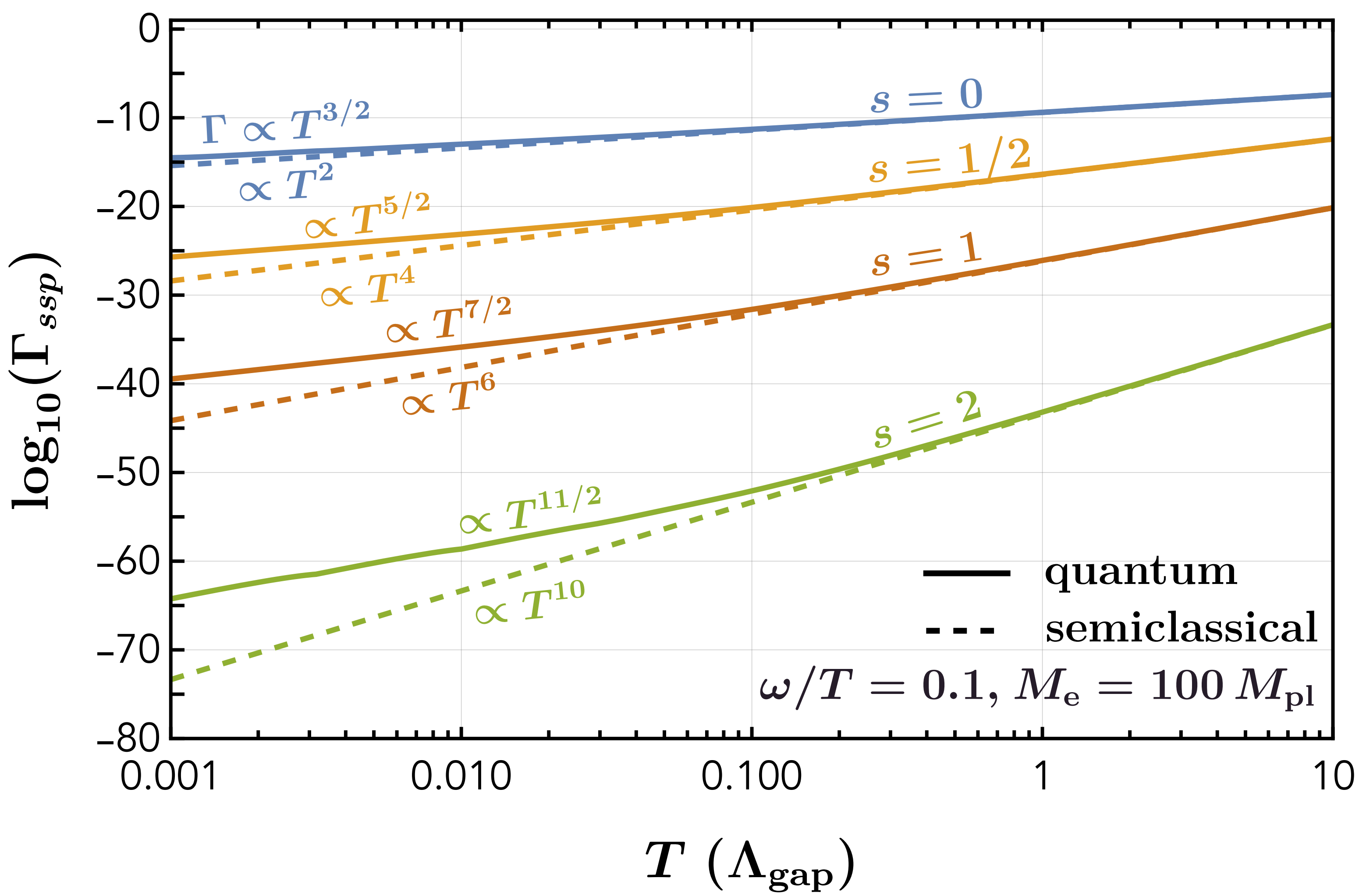
- **Universal AdS$_2 \times S^2$ geometry; matched to the Schwarzian action; obtain the one-loop exact partition function**

$$Z(T) = \left(M_{pl}^2\, r_e^3\, T\right)^{3/2}\ e^{S_0 - M_e/T + 2\pi^2 M_{pl}^2 r_e^3\, T}$$

Including Quantum Effect

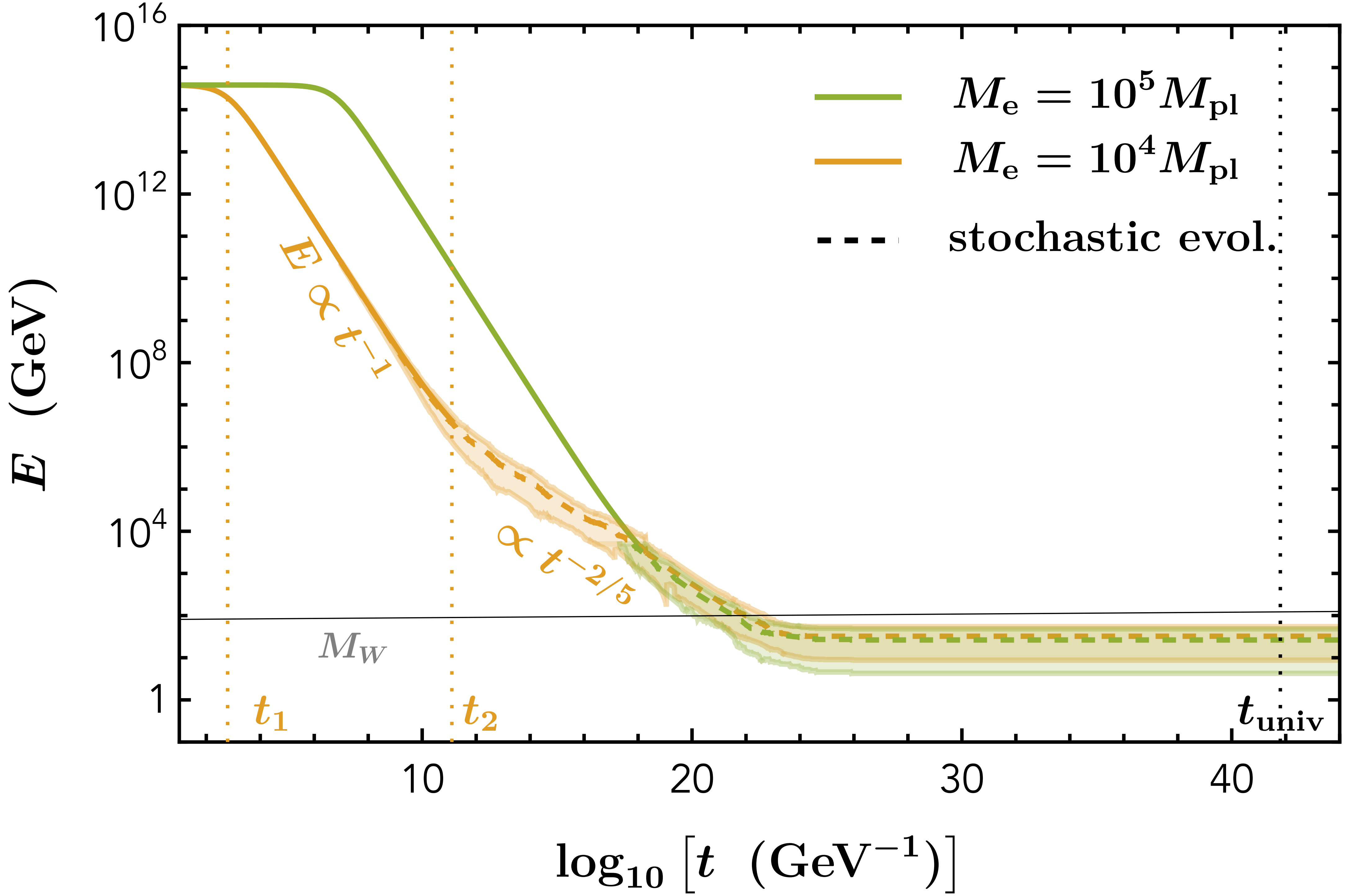
- Using AdS$_2$/CFT$_1$, we calculated the greybody factors

$$\langle N_{slp}(\omega)\rangle = \frac{\Gamma_{slp}(\omega, T, r_e)}{2\pi}\frac{1}{e^{\omega/T} \mp 1}$$



YB, Korwar, 2301.07739

Including Quantum Effect



YB, Korwar, 2301.07739

What are the charges?

GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking

(Communicated by M. J. Rees)

(Received 1970 November 9)

SUMMARY

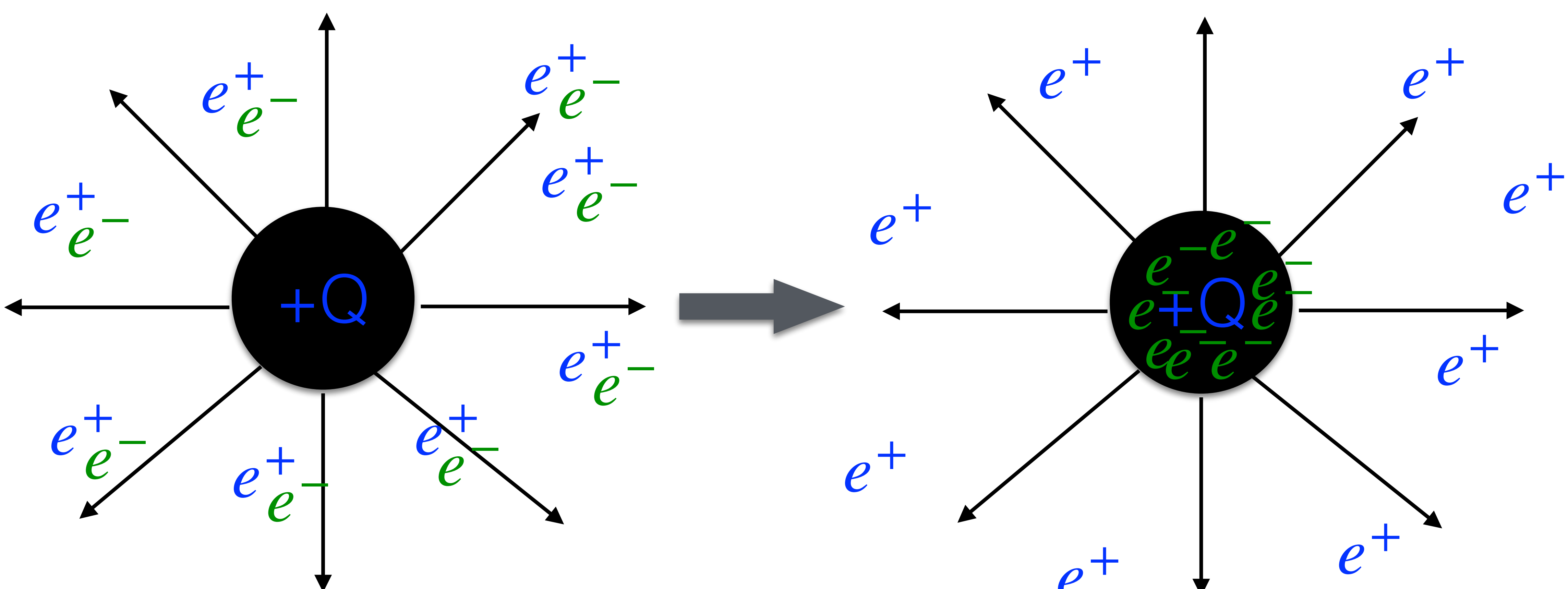
It is suggested that there may be a large number of gravitationally collapsed objects of mass 10^{-5} g upwards which were formed as a result of fluctuations in the early Universe. They could carry an electric charge of up to ± 30 electron units. Such objects would produce distinctive tracks in bubble chambers and could form atoms with orbiting electrons or protons. A mass of 10^{17} g of such objects could have accumulated at the centre of a star like the Sun. If such a star later became a neutron star there would be a steady accretion of matter by a central collapsed object which could eventually swallow up the whole star in about ten million years.

Electrically-Charged BH in SM

- **The charged BH has a large electric field close to the event horizon**

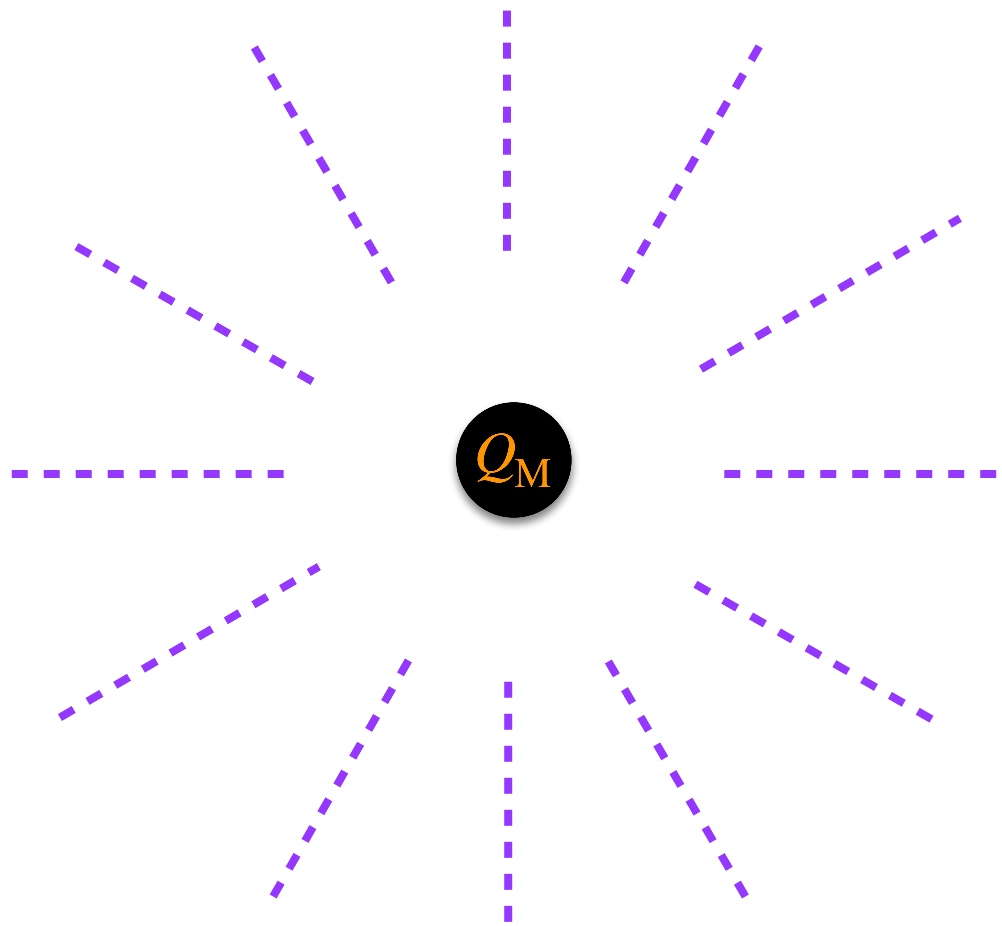
$$E = \frac{M_{\rm pl}^3}{\sqrt{4\pi}\, M_{\rm eBH}} > m_e^2 \qquad \text{for } M_{\rm eBH} < 10^8\, M_\odot$$

- **The Schwinger effects can generate electrons and positrons from vacuum and discharge the eBH**



Magnetically-Charged BH in SM

- **Since there is no finite-energy magnetic monopole in the SM, no worry about Schwinger discharge**
- **If the GUT exists, one may worry its emission of GUT monopole, which is very heavy**



$$B(R_{\rm eBH}) = \frac{Q}{2\,e\,R_{\rm eBH}^2} \approx \frac{e\,M_{\rm pl}^2}{2\,\pi\,Q} \lesssim M_{\rm GUT}^2$$

$$Q \gtrsim 10^6$$

EW Symmetry Restoration in B Field

- **In a large B field background, the electroweak symmetry is restored**

Salam and Strathdee, NPB90 (1975) 203

Ambjorn and Olesen, NPB330 (1990) 193

$$\mathscr{E} \supset \frac{1}{2}|D_iW_j - D_jW_i|^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{4}Z_{ij}^2 + \frac{1}{2}\, g^2\varphi^2 W_iW_i^\dagger + (g^2\varphi^2/4\cos^2\theta_W)Z_i^2$$
$$+ig(F_{ij}\sin\theta_W + Z_{ij}\cos\theta_W)W_i^\dagger W_j + \frac{1}{2}g^2\left[(W_iW_i^\dagger)^2 - (W_i^\dagger)^2(W_j)^2\right]^2$$
$$+(\partial_i\varphi)^2 + \lambda(\varphi^2 - \varphi_0^2)^2$$

$$(W_1^\dagger, W_2^\dagger)\begin{pmatrix} \frac{1}{2}g^2\varphi_0^2 & i\,e\,F_{12} \\ -i\,e\,F_{12} & \frac{1}{2}g^2\varphi_0^2 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$$

- **For a large $|F_{12}|$, a negative determinant leads to W-condensation and electroweak restoration. This happens when**

$$e\,B \gtrsim m_h^2$$

Electroweak Symmetry Restoration

$$B(R_{\rm eBH}) = \frac{Q}{2\,e\,R_{\rm eBH}^2} \approx \frac{e\,M_{\rm pl}^2}{2\,\pi\,Q} \qquad\qquad e\,B(R_{\rm eBH}) \gtrsim m_h^2$$

- **Electroweak symmetry restoration happens for**

$$Q \lesssim Q_{\rm max} \equiv \frac{e^2\,M_{\rm pl}^2}{2\pi\,m_h^2} \approx 1.4 \times 10^{32}$$

Lee, Nair, Weinberg, PRD45(1992) 2751

Maldacena, arXiv:2004.06084

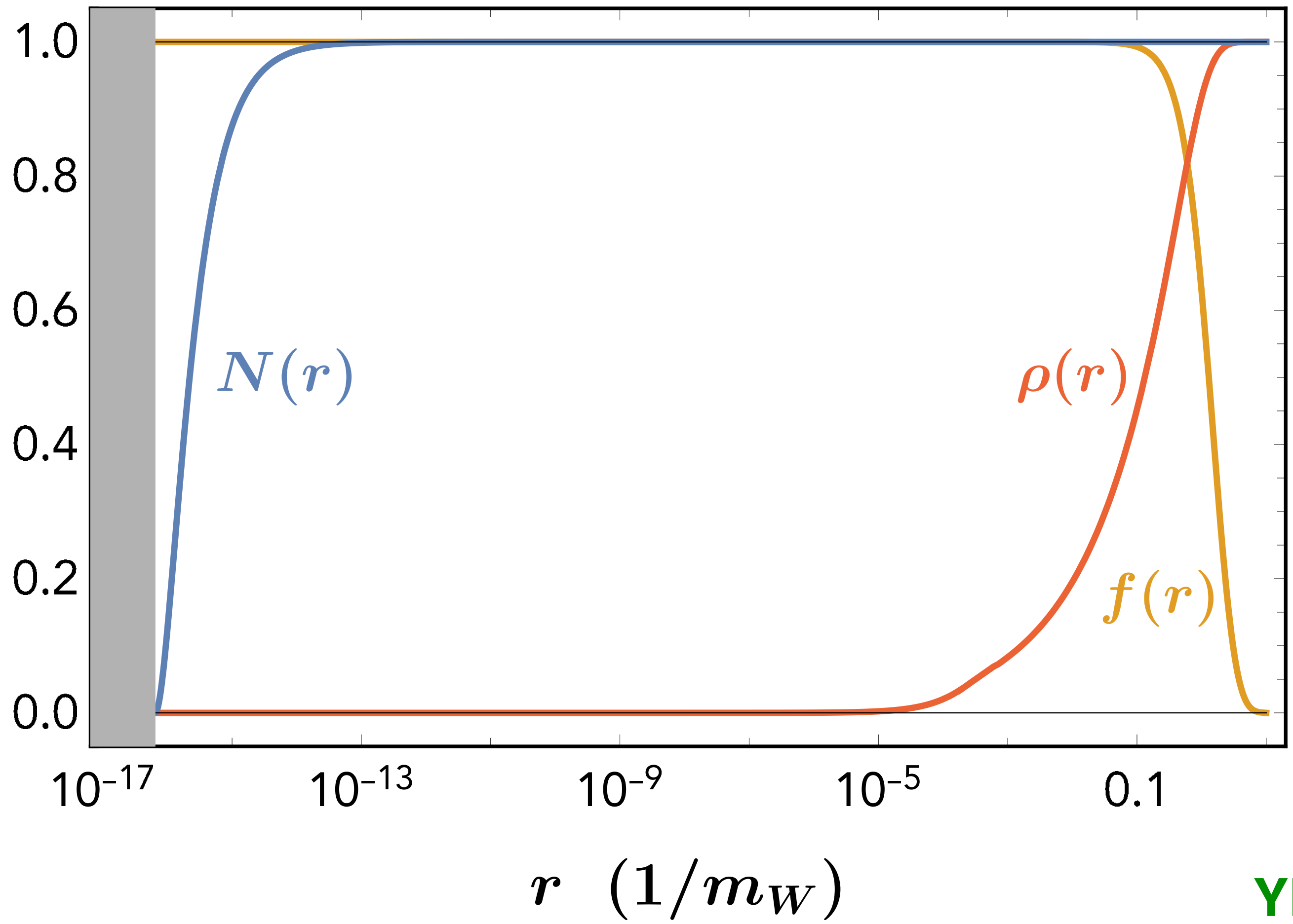
- **For Q=2, one can obtain the spherically symmetric configuration**

- **For Q > 2, a non-spherically symmetric configuration is anticipated, and requires complicated numerical calculations**

Guth, Weinberg, PRD14(1976) 1660

Q=2: spherical

extremal hMBH



$N(r)$: metric

$\rho(r)$: Higgs

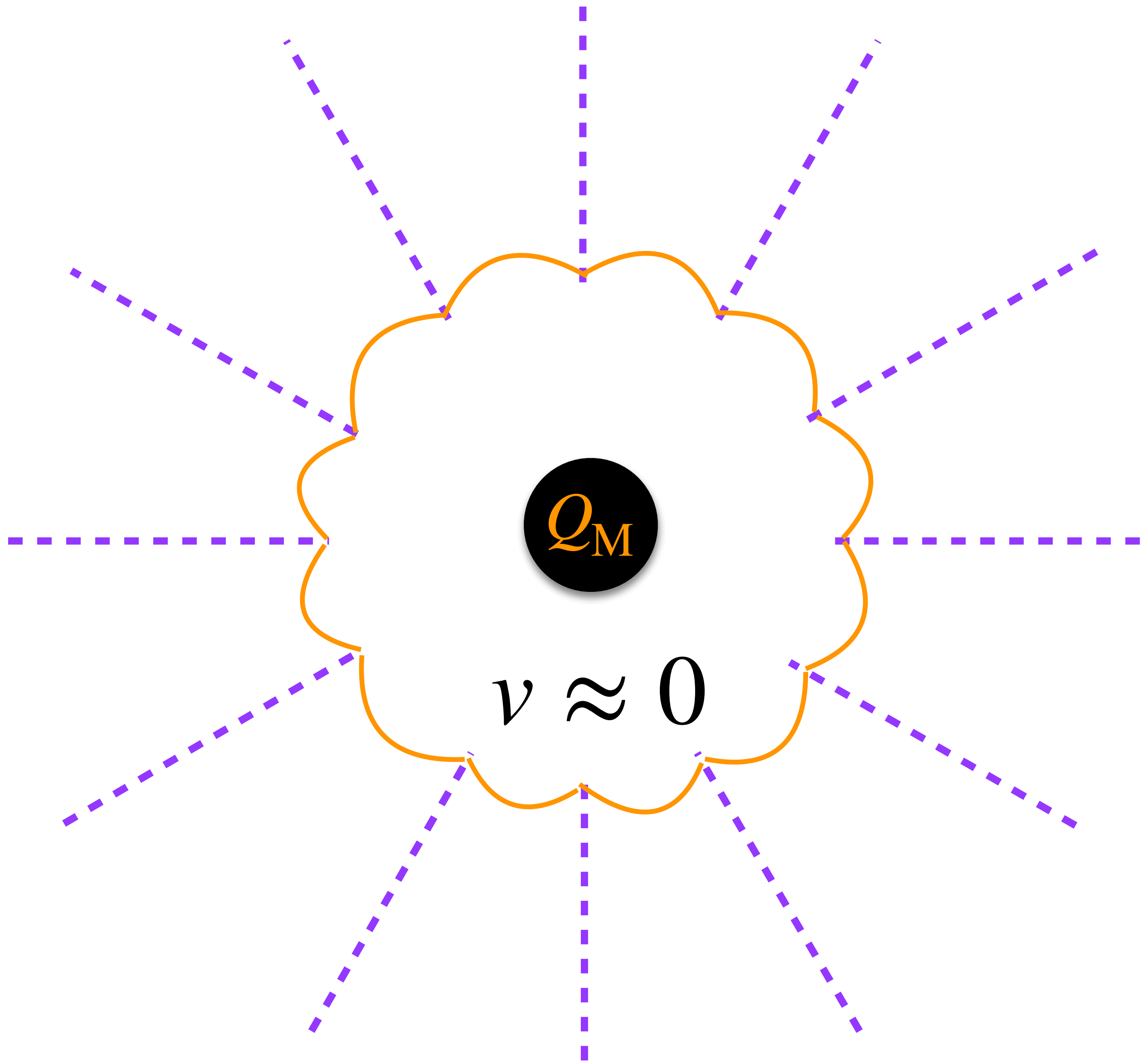
$f(r)$: gauge

YB, Korwar, 2012.15430

$$M_{\text{ehMBH}} \approx \cos\theta_W \frac{\sqrt{4\pi}\,M_{\text{pl}}}{e} + 0.75 \times \frac{2\pi\, v^2}{m_W} = (1.2 \times 10^{20} + 3.6 \times 10^3)\,\text{GeV}$$

- **Hypercharged black hole in the core with EW hair**
- **Electroweak symmetry is restored in the core region**

Q>2: non-spherical



$v = 246\,\mathrm{GeV}$

$R_{\rm EW} \simeq \sqrt{\frac{Q}{2}}\frac{1}{m_h}$

- To have the electroweak hair, $r_H < R_{\rm EW}$

$Q < Q_{\rm max} \simeq 10^{32}$ $M_{\star} \lesssim 9 \times 10^{51}\,\mathrm{GeV} \sim M_{\oplus}$ $R_{\rm EW}^{\rm max} \sim 1\,\mathrm{cm}$

Primordial magnetic black holes for all dark matter?

Parker Limits

- **Requiring the domains of coherent magnetic field are not drained by magnetic monopoles**

- **PMBH flux:** $F_{\bullet} \approx (9.5 \times 10^{-21}\,\mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1})\, f_{\bullet} \left(\frac{10^{26}\,\mathrm{GeV}}{M_{\bullet}}\right) \left(\frac{\rho_{\mathrm{DM}}}{0.4\ \mathrm{GeV\,cm}^{-3}}\right) \left(\frac{v}{10^{-3}}\right)$

- **Mean energy gained by PMBHs for the regeneration time is smaller than the energy stored in B**

Turner, Parker, Bogdan, PRD26(1982) 1296

$$\Delta E \times F_{\bullet} \times (\pi \ell_c^2) \times (4\pi\ \mathrm{sr}) \times t_{\mathrm{reg}} \lesssim \frac{B^2}{2}\,\frac{4\pi\,\ell_c^3}{3}$$

$$\Delta E \simeq M_{\bullet} \Delta v^2/2 \qquad \Delta v \simeq B\, h_Q\, \ell_c/(M_{\bullet} v)$$

$$\rho_{0.4} = \rho_{\mathrm{DM}}/(0.4\,\mathrm{GeV\,cm}^{-3})$$

$$v_{-3} = v/(10^{-3})$$

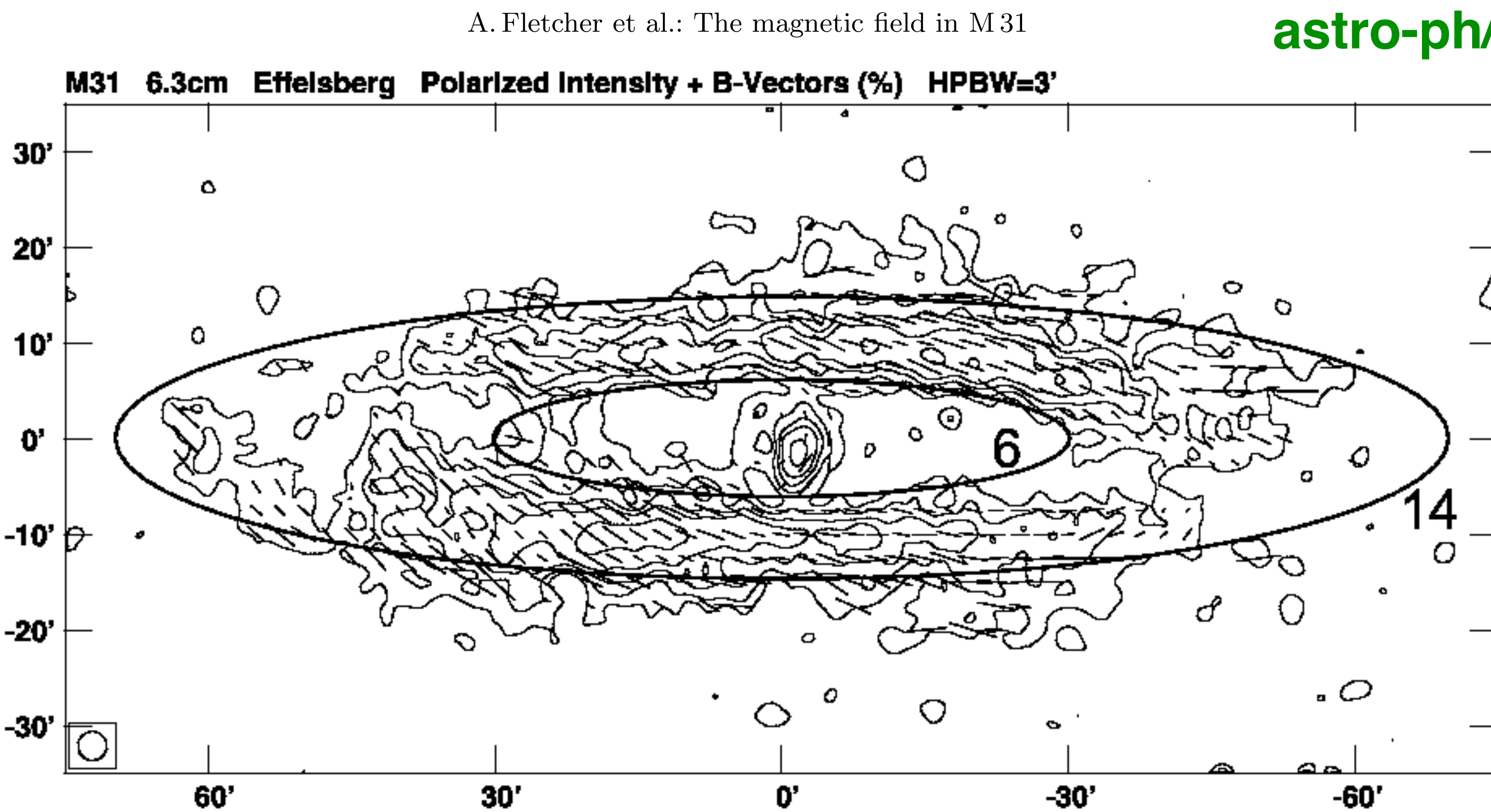
$$t_{15} = t_{\mathrm{reg}}/(10^{15}\,\mathrm{s})$$

$$\ell_{21} = \ell_c/(10^{21}\ \mathrm{cm})$$

$$f_{\bullet} \lesssim 50 \times \frac{v_{-3}}{\rho_{0.4}\,\ell_{21}\,t_{15}}$$

Parker Limit from M31

astro-ph/0310258

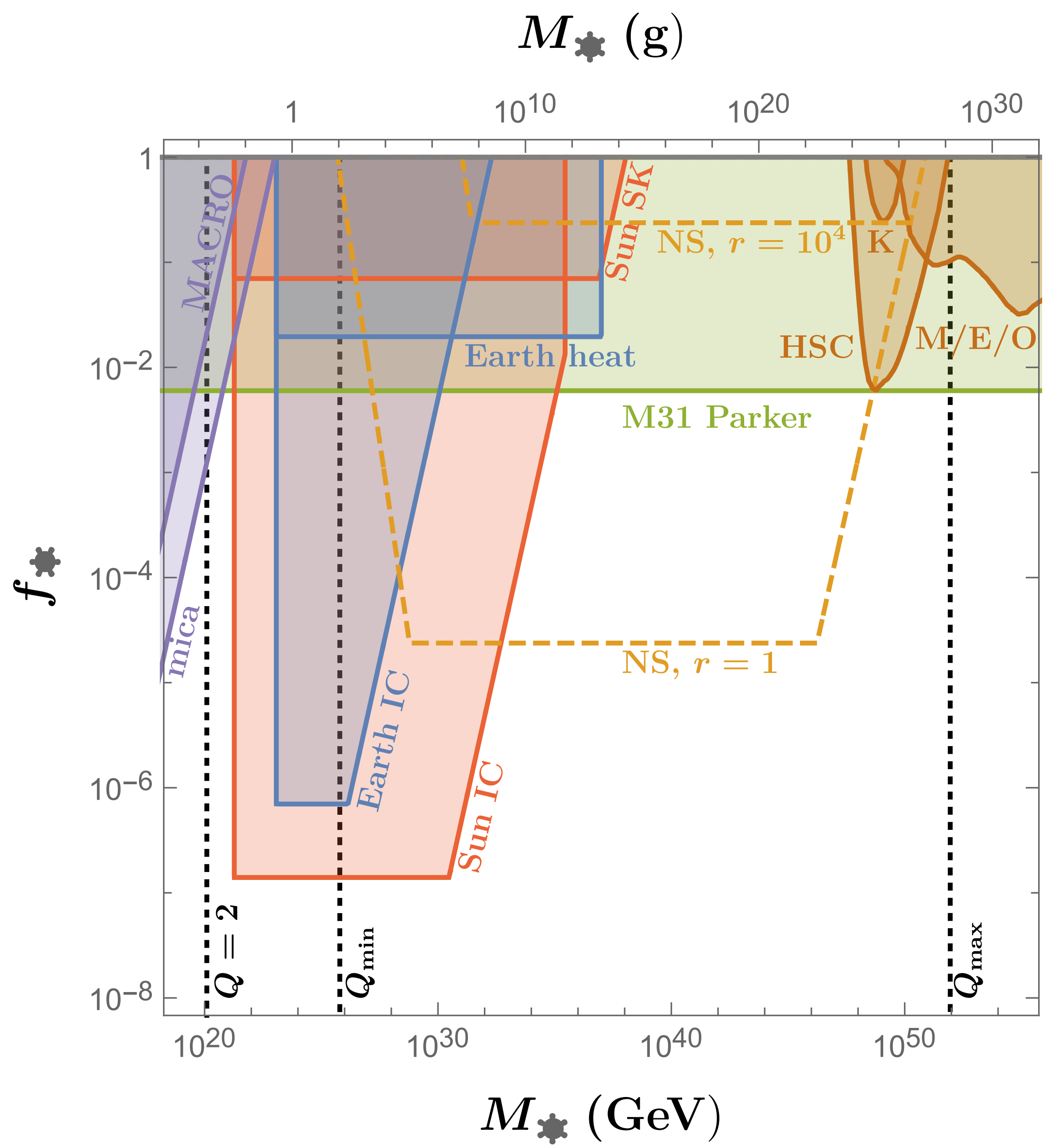


$$\ell_c \sim 10 \text{ kpc} \Rightarrow \ell_{21} \sim 30 \text{ and } t_{\text{reg}} \sim 10 \text{ Gyr} \Rightarrow t_{15} \sim 300$$

$$f_{\bullet} \lesssim 6 \times 10^{-3}$$

which is independent of PMBH mass

Fraction of PMBH over dark matter



YB, Berger, Korwar, Orlofsky 2007.03703

Other searches: Ghosh, Thalapillil, Ullah, 2009.03363

Diamond and Kaplan, 2103.01850

Magnetic Monopoles Inside Earth?

Carl Friedrich Gauss – *General Theory of Terrestrial Magnetism* – a revised translation of the German text

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Abstract. This is a translation of the *Allgemeine Theorie des Erdmagnetismus* published by Carl Friedrich Gauss in 1839 in the *Resultate aus den Beobachtungen des Magnetischen Vereins im Jahre 1838*. The current translation is based on an earlier translation by Elizabeth Juliana Sabine published in 1841. This earlier translation has been revised, corrected, and extended. Numerous biographical comments on the scientists named in the original text have been added as well as further information on the observational material used by Carl Friedrich Gauss. An attempt is made to provide a readable text to a wider scientific community, a text laying the foundation of today's understanding of planetary magnetic fields.

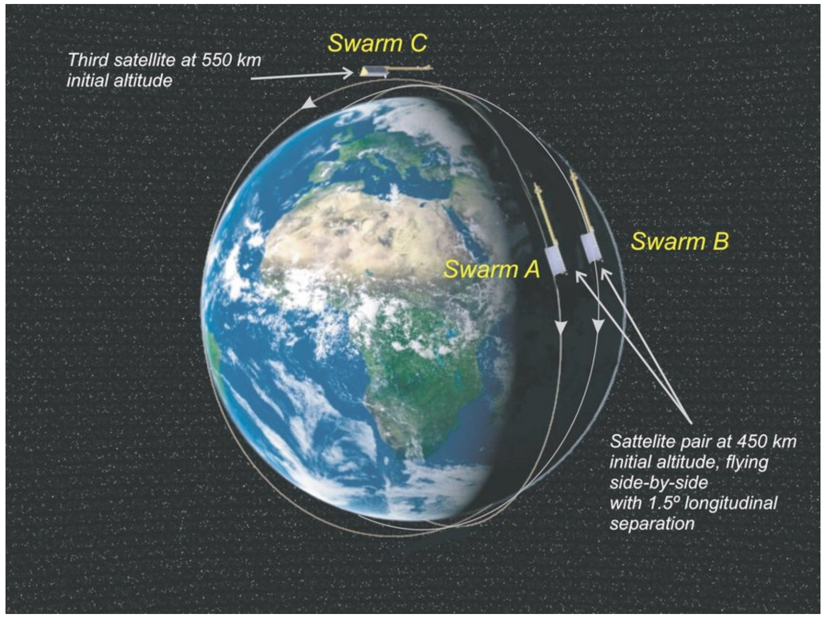
Y components would not at all be affected. Once the future has provided a more extensive opulence of precise observations than currently offered, one might determine whether their precise representation requires a non-vanishing value of P^0 or not[74]. Based on the current state of the data, such an undertaking would be completely unsuccessful.

[74]T: Gauss is discussing here the possible existence of magnetic monopoles. It is remarkable how important experimental results are for this mathematician.

Monopole Moment of Earth Magnetic Field

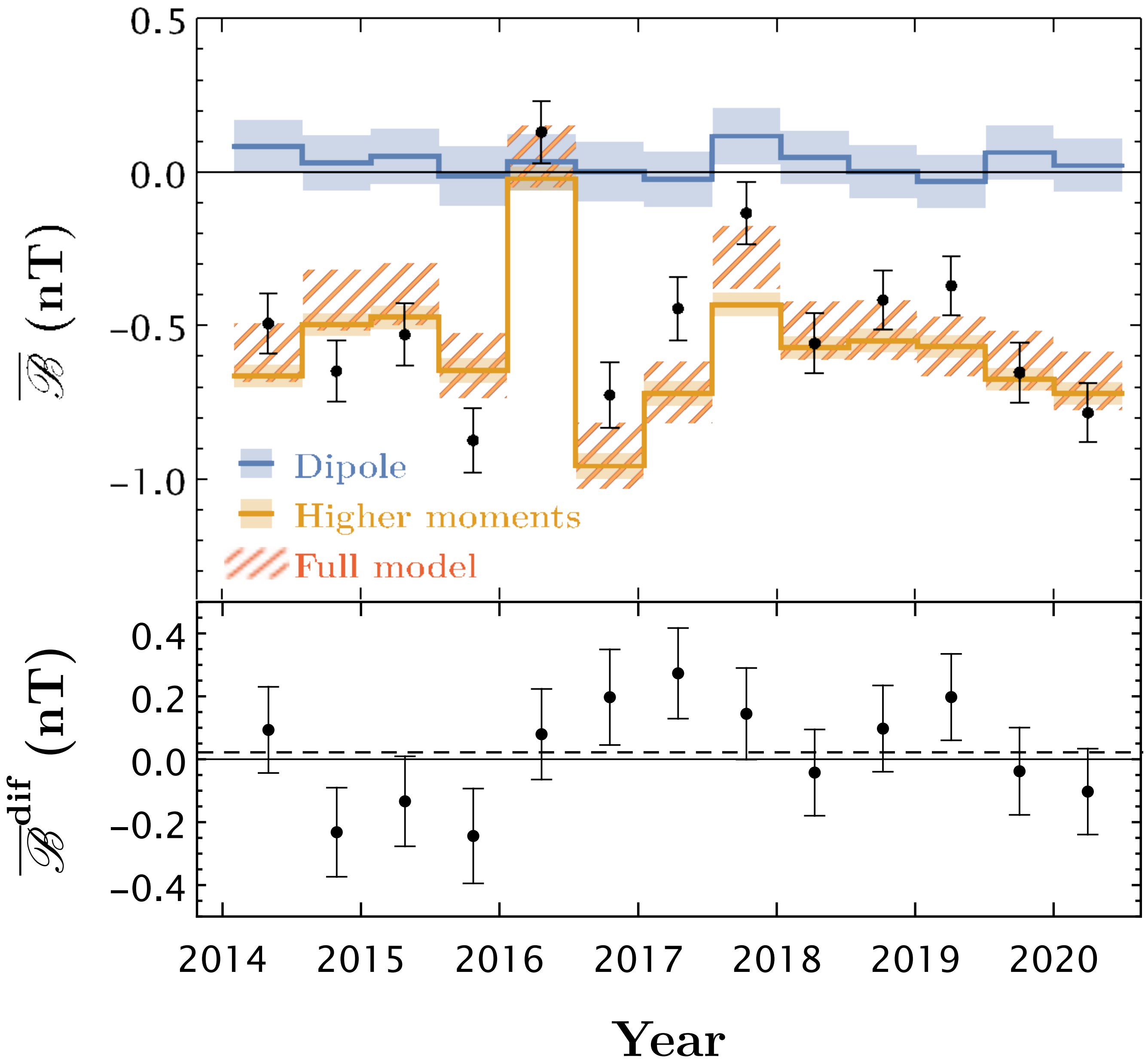
- Using Gauss law to search for monopoles

$$\overline{B}_{\rm m} \equiv \frac{1}{4\pi} \oint \boldsymbol{B}_{\rm m}(r,\theta,\phi)\cdot \hat{\mathbf{n}}\, d\Omega = Q\, h\, \frac{1}{4\pi\, R^2}$$



$$\overline{\mathscr{B}} = \frac{1}{4\pi} \int \left[\frac{r(\theta,\phi)}{R_{\rm ref}}\right]^3 \boldsymbol{B}(r,\theta,\phi)\cdot \hat{\mathbf{r}}\, d\Omega$$

Monopole Moment of Earth Magnetic Field



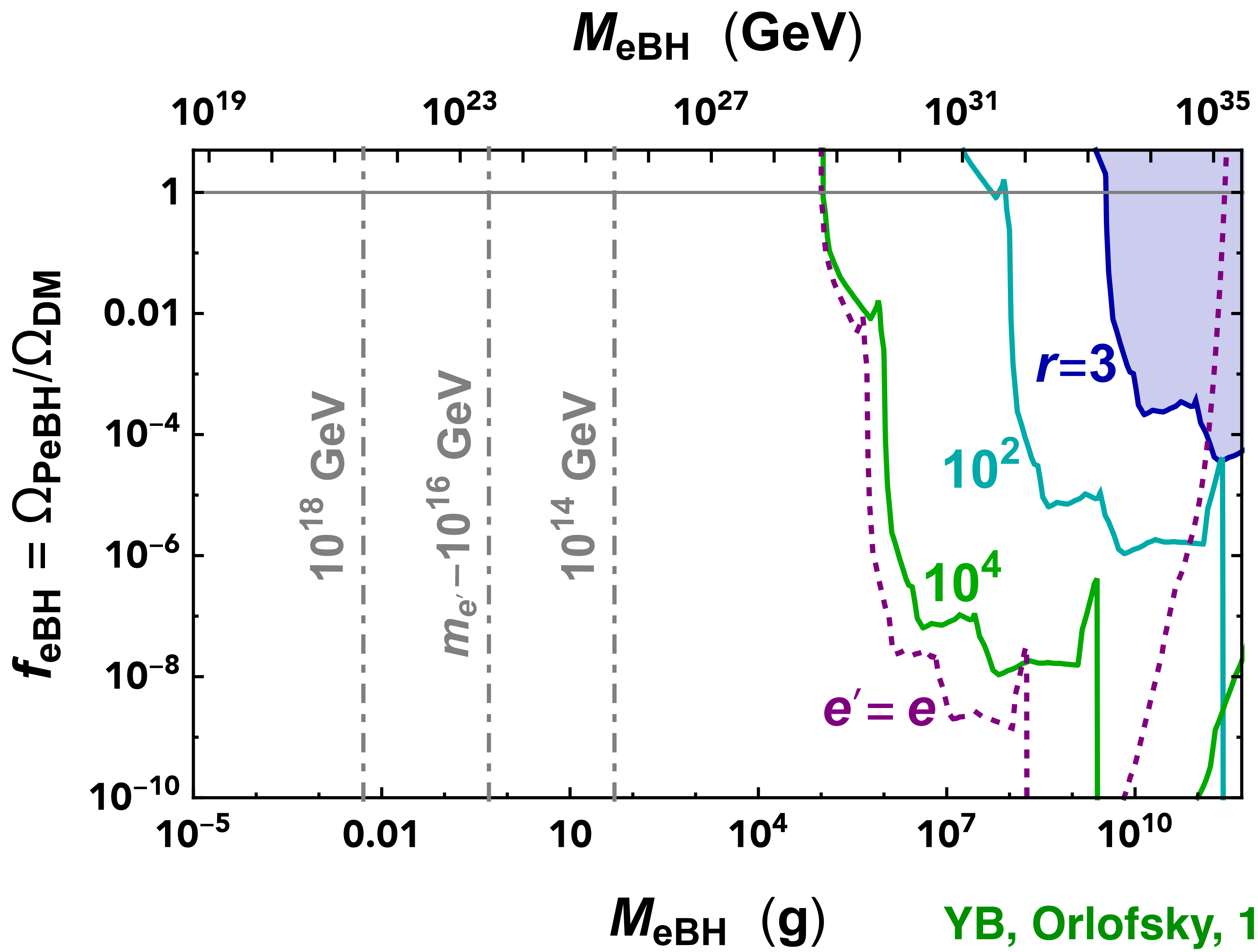
$|B_m(r = R_\oplus)| < 0.13\,\text{nT}$ or $|Q_{\rm net}| < 1.6 \times 10^{19}$

YB, Lu, Orlofsky, 2103.06286

Primordial dark-charged black holes for all dark matter!

Dark QED Model

- **To have a very heavy dark electron mass to suppress Schwinger discharge**



$r \equiv M_{\rm BH}^{\rm init}/M_{\rm eBH}$

YB, Orlofsky, 1906.04858

Pacheco, Kiritsis, Lucca, Silk, 2301.13215

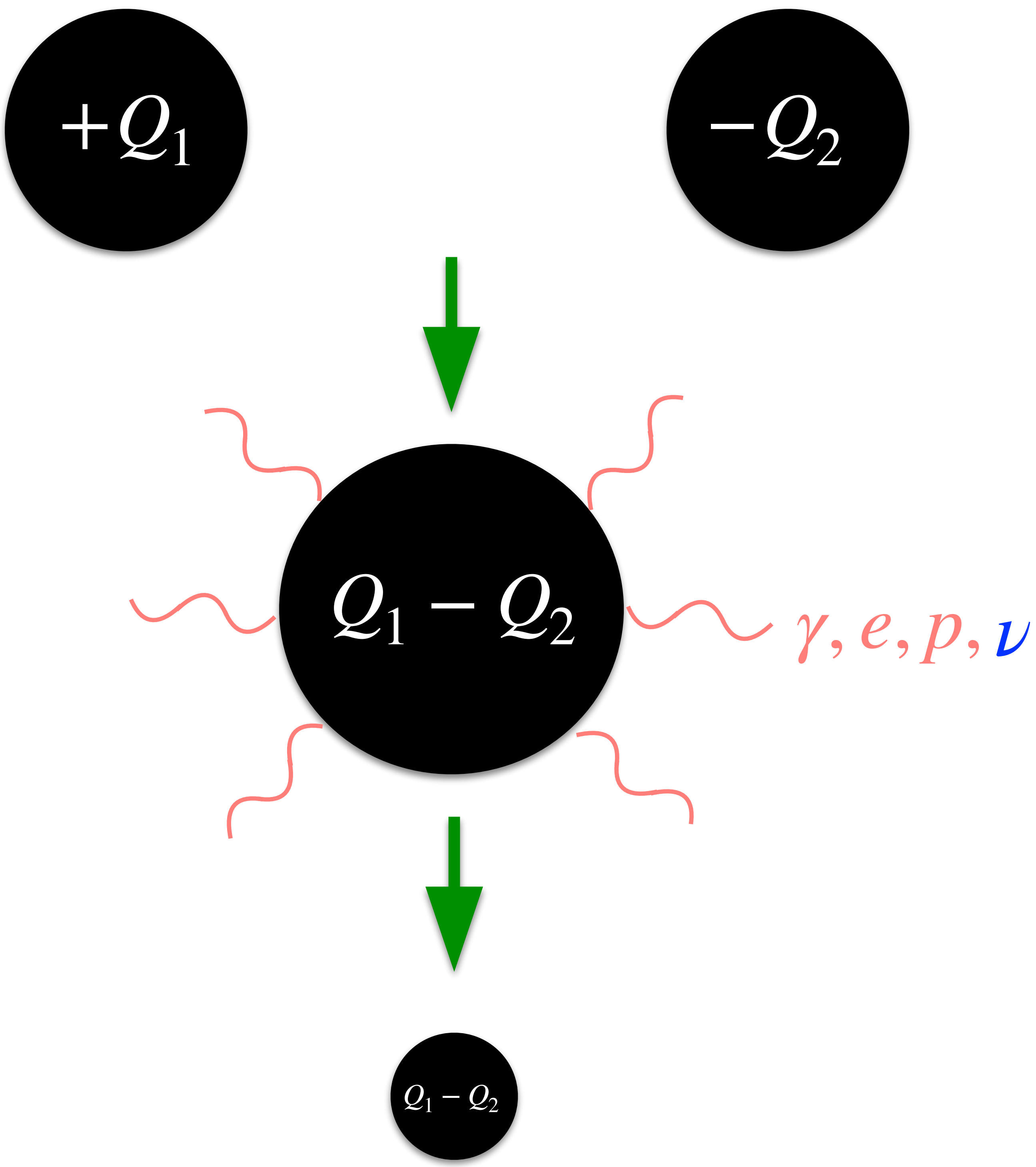
Formation

- There are various ways to form primordial black holes
  - Large primordial fluctuations
  - Phase transitions, boson stars, ……
- The formation of black holes eat totally N objects with a mean total zero charge but $\sqrt{N}$ variance non-zero charge
- Anticipate the net black hole charge: $\sim \sqrt{N}$

YB, Orlofsky, 1906.04858   Stojkovic, Freese, hep-ph/0403248

- Produce large number of monopoles and anti-monopoles (maybe Nambu's dumbbell configurations)
- To be studied more

Signature



Conclusions

- Magnetic black hole exists in SM+GR. It is an interesting magnetic monopole object.

- Magnetic black holes have an electroweak-symmetric hair for $Q < 10^{32}$. They could compose of a subcomponent of dark matter.

- Primordial extremal black holes with a mass in ($M_{\rm pl}, 10^8$ g) could still account for all dark matter, if they are charged under some hidden or dark charge.

Thanks!

Q=2: spherical solution

$$ds^2 = P^2(r)\, N(r)\, dt^2 - N(r)^{-1}\, dr^2 - r^2\, d\theta^2 - r^2\, \sin^2\theta\, d\phi^2$$

$$S_{\rm E} = -\frac{1}{16\pi\, G}\int d^4x\sqrt{-g}\, R = -\frac{1}{2\, G}\int dt\, dr\, r\, P'(1-N)$$

$$S_{\rm matter} \supset \int d^4x\sqrt{-g}\, \mathcal{L}_{\rm EW}$$

$$N(r) = 1 - \frac{2\, G\, F(r)}{r} + \frac{4\,\pi\, G}{g_{\rm Y}^2\, r^2}$$

- **The asymptotic mass of the system has**

$$M = F(\infty)$$

Q=2: spherical solution

$$\mathcal{L}_{\mathrm{SM}} \supset \mathcal{L}_{\mathrm{EW}} = -\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + |D_\mu H|^2 - \frac{\lambda}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2$$

$$D_\mu H = \left(\partial_\mu - i\,\frac{g}{2}\,\sigma^a\,W^a_\mu - i\,\frac{g_Y}{2}\,Y_\mu\right) H$$

$$H = \frac{v}{\sqrt{2}}\,\rho(r)\,\xi \qquad\qquad \xi = i\begin{pmatrix} \sin\left(\frac{\theta}{2}\right) e^{-i\,\phi} \\ -\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$W^a_i = \epsilon^{aij}\,\frac{r^j}{r^2}\left(\frac{1-f(r)}{g}\right) \qquad\qquad Y_i = -\frac{1}{g_Y}(1-\cos\theta)\,\partial_i\phi$$

- **Change from the hedgehog gauge to the unitary gauge**

$$\xi \longrightarrow U\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{with} \qquad U = -i\begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) e^{-i\phi} \\ \sin\left(\frac{\theta}{2}\right) e^{i\phi} & -\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$A_\mu = -\frac{1}{e}(1-\cos\theta_W)\partial_\mu\phi$$

$$Z_\mu = 0$$

Cho and Maison, hep-th/9601028

Q=2: EOMs and BCs

$$S_{\rm matter} \;=\; -4\pi \int dt\, dr\, r^2\, \left[P(r)\, N(r)\, \mathcal{K} + P(r)\, \mathcal{U}\right]$$

$$\begin{aligned}
\mathcal{K} &= \frac{v^2\,\rho'^2}{2} + \frac{f'^2}{g^2\, r^2}\,, \\
\mathcal{U} &= \frac{v^2\, f^2\, \rho^2}{4\, r^2} + \frac{(1-f^2)^2}{2\, g^2\, r^4} + \frac{\lambda}{8}\, v^4\, (\rho^2-1)^2 + \frac{1}{2\, g_{_Y}^2\, r^4} \equiv \mathcal{U}_1 + \frac{1}{2\, g_{_Y}^2\, r^4}
\end{aligned}$$

EOM's

$$\begin{aligned}
F' &= 4\pi\, r^2\, (\mathcal{U}_1 + N\, \mathcal{K})\ , \\
(N\, f')' + 8\pi\, G\, r\, N\, f'\, \mathcal{K} &= \frac{f(f^2-1)}{r^2} + \frac{g^2}{4}\, v^2\, f\, \rho^2\ , \\
\left(r^2\, N\, \rho'\right)' + 8\pi\, G\, r^3\, N\, \rho'\, \mathcal{K} &= \frac{1}{2}\, \rho\, f^2 + \frac{\lambda\, v^2}{2}\, r^2\, \rho(\rho^2-1)\ .
\end{aligned}$$

BC's

$$N' = \frac{1}{r} - 8\pi\, G\, r\, \mathcal{U}\,, \qquad \text{at } r = r_H$$

$$N'\, f' = \frac{f(f^2-1)}{r^2} + \frac{g^2}{4}\, v^2\, f\, \rho^2\,, \qquad \text{at } r = r_H\ ,$$

$$N'\, \rho' = \frac{1}{2}\, \frac{f^2\, \rho}{r^2} + \frac{\lambda\, v^2}{2}\, \rho(\rho^2-1)\,, \qquad \text{at } r = r_H\ .$$

$$f(\infty) = 0$$

$$\rho(\infty) = 1$$

Q=2: solutions

- **Setting $f(r) = 0$ and $\rho(r) = 1$, one has the ordinary RN magnetic black hole solution**

$$M_{\rm BH}^{\rm RN} = \frac{r_H}{2\,G} + \frac{2\pi}{e^2\, r_H} \qquad \geq \qquad M_{\rm eBH}^{\rm RN} = \frac{\sqrt{4\pi}\, M_{\rm pl}}{e}$$

- **For the hairy magnetic black hole solution:**

$$M_{\rm hMBH} = F(\infty) = \int_{r_H}^{\infty} dr' 4\pi\, r'^2\, [\mathcal{K}(r') + \mathcal{U}_1(r')] + \frac{r_H}{2\,G} + \frac{2\pi}{g_Y^2\, r_H}$$

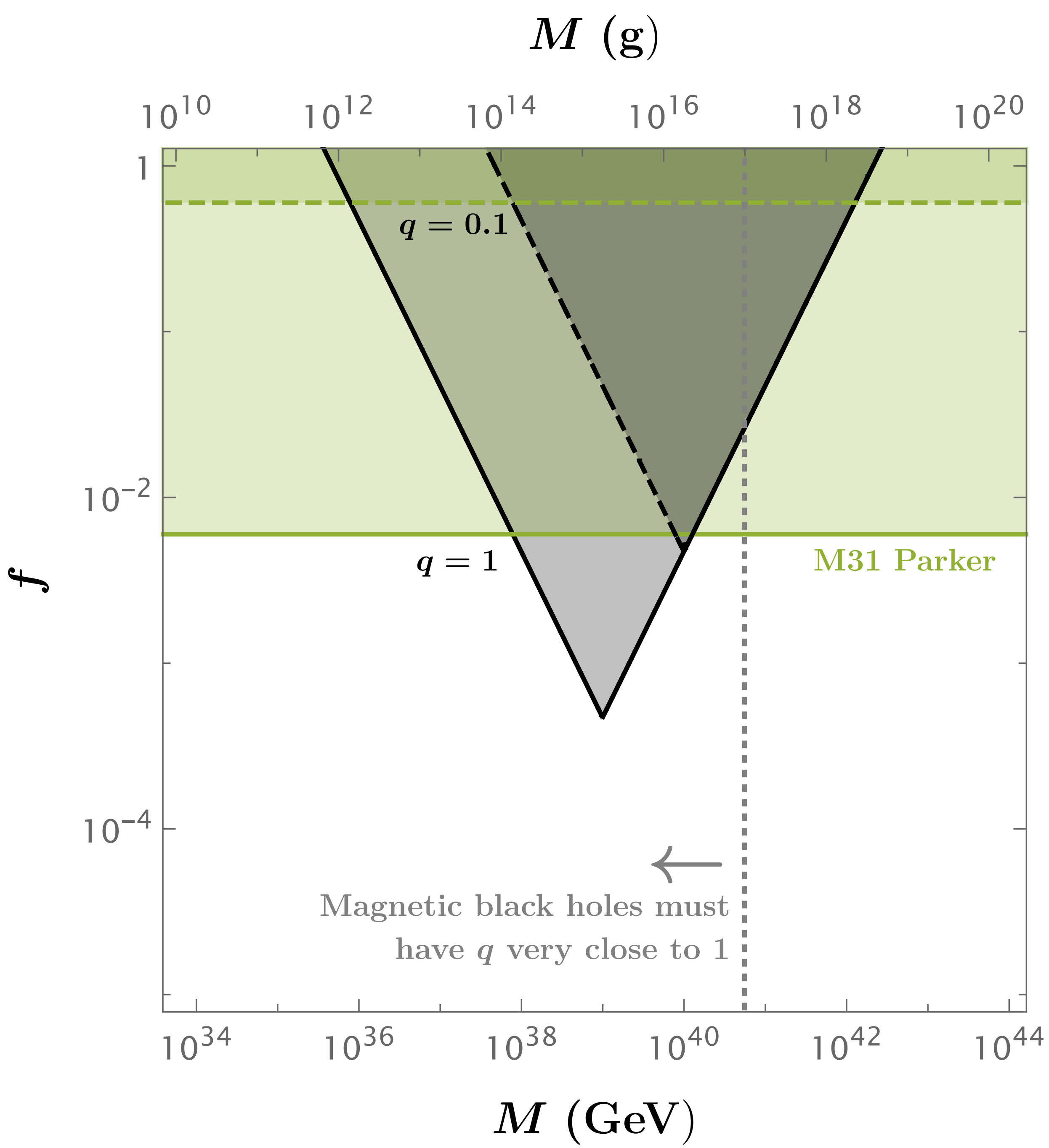
Hair mass $\ll$ **Black hole mass**

- **Ignoring the hair mass, one has** $e = g_Y \cos\theta_W$

$$M_{\rm hMBH} \approx \frac{r_H}{2\,G} + \frac{2\pi}{g_Y^2\, r_H} \geq M_{\rm ehMBH} = \cos\theta_W \frac{\sqrt{4\pi}\, M_{\rm pl}}{e}$$

Hyper-magnetic black hole!

Monopole Moment of Earth Magnetic Field



YB, Lu, Orlofsky, 2103.06286