Primordial black holes and gravitational waves induced by exponential-tailed perturbations

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JCAP05(2023)044

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NEHOP 2023 Contribution ID:13

Primordial black hole(PBH)

B.J. Carr and S.W. Hawking ' 1974, Norbert Duchting' 20,
Jakub Scholtz and James Unwin' 20, R.Saito et al. '11

Candidate of DM

Seeds of SMBH ?

Planet nine ?

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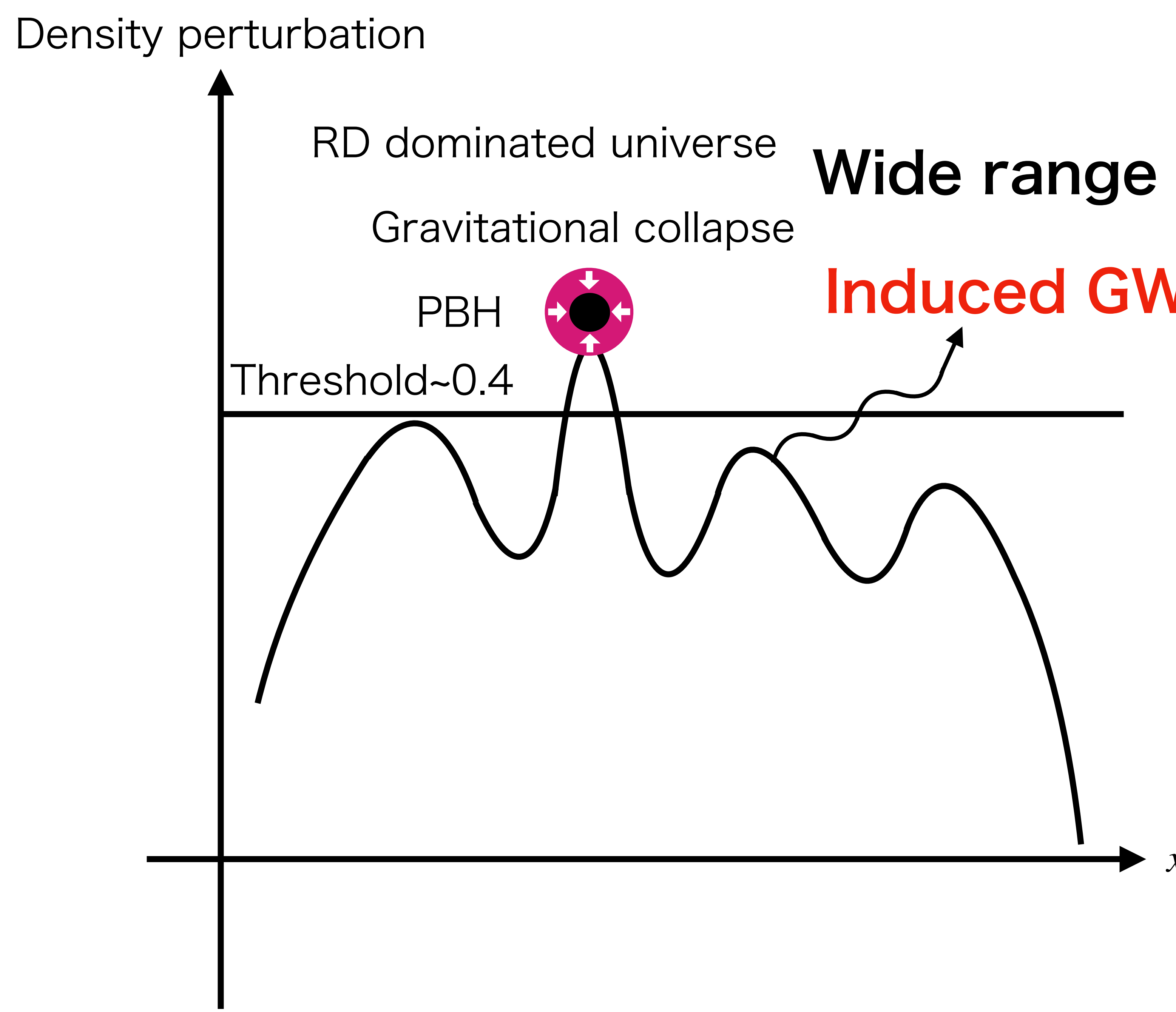
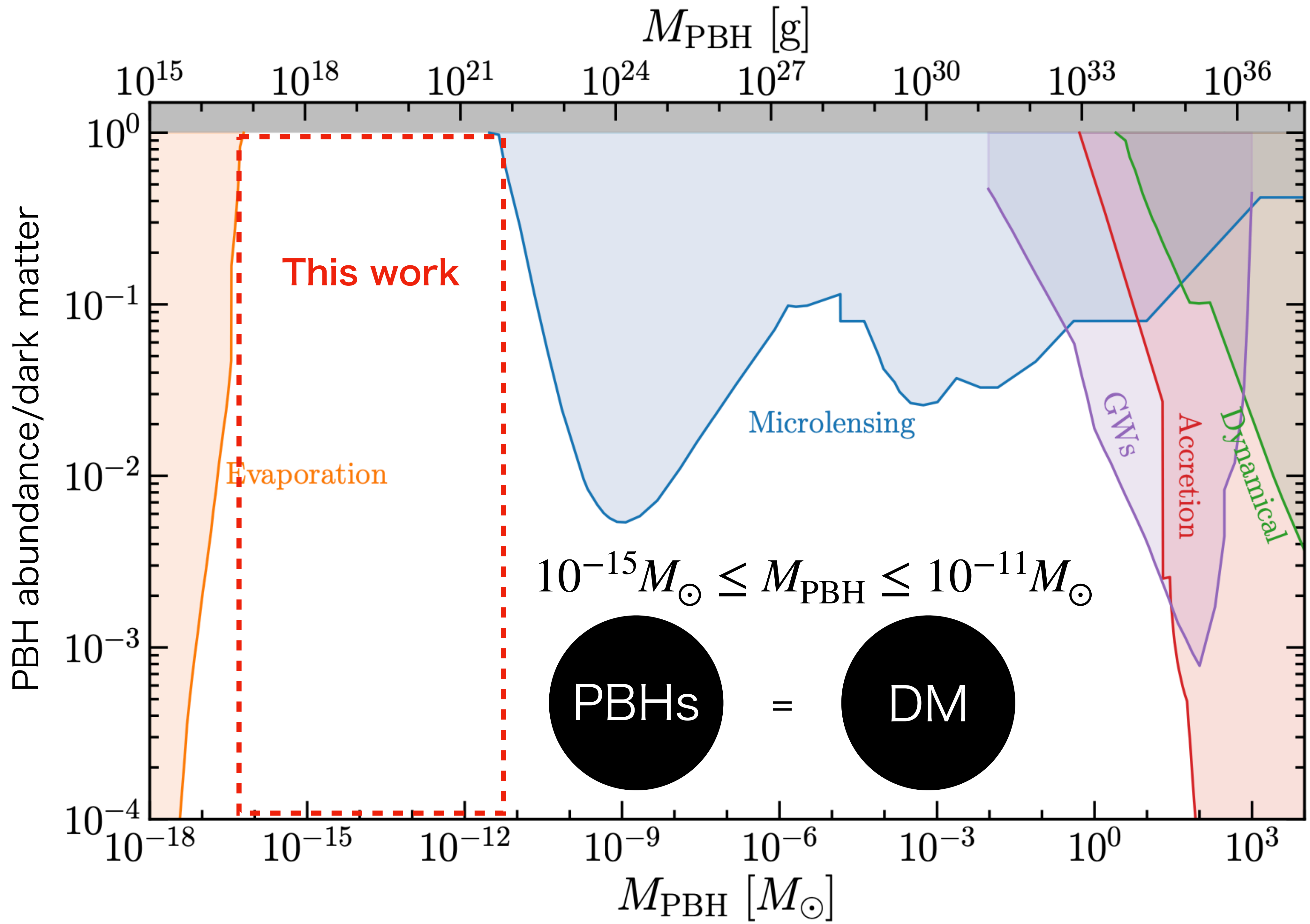


Wide range of mass

Induced GWs

Constraints of PBH abundance

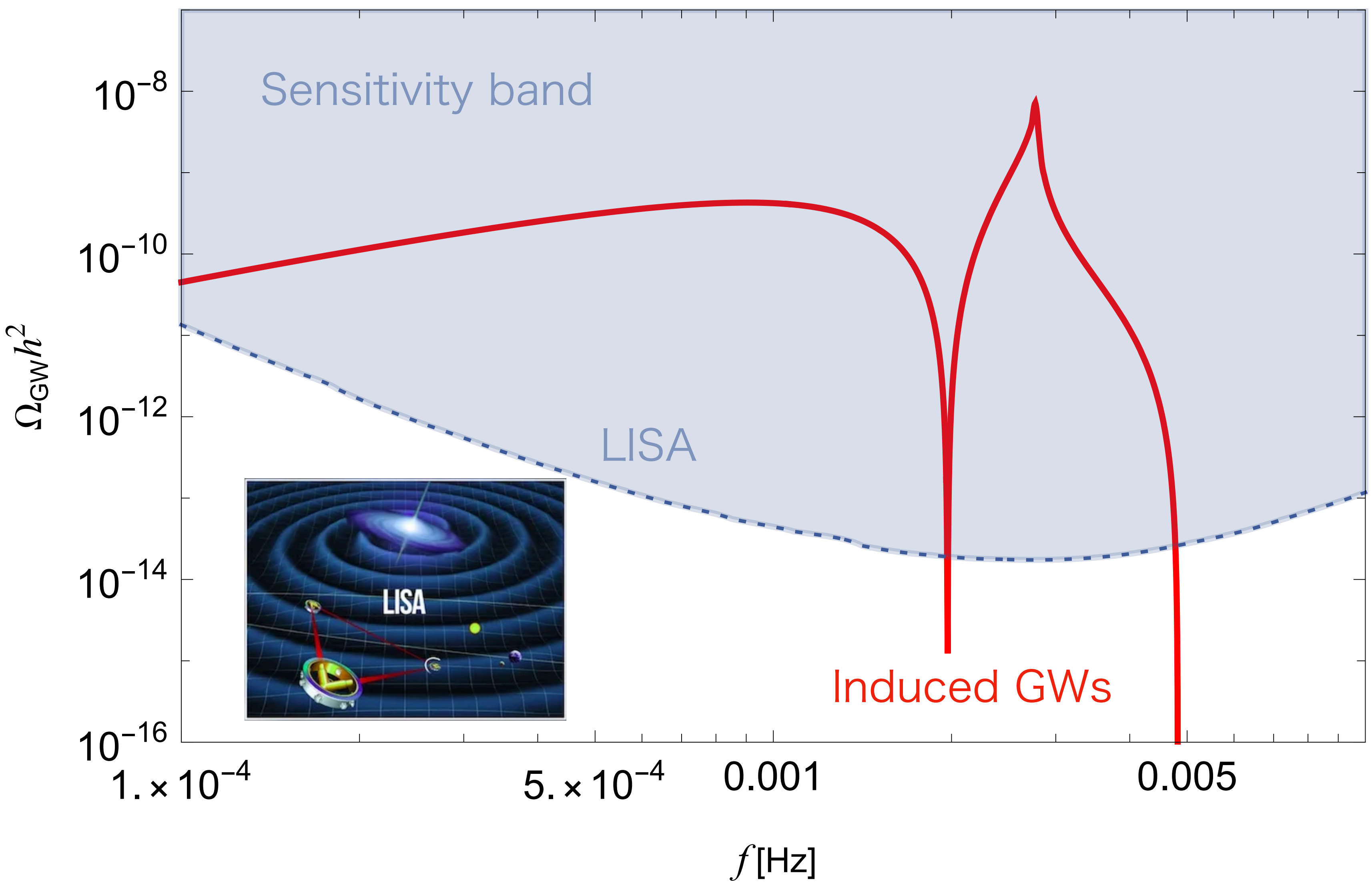
M.Green and J.Kavanagh '20, B.J Carr et.al' 20



Test of PBH DM scenario by induced GWs

N.Bartolo et al. '19, R.Saito et al. '11

Assumption : density perturbation follows Gaussian distribution



Indirect evidence of PBH=DM scenario

Non-Gaussianity of primordial perturbation

Cai, Chen, Namjoo, Sasaki, Wang and Wang '18, Atal, Cid, Escriv`a and Garriga '19,

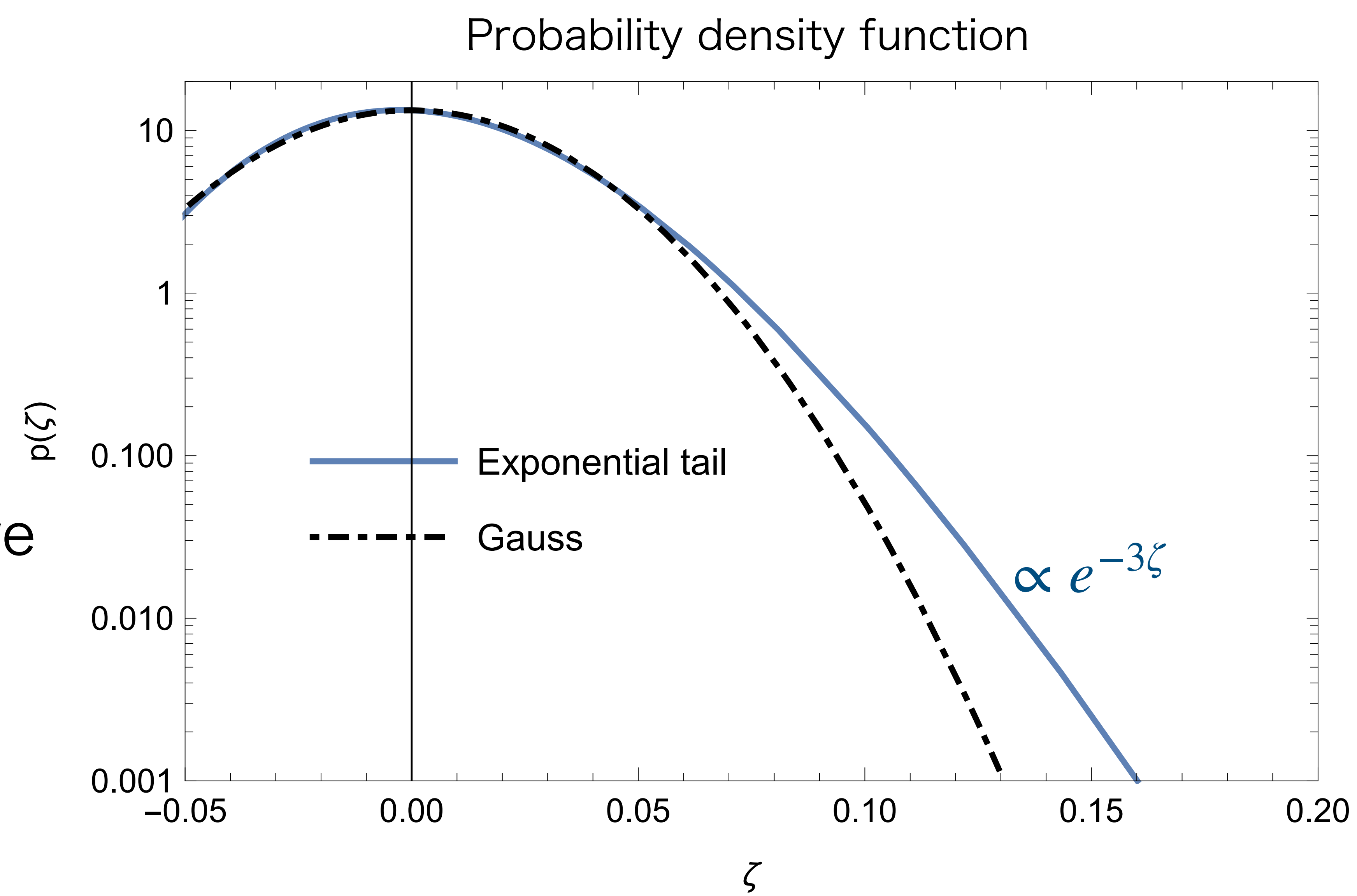
G.Figuroa, Raatikainen, Rasanen, Tomberg '21

Exponential tail

$$\zeta = -\frac{1}{3}\ln(1-3\zeta_g)$$

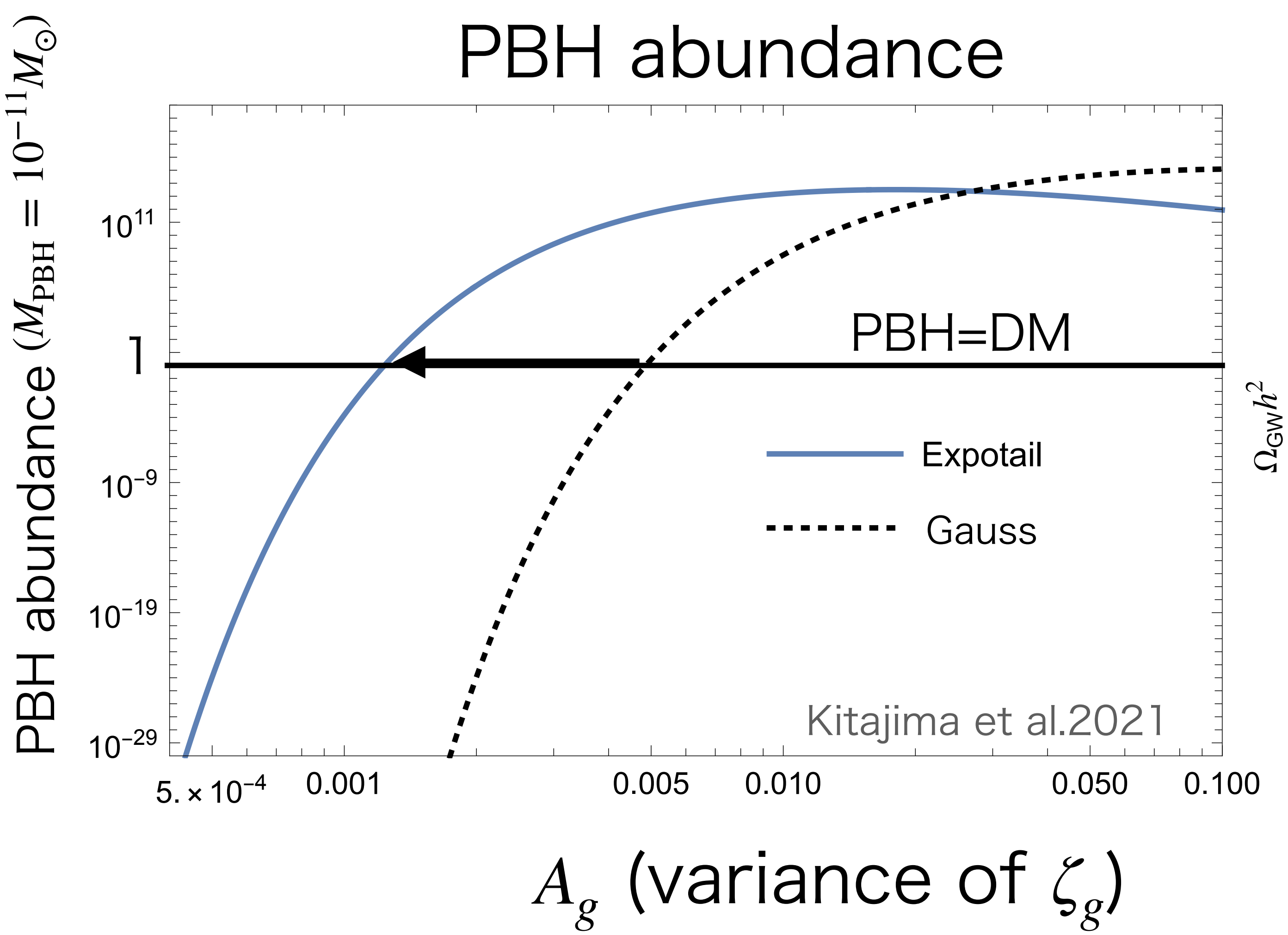
ζ : primordial curvature perturbation

ζ_g : Random field

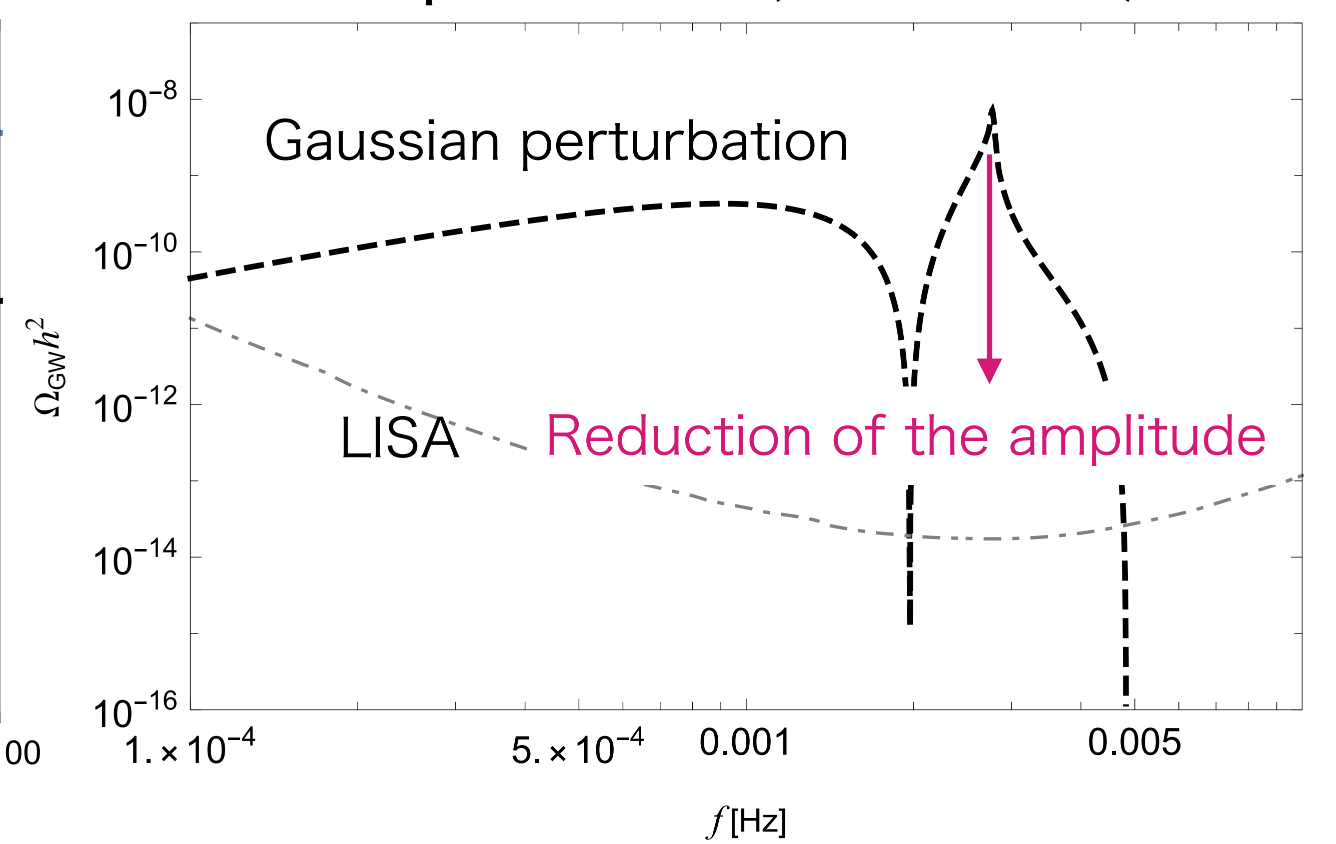


The aim of this work

PBH abundance



GWs spectrum (PBH=DM)



Test the detectability of GWs Induced by Expotail in LISA

Formulation of scalar induced GWs

Equation of motion for GWs

$$\Box h_\lambda(\tau,\boldsymbol{k}) + 2\mathscr{H} h'_\lambda(\tau,\boldsymbol{k}) = 4S_\lambda(\tau,\boldsymbol{k}) \propto \zeta(q_1)\zeta(|\boldsymbol{k}-\boldsymbol{q}_1|)$$

$(\lambda = +, \times)$ Source → Solution: $h_\lambda(\tau,\boldsymbol{k}) \propto \zeta(q_1)\zeta(|\boldsymbol{k}-\boldsymbol{q}_1|)$

Power spectrum of GWs

$$(2\pi)^3 P_{\lambda\lambda}(\tau,k)\delta^{(3)}(\mathbf{k}+\mathbf{k}') = \left\langle h_\lambda(\tau,\mathbf{k}) h_\lambda(\tau,\mathbf{k}') \right\rangle \propto \left\langle \zeta(\boldsymbol{q}_1)\zeta(\boldsymbol{k}-\boldsymbol{q}_1)\zeta(\boldsymbol{q}_2)\zeta(\boldsymbol{k}-\boldsymbol{q}_2) \right\rangle$$

Ex) : $\zeta = \zeta_g$ (Gaussian perturbation)

$$\left\langle \zeta(\boldsymbol{q}_1)\zeta(\boldsymbol{k}-\boldsymbol{q}_1)\zeta(\boldsymbol{q}_2)\zeta(\boldsymbol{k}-\boldsymbol{q}_2) \right\rangle = \left\langle \underbrace{\zeta_g(\boldsymbol{q}_1)\zeta_g(\boldsymbol{k}-\boldsymbol{q}_1)}_{\sim O(A_g)} \zeta_g(\boldsymbol{q}_2)\zeta_g(\boldsymbol{k}-\boldsymbol{q}_2) \right\rangle \sim O(A_g^2)$$

Formulation of scalar induced GWs

Non-Gaussian perturbation

$$\zeta = -\frac{1}{3}\ln\left(1 - 3\zeta_g\right) = \zeta_g + \underline{F_{\rm NL}\zeta_g^2 + G_{\rm NL}\zeta_g^3 + H_{\rm NL}\zeta_g^4 + I_{\rm NL}\zeta_g^5} \cdots$$

Corrections of non-Gaussianities

Exponential tail

$F_{\rm NL} = 3/2,\ G_{\rm NL} = 3,\ H_{\rm NL} = 27/4,\ I_{\rm NL} = 81/5$

Ex) :

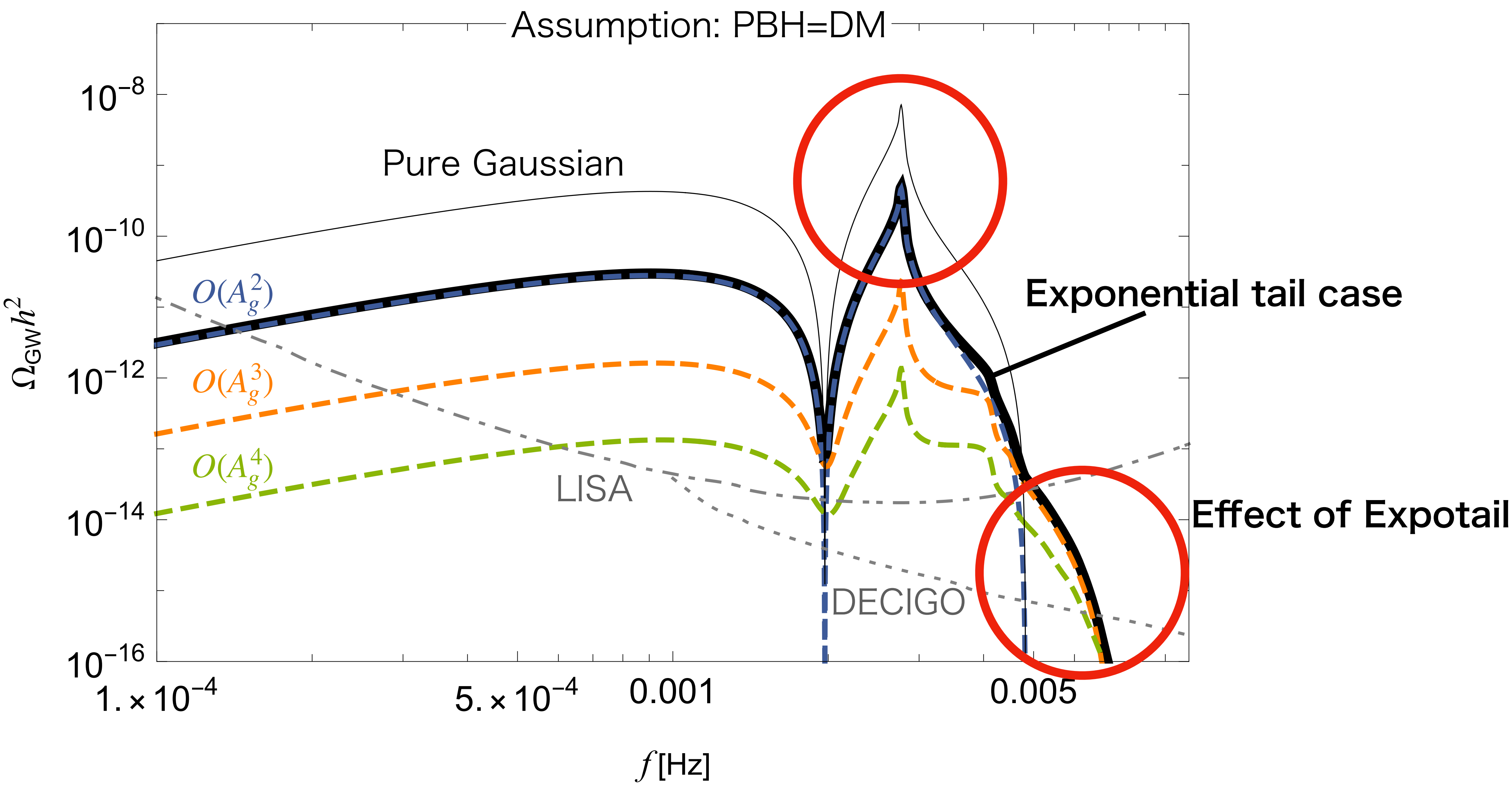
Effects of Exponential tail

$$\langle\zeta\zeta\zeta\zeta\rangle \propto \underbrace{\left\langle \zeta_g\zeta_g\zeta_g\zeta_g \right\rangle}_{O(A_g^2)} + \underbrace{F_{\rm NL}^2\left\langle \zeta_g\zeta_g\zeta_g^2\zeta_g^2 \right\rangle + G_{\rm NL}\left\langle \zeta_g\zeta_g\zeta_g\zeta_g^3 \right\rangle + \cdots}_{O(A_g^3)}$$

$$+ \underbrace{F_{\rm NL}^4\left\langle \zeta_g^2\zeta_g^2\zeta_g^2\zeta_g^2 \right\rangle + F_{\rm NL}^2 G_{\rm NL}\left\langle \zeta_g\zeta_g^2\zeta_g^2\zeta_g^3 \right\rangle + \cdots}_{O(A_g^4)} + \cancel{O(A_g^5)} + \cdots$$

Result

- GWs can be detectable in LISA
- DECIGO might detect the footprint of Exponential tail



Summary

- Can GWs test the scenario where PBH = 100% DM?
- We investigated the detectability of GWs induced by Exponential tail

Exponential tail-type

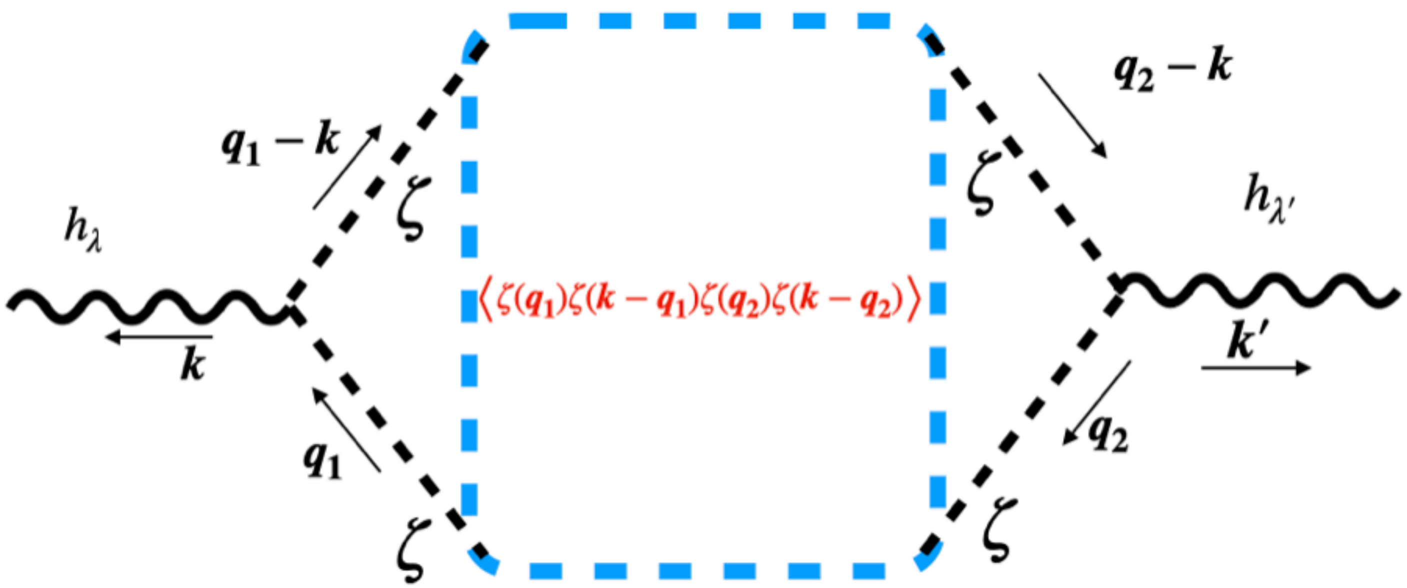
GWs are detectable in LISA

DECIGO might detect the footprint of Exponential tail

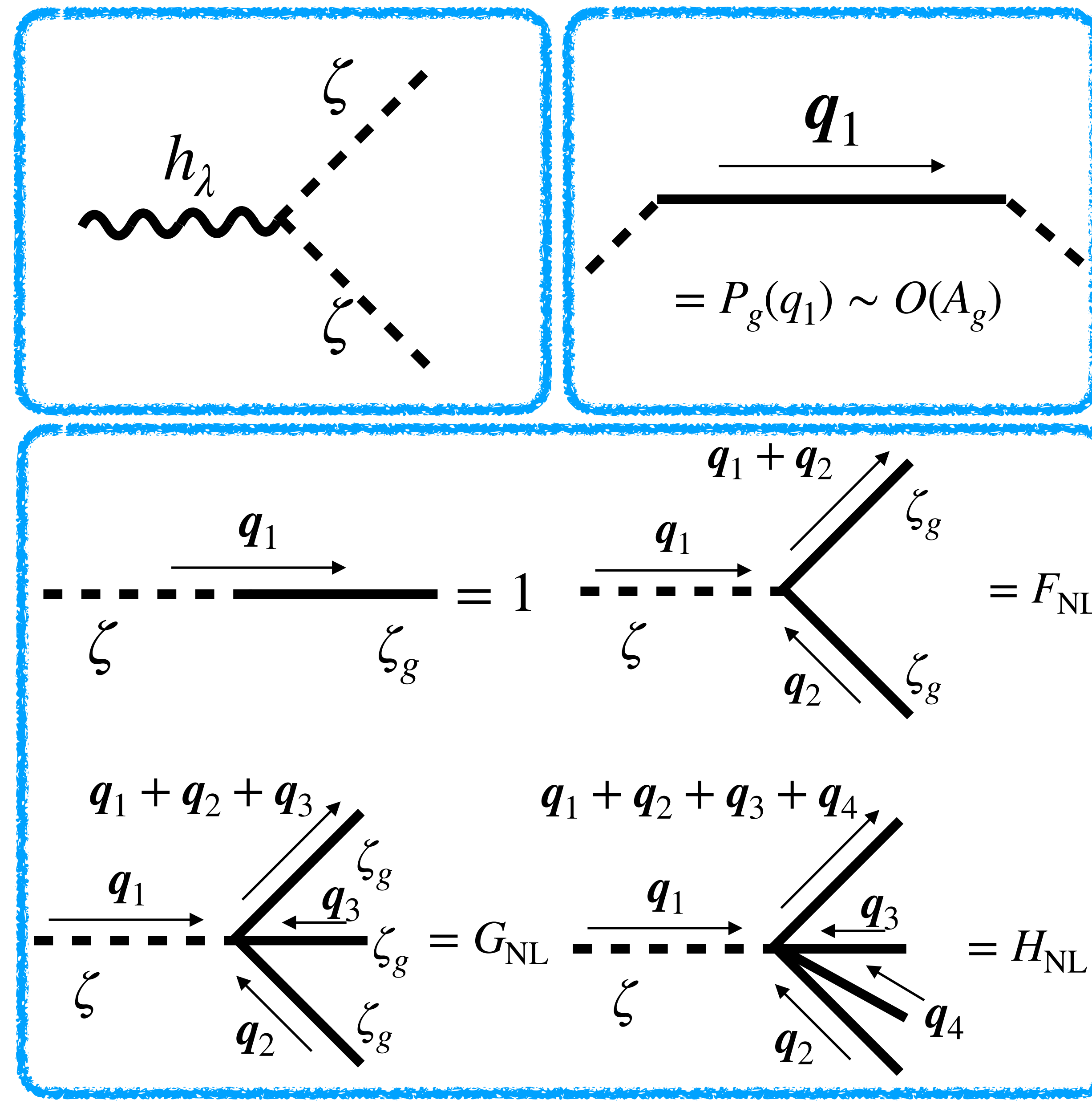
Appendix

Feynman diagram

Diagram of $\langle h_\lambda(\tau, \boldsymbol{k}) h_{\lambda'}(\tau, \boldsymbol{k}') \rangle$

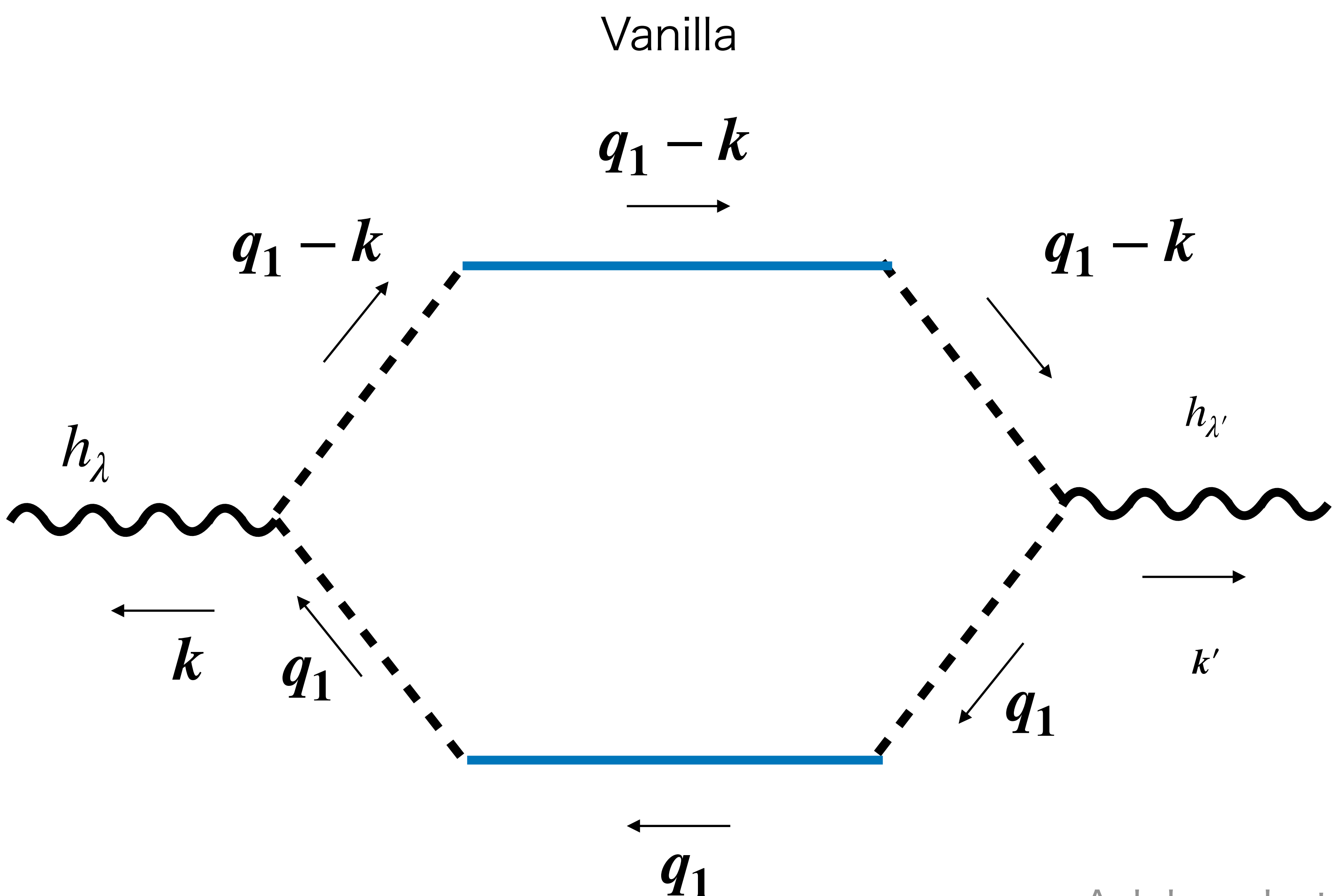


Adshead et al 2105.01659



Contribution of Gaussian perturbation

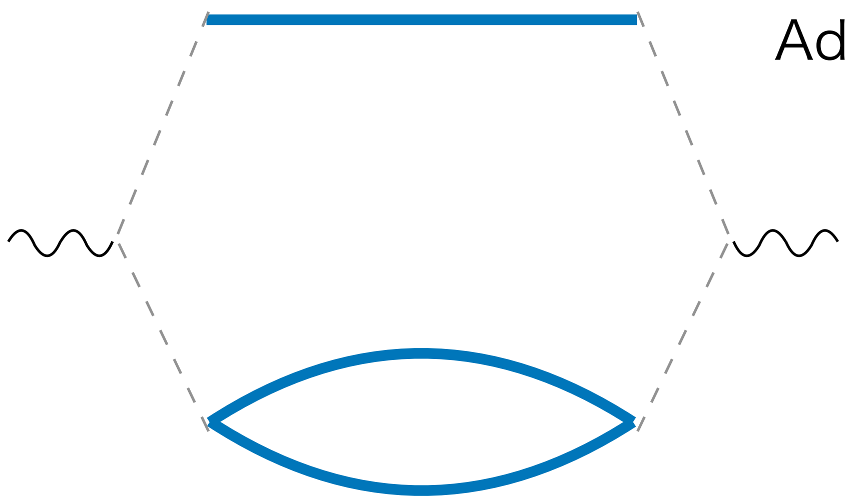
$$\left\langle \zeta_g(\boldsymbol{q_1})\zeta_g(\boldsymbol{k}-\boldsymbol{q_1})\zeta_g(\boldsymbol{q_2})\zeta_g(\boldsymbol{k}-\boldsymbol{q_2}) \right\rangle \sim O(A_g^2)$$



Adshead et al' 21

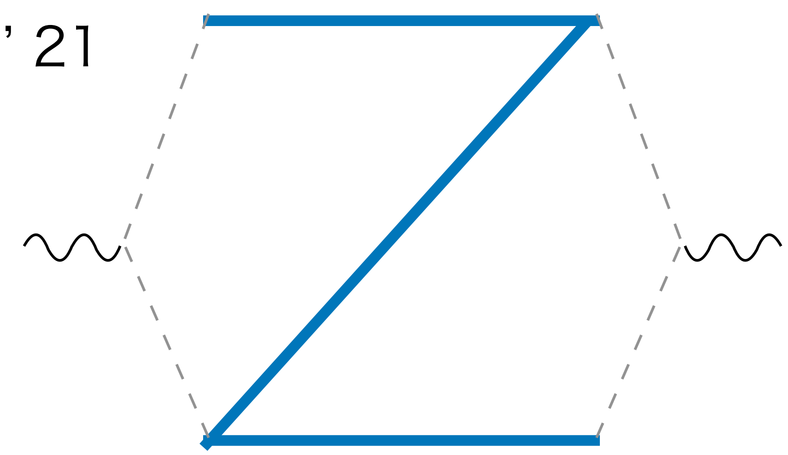
Contributions of Exponential tail

$O(A_g^3)$

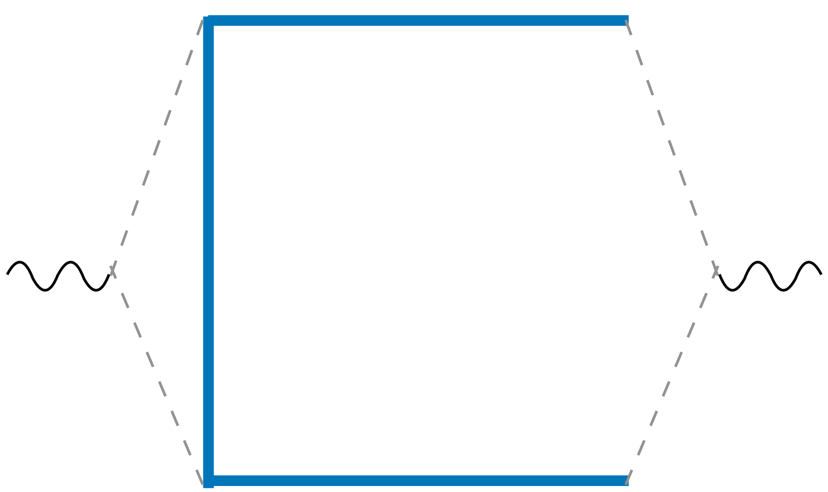


1-convolution

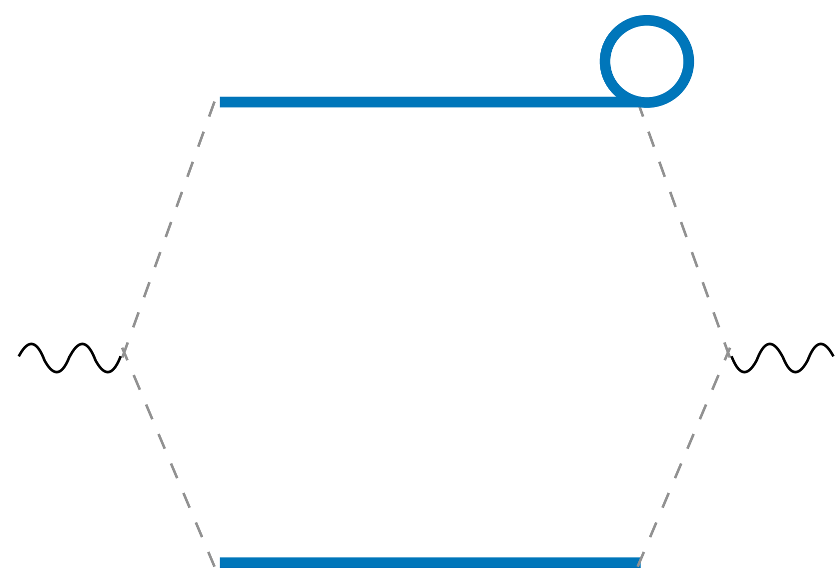
Adshead et al' 21



Z



C



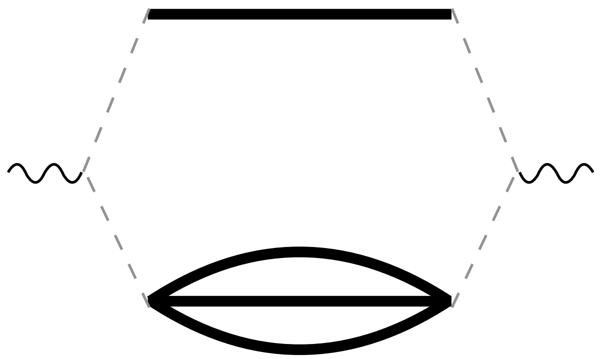
1 loop

C. Yuan and Q.-G. Huang' 20

Contributions of Exponential tail

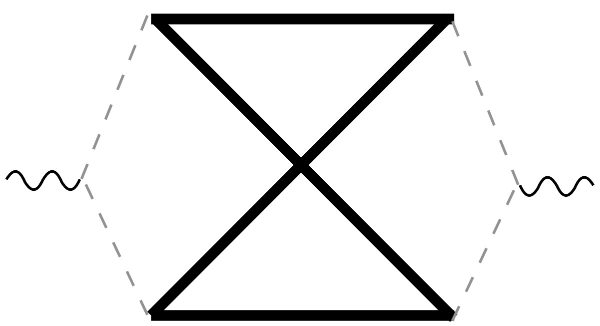
$O(A_g^4)$

Yuan and Huang, 2007.10686

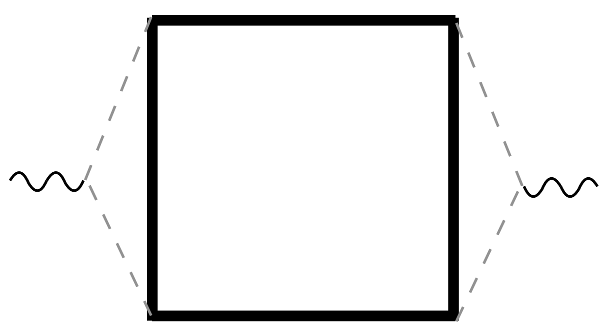


2-conv

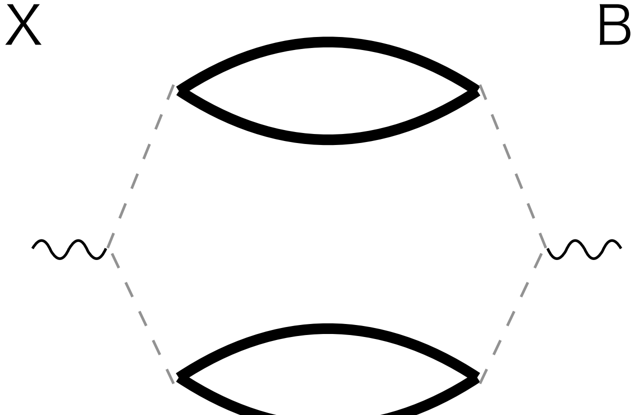
Adshead et al 2105.01659



X



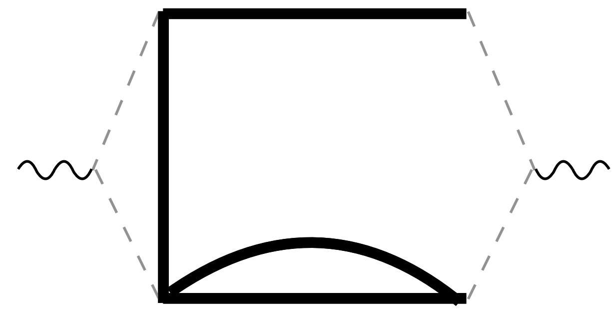
Box



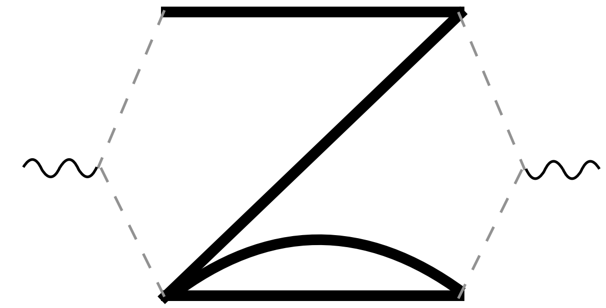
(1,1)-conv

New

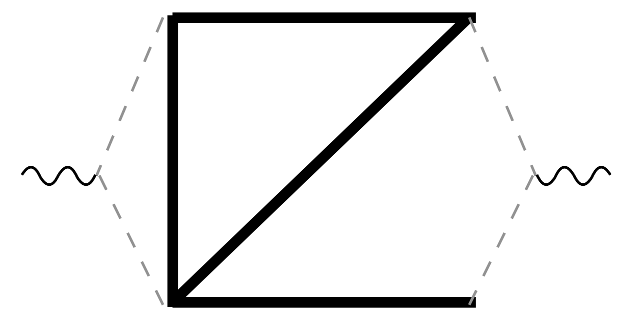
Abe, **RI**, Tada, Yokoyama



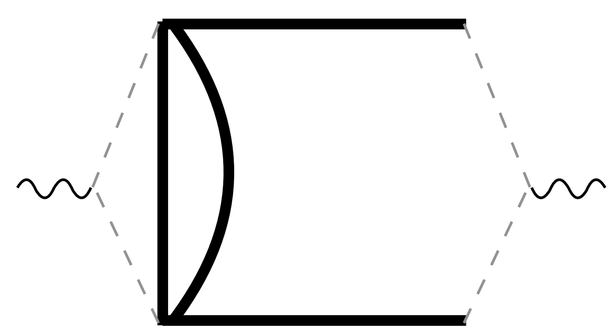
1-conv. C 1



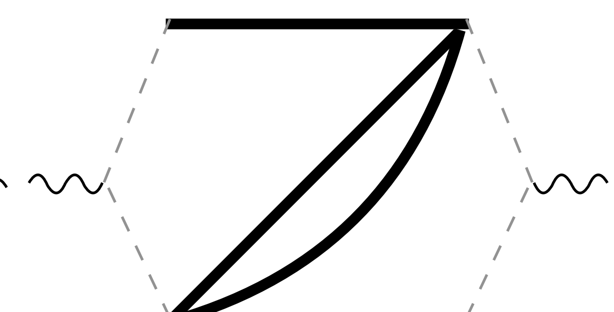
1-conv. Z 1



CZ



1-conv. C 2



1-conv. Z 2

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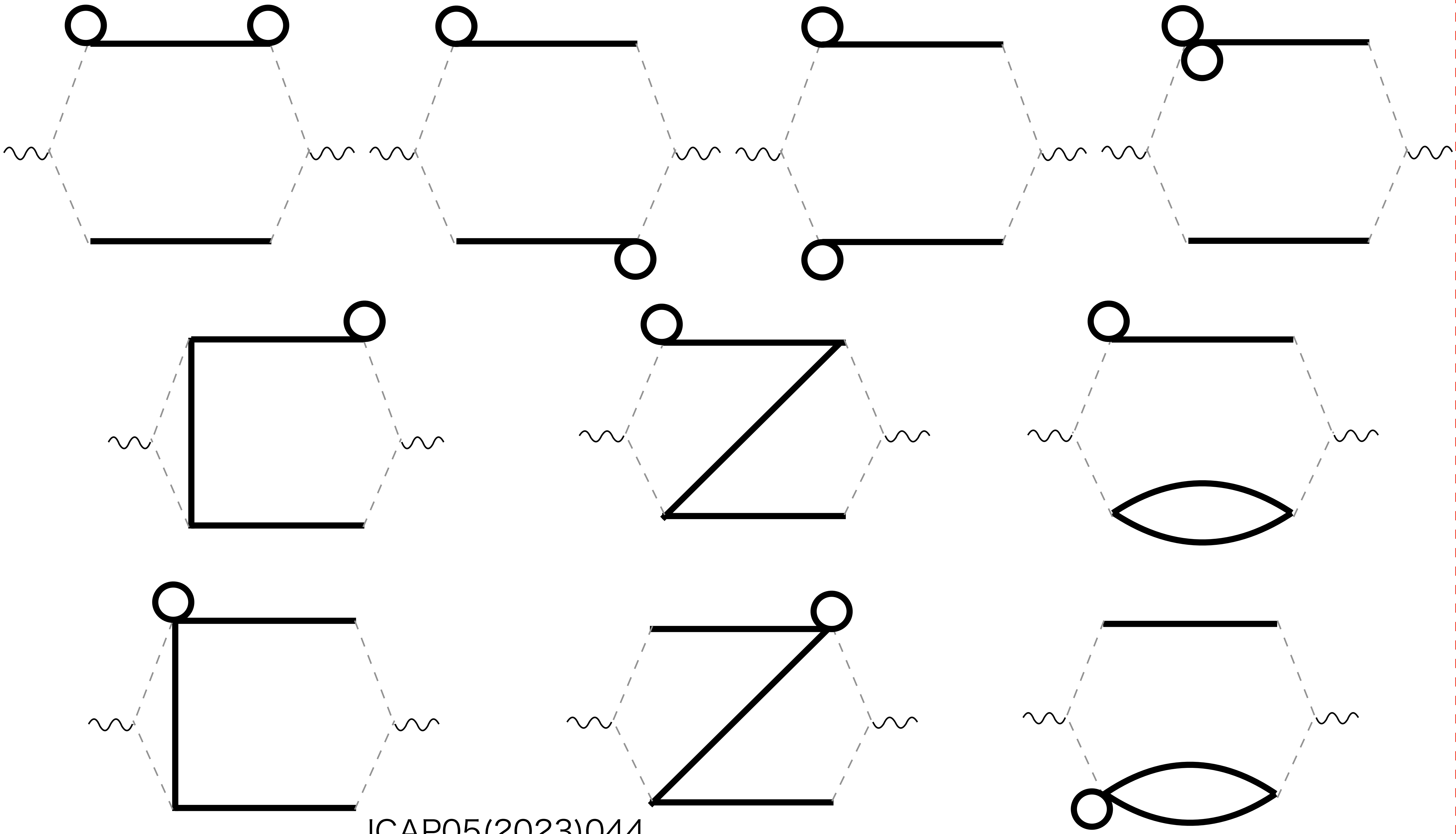
Contributions of Exponential tail

$O(A_g^4)$

Abe, **RI**, Tada, Yokoyama

New



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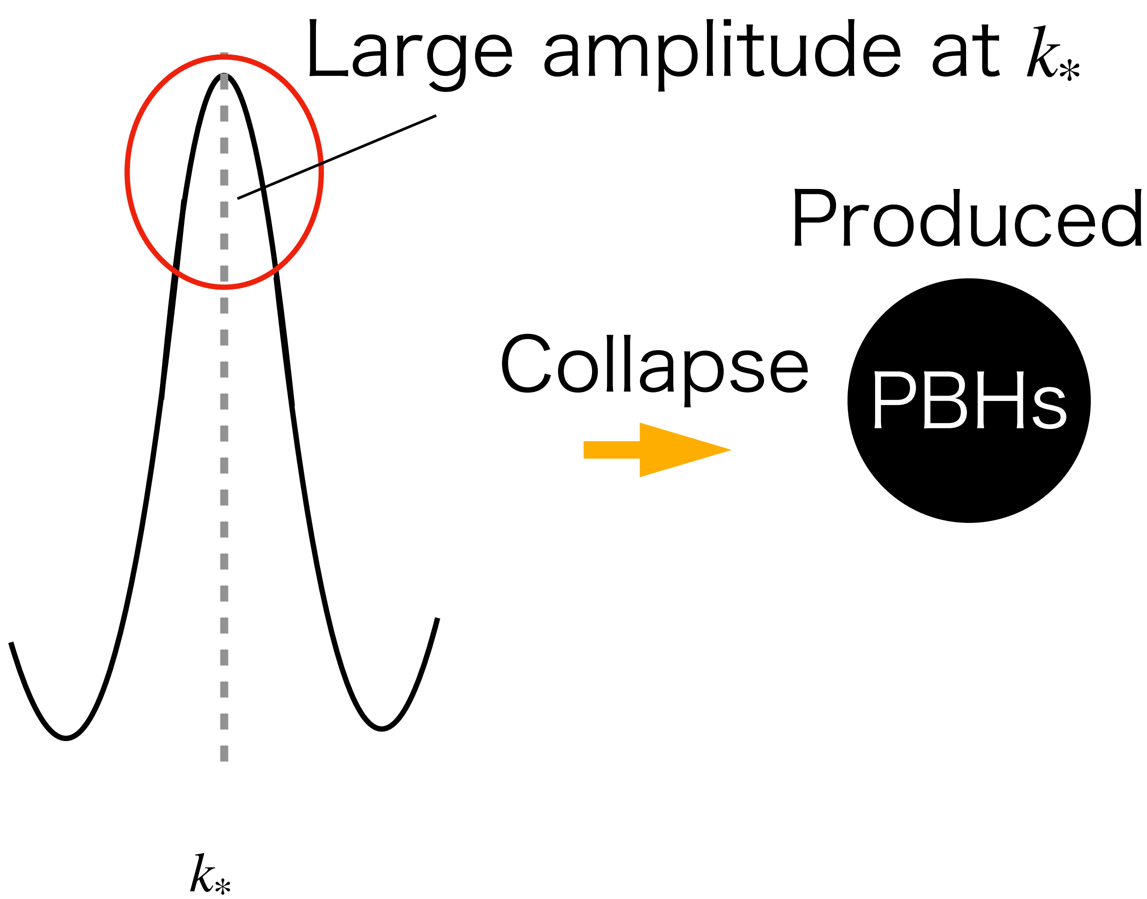
USR (Ultra slow-roll models)

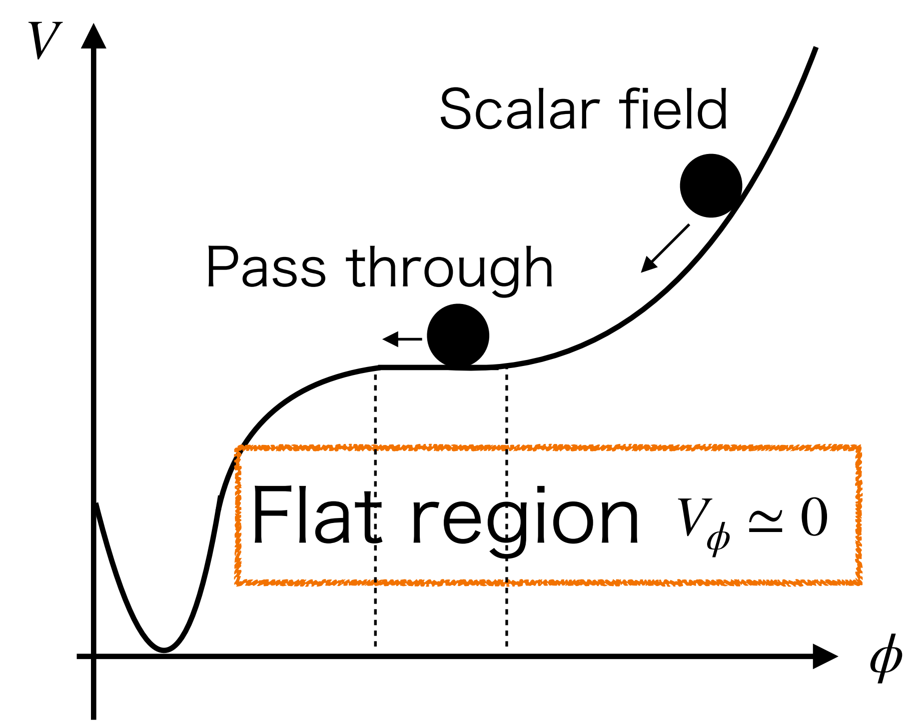
PBH can be realized in USR

Primordial density perturbation



High peak spectrum

$$P_{\zeta_g}(k) \propto \frac{H^2}{M_{\rm pl}^2 \epsilon k^3} \qquad \epsilon \equiv \frac{M_{\rm pl}}{2}\left(\frac{V_\phi}{V}\right)^2$$

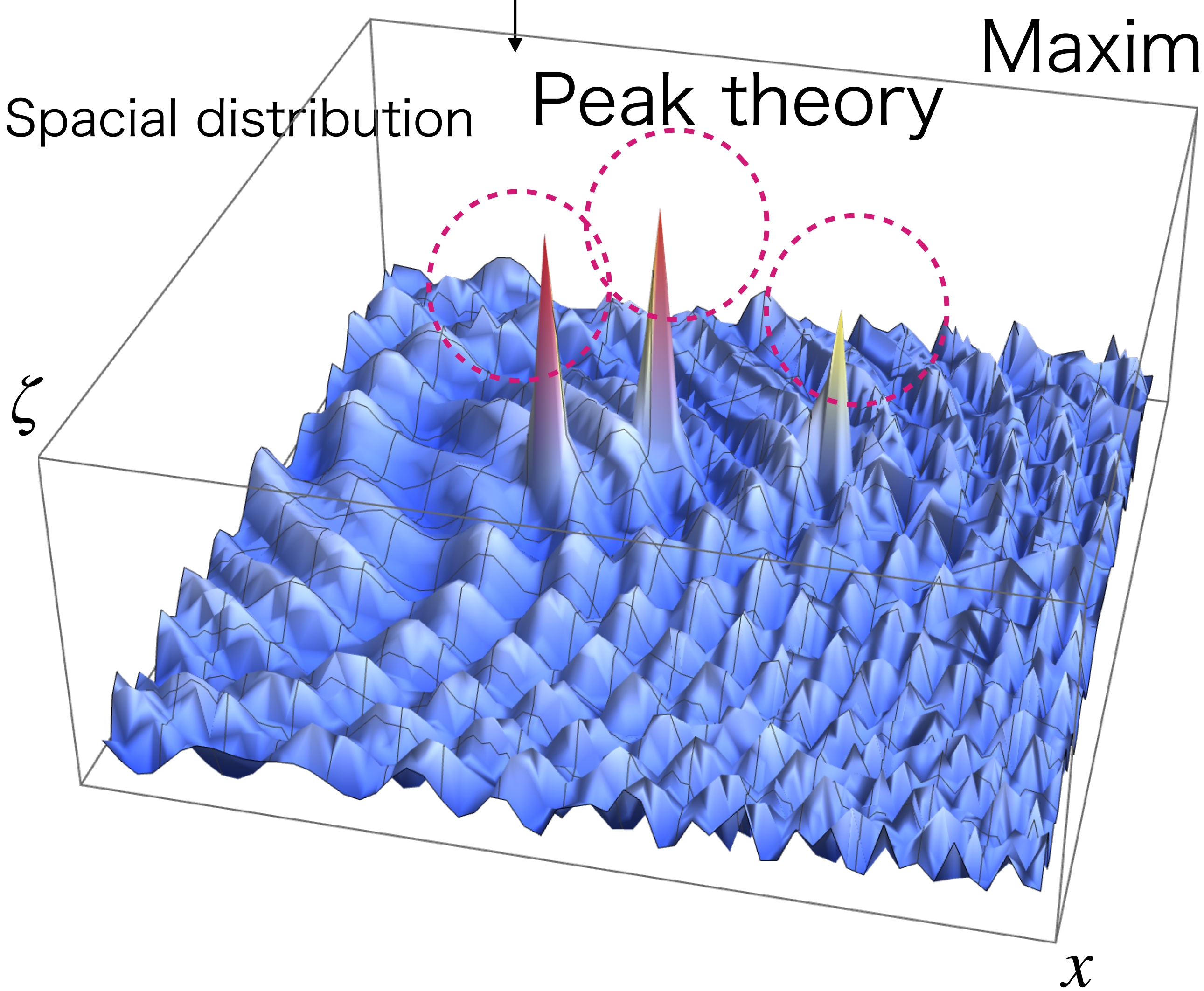


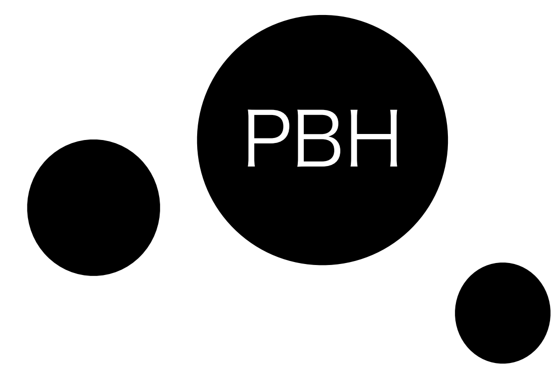
PBH abundance

Primordial perturbation : $\zeta = \frac{1}{\beta}\ln(1-\beta\zeta_g)$

Determination of PBH formation

Maximum mean compaction > threshold



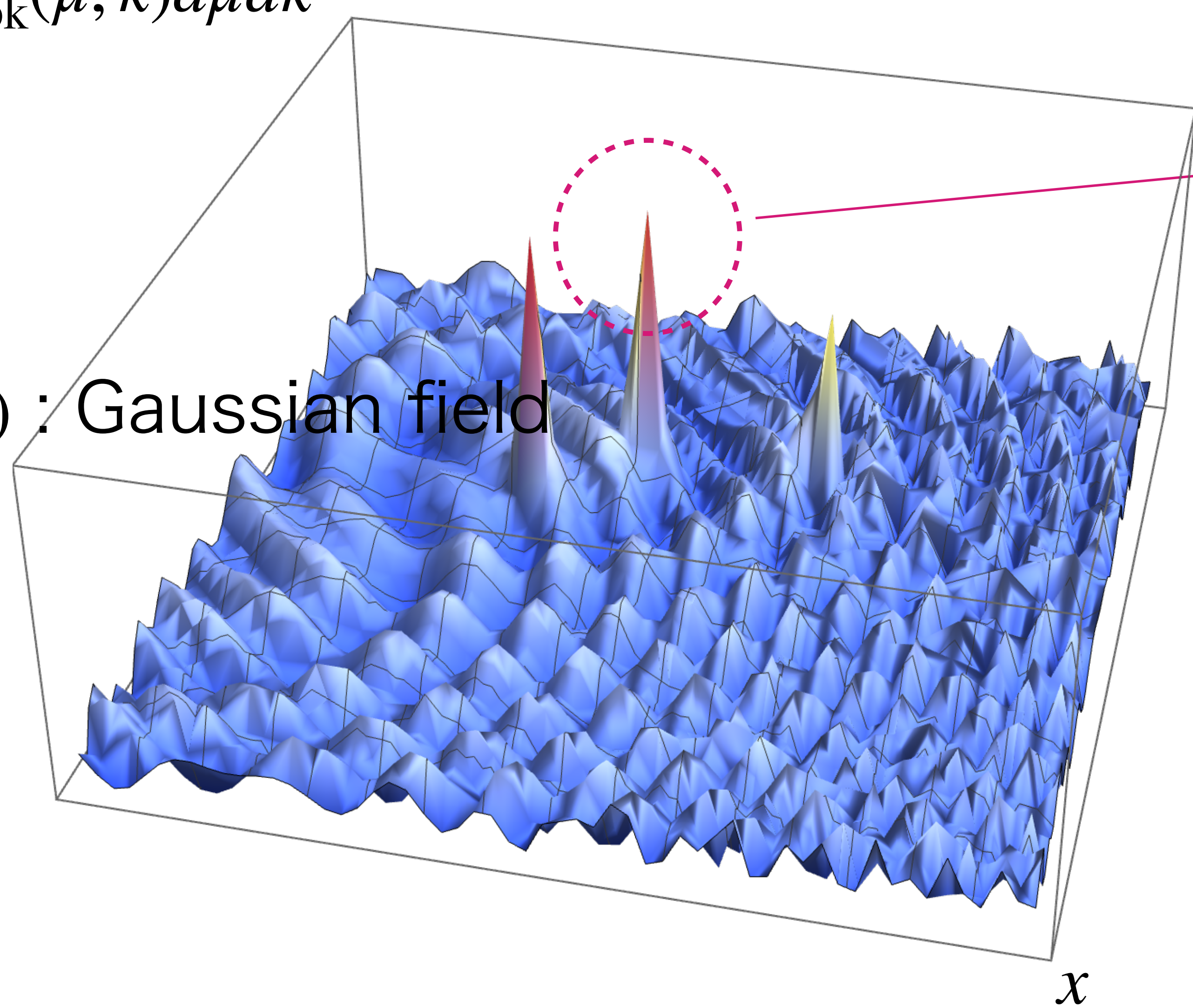


PBH abundance

Peak theory

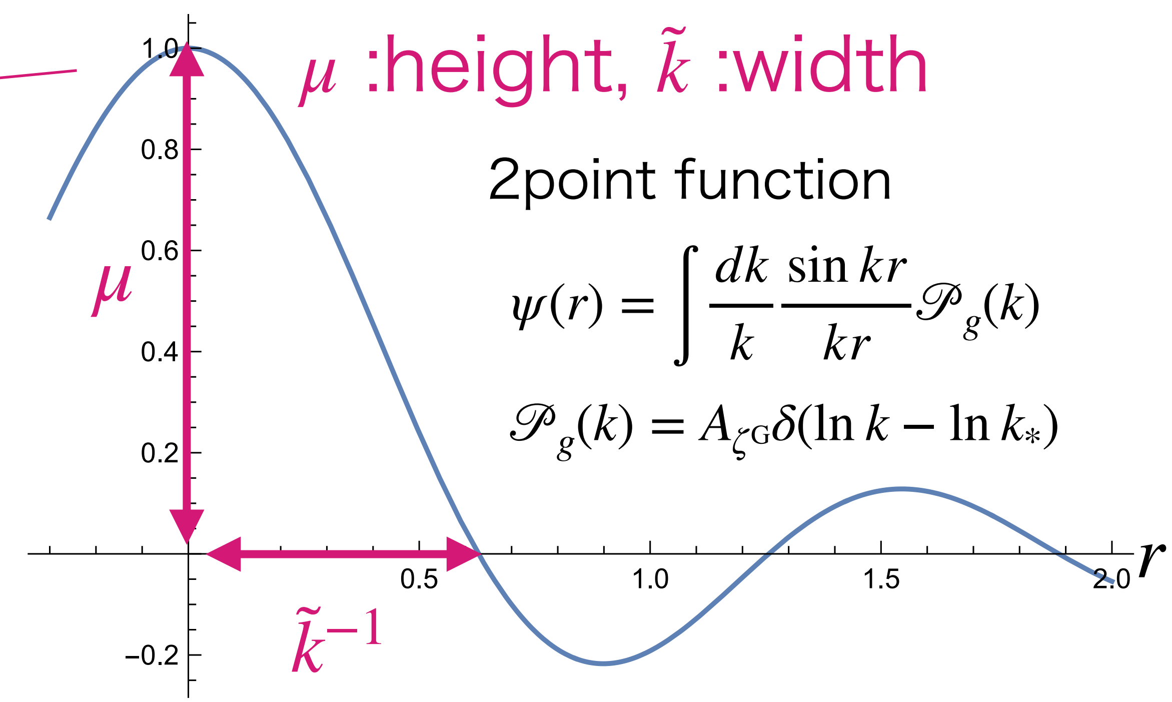
Peak number density

$n_{\rm pk}(\mu, \tilde{k})d\mu d\tilde{k}$



Around peak : spherical symmetric

$$\hat{\zeta}_g(r) = \hat{\zeta}_g(\psi(r), \mu, \tilde{k}) = \mu \frac{\sin k_* r}{k_* r}$$



2point function

$$\psi(r) = \int \frac{dk}{k} \frac{\sin kr}{kr} \mathscr{P}_g(k)$$

$$\mathscr{P}_g(k) = A_{\zeta \rm G} \delta(\ln k - \ln k_*)$$

Yoo, Harada, Garriga, and Kohri ’18

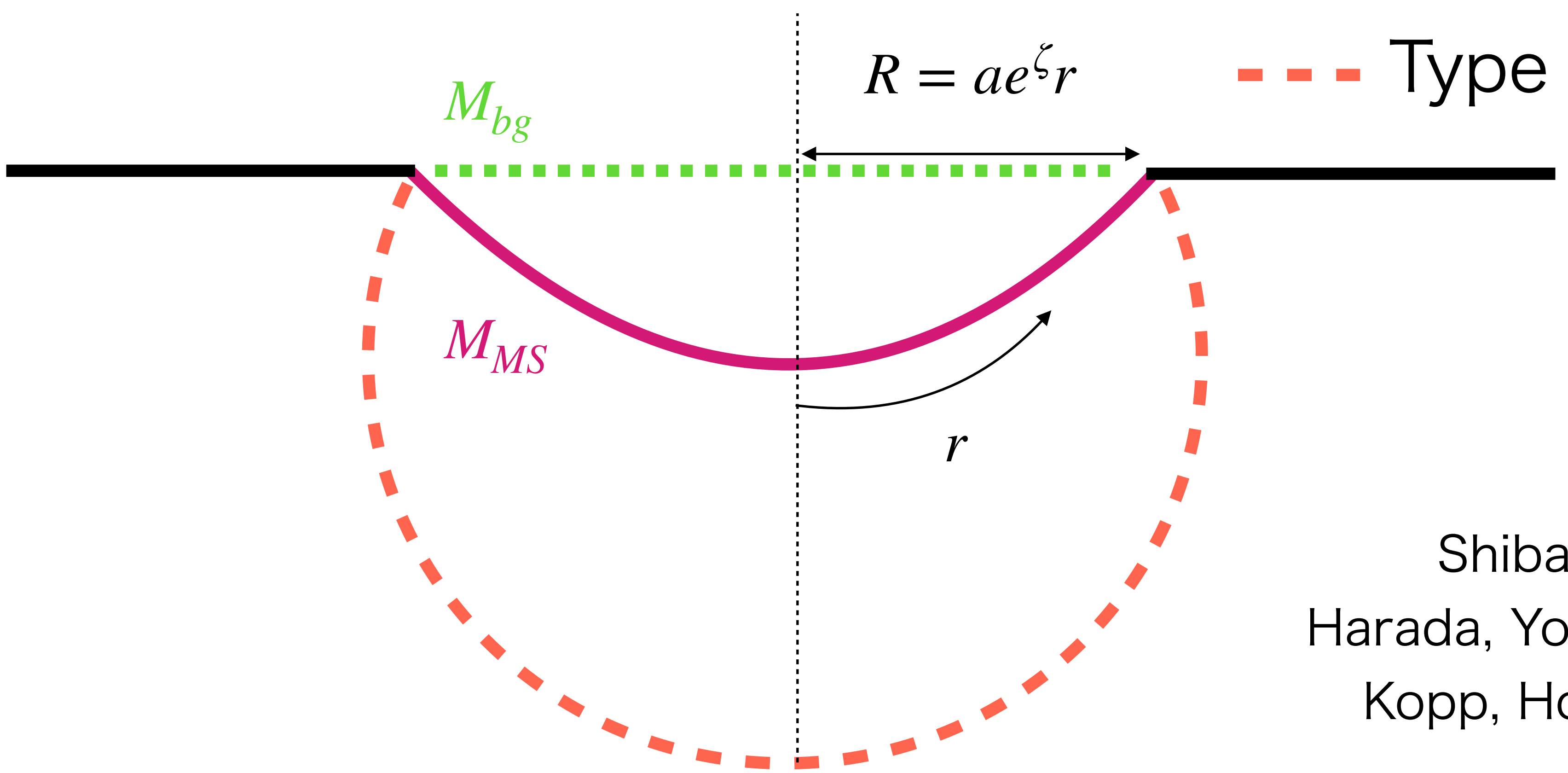
Compaction function

Overdensity | Spacial mean value | Conserved at super horizon

$$C = \frac{M_{MS} - M_{bg}}{4\pi M_{\rm pl}^2 R} = \frac{1}{V(R)} \int_0^R \delta 4\pi R^2 dR \bigg|_{R=H^{-1}} = \frac{2}{3}[1-(1+r\zeta')^2] \overset{?}{>} C_{\rm th}$$

Type I : $R =$ monotonic

Type II : $R \neq$ monotonic



Shibata & Sasaki '99
Harada, Yoo, Nakama, Koga '15
Kopp, Hofmann, Weller '11

Mean compaction function

Threshold value of the compaction function

$$\frac{1}{5} \leq C_{\rm th} \leq \frac{1}{3}$$

Changed by the peak profile

Mean compaction function

$$\bar{C} = \frac{1}{V(R)} \int_0^R C(R) \times 4\pi R^2 dR$$

Almost universal

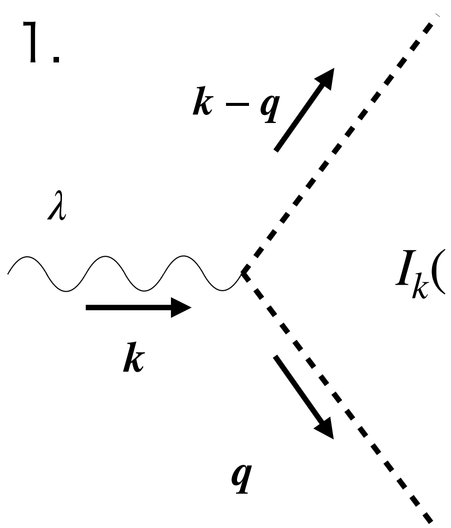
$$\bar{C}_{\rm m} > \boxed{\bar{C}_{\rm th} \simeq \frac{2}{5}}$$

Atal, Cid, Escrivà, Garriga '19
Escriv`a, Germani, Sheth '19

Diagrammatic approach

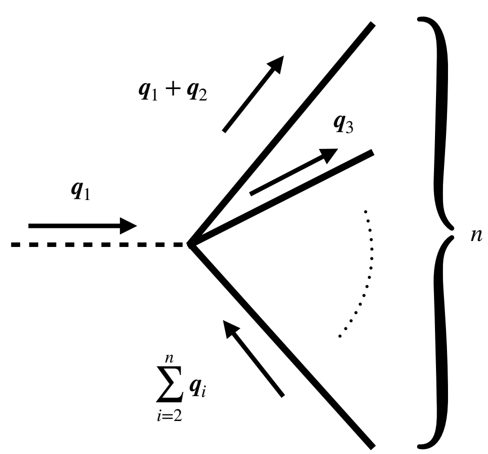
Diagrammatic rules

1.



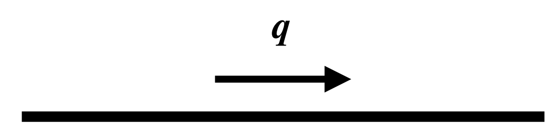
$I_k(|\boldsymbol{k}-\boldsymbol{q}|, q, k)Q(\boldsymbol{k}, \boldsymbol{q})$

2.



$n!F_{\mathrm{NL}}^{(n-1)}$

3.



$P_g(q)$

4. Integrate over each undetermined momentum

$$\int \frac{d^3q}{(2\pi)^3}$$

5. Divide by the symmetric factor

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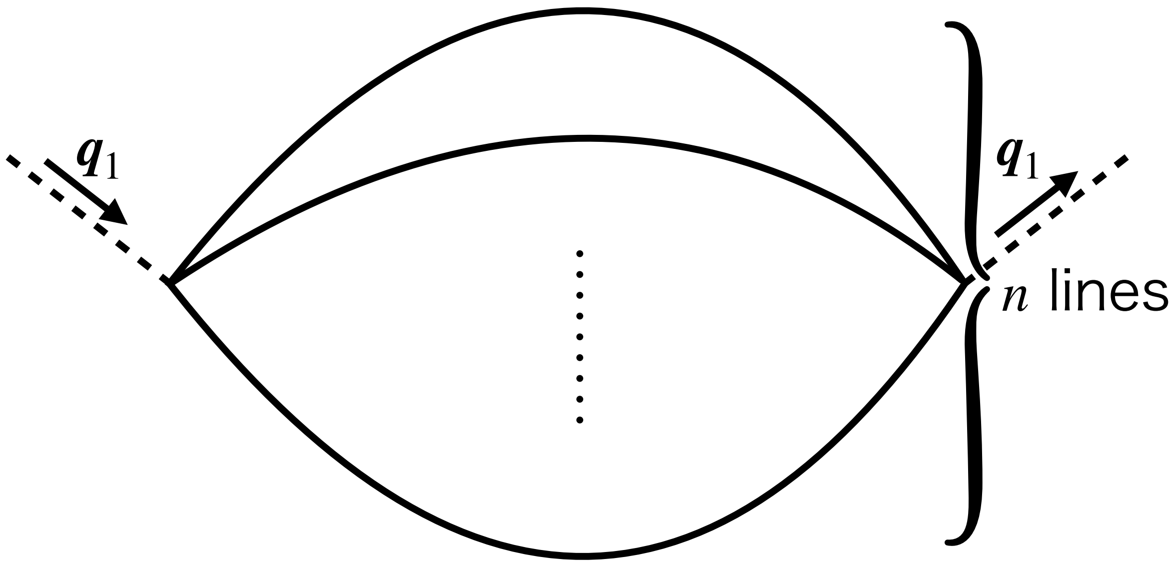
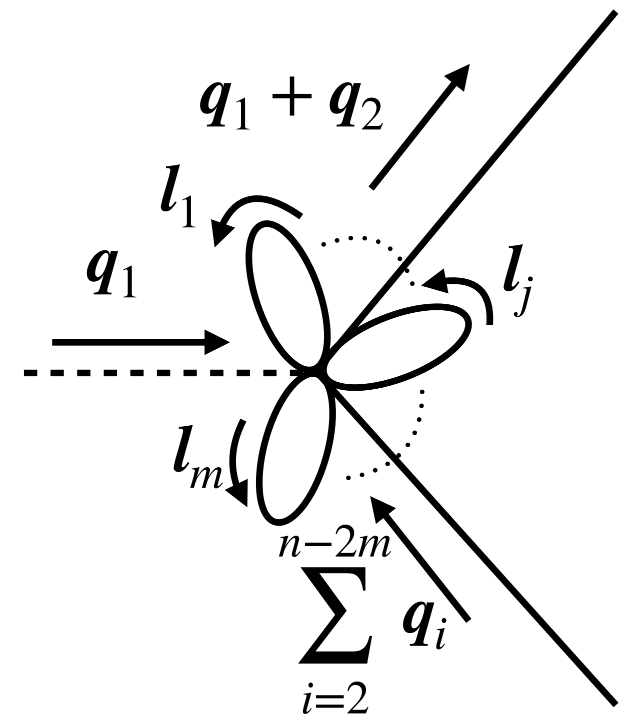
Diagrammatic approach

Loop structures



Symmetric factor $n!$



Symmetric factor $2^m m!$

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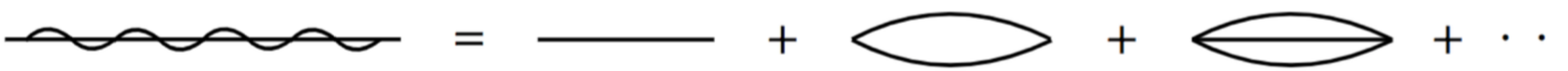
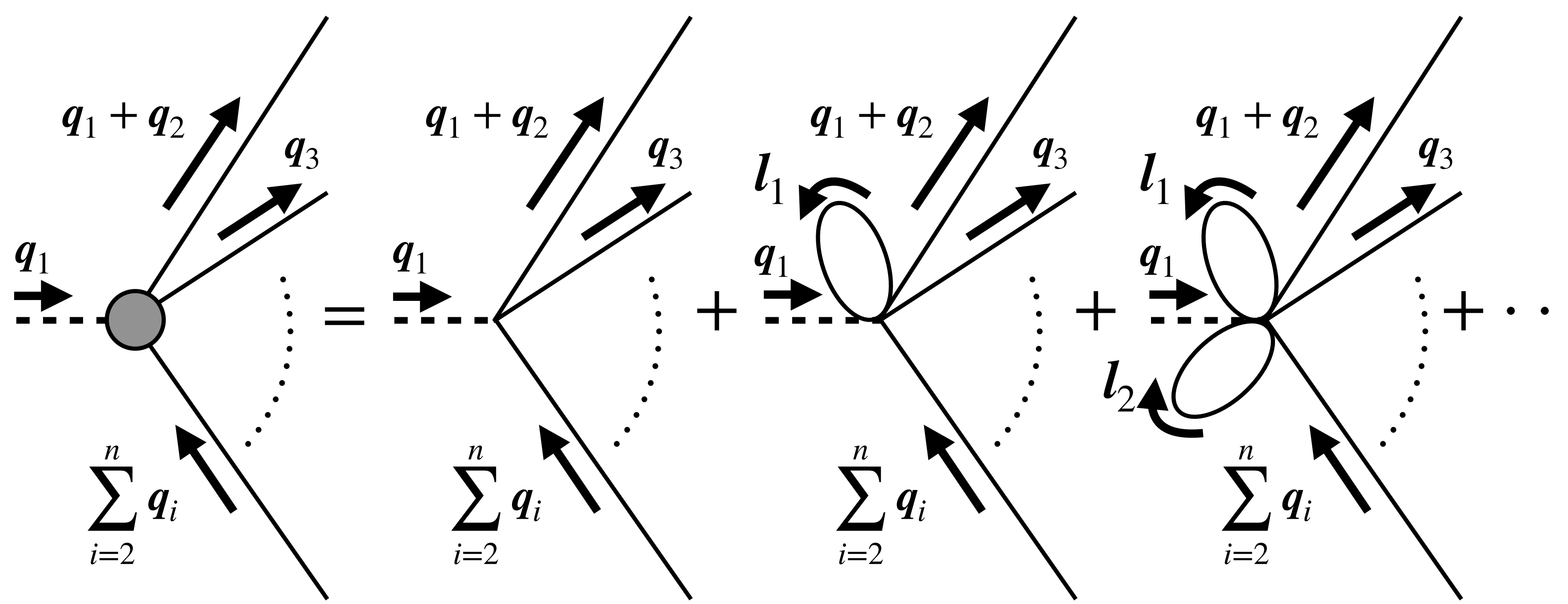
Diagrammatic approach

Renormalized propagator



Renormalized vertex



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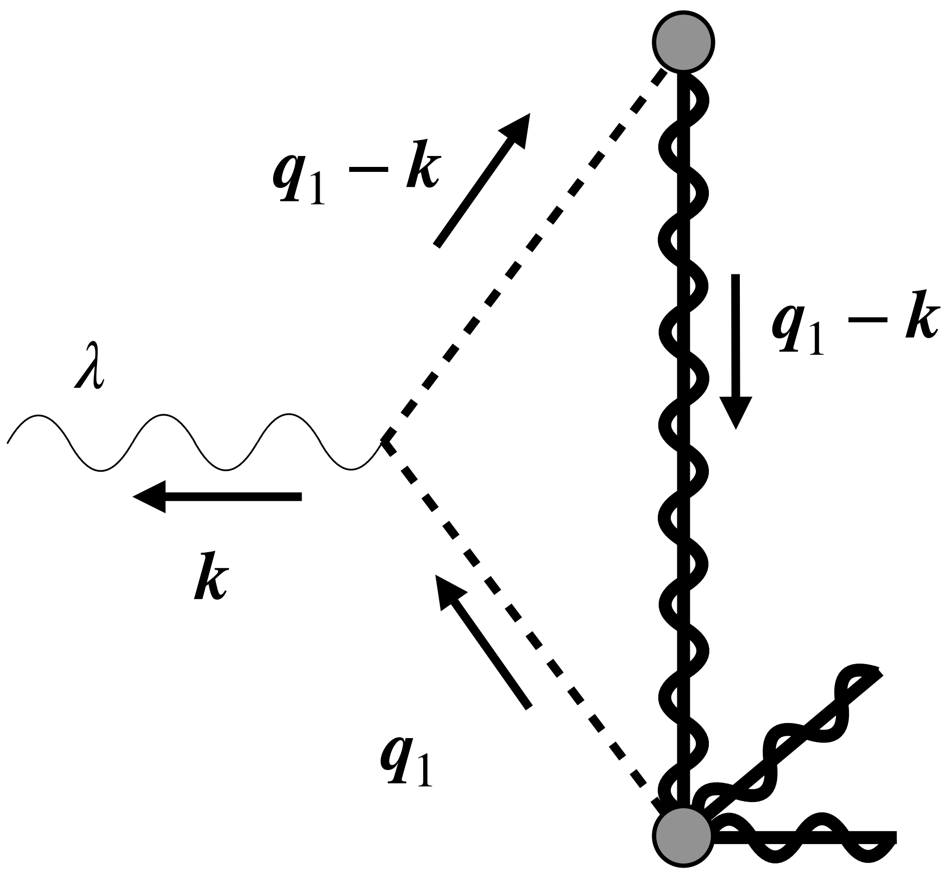
Diagrammatic approach

Prohibited structure



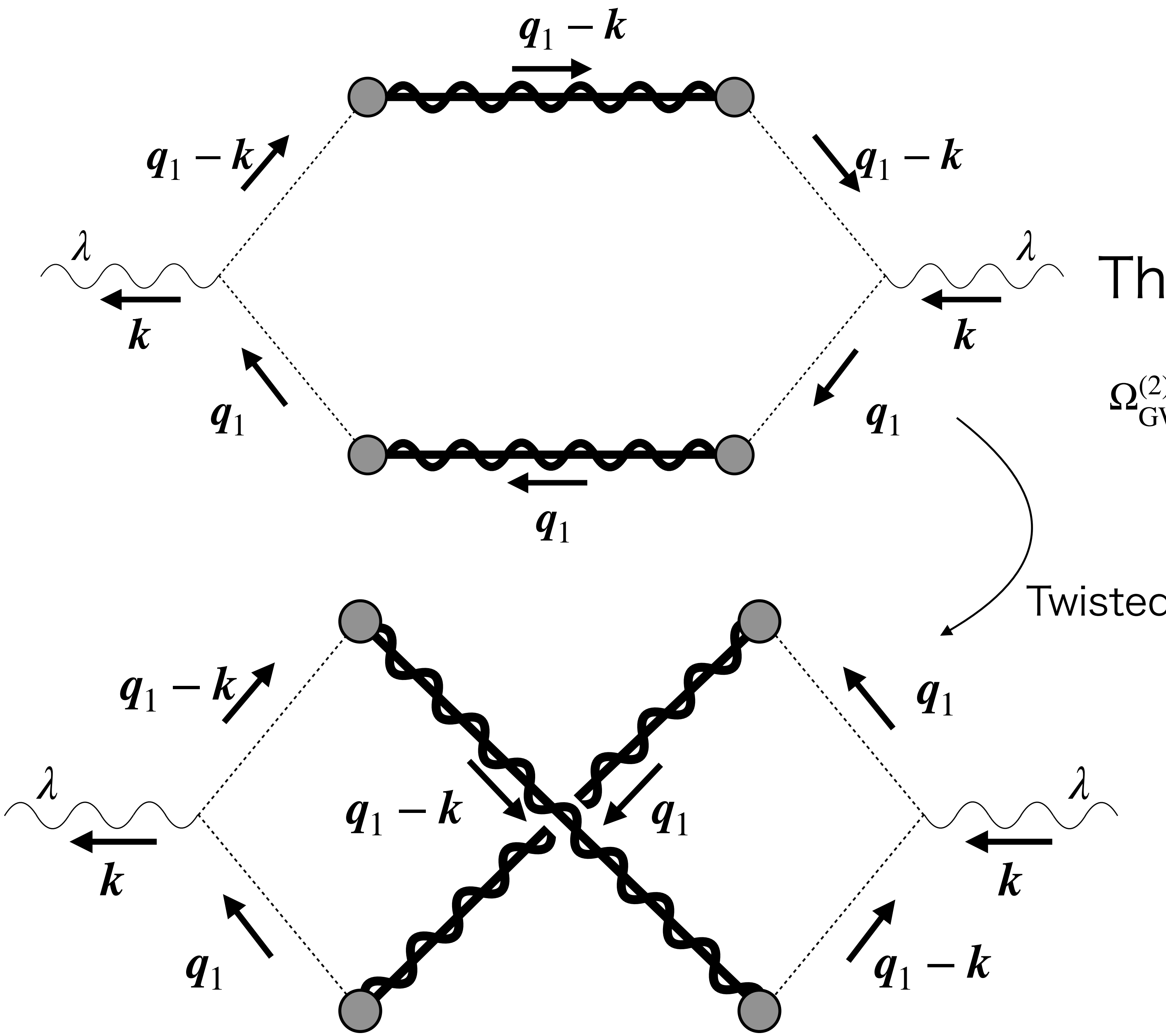
$$\int \frac{d^3q}{(2\pi)^3}\, Q_\lambda(\mathbf{k}, \mathbf{q})\, I(|\mathbf{k}-\mathbf{q}|, \mathbf{q}, \tau)\, P_g(q)$$

$$= \int_0^{2\pi} d\phi \begin{cases} \cos 2\phi & (\lambda = +) \\ \sin 2\phi & (\lambda = \times) \end{cases} \times \mathscr{F}(k, q, \theta, \tau)$$

$$= 0$$

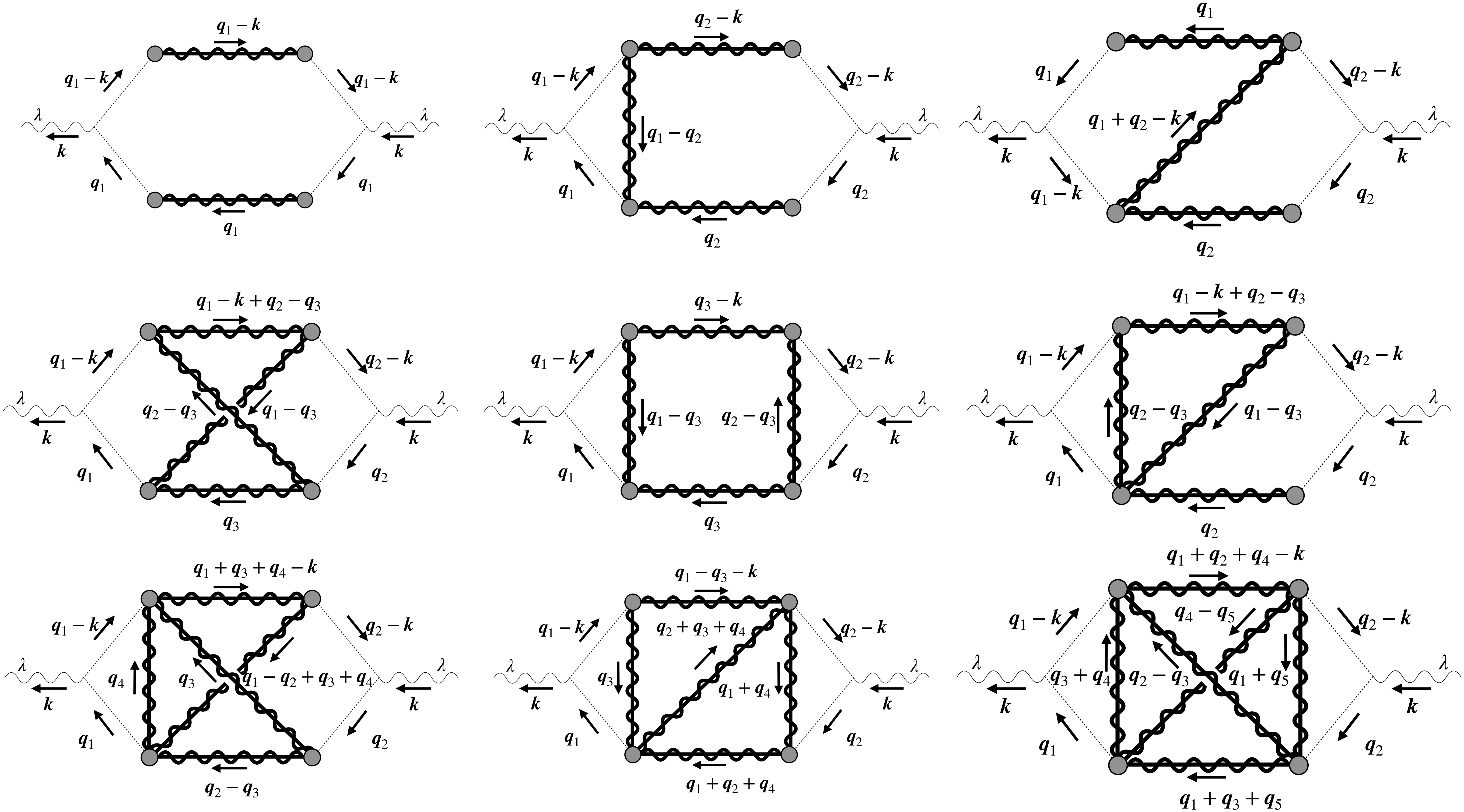
Diagrammatic approach

Vanilla term



They give the same contribution

$$\Omega_{\mathrm{GW}}^{(2)}(k) = 2 \times \frac{1}{48}\left(\frac{k}{aH}\right)^2 \sum_{\lambda=+,\times} \overline{P_{\lambda\lambda}^{\mathrm{Vanilla}}(\tau \to \infty, k)}$$


q_1-k
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q_1+q_2-k
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q_4-q_5
q_2-k
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q_2
$q_1+q_3+q_5$