19th Jun. 2023 @ NEHOP

Statistics of coarse-grained cosmological fields in Stochastic inflation

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w/ M. Yamada 2304.01249, 2306.07324

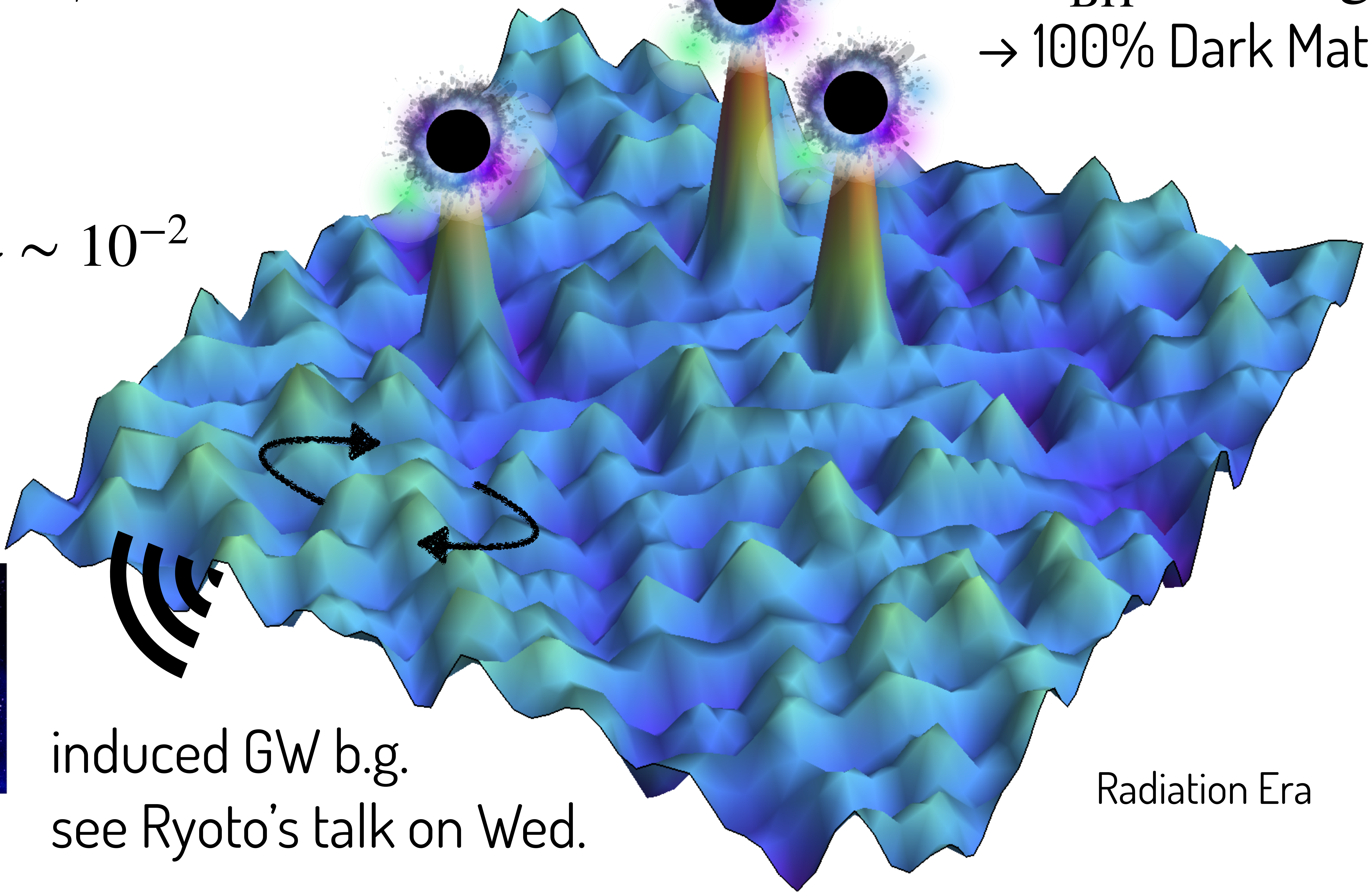
Primordial BH

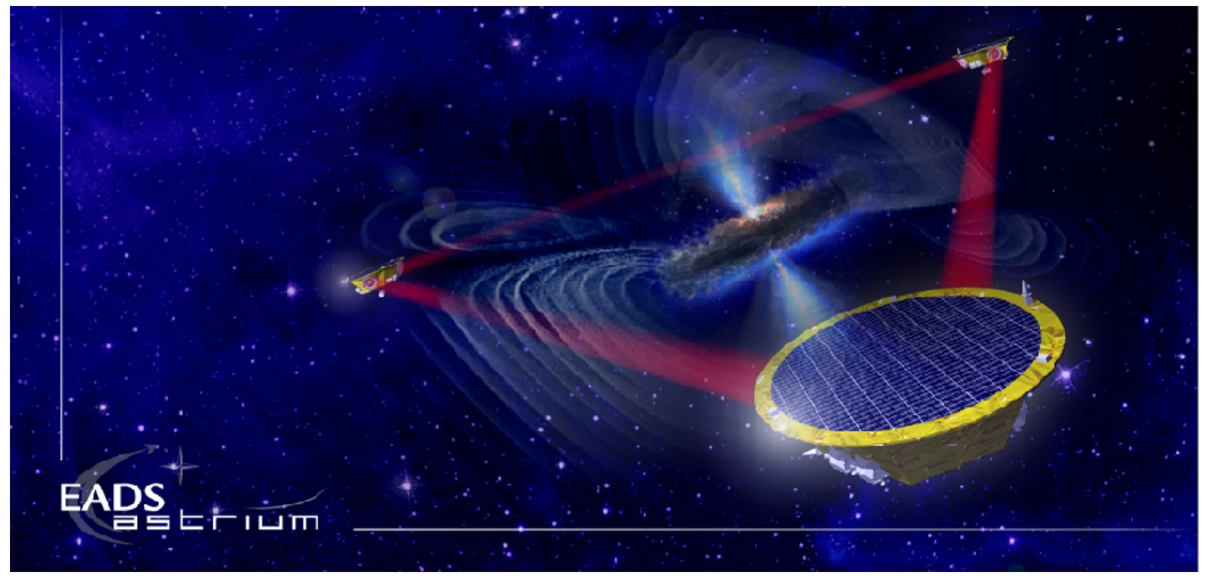
Carr & Hawking '74 (cf. Escrivà, Kuhnel, YT 22)

$\frac{\delta\rho}{\rho} \sim 1$

if $M_{\mathrm{BH}} \sim 10^{20}\,\mathrm{g}$
→ 100% Dark Matter

$\mathscr{P}_\zeta \sim 10^{-2}$





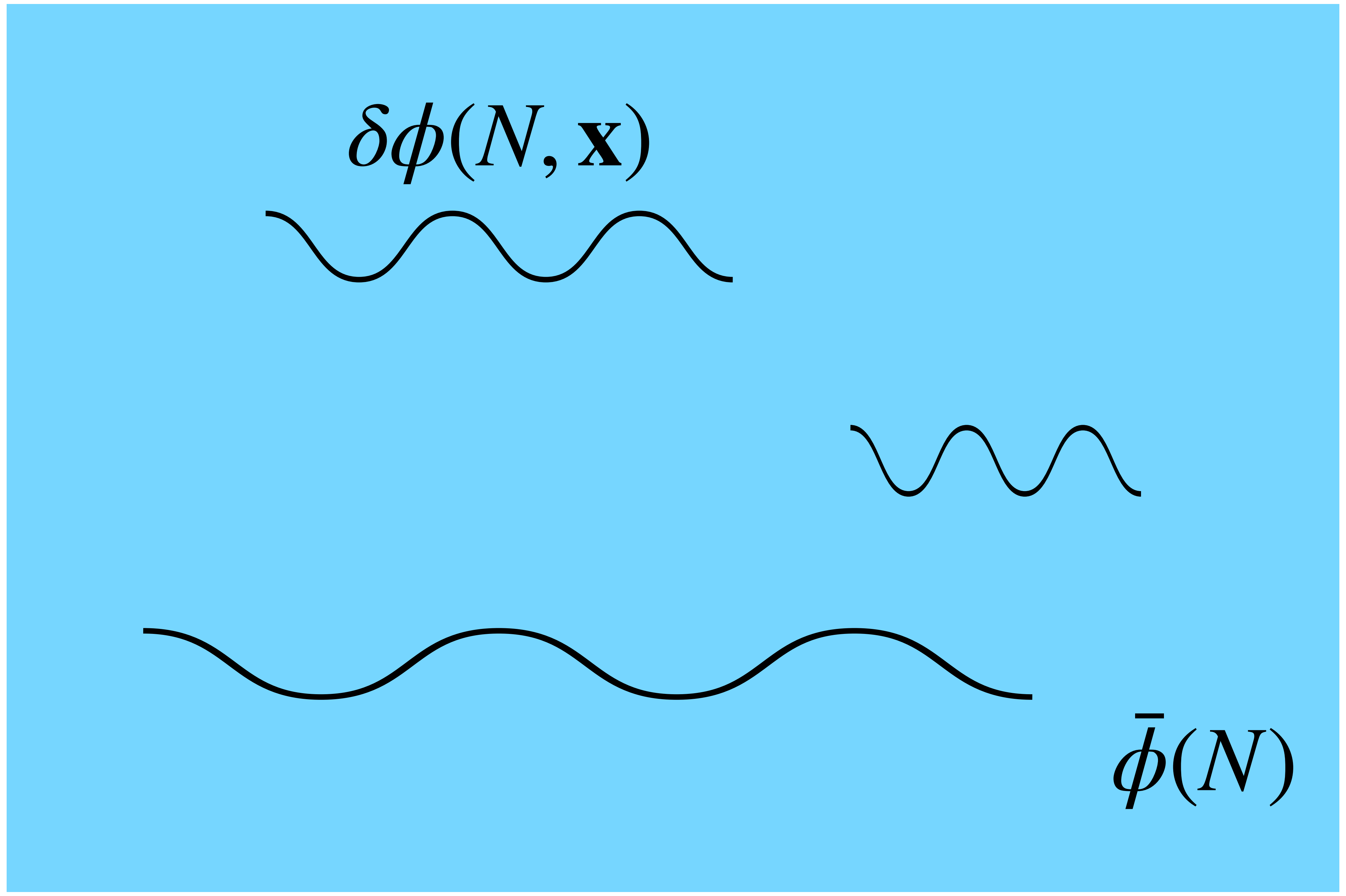
LISA

induced GW b.g.
see Ryoto's talk on Wed.

Radiation Era

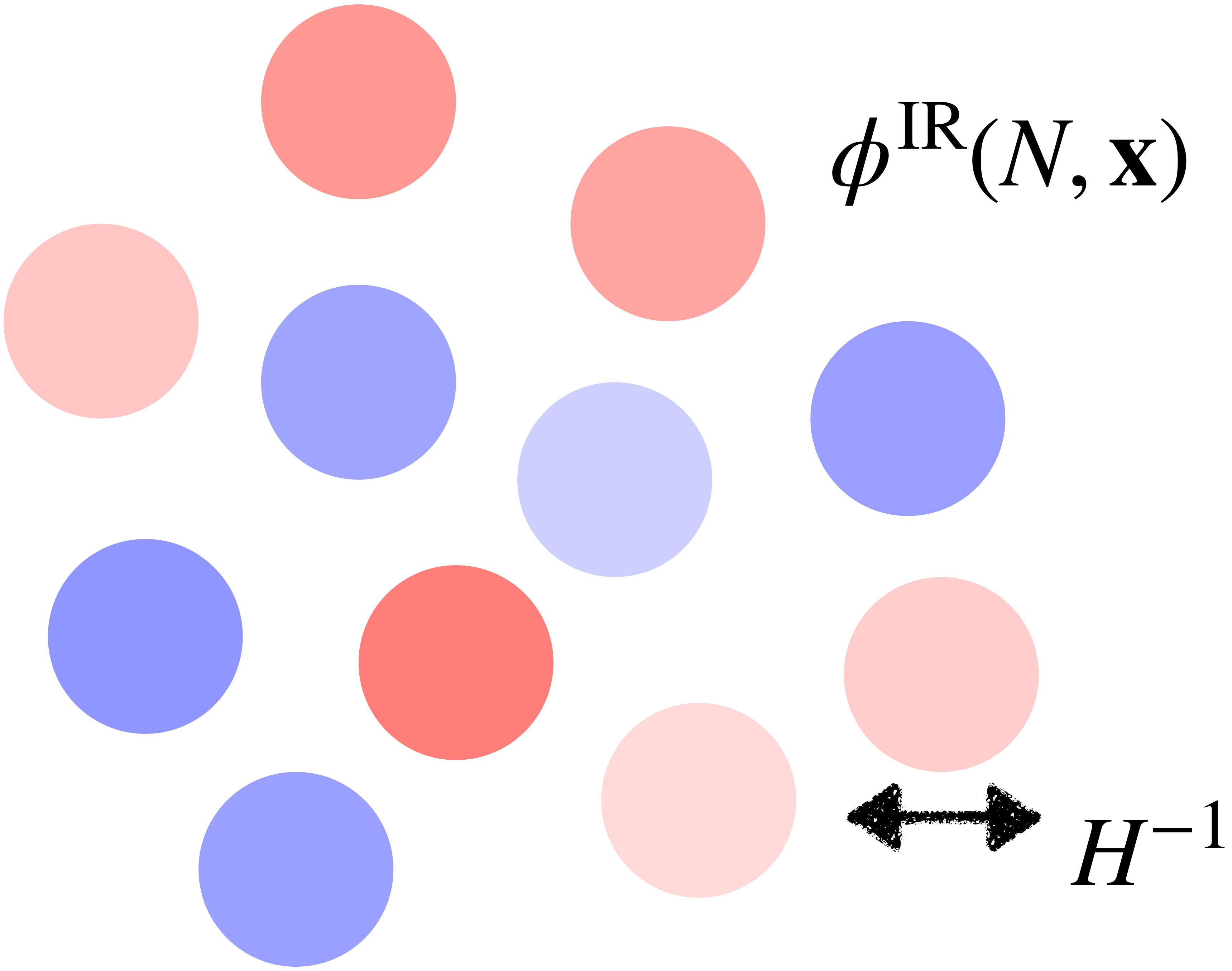
Large PTB?

- PTB theory for small PTB



$$\begin{cases} \phi(N,\mathbf{x}) = \bar{\phi}(N) + \delta\phi(N,\mathbf{x}) \\ g_{\mu\nu}(N,\mathbf{x}) = \bar{g}_{\mu\nu}(N) + \delta g_{\mu\nu}(N,\mathbf{x}) \end{cases}$$

- Stochastic approach Starobinsky '86
  - even for large PTB
  - only for superH



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Integrate out

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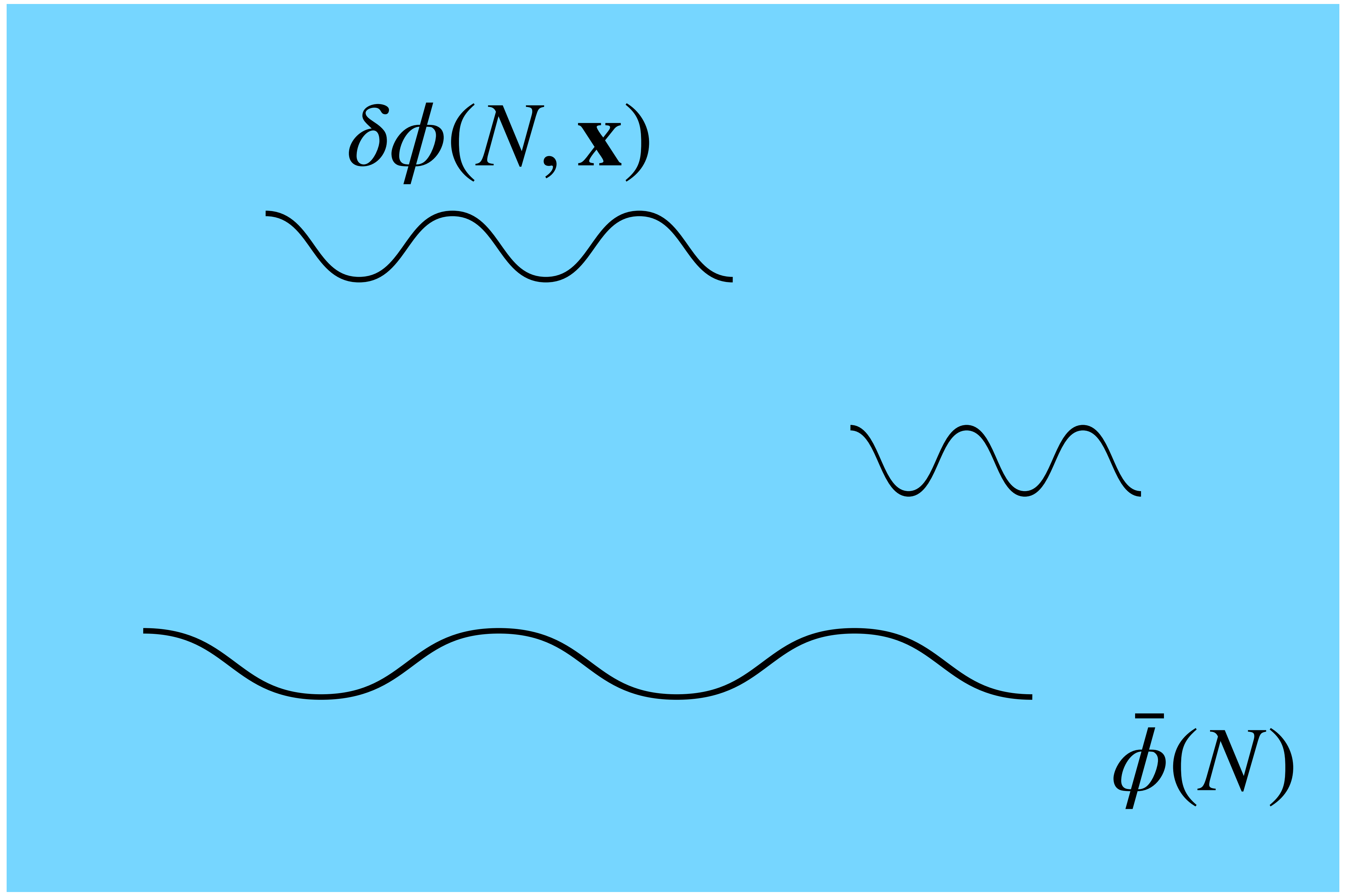

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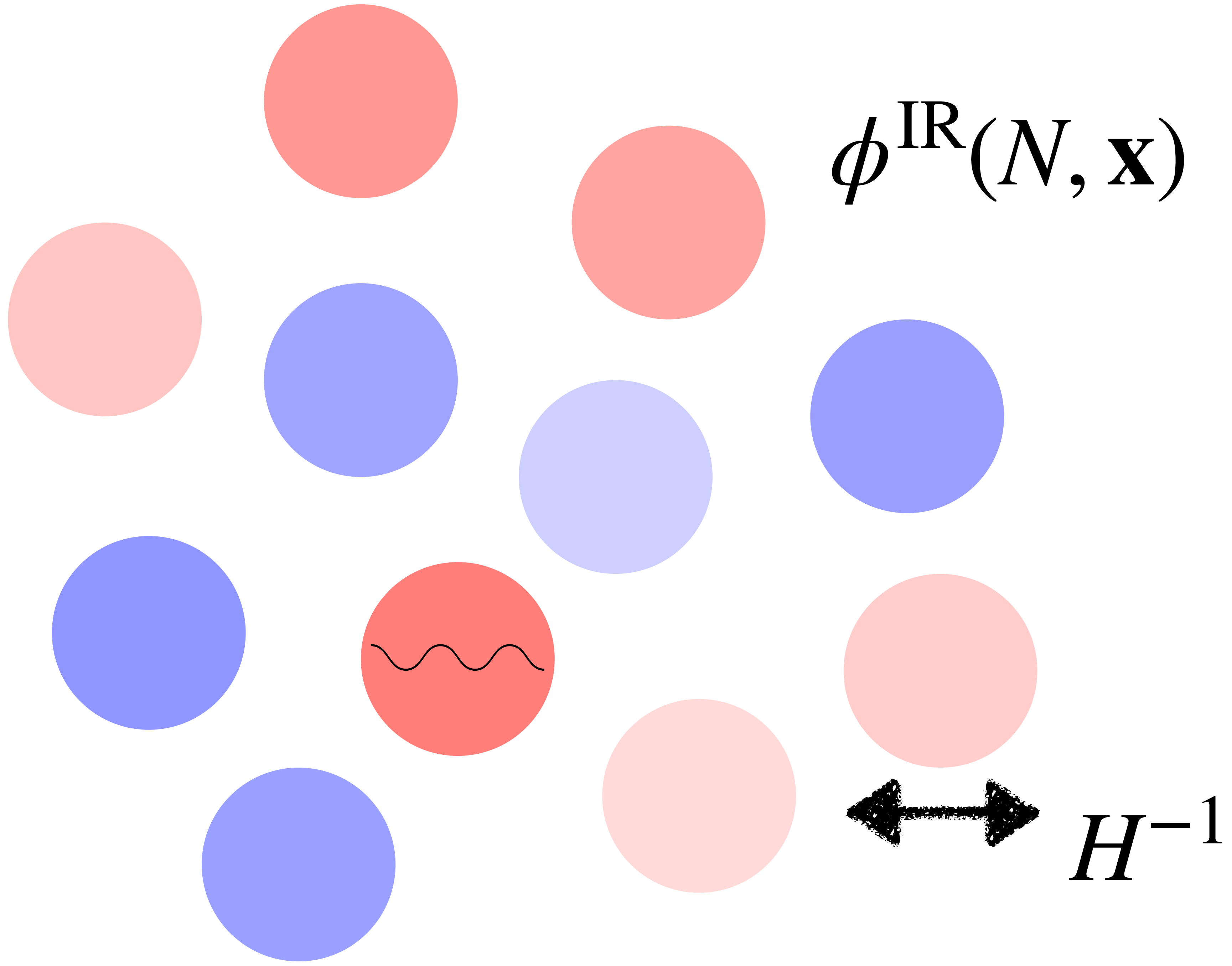

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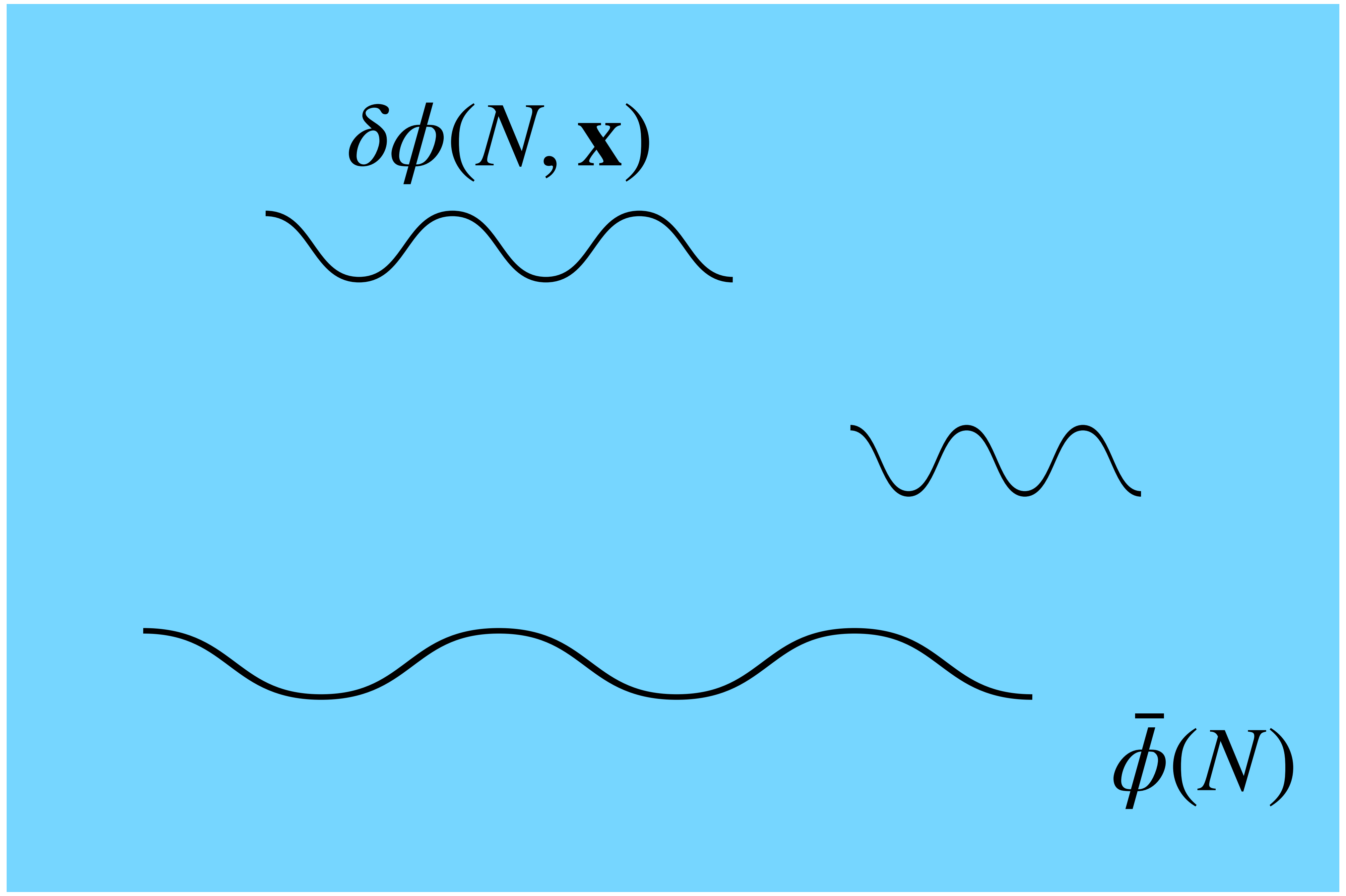

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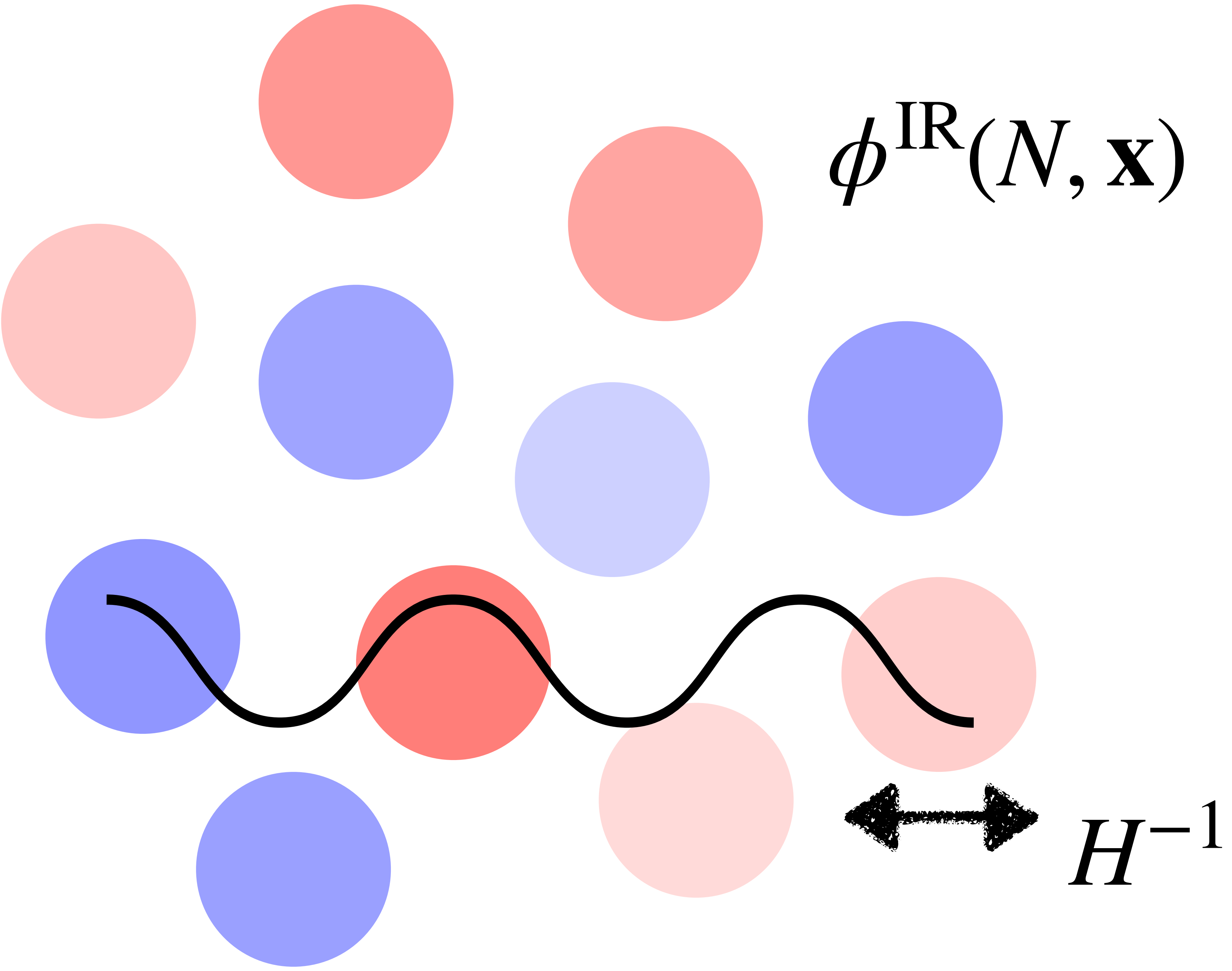

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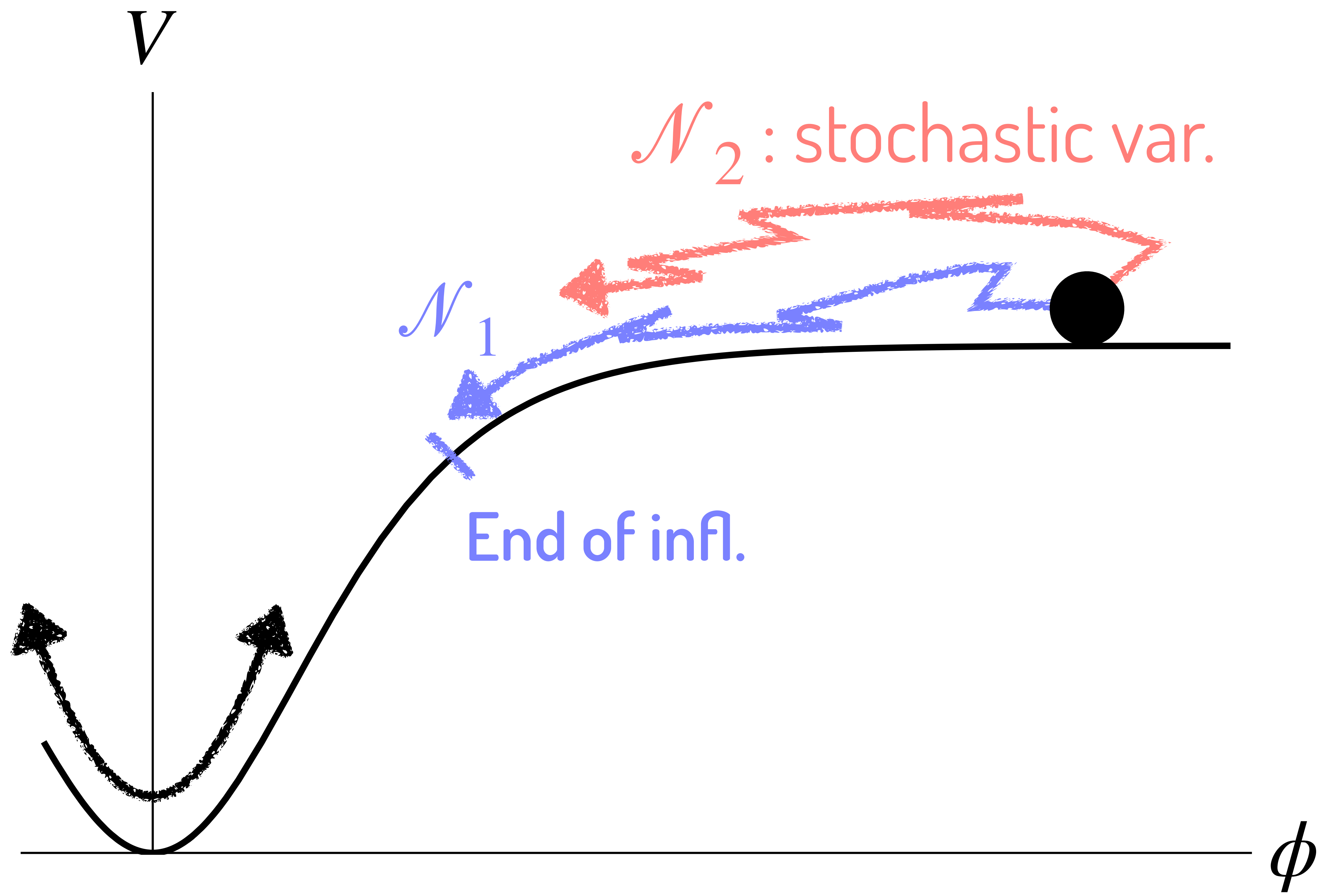
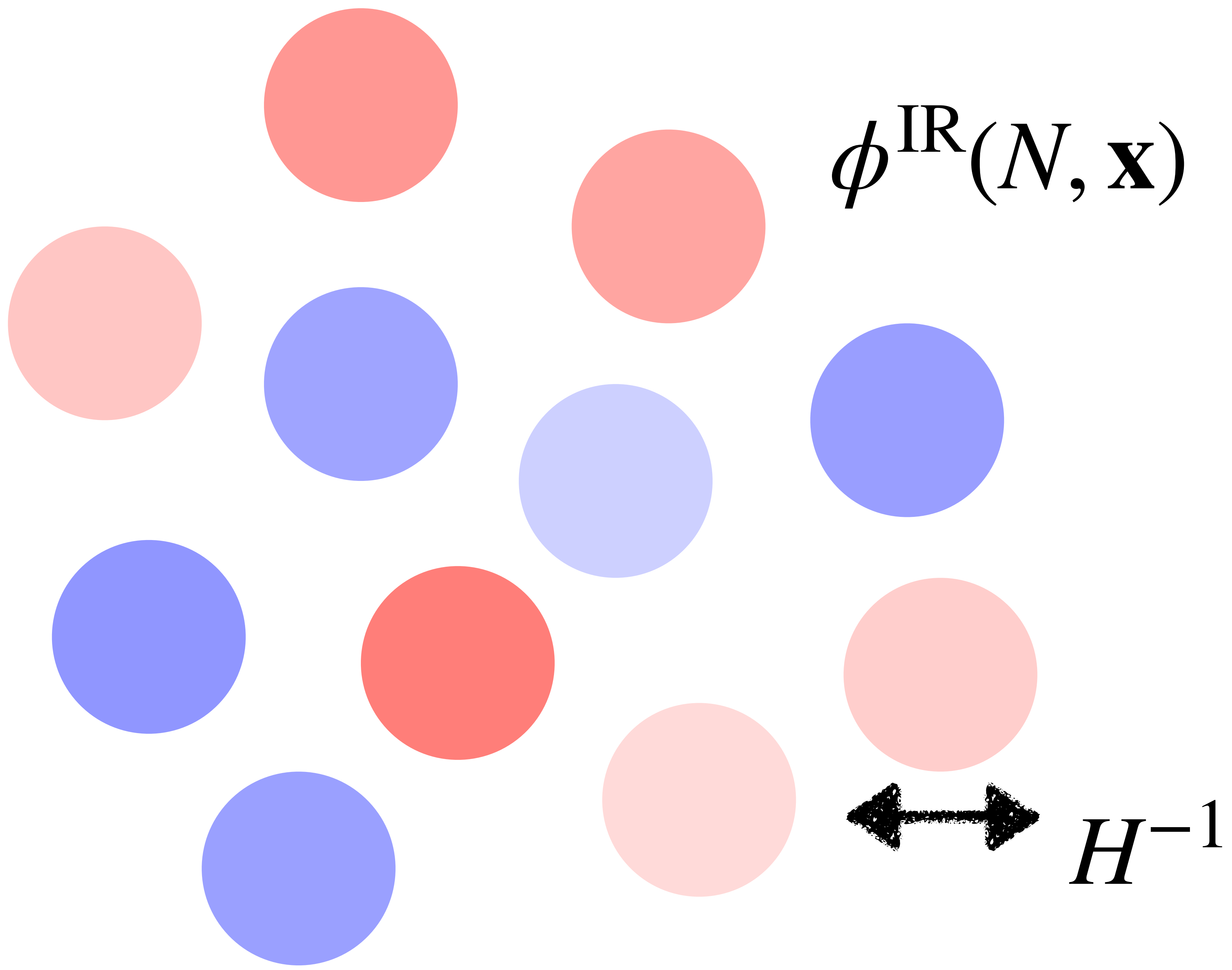


$$\begin{cases} \dfrac{\mathrm{d}\phi^{\mathrm{IR}}(N,\mathbf{x})}{\mathrm{d}N} = -\dfrac{V'}{3H^2}(N,\mathbf{x}) + \dfrac{H(N,\mathbf{x})}{2\pi}\xi(N,\mathbf{x}) \\ \langle \xi(N,\mathbf{x})\xi(N',\mathbf{x}')\rangle = \delta(N-N')\dfrac{\sin(aHr)}{aHr} \\ 3M_{\mathrm{Pl}}^2 H^2(N,\mathbf{x}) = \rho(N,\mathbf{x}) \end{cases}$$

Stoc. Noise

- even for large PTB
- only for superH

▶ Stochastic approach Starobinsky '86



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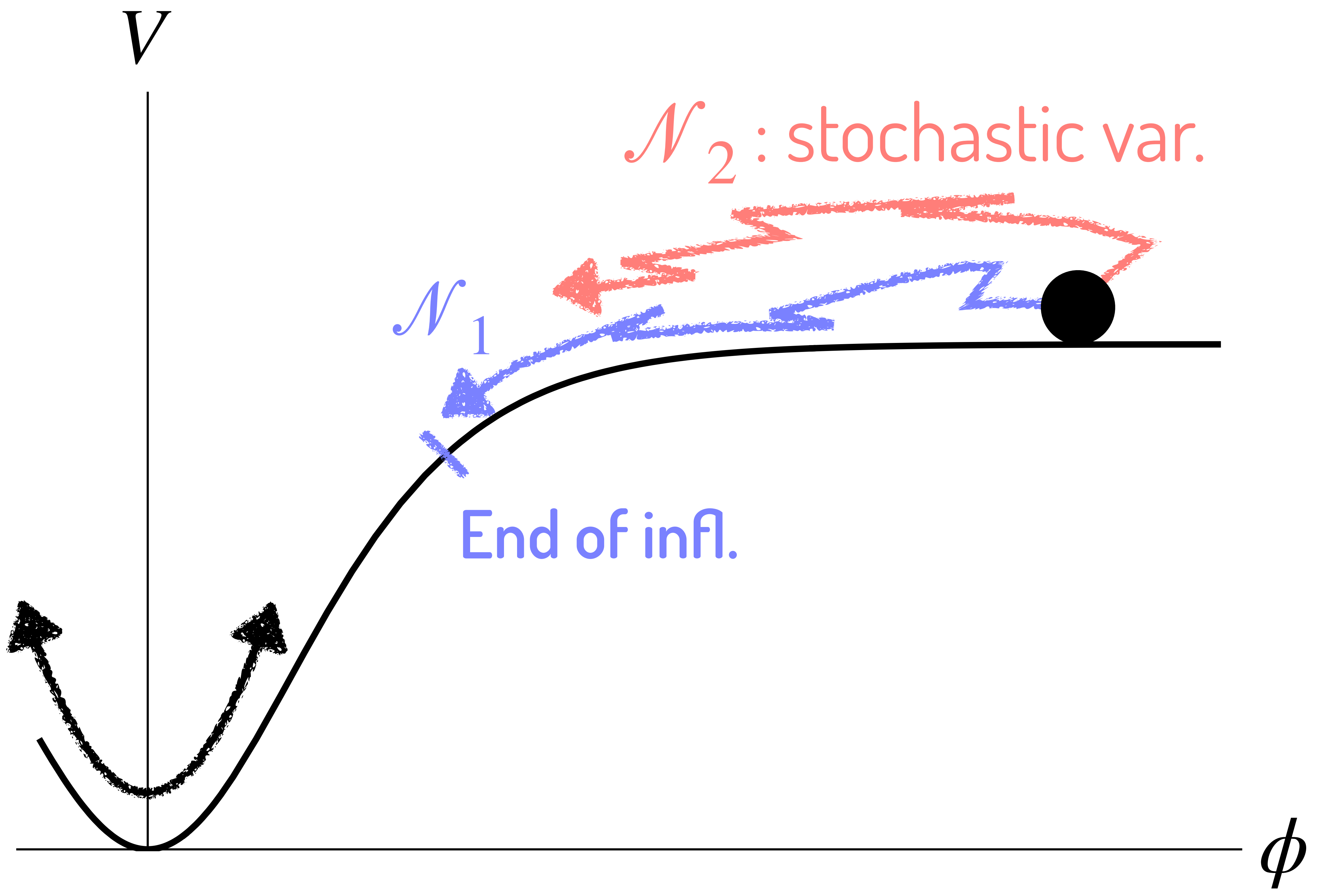
Integrate out

Stochastic-δN

Kawasaki, YT '15 + Fujita '14 + Takesako '13



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Stoc. Noise

Lyth, Malik, Sasaki '05 : the time difference δN is ...

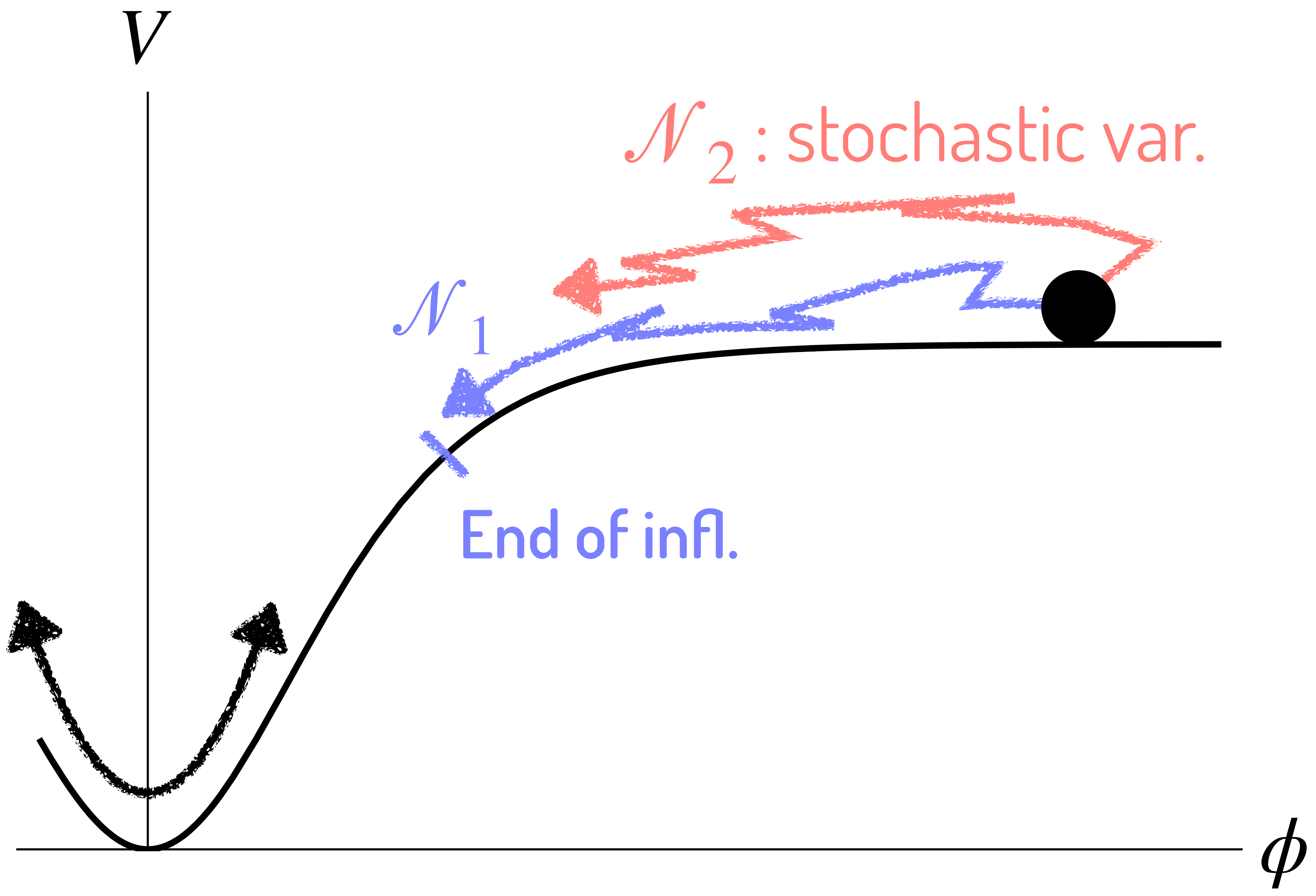
- conserved on superH after inflation
- $\delta\rho \simeq -\dfrac{\dot{\rho}}{H}\delta N$
- equivalent to the curv. ptb. ζ

In the stochastic form.

$$\zeta_{H_{\mathrm{inf}}^{-1}}(\mathbf{x}) = \delta\mathcal{N}(\mathbf{x}) = \mathcal{N}(\mathbf{x}) - \langle\mathcal{N}\rangle$$

Stochastic-δN

Kawasaki, YT '15 + Fujita '14 + Takesako '13



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In the stochastic form.

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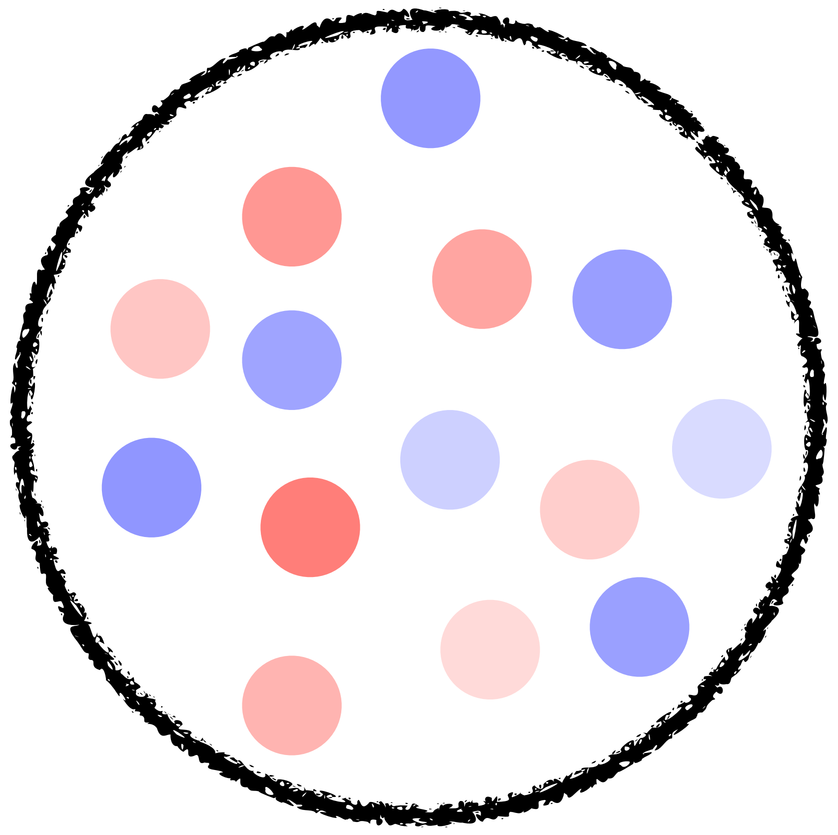
How probabilistically distributed?

How spatially correlated?



Adjoint FP

Vennin & Starobinsky '15

Langevin eq. : $\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi$

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with the absorption b.c. $P(\phi = \phi_{\mathrm{f}} \mid N) = 0$ at the end of inflation ϕ_{f}

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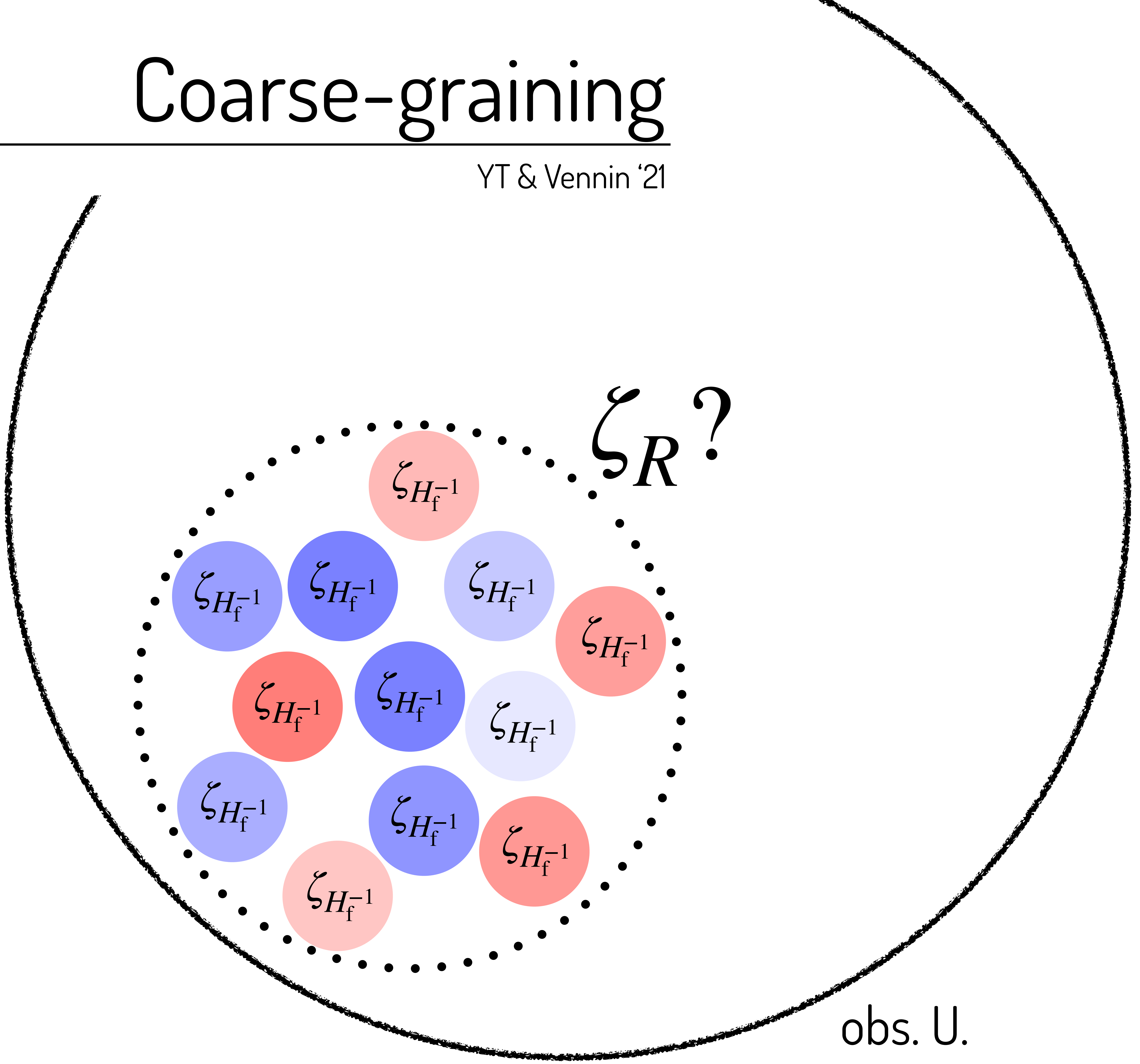
$\Leftrightarrow$ Series of PDE : $\mathscr{L}_{\mathrm{FP}}^\dagger \cdot \langle \mathcal{N}^n(\phi)\rangle = -n\langle \mathcal{N}^{n-1}(\phi)\rangle, \quad \mathscr{L}_{\mathrm{FP}}^\dagger \cdot \langle \delta\mathcal{N}^2(\phi)\rangle = -\left(\frac{H}{2\pi}\right)^2 \left(\partial_\phi \langle \mathcal{N}(\phi)\rangle\right)^2$

with the b.c. $\langle \mathcal{N}^n(\phi_{\mathrm{f}})\rangle = 0$

Coarse-graining

YT & Vennin '21



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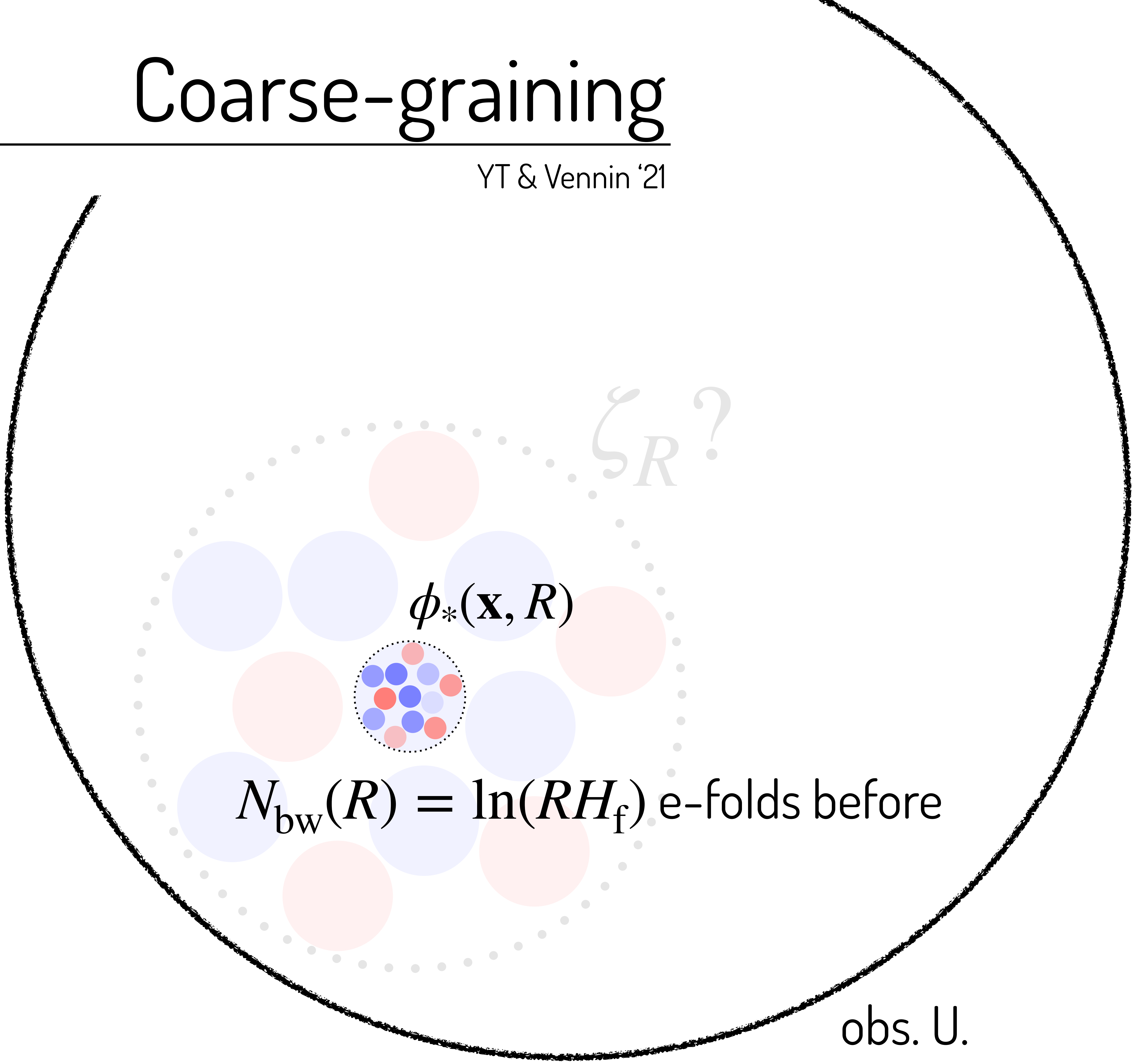

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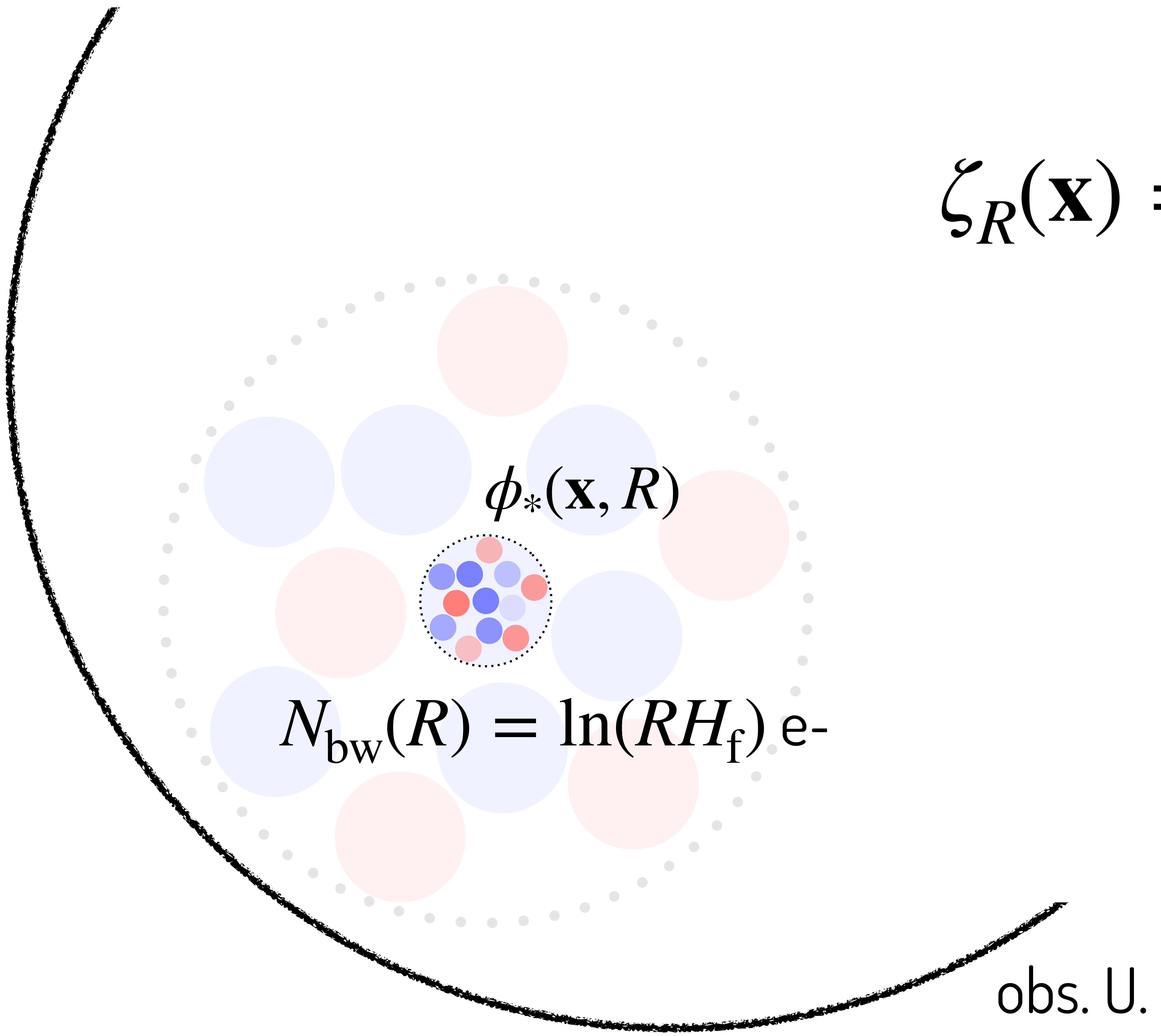

same dyn. until ϕ_*

$$\zeta_R(\mathbf{x}) = \mathcal{N}(\phi_\mathrm{i} \to \phi_*(\mathbf{x}, R)) + \langle \mathcal{N}(\phi_*(\mathbf{x}, R)) \rangle - \langle \mathcal{N}(\phi_\mathrm{i}) \rangle$$

independent after ϕ_*

Coarse-graining

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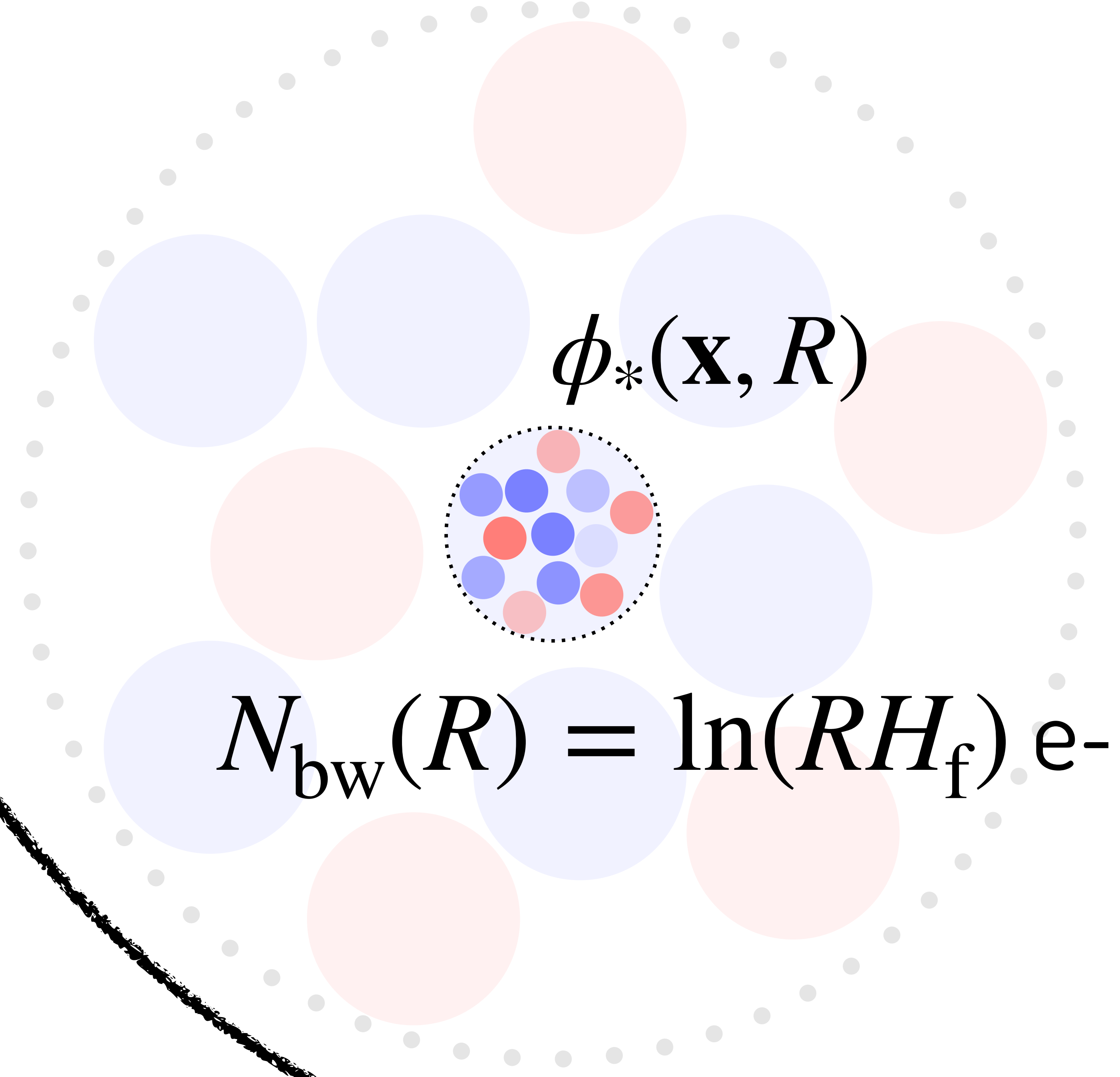

obs. U.

same dyn. until ϕ_*

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independent after ϕ_*

$$P(\zeta_R) = \int \mathrm{d}\phi_* \, \mathbb{P}\left(\mathcal{N}_{\mathrm{i}\to *} = \langle \mathcal{N}_\mathrm{i} \rangle - \langle \mathcal{N}_* \rangle + \zeta_R\right) \times \mathbb{P}\left(\phi = \phi_* @ - N_{\mathrm{bw}}(R)\right)$$

Power spectrum

Ando & Vennin '20, YT & Yamada '23b

$$\int^{-\ln\frac{R}{a}} \mathrm{d}\ln k\,\mathcal{P}_\zeta(k) = \langle \zeta_R^2 \rangle = \int \mathrm{d}\zeta_R\,\zeta_R^2 P(\zeta_R)$$

$$\Rightarrow \mathcal{P}_\zeta(k) = -\left.\frac{\mathrm{d}\langle\zeta_R^2\rangle}{\mathrm{d}\ln R}\right|_{R=a/k} = -\int \mathrm{d}\phi_*\mathrm{d}\zeta_R\,\zeta_R^2\,\mathbb{P}\left(\mathcal{N}_{\mathrm{i}\to *} = \langle\mathcal{N}_\mathrm{i}\rangle - \langle\mathcal{N}_*\rangle + \zeta_R\right)$$

$$\times \left.\frac{\partial\mathbb{P}\left(\phi = \phi_*@-N_\mathrm{bw}\right)}{\partial N_\mathrm{bw}}\right|_{N_\mathrm{bw}=\ln\frac{a_\mathrm{f}H_\mathrm{f}}{k}}$$

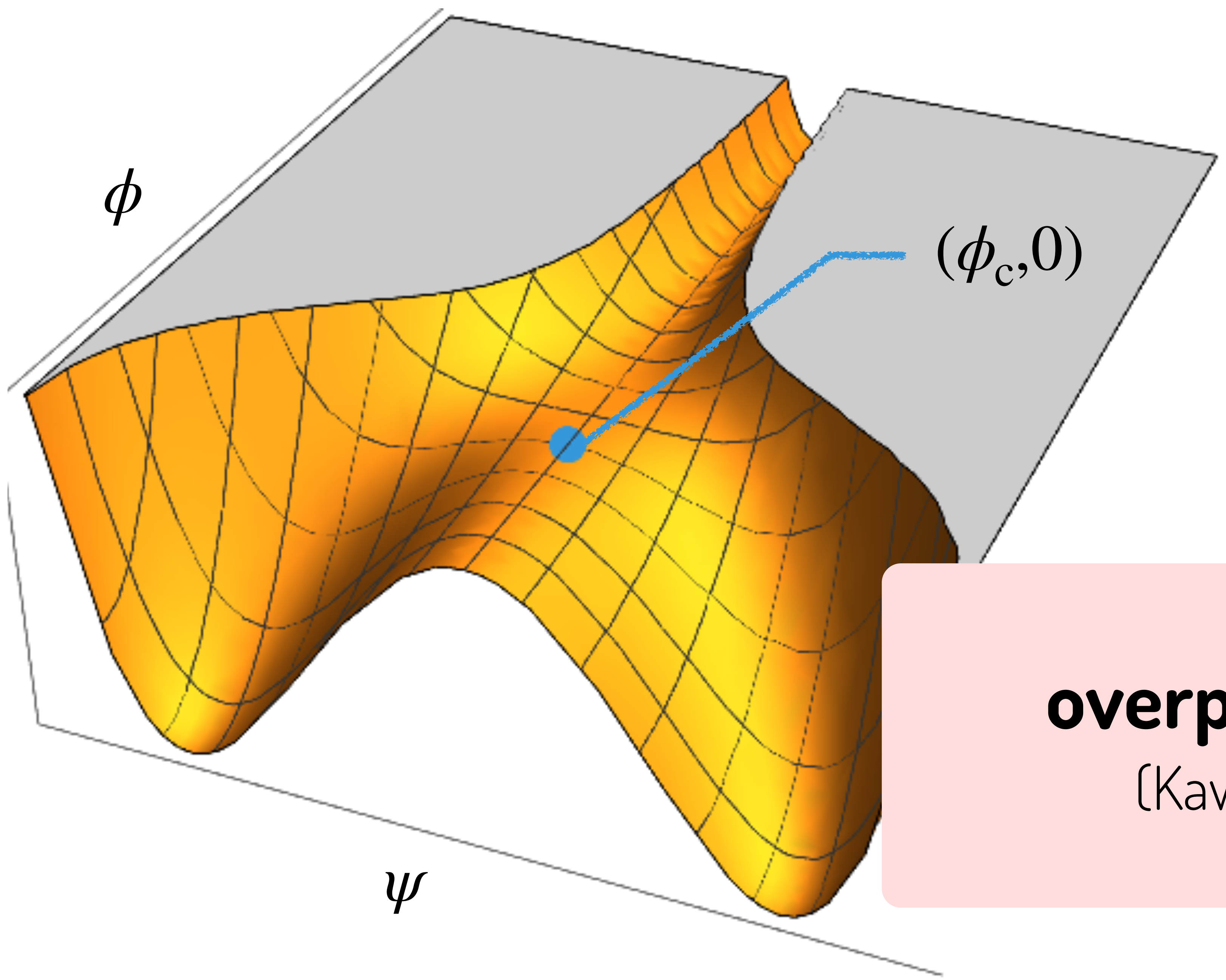
$$\cdots \approx \frac{1}{S}\sum_{i=1}^{S}\frac{\left\langle\delta\mathcal{N}^2(\phi_i^-)\right\rangle - \left\langle\delta\mathcal{N}^2(\phi_i^+)\right\rangle}{\Delta N}$$

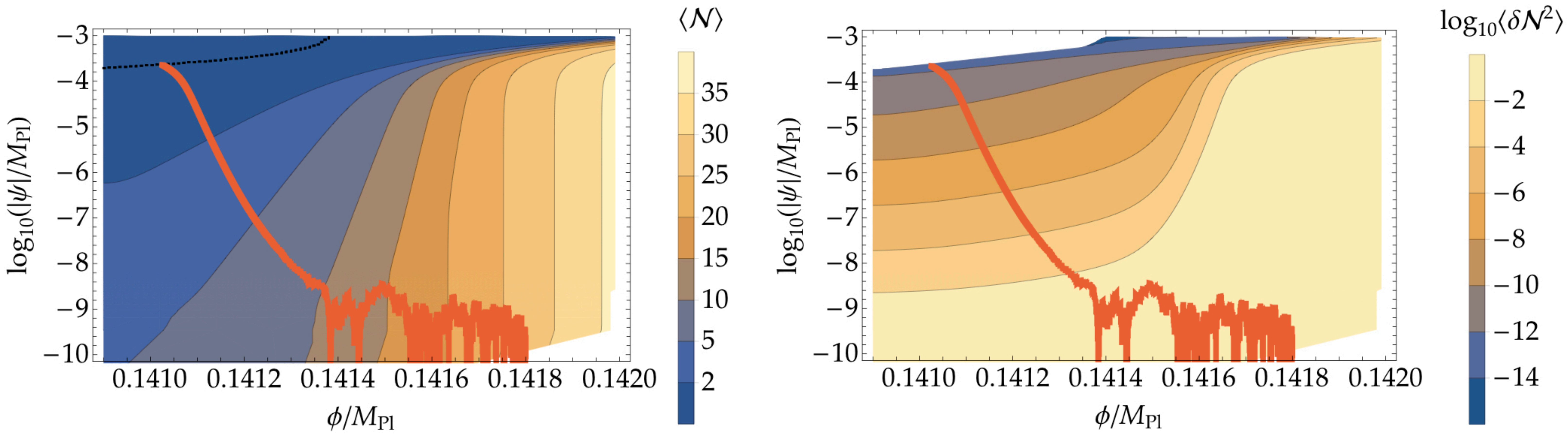
stat. average of der. of variance

Hybrid inflation

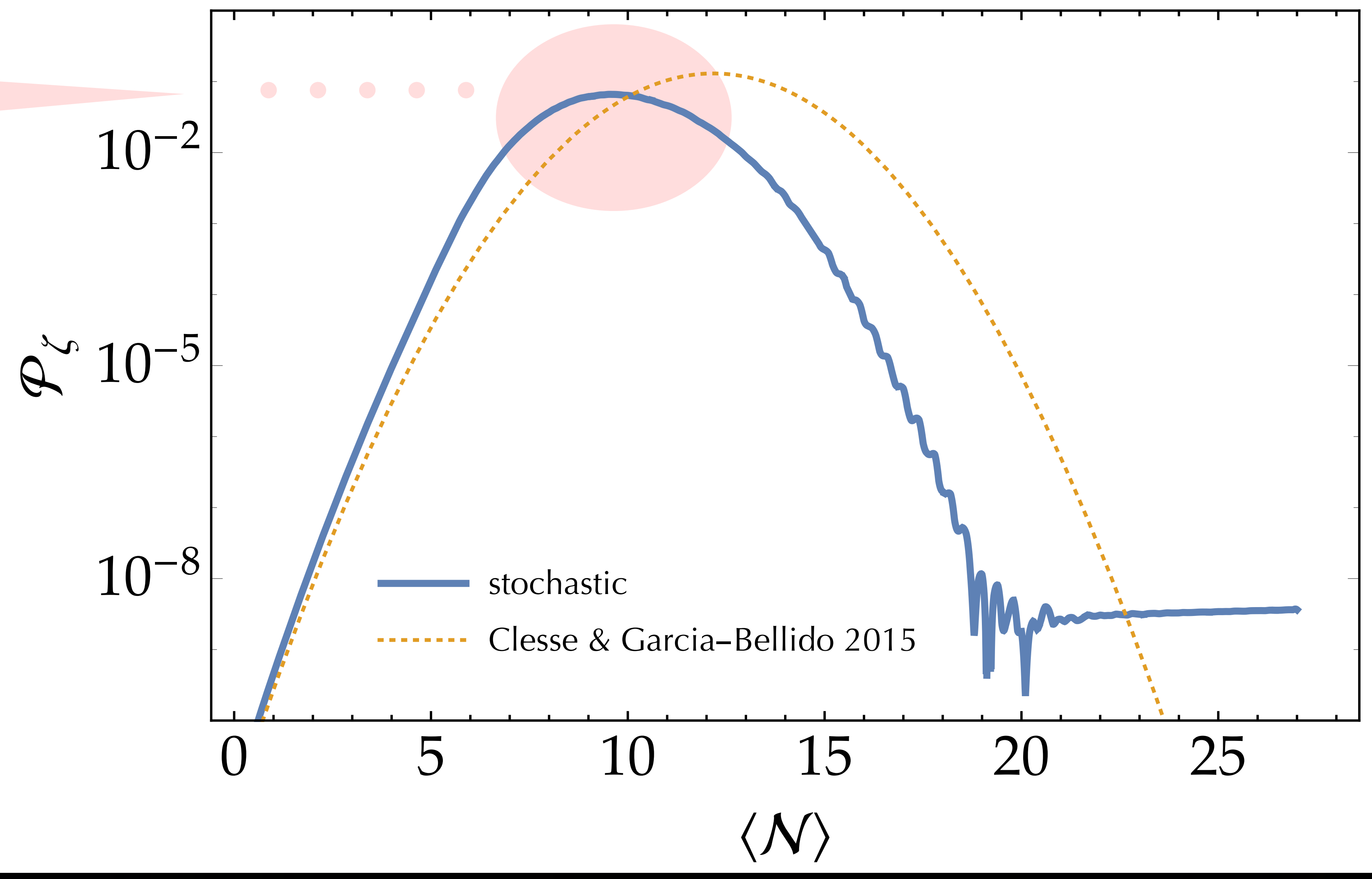
Kawasaki & YT '15.

$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2+2\frac{\phi^2\psi^2}{\phi_c^2M^2}+\frac{\phi-\phi_c}{\mu_1}-\frac{(\phi-\phi_c)^2}{\mu_2^2}\right]$$





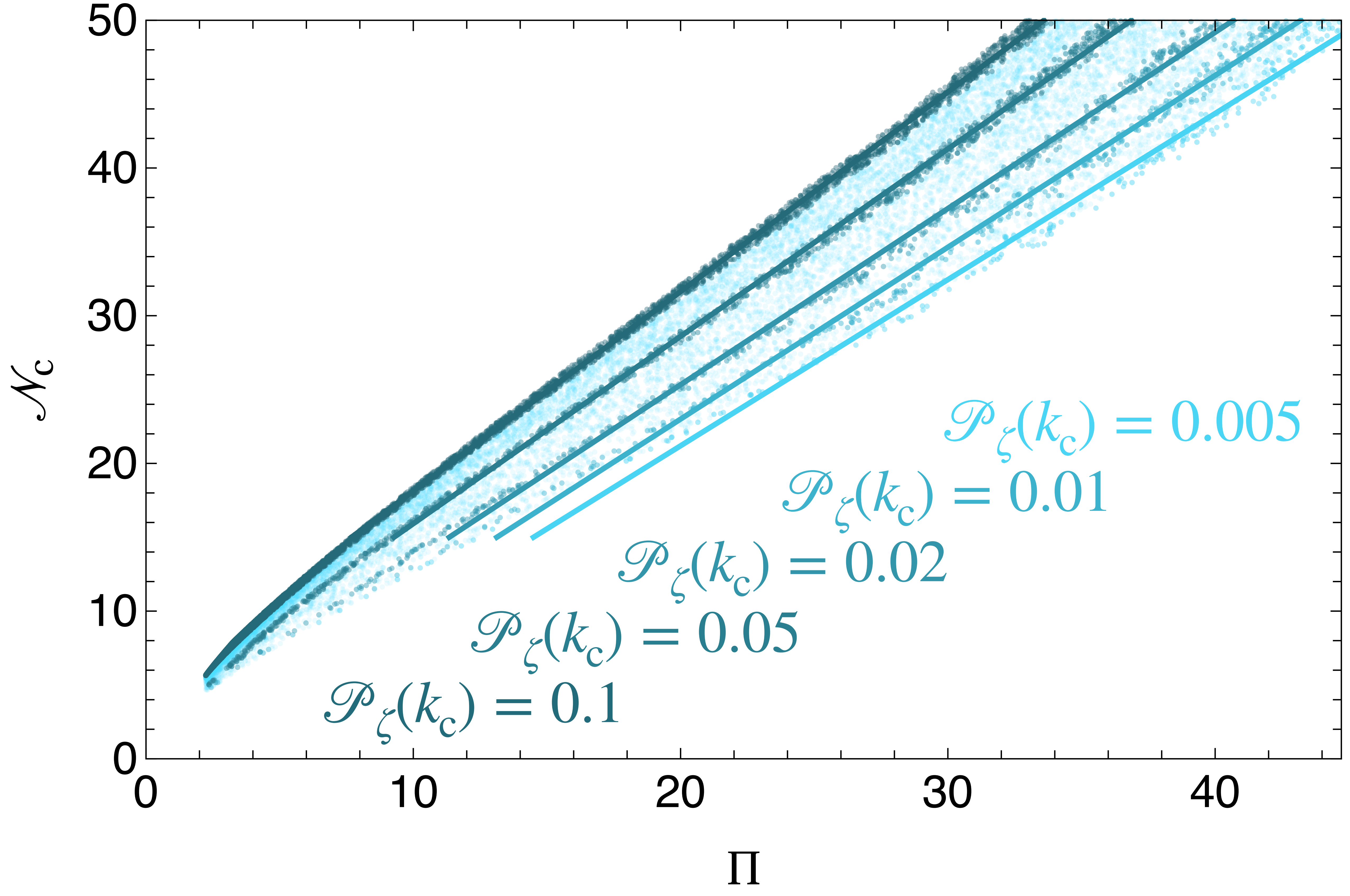
overproduce PBHs
(Kawasaki & YT '15)



~~No-Go~~ 1 : ϕ^3

YT & Yamada '23a

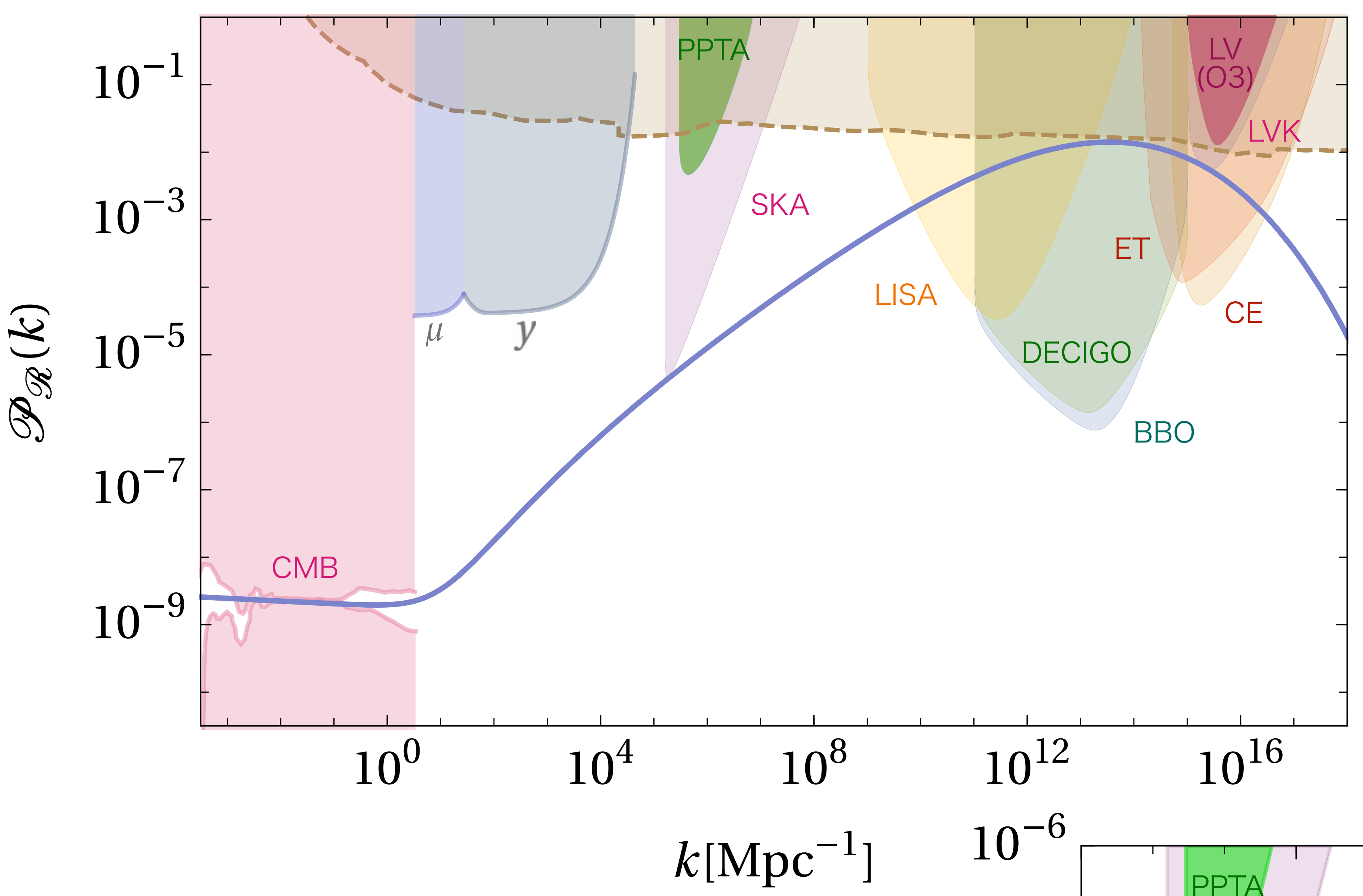
$$V(\phi,\psi) = \Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} + \frac{\phi-\phi_c}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2} + \frac{(\phi-\phi_c)^3}{\mu_3^3}\right]$$

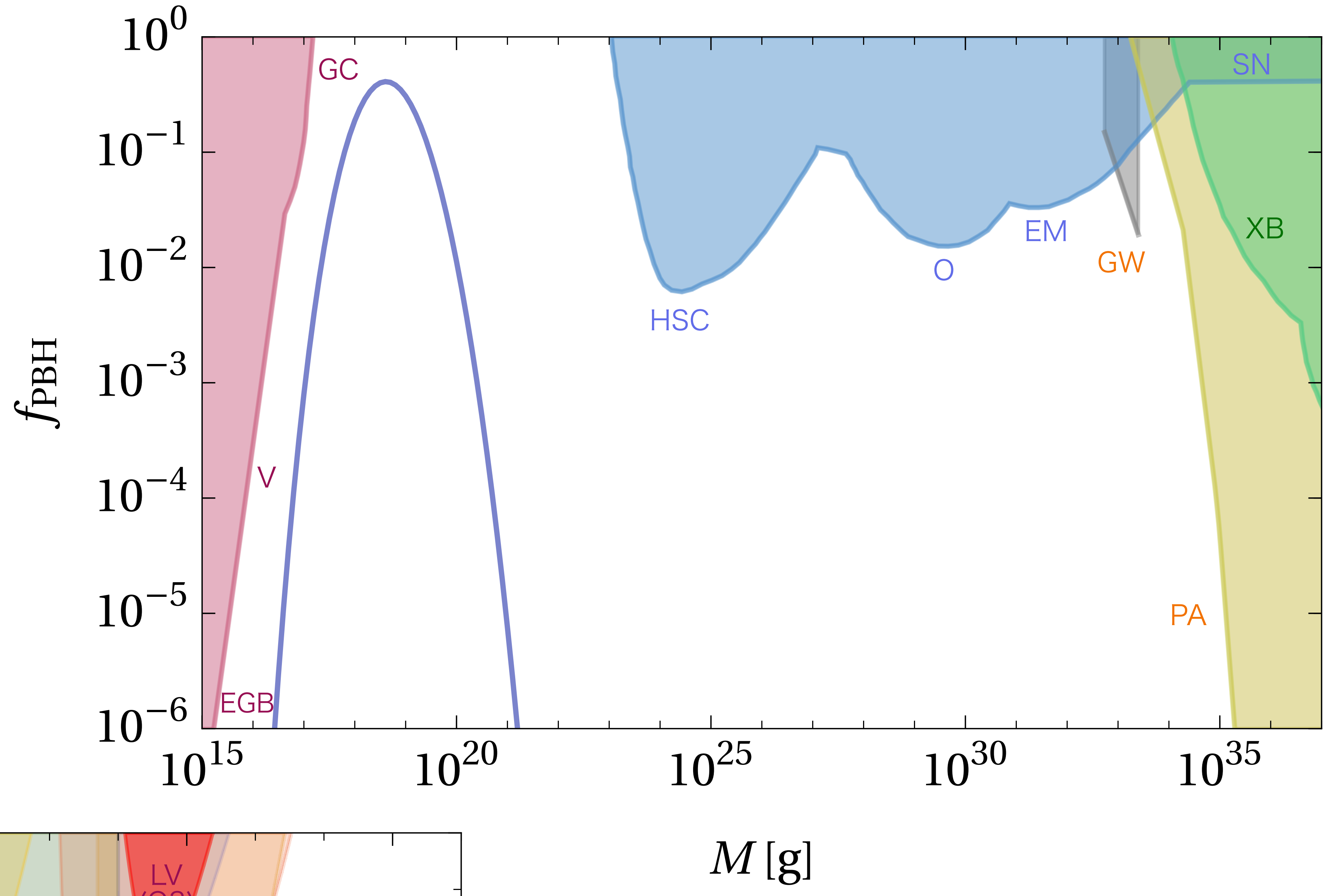


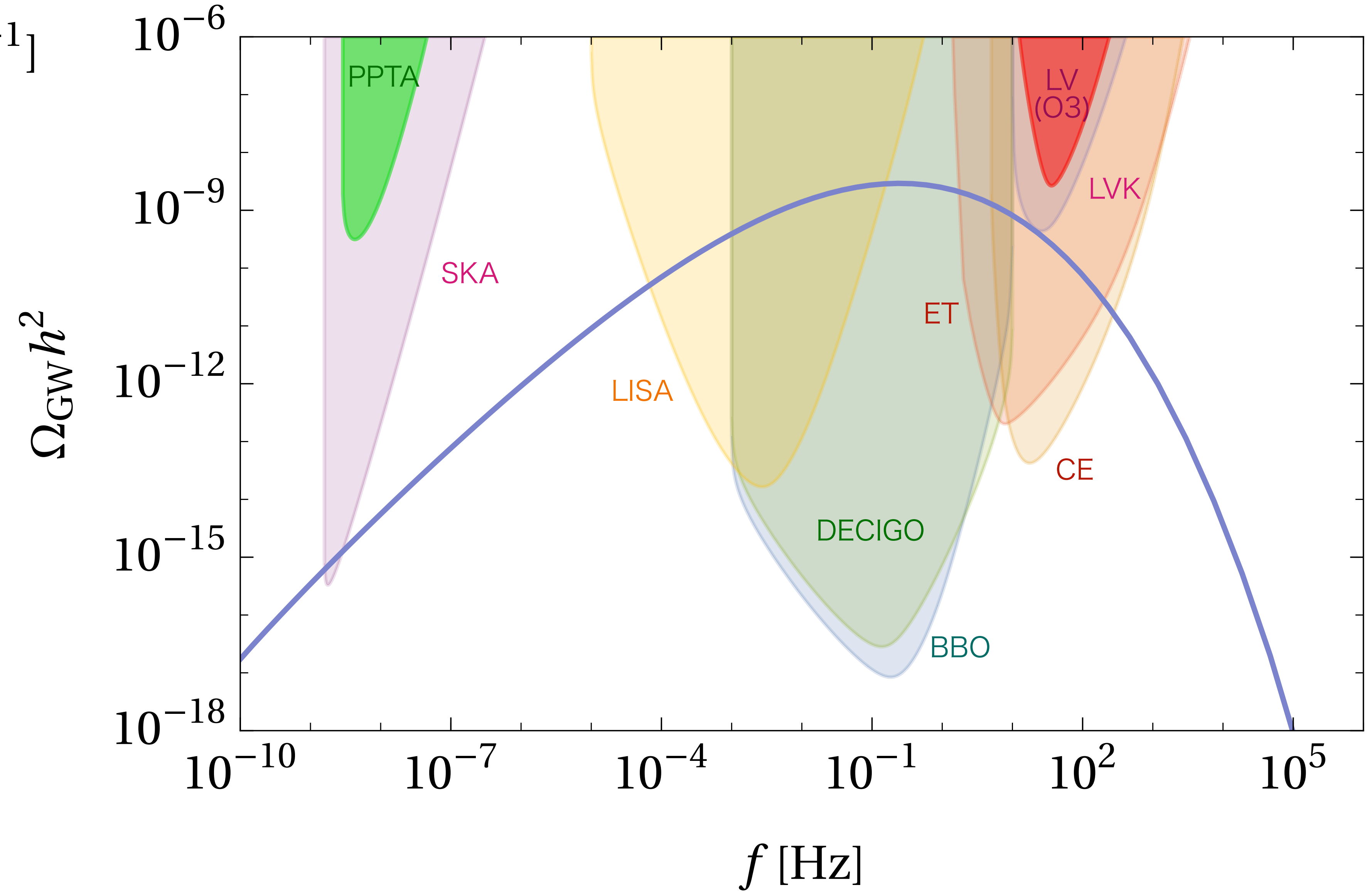
$$\left|\frac{\mathrm{d}\ln\mathcal{P}_\zeta(k_c)}{\mathrm{d}\ln\mu_3}\right| \sim 1$$ mild tuning

~~No Go~~ 1 : ϕ^3

YT & Yamada '23a







$M = \phi_c/\sqrt{2} = M_{\mathrm{Pl}}/10$

$\Pi^2 = 185$

$\mu_2 = 4.21 M_{\mathrm{Pl}}$

$\mu_3 = 0.182 M_{\mathrm{Pl}}$

~~No Go~~ 2 : multi-ψ

Halpern+ '14, YT & Yamada '23b

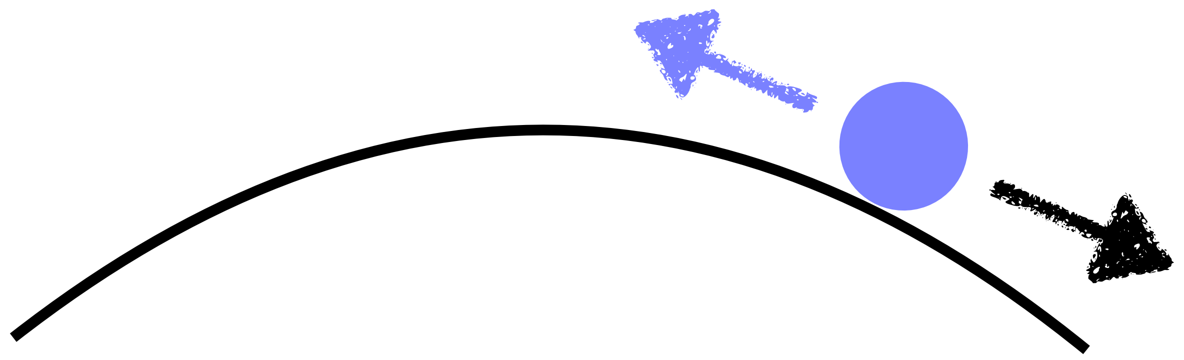
$$\psi \to \vec{\psi} = (\psi_1, \psi_2, \cdots, \psi_{\mathscr{D}})$$

$$\mathscr{L}_{\mathrm{FP}}^{\dagger} \cdot P_{\mathrm{FPT}}(\mathcal{N} \mid \vec{\psi}) = -M_{\mathrm{Pl}}^2 \frac{V_i}{V} \partial_i P_{\mathrm{FPT}}(\mathcal{N} \mid \vec{\psi}) + \frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \partial_i^2 P_{\mathrm{FPT}}(\mathcal{N} \mid \vec{\psi})$$

$$= \left[-M_{\mathrm{Pl}}^2 \frac{V_i}{V} + \frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \frac{\mathscr{D}-1}{\psi_r}\right] \partial_{\psi_r} P_{\mathrm{FPT}}(\mathcal{N} \mid \psi_r) + \frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \partial_{\psi_r}^2 P_{\mathrm{FPT}}(\mathcal{N} \mid \psi_r)$$

Centrifugal force

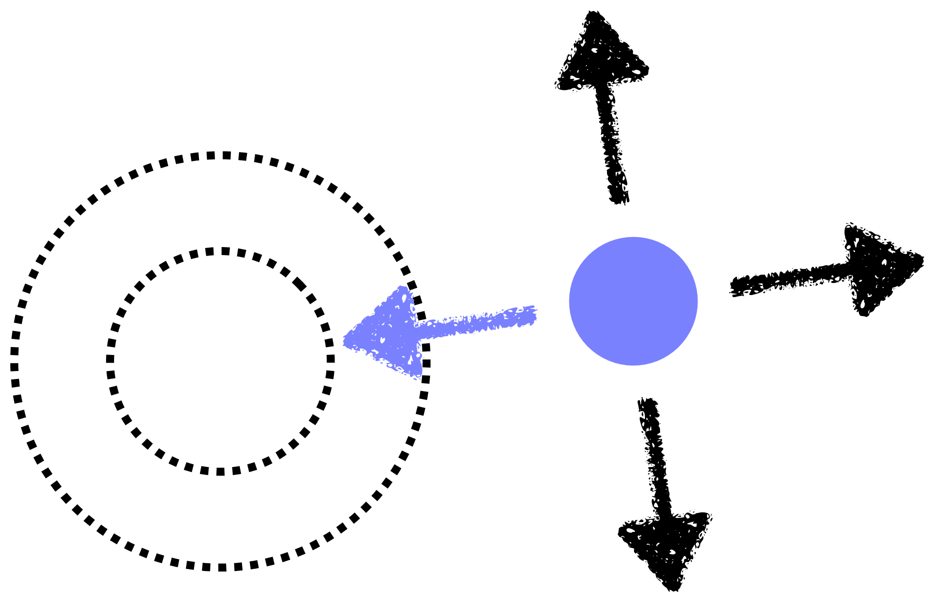
$\mathscr{D} = 1$:



$p = \frac{1}{2}$

$$\Rightarrow \langle \psi_r^2 \rangle_{\mathrm{c}} = \mathscr{D} \langle \psi^2 \rangle_{\mathrm{c}} \Big|_{\mathscr{D}=1}$$

$\mathscr{D} = 2$:



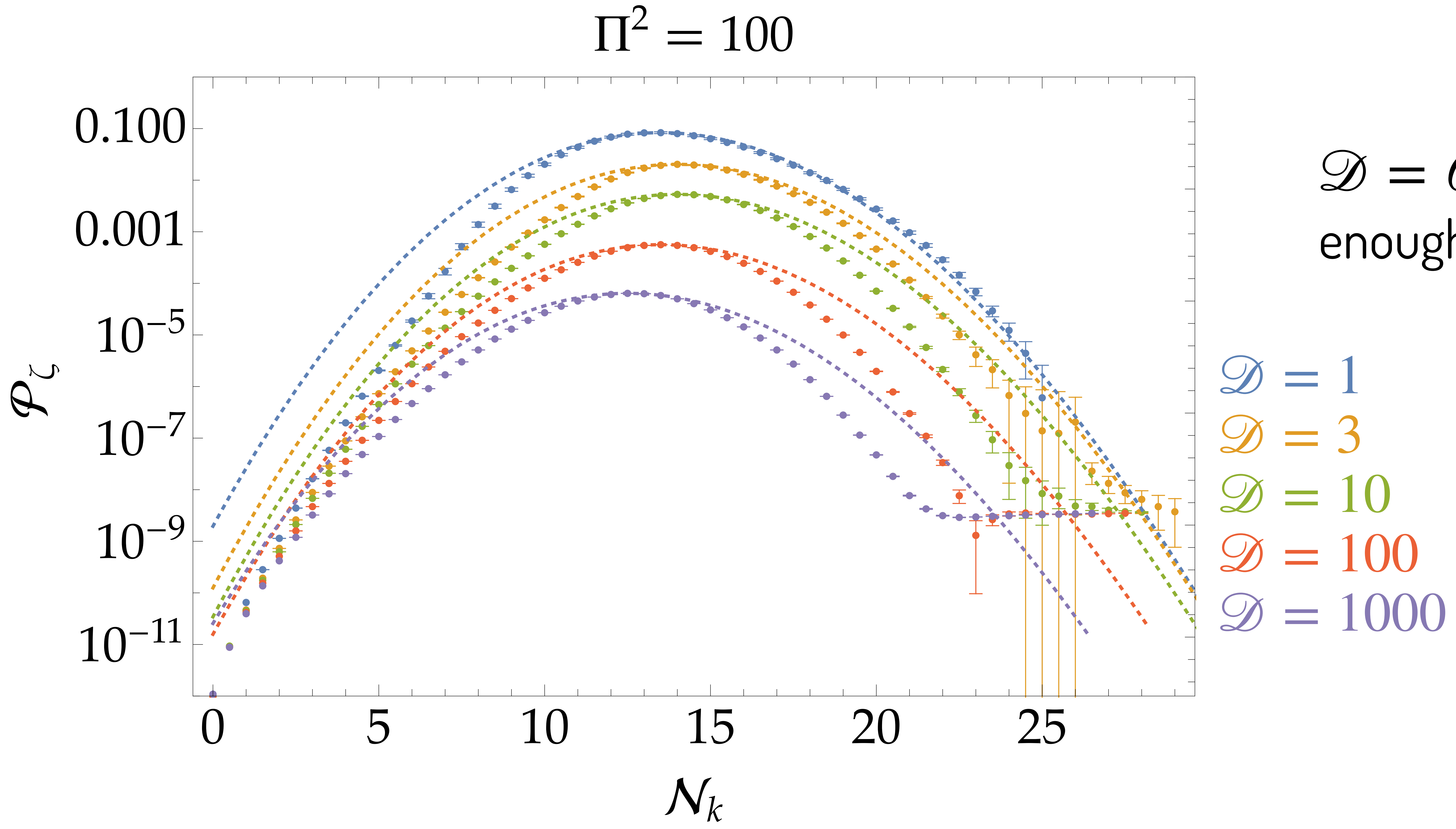
$p = \frac{1}{4}$

$$\Rightarrow \mathscr{P}_\zeta^{\max} \Big|_{\mathscr{D}} = \frac{1}{\mathscr{D}} \mathscr{P}_\zeta^{\max} \Big|_{\mathscr{D}=1}$$

~~No-Go~~ 2 : multi-ψ

YT & Yamada '23b

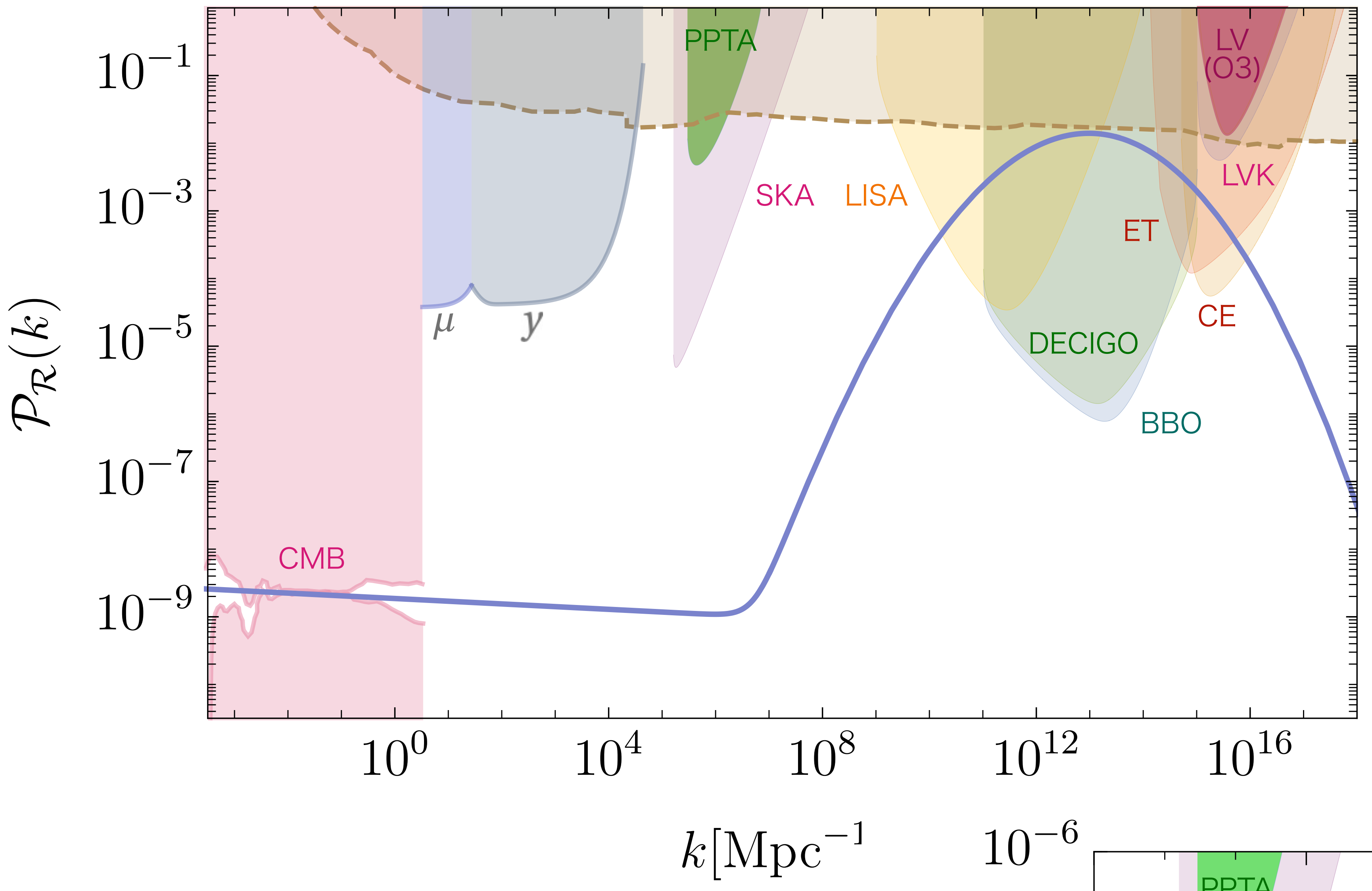
$$\psi \to \overrightarrow{\psi} = (\psi_1, \psi_2, \cdots, \psi_{\mathscr{D}})$$

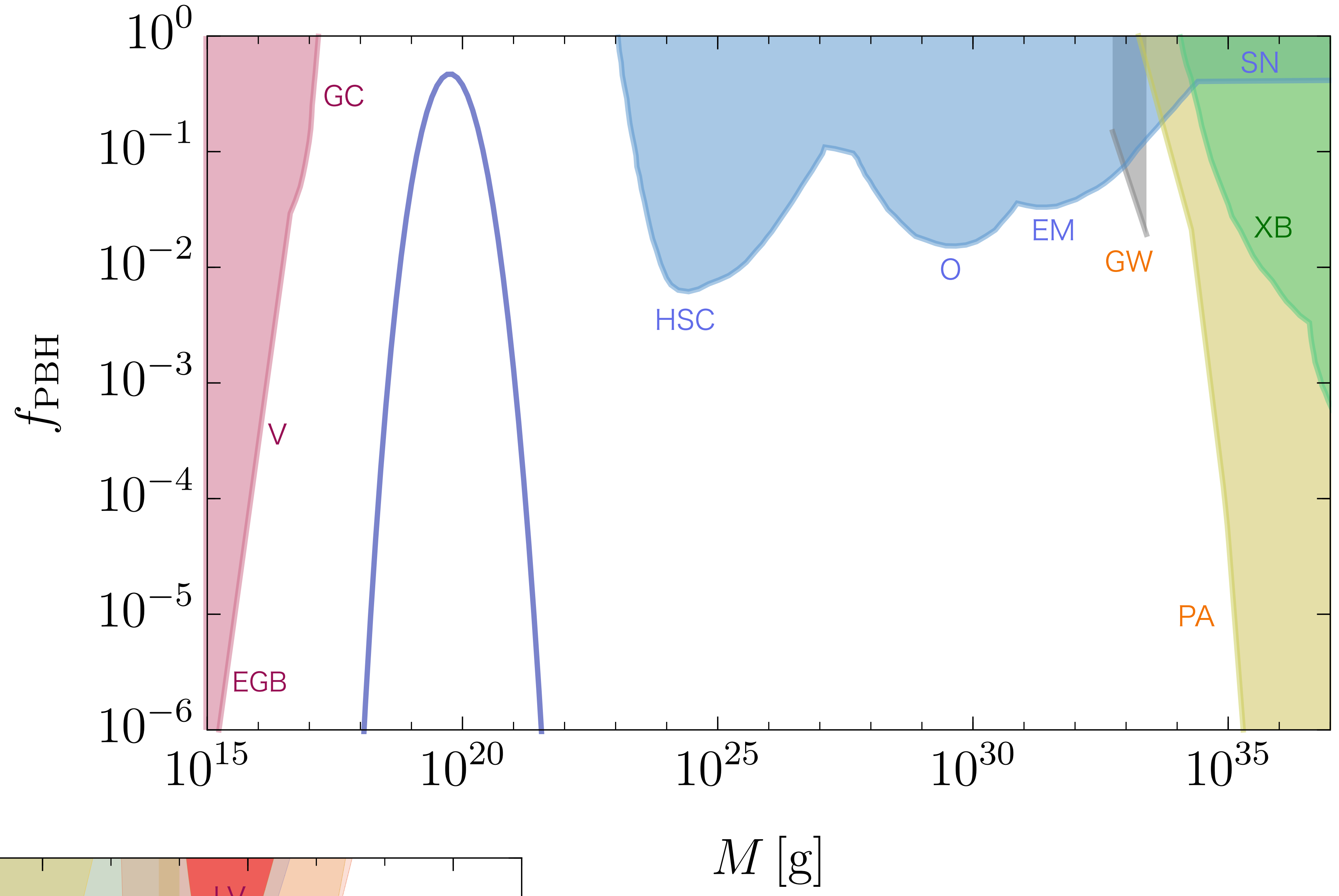


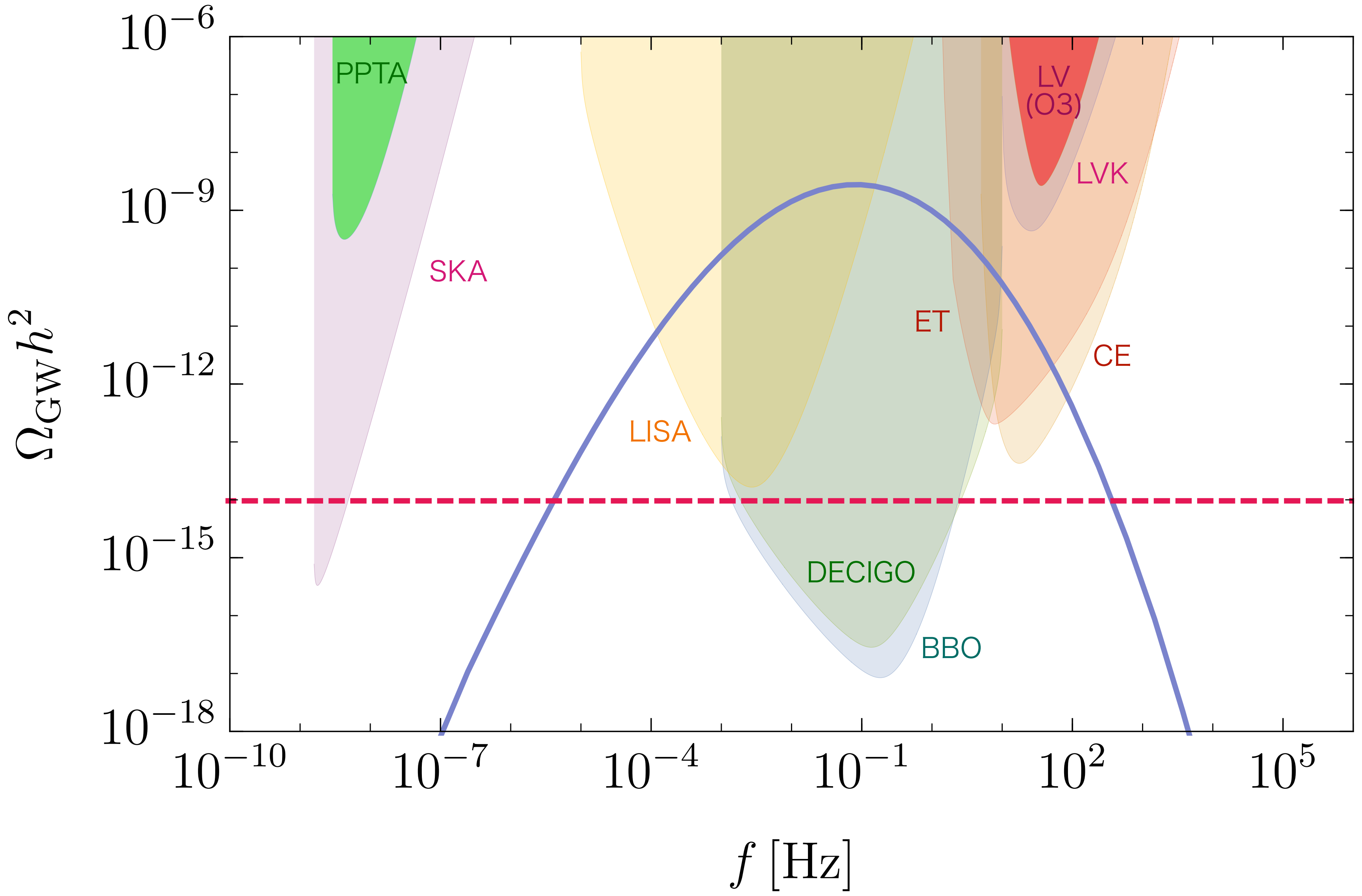
$\mathscr{D} = \mathcal{O}(1)$ would be enough for PBHs

~~No Go~~ 2 : multi-ψ

YT & Yamada '23b







$M = \phi_c/\sqrt{2} = 10^{16}\,\mathrm{GeV}$

$\Pi^2 = 100$

$\mu_2 = 10 M_{\mathrm{Pl}}$

$\mathscr{D} = 5$

δ as coarse-shelled ζ

YT & Vennin '21

- (linear) density contrast

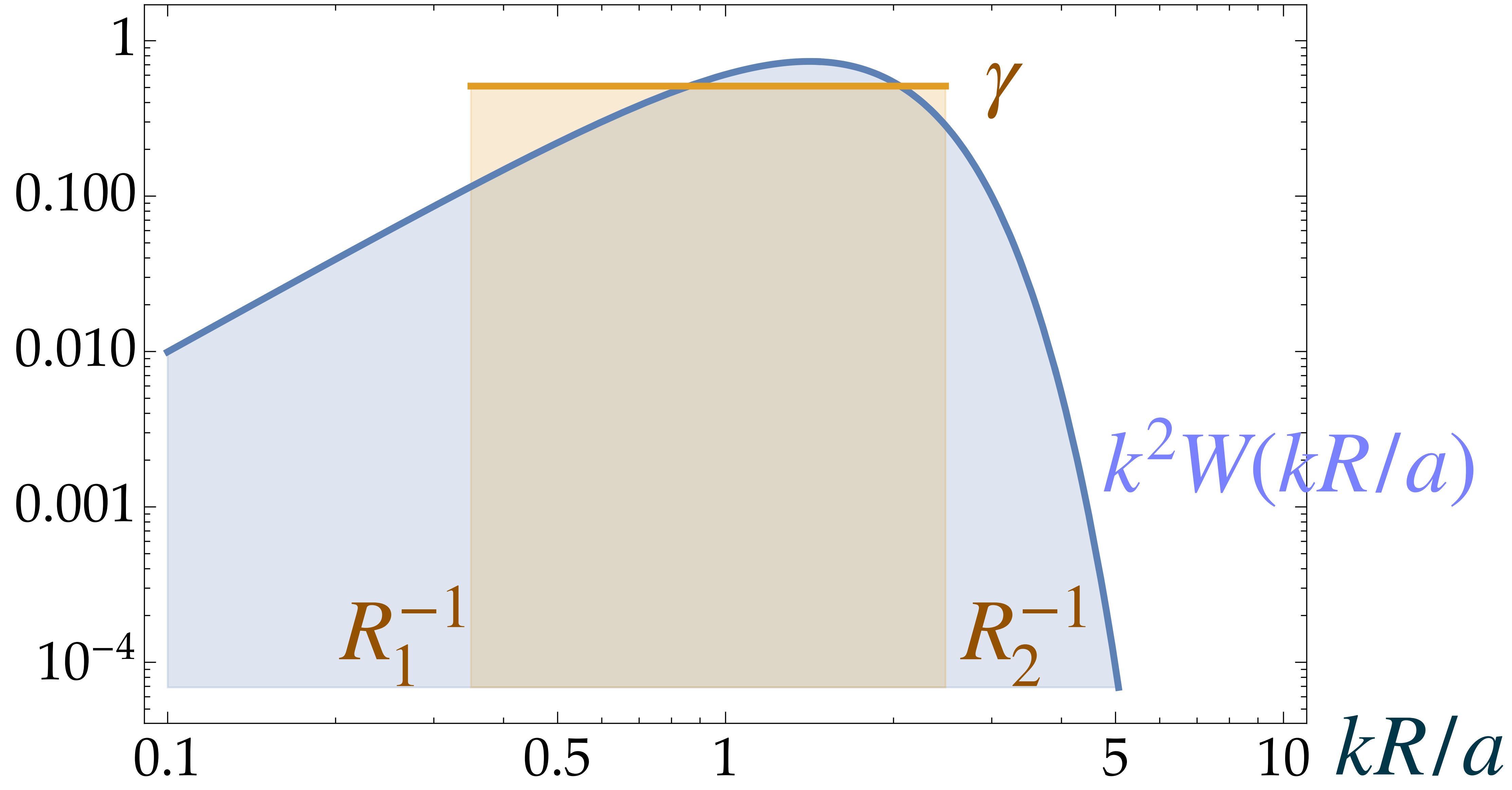
$$\delta = -\frac{9}{4}\frac{1}{a^2H^2}\nabla^2\zeta$$

- (nonlinear) compaction func.

$$\mathscr{C}(r) = \frac{2}{3}\left[1 - \left(1 + r\zeta'(r)\right)^2\right]$$

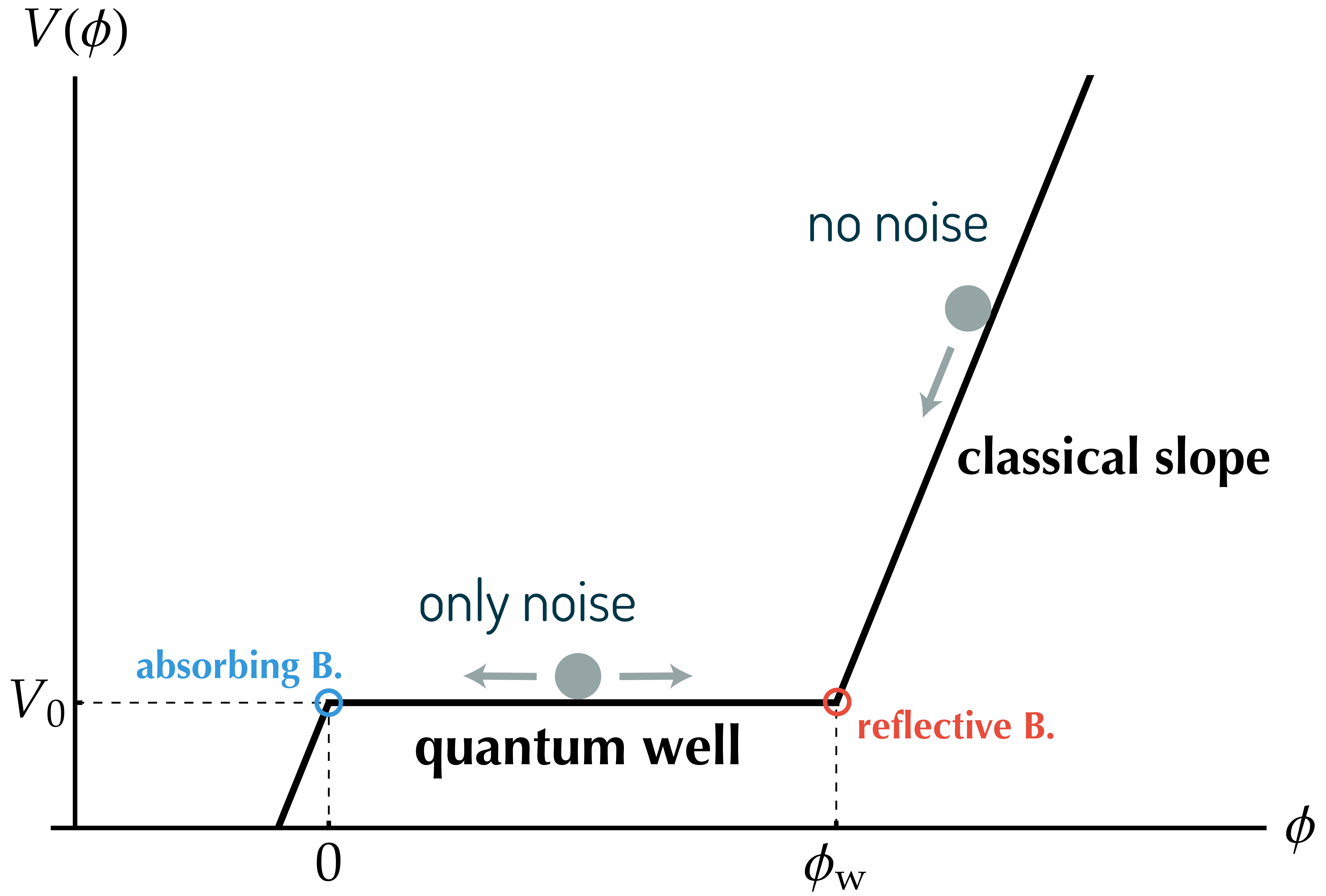
$$(-\nabla^2\zeta_R)(\mathbf{k}) = k^2 W\left(\frac{kR}{a}\right)\zeta(\mathbf{k}) \approx \gamma\left(\zeta_{R_2} - \zeta_{R_1}\right) =: \Delta\zeta$$

w/ appropriate R_1, R_2, γ



Quantum well

YT & Vennin '21

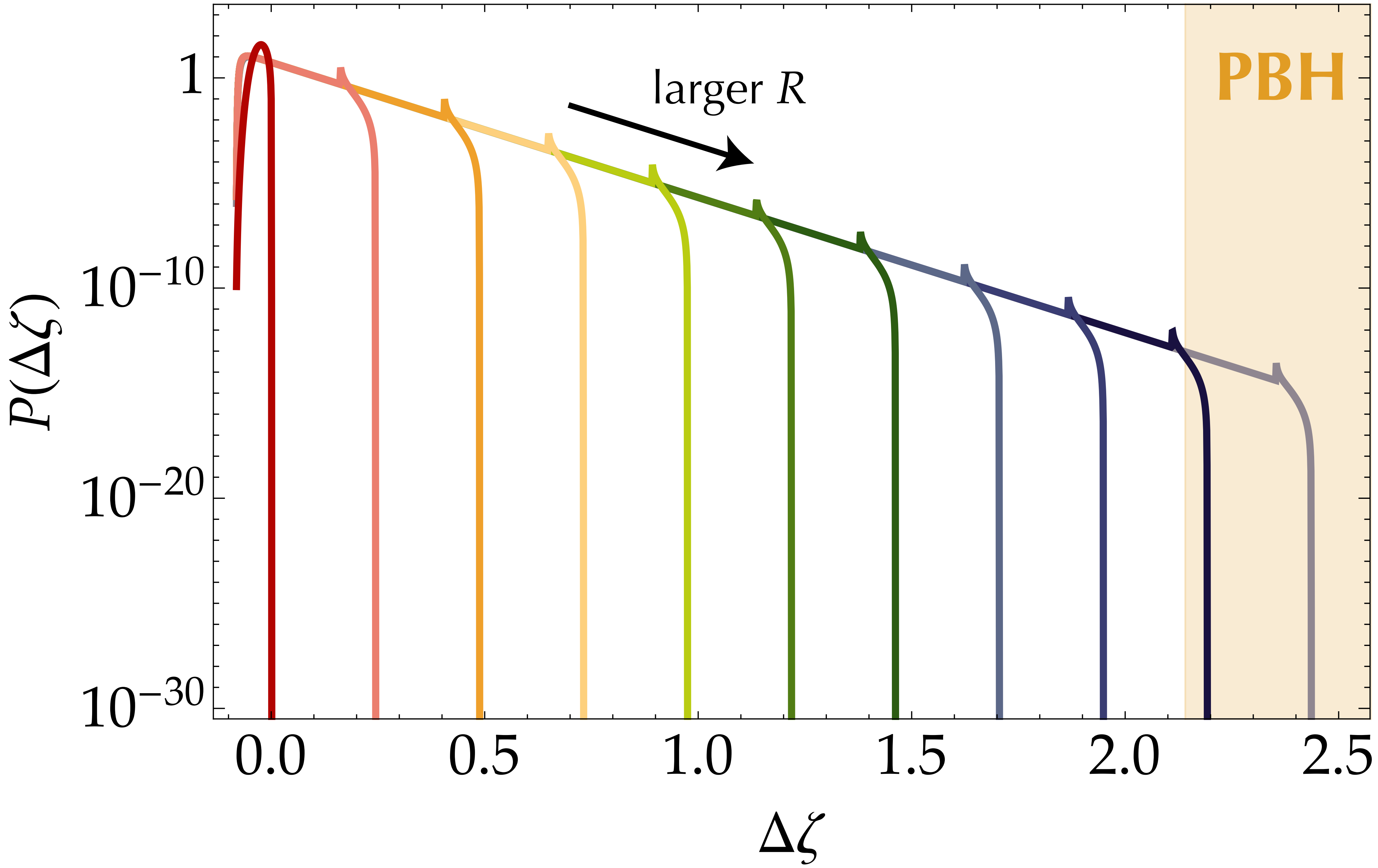


$$\mu = \sqrt{2}\frac{\phi_{\rm w}}{H/2\pi}$$

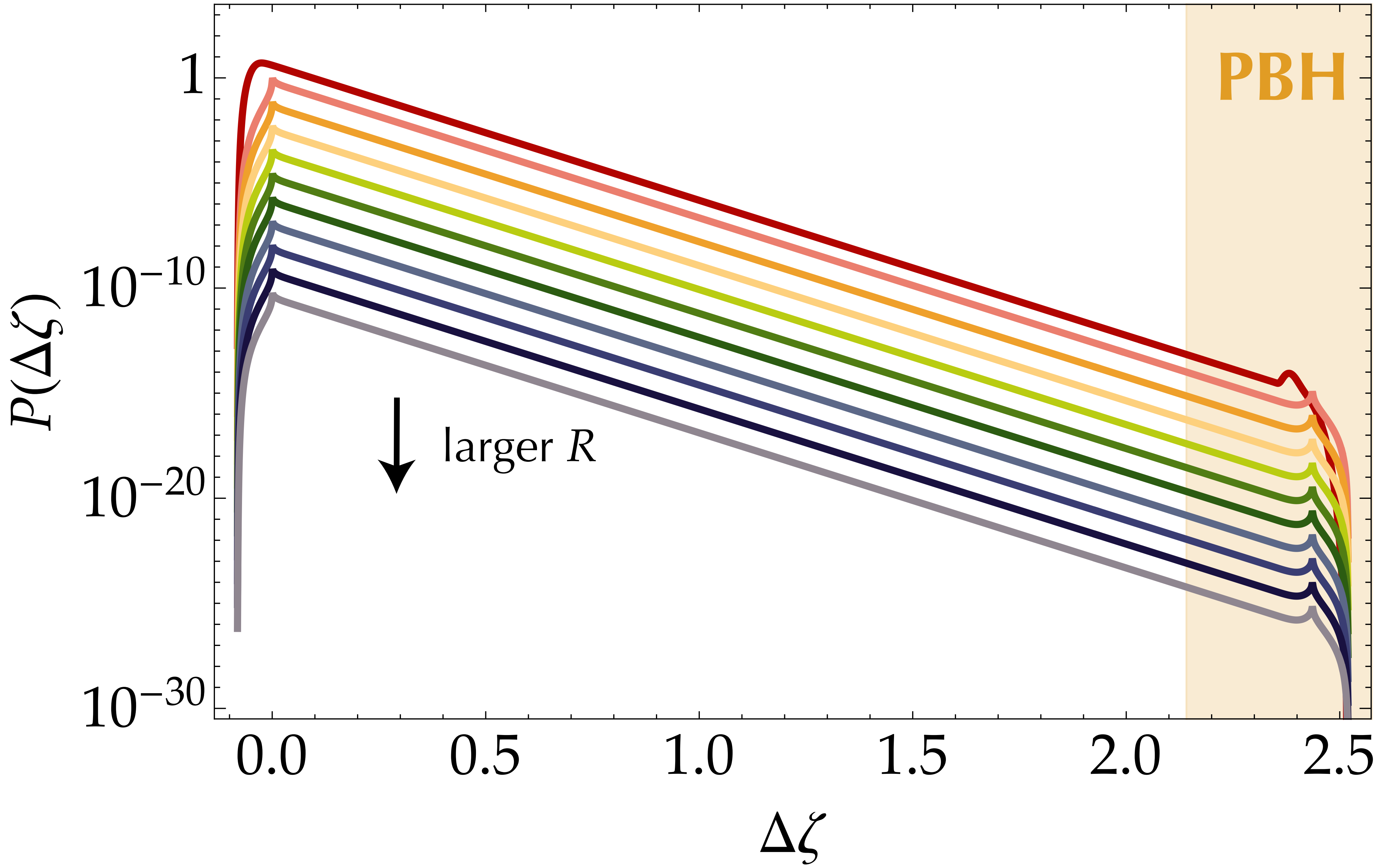
Quantum well

YT & Vennin '21

$\mu = \frac{1}{\sqrt{6}}, N_{\rm bw}^{(2)} < 0$



$\mu = \frac{1}{\sqrt{6}}, N_{\rm bw}^{(2)} > 0$



Quantum well

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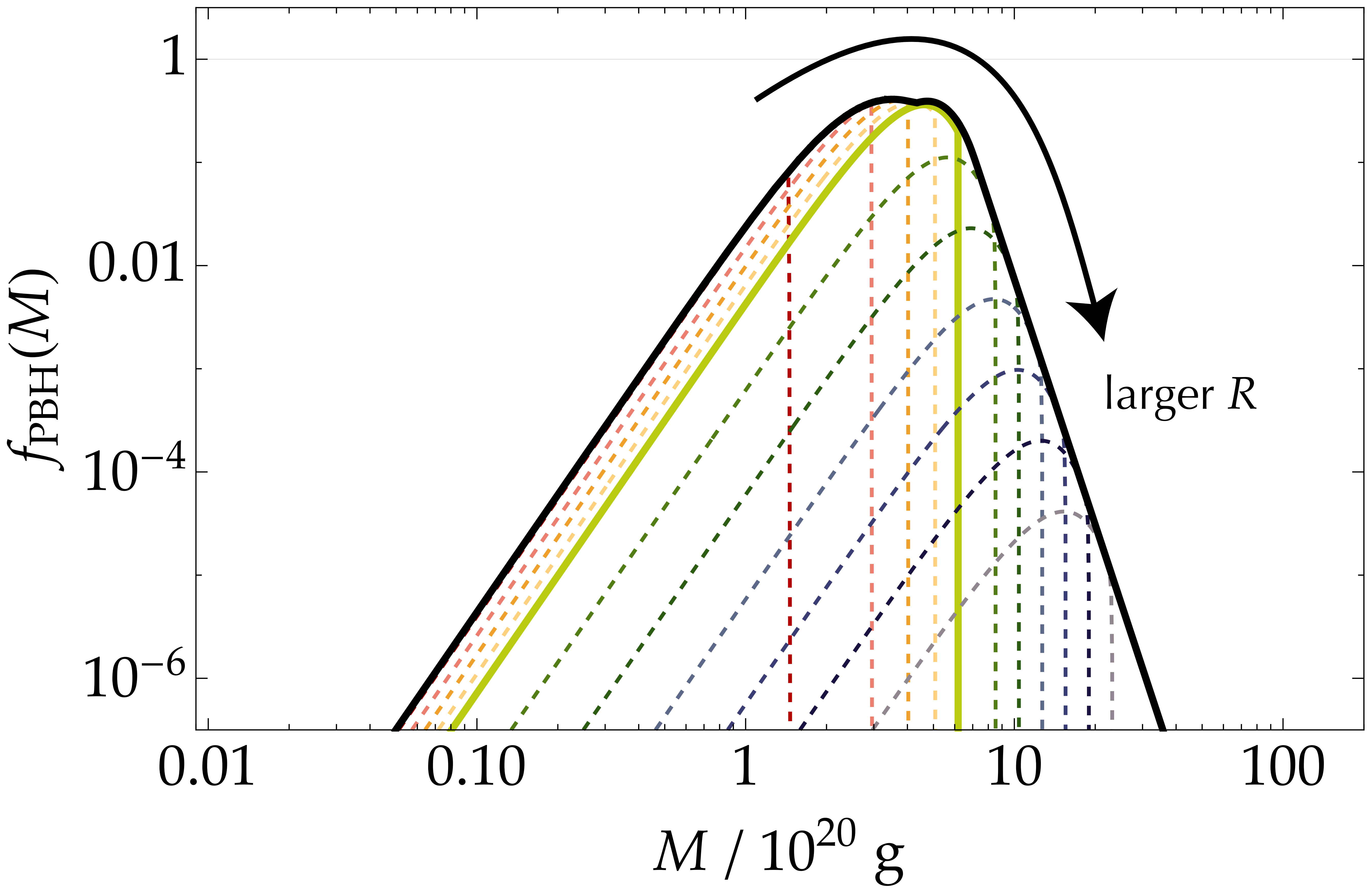
$$\mu = \frac{1}{\sqrt{6}}$$



Quantum well

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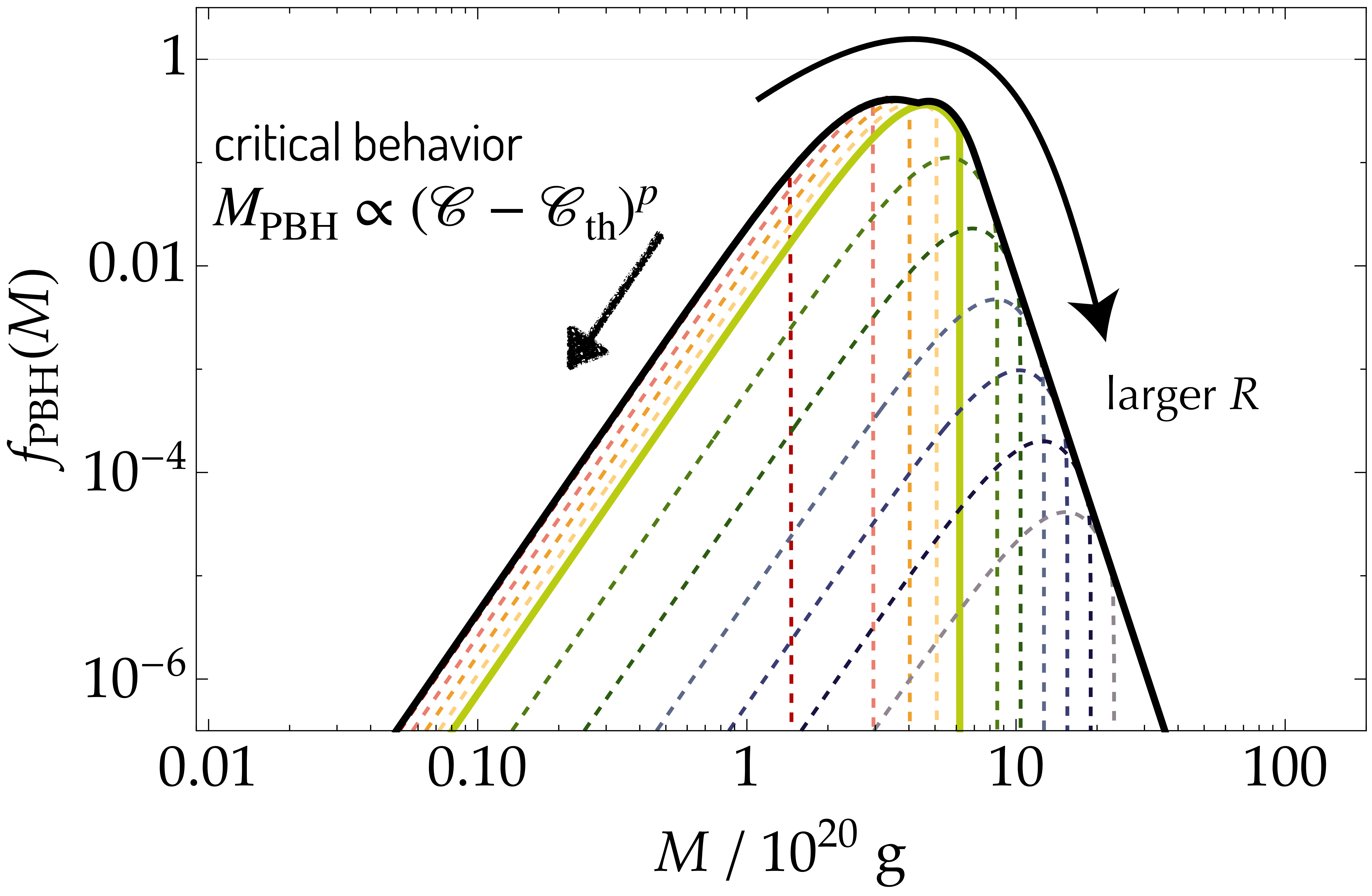
$$\mu = \frac{1}{\sqrt{6}}$$



Quantum well

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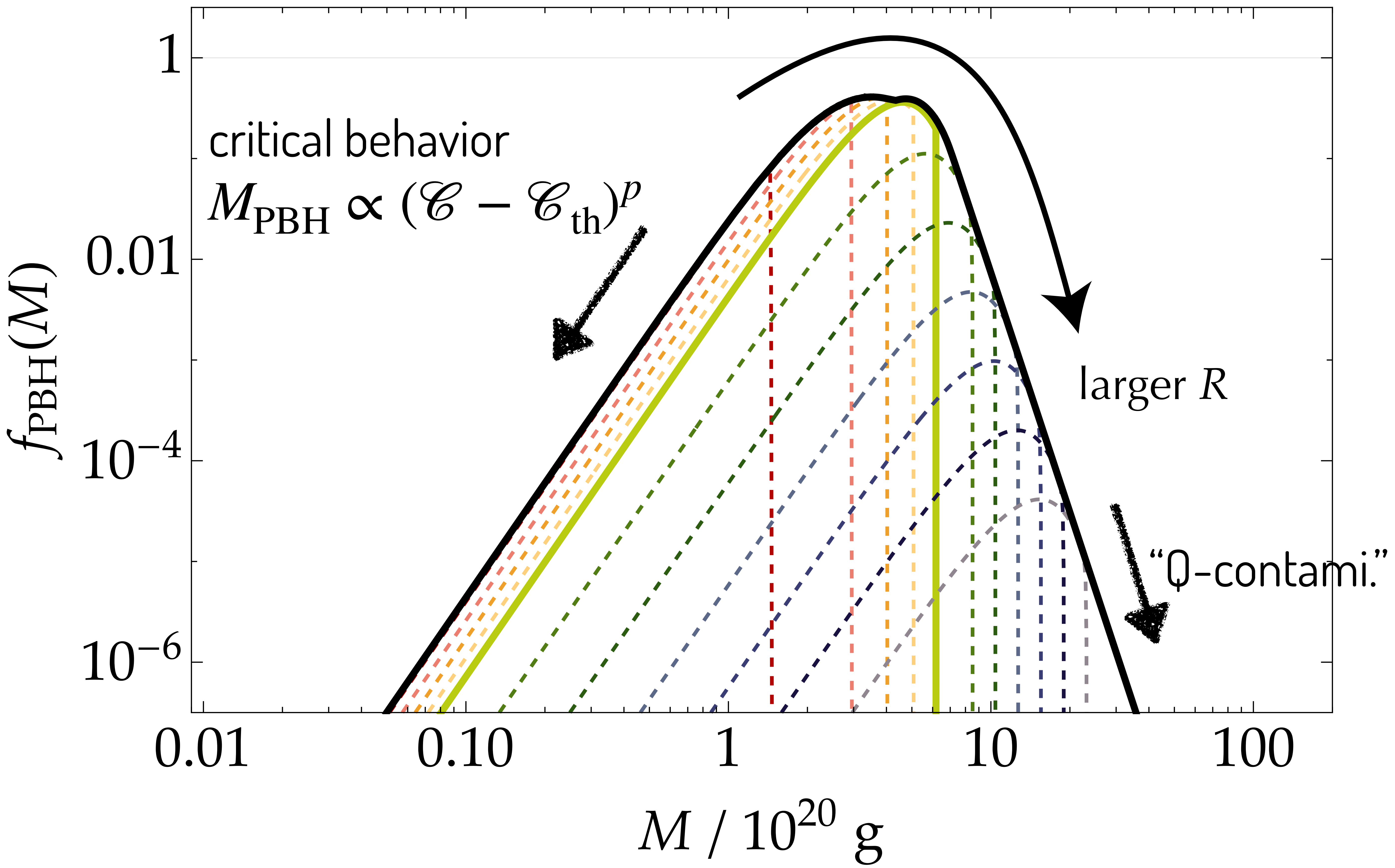
$$\mu = \frac{1}{\sqrt{6}}$$



Summary

Stochastic formalism, EFT of superH fields, is a powerful tool

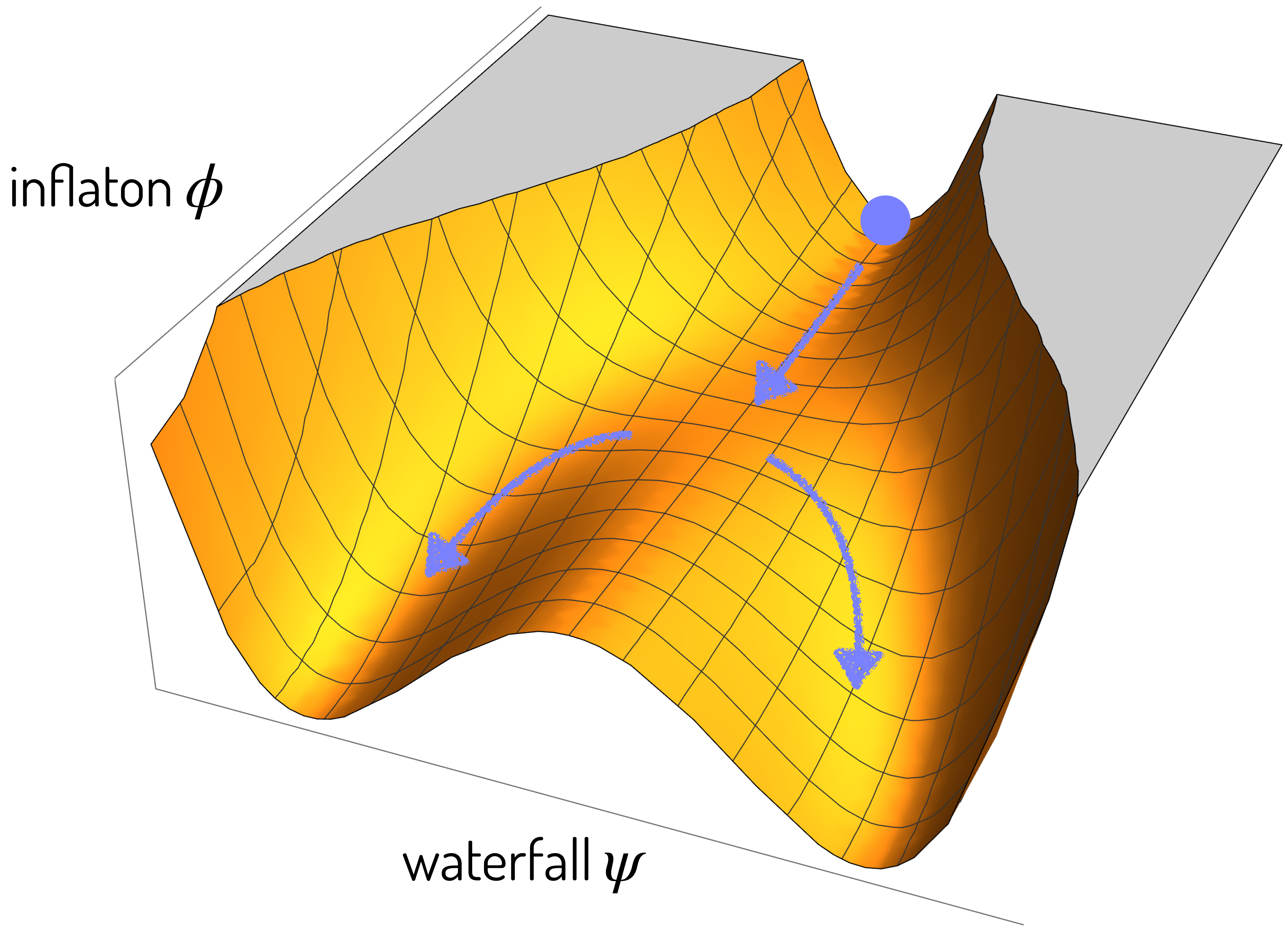
- w/ δN-formalism → stats. of ζ
- Coarse-graining → spatial correlation
- More direct num. calc.? → See Yurino's talk! (this afternoon)

Appendices

Hybrid Inflation

Linde '94

$$V(\phi,\psi) = \frac{1}{4\lambda}(M^2 - \lambda\psi^2)^2 + \frac{g^2}{2}\phi^2\psi^2 + V(\phi)$$



- ψ's ptb. determines the whole dynamics during/after phase trs.
- Flat potential can extend the waterfall phase $\mathcal{N}_{\rm water} \simeq \mathcal{N}_{\rm BH}$

Realise Massive PBH??

García-Berido, Linde, Wands '96
Clesse & García-Berido '15

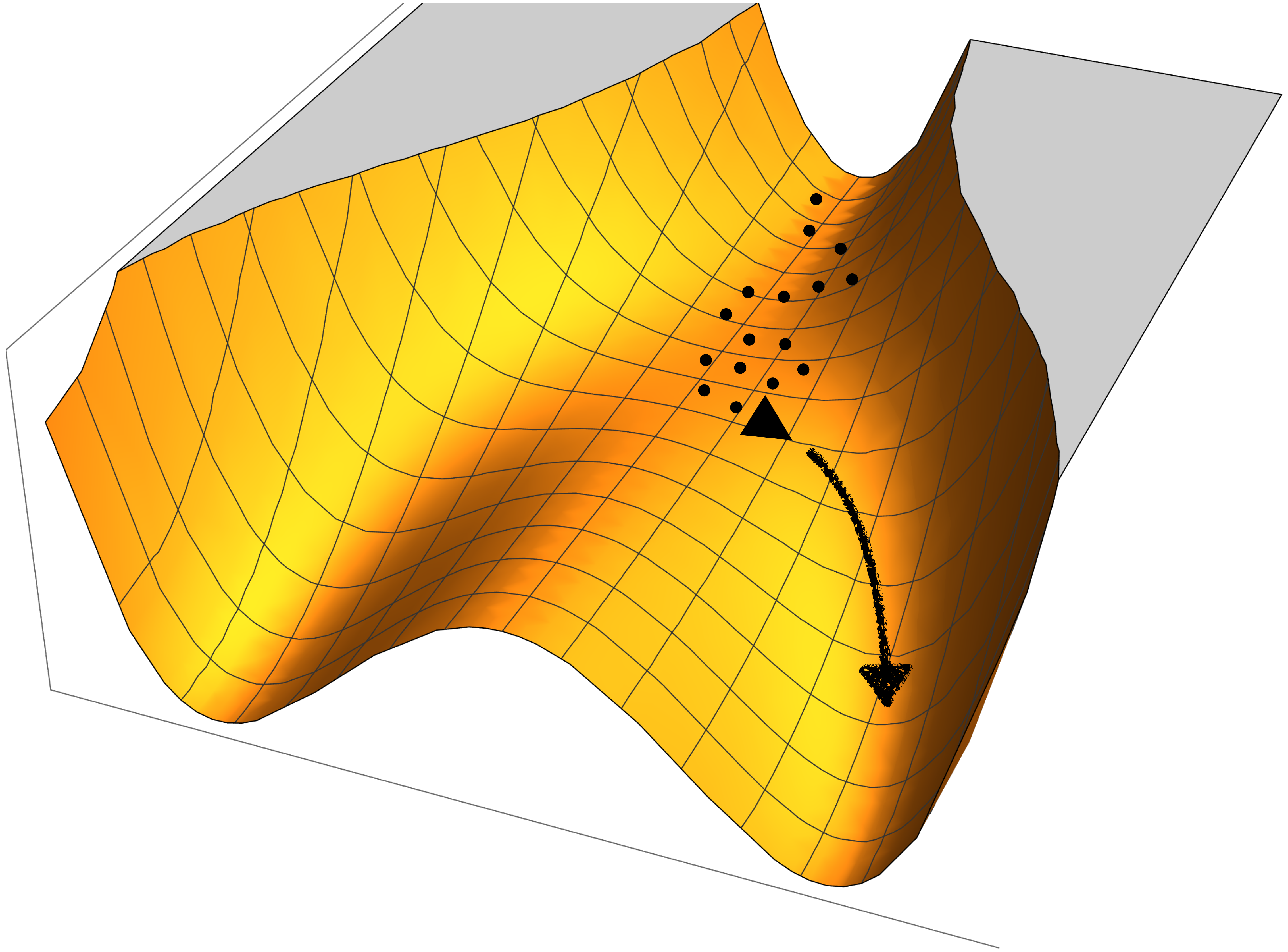
No-Go on Massive PBHs

Kawasaki & YT '15

$$V(\phi,\psi) = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2}\right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} + \frac{\phi - \phi_c}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2} \right]$$

$$\mathscr{P}_\zeta(k_{\mathrm{CMB}}) \simeq \frac{\Lambda^4}{24\pi^2\epsilon} \simeq 2\times 10^{-9}$$

$$n_s \simeq 1 + 2\frac{V_{\phi\phi}}{V} \simeq 0.96$$



- Valley phase

$$\frac{\mathrm{d}\langle\psi^2\rangle}{\mathrm{d}N} \simeq -\frac{V_{\psi\psi}}{V}(N)\bigg|_{\psi=0} \langle\psi^2\rangle + \left(\frac{H}{2\pi}\right)^2$$

$$\phi \simeq \phi_c - \frac{N - N_c}{\mu_1}$$

- Waterfall phase

$(\phi_{\mathrm{b.g.}}, \psi_{\mathrm{b.g.}})$ from $\phi = \phi_c, \quad \psi = \sqrt{\langle\psi^2\rangle}\Big|_{\phi_c}$

linear ptb. around $(\phi_{\mathrm{b.g.}}, \psi_{\mathrm{b.g.}})$

No-Go on Massive PBHs

Kawasaki & YT '15

$$V(\phi,\psi) = \Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2M^2} + \frac{\phi-\phi_c}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2}\right]$$

$$\mathcal{P}_\zeta(k_{\mathrm{CMB}}) \simeq \frac{\Lambda^4}{24\pi^2\epsilon} \simeq 2\times10^{-9}$$

$$n_s \simeq 1 + 2\frac{V_{\phi\phi}}{V} \simeq 0.96$$

Clesse & García-Bellido '15

$$\begin{cases} \mathcal{N}_{\mathrm{BH}} \simeq \mathcal{N}_c \propto \Pi & \Pi = M\sqrt{\phi_c\mu_1} \\ \mathcal{P}_{\zeta,\max} \simeq \mathcal{P}_\zeta(k_c) \propto \Pi & \end{cases}$$

- Valley phase

$$\frac{\mathrm{d}\langle\psi^2\rangle}{\mathrm{d}N} \simeq -\left.\frac{V_{\psi\psi}}{V}(N)\right|_{\psi=0}\langle\psi^2\rangle + \left(\frac{H}{2\pi}\right)^2$$

$$\phi \simeq \phi_c - \frac{N-N_c}{\mu_1}$$

- Waterfall phase

$(\phi_{\mathrm{b.g.}},\psi_{\mathrm{b.g.}})$ from $\phi=\phi_c$, $\psi = \left.\sqrt{\langle\psi^2\rangle}\right|_{\phi_c}$

linear ptb. around $(\phi_{\mathrm{b.g.}},\psi_{\mathrm{b.g.}})$

$$\therefore\ \mathcal{P}_\zeta(k_c) \simeq 0.01\mathcal{N}_{\mathrm{BH}}$$

e.g. $\mathcal{P}_\zeta(k_c) \simeq \mathcal{O}(0.1)$ for $\mathcal{N}_{\mathrm{BH}} = \mathcal{O}(10)$

Massive PBHs will be inevitably overproduced!