Impact of non gaussianities on the primordial black hole abundance

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Based on: arXiv:2211.01728
arXiv:2305.13382

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NEHOP
NEW HORIZONS IN
PRIMORDIAL BLACK HOLE PHYSICS

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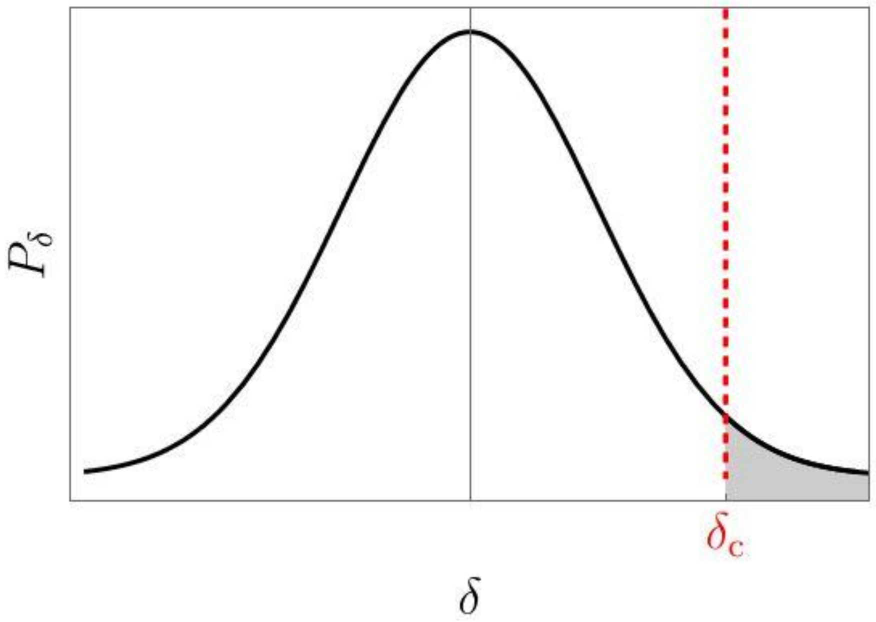
Abundance of PBHs

Mass Fraction

$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^{\gamma}\, \mathrm{P}_{\delta}(\delta) d\delta$$



Mass distribution

$$f_{\mathrm{PBH}}(M_{\mathrm{PBH}}) \equiv \frac{1}{\Omega_{\mathrm{DM}}} \frac{d\Omega_{\mathrm{PBH}}}{d \log M_{\mathrm{PBH}}}\,, \qquad \Omega_{\mathrm{PBH}} = \int d\log M_H \left(\frac{M_{\mathrm{eq}}}{M_H}\right)^{1/2} \beta_{\mathrm{NG}}(M_H)\,,$$

Abundance of PBHs: The role of Non-Gaussianities (NG).

An exact formalism for the computation of PBHs mass fraction abundance (or, equivalently their mass distribution) in the presence of local non gaussianity (NG) in the curvature perturbation field ζ by including:

NON-LINEARITIES (NL)

$$\delta(r,t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(r)}\left[\zeta''(r) + \frac{2}{r}\zeta'(r) + \frac{1}{2}\zeta'(r)^2\right]$$

T. Harada, C. M. Yoo, T. Nakama and Y. Koga,.– arXiv:1503.03934

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$

$$\zeta = F(\zeta_{\mathrm{G}})$$

power-series expansion

$$\zeta = \zeta_{\mathrm{G}} + \tfrac{3}{5} f_{\mathrm{NL}}\zeta_{\mathrm{G}}^2 + \tfrac{9}{25} g_{\mathrm{NL}}\zeta_{\mathrm{G}}^3 + \dots$$

ultra slow-roll with an upward step

$$\zeta = -\tfrac{2}{|h|}\left[\sqrt{1 - |h|\zeta_{\mathrm{G}}} - 1\right]$$

curvaton

$$\boxed{\zeta = \log\left[X(r_{\mathrm{dec}}, \zeta_{\mathrm{G}})\right]}$$

ultra slow-roll

$$\zeta = -\left(\tfrac{6}{5} f_{\mathrm{NL}}\right)^{-1} \log\left(1 - \tfrac{6}{5} f_{\mathrm{NL}}\zeta_{\mathrm{G}}\right)$$

Threshold statistics on the Compaction: Mathematical formulation

By integrating δ over the radial coordinate *r* we get the compaction function *C*

$$\mathcal{C}(r) = -2\Phi\, r\, \zeta'(r)\left[1 + \frac{r}{2}\zeta'(r)\right] = \mathcal{C}_1(r) - \frac{1}{4\Phi}\mathcal{C}_1(r)^2\,, \qquad \mathcal{C}_1(r) := -2\Phi\, r\, \zeta'(r)$$

In the presence of NG C_1 takes the form

$$\mathcal{C}_1(r) = -2\Phi\, r\, \zeta_{\rm G}'(r)\,\frac{dF}{d\zeta_{\rm G}} = \mathcal{C}_{\rm G}(r)\,\frac{dF}{d\zeta_{\rm G}}\,, \qquad \text{with} \quad \mathcal{C}_{\rm G}(r) := -2\Phi\, r\, \zeta_{\rm G}'(r)$$

From the two-dimensional joint PDF of ζ_G and C_G, called P_G

Later on confirmed also by A.Gow et al arXiv:2211.08348

NG PBH mass fraction-distribution adopting threshold statistics on the compaction function

$$\beta_{\rm NG} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C} - \mathcal{C}_{\rm th})^{\gamma}\,{\rm P}_{\rm G}(\mathcal{C}_{\rm G}, \zeta_{\rm G})\,d\mathcal{C}_{\rm G}\,d\zeta_{\rm G}\,,$$

$$\mathcal{D} = \{\mathcal{C}_{\rm G},\, \zeta_{\rm G} \in \mathbb{R} \;:\; \mathcal{C}(\mathcal{C}_{\rm G}, \zeta_{\rm G}) > \mathcal{C}_{\rm th} \;\wedge\; \mathcal{C}_1(\mathcal{C}_{\rm G}, \zeta_{\rm G}) < 2\Phi\}\,,$$

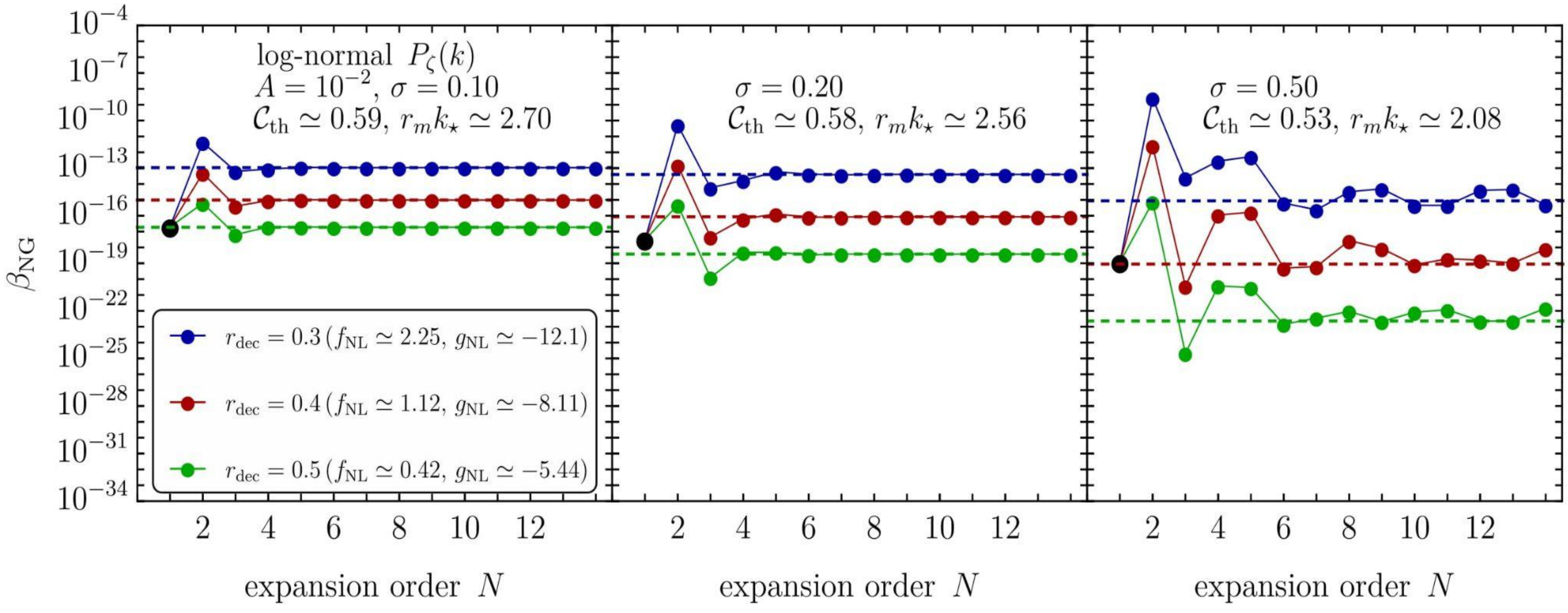
$$f_{\rm PBH}(M_{\rm PBH}) = \frac{1}{\Omega_{\rm DM}} \int_{M_{\rm H}^{\rm min}(M_{\rm PBH})} d\log M_{\rm H} \left(\frac{M_{\rm eq}}{M_{\rm H}}\right)^{1/2} \left[1 - \frac{\mathcal{C}_{\rm th}}{\Phi} - \frac{1}{\Phi}\left(\frac{M_{\rm PBH}}{\mathcal{K} M_H}\right)^{1/\gamma}\right]^{-1/2} \frac{\mathcal{K}}{\gamma}\left(\frac{M_{\rm PBH}}{\mathcal{K} M_{\rm H}}\right)^{\frac{1+\gamma}{\gamma}}$$
$$\times \int d\zeta_{\rm G}\, P_{\rm G}(\mathcal{C}_{\rm G}(M_{\rm PBH}, \zeta_{\rm G}), \zeta_{\rm G} | M_{\rm H}) \left(\frac{dF}{d\zeta_{\rm G}}\right)^{-1}.$$

Application to the curvaton model (1)

Failure of the perturbative approach (Narrow)

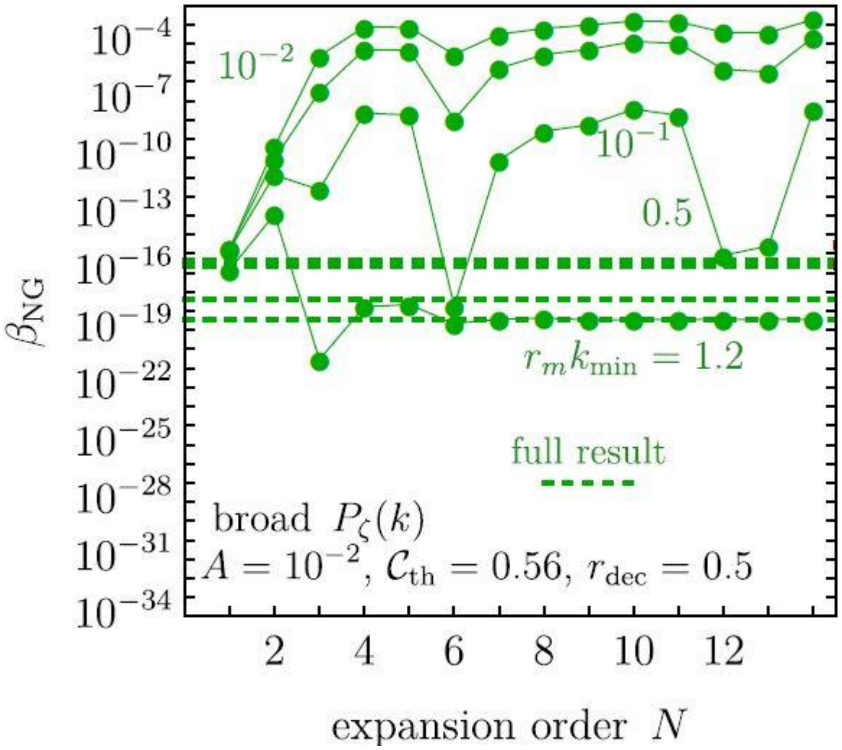
- - - - $\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$

● $\zeta_N = \sum_{n=1}^{N} c_n(r_{\rm dec})\zeta_{\rm G}^n$



Application to the curvaton model (1)

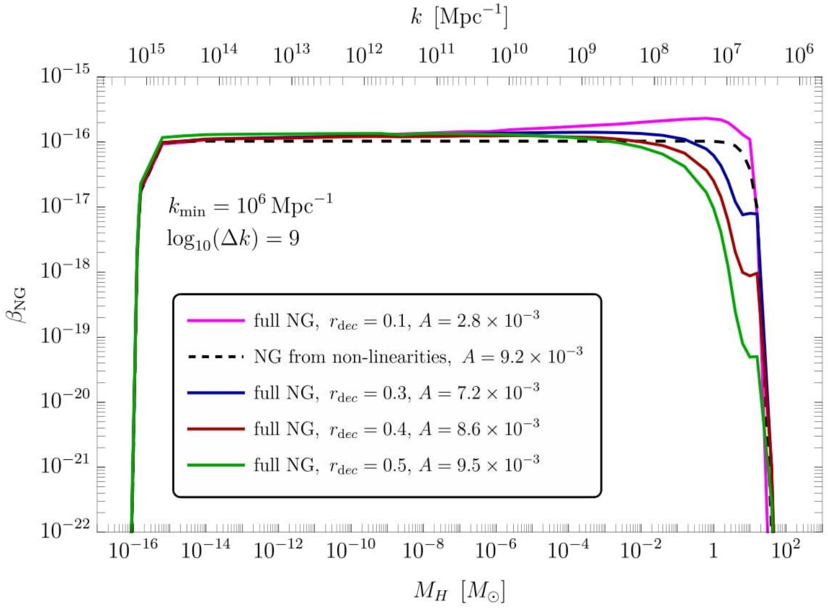
Failure of the perturbative approach (Broad)

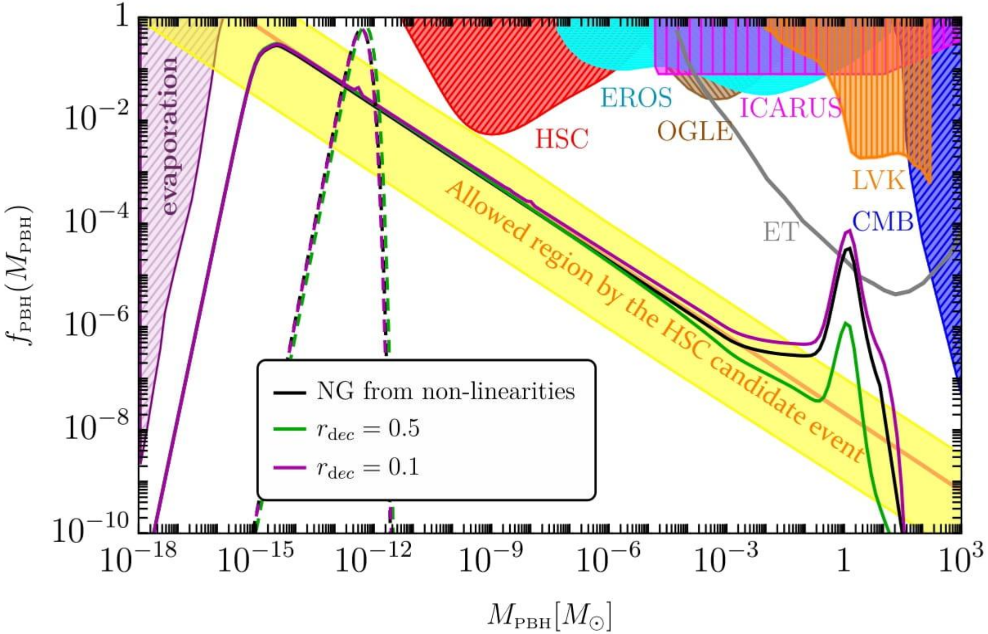


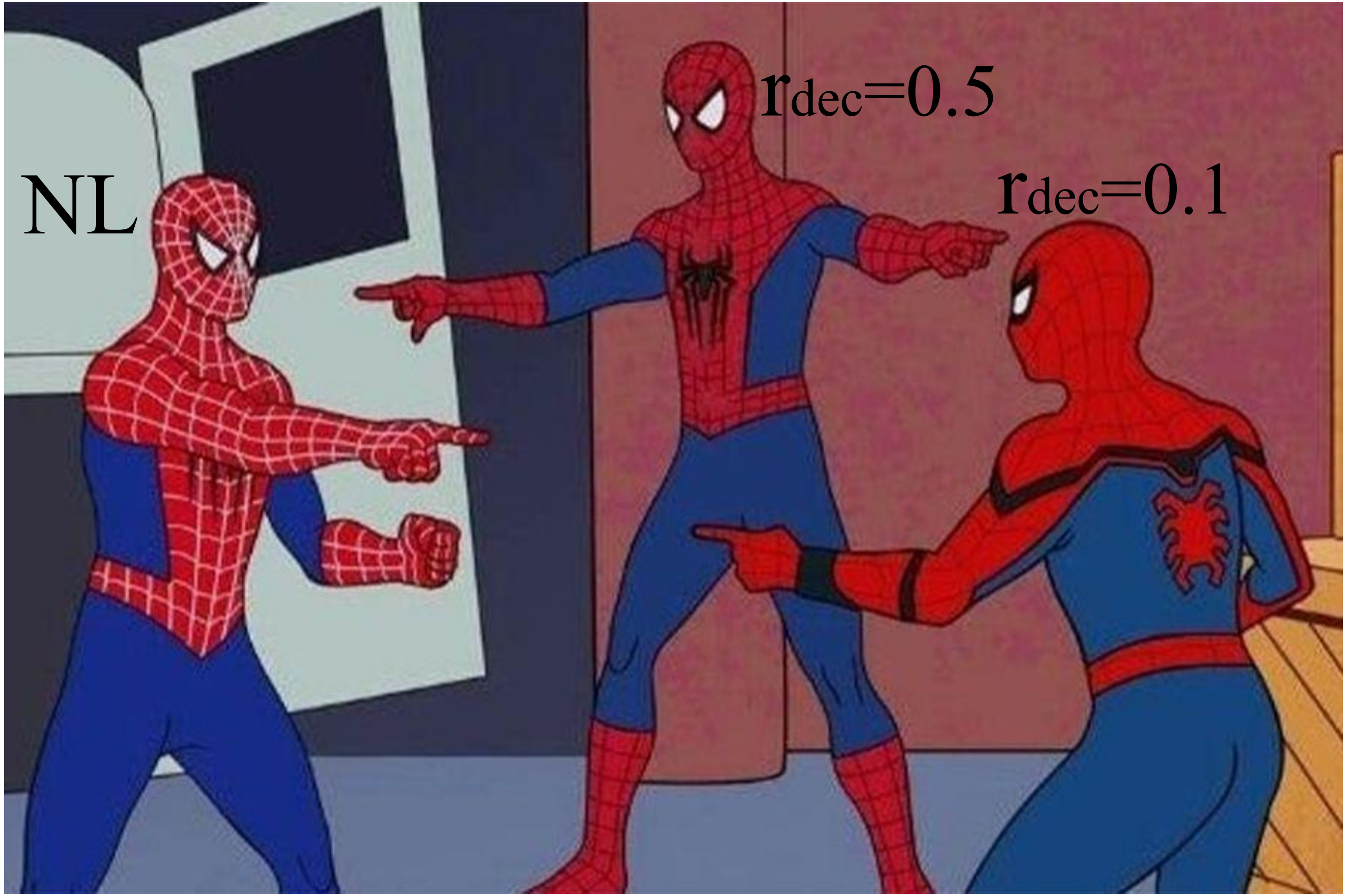
For a broad Power spectrum the power-series expansion is simply wrong (the series does not converge for whatever N) and one is forced to use the full result NG.

Application to the curvaton model (2)

Breaking of M_H-Independence





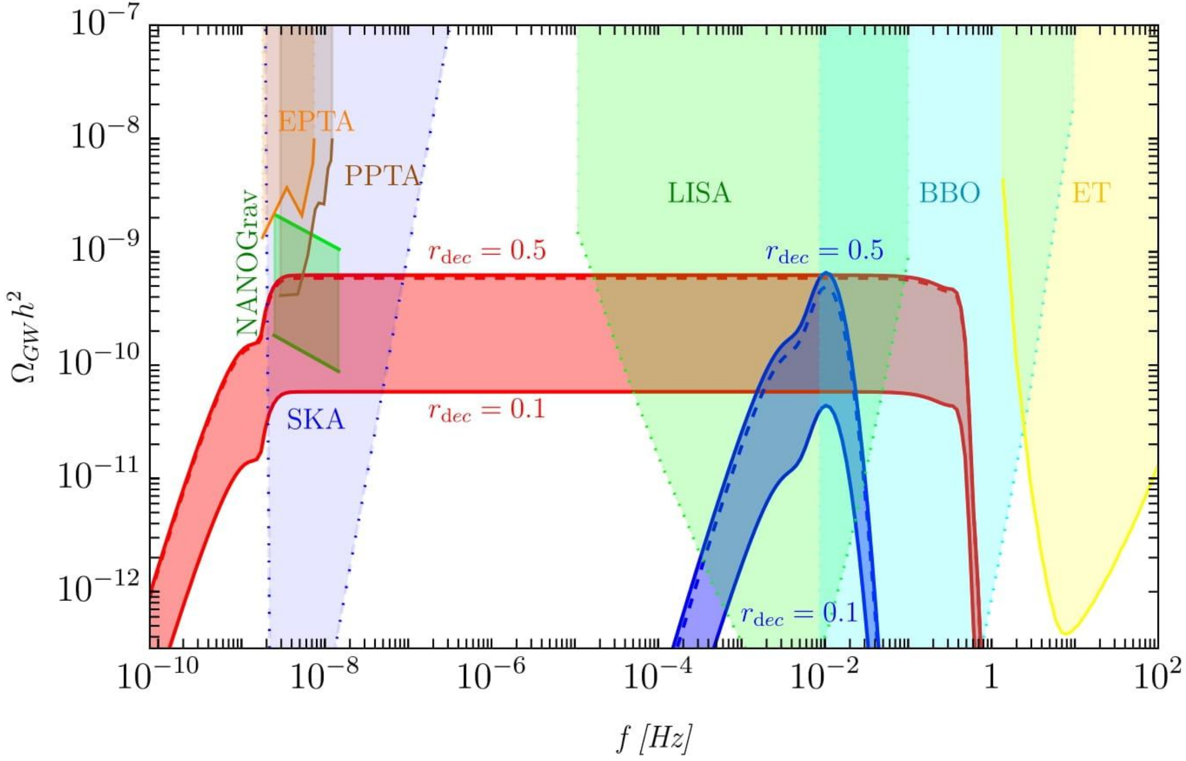


We need another observable: The induced Gravitational waves

Application to the curvaton model (3)

$$\Omega_{\rm GW}(k) \simeq \Omega_{\rm r0} P_\zeta^2(k)$$

Need for another observable



Axion-like Curvaton model

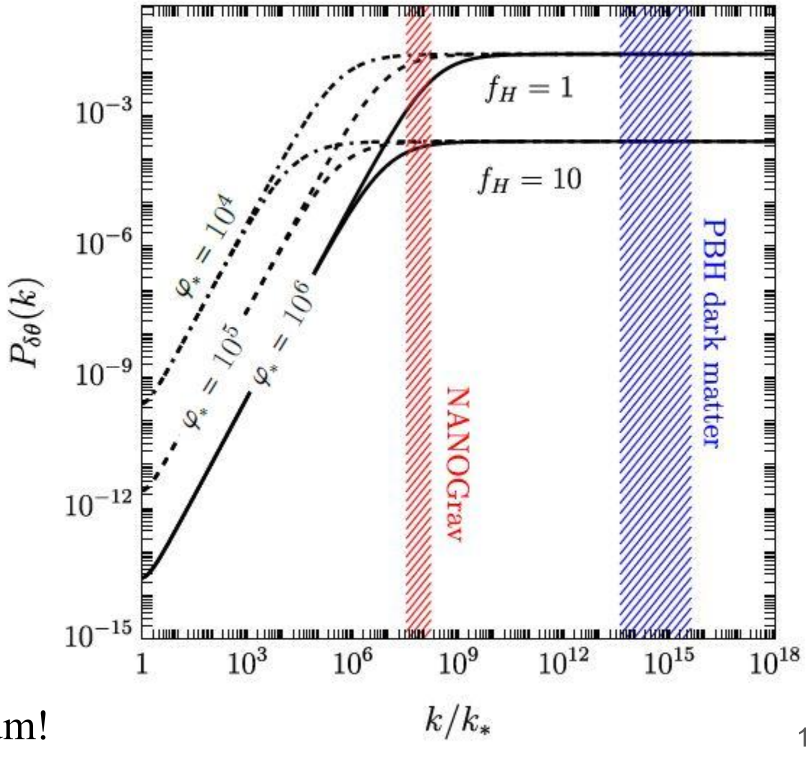
M. Kawasaki, N. Kitajima, and T. T. Yanagida – arXiv:1207.2550
K. Ando, K. Inomata, M. Kawasaki, K. Mukaida, and T. T. Yanagida, –arXiv:1711.08956
K. Inomata, M. Kawasaki, K. Mukaida, and T. T. Yanagida,-- arXiv:2011.01270

A complex scalar field with an angular component ϑ dubbed curvaton. $\Phi = \varphi e^{i\vartheta}$
We assume the following potential, and so the EoM:

$$V(\varphi) = \frac{c}{2} H_{\rm inf}^2 (\varphi - f)^2$$

So we get an enhancement of $\delta\vartheta$

$$k^{3/2} |\delta\vartheta_k| = \frac{1}{\sqrt{2}\varphi_H(N_k)}$$



In order to make the perturbations $\delta\vartheta_k$ phenomenologically relevant, we suppose decay into photons after inflation.

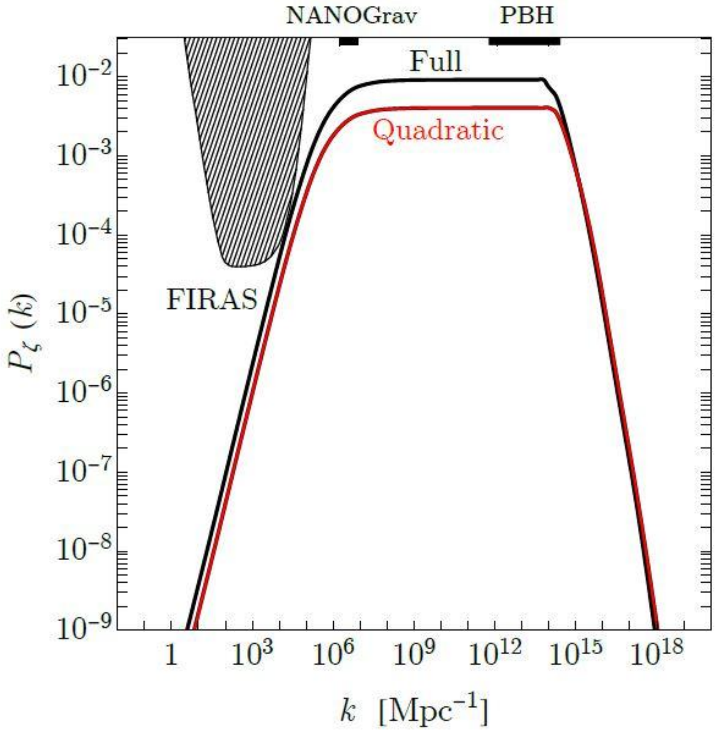
We find for the first time in a particle model a Broad spectrum!

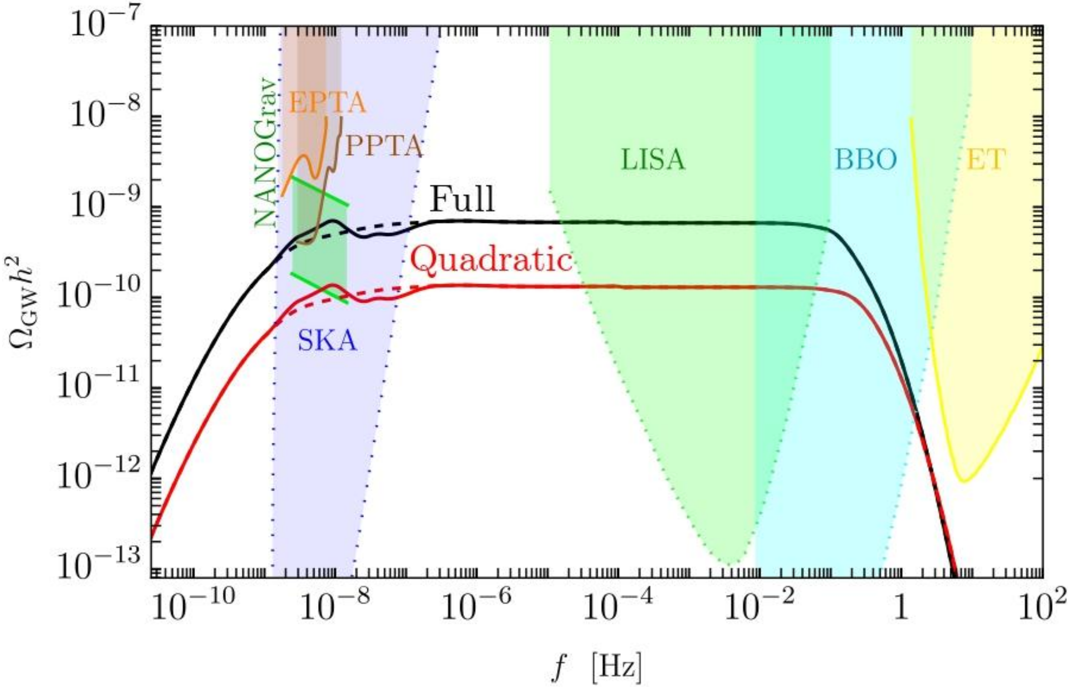
Phenomenological consequences: SIGWs

Impact of QCD phase transition as:
K. T. Abe, Y. Tada and I Ueda, – arXiv:2010.06193

$$\zeta_2 \equiv \zeta_G + \frac{3}{5} f_{NL}(r_{dec}) \zeta_G^2 \,, \qquad \text{with} \qquad f_{NL}(r_{dec}) \equiv \frac{5}{3}\left(\frac{3}{4 r_{dec}} - 1 - \frac{r_{dec}}{2}\right)$$

$$\zeta = \log\left[X(r_{dec}, \zeta_G)\right]$$





A Myriad of GWs experiment and data are coming

Take home message

- *Fundamental to take into account both kind of NGs computing the abundance of PBHs.*
- *Quadratic approximation leads to wrong phenomenologically results.*

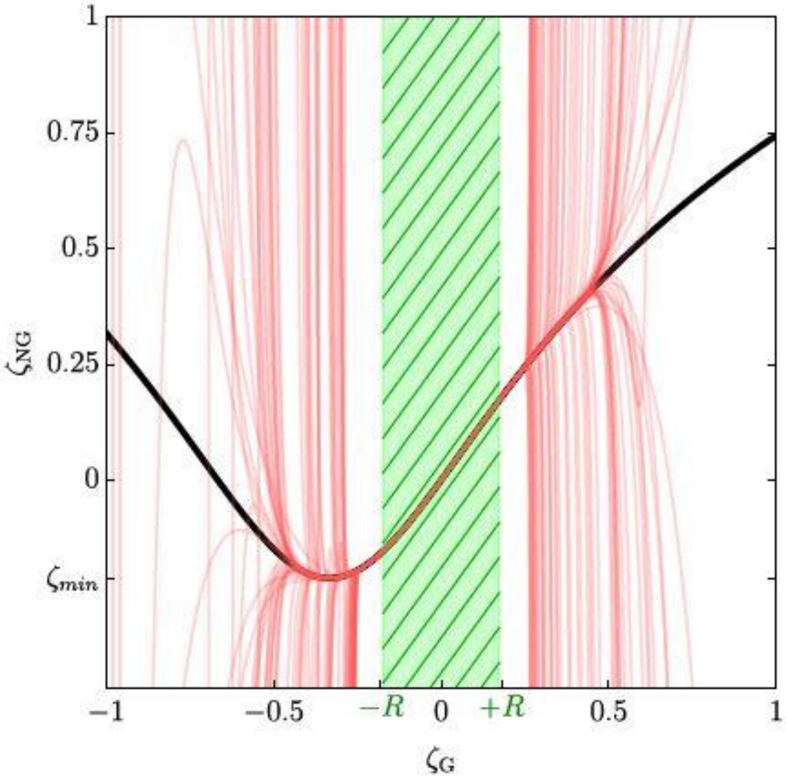
Backup Slides

Failure of perturbative approach

$$\sum_{n=1}^{\infty} c_n(r_{\rm dec})\zeta_{\rm G}^n = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]$$

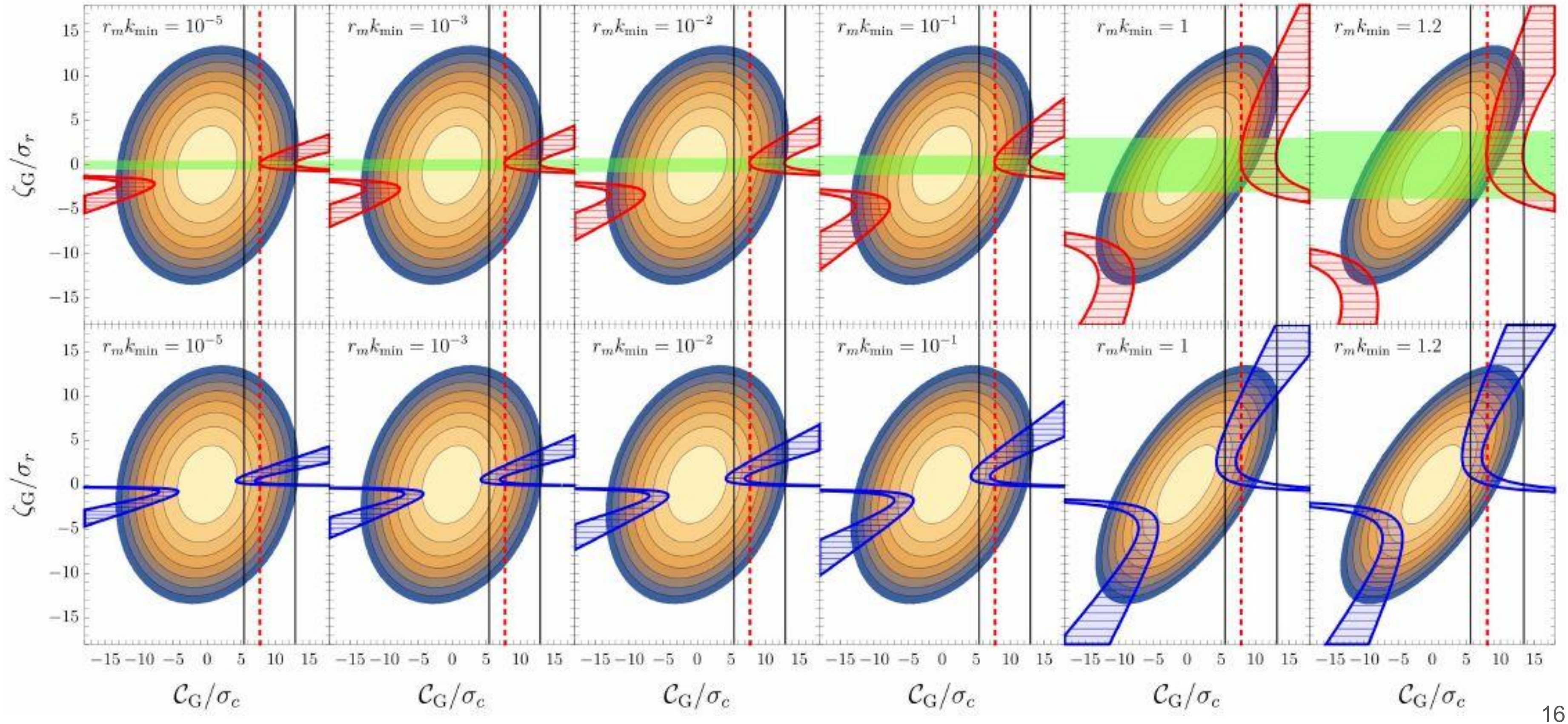
$$c_n(1) = \frac{(-1)^{n+1}}{n(2/3)^{n-1}}\,,$$

$$R = \lim_{n\to\infty}\left|\frac{c_n(1)}{c_{n+1}(1)}\right| = \frac{2}{3}\,,$$



$$\frac{4(1-\sqrt{1-3\mathcal{C}_{\rm th}/2})}{3} < \mathcal{C}_{\rm G}\frac{dF}{d\zeta_{\rm G}} < \frac{4}{3}$$

Breaking of scale invariance



SIGW

In a pure RD universe with cg fixed

$$\Omega_{\mathrm{GW}} = \frac{c_g \Omega_r}{36} \int_0^{\frac{1}{\sqrt{3}}} dt \int_{\frac{1}{\sqrt{3}}}^{\infty} ds \left[\frac{\left(t^2 - 1/3\right)\left(s^2 - 1/3\right)}{t^2 - s^2} \right]^2 \left[\mathcal{I}_c(t,s)^2 + \mathcal{I}_s(t,s)^2 \right] P_\zeta \left[\frac{k\sqrt{3}}{2}(s+t) \right] P_\zeta \left[\frac{k\sqrt{3}}{2}(s-t) \right]$$

$$\mathcal{I}_c(t,s) = 4 \int_0^{\infty} d\tau \tau (\cos\tau) [2T(t,\tau)T(s,\tau) + \left(T(t,\tau) + t\tau T'(t,\tau)\right)\left(T(s,\tau) + s\tau T'(s\tau)\right)] ,$$

$$\mathcal{I}_s(t,s) = -4 \int_0^{\infty} d\tau \tau (\sin\tau) [2T(t,\tau)T(s,\tau) + \left(T(t,\tau) + t\tau T'(t,\tau)\right)\left(T(s,\tau) + s\tau T'(s,\tau)\right)] ,$$