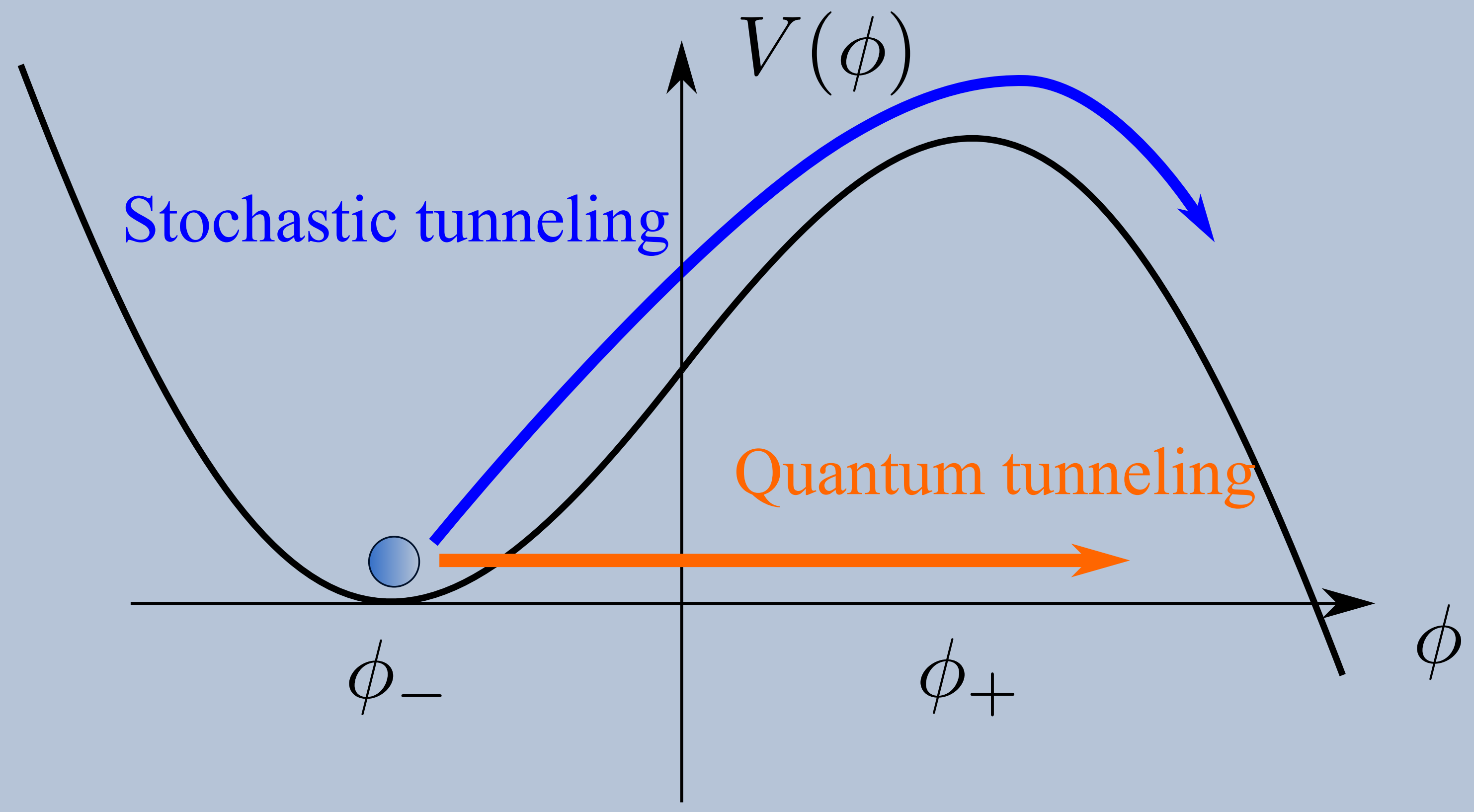
LABORATOIRE DE PHYSIQUE
DE L'ÉCOLE NORMALE SUPÉRIEURE



Primordial black holes from stochastic tunnelling

Chiara Animali

in collaboration with Vincent Vennin JCAP02(2023)043

19th June 2023
New Horizons in Primordial Black Holes, Napoli

Primordial Black Holes

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- Black holes which could have formed in the early Universe through a non-stellar way

  Hawking [1971] : *Gravitationally collapsed objects of very low mass*

  Carr & Hawking [1974]: *Black holes in the early Universe*

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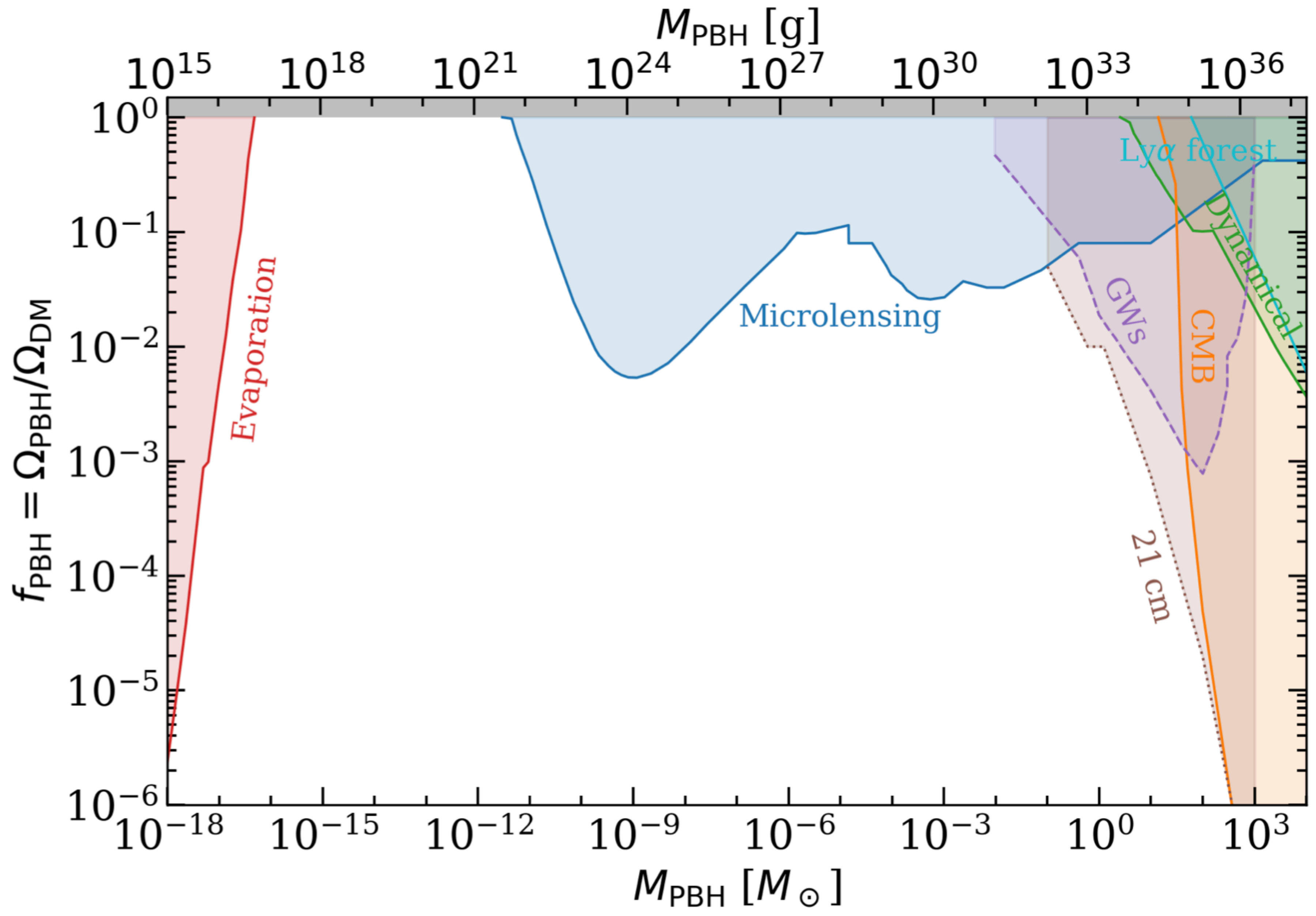

P. Villanueva-Domingo, O. Mena, S. Palomares-Ruiz [2021]
A brief review on primordial black holes as dark matter

- They could solve several conundrums in astrophysics and cosmology

  - They could be the totality, or a fraction, of the Dark Matter

  - They may explain the existence of progenitors for the merging events observed by LIGO/VIRGO

  - They could be the seeds of supermassive black holes in galactic nuclei

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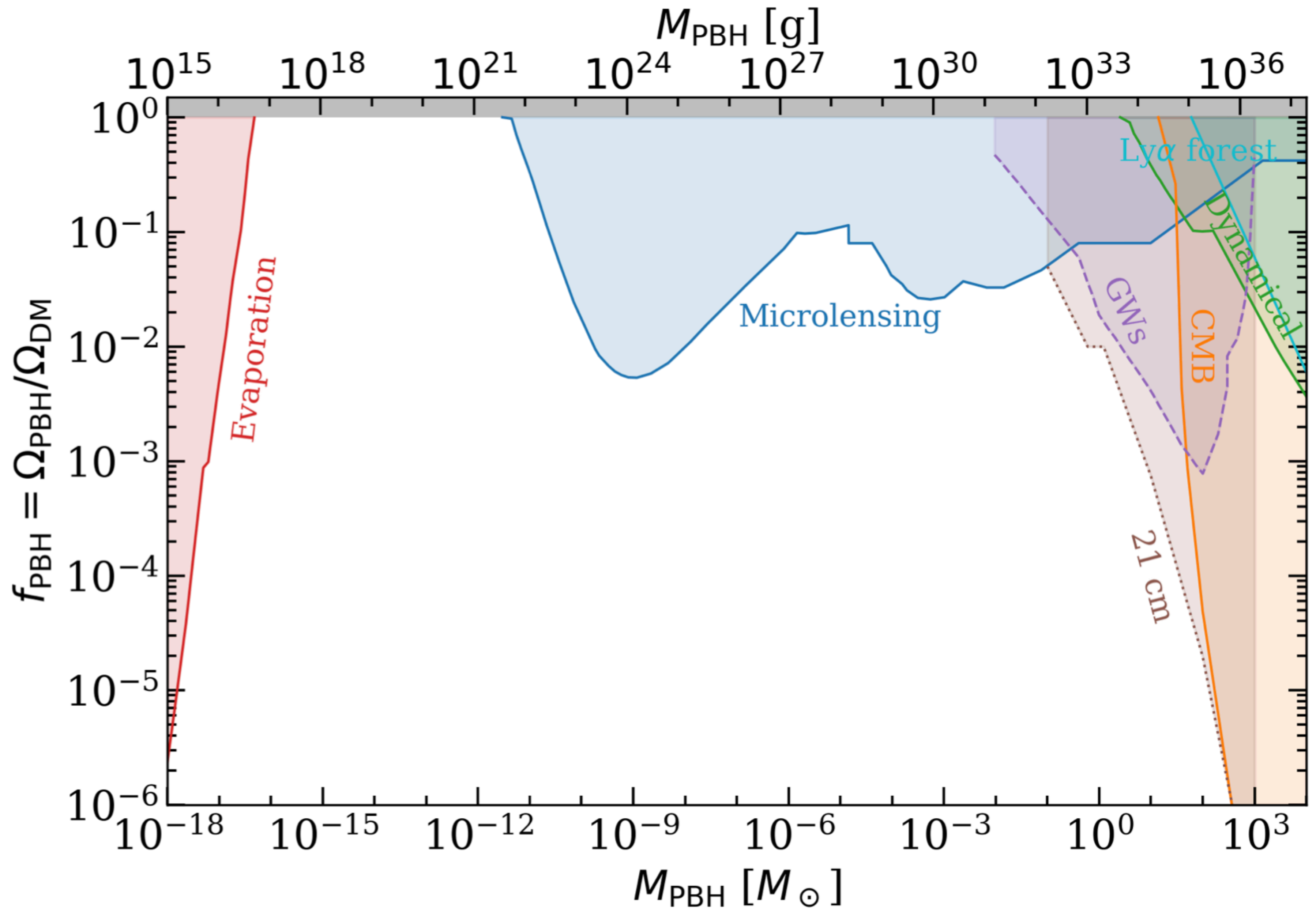

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- They could extend the region of the inflationary potential we can probe

- They provide a place to look for quantum effects (such as quantum diffusion)

Probing the missing scales of inflation

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- Simplest realisation: slow-roll inflation

scalar field ϕ (inflaton) slowly rolling towards the minimum of its potential

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\,\pi\,G} \left(\frac{V_{,\phi}}{V} \right)^2$$

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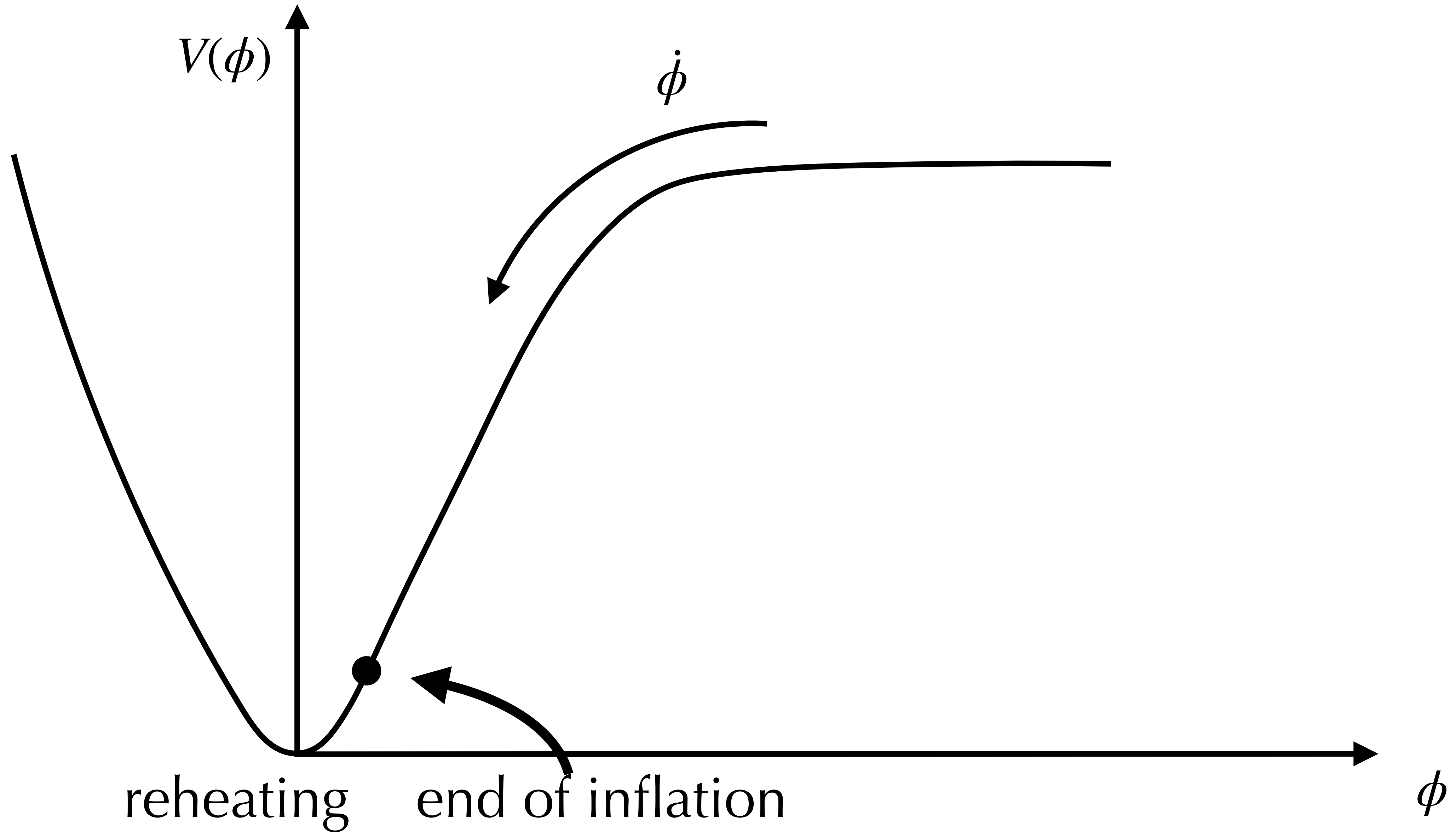

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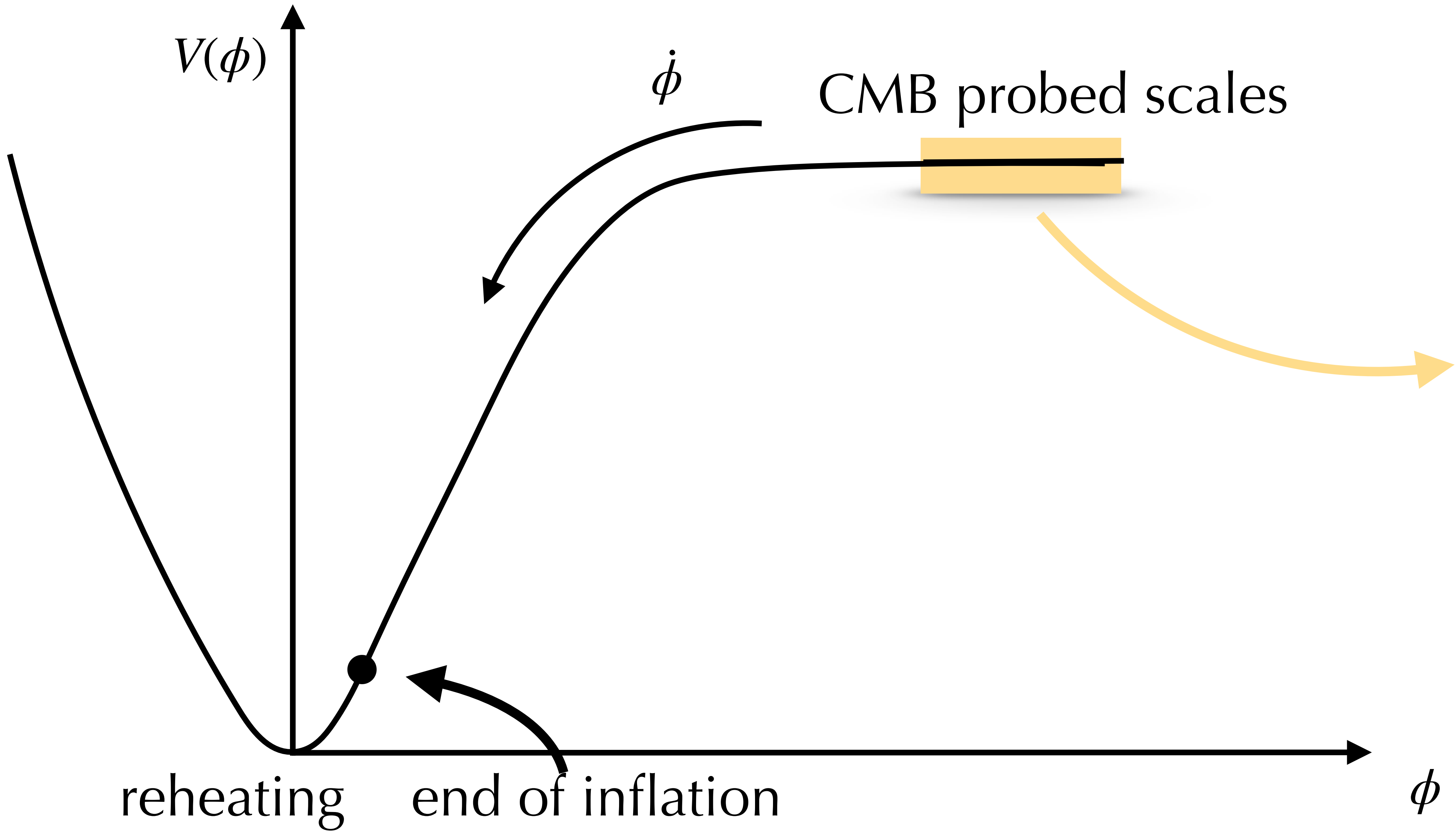
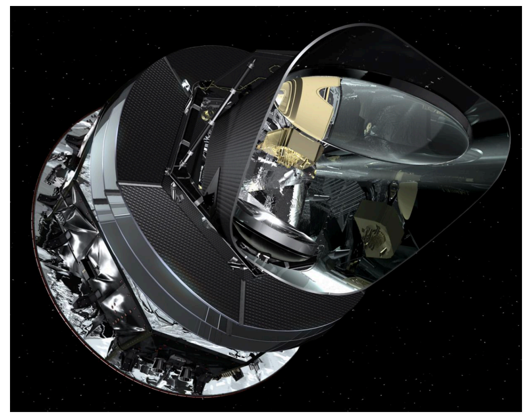


small perturbations $\zeta \simeq 10^{-5}$

quasi-Gaussian

almost scale invariant

Constrained window ~ 7 e-folds



Planck 2018 results. X. *Constraints on inflation*

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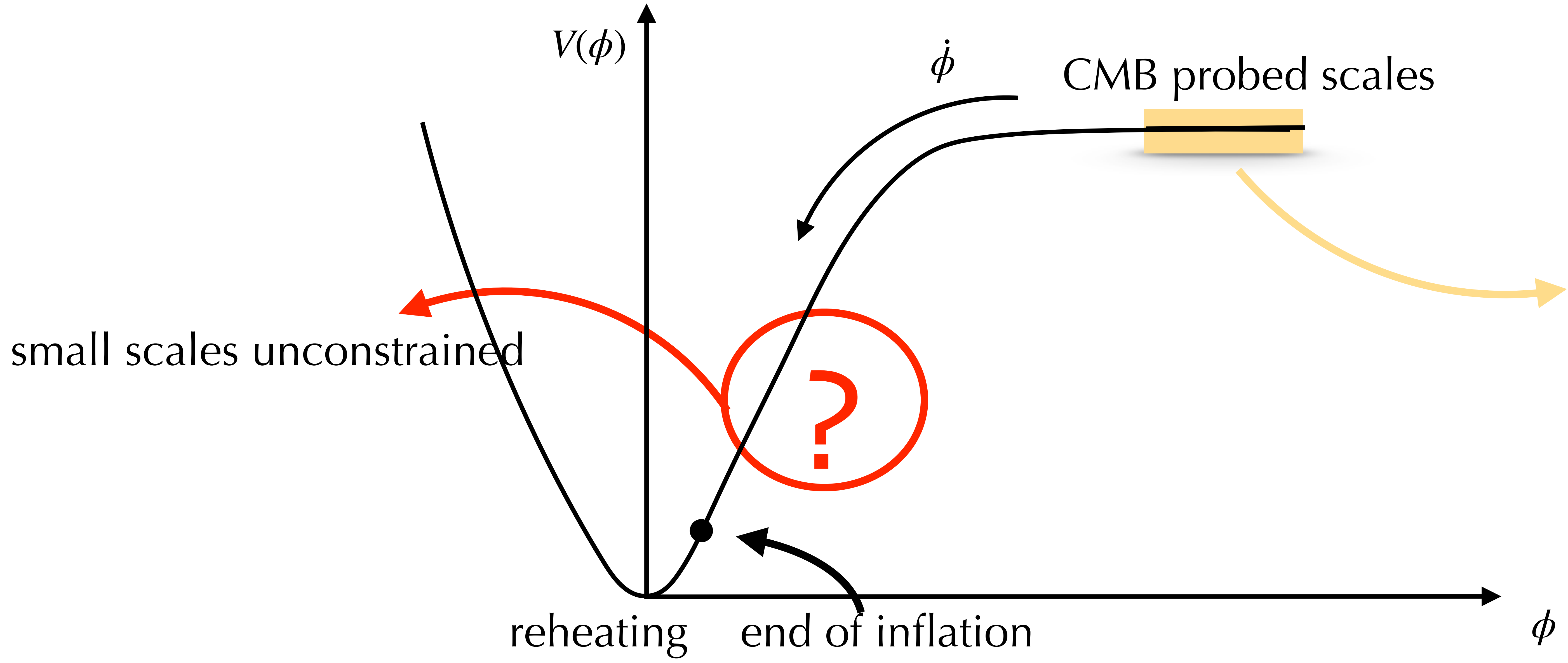

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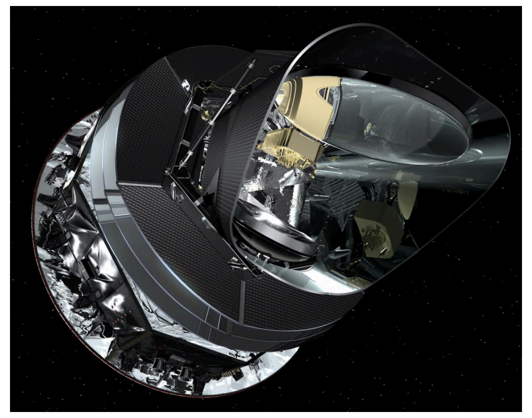

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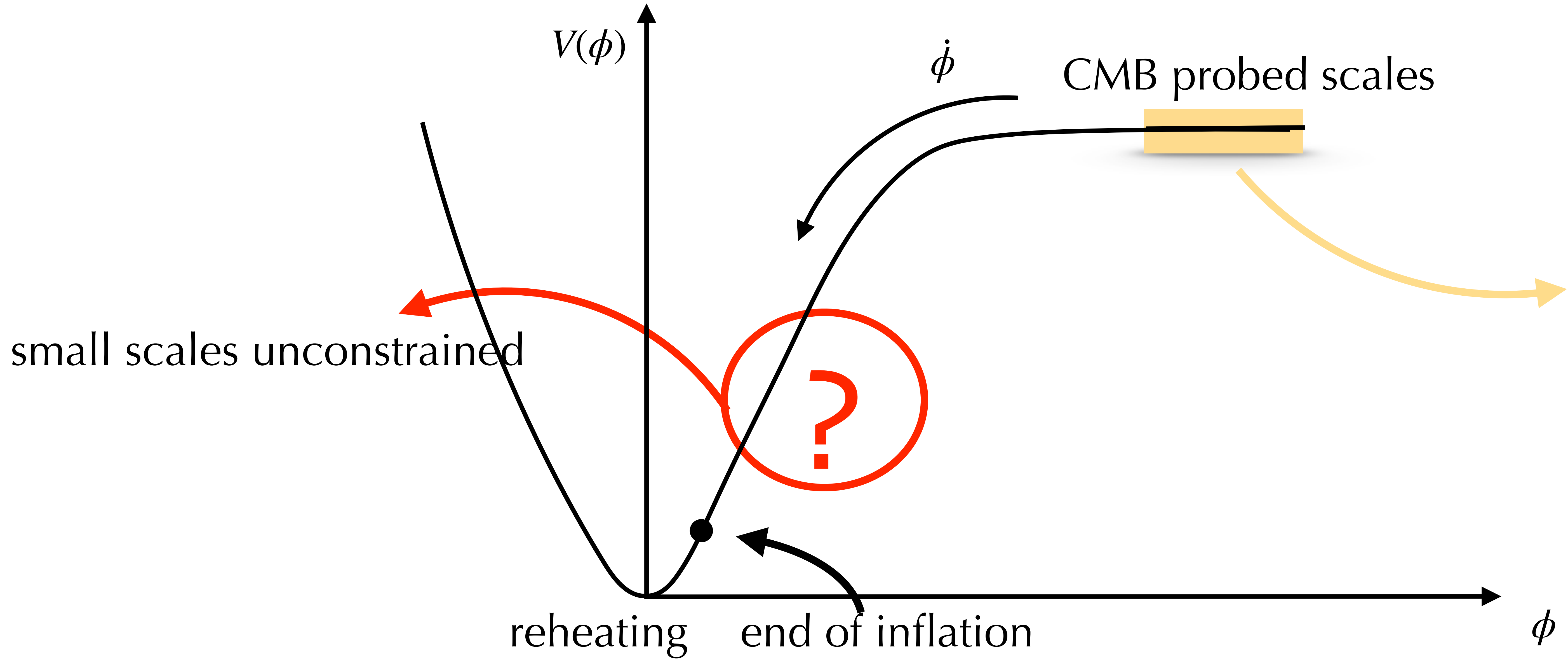

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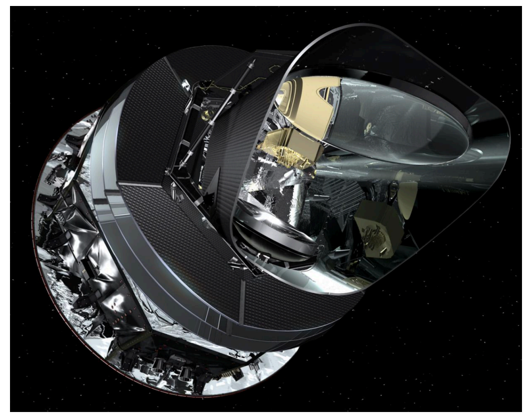

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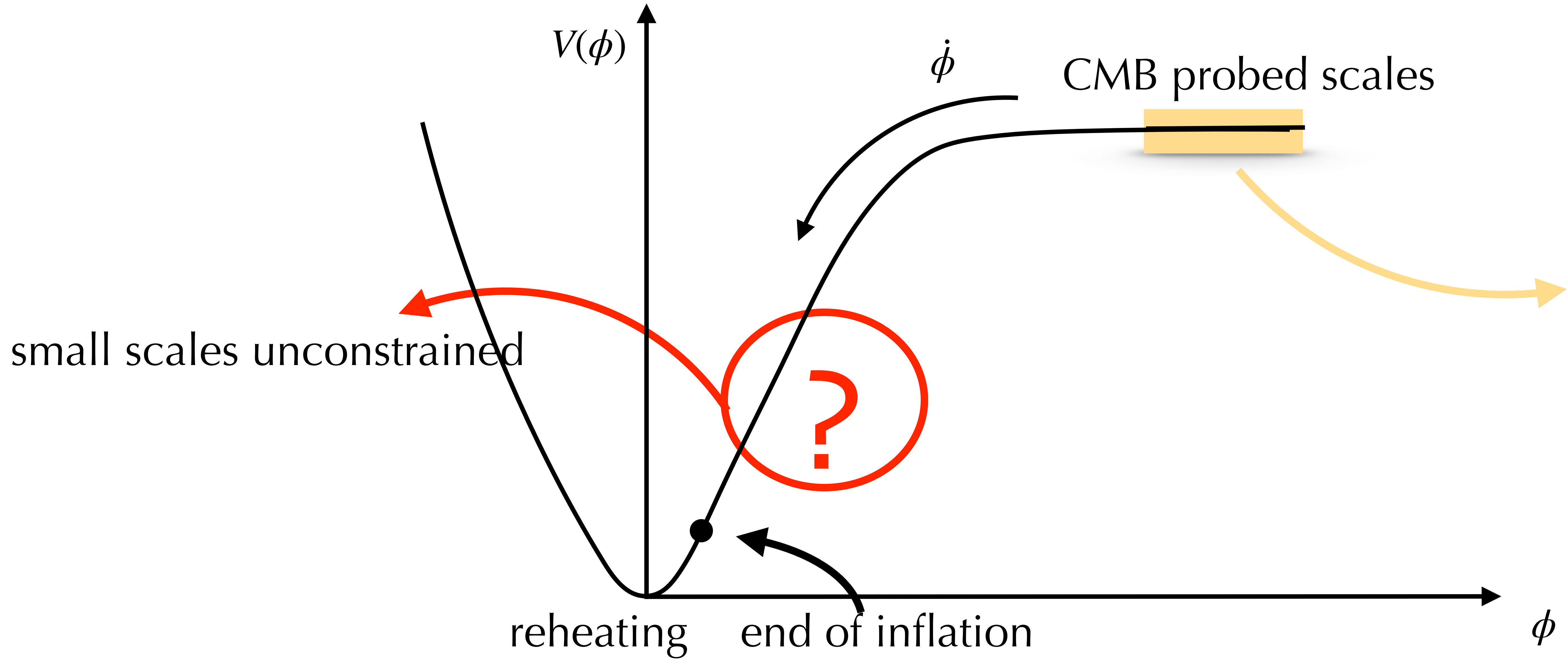

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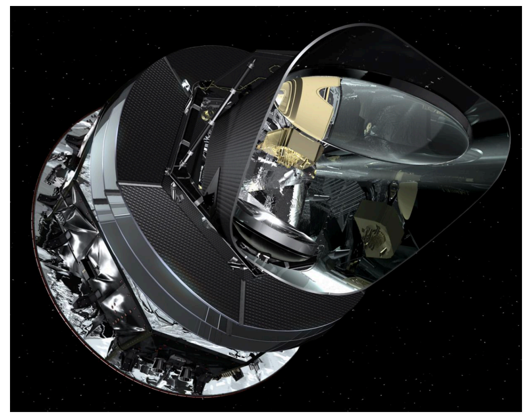

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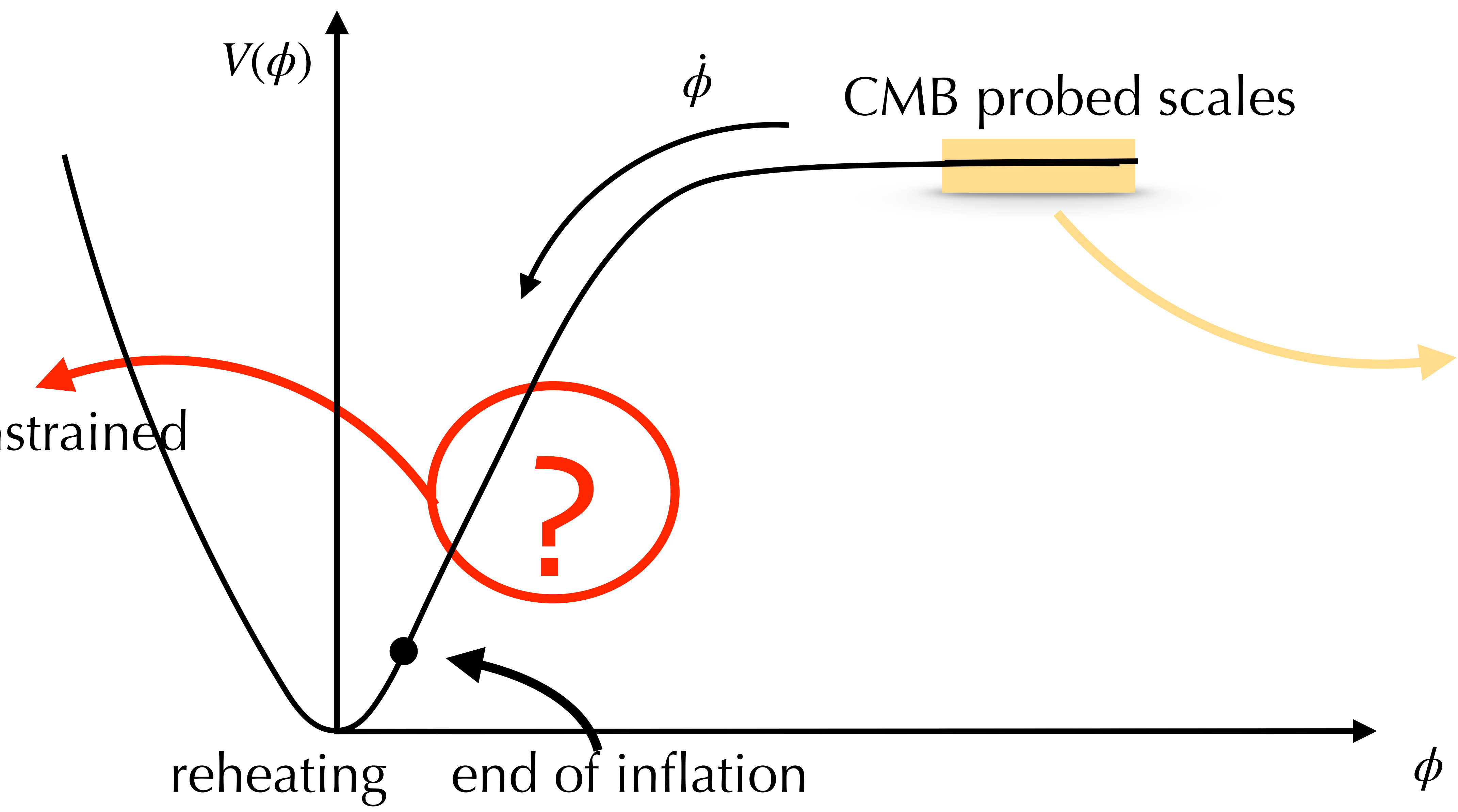

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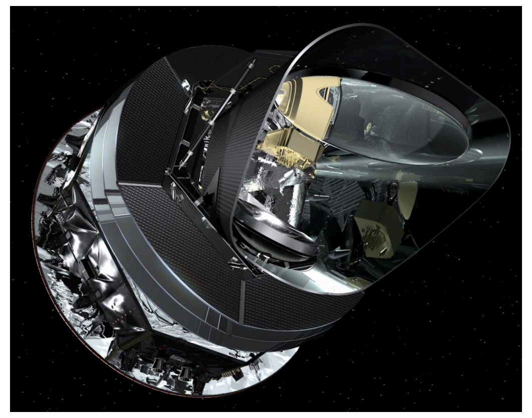
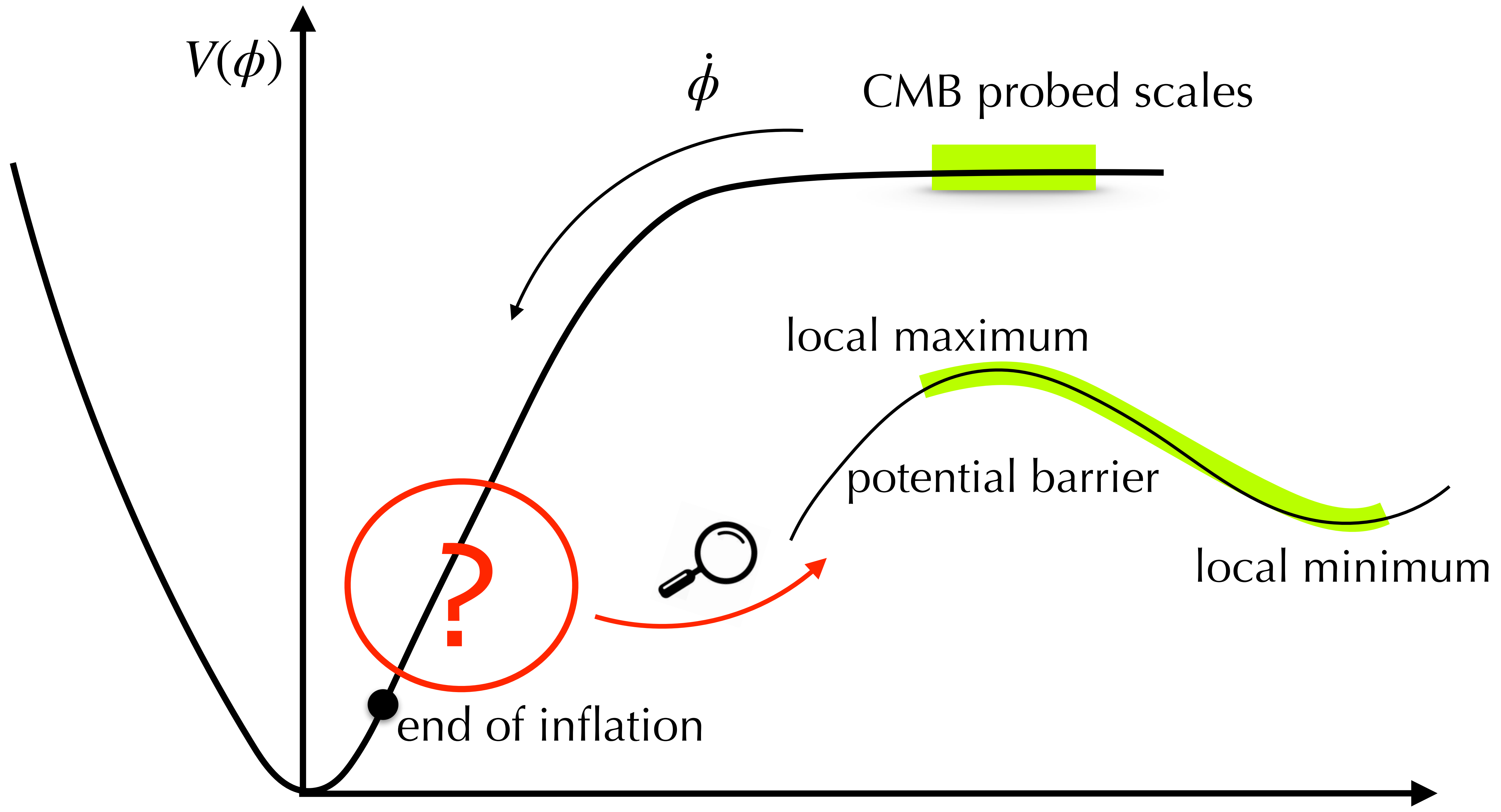


Planck 2018 results. X. *Constraints on inflation*

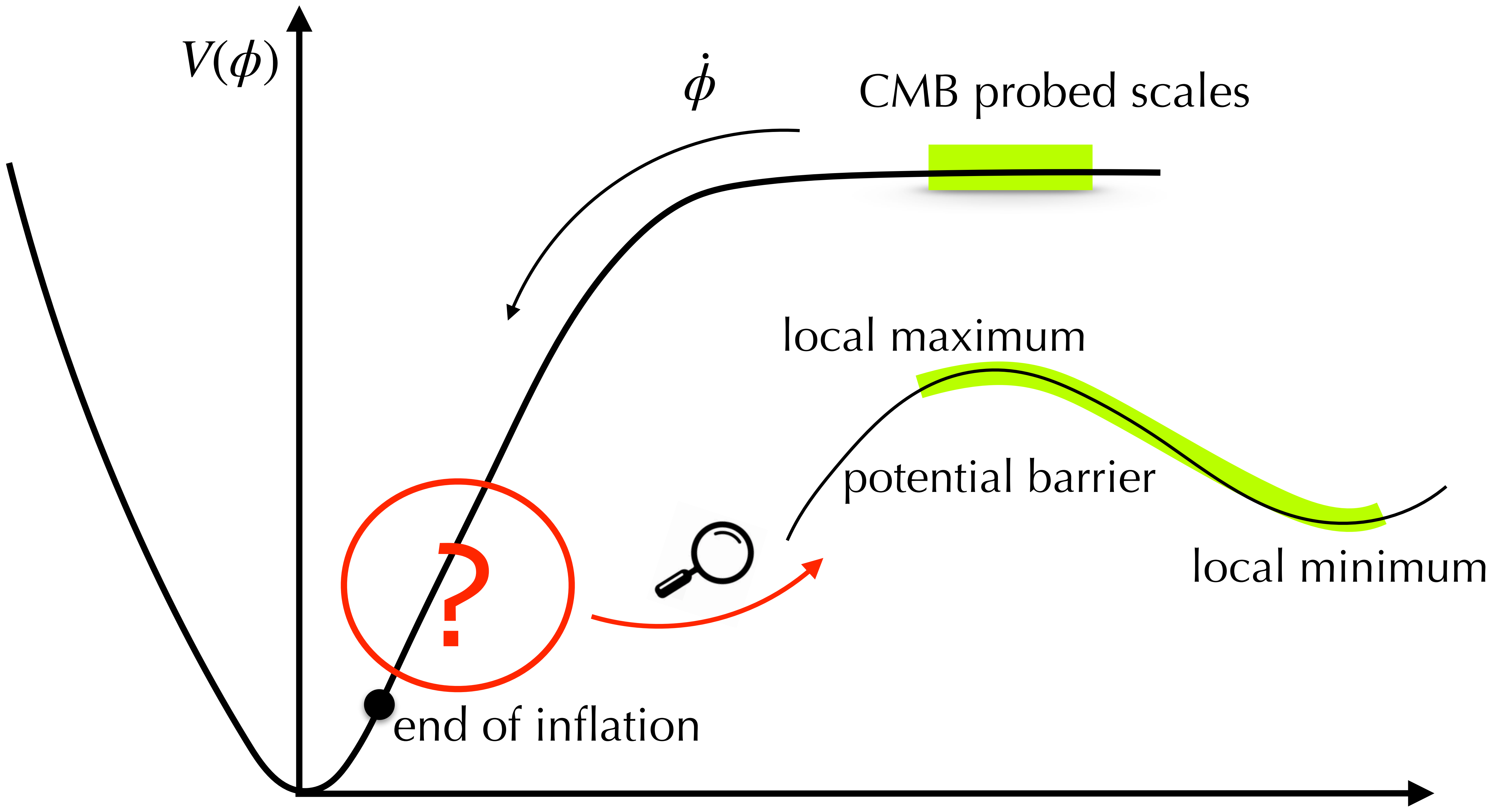
- New observational windows at small scales
- Open view about possible deviations from “vanilla inflation” outside the constrained range
- Investigation of possible phenomenological consequences that might be looked for

Stochastic tunnelling



Stochastic tunnelling

- False vacuum state



Stochastic tunnelling

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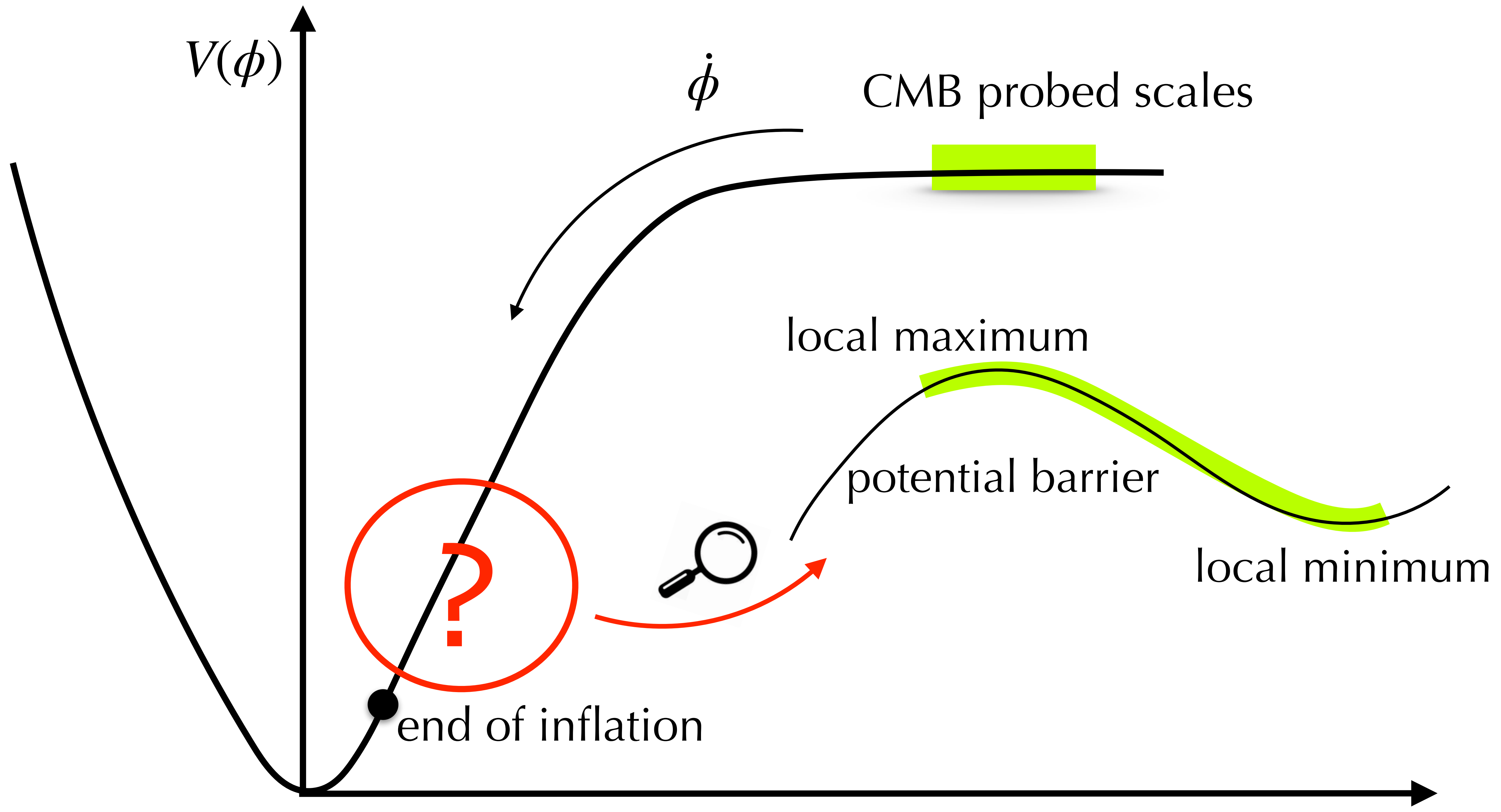
- Local minima naturally appear in various contexts: - high energy constructions (supersymmetry, supergravity)
  - breaking of flat-inflection point condition through radiative corrections
  -specific inflationary models (critical Higgs)
  - etc.



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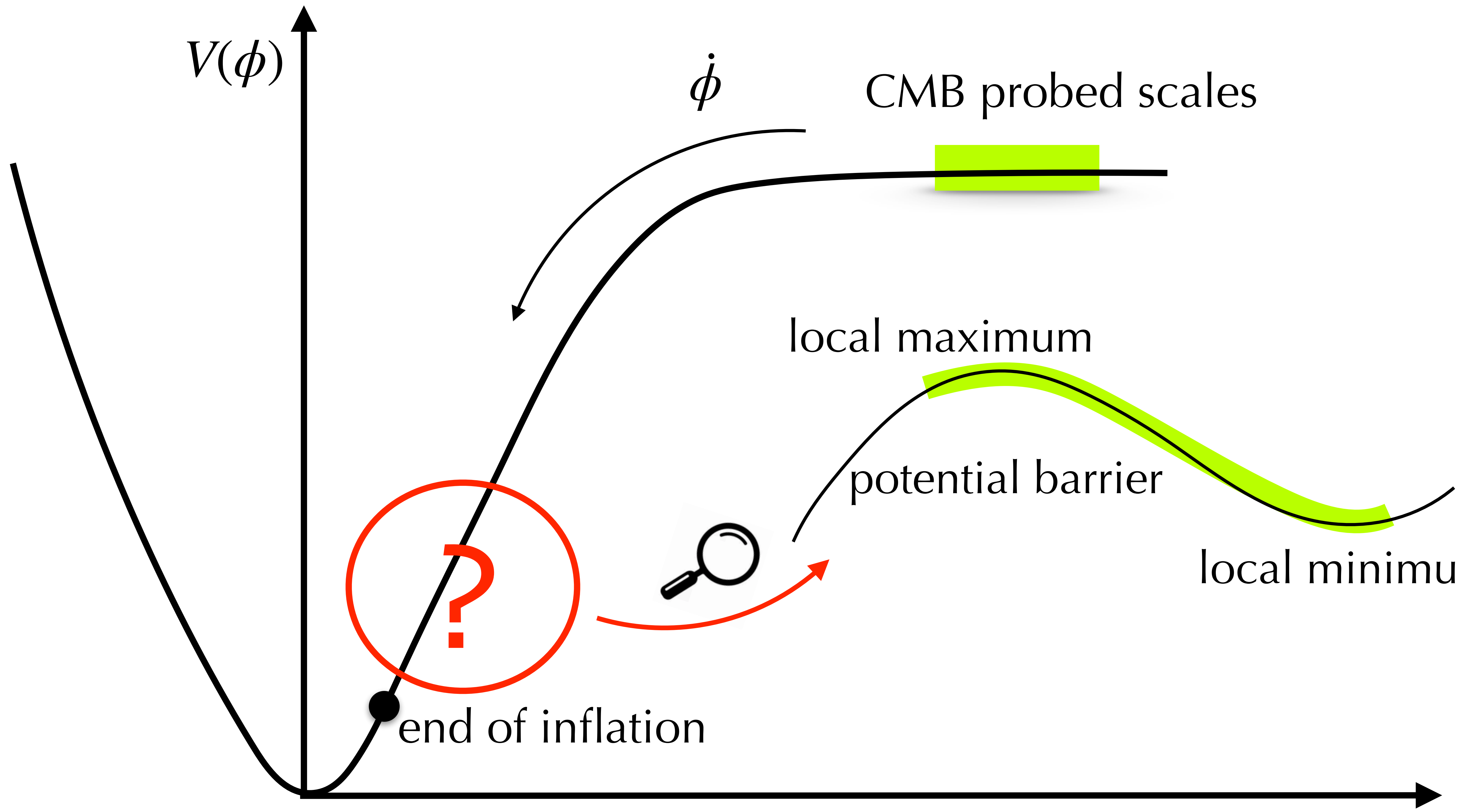
  1) Large classical velocity

  2) "Stochastic tunnelling": quantum fluctuations jiggle the inflaton and push it outwards



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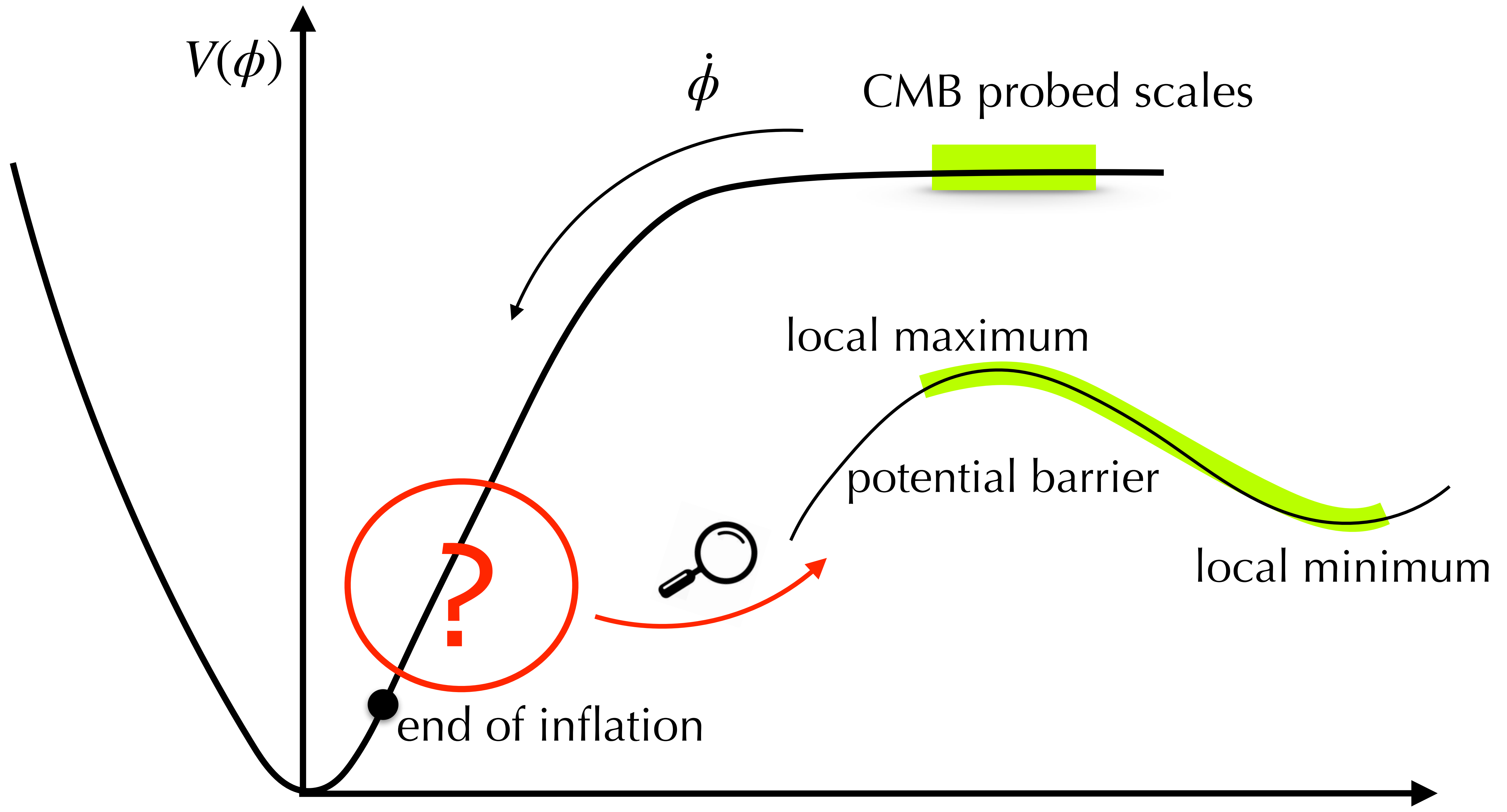

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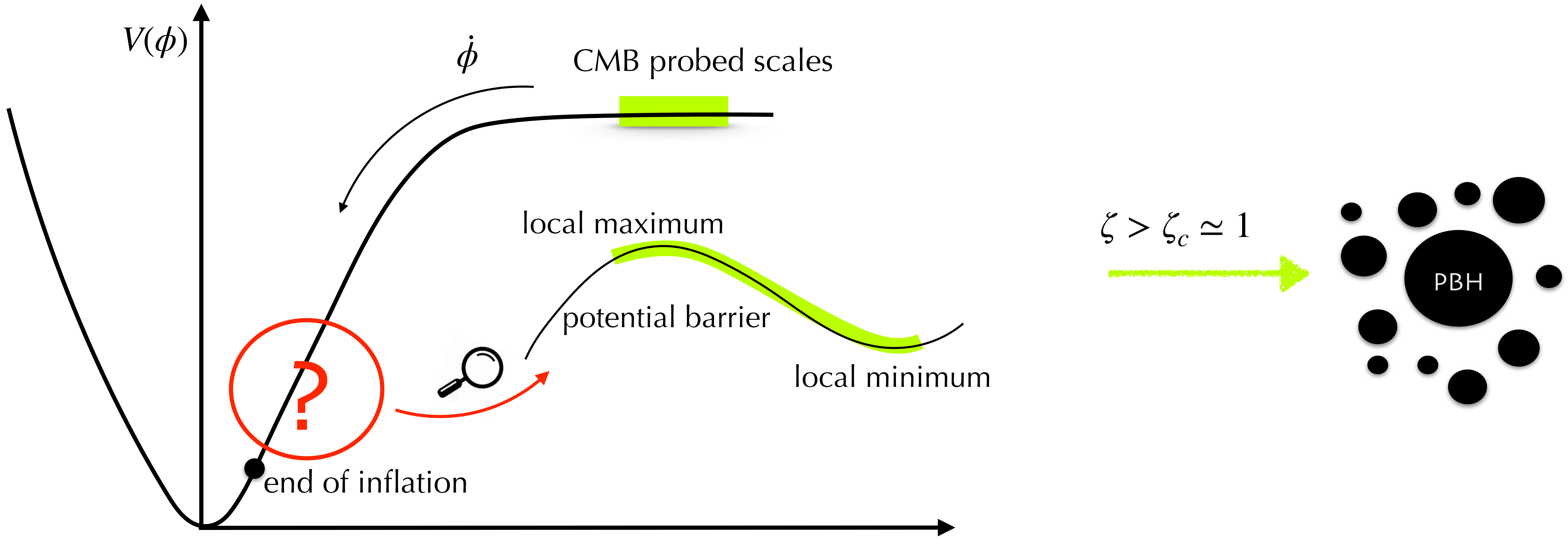

Quantum diffusion during inflation: stochastic tunnelling

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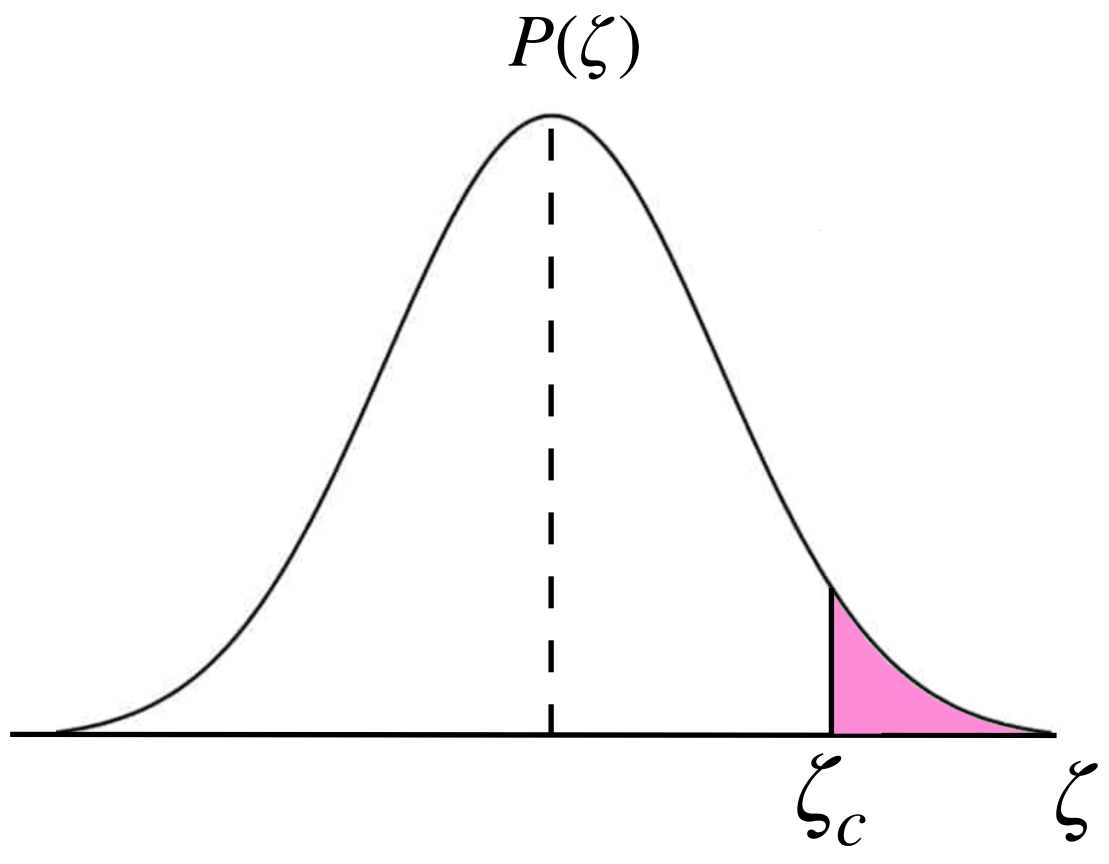
- Abundance of PBHs $\beta(M) = \int_{\zeta_c}^{\infty} P(\zeta)\, d\zeta$

"crude" Press-Schechter estimate



Quantum diffusion during inflation: stochastic formalism

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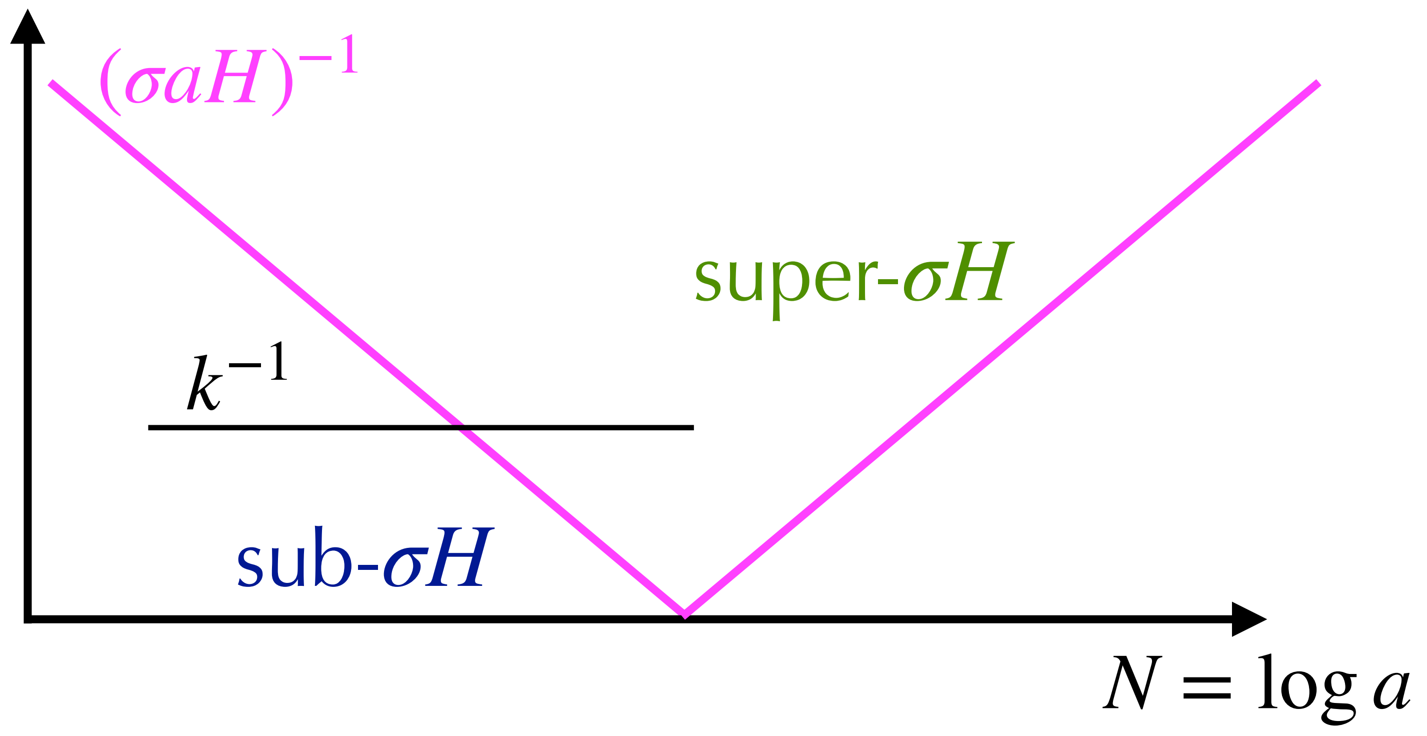
Splitting fields into UV and IR part: coarse-graining scale $k_{cg} = \sigma \, a \, H$

$$\phi(x) = \phi_{cg} + \int \frac{dk}{(2\pi)^{3/2}} W\left(\frac{k}{\sigma a H}\right) \left[\phi_k(N)\, e^{-i\vec{k}\cdot\vec{x}}\, \hat{a}_{\vec{k}} + h.c.\right]$$



Quantum subhorizon fluctuations source the background

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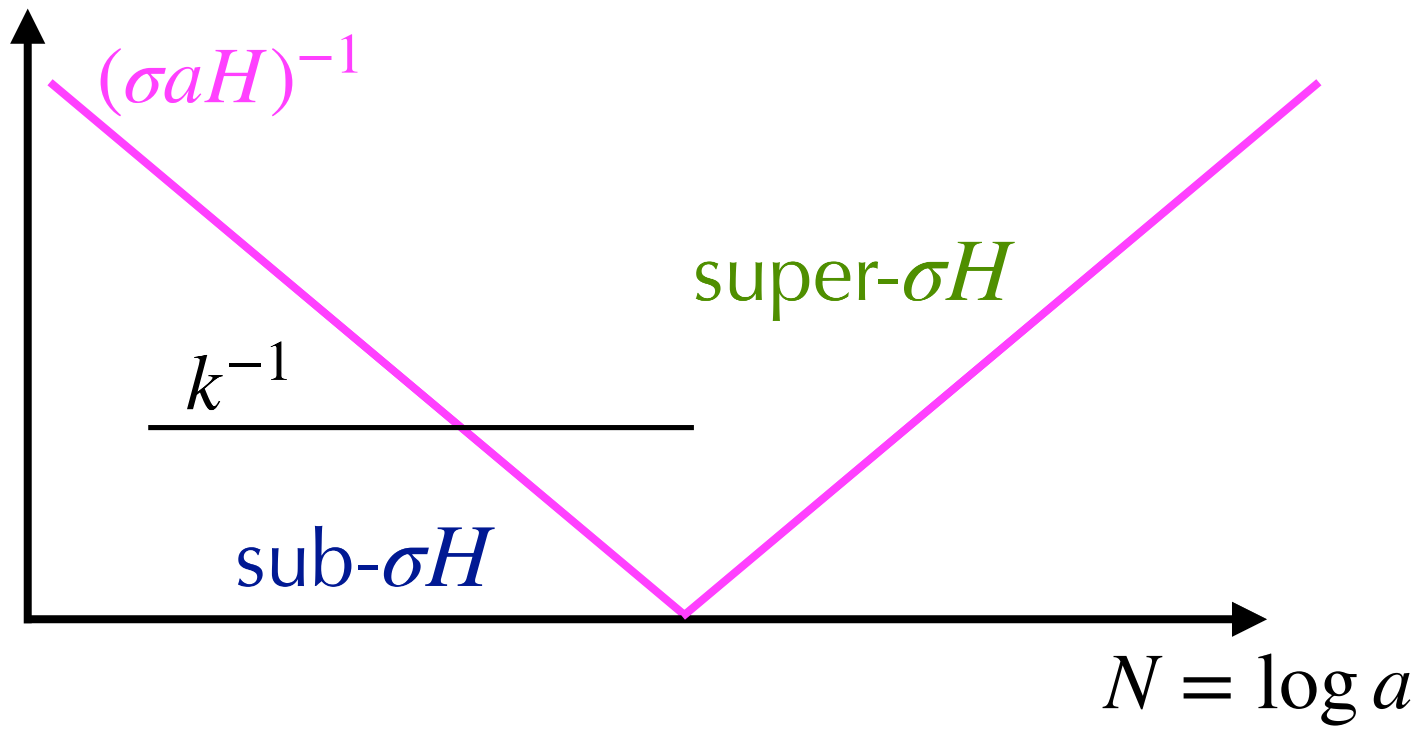
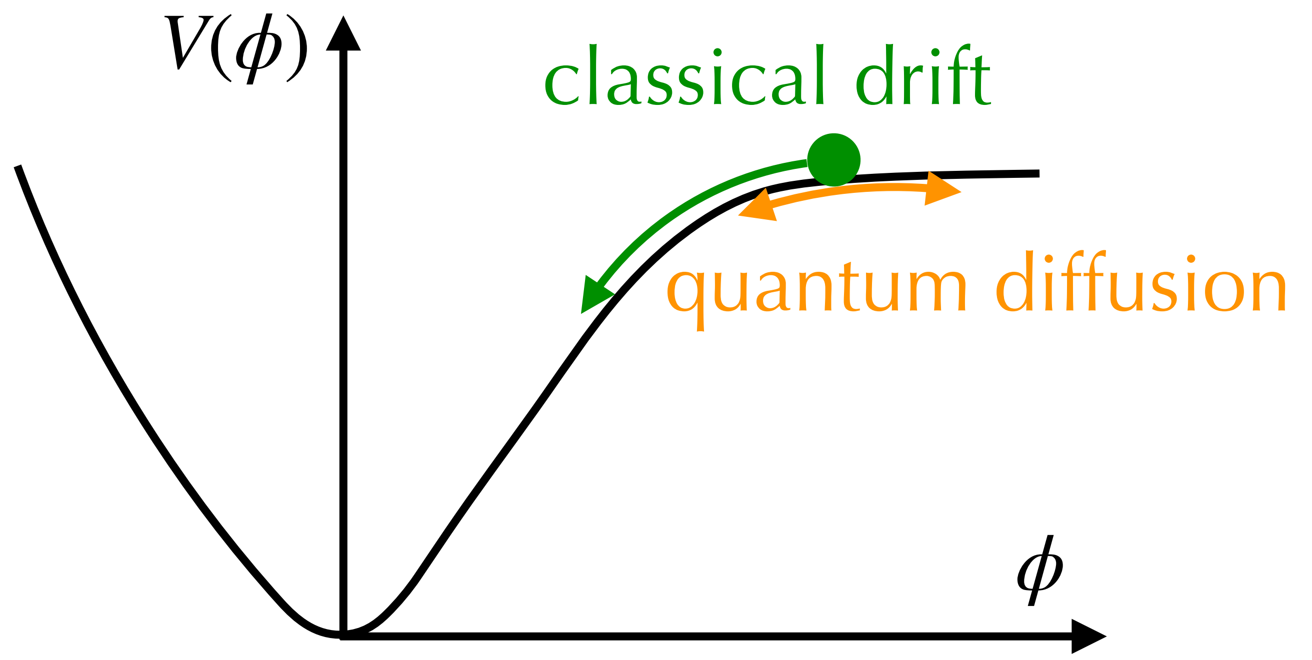


Quantum subhorizon fluctuations source the background

Dynamics at leading order in slow roll:

$$\frac{d}{dN}\phi_{cg} = -\frac{V_{,\phi}(\phi_{cg})}{3H^2(\phi_{cg})} + \frac{H(\phi_{cg})}{2\pi}\xi(N)$$



Quantum diffusion during inflation: properties of perturbations

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- How to reconstruct the statistics of ζ in presence of quantum diffusion?

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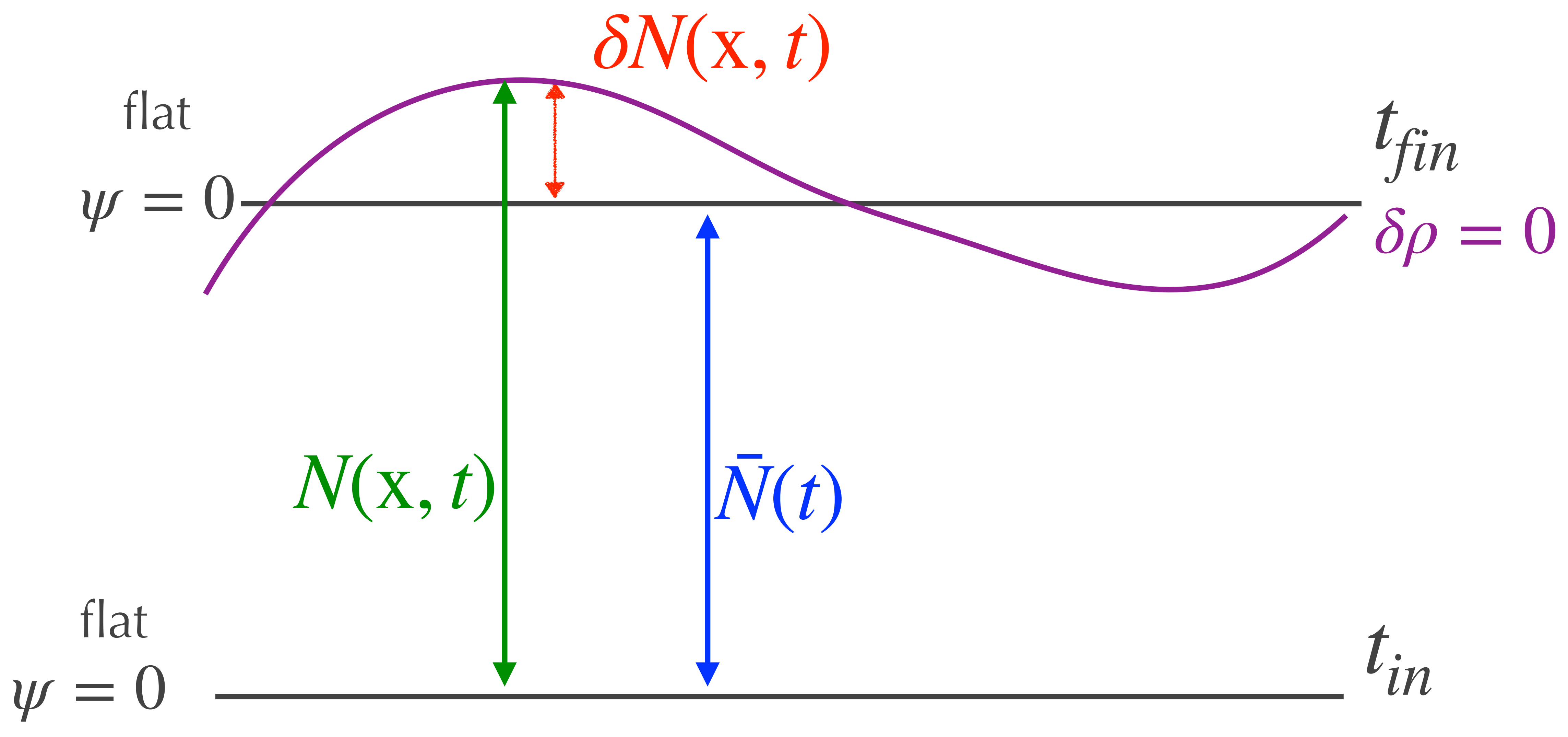
- δN formalism

Lifshitz, Khalatnikov [1960]

Starobinsky [1983]

Wands, Malik, Lyth, Liddle [2000]



$$\zeta(t, \mathrm{x}) = N(t, \mathrm{x}) - \bar{N}(t) \equiv \delta N$$

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- Stochastic-δN formalism

[Enqvist, Nurmi, Podolsky, Rigopoulos [2008]]

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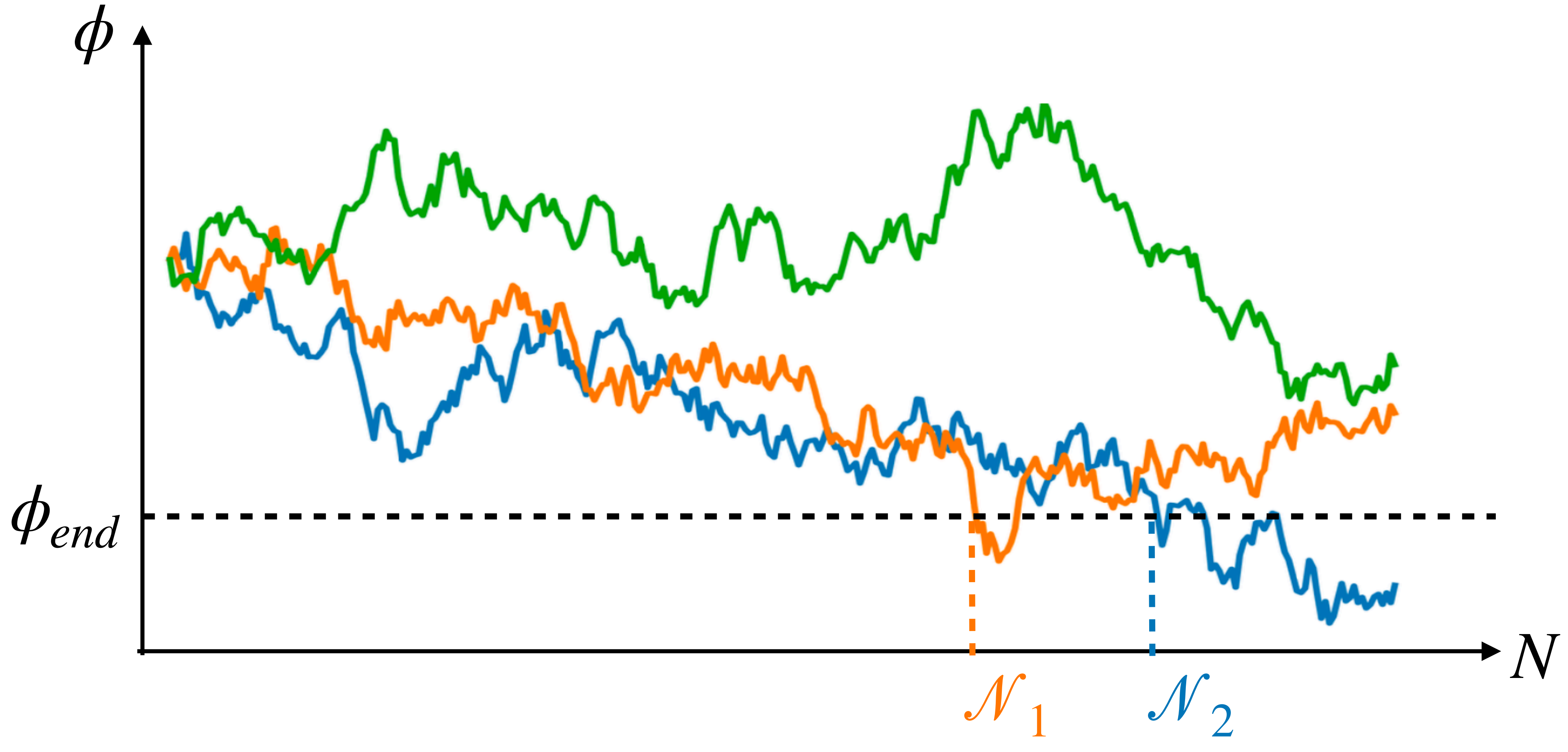
Number of e-folds is a stochastic variable $\mathcal{N}$



Statistics of ζ from the statistics of $\mathcal{N}$

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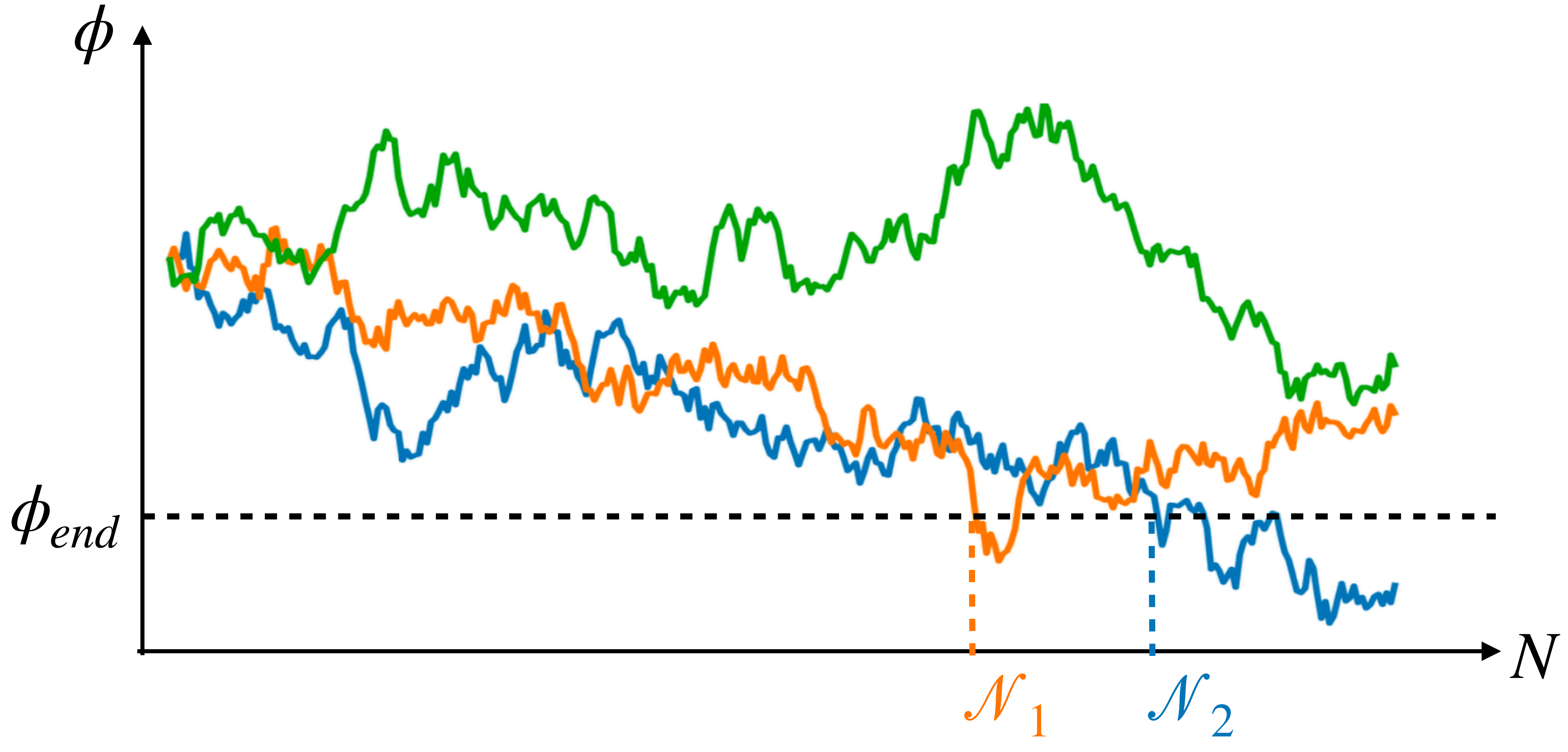

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Distribution function for the duration of inflation (first passage time)

$$\frac{\partial}{\partial \mathcal{N}} P(\mathcal{N}, \phi) = \mathcal{L}^{\dagger}_{FP}(\phi) \cdot P(\mathcal{N}, \phi) \qquad \frac{1}{M_{Pl}^2} \mathcal{L}^{\dagger}_{FP}(\phi) = -\frac{v'(\phi)}{v(\phi)} \frac{\partial}{\partial \phi} + v(\phi) \frac{\partial^2}{\partial \phi^2} \qquad v = \frac{V}{24\pi^2 M_{Pl}^4}$$

Stochastic-δN formalism: exponential tails

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- Full PDF of the first passage time

Pattison, Vennin, Assadullahi, Wands [2017]

Characteristic function (includes all moments)

$$\chi(t,\phi) \equiv \langle e^{it\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{it\mathcal{N}} P(\mathcal{N},\phi)\, d\mathcal{N}$$

→

Obeys differential equation

$$\mathcal{L}^{\dagger}_{FP} \cdot \chi(t,\phi) = -\, i\, t\, \chi(t,\phi)$$

→

Full PDF given by inverse Fourier transform

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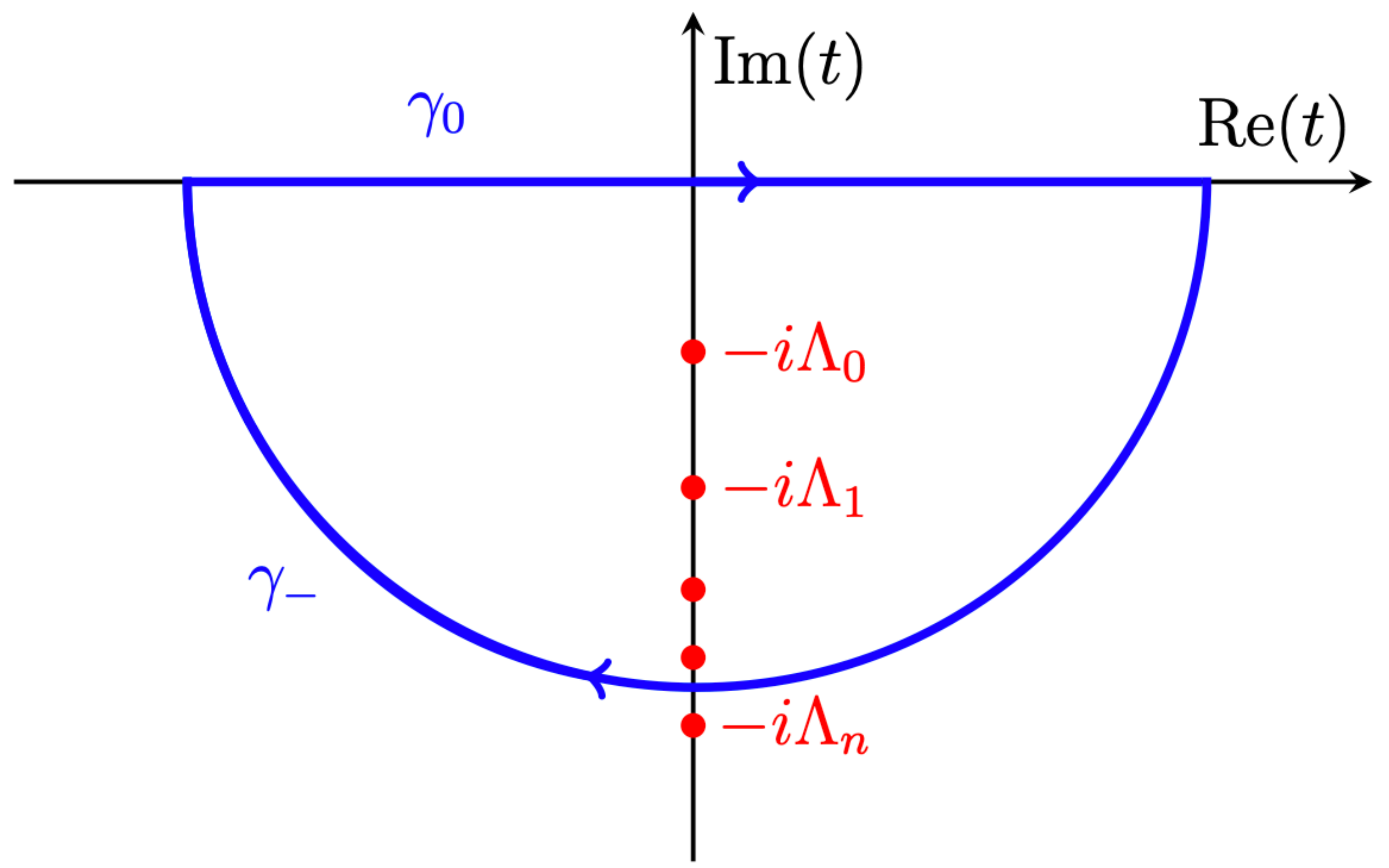

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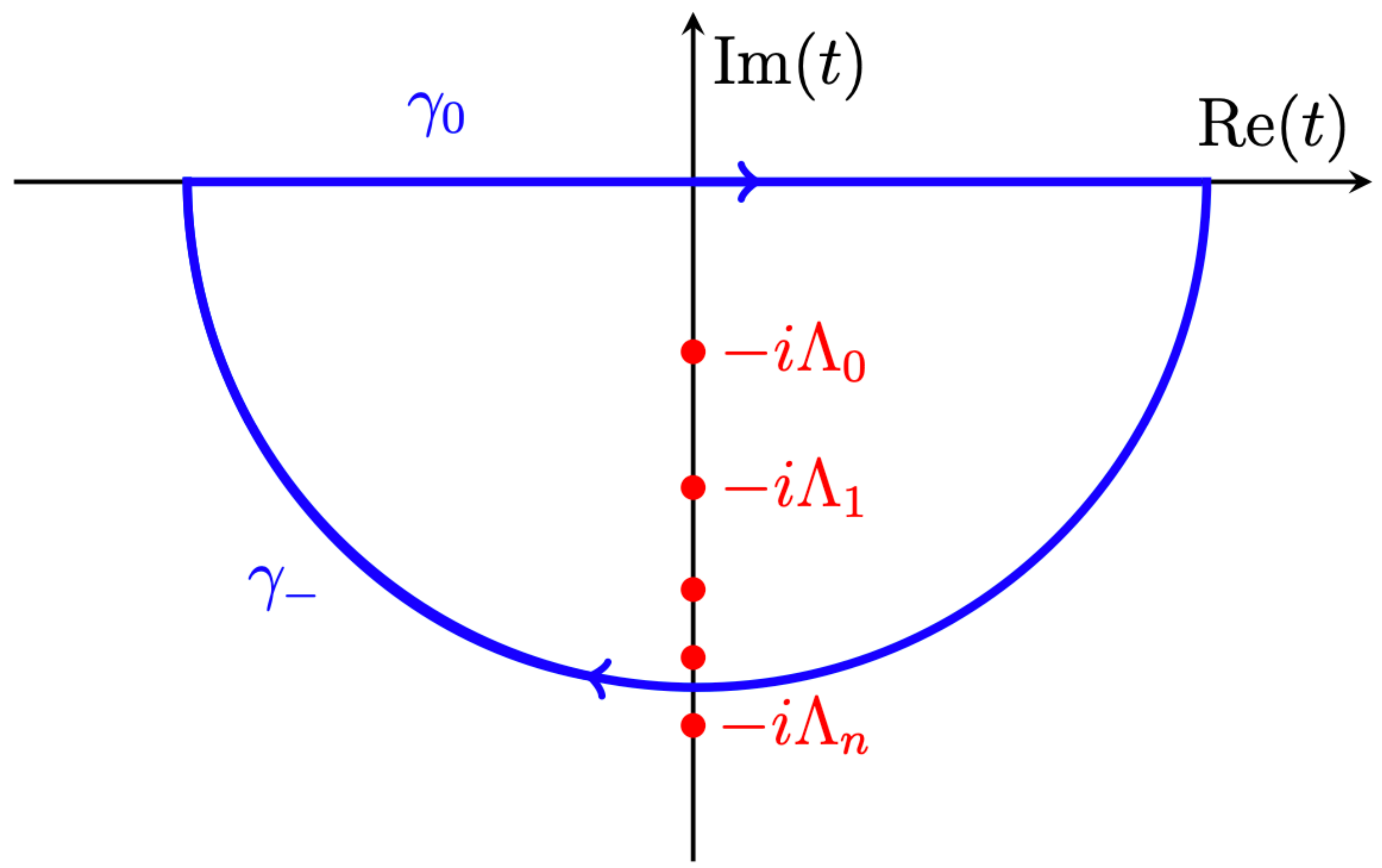

Tail of the PDF for ζ has an exponential fall-off behaviour

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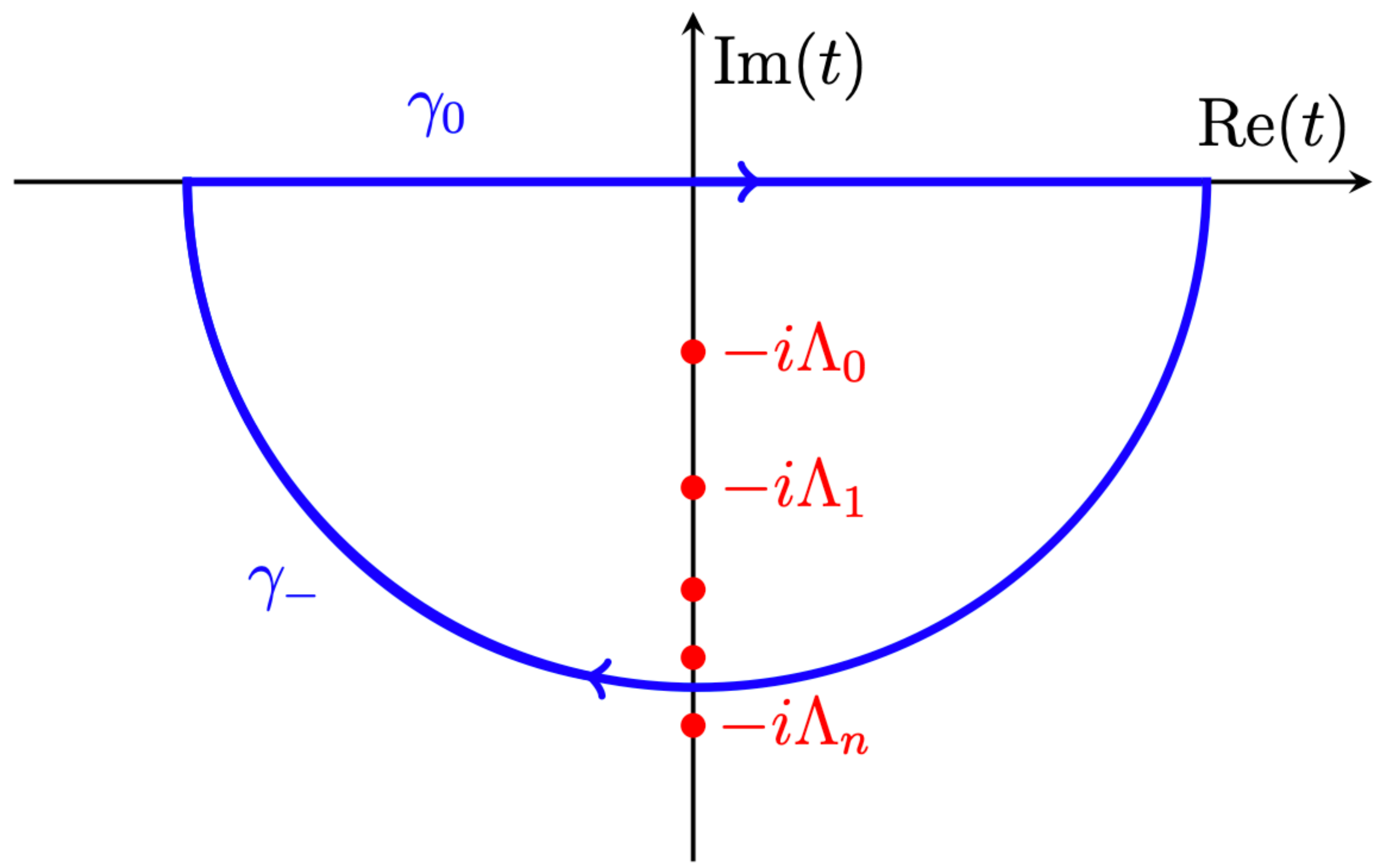

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This type of non-Gaussianities cannot be captured by perturbative parametrisations (such as the f_{NL} expansion) !

False vacuum: simple toy models

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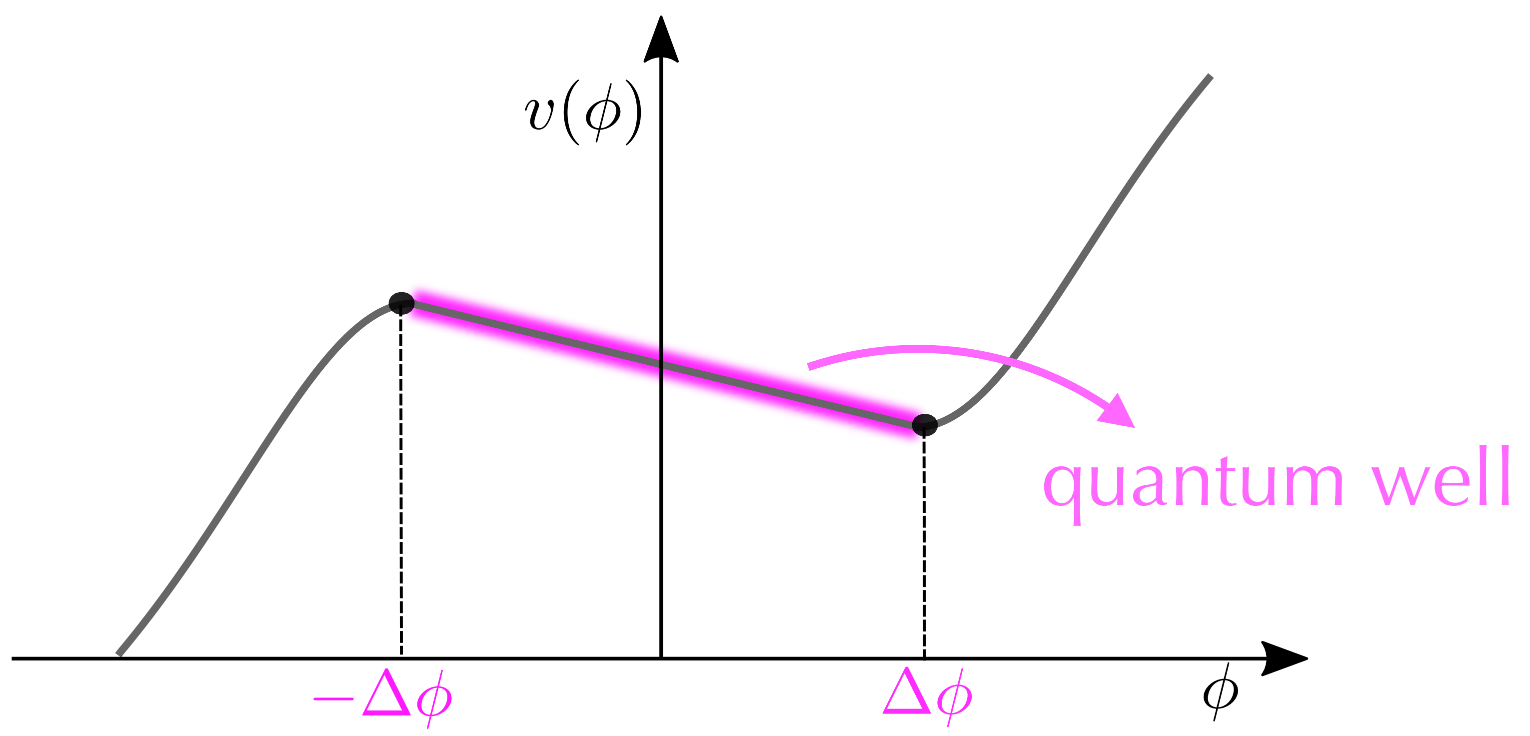
- Linear model

$$v(\phi) = v_0 \left(1 - \alpha \frac{\phi}{\Delta\phi} \right)$$



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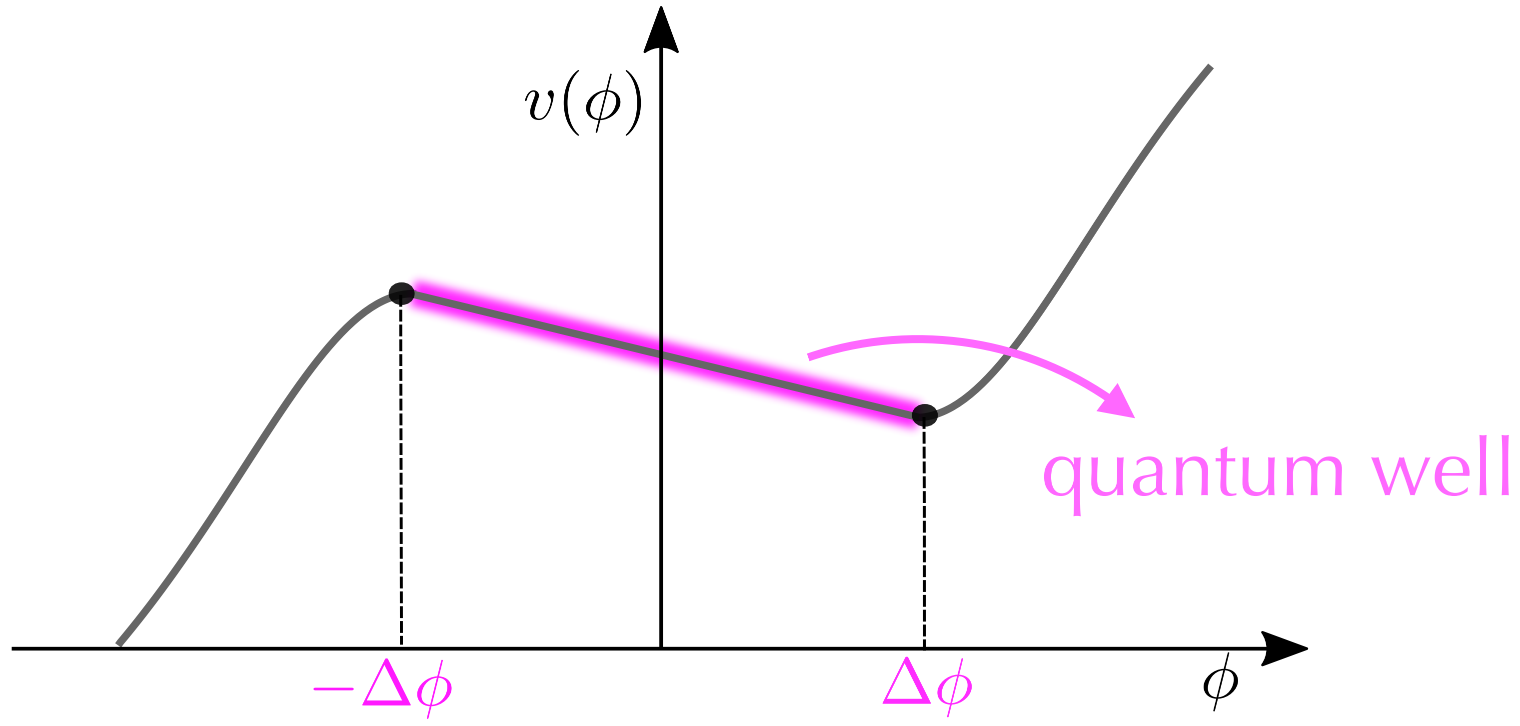

- Quadratic model ("two-parabola approximation")

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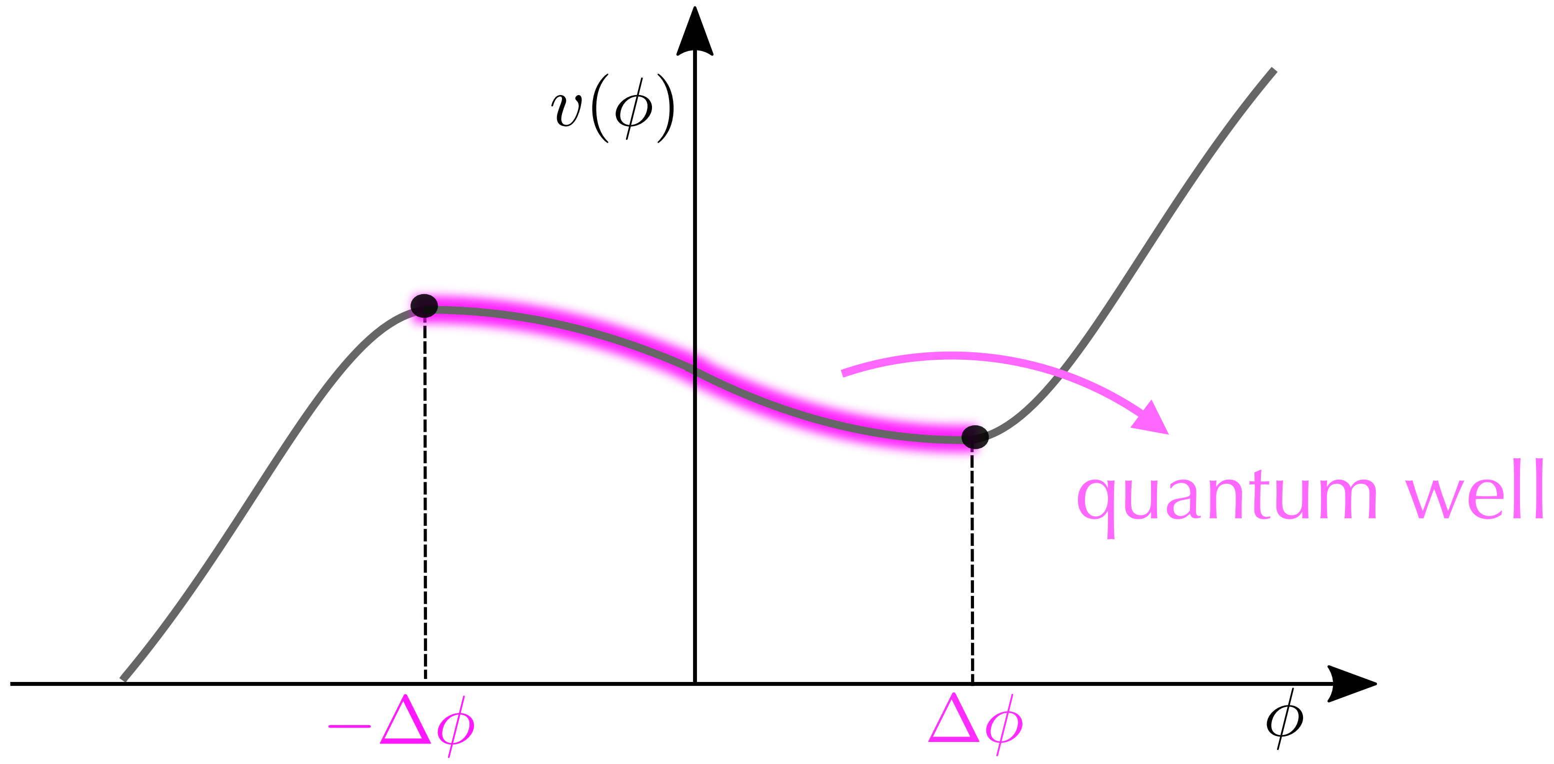

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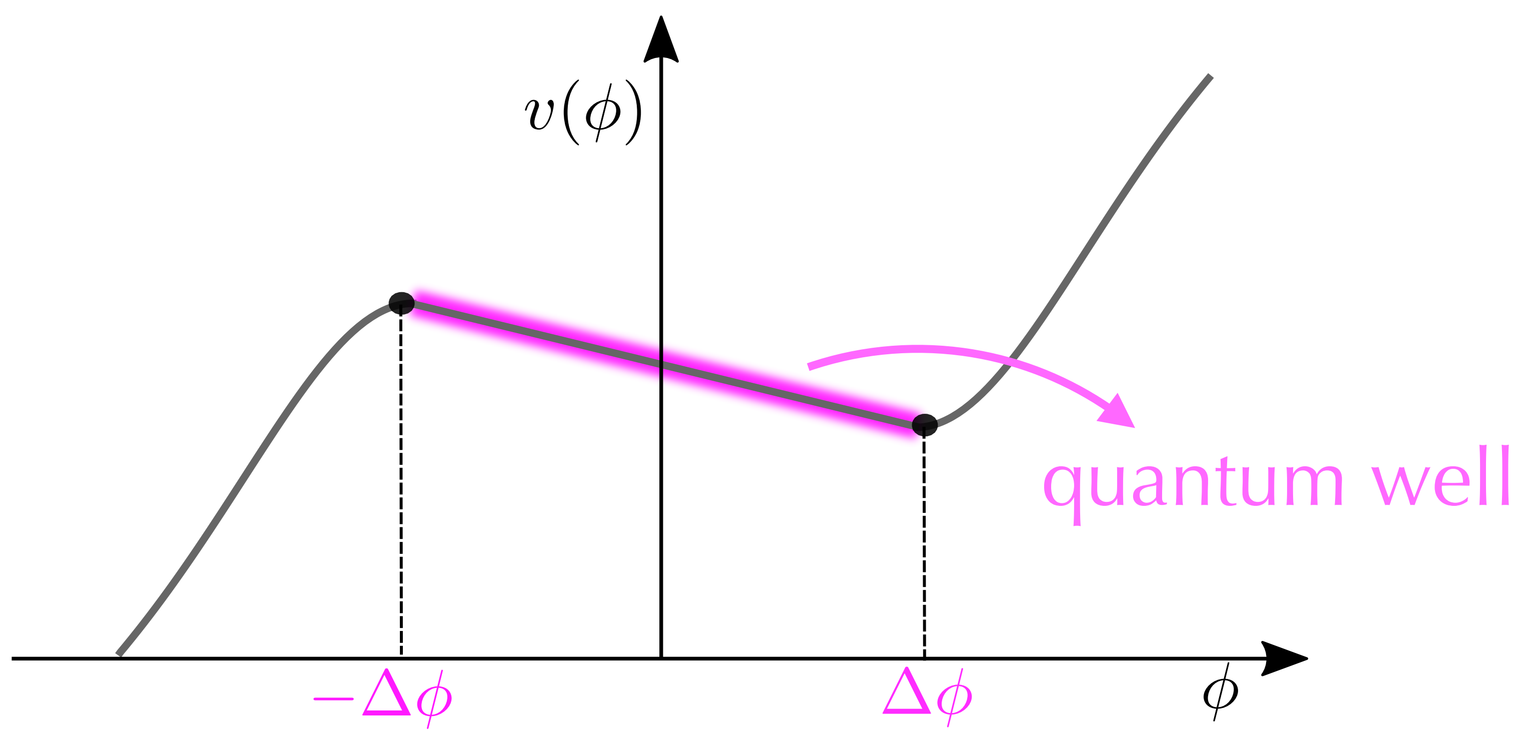

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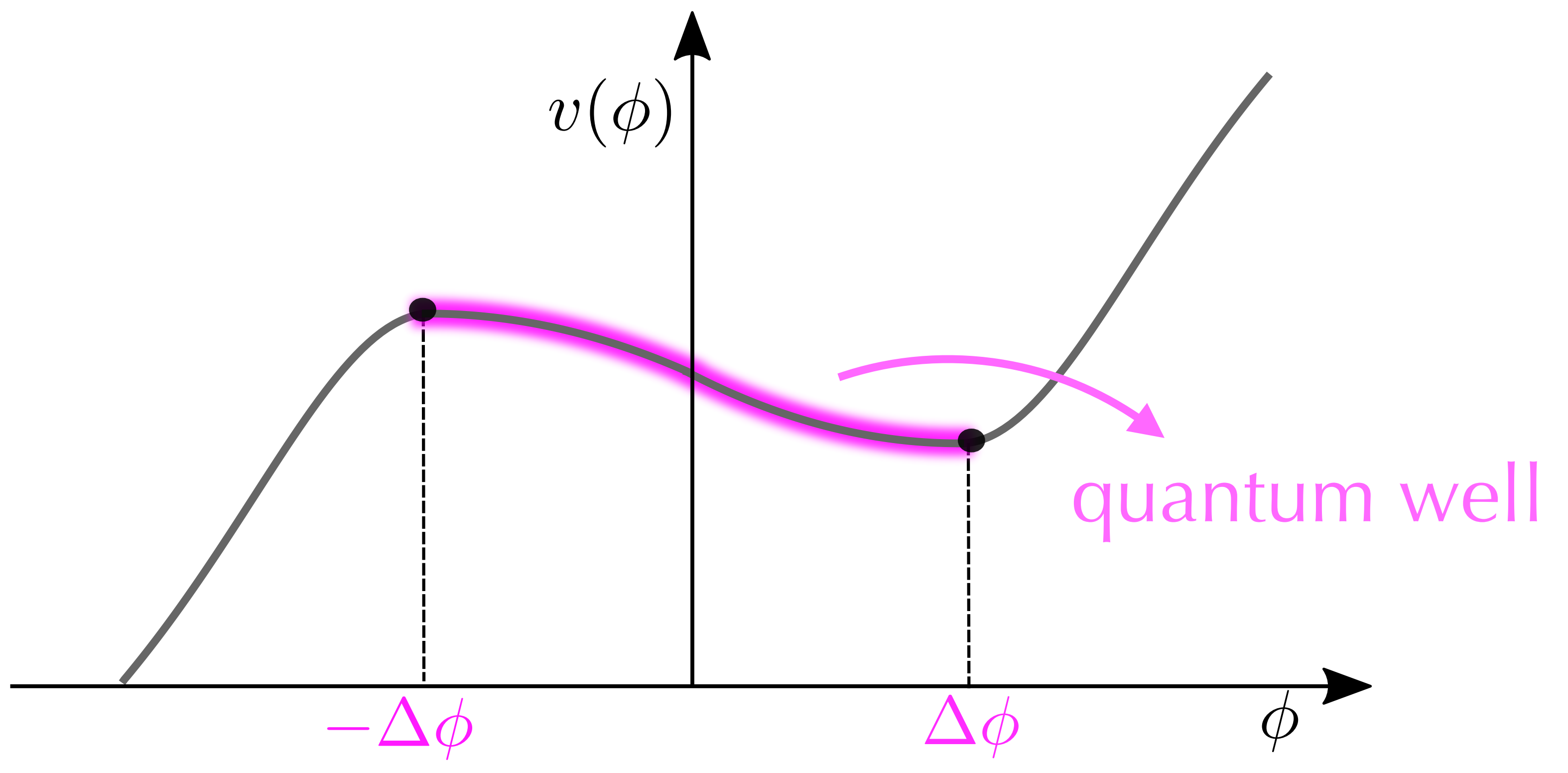

Quantum diffusion in highlighted regions, potential gradient elsewhere

Slow roll preserved: $\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{v'}{v}\right)^2 \ll 1, \ |\eta| = \left|M_{Pl}^2 \frac{v''}{v}\right| \ll 1$

$\langle \mathcal{N} \rangle$ smaller than ~ 50: $\Delta v = v(-\Delta\phi) - v(\Delta\phi) \ll v_0$

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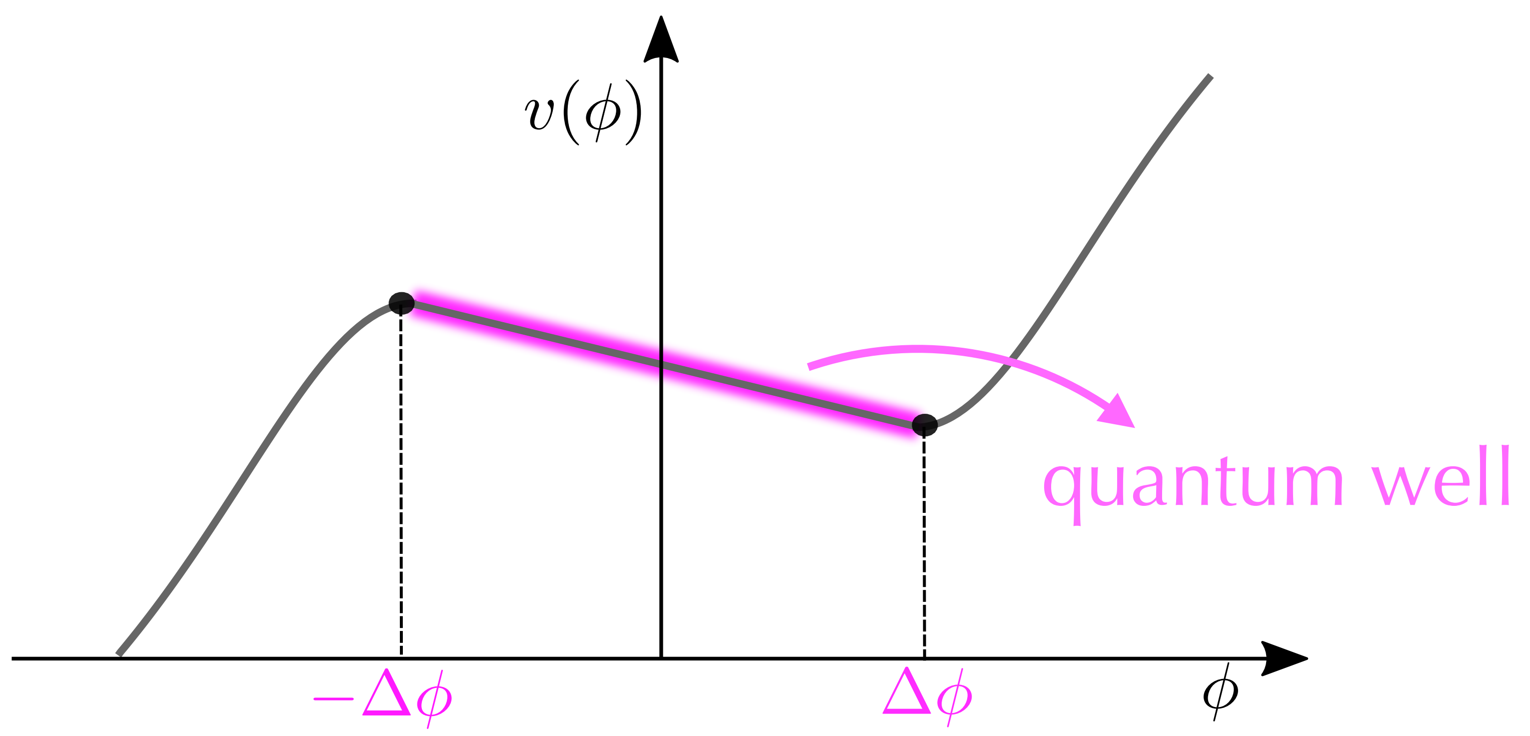

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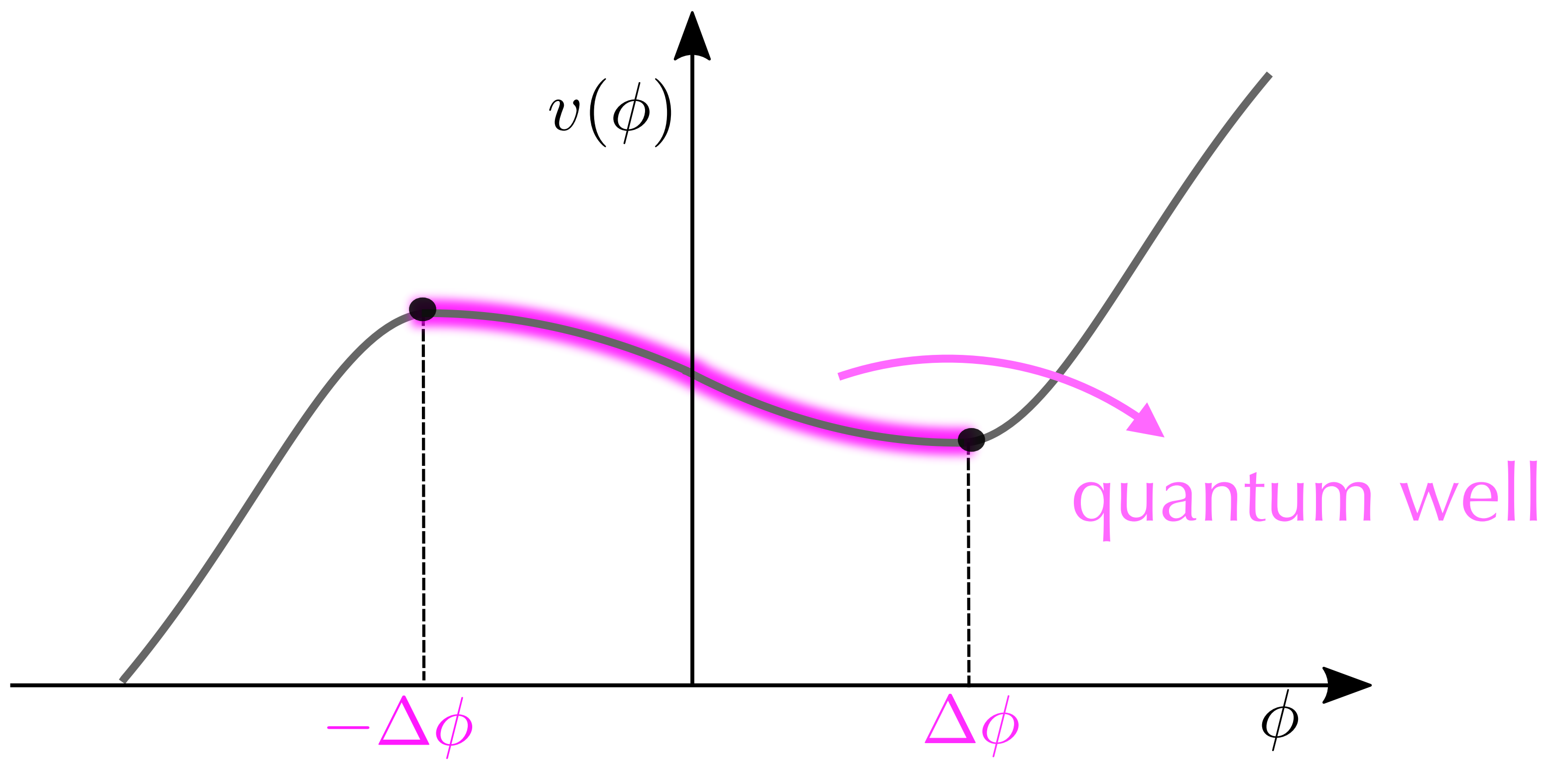

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$$a = \frac{\alpha}{v_0} \propto \frac{M_{Pl}^4\, \Delta V}{V^2}$$

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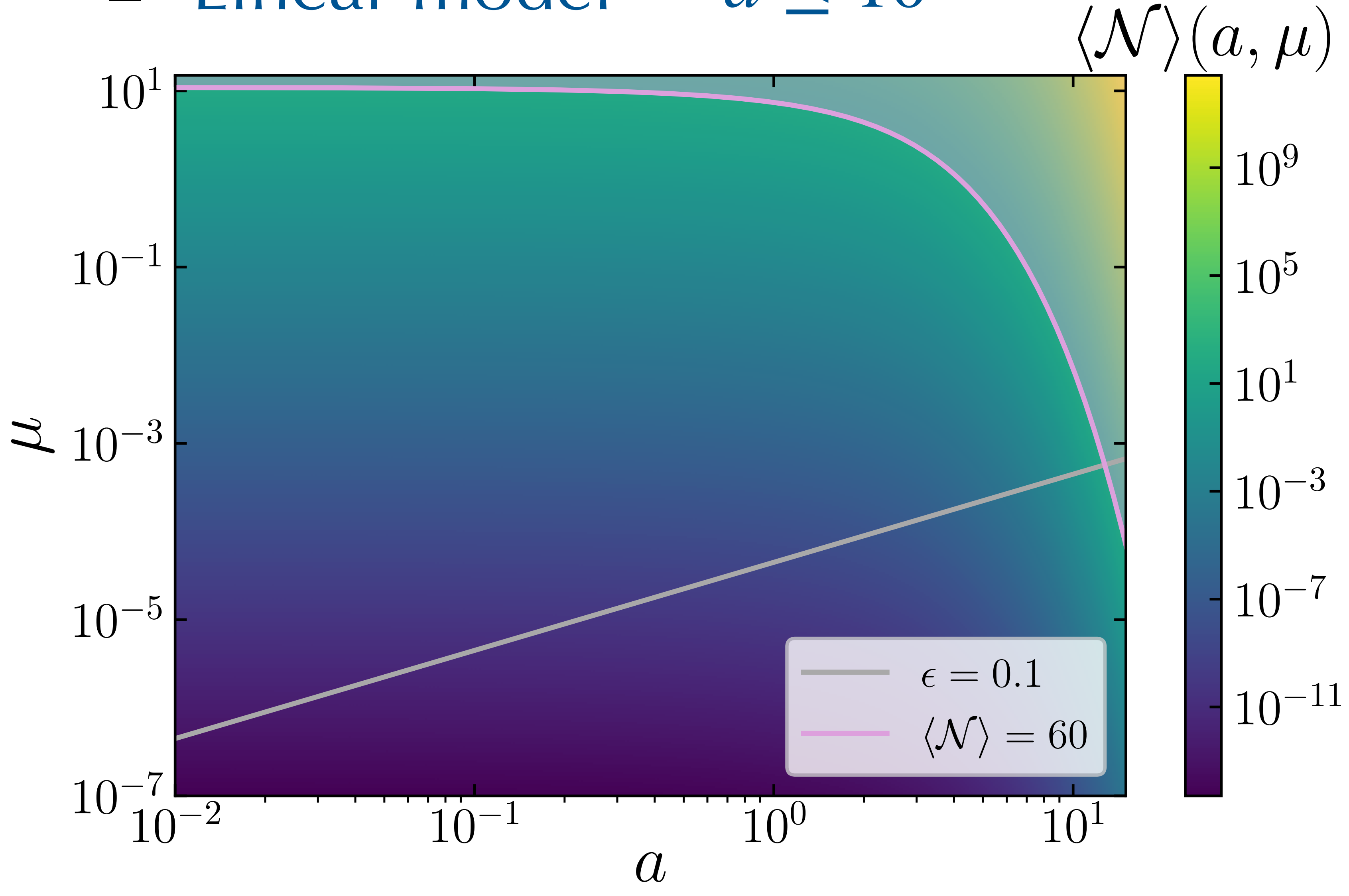

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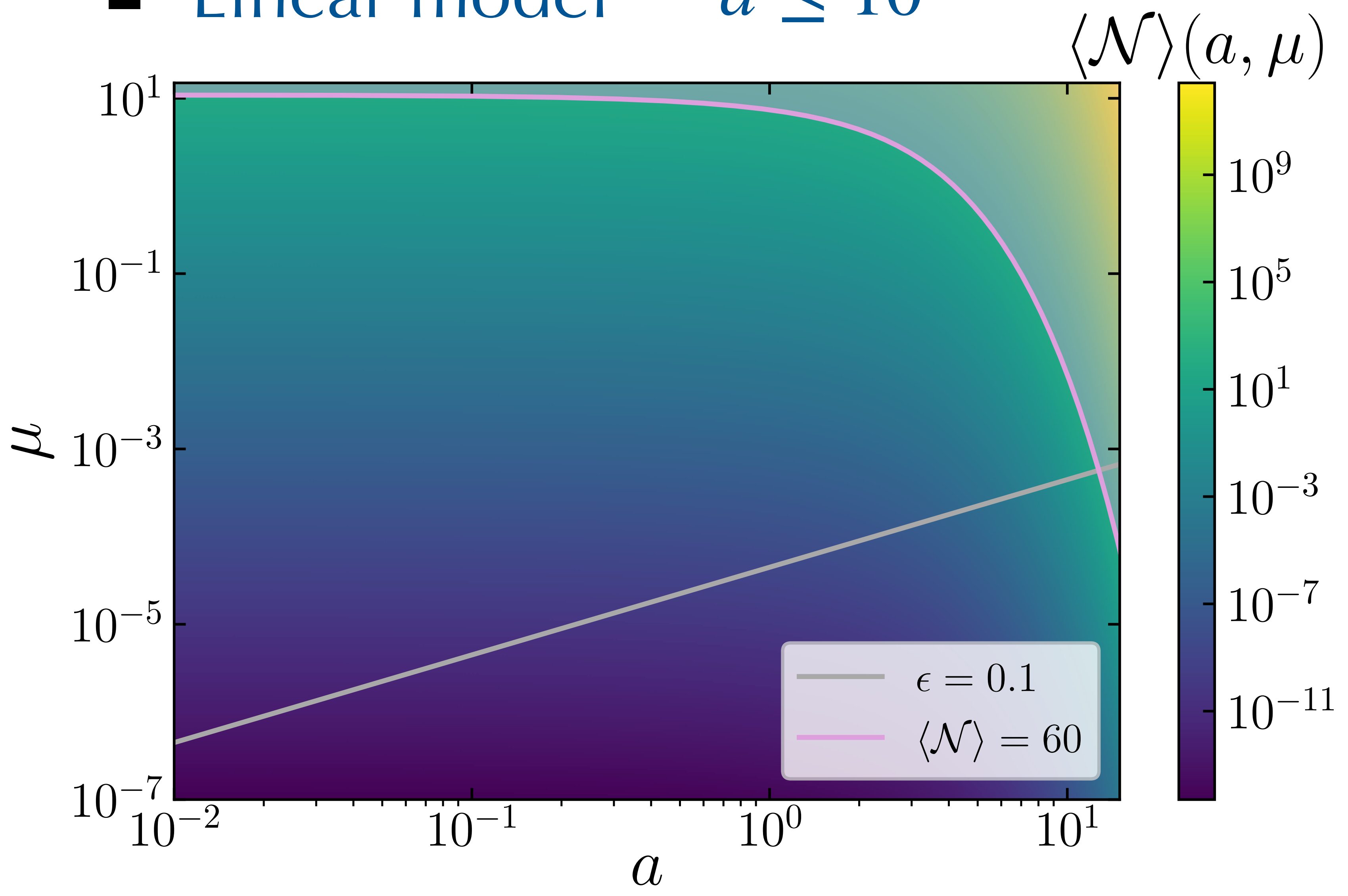

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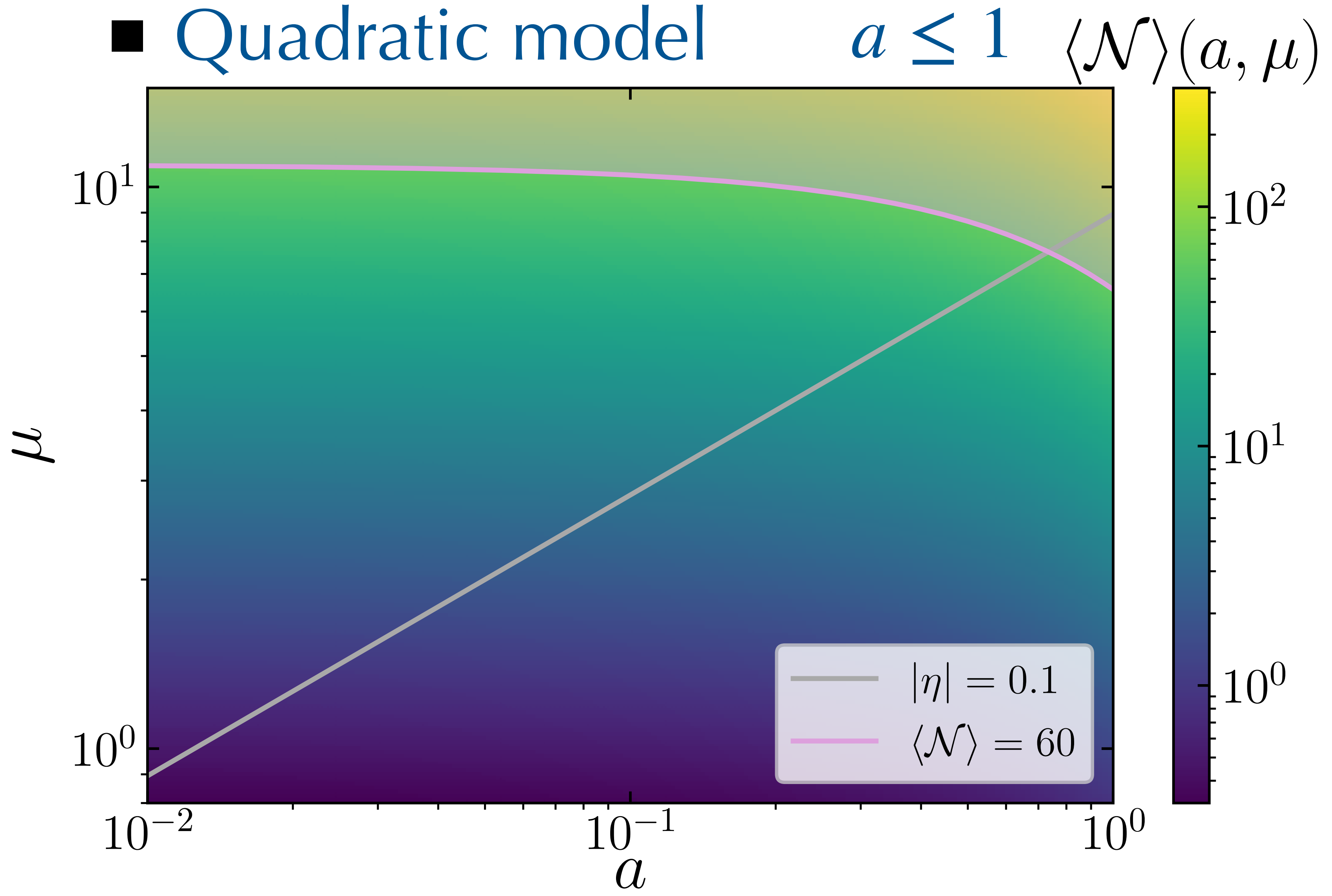

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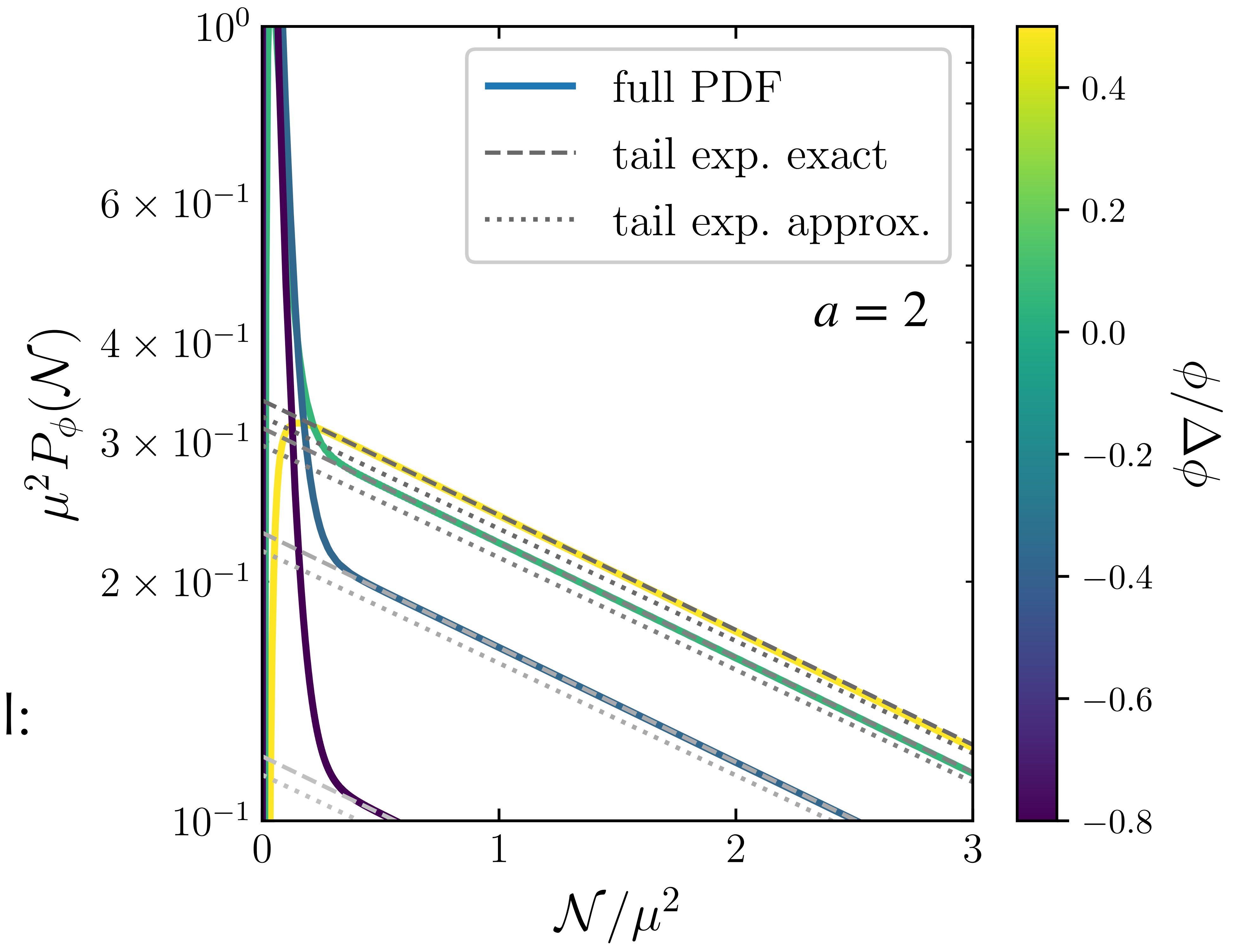

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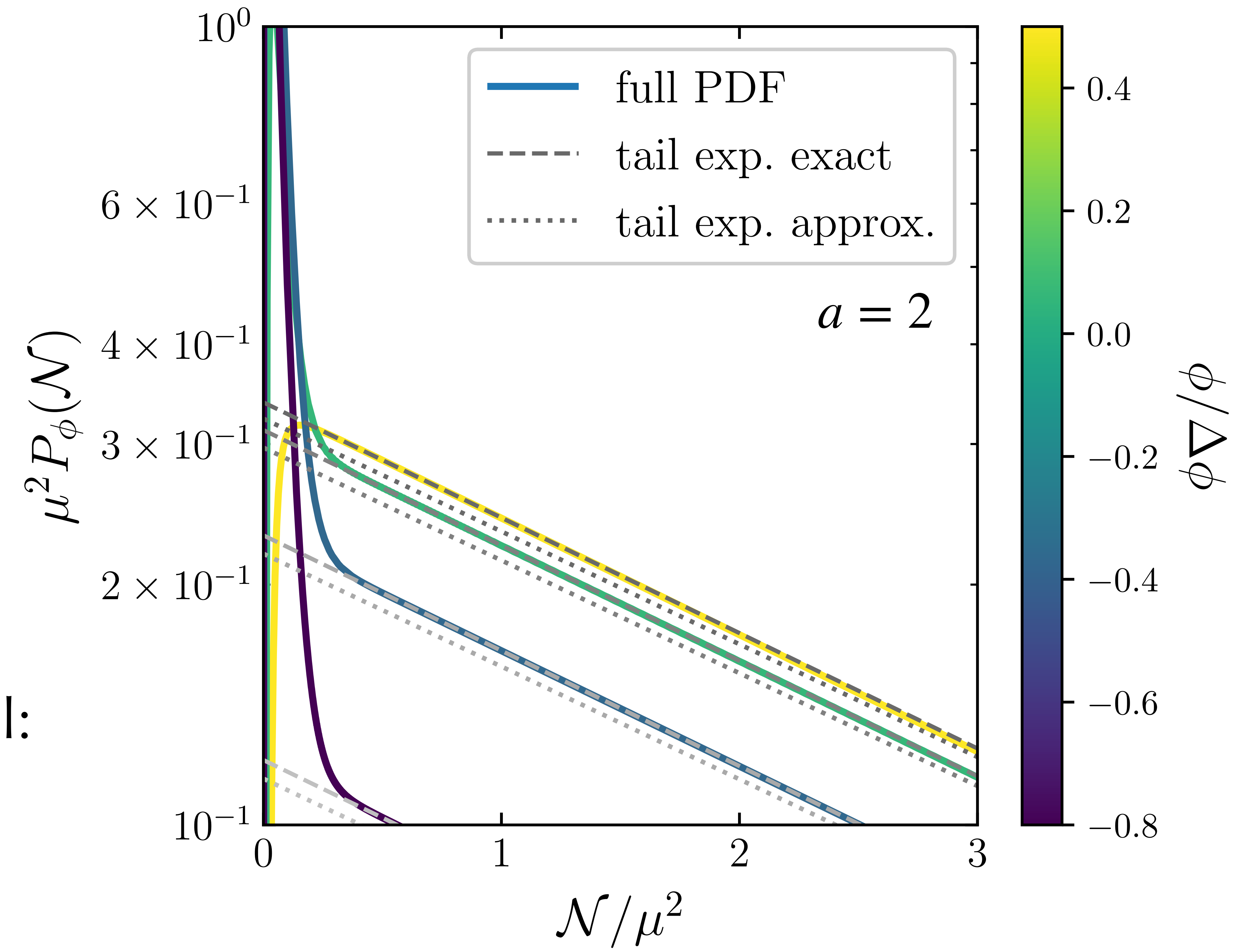

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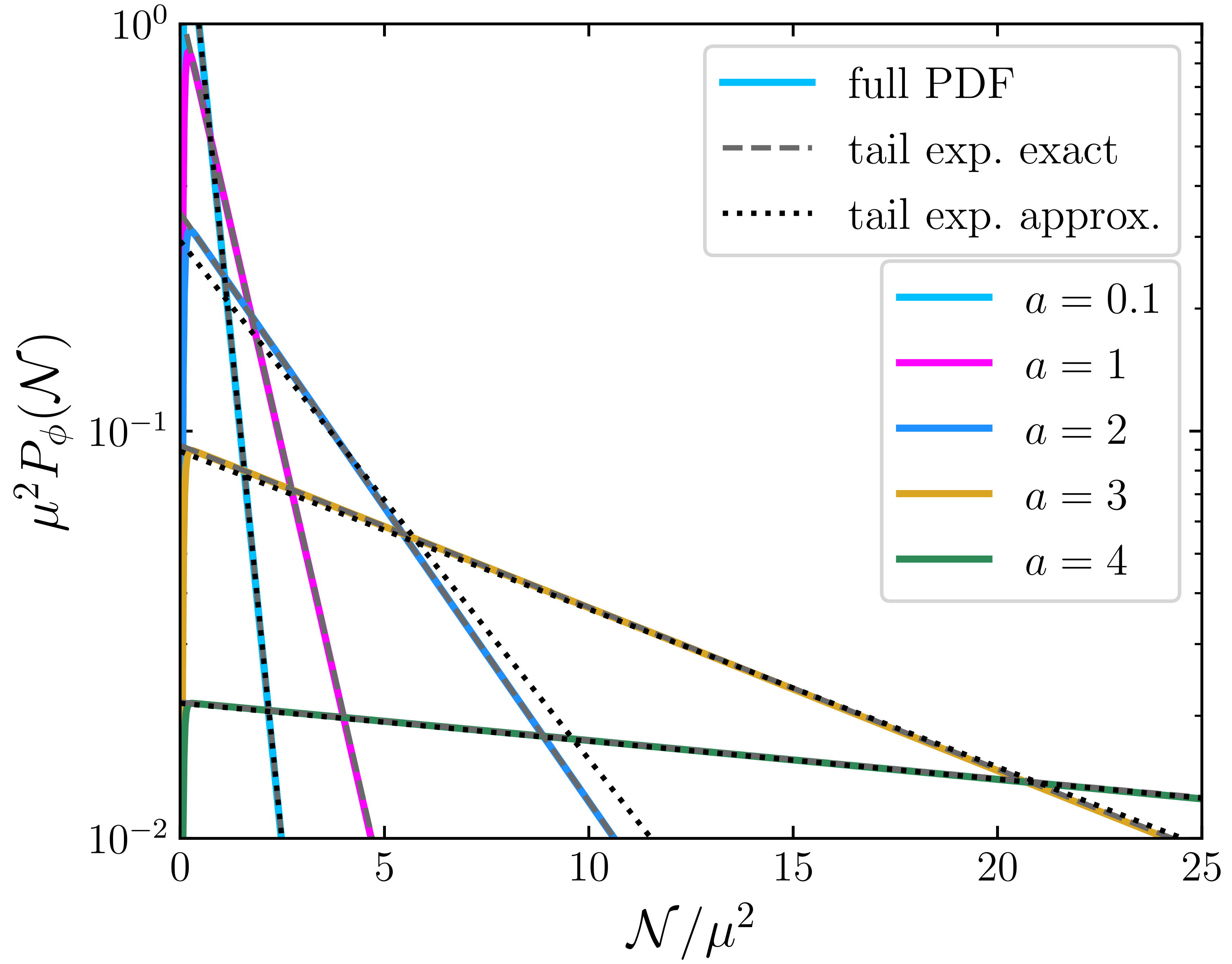

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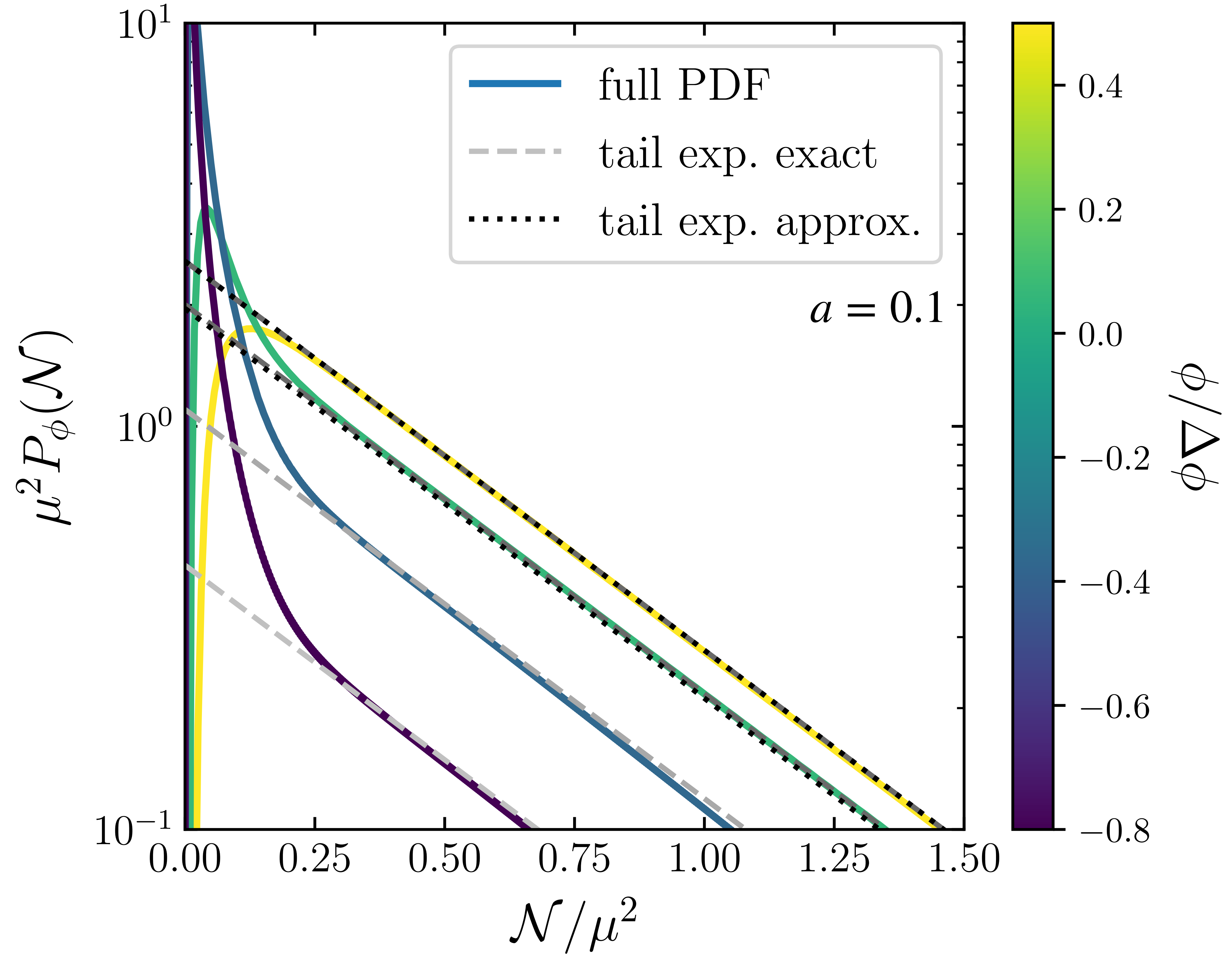

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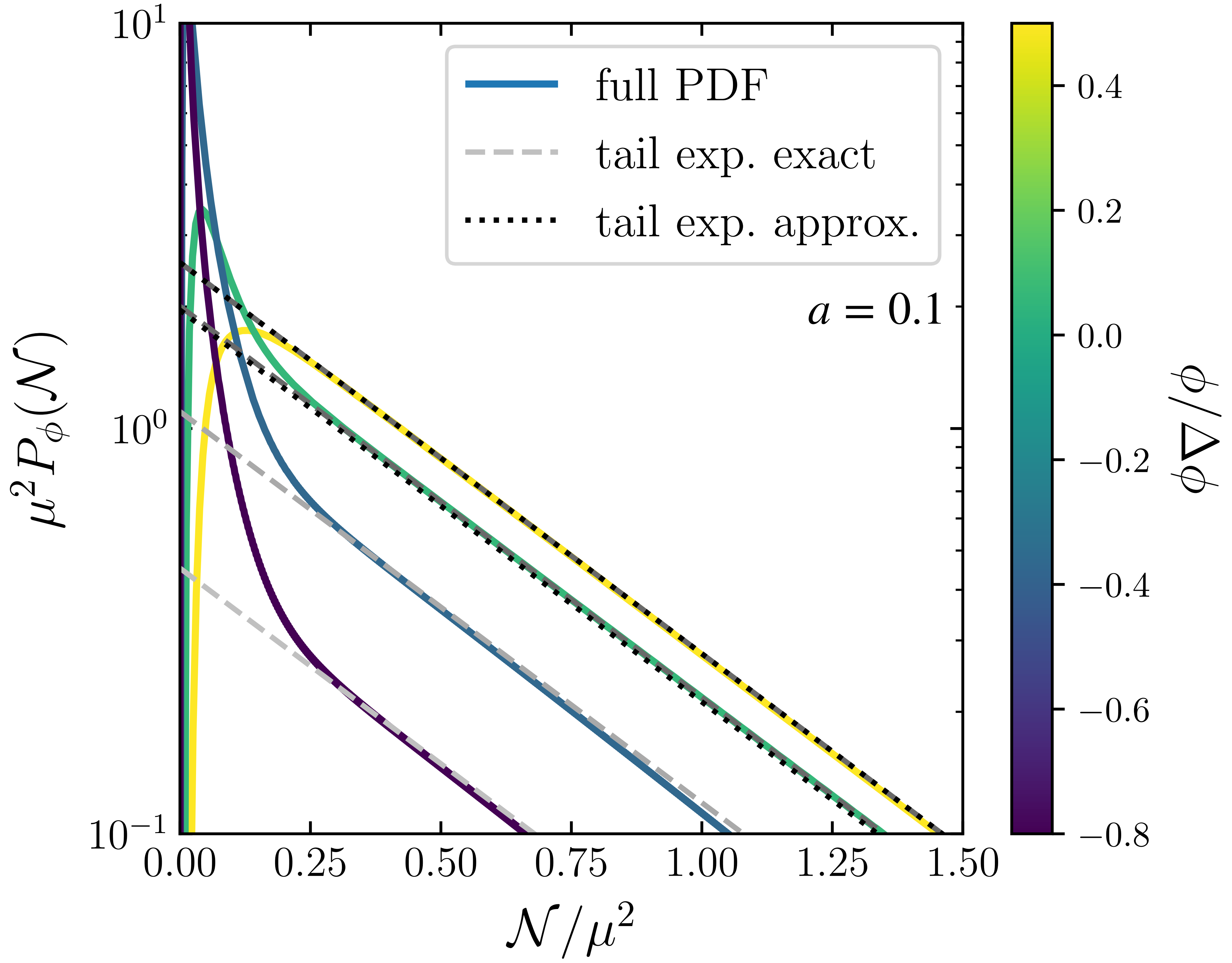

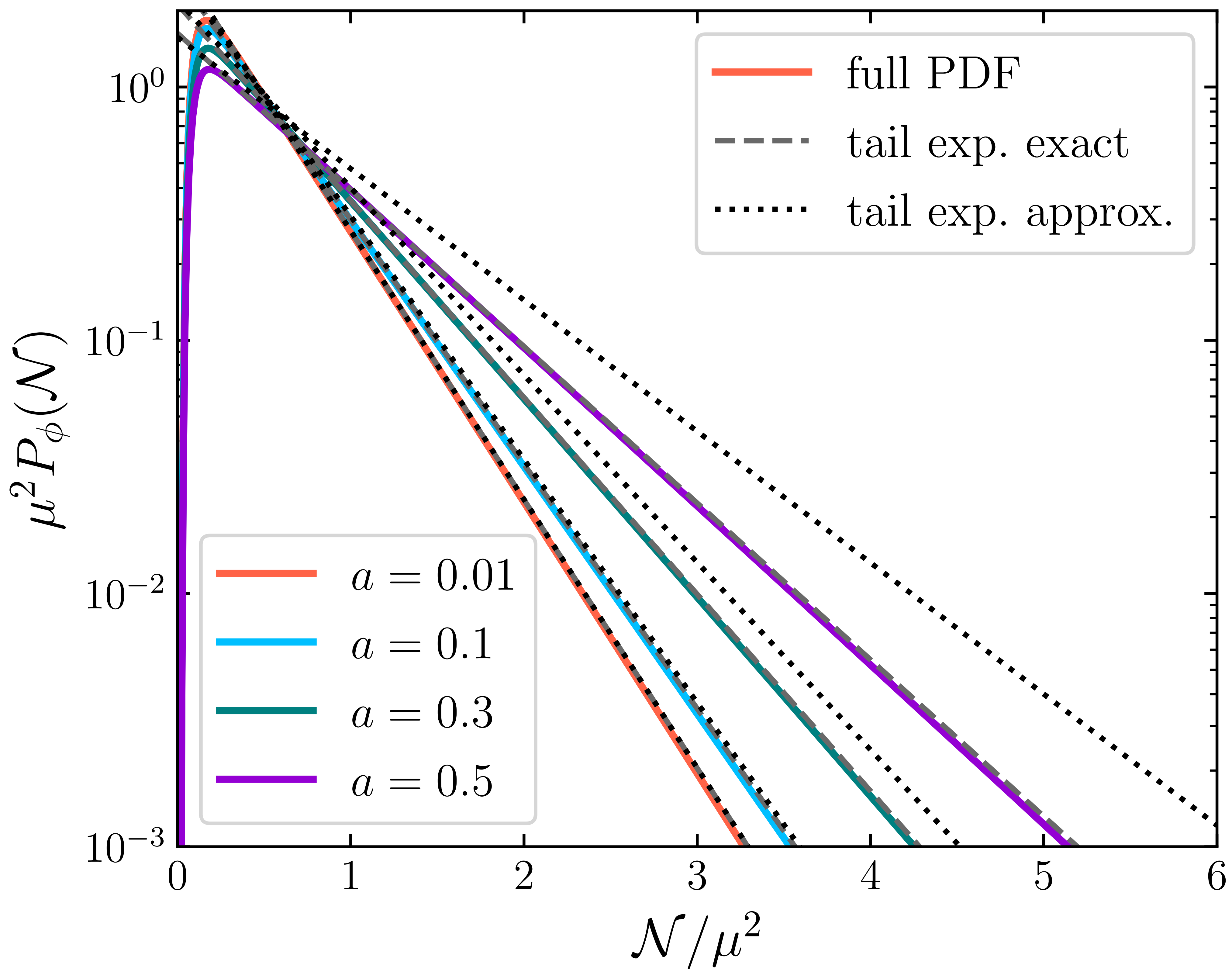

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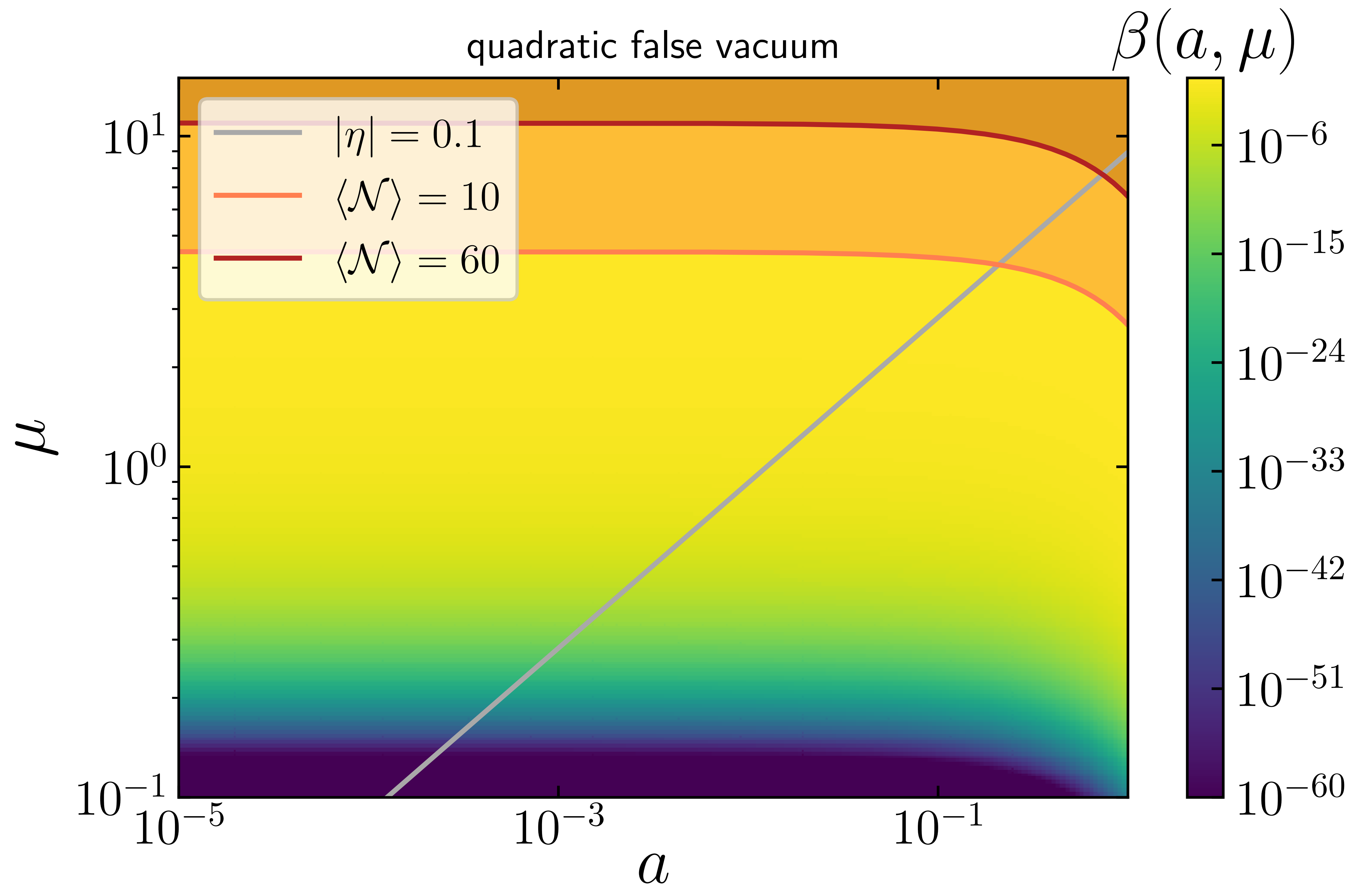
⟶ super-exponential dependence on a PBHs are overproduced when $a \gtrsim 8$

False vacuum: implications for Primordial Black Holes

Quadratic model

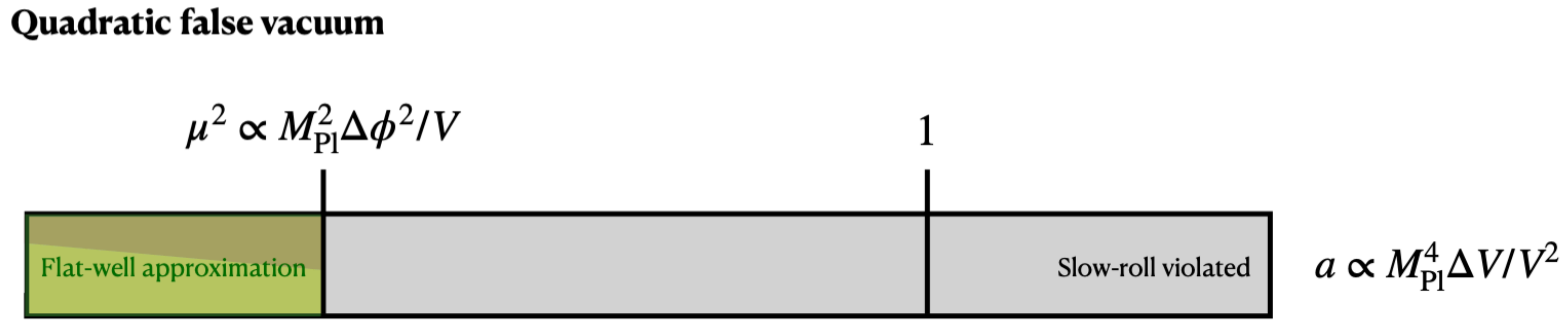


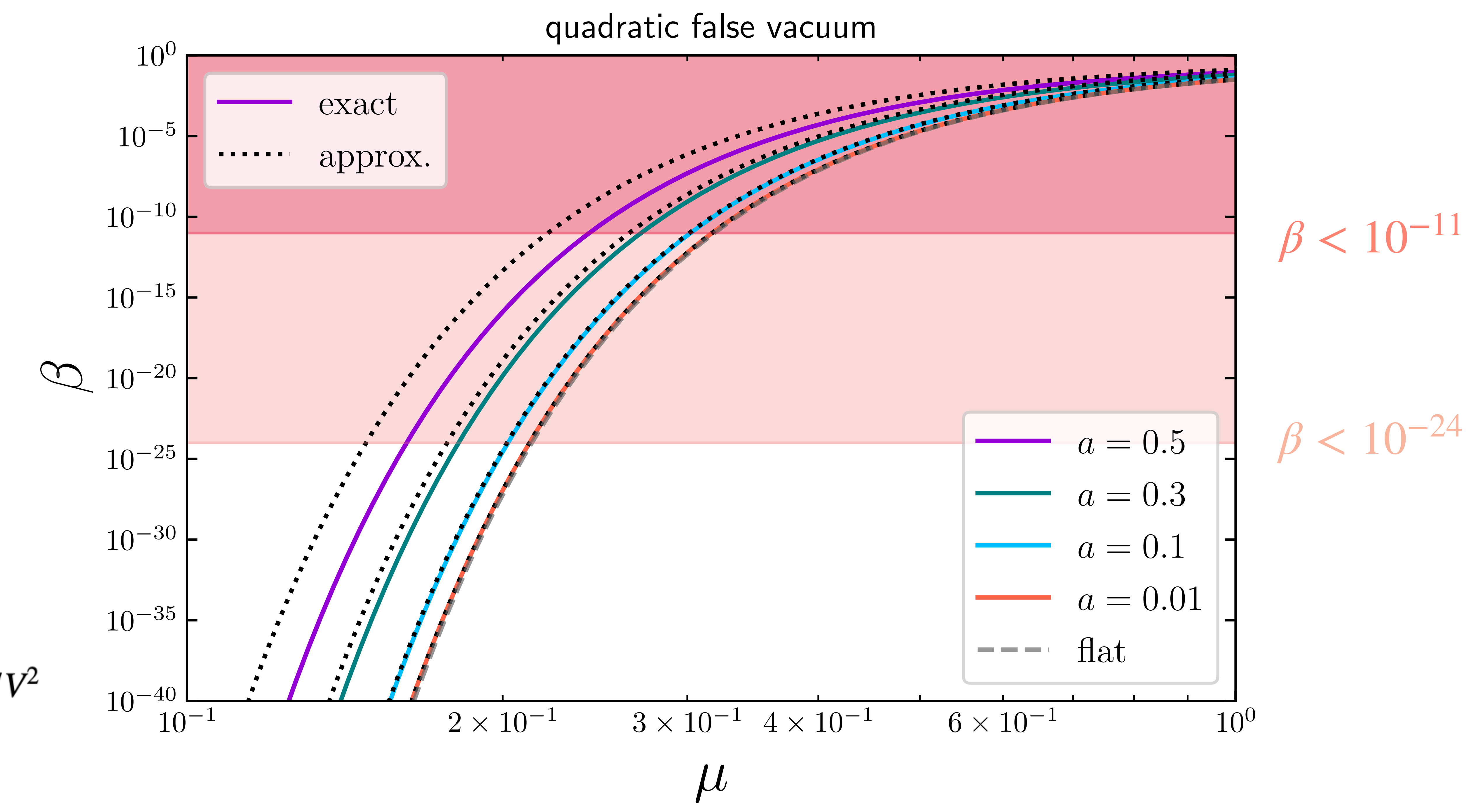
PBH abundance well captured by a flat-well limit ($a = 0$)

If $\mu \ll 1$, tiny amount of PBHs produced

If $\mu \lesssim 1$, PBHs produced with sizable abundance

If $\mu \gtrsim 1$, PBHs overproduced





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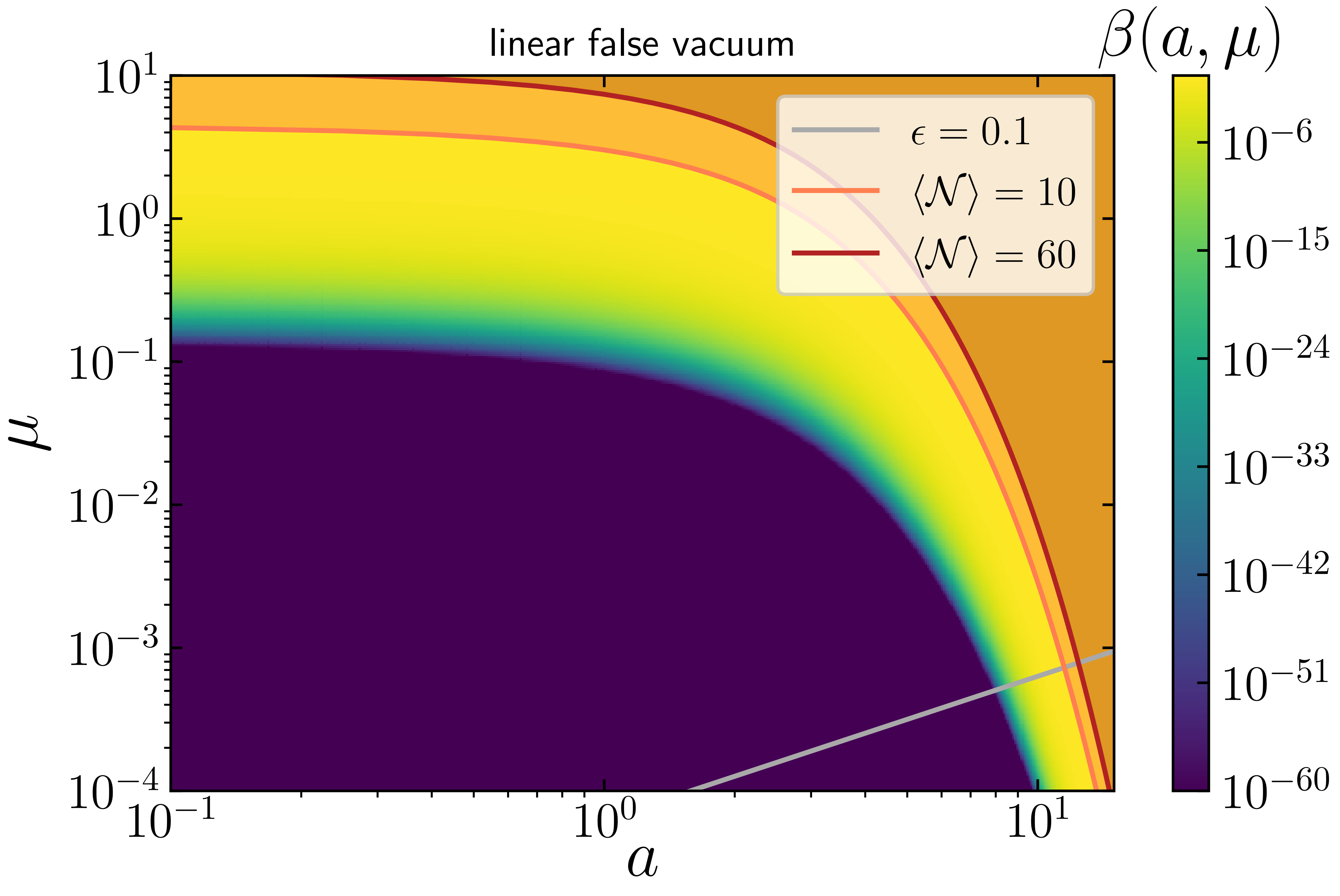
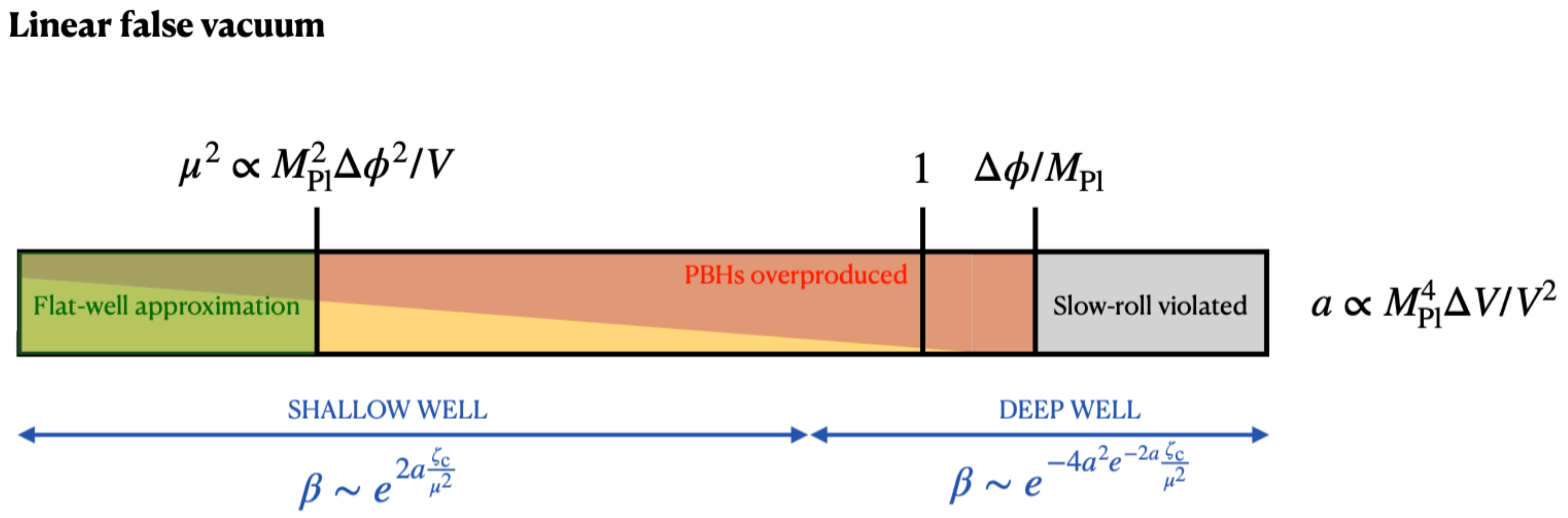
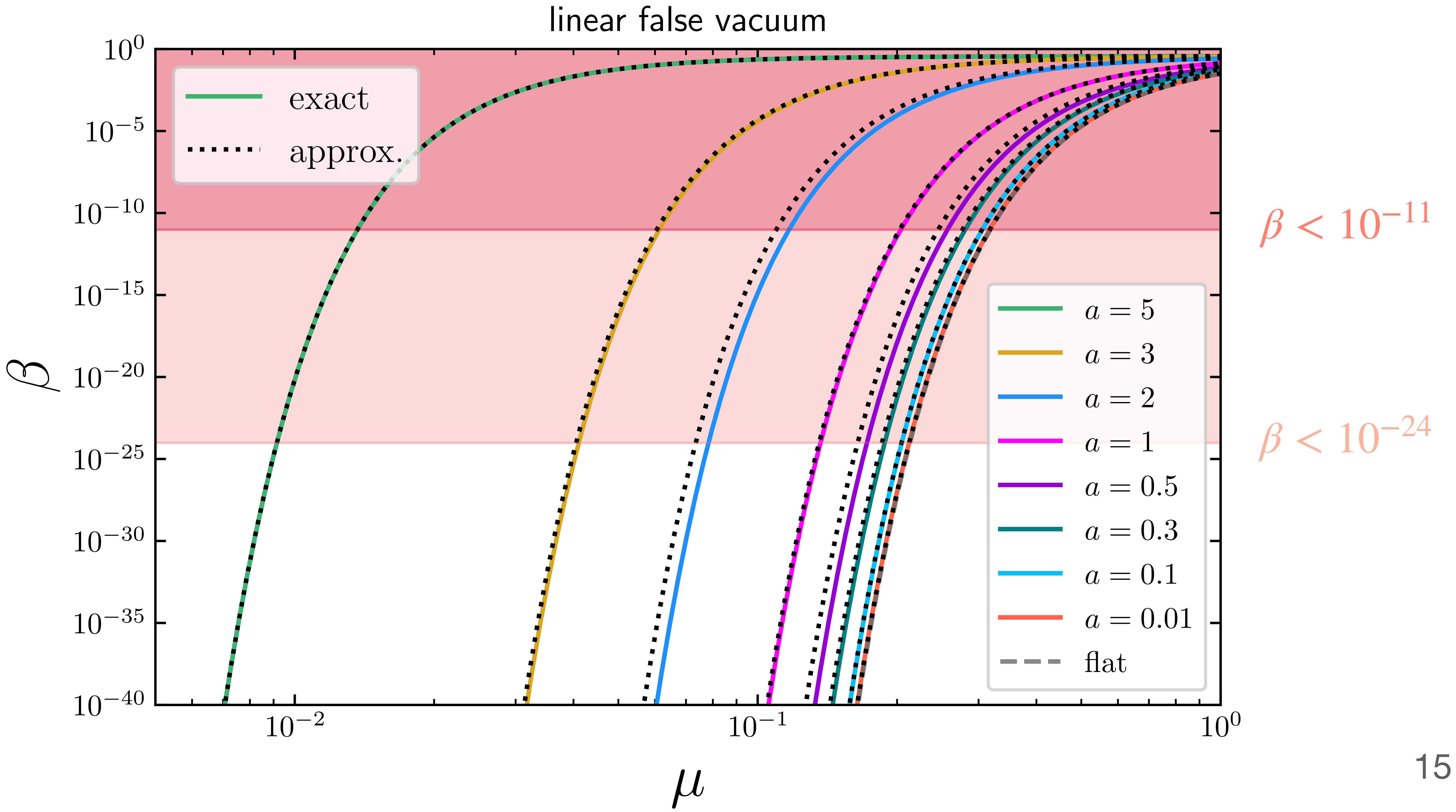


Additional regimes:

If $\mu^2 \ll a \ll 1$ (μ small):
large deviations from flat-well, still shallow-well domain;
non-trivial imprint of the false-vacuum profile

If $a \sim \mathcal{O}(1)$: large PBH production





Summary

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- PBHs are interesting objects to learn more about inflation beyond the CMB probed regime

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- Numerical simulations

LPENS
LABORATOIRE DE PHYSIQUE
DE L'ÉCOLE NORMALE SUPÉRIEURE

Many thanks for the attention!

chiara.animali@phys.ens.fr

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Cosmic Inflation

Cosmic Inflation

- High energy phase of accelerated expansion of spacetime $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2$ $\dot{a}, \ddot{a} > 0$

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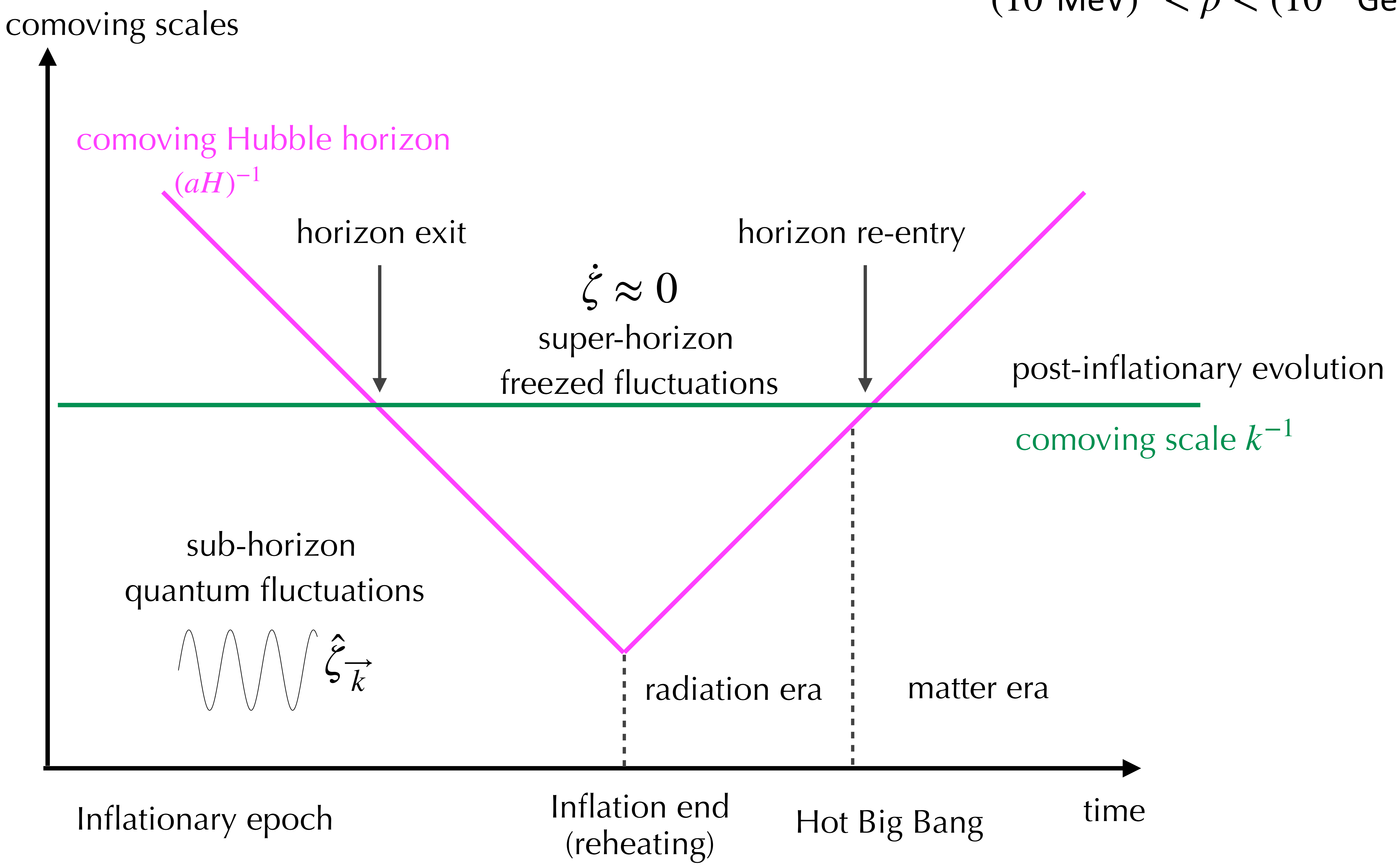

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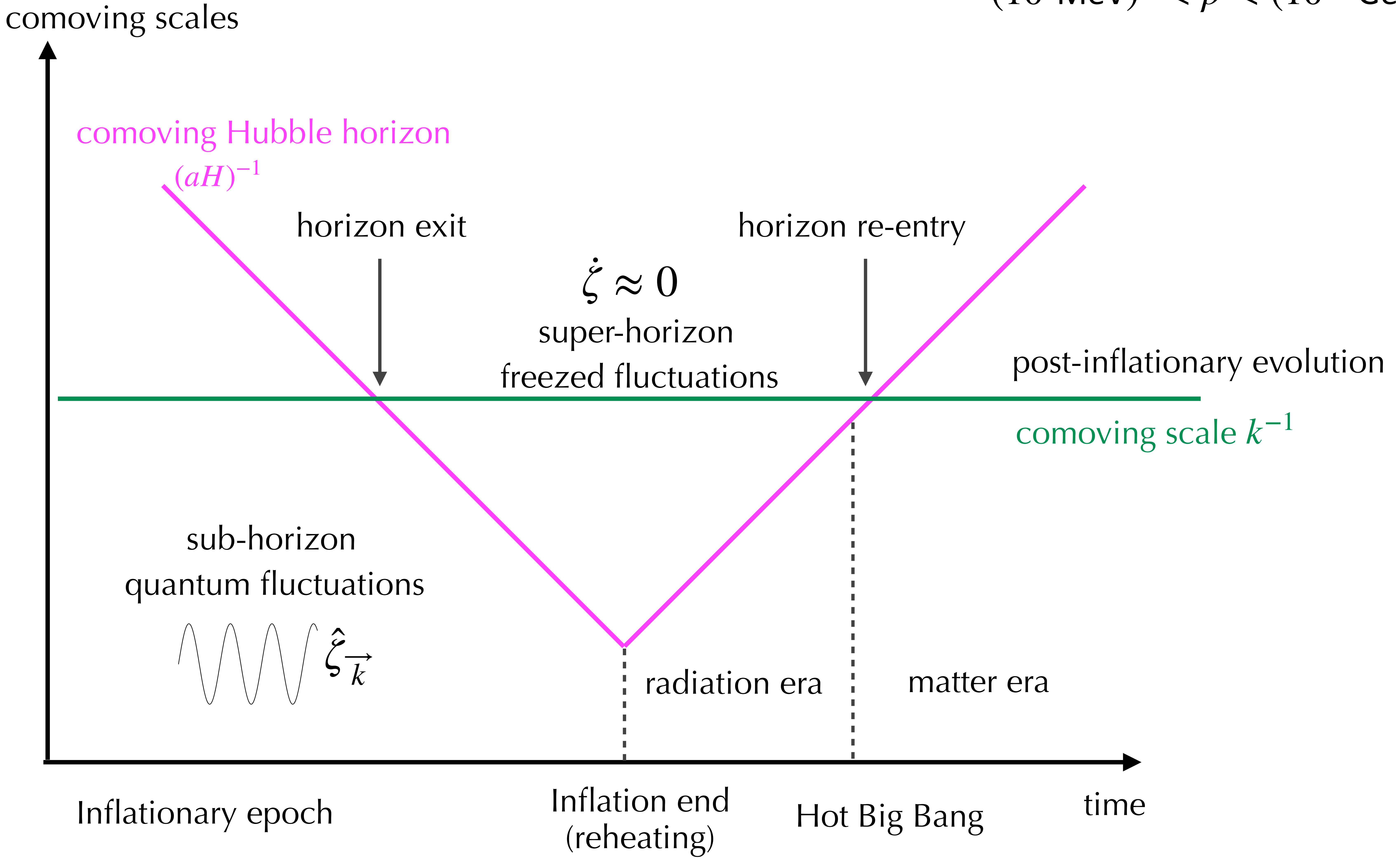
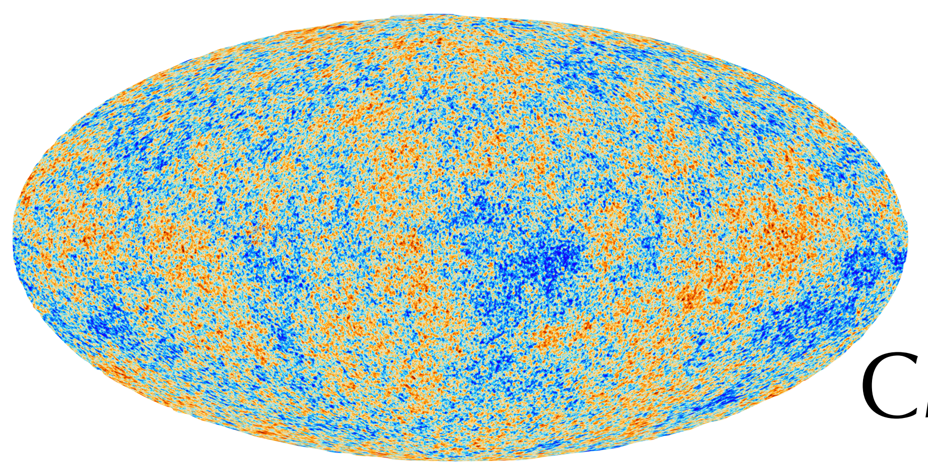
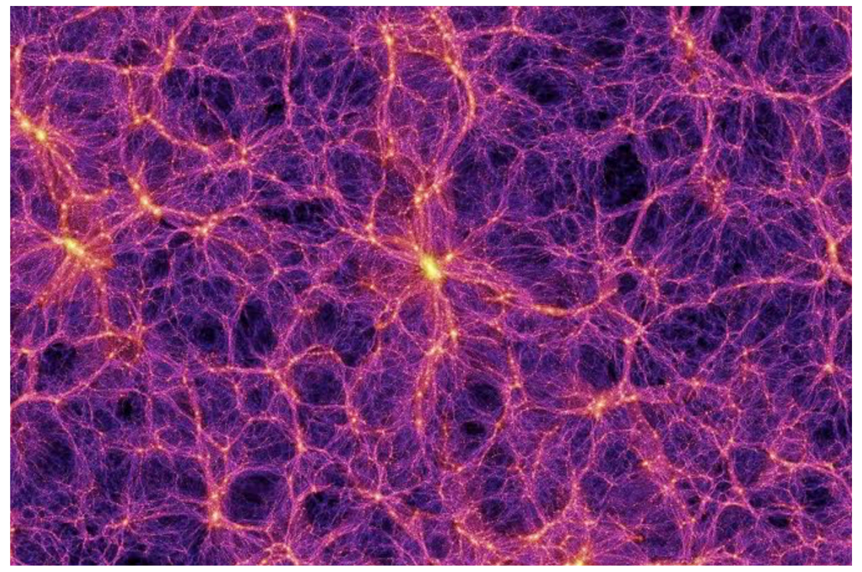




CMB



LSS

Primordial Black Holes

LIGO SCIENTIFIC, VIRGO collaboration [2016]:
Observation of gravitational waves from a binary black hole merger

Primordial Black Holes

- Black holes which could have formed in the early Universe through a non-stellar way

Hawking [1971] : *Gravitationally collapsed objects of very low mass*

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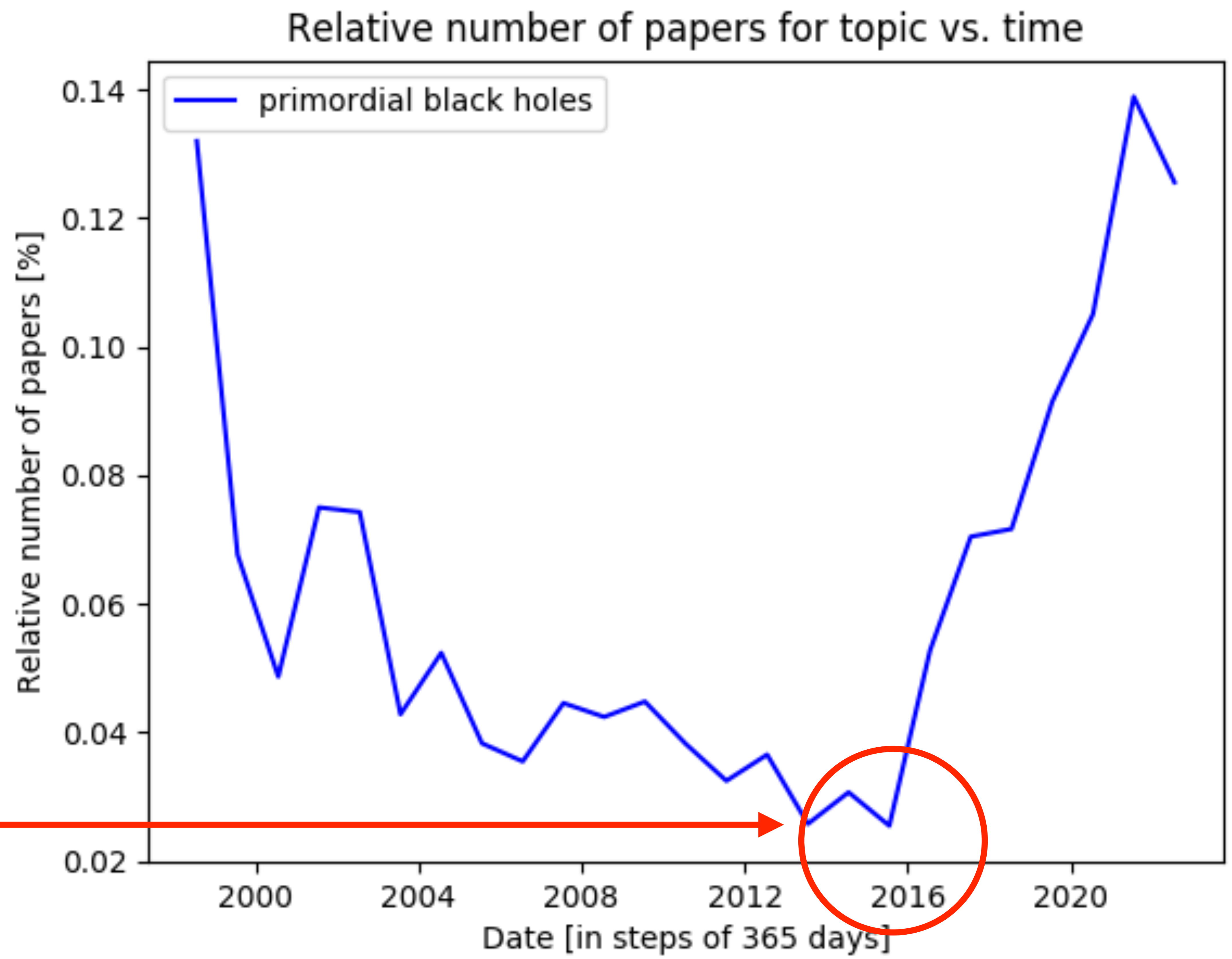

[Plot realised via www.benty-fields.com/trending]

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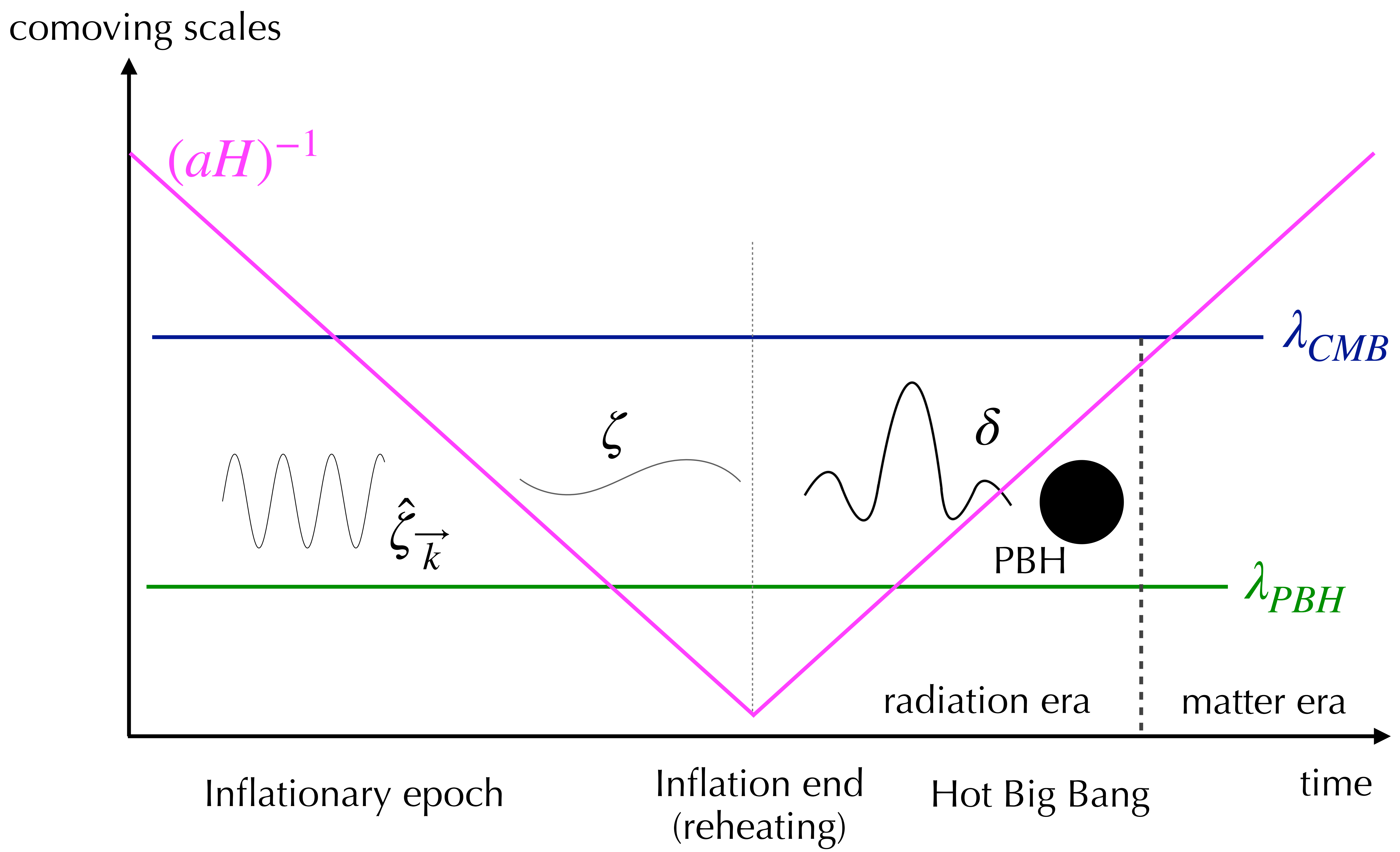
S. Bird, I. Cholis, J.B. Muñoz, Y. Ali-Haïmoud,
M. Kamionkowski, E. D. Kovetz, A. Raccanelli, A. G. Riess [2016]:
Did LIGO detect dark matter?

Primordial Black Holes : How?

- PBHs may be originated from peaks of the density perturbations generated in the early universe



$$\delta \sim \left.\frac{\delta\rho}{\rho}\right|_{k=aH} \sim \zeta > \zeta_c \sim 1$$

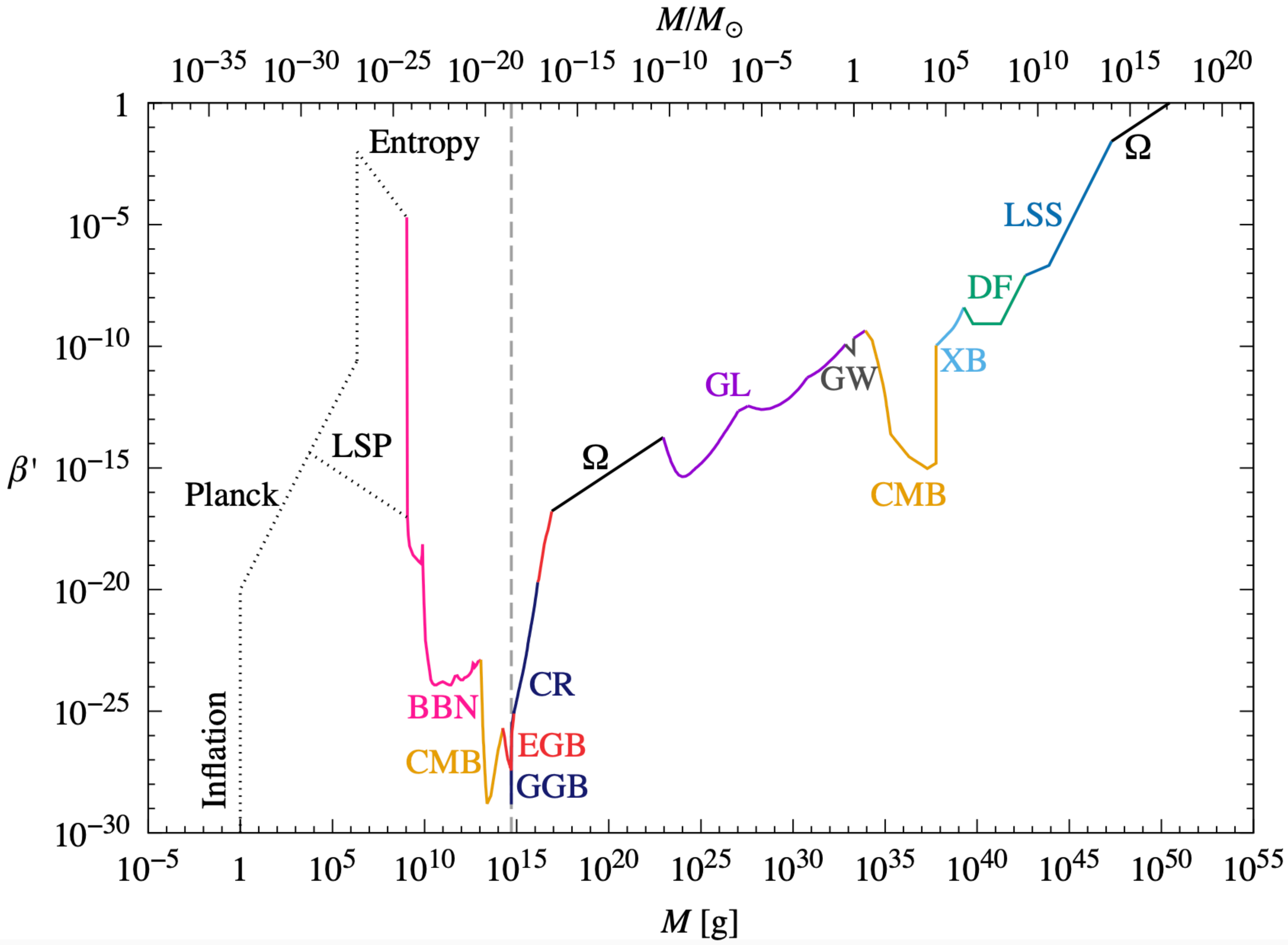
Primordial black holes: observational constraints

Depends on the mass at which PBHs form

$10^9 g < M_{PBH} < 10^{16} g \longrightarrow$ from $\beta < 10^{-24}$ to $\beta < 10^{-17}$ — PBH Hawking evaporation on Big Bang Nucleosynthesis and on the extragalactic photon background

$10^{16} g < M_{PBH} < 10^{50} g \longrightarrow$ from $\beta < 10^{-11}$ to $\beta < 10^{-5}$ — Gravitational and astrophysical effects

$M_{PBH} < 10^9 g$ — Not yet evaporated: no direct observational constraints



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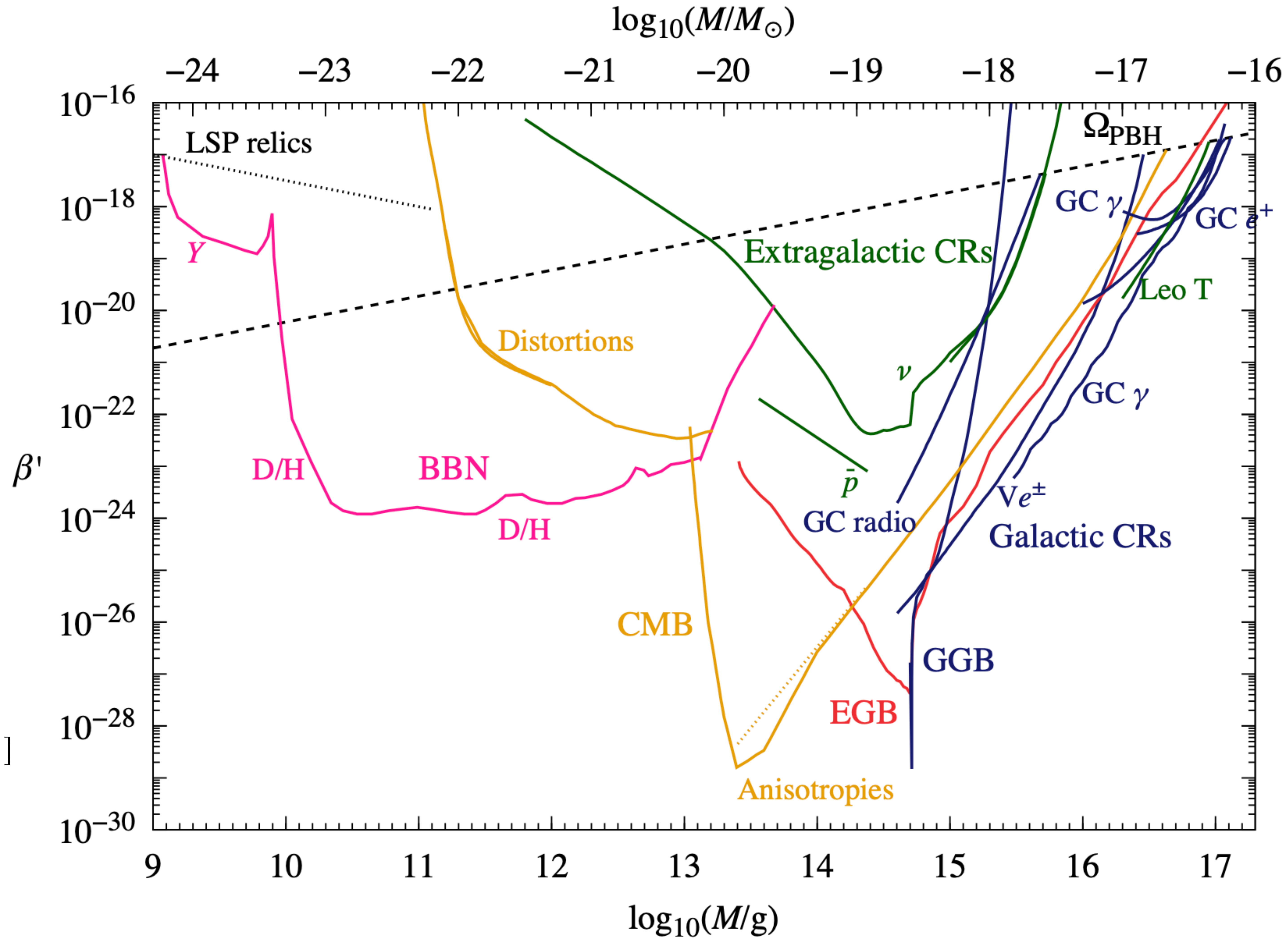

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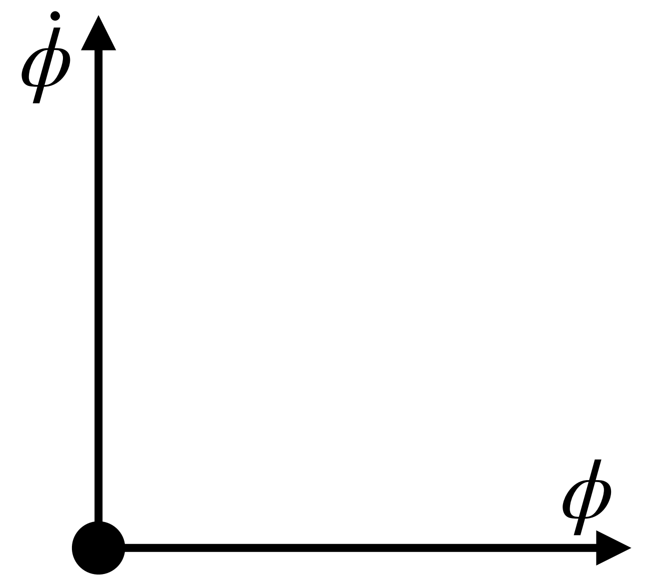
False vacuum: preserving slow roll

Slow roll requires: $|\ddot{\phi}| \ll 3H|\dot{\phi}|, |V_{,\phi}|$

What happens if $|V_{,\phi}| = 0$?



minimum $(\phi = 0, \dot{\phi} = 0)$

$V(\phi) \simeq V_0 + m^2\phi^2/2$

$$\ddot{\phi} + 3\,H(\phi,\dot{\phi})\,\dot{\phi} + V_{,\phi} = 0 \qquad H^2(\phi,\dot{\phi}) = \frac{1}{3M_{Pl}^2}\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right)$$

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$m \gg 3H_0/2$: damped oscillations, friction term $3H\dot{\phi}$ subdominant: far from slow-roll regime

$m \ll 3H_0/2$ $\qquad \phi \simeq A\exp\left(-3H_0t\right) + B\exp\left(-\frac{1}{3}\frac{m^2}{H_0^2}H_0t\right) \simeq B\exp\left(-\frac{m^2t}{3H_0}\right)$

$$3H\dot{\phi} \simeq -m^2\phi = -V_{,\phi}(\phi) \qquad \ddot{\phi} \simeq \frac{m^4}{9\,H_0^2}\phi = \frac{m^2}{9\,H_0^2}V_{,\phi} \ll V_{,\phi}(\phi)$$

slow-roll regime: acceleration term subdominant

(m^2/H_0^2 - suppressed)

δN formalism

FLRW metric: $ds^2 = -\,dt^2 + a^2(t)\,\delta_{ij}\,dx^i\,dx^j$

deviations from homogeneity and isotropy: $ds^2 = -\,dt^2 + a^2(t)\,e^{2\zeta(t,\vec{x})}\,\gamma_{ij}$

$t-$slices of uniform energy density

$x-$worldlines comoving

local scale factor: $\tilde{a}(t,\vec{x}) = a(t)\,e^{\zeta(t,\vec{x})}$

expansion from flat slice at time t_{in} to a slice of uniform energy density:

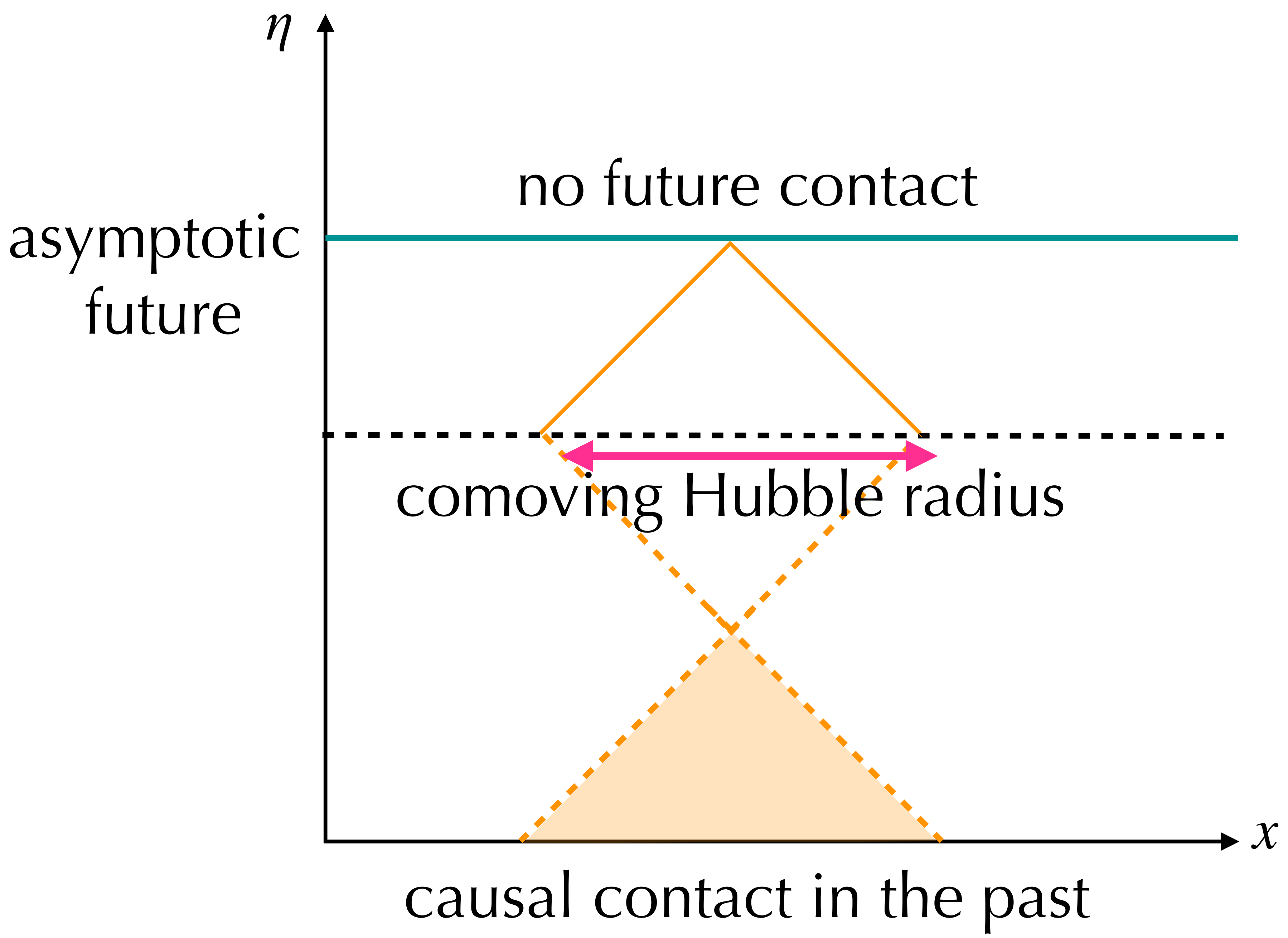
$$N(t,\vec{x}) = \log\left[\frac{\tilde{a}(t,\vec{x})}{a(t_{in})}\right] \qquad \zeta(t,\vec{x}) = N(t,\vec{x}) - N_0(t) \equiv \delta N \qquad N_0(t) = \log\left[\frac{a(t)}{a(t_{in})}\right]$$

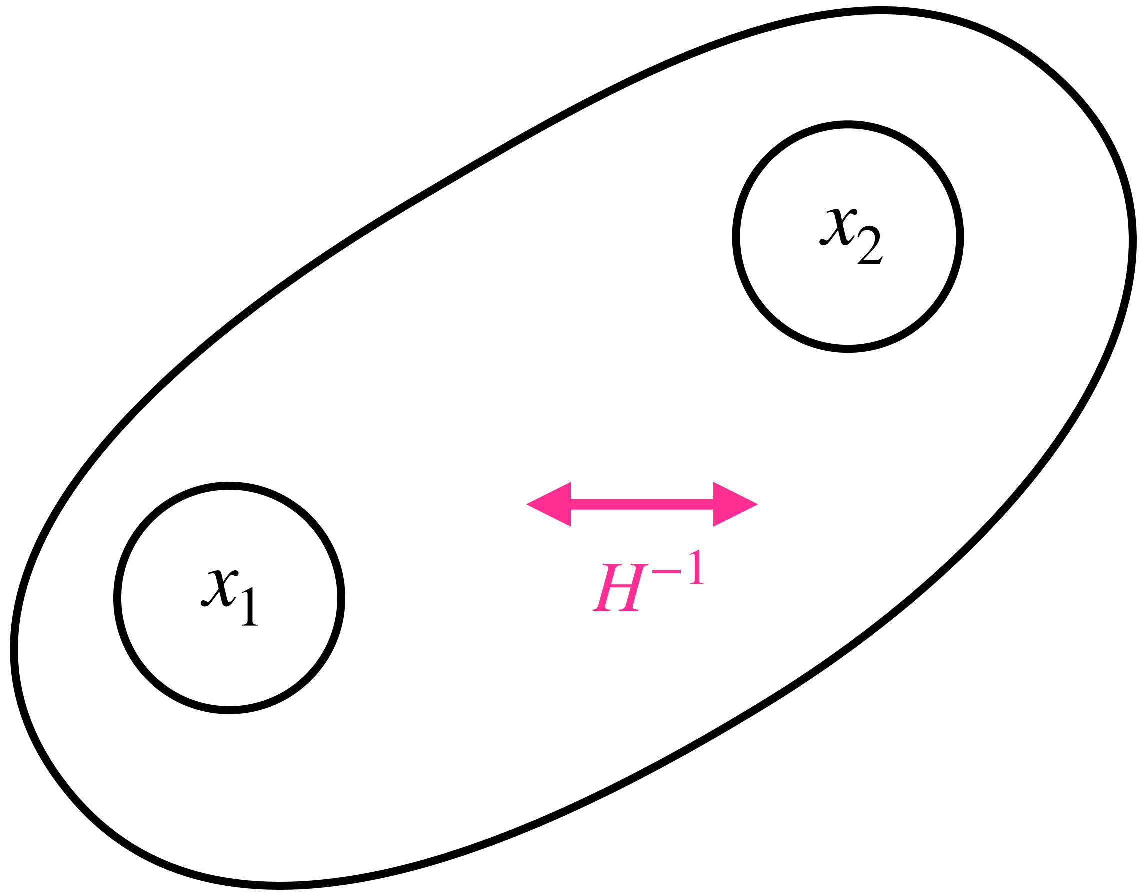
Separate universe approach

On super-Hubble scale, the evolution of the universe at each position is independent
(and follows the same evolution as the background)

$N(t, \vec{x})$: amount of expansion in unperturbed universe

ζ: known from the evolution of a family of unperturbed universes





[Salopek and Bond; Sasaki and Stewart; Wands; Lyth and Liddle]

Stochastic-δN formalism

Phase space field vector: $\Phi = (\phi_1, \pi_1, \cdots \phi_n, \pi_n)$

$$\Phi_{cg} = \frac{1}{(2\pi)^{3/2}} \int_{k<k_\sigma} d^3k \Phi_k e^{-ik\vec{x}}$$

$$\frac{d\Phi_{cg}}{dN} = F(\Phi_{cg}) + G(\Phi_{cg}) \cdot \xi$$

$$\langle \xi_i(\vec{x}_i, N_i)\, \xi_i(\vec{x}_j, N_j) \rangle = \delta_{ij}\, \delta(N_i - N_j)$$

$$(G^2)_{ij} = \frac{d \log k_\sigma}{dN} \mathscr{P}_{\Phi_i, \Phi_j} \left[k_\sigma(N), N \right]$$

$$\delta N_{cg}(\vec{x}) = \mathcal{N}(\vec{x}) - \langle \mathcal{N} \rangle = \zeta_{cg}(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int_{k_{in}}^{k_{end}} d\vec{k}\, \zeta_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}}$$

Curvature perturbation coarse grained between:
the scale that crosses the Hubble radius at initial time (k_{in}) and the scale that crosses the Hubble radius at final time k_{end}

Fokker-Planck equation

Evolution given by the Langevin equation: $$\Phi(N+\delta N) = \Phi(N) + F(\Phi)\delta N + G(\Phi)\cdot\int_N^{N+\delta N} d\tilde{N}\,\xi(\tilde{N})$$

Where to evaluate F and G? At $\Phi(N)$ or at $\Phi(N+\delta N)$?

$$\Phi_\alpha(N) = (1-\alpha)\Phi(N) + \alpha\Phi(N+\delta N) \qquad 0 \le \alpha \le 1$$

Itô prescription: $\alpha = 0$

Stratonovitch prescription: $\alpha = \dfrac{1}{2}$

$$\Phi(N+\delta N) = \Phi(N) + F[\Phi_\alpha(N)]\delta N + G[\Phi_\alpha(N)]\cdot\int_N^{N+\delta N} d\tilde{N}\,\xi(\tilde{N})$$

Fokker-Planck equation: $$\frac{\partial}{\partial N}P(\Phi, N\,|\,\Phi^{in}, N_{in}) = \mathscr{L}_{FP}(\Phi)P(\Phi, N\,|\,\Phi^{in}, N_{in})$$

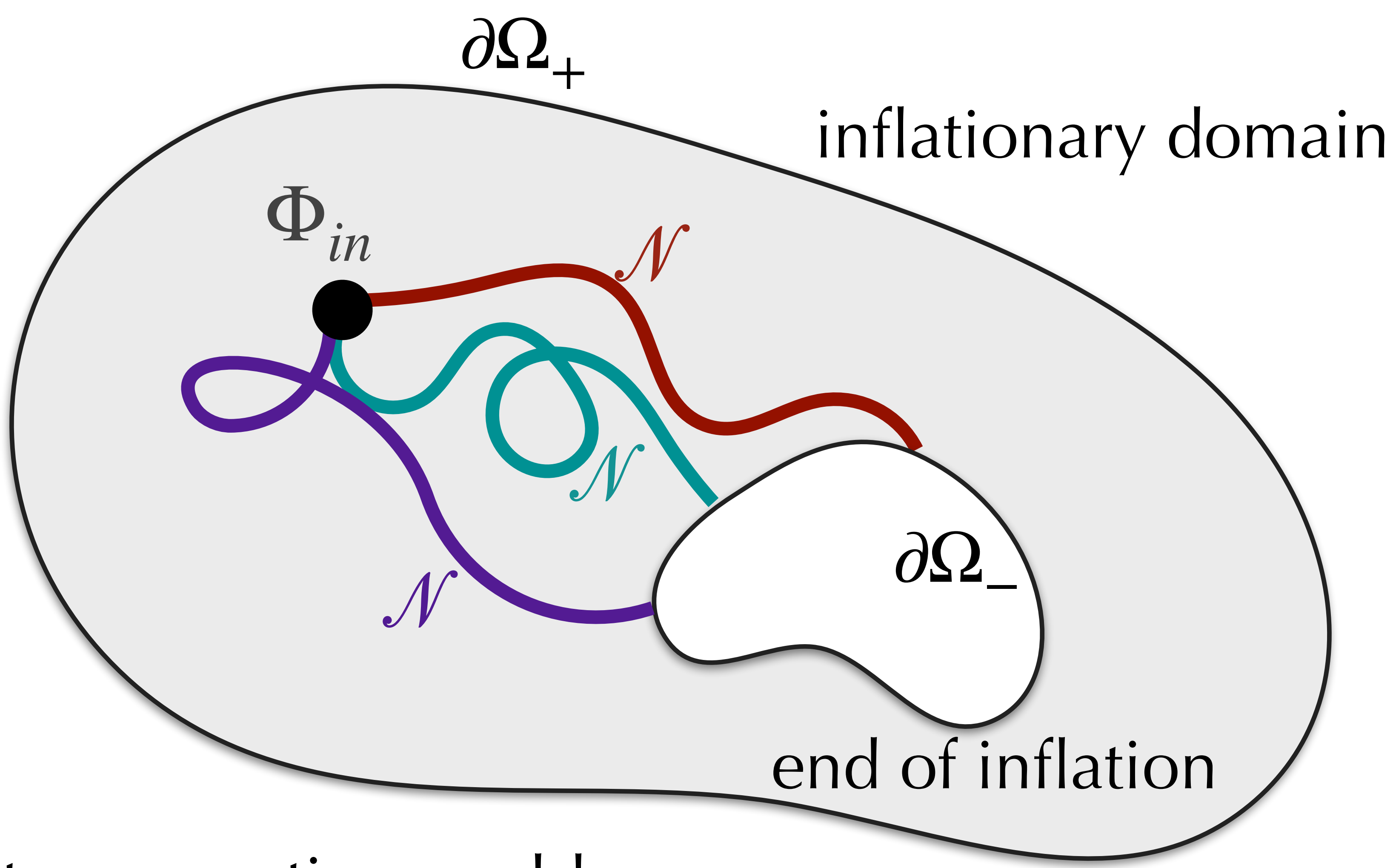
$$\mathscr{L}_{FP}(\Phi) = -\frac{\partial}{\partial\Phi_i}\left[F_i(\Phi) + \alpha G_{lj}(\Phi)\frac{\partial G_{ij}(\Phi)}{\partial\Phi_l}\right] + \frac{1}{2}\frac{\partial^2}{\partial\Phi_i\Phi_j}G_{il}(\Phi)G_{jl}(\Phi)$$

First passage time distribution

$$\mathcal{L}^{\dagger}_{FP}(\Phi) = F_i(\Phi)\frac{\partial}{\partial \Phi_i} + \alpha G_{il}(\Phi)\frac{\partial G_{lj}(\Phi)}{\partial \Phi_l}\frac{\partial}{\partial \Phi_i} + \frac{1}{2}G_{il}(\Phi)G_{jl}(\Phi)\frac{\partial^2}{\partial \Phi_i \Phi_j}$$

$$\frac{\partial}{\partial \mathcal{N}}P(\mathcal{N},\Phi) = \mathcal{L}^{\dagger}_{FP}(\Phi)\cdot P(\mathcal{N},\Phi)$$

$$\int d\Phi f_1(\Phi)\left[\mathcal{L}_{FP}(\Phi)\cdot f_2(\Phi)\right] = \int d\Phi \left[\mathcal{L}^{\dagger}_{FP}(\Phi)\cdot f_1(\Phi)\right] f_2(\Phi)$$



first passage time problem

Boundary conditions $\quad \partial\Omega = \partial\Omega_- \cup \partial\Omega_+$

$\partial\Omega_-$: all moments of the FPT vanish on $\partial\Omega_-$ (absorbing boundary)

Sometimes additional conditions required on $\partial\Omega_+$: absorbing or reflective boundary (gradients of moments projected onto the orthogonal direction to the tangent surface of $\partial\Omega_+$ vanish)

hierarchy of coupled differential equations: $\quad \mathcal{L}^{\dagger}_{FP}(\Phi^{in})\cdot\langle\mathcal{N}^n\rangle(\Phi^{in}) = -\,n\langle\mathcal{N}^{n-1}\rangle(\Phi^{in})$

Characteristic function

$\chi_{\mathcal{N}}(t,\Phi) \equiv \langle e^{it\mathcal{N}} \rangle$

Taylor expansion around $t = 0$: $\chi_{\mathcal{N}}(t,\Phi) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \langle \mathcal{N}^n(\Phi) \rangle$

Act with the Fokker-Planck operator: $\mathcal{L}^{\dagger}_{FP} \cdot \chi_{\mathcal{N}}(t,\Phi) = -it\chi_{\mathcal{N}}(t,\Phi)$ set of uncoupled differential equations

$\chi_{\mathcal{N}}$ is the Fourier transform of the first passage time distribution: $\chi_{\mathcal{N}}(t,\Phi) = \int_{-\infty}^{+\infty} e^{it\mathcal{N}} P(\mathcal{N},\Phi) d\mathcal{N}$

Using: $\zeta_{cg} = \delta N_{cg} = \mathcal{N} - \langle \mathcal{N} \rangle$: $P(\zeta_{cg},\Phi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\left[\zeta_{cg} + \langle \mathcal{N} \rangle(\Phi)\right]} \chi_{\mathcal{N}}(t,\Phi)\, dt$

$\langle \mathcal{N} \rangle(\Phi) = -i \frac{\partial \chi_{\mathcal{N}}(t,\Phi)}{\partial t} \Big|_{t=0}$

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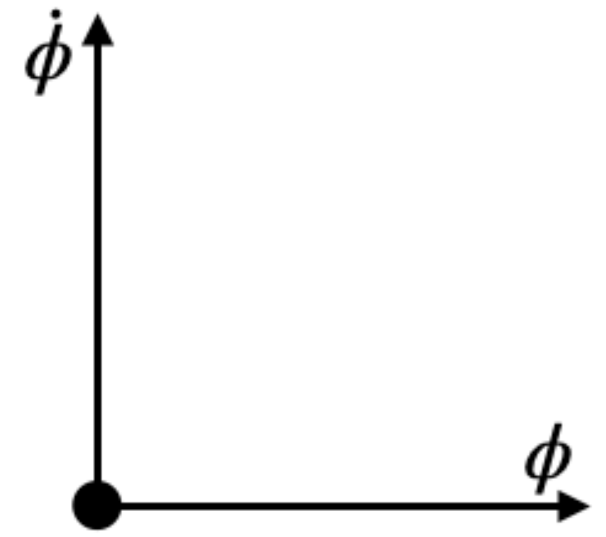

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