

$$\beta^{lin,shallow} \simeq \frac{4}{\pi} \left[ 1 + \left( \frac{8}{\pi^2} - \frac{\pi^2}{12} \right) a \right] e^{-\frac{\pi^2}{8} - \left( \frac{\pi^4}{4} - 2a \right) \frac{\zeta_c}{\mu^2}}$$