

$$\beta^{quad,shallow} \simeq \frac{4}{\pi} \left[1 + \left(\frac{32}{\pi^3} + \frac{4}{\pi} - \frac{5\pi^2}{48} - 1 \right) a \right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - \frac{8}{\pi} a \right) \frac{\zeta_c}{\mu^2}}$$