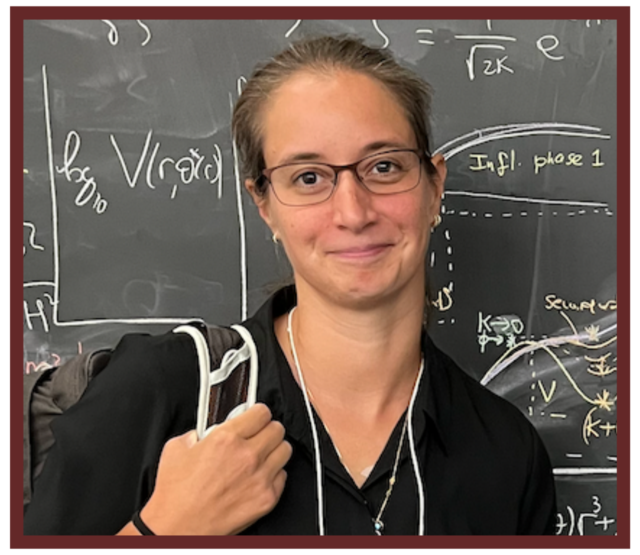
Planck Constraints and Gravitational Wave Forecasts for PBH Dark Matter seeded by Multifield Inflation *

Sarah Geller, *Center for Theoretical Physics, MIT*

New Horizons in Primordial Black Hole Physics (NEHOP) June 2023

The Multifield Inflation/PBH research group:

Shyam Balaji,
CNRS &LPTHE



Sarah Geller, **MIT**

David Kaiser, **MIT**

Evan McDonough
University of Winnipeg

Wenzer Qin, **MIT**

*(See: 2205.04471, 2303.02168)

The Big Picture

Larger question: What is dark matter *made of?*

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Idea: Dark matter could be made up of primordial black holes (PBHs) !

PBHs form in the very early universe ($\lesssim \mathcal{O}(1)$ second) from direct collapse of overdensities.

Primordial Black Holes as Dark Matter

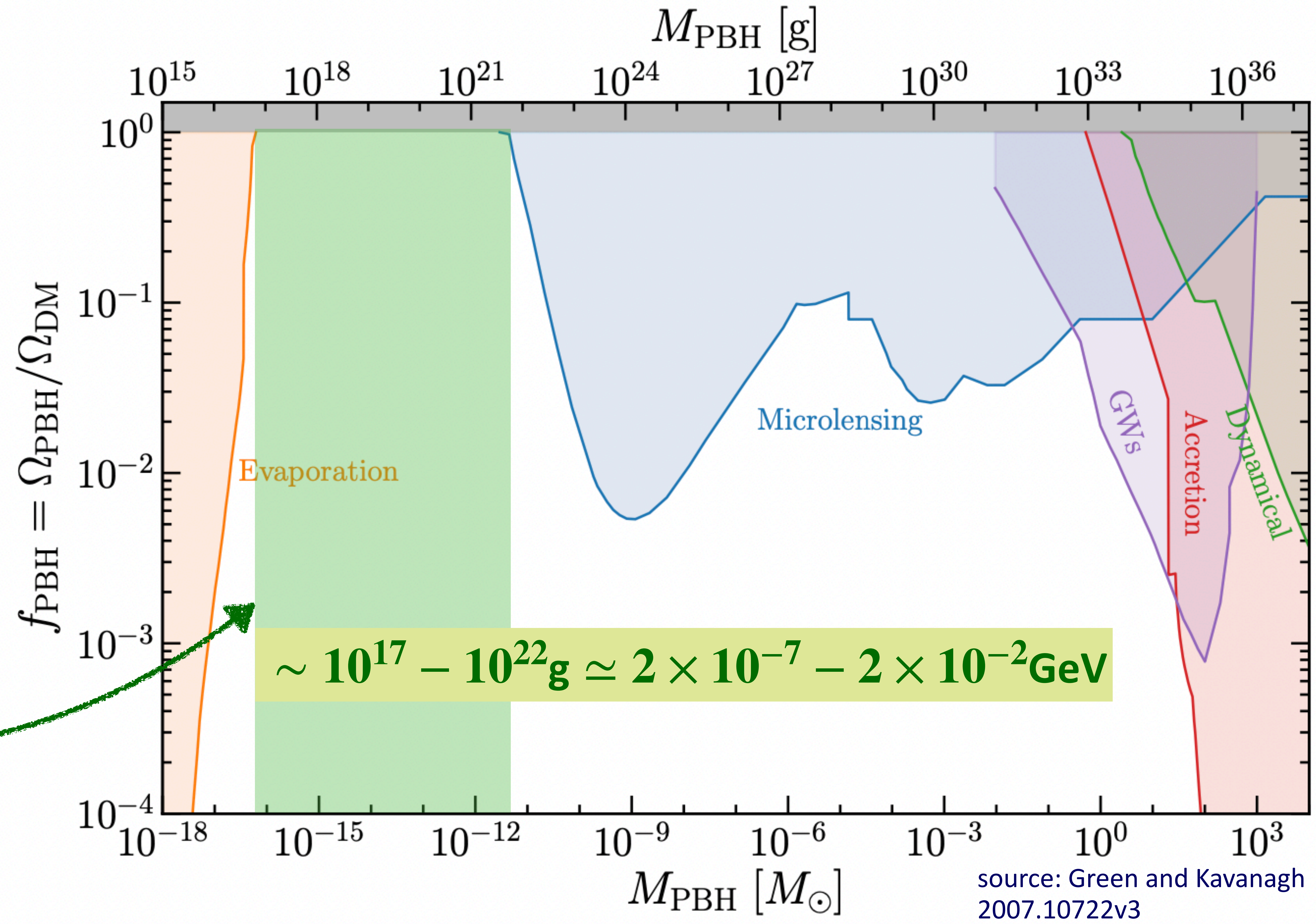
Non-interacting to good approximation

Massive Compact Halo Objects (MACHOs)

Wide range of possible PBH masses allowed from collapse of primordial over-densities

Avoid need to posit one or more BSM fields (aside from inflaton)

Parameter space has been heavily constrained by multiple experiments and calculations!



source: Green and Kavanagh 2007.10722v3

PBHs in this mass range could constitute $\mathcal{O}(1)$ fraction of Dark Matter

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Larger question: What is dark matter *made of?*

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Problem:

There are both observational and theoretical constraints on the mass ranges of PBHs that could account for any sizable fraction of the dark matter.

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Our Specific Questions:

1. Are primordial black holes a generic prediction of inflationary models?

2. What is the predicted gravitational wave (GW) spectrum from this PBH production and is it observable with current or forthcoming detectors?

Primordial Black Holes from Critical Collapse

Curvature perturbations decompose into modes with freq. k

↓

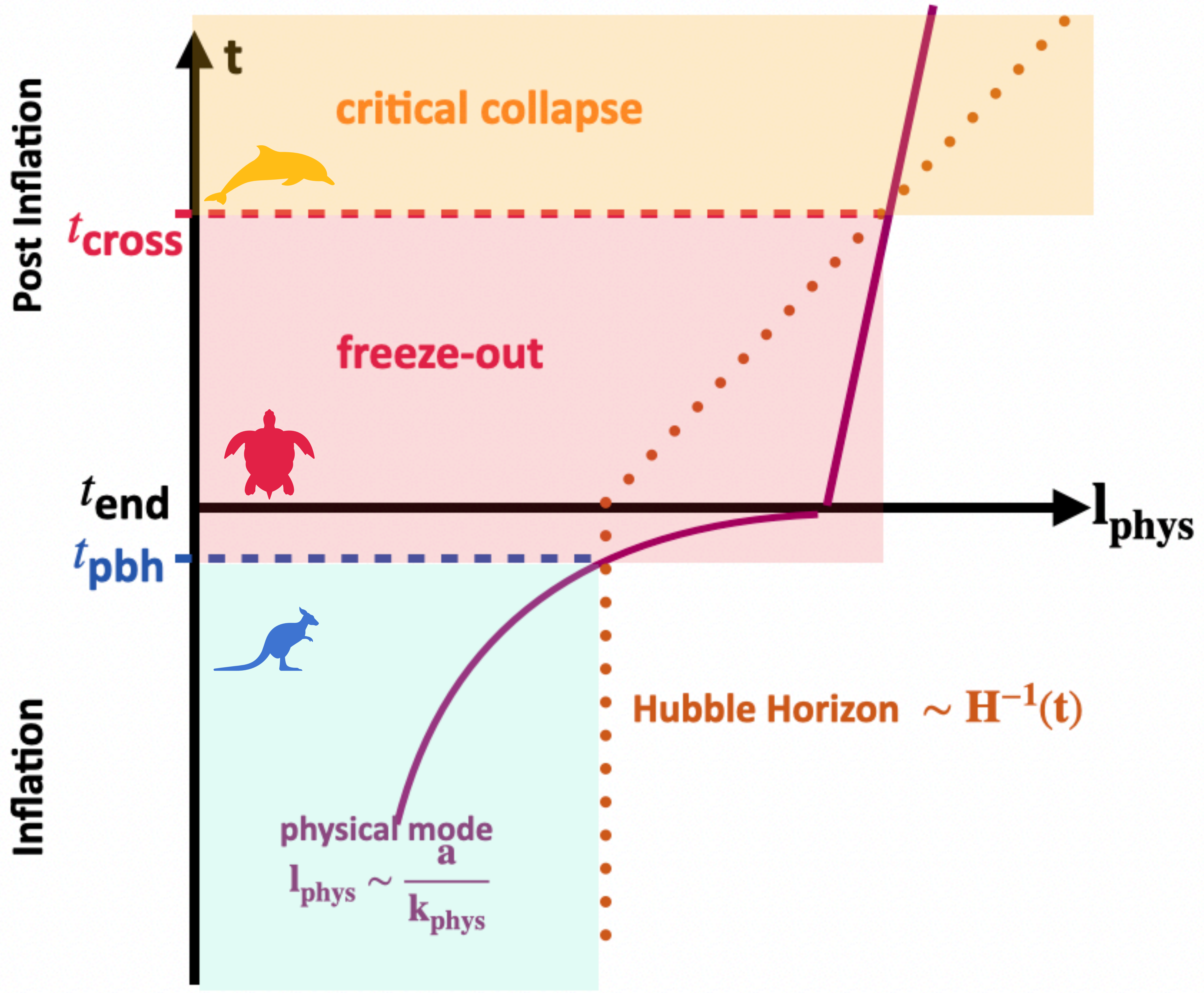
Cross outside Hubble horizon before end of inflation k<aH ("Super-Hubble")

↓

"freeze out"

↓

Cross back into Hubble patch when k=aH
k>aH "Sub-Hubble"



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"freeze out"

Cross back into Hubble patch when k=aH k>aH "Sub-Hubble"

For $\delta = \frac{\delta\rho}{\rho} > \delta_c$, PBH will form at time t_c for mode with wavenumber

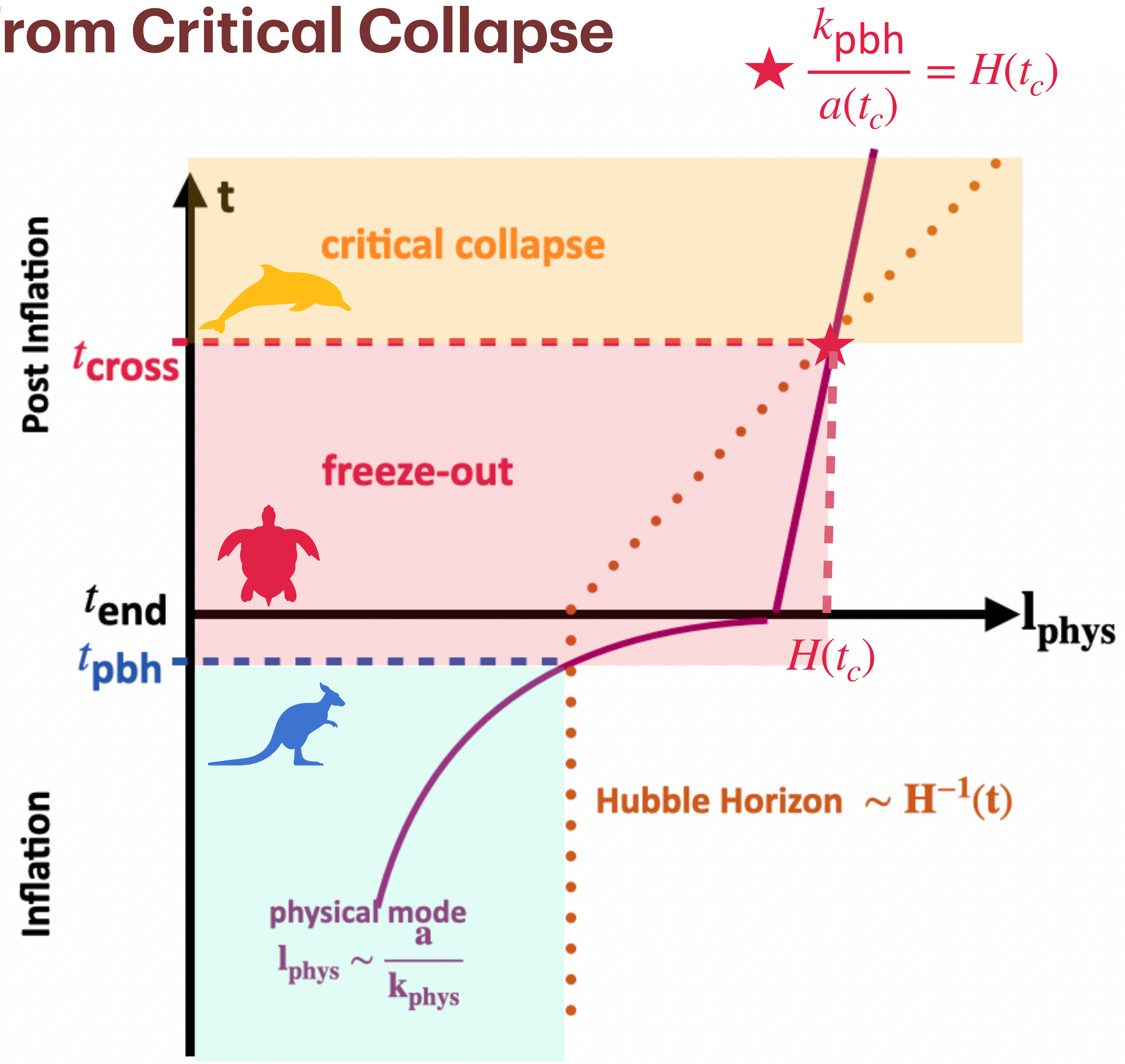
$k_{PBH} = a(t_c)H(t_c)$

Corresponds to threshold for

$\mathscr{P}_R(k_{PBH}) \geq 10^{-3}$

Mass distribution centered around $\bar{M} = \gamma M_H(t_c)$, $\gamma \sim .2$

$M_H(t_c) \equiv$ mass within Hubble volume at t_c



Model and Methods

Model: A generic inflationary potential with multiple (2) scalar fields and non-minimal couplings to gravity.

Multifield action

$$\tilde{S} = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[f\left(\phi^I\right)\tilde{R} - \frac{1}{2}\delta_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}\left(\phi^I\right)\right]$$

Non-minimal coupling

$$f(\phi^I) = \frac{1}{2}\left[M_{\mathrm{pl}}^2 + \sum_{I=1}^{N}\xi_I(\phi^I(x^\mu))^2\right]$$

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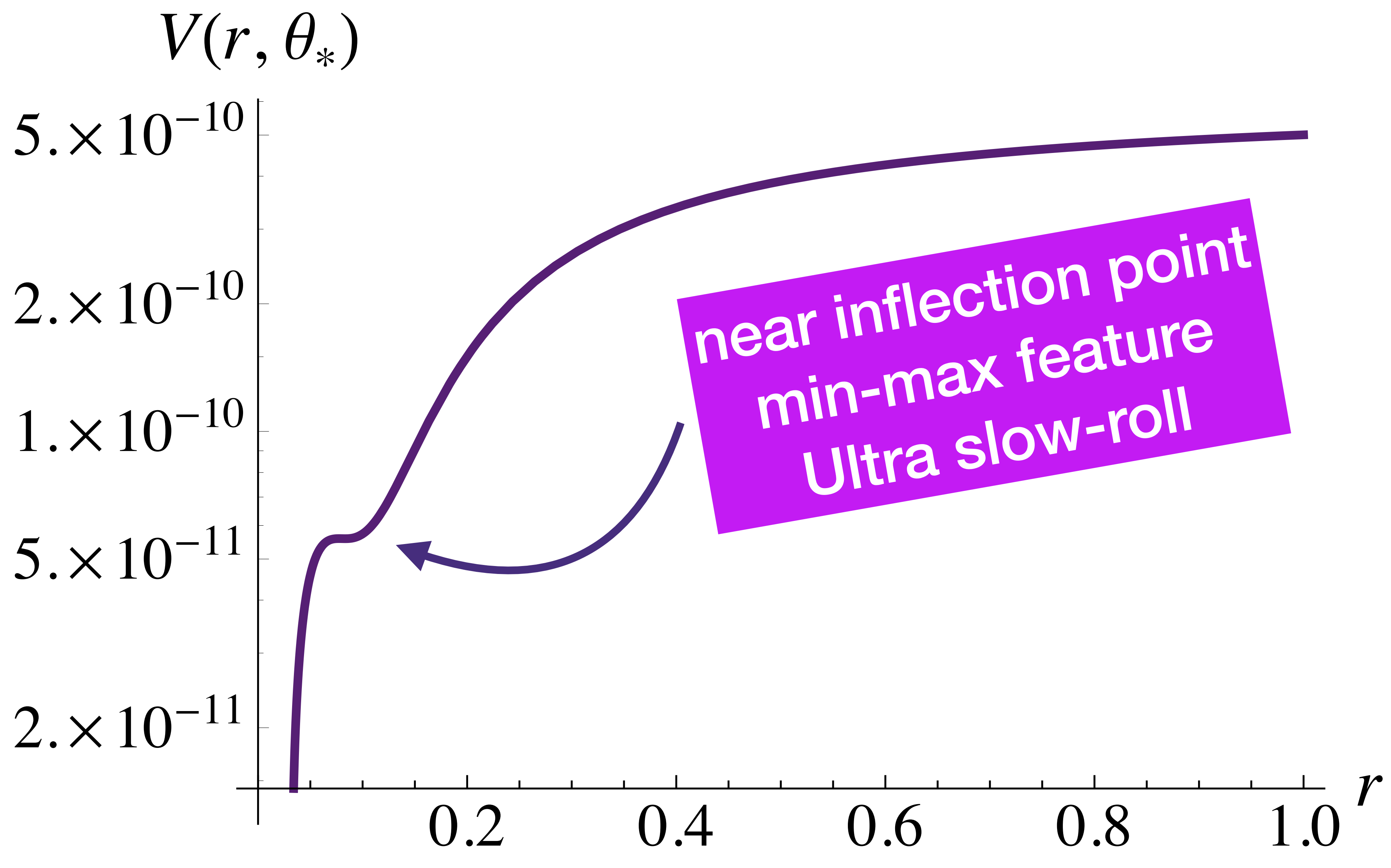
Impose a few additional symmetries to limit number of degrees of freedom in field space.

Potential is characterized by functions $\mathscr{B}, \mathscr{C}, \mathscr{D}$ depending on 5 parameters: ξ, b, c_1, c_2, c_4

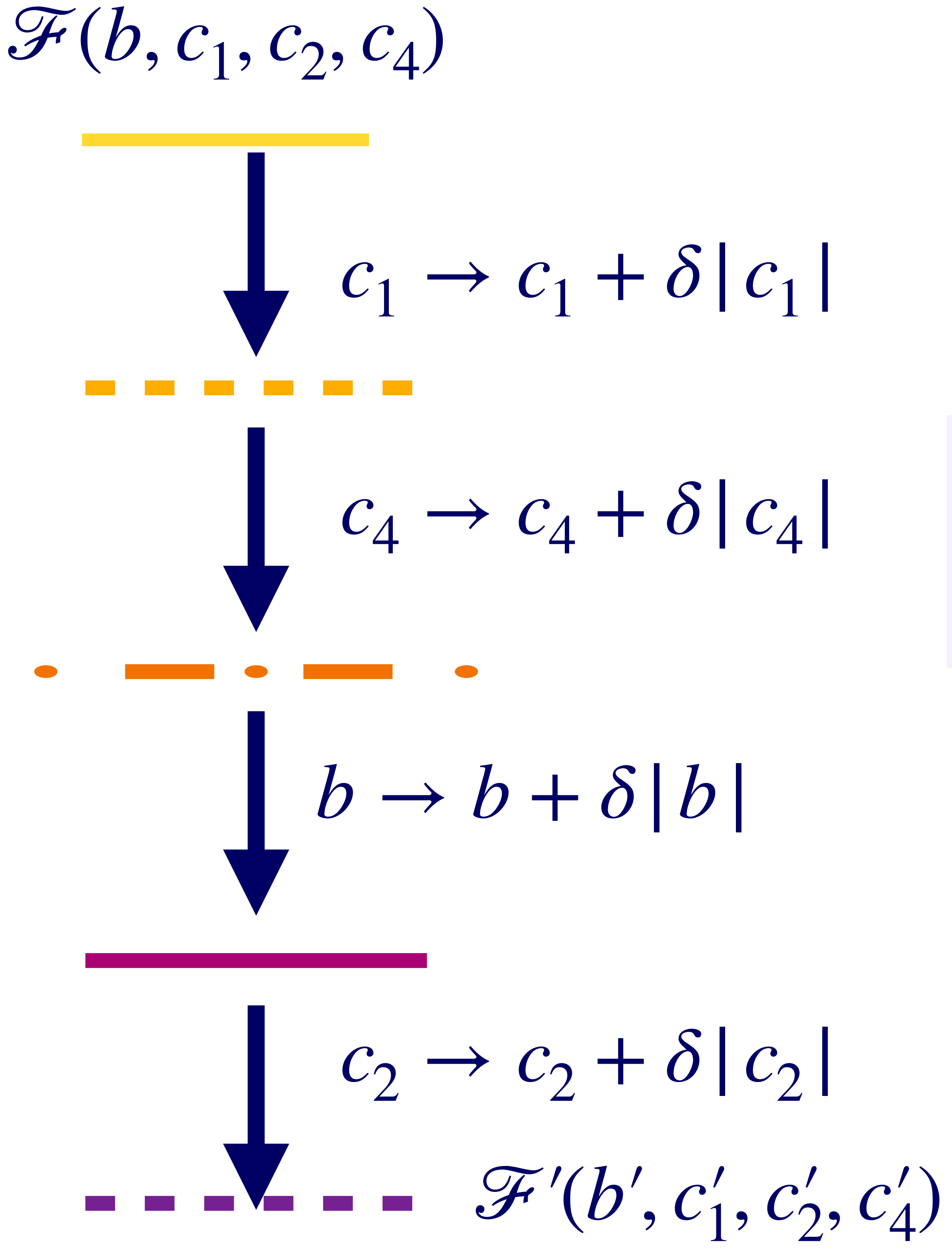
$$V(r,\theta) = \frac{1}{4f^2(r,\theta)}\left(\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right)$$



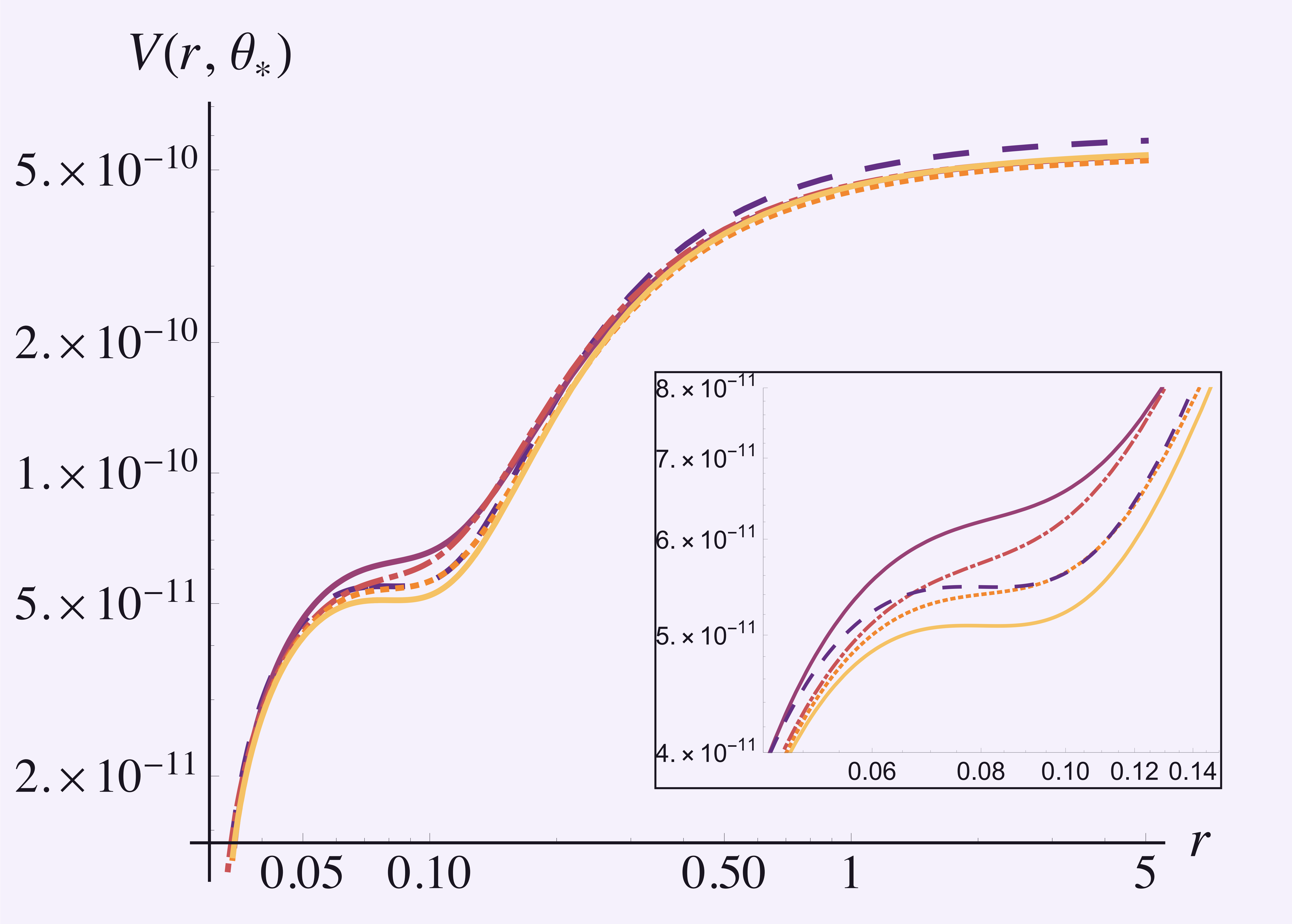
Parameter Space Degeneracy Directions

Interplay of parameters leads to degeneracies

Vary one parameter at time,
get back to a self-similar potential with
 different values of the parameters:

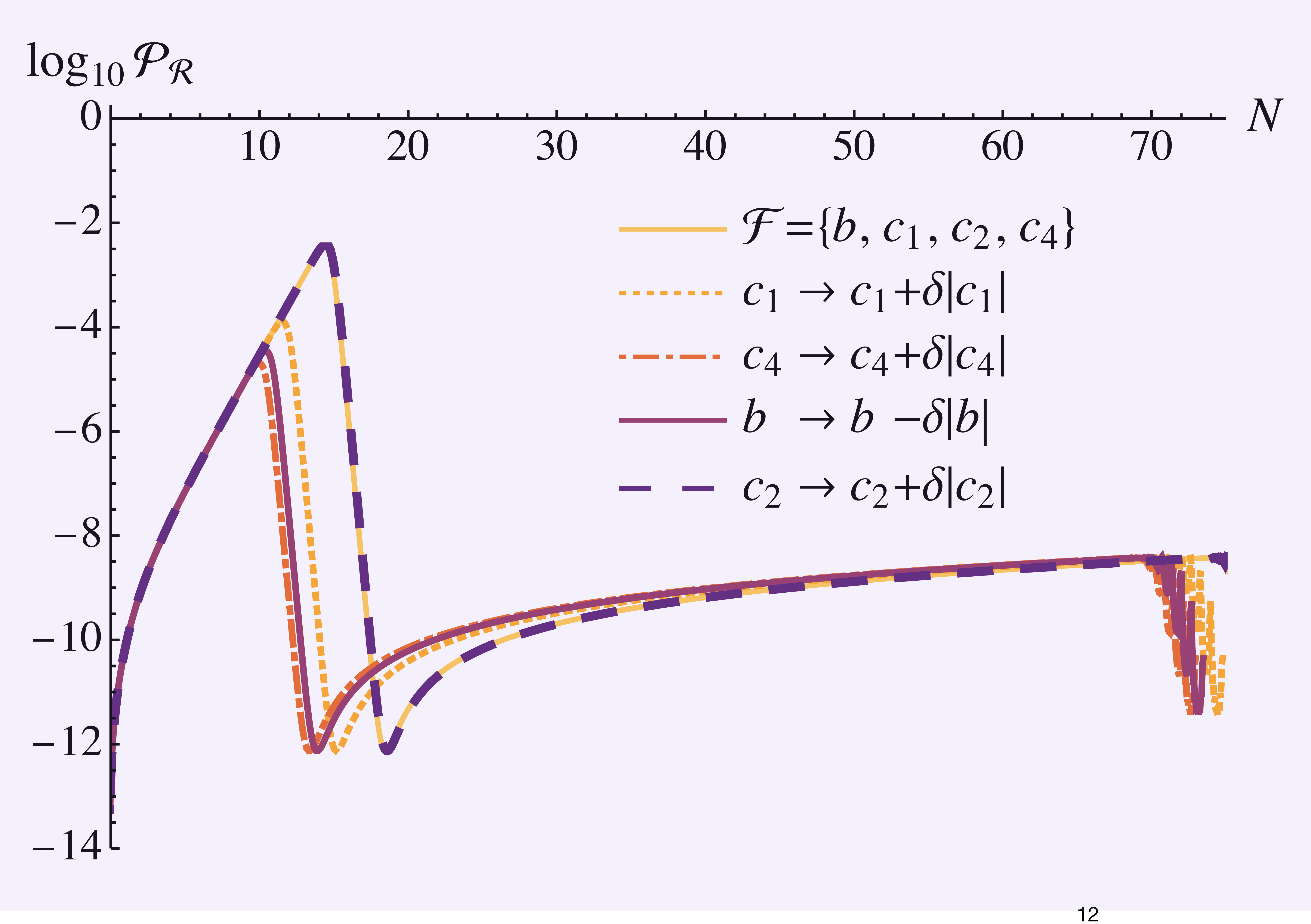


$\mathscr{F} \simeq \mathscr{F}'$



Parameter Space Degeneracy Directions

Why does it matter to find degenerate regions of parameter space?
Essentially: helps to answer question of how generic a feature DM PBHs are in these models



Degeneracy is a statement about observables, determined by the power spectrum

Define as degenerate if total $\Delta\chi^2 < .01$

Multifield models allow for degeneracies- less well-constrained.

Degeneracies aren't perfect: have finite extent—so they do impact the likelihoods.

Methods: MCMC simulations

(Degenerate $\equiv$ total $\Delta\chi^2$<.01)

(Sky)walkers conduct random walks to map degenerate regions of the ~~parameter space~~ Tatooine.

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Compare with Planck 2018 CMB temperature & polarization data and constraints on PBHs as Dark Matter.

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CMB constraints at pivot scale, k_* . Assume a ***Gaussian likelihood*** over Planck/BICEP/Keck $A_s(k_*), n_s(k_*), \alpha(k_*), r(k_*)$

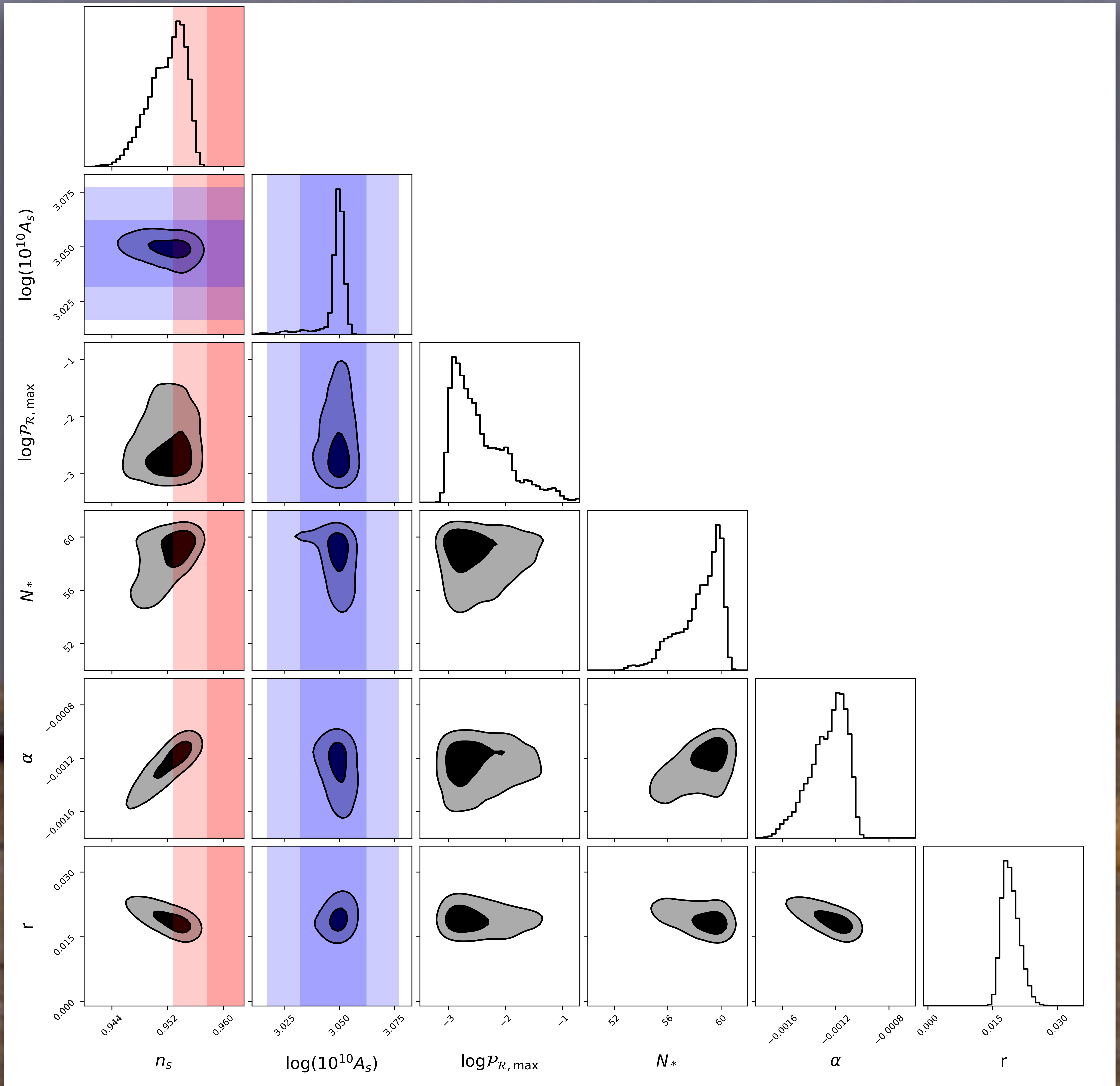
PBH constraints at Hubble crossing during USR: $\mathscr{P}_R(k_{PBH}), \Delta N$. Assume uniform likelihood for $\mathscr{P}_R(k_{\text{PBH}}) \geq 10^{-3}$, $14 \leq \Delta N \leq 25$

Methods: MCMC simulations

(Degenerate $\equiv$ total $\Delta\chi^2$<.01)

Constraints from requiring PBH DM and satisfying *Planck* 2018 data

Parameter	Constraint
$\boldsymbol{b}$	$-1.87\,(-1.73)^{+0.09}_{-0.11}\times 10^{-4}$
$\boldsymbol{c_1}$	$2.61\,(2.34)^{+0.24}_{-0.17}\times 10^{-4}$
$\boldsymbol{c_2}$	$3.69\,(3.42)^{+0.22}_{-0.16}\times 10^{-3}$
$\boldsymbol{c_4}$	$4.03\,(3.75)^{+0.24}_{-0.17}\times 10^{-3}$
$n_s(k_*)$	$0.952\,(0.956)^{+0.002}_{-0.003}$
$\log(10^{10}A_s)$	$3.049\,(3.048)^{+0.001}_{-0.001}$
N_*	$58.8\,(60.0)^{+1.2}_{-2.2}$
$\alpha(k_*)$	$-0.0012\,(-0.0010)^{+0.0001}_{-0.0002}$
$r(k_*)$	$0.019\,(0.016)^{+0.002}_{-0.001}$
b/c_2	$-5.04(-5.05)^{+0.03}_{-0.05}\times 10^{-2}$
c_1/c_2	$7.07(6.84)^{+0.32}_{-0.26}\times 10^{-2}$
c_4/c_2	$1.091(1.096)^{+0.009}_{-0.008}$



Gravitational Wave Forecasts from PBH formation

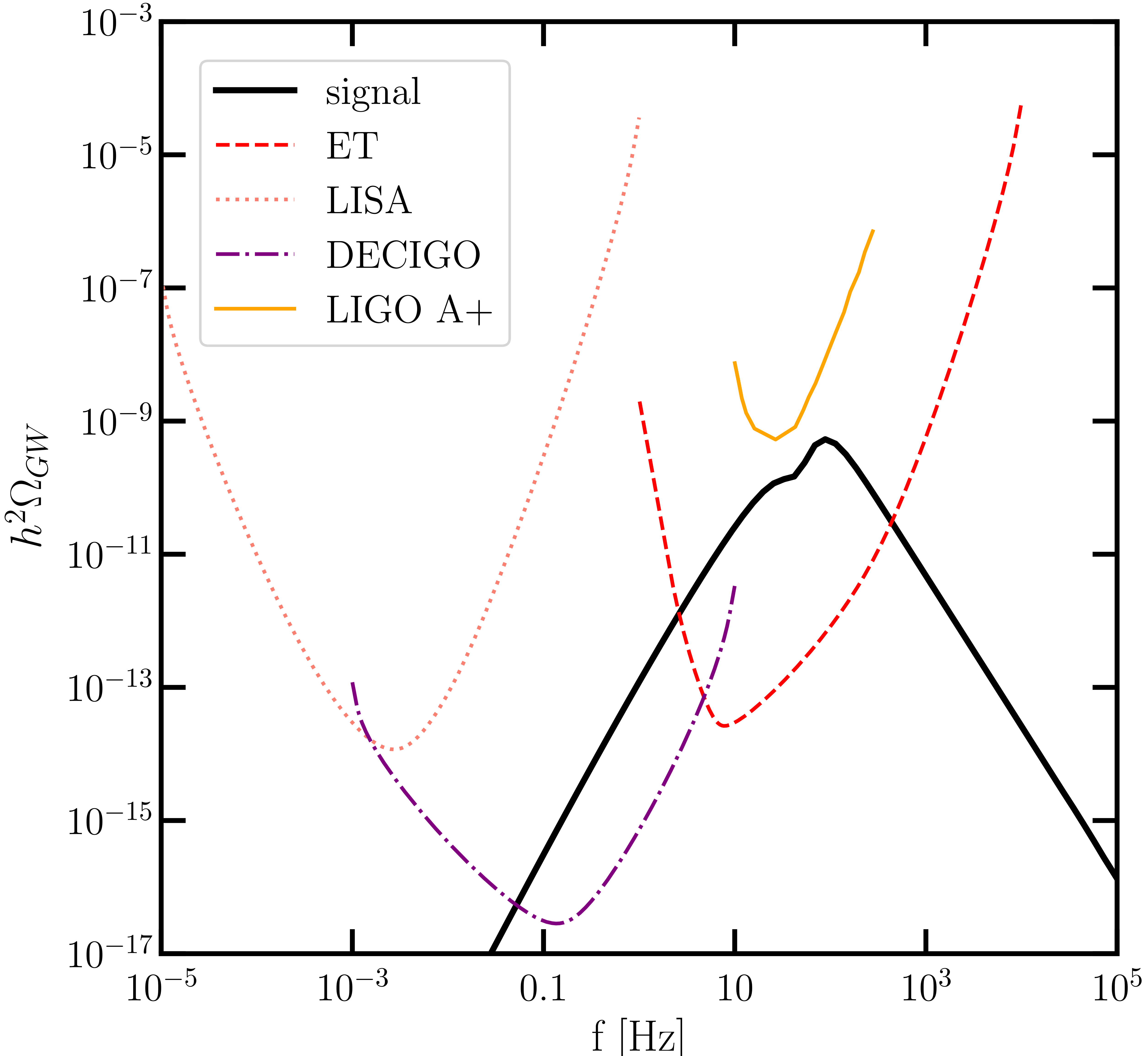
Scalar mode perturbations that give rise to PBHs will contribute to the GW spectrum at **second order**

Gravitational waves induced by linear scalar modes at second order have dimensionless spectral density today:

$$\Omega_{GW,0}h^2 \approx 1.62\times 10^{-5}\left(\frac{1}{24}\left(\frac{k}{aH}\right)^2\overline{P_h(k,\eta)}\right)$$

$\xi = 100, b = -1.8\times 10^{-4}, c1 = 2.5\times 10^{-4},$
$c_2 = 3.570913\times 10^{-3}, c_4 = 3.9\times 10^{-3}$

Spectral Density v Power Integrated Sensitivity



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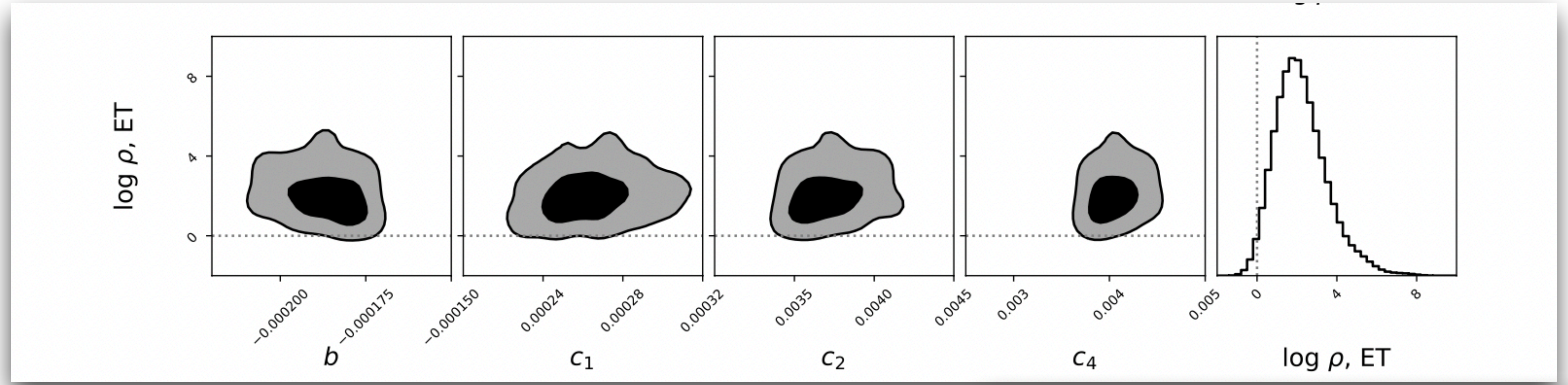
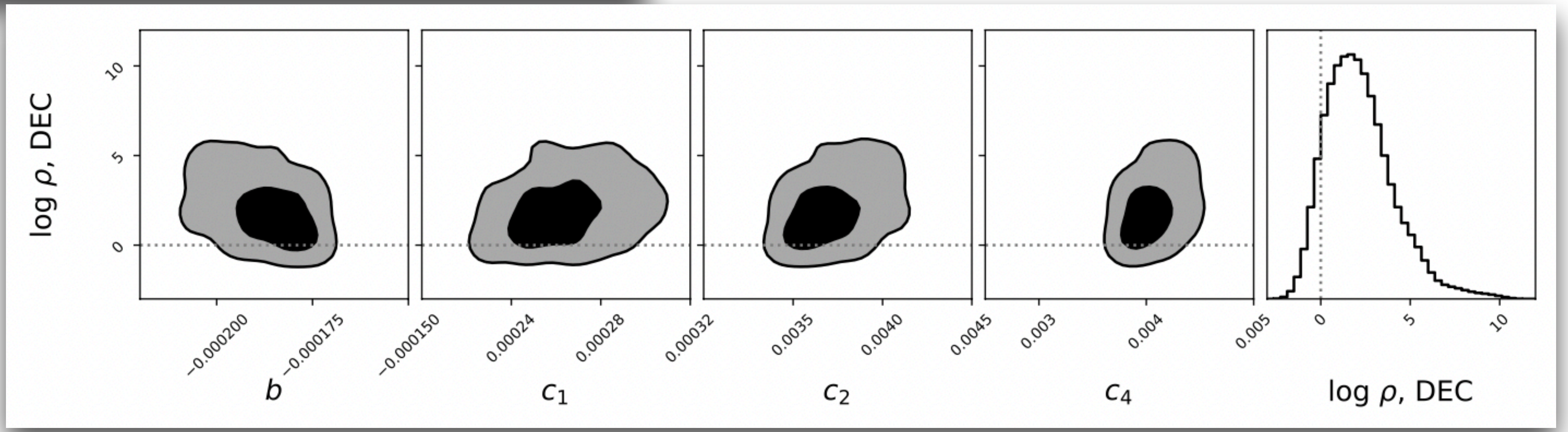




$\xi = 100, b = -1.8 \times 10^{-4}, c1 = 2.5 \times 10^{-4},$
$c_2 = 3.570913 \times 10^{-3}, c_4 = 3.9 \times 10^{-3}$

Conclusions: What can we say about how likely PBHs in DM range are in these models?

Beginning with a generic multifield inflation model and allowing for non-minimal gravitational couplings, **we find a robust region of the parameter space in our model that is compatible with *Planck* data and can produce PBHs in the light asteroid-mass range.**

Constraints on allowed regions in parameter space are driven mostly by fitting n_s (Gaussian tail on one end of posterior distribution) and N_* (sharp cutoff at other end) .

Parameters of model are constrained at $\approx 10\,\%$ but degeneracy direction leads to fine-tuned ratios of parameters at percent level.

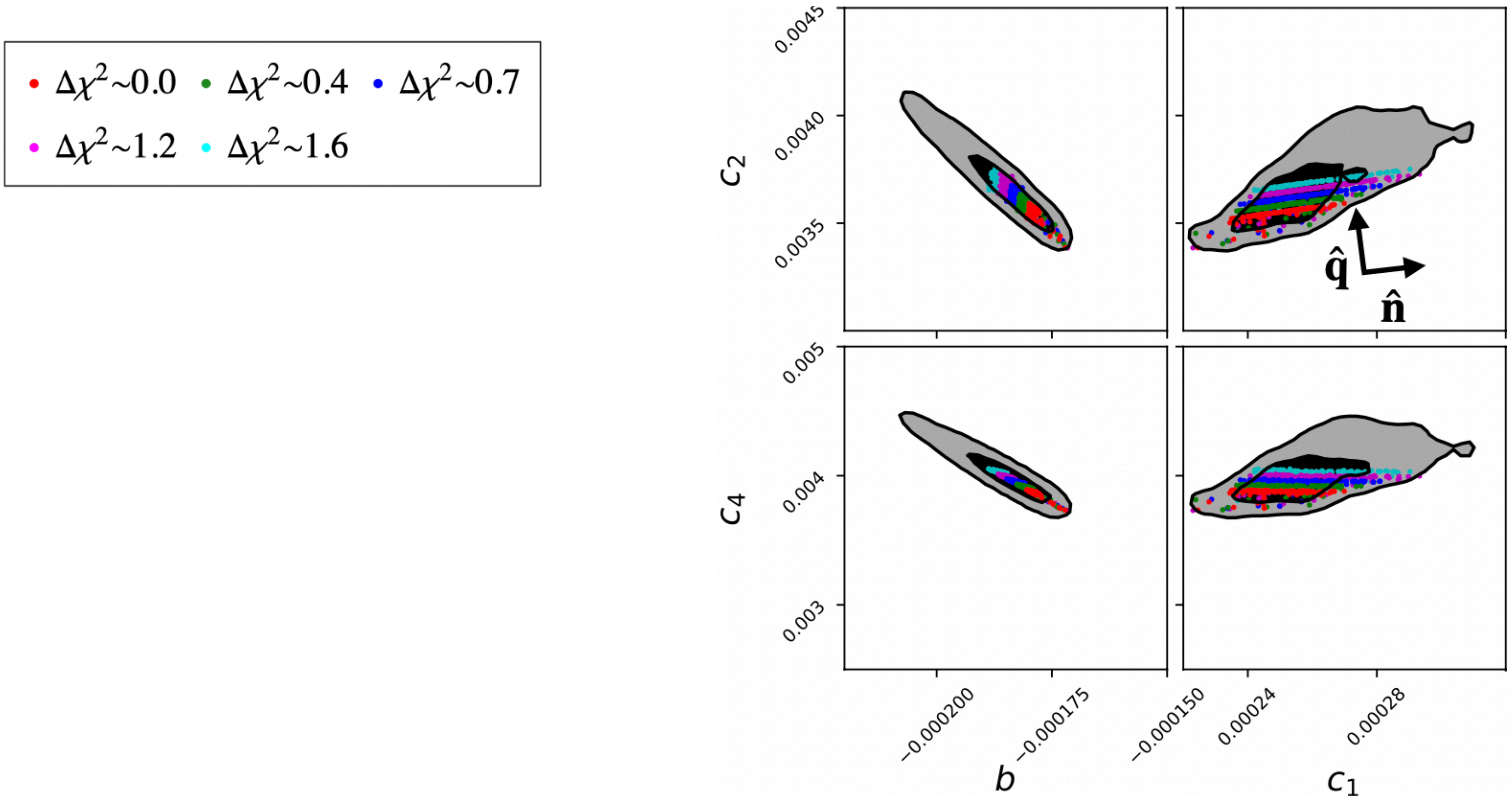
Most constraining quantity is n_s with error bars at $\sim 1\,\%$ thus relative fine-tuning of parameters to match both PBH and CMB constraints is $\mathcal{O}\left(10^{-5}\right)$.

Questions and Answers

1.how does the conformal transformation on field space actually work?
2. what are other possible/removed constraints on the parameter space of \omega_{PBH DM
3. what is the UV (SUGRA) embedding for this class of models? how is the EFT derived?
4. Does inflation itself require fine tuning of initial conditions?
5. more about the motivation for non-minimal couplings.
6. when is quantum diffusion a problem during USR?
7. say more about reheating in MFI models?
8.what are the non-Gaussianities in your models like? what is f_{NL}?
9.how many observables vs dof do you have (ignoring the GWs for the moment?)
10. Did you marginalize over the reheating histories? How do you fit N_{*}?

Parameter Space Orthogonal Directions

5 super-sets: same color = degeneracy direction, changing colors= orthogonal direction



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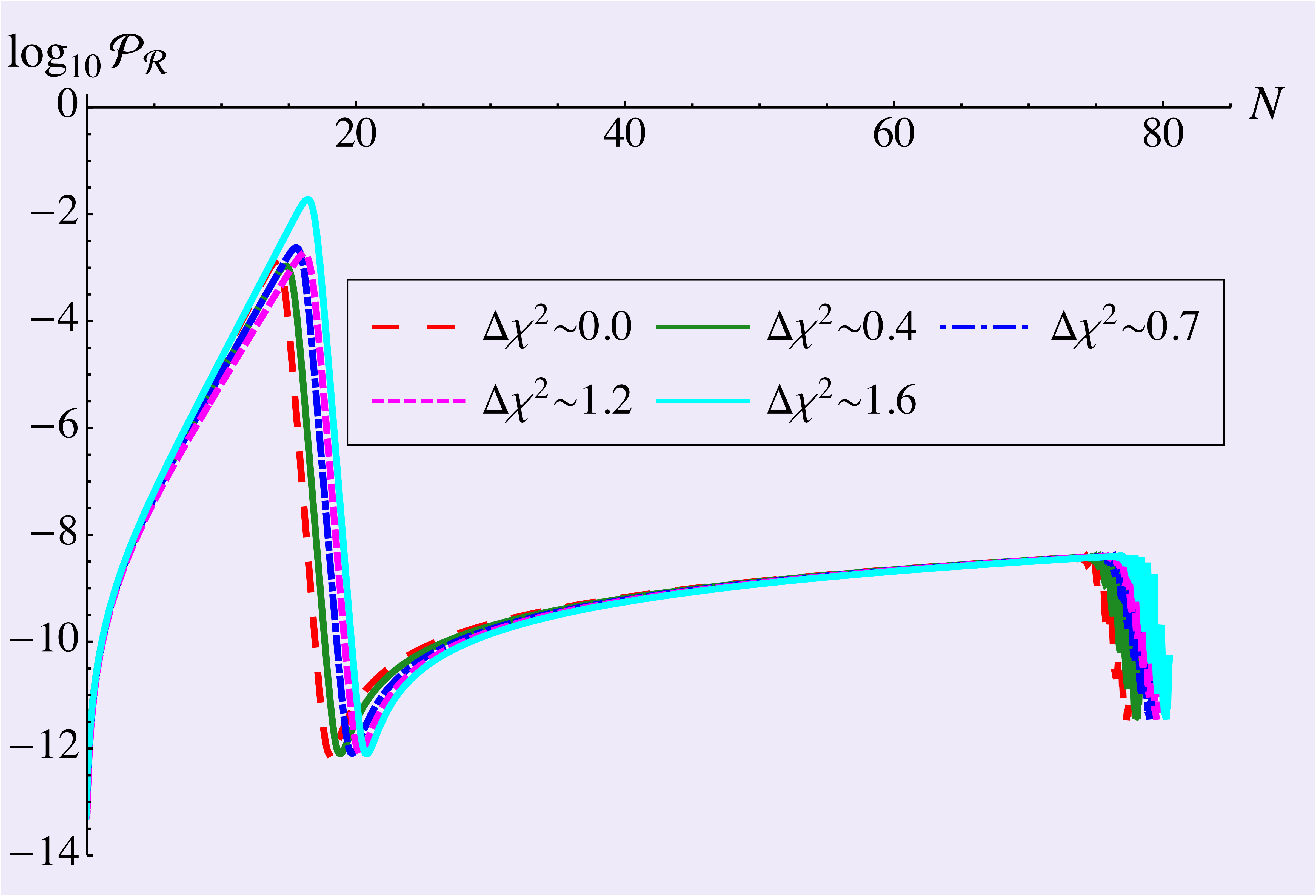
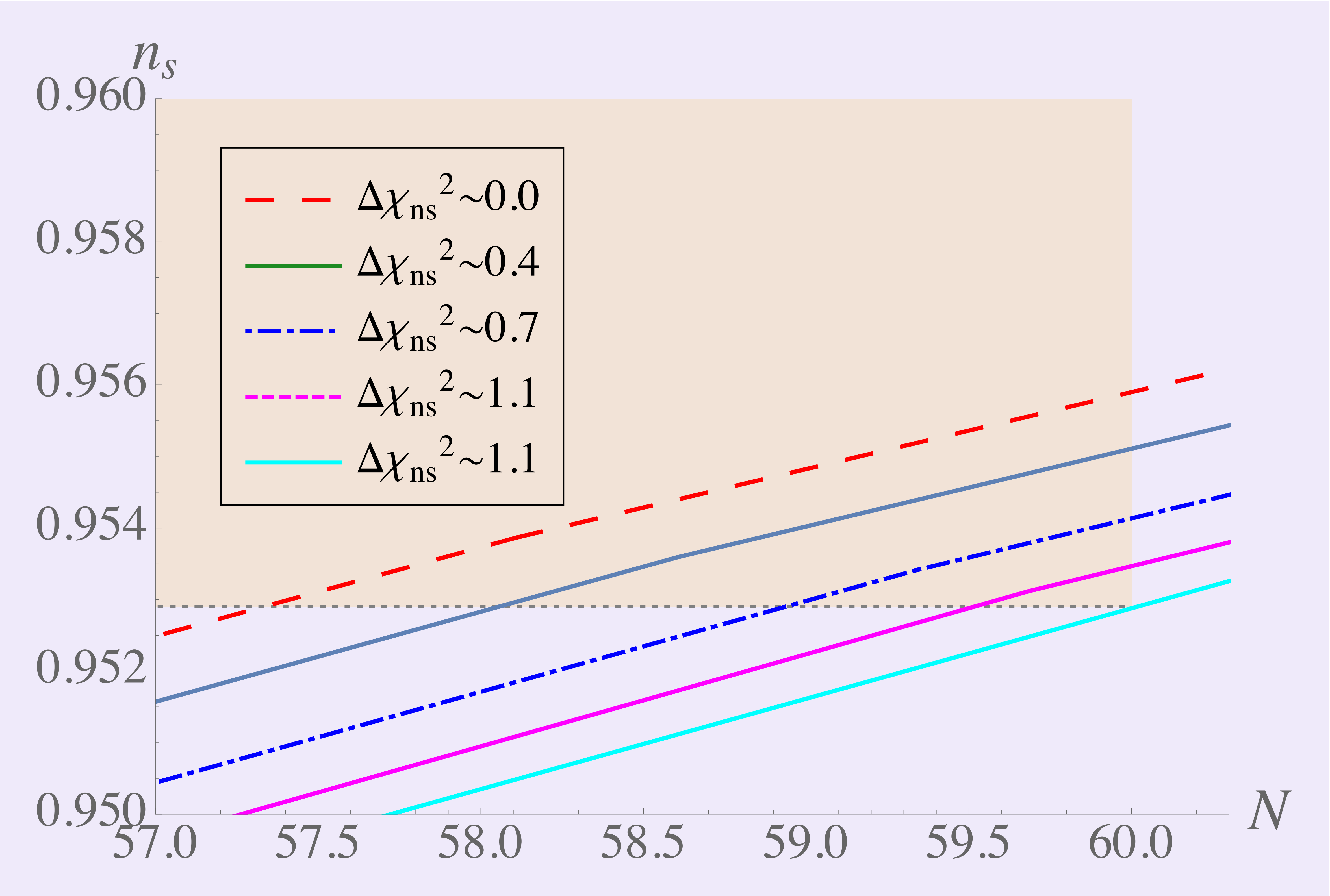




The D-dim Conformal Transformation from Jordan Frame → Einstein Frame (1)

Jordan Frame Action: $\tilde{S} = \int \mathrm{d}^D x \sqrt{-\tilde{g}} \left[f(\phi^1 \ldots \phi^N)\tilde{R} - \frac{1}{2}\delta_{IJ}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu \phi^I \tilde{\nabla}_\nu \phi^J - \tilde{V}(\phi^1 \ldots \phi^N) \right]$, $\quad f(\phi) = \frac{1}{2}\left[M_0^{D-2} + \xi_I (\phi^I)^2\right]$

Conformal transformation: $\boxed{\mathbf{\Omega^{D-2}(x) = \frac{2}{M_{(D)}^{D-2}} f[\phi(x)]}}$ Transforms metric as: $\tilde{g}_{\mu\nu} \to g_{\mu\nu} = \Omega^2(x) g_{\mu\nu} \implies$

$g^{\mu\nu} = \Omega^{-2}\tilde{g}^{\mu\nu}$ and $\sqrt{-g} = \Omega^D(x)\sqrt{-\tilde{g}}$

(in our 2-field model, $M_{(2)} = M_{\mathrm{pl}}$)

$$\Gamma^a_{bc} = \tilde{\Gamma}^a_{bc} + \frac{1}{\Omega}\left[\delta^a_b \nabla_c \Omega + \delta^a_c \nabla_b \Omega - g_{bc}\nabla^a \Omega\right], \qquad \Box\Omega = \frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\, g^{\mu\nu}\partial_\nu \Omega\right] \qquad R = \frac{1}{\Omega}\left[\tilde{R} - \frac{2(D-1)}{\Omega}\Box\Omega - (D-1)(D-4)\frac{1}{\Omega^2} g^{\mu\nu}\nabla_\mu \Omega \nabla_\nu \Omega\right]$$

combine to form $\mathcal{G}_{IJ}$

Einstein Frame Action:

E-H term: $\int \mathrm{d}^D x \sqrt{-g}\left[\boxed{\frac{M_{(D)}^{D-2}}{2} R} - \frac{1}{2}\frac{D-1}{D-2} M_{(D)}^{D-2} \frac{1}{f^2} g^{\mu\nu}\nabla_\mu f \nabla_\nu f\right]$

Kinetic terms: $\int \mathrm{d}^D x \sqrt{-g}\left[-\frac{1}{4f} M_{(D)}^{D-2}\delta_{IJ} g^{\mu\nu}\nabla_\mu \phi^I \nabla_\nu \phi^J\right]$

$V(\phi^I) = \frac{\tilde{V}(\phi^I)}{\Omega^D}$

Kaiser 1003.1159v2

The Conformal Transformation from Jordan Frame → Einstein Frame (2)

Jordan Frame Action: $\tilde{S} = \int \mathrm{d}^D x \sqrt{-g} \left[f(\phi^1 \ldots \phi^N) \tilde{R} - \frac{1}{2} \delta_{IJ} g^{\mu\nu} \nabla_\mu \phi^I \nabla_\nu \phi^J - \tilde{V}(\phi^1 \ldots \phi^N) \right]$, $\quad f(\phi) = \frac{1}{2}\left[M_0^{D-2} + \xi_I (\phi^I)^2 \right]$

$\mathcal{G}_{IJ}$ can be put in form δ_{IJ} only if R^I_{JKL} (field space Riemann tensor) vanishes identically

To show that a $\mathcal{G}_{IJ}$ can't be put in form δ_{IJ} it suffices to show that the Einstein frame Ricci scalar is nonzero: $R \neq 0$

By computing the Ricci scalar, can show every term in it depends on ϕ^I and the Riemann tensor would have to vanish everywhere in field space (OR can happen if only one of the fields is non-minimally coupled, but then potential gets new interactions).

Kaiser 1003.1159v2

PBHs as Dark Matter: The Available Parameter Space

Constraints from Femto-lensing?

A Gould (1992) proposed gamma-ray bursts could be used to constrain PBHs in the range $10^{17} \sim 10^{20}$ g via interference fringes. Later work (Katz et al.) showed constraints should be discounted because 1. gamma ray bursts too large for point sources and 2. need to consider wave optics
(Source: Green and Kavanagh 2020)

Subaru HSC Constraints?

"High cadence optical observation of M31 constraints…are weaker than initially found due to finite sources and wave optics effects."
(Source: Green and Kavanagh 2020)

SUGRA and SUSY Background of Inflaton Potential (1)

Start with $\mathcal{N} = 1$ 4-dimensional supergravity with 2 chiral superfields

$$\boldsymbol{\Phi}(y)^I = \underbrace{\Phi(y)}_{\text{complex scalar field}} + \sqrt{2}\underbrace{\theta\psi(y)}_{\text{fermion}} + \underbrace{\theta\theta F(y)}_{\text{auxiliary field}}$$

One next integrates out the auxiliary fields, get the Lagrangian we started with: $\mathscr{L} = \mathscr{G}_{IJ} g^{\mu\nu}\partial_\mu \Phi^I \partial_\nu \bar{\Phi}^{\bar{J}} - V(\Phi, \bar{\Phi})$ With a generic choice of superpotential (linear terms dropped - unless Φ^I is gauge singlet.)

$$\tilde{W} = \mu b_{IJ}\Phi^I\Phi^J + c_{IJK}\Phi_I\Phi_J\Phi_K + \mathcal{O}\left(\frac{\Phi_I^4}{M_{\text{pl}}}\right)$$

$$= b_1(\Phi_1)^2 + b_2(\Phi_2)^2 + c_1(\Phi_1)^3 + c_2(\Phi_1)^2\Phi_2 + c_3\Phi_1(\Phi_2)^2 + c_4(\Phi_2)^3 + \mathcal{O}\left(\frac{\Phi_I^4}{M_{\text{pl}}}\right)$$

In (local) SUGRA we also choose a Kähler potential (such that imaginary part of Φ^I remains heavy/decoupled)

$$K(\Phi, \bar{\Phi}) = \sum_{I,J} (\Phi^I - \bar{\Phi}^I)^2$$

The potential for the scalar field part of $W(\boldsymbol{\Phi}, \bar{\boldsymbol{\Phi}})$ is:

$$V(\Phi, \bar{\Phi}) = \exp\left(\frac{K(\Phi, \bar{\Phi})}{M_{\text{pl}}^2}\right)\left(\mathscr{G}^{I\bar{J}}\nabla_I W(\Phi)\nabla_{\bar{J}}\bar{W}(\bar{\Phi}) - \frac{3}{M_{\text{pl}}^2}W(\Phi)\bar{W}(\bar{\Phi})\right)$$

where $\nabla_I = \partial_I + \frac{1}{M_{\text{pl}}^2}K_{,I}$

(McDonough,Long,Kolb), (Linde),(Bertolami, Ross)

SUGRA and SUSY Background of Inflaton Potential (2)

$$V(\Phi, \bar{\Phi}) = \exp\left(\frac{K(\Phi, \bar{\Phi})}{M_{\text{pl}}^2}\right)\left(\mathcal{G}^{I\bar{J}}\nabla_I W(\Phi)\nabla_{\bar{J}}\bar{W}(\bar{\Phi}) - \frac{3}{M_{\text{pl}}^2}W(\Phi)\bar{W}(\bar{\Phi})\right) \quad \text{where} \quad \nabla_I = \partial_I + \frac{1}{M_{\text{pl}}^2}K_{,I}$$

Take the limit of $V(\Phi, \bar{\Phi})$ as $\frac{|\Phi^I|^2}{M_{\text{pl}}^2} \to 0$ to get the expression for $V(\phi)$. The ψ dependence drops out because of the choice of Kähler potential which makes the imaginary part of the complex scalar field heavy- it decouples for all of inflation.

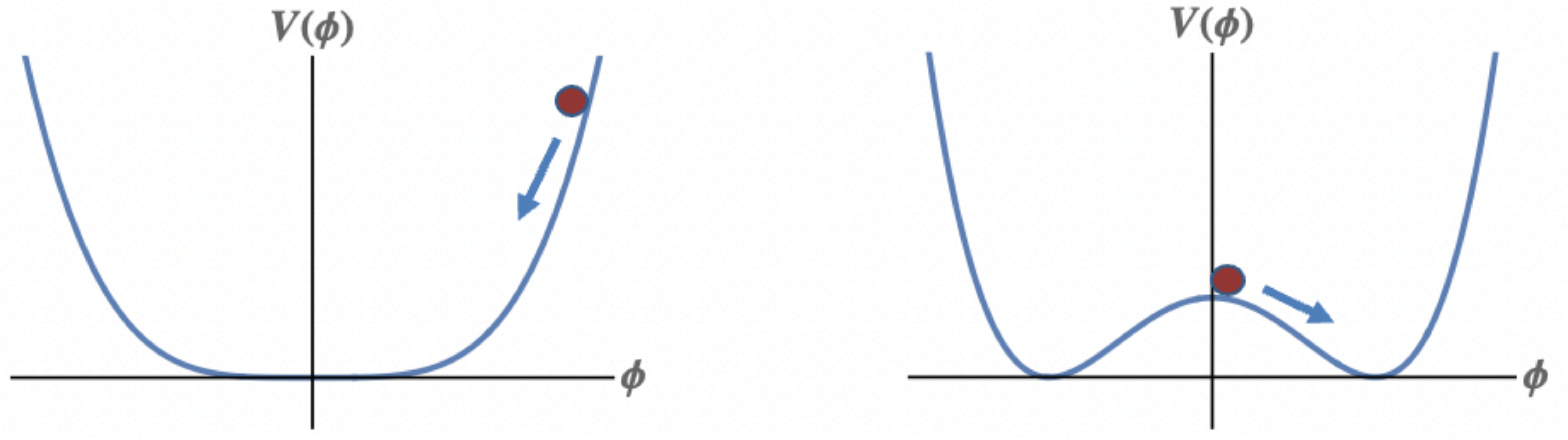
(McDonough,Long,Kolb), (Linde),(Bertolami, Ross)

Does Inflation Itself Require Fine-Tuning of the Initial Conditions?

eg. a smooth patch of size $r > r_H \sim \dfrac{1}{H}$? Numerical simulations have been done but are limited by difficulty of putting these simulations onto computers.
Most are 1+1 dimensional.

For recent review of Inflation see:
Inflation after Planck: Judgement Day Chowdhury, Martin, Ringeval, Vennin

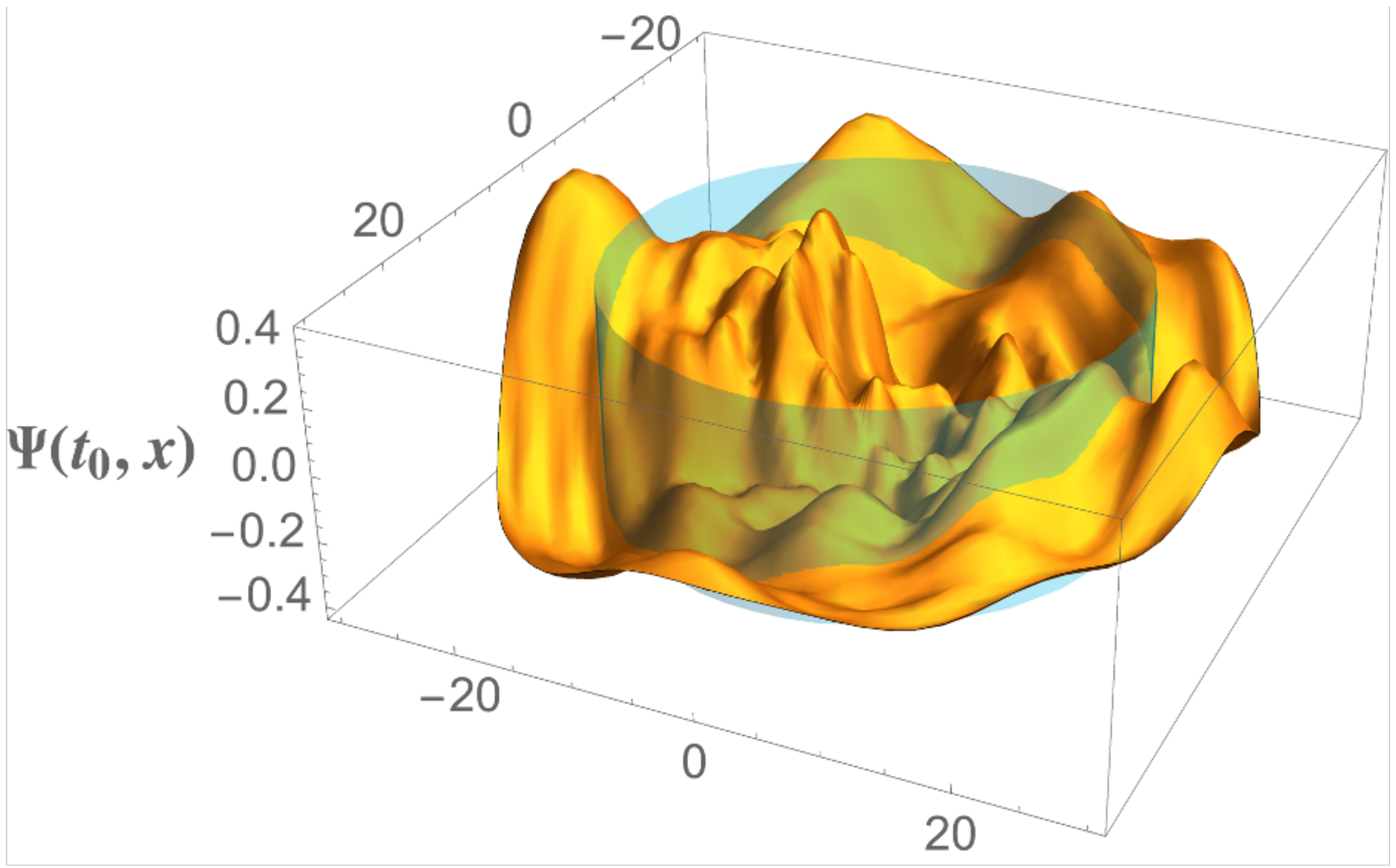
Some 3+1 dimensional Numerical Relativity Sims have been done recently e.g. Clough, Lim, Flauger 1712.07352



Large-field inflation is robust even amid large initial inhomogeneities; small-field inflation requires more special initial conditions, but is still more robust than analytic estimates had suggested.

Source: David Kaiser Jan. 2021

Work by Kaiser, Fitzpatrick, Bloomfield, Hilbert (arXiv:1906.08651) simulated



More on the non-minimal couplings...

1. Why isn't $\xi = -1/6$?

$-1/6$ is a fixed point of the β-function, but any nonzero value will work for renormalization. If we start with $\xi \neq -1/6$ then the RG $\Longrightarrow \xi$ will run to higher values in the UV. If at tree level, $\xi = -1/6$, it will stay there for any energy scale.

2. How does renormalization work in this context?

Renormalization of a QFT is possible in a **fixed** curved background, not in dynamical curved background.

IF we set aside renormalization of the gravitational sector, and consider an EFT for self interacting scalar fields in 3+1 dimensions, then we must include the $f(\phi)\tilde{R} \in \mathscr{L}$ and ξ can be any dimensionless free parameter

$$\boxed{\mathscr{L} \ni f(\phi)\tilde{R} \sim \left(M^2 + \sum_I \xi_I (\phi^I)^2\right)\tilde{R}}$$

More on the non-minimal couplings...

3. Why do we only consider non-negative values of ξ_ϕ, ξ_χ in our models?

When we perform the conformal transformation, the conformal factor is $\Omega^2 \sim f(\phi^I) \sim \left[M^2 + \sum_I \xi_I (\phi^I)^2\right]$.

If we allowed one coupling ξ_K to have sign$(\xi_K) \neq$ sign(ξ_I) for all $I \neq K$,
then there exists a value of $\Omega(x)$ such that $\Omega(x) = 0$ for $\phi \neq 0 \implies$ conformal transformation is not everywhere well defined.

Quantum Diffusion During Ultra-Slow Roll Phase

Main idea:

1. During Ultra Slow-roll, quantum fluctuations must not make field zoom past the min/max feature ($V_{,\sigma} \simeq 0$) too quickly or $\mathscr{P}_R$ will not get large enough for PBH formation.
2. Also can't have insufficient kinetic energy for the field to classically pass through the local minimum or quantum diffusion effects become dominant

The condition that must be satisfied for us to ignore quantum diffusion effects during slow roll is:

$$\mathscr{P}_R(k) < 1/6$$

Approach: Back-reaction from quantum fluctuations → variance in kinetic energy density:

$$\langle(\Delta K)^2\rangle \simeq \frac{3H^4}{4\pi^2}\rho_{\text{kin}} \qquad (\rho_{\text{kin}} = \dot{\sigma}^2/2)$$

Classical evolution >> Quantum diffusion during ultra slow-roll **IF** $\rho_{\text{kin}} > \sqrt{\langle(\Delta K)^2\rangle}$. Equivalent to

Idea: Use $\Delta E \Delta t \leq \hbar/2$ as bound to determine when system will tunnel. Tunnel to right →restart inflation, tunnel left → first order phase transition ends inflation.

Reheating in Multifield Models with Non-minimal couplings

Reheating has been studied in such models using lattice simulations

Our model $N_{\text{reh}} \sim \mathcal{O}(1)$ e-folds.
Between t_{end} and t_{rd}, energy red-shifts as

$$\rho(t_{\text{rd}}) = \rho(t_{\text{end}})e^{-3N_{\text{reh}}}$$

$$\Delta N = \frac{1}{2}\log\left[\frac{2H^2(t_{\text{pbh}})}{H(t_{\text{end}})}e^{-N_{\text{reh}}/4}t_c\right]$$

Radiation domination ($w \simeq 1/3$) within 1-3 e-folds $\implies 18 \lesssim \Delta N \lesssim 25$

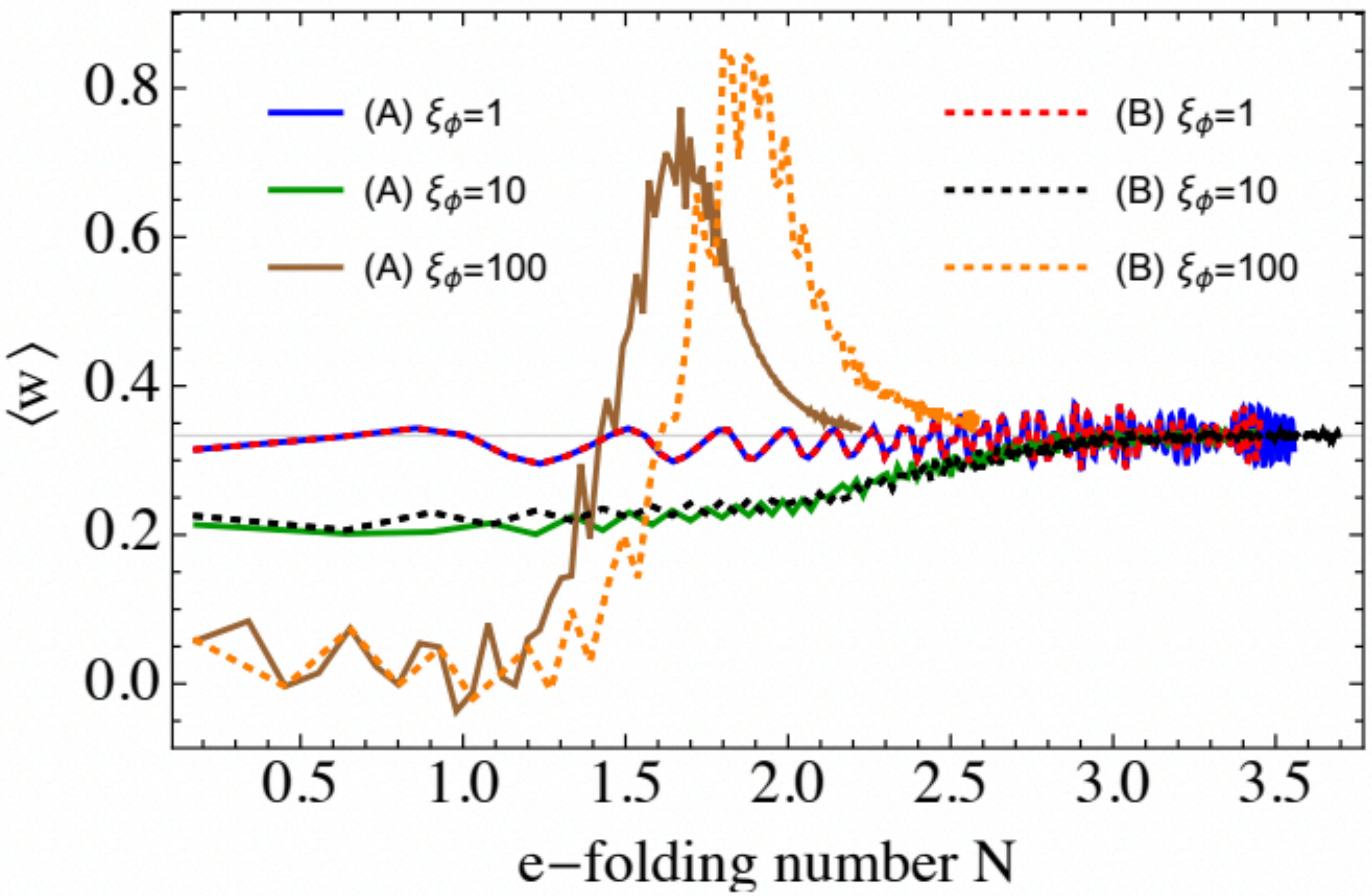


FIG. 5. The averaged effective equation of state $\langle w \rangle$ for $\xi_\phi = 1, 10, 100$ and the two representative cases, "generic" (A) and symmetric (B).

1905.12562v2 Nguyen, van de Vis, Sfakianakis, Giblin, Kaiser 2019

Non-Gaussianities: constraints and our model

Equation of motion for the Adiabatic Modes:

$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[\frac{k^2}{a^2} + \mathcal{M}_{\sigma\sigma} - \omega^2 - \frac{1}{M_{\rm pl}^2 a^3}\frac{d}{dt}\left(\frac{a^3\dot{\sigma}^2}{H}\right)\right]Q_\sigma = 2\frac{d}{dt}(\omega Q_s) - 2\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)(\omega Q_s)$$

Modes couple only when $\omega \neq 0$!
Scalar turn rate acts as a source term

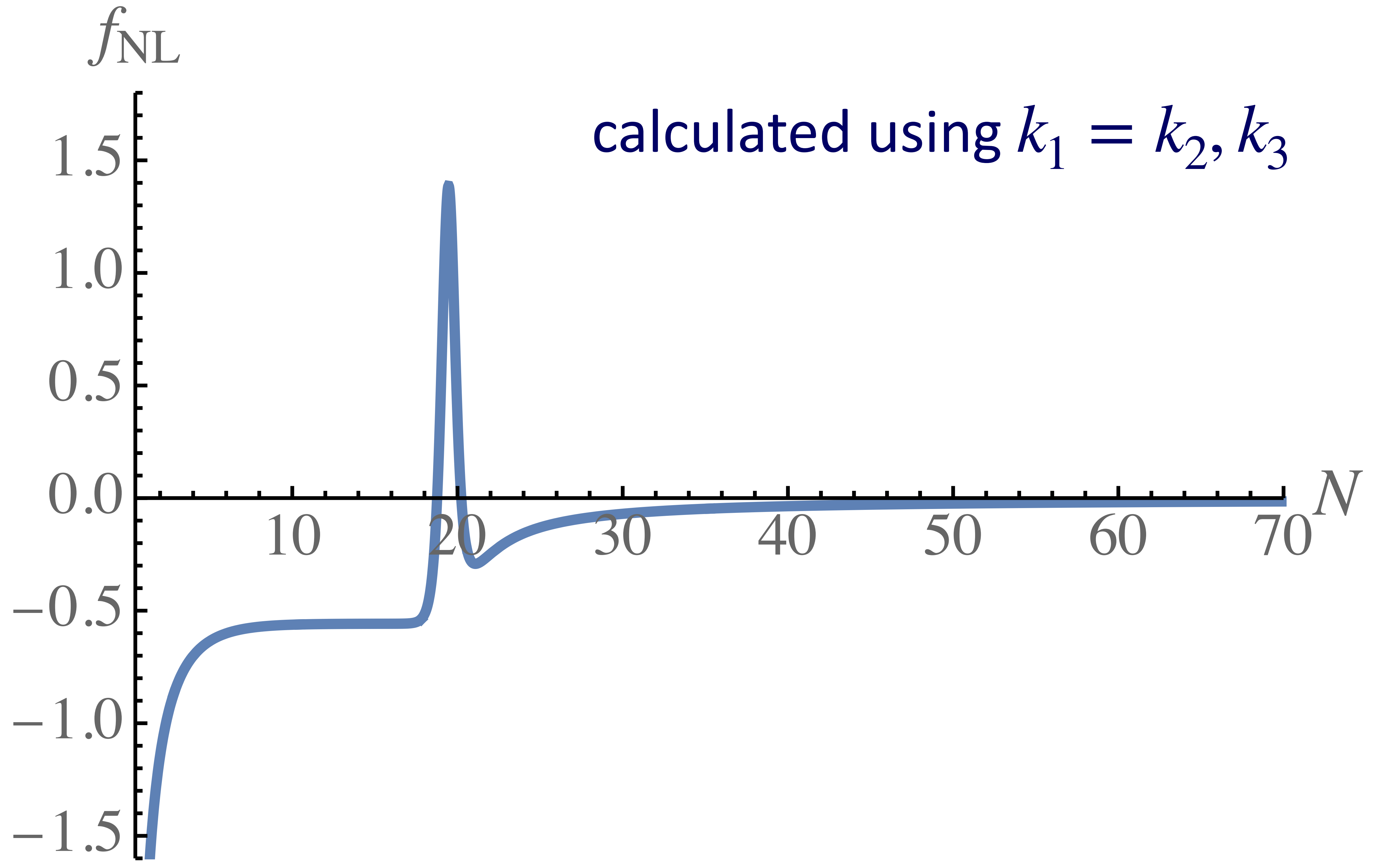
Equation of motion for the Isocurvature Modes:

$$\ddot{Q}_s + 3H\dot{Q}_s + \left[\frac{k^2}{a^2} + \mu_s^2\right]Q_s = 4M_{\rm pl}^2\frac{\omega}{\dot{\sigma}}\frac{k^2}{a^2}(\psi + a^2H(\dot{E} - Ba^{-1}))$$

$f_{\rm NL}$ is defined in terms of power spectrum and bispectrum:

$$f_{\rm NL}(k_1,k_2,k_3) = \frac{5}{6}\frac{\mathcal{B}_\zeta(k_1,k_2,k_3)}{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + \mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3) + \mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3)}$$

where $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho$



$\mu = M_{\rm pl}, b_1 = b_2 = -1.8\times10^{-4}, c_1 = 2.5\times10^{-4}, c_4 = 3.9\times10^{-3}, \xi_\phi = \xi_\chi = 100,$ $c_2 = c_3 = 3.570193\times10^{-3}$

Observables and Parameters

"With four parameters I can fit an elephant and with five I can make him wiggle his trunk"
Enrico Fermi to John Von Neumman (https://www.nature.com/articles/427297a)

	Observables
Ω_k	spatial curvature energy density
$n_s(k_*)$	spectral index
$\alpha(k_*)$	running of spectral index
$r(k_*)$	tensor/scalar ratio
$\beta_{\text{iso}}(k_*)$	isocurvature fraction
f_{NL}	local non-Gaussianity
$\mathcal{P}_R(k_{\text{pbh}})$	peak amplitude of $\mathcal{P}_R$
ΔN	e-folds before t_{end} when peak first passes outside $\dfrac{1}{H(k_{\text{pbh}})}$

Observables and Parameters

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$$V(r,\theta) = \frac{1}{\left(1 + r^2\left(\xi_\phi \cos^2\theta + \xi_\chi \sin^2\theta\right)\right)^2}\left[\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right]$$

Observables

Ω_k	spatial curvature energy density
$n_s(k_*)$	spectral index
$\alpha(k_*)$	running of spectral index
$r(k_*)$	tensor/scalar ratio
$\beta_{\rm iso}(k_*)$	isocurvature fraction
$f_{\rm NL}$	local non-Gaussianity
$\mathscr{P}_R(k_{\rm pbh})$	peak amplitude of $\mathscr{P}_R$
ΔN	e-folds before $t_{\rm end}$ when peak first passes outside $\dfrac{1}{H(k_{\rm pbh})}$

Parameters	**Deg. of Freedom**
Non-minimal couplings: ξ_ϕ, ξ_χ	2
(dimensionless) mass matrix elements: $b_1, b_2, b_3(=b_{12})$	3
"Yukawa" couplings: c_1, c_2, c_3, c_4	4
Initial conditions: $r(t_i), \theta(t_i), \dot{r}(t_i), \dot{\theta}(t_i)$	4

Observables and Parameters

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	Observables
Ω_k	spatial curvature energy density
$n_s(k_*)$	spectral index
$\alpha(k_*)$	running of spectral index
$r(k_*)$	tensor/scalar ratio
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f_{NL}	local non-Gaussianity
$\mathscr{P}_R(k_{\text{pbh}})$	peak amplitude of $\mathscr{P}_R$
ΔN	e-folds before t_{end} when peak first passes outside $\dfrac{1}{H(k_{\text{pbh}})}$

$$V(r,\theta) = \frac{1}{\left(1 + r^2\left(\xi_\phi \cos^2\theta + \xi_\chi \sin^2\theta\right)\right)^2}\left[\mathscr{B}(\theta)r^2 + \mathscr{C}(\theta)r^3 + \mathscr{D}(\theta)r^4\right]$$

Parameters	Deg. of Freedom		
Non-minimal couplings: ξ_ϕ, ξ_χ	$\cancel{2}$	$\xi_\phi = \xi_\chi$	1
(dimensionless) mass matrix elements: $b_1, b_2, b_3 (= b_{12})$	$\cancel{3}$	$b_1 = b_2, b_3 = 0$	1
"Yukawa" couplings: c_1, c_2, c_3, c_4	$\cancel{4}$	$c_2 = c_3$	3
Initial conditions: $r(t_i), \theta(t_i), \dot{r}(t_i), \dot{\theta}(t_i)$	$\cancel{4}$	only $r(t_i)$	1

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Determining the Value of N_{CMB}

We do not marginalize over reheating histories due to computational costs.

We allow $N_{\text{CMB}} = 55 \pm 5$ e-folds to account for the usual uncertainty in reheating. Multifield inflationary models such as those we consider typically have efficient reheating $N_{\text{reh}} \sim \mathcal{O}(1)$.

N_{CMB} is thus a derived value rather than a parameter.

We allow the MCMC to optimize the value within the given window to fit the CMB+ PBH constraints