Supermassive Black Holes from Direct Inflationary Collapse

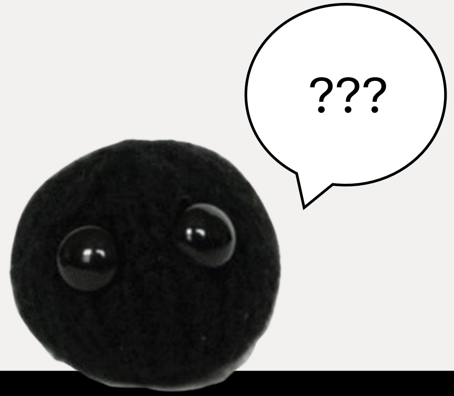
Aurora Ireland (University of Chicago)
New Horizons in Primordial Black Hole Physics
19.6.23

Based on upcoming work with
Dan Hooper, Gordan Krnjaic,
& Albert Stebbins

Motivations

- Quasars powered by supermassive black holes (SMBH) with masses $M{\sim}10^{8-10}M_{\odot}$ found in ***excess*** in the high-redshift universe
- Generally thought that SMBH grow from lower mass seeds (possibly Population III stars) through accretion
- Eddington limited accretion rate $\Rightarrow M{\sim}10^{2}M_{\odot} \rightarrow 10^{10}M_{\odot}$ in ${\sim}0.8$ Gyr
  - SMBH must grow continuously for first ~Gyr of universe's history
- Most SMBH do not seem to have grown much since ~1 Gyr after the Big Bang
  - Comoving number density of $M \sim 10^{10}M_{\odot}$ SMBH has remained approx. ***constant*** since $z{\sim}5$

Motivations



1) How did these SMBH come to be so massive on such a short time scale?

2) Why did their growth rate dramatically slow during subsequent ~ 13 Gyr?

Possibility: Predominantly ***primordial*** rather than astrophysical in origin?

PBH

- Form from collapse of large primordial density fluctuations
- Requires significant enhancement of $\mathcal{P}_{\mathcal{R}}$ on small scales

$$\sigma_\delta^2 = \int_0^\infty \frac{dk}{k} W^2(k,R)\,\mathcal{P}_\delta(k) \qquad P_\delta^{\mathrm{G}} = \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp\left[-\frac{\delta^2}{2\sigma_\delta^2}\right] \qquad \beta \simeq 2\int_{\delta_{\mathrm{th}}}^\infty d\delta P_\delta$$

$\Rightarrow$ Presuming Gaussian statistics for $\mathcal{R}$, need $\mathcal{P}_{\mathcal{R}} \sim \mathcal{O}(10^{-2})$

- Pretty easy to engineer in inflationary models
  - *Single field:* Inflection point, USR plateau, localized features (bumps, dips, steps), etc.
  - *Multifield:* Instabilities in scalar sector (hybrid inflation), non-canonical kinetic terms, non-minimal couples to R, trajectories deviating from geodesics in field space, etc.

PBH

- Issue: SMBH form ***late***! (size set by size of horizon at time of collapse)
  - After BBN: $T \sim \text{MeV} \Rightarrow M_{\rm hor} \sim 10^5 M_\odot$
  - Before matter domination, recombination: $T \sim \text{eV} \Rightarrow M_{\rm hor} \sim 3\times10^{17} M_\odot$
- Such large amplification at late times inevitably leads to spectral distortions of the CMB
  - $T \sim$ 10-400 keV $\Rightarrow$ double Compton scattering, thermal Bremmstrahlung inefficient
    $\Rightarrow$ μ-type distortions!
  - $T \lesssim$ 10 keV $\Rightarrow$ Compton scattering inefficient
    $\Rightarrow$ y-type distortions

Spectral distortion constraints

- For a sharply peaked $\mathcal{P}_\mathcal{R} = \sigma_\mathcal{R}^2\, k\, \delta(k - k_{BH})$, μ- and y-type spectral distortions can be estimated as [Chluba et al, 2012]:

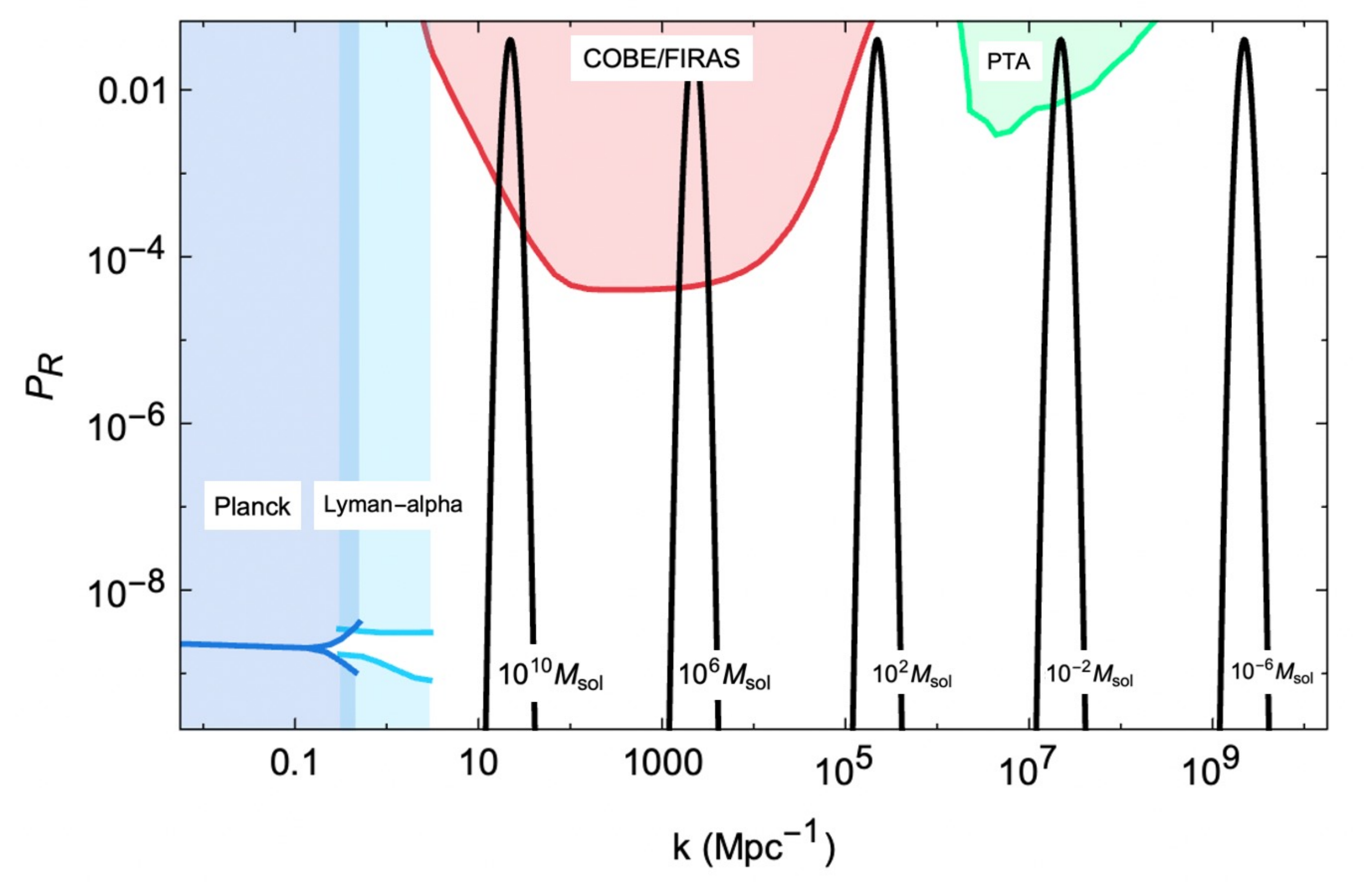
$$\mu \approx 2.2\sigma_\mathcal{R}^2 \left\{ \exp\left[-\left(\frac{1.5 \times 10^5\, M_\odot}{M_{\rm BH}} \right)^{1/2} \left(\frac{\gamma}{0.2} \right)^{1/2} \left(\frac{3.91}{g_{\star,S}(T)} \right)^{1/3} \left(\frac{g_{\star,S}(T)}{3.36} \right)^{1/4} \right] - \exp\left[-\left(\frac{4.5 \times 10^9\, M_\odot}{M_{\rm BH}} \right) \left(\frac{\gamma}{0.2} \right) \left(\frac{3.91}{g_{\star,S}(T)} \right)^{2/3} \left(\frac{g_{\star,S}(T)}{3.36} \right)^{1/2} \right] \right\}$$

$$y \approx 0.4\, \sigma_\mathcal{R}^2\, \exp\left[-\left(\frac{4.5 \times 10^9\, M_\odot}{M_{\rm BH}} \right) \left(\frac{\gamma}{0.2} \right) \left(\frac{3.91}{g_{\star,S}(T)} \right)^{2/3} \left(\frac{g_{\star,S}(T)}{3.36} \right)^{1/2} \right]$$

- Constraints from COBE/FIRAS:
  - $|\mu| \lesssim 9.0 \times 10^{-5}$
  - $|y| \lesssim 1.5 \times 10^{-5}$

Spectral distortion constraints

- Tension:
  - Need small σ^2 for consistency with spectral distortions
  - Need large σ^2 for non-vanishing β
- Naively restricted to PBH with $M \lesssim 10^2 M_\odot$



$$\sigma_{\mathcal{R}}^2 = 0.01 \quad \Rightarrow \quad \beta \simeq 10^{-20}$$

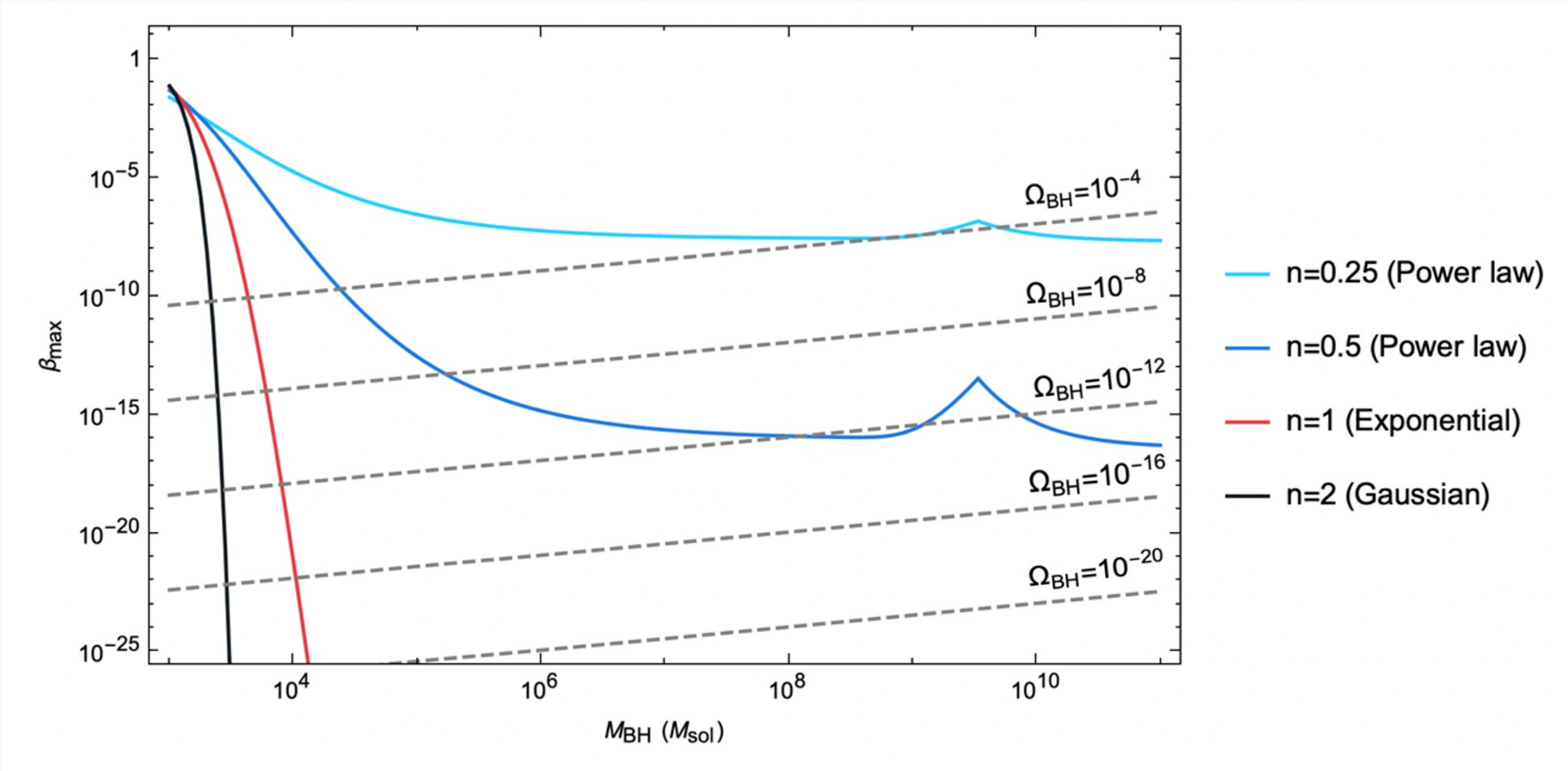
Departures from Gaussianity

- Estimate $\sigma_{\mathcal{R}}^2 \sim \mathcal{O}(10^{-2})$ presumed Gaussian probability distribution function (pdf)
  $\Rightarrow$ Less amplification required for heavier tailed distributions!
- Assumption of Gaussian statistics for δ is generically false
  - Non-linear mapping between curvature perturbation $\mathcal{R}$ and density contrast δ
  - Statistics of $\mathcal{R}$ in models with local amplification are generally non-Gaussian
- To quantify degree of non-Gaussianity required, consider fiducial pdf:

$$P_\delta^{(n)} = \frac{1}{2\sqrt{2}\,\Gamma(1+1/n)\sigma_0} \exp\left(-\left[\frac{|\delta|}{\sqrt{2}\sigma_0}\right]^n\right)$$

Gaussian: $n = 2$
Exponential: $n = 1$
Power law: $n < 1$

Departures from Gaussianity



Max PBH mass fraction at formation for $\sigma^2 = \int d\delta\ \delta^2 P_\delta^{(n)}$ saturating spectral distortion constraints

Non-Minimal Self-Interacting Curvaton

- Plenty of inflationary models capable of producing heavy exponential tails, but we need to do **better** than exponential
- Claim: Non-minimal self-interacting curvaton model can produce sufficiently heavy power law tail
- Curvaton χ
  - Light ($m_\chi^2 \ll H$) spectator during inflation with subdominant ρ_χ
  - Responsible for generating curvature perturbation
  - Initially isocurvature perturbations converted to adiabatic upon decay
  - Non-Gaussianity from inefficient conversion

~~Non-Minimal Self-Interacting~~ Curvaton

- Consider standard curvaton scenario with $V(\chi) = \frac{1}{2} m_\chi^2 \chi^2$
- During inflation:
  - Background value “frozen-in” at χ_*
  - Receives perturbations $\delta\chi_* \simeq H_*/2\pi$ (initially Gaussian)
- After inflation:
  - Starts to oscillate about minimum when $H \simeq m_\chi$
  - Decays when $H \sim \Gamma_\chi$ (isocurvature → adiabatic)
- Non-linear mapping between ζ and ζ_χ ⇒ non-Gaussian pdf
- δN formalism allows for fully non-perturbative calculation of ζ and its statistics

~~Non-Minimal Self-Interacting~~ Curvaton

- δN formalism
  - Compute non-linear evolution of cosmological perturbations on super-Hubble scales
  - Curvature perturbation = difference between perturbed vs unperturbed amount of expansion: $\zeta = N(\bar{\chi} + \delta\chi) - N(\bar{\chi})$
- First used to investigate NG in curvaton model in [Sasaki et al, 2006]
  - Total obeys: $e^{4\zeta_b} - \frac{4r}{3+r}\left(e^{3\zeta_\chi}\right)e^{\zeta_b} + \frac{3r-3}{3+r} = 0$ with $r = \frac{3\Omega_{\chi,\mathrm{dec}}}{4 - \Omega_{\chi,\mathrm{dec}}}$
  - Solution: $\zeta_b = \ln X$ with $X = \frac{B^{1/2} + \sqrt{ArB^{-1/2} - B}}{(3+r)^{1/3}}$

$A = e^{3\zeta_\chi} = (1+\delta_\chi)^2 = \left(1 + \frac{\delta\chi_*}{\chi_*}\right)^2$ $B = \frac{1}{2}\left[C^{1/3} + (r-1)(3+r)^{1/3}C^{-1/3}\right]$ $C = (Ar)^2 + \sqrt{(Ar)^4 + (3+r)(1-r)^3}$

~~Non-Minimal Self-Interacting~~ Curvaton

- This solution gives us a mapping between ζ and the Gaussian reference variable $\delta_\chi = \delta\chi_*/\chi_*$, with pdf: $$P_{\delta_\chi}[\delta_\chi] = \frac{1}{\sqrt{2\pi}\sigma_0}\exp\left[-\frac{\delta_\chi^2}{2\sigma_0^2}\right]$$

- By conservation of probability: $$P_\zeta[\zeta] = P_{\delta_\chi}\left[\delta_\chi^+(\zeta)\right]\left|\frac{d\delta_\chi^+}{d\zeta}\right| + P_{\delta_\chi}\left[\delta_\chi^-(\zeta)\right]\left|\frac{d\delta_\chi^-}{d\zeta}\right|$$

  where $$\delta_\chi^\pm = -1 \pm \sqrt{\frac{3+r}{4r}e^{3(\zeta+\langle\zeta_b\rangle)} + \frac{3r-3}{4r}e^{-(\zeta+\langle\zeta_b\rangle)}}$$

- PBH mass fraction at formation:

Evaluate roots at $\zeta_{th} \simeq 0.932$

$$\beta = 2\int_{\delta_{\chi,\mathrm{th}}^+}^{\infty} d\delta_\chi\, P_{\delta_\chi}[\delta_\chi] + 2\int_{-\infty}^{\delta_{\chi,\mathrm{th}}^-} d\delta_\chi\, P_{\delta_\chi}[\delta_\chi] \quad \Rightarrow \quad \beta = \mathrm{erfc}\left(\frac{\delta_{\chi,\mathrm{th}}^+}{\sqrt{2}\sigma_0}\right) + \mathrm{erfc}\left(\frac{|\delta_{\chi,\mathrm{th}}^-|}{\sqrt{2}\sigma_0}\right)$$

Non-Minimal ~~Self-Interacting~~ Curvaton

- Recall: For PBH formation, need localized amplification of $\mathcal{P}_\zeta$ on small scales
  - σ_0^2 computed from a knowledge of the primordial power spectrum
- [Pi & Sasaki, 2022] accomplish by introducing a non-trivial kinetic term:

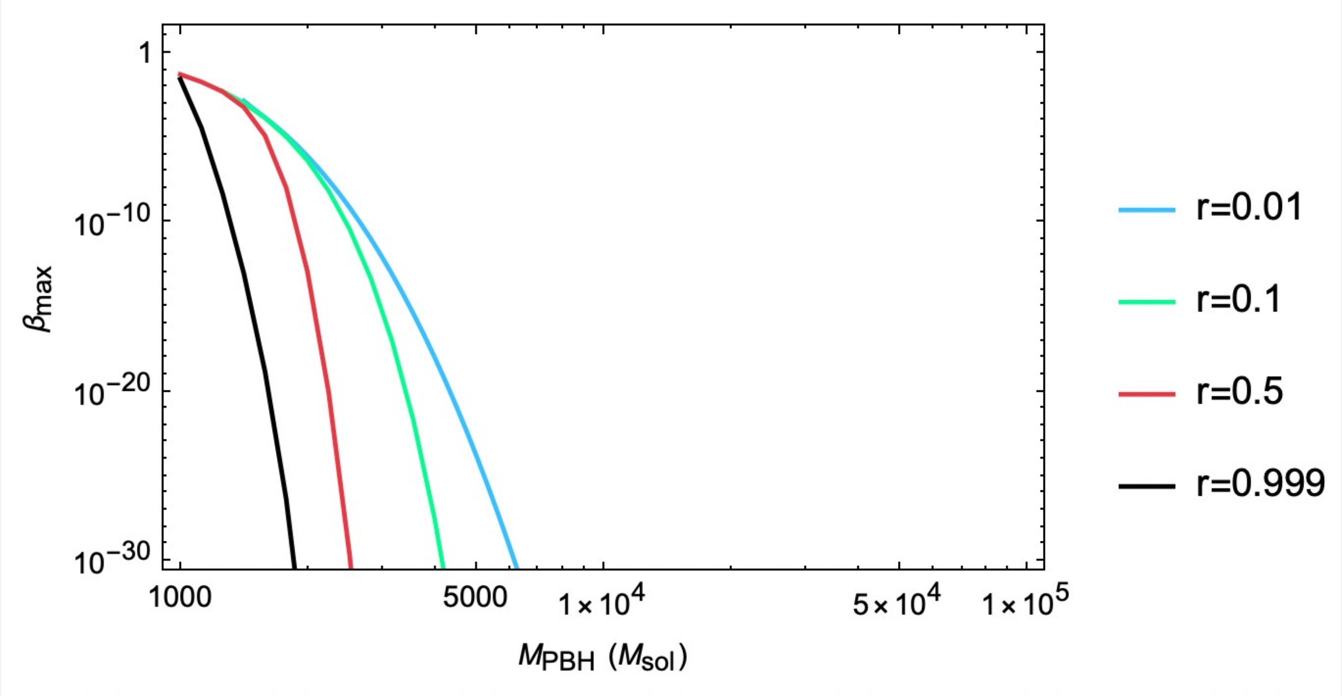
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{2}f(\phi)^2(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2$$

- Choose $f(\phi)$ such that kinetic term is suppressed on scale $k_{BH} \Rightarrow$ peak in $\mathcal{P}_\zeta$!

$$\mathcal{P}_{\delta_\chi}(k) = \frac{k^3}{2\pi^2}\left|\frac{\delta\chi_k}{\chi}\right|^2 = \frac{1}{\chi(t_k)^2}\left(\frac{H(t_k)}{2\pi f(\phi_k)}\right)^2$$

Non-Minimal ~~Self-Interacting~~ Curvaton

- Compare against spectral distortion constraints: $$\sigma_\zeta^2 = \int d\delta_\chi \, \zeta_b^2 \, P_{\delta_\chi}[\delta_\chi] - \left(\int d\delta_\chi \, \zeta_b \, P_{\delta_\chi}[\delta_\chi]\right)^2$$
- Result:



Largest deviation from Gaussianity when $r \ll 1$ (curvaton very subdominant at decay)

Non-Minimal Self-Interacting Curvaton

- Quadratic potential:
  - χ and $\delta\chi$ obey same eqs on superhorizon scales $\Rightarrow \chi \sim \delta\chi \ \Rightarrow \ \frac{\delta\chi}{\chi} = \frac{\delta\chi_*}{\chi_*}$
  - ***No non-linear evolution*** for curvaton contrast δ_χ following horizon exit
  - Exact relation: $e^{3\zeta_\chi} = \left(1 + \delta_\chi\right)^2$
- Introduce self-interactions $\Rightarrow$ mapping between ζ_χ and initial Gaussian perturbations $\delta\chi_*$ becomes even more dramatically non-linear!
- Schematic evolution:
  - 1) $t_* \lesssim t \lesssim t_{\mathrm{int}}$: slow-roll, $\chi \simeq \chi_*$ frozen-in
  - 2) $t_{\mathrm{int}} \lesssim t \lesssim t_{\mathrm{osc}}$: non-quadratic interaction regime (begins when $V'' \sim H^2$)
  - 3) $t_{\mathrm{osc}} \lesssim t \lesssim t_{\mathrm{dec}}$: quadratic field oscillations

Non-Minimal Self-Interacting Curvaton

- Gaussian reference variable: $\delta\chi_*$
- Mapping between ζ, ζ_χ: $e^{4\zeta_b} - \frac{4r}{3+r}\left(e^{3\zeta_\chi}\right)e^{\zeta_b} + \frac{3r-3}{3+r} = 0$
- Need mapping between ζ_χ and $\delta\chi_*$; generically $\delta\chi_*^j = g_j(\zeta_\chi)$
- For weak interactions, can demonstrate:

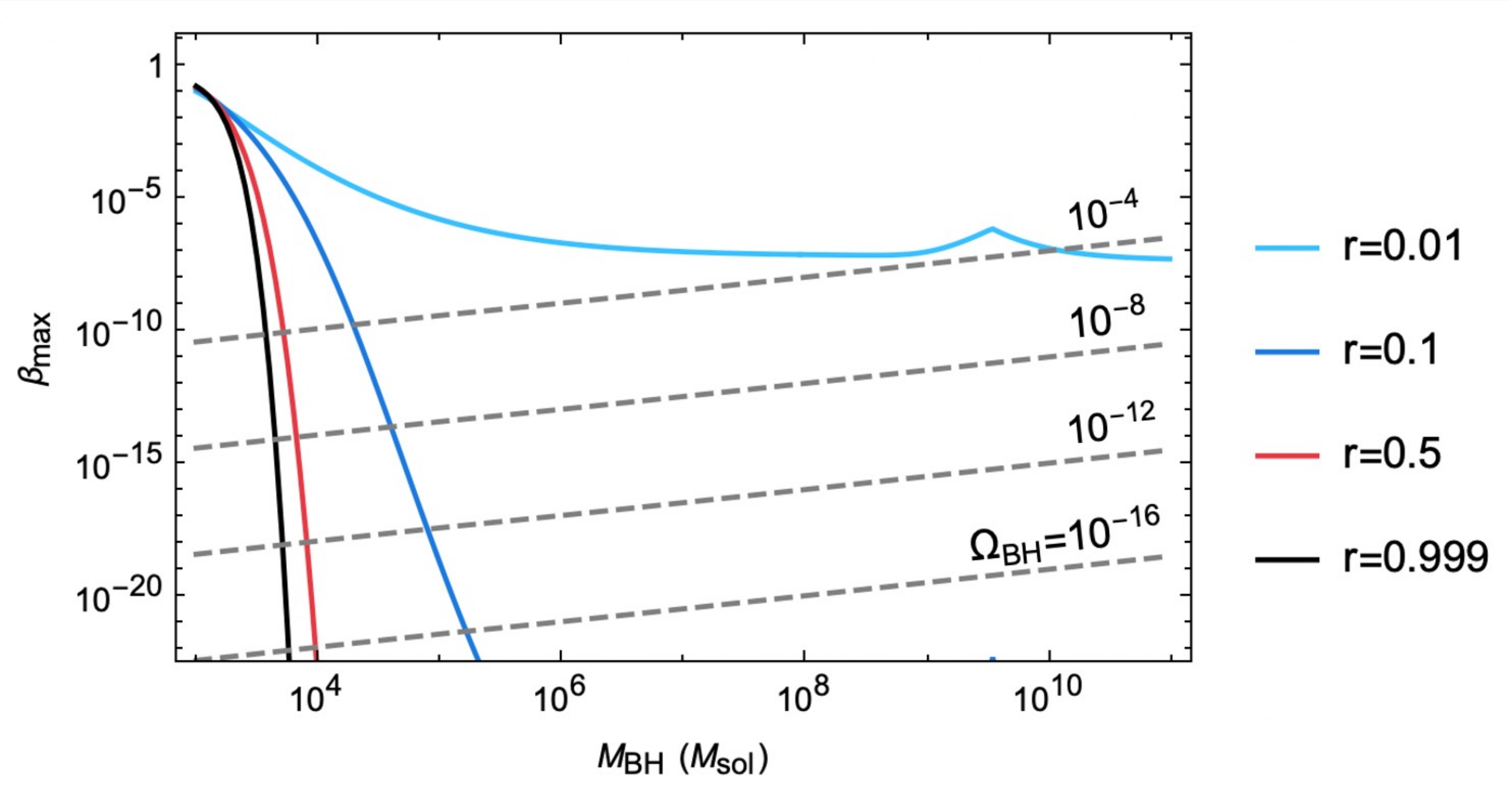
$$\zeta_\chi(\delta\chi_*) = \frac{2}{3}\frac{\chi'_{\rm osc}}{\chi_{\rm osc}}\delta\chi_* + \frac{1}{3}\left(\frac{\chi_{\rm osc}\chi''_{\rm osc}}{\chi'^2_{\rm osc}}\right)\left(\frac{\chi'_{\rm osc}}{\chi_{\rm osc}}\right)^2\delta\chi_*^2 + \frac{2}{9}\left(1 - \frac{3}{2}\frac{\chi_{\rm osc}\chi''_{\rm osc}}{\chi'^2_{\rm osc}} + \frac{1}{2}\frac{\chi^2_{\rm osc}\chi'''_{\rm osc}}{\chi'^3_{\rm osc}}\right)\left(\frac{\chi'_{\rm osc}}{\chi_{\rm osc}}\right)^3\delta\chi_*^3$$

- Pdf: $P_\zeta[\zeta] = \sum_j \left|\frac{g_j(\zeta)}{d\zeta}\right| P_{\delta\chi_*}[g_j(\zeta)]$

Non-Minimal Self-Interacting Curvaton

- Of course, can also implement δN formalism numerically
- Following inflation: $\ddot{\chi} + 3H\dot{\chi} + \frac{dV}{d\chi} = -\Gamma_\chi \dot{\chi}$ $\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} = \Gamma_\chi \dot{\chi}^2$ $H^2 = \frac{8\pi}{3M_{\rm Pl}^2}(\rho_{\rm rad} + \rho_\chi)$
- Solve until curvaton has completely decayed (t_f s.t. $H_f \ll \Gamma_\chi$)
- Compute # e-folds elapsed: $N(t_f) = \ln a_f / a_i$
- Now perturb initial conditions, $\chi_* \to \chi_* + \delta\chi_*$ with $\delta\chi_* = H_*/2\pi$, and evolve to the same final hypersurface of fixed energy density
- $\zeta = N(\chi_* + \delta\chi_*) - N(\chi_*)$
- Repeat for many different $\delta\chi_*$ for statistics

Non-Minimal Self-Interacting Curvaton



Non-linear growth between horizon exit and onset of oscillations *dramatically* boosts non-Gaussianity

Conclusions

- We observe evidence of an excess of SMBH at high redshift
  - Can’t be readily explained by accretion or mergers
- Possibility: Primordial origin?
- Amplification of $\mathcal{P}_{\mathcal{R}}$ required is naively in tension with constraints on CMB spectral distortions
- Can circumvent bounds and produce PBH with smaller peak provided a sufficiently non-Gaussian pdf
- The *non-minimal self-interacting curvaton model* is a physical model capable of producing SMBH via such a dramatically non-Gaussian pdf!