New Horizons in
Primordial Black Hole Physics

highlight talk on

The Effective Theory of Quantum Black Holes

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E. Binetti, M. Del Piano, S. Hohenegger, F. Pezzella, F. Sannino
Phys.Rev.D 106 (2022) 4, 046006 2203.13515 [gr-qc]

Main idea:
Encode modifications to the Schwarschild metric while preserving the underlying symmetries

1. Take the Schwarzschild metric $f_0(r) = 1 - \frac{2M}{r}$
2. Introduce quantum corrections $f(r) = 1 - \frac{2M}{r}\mathcal{G}(d)$
3. Let the physical quantity d be the proper distance

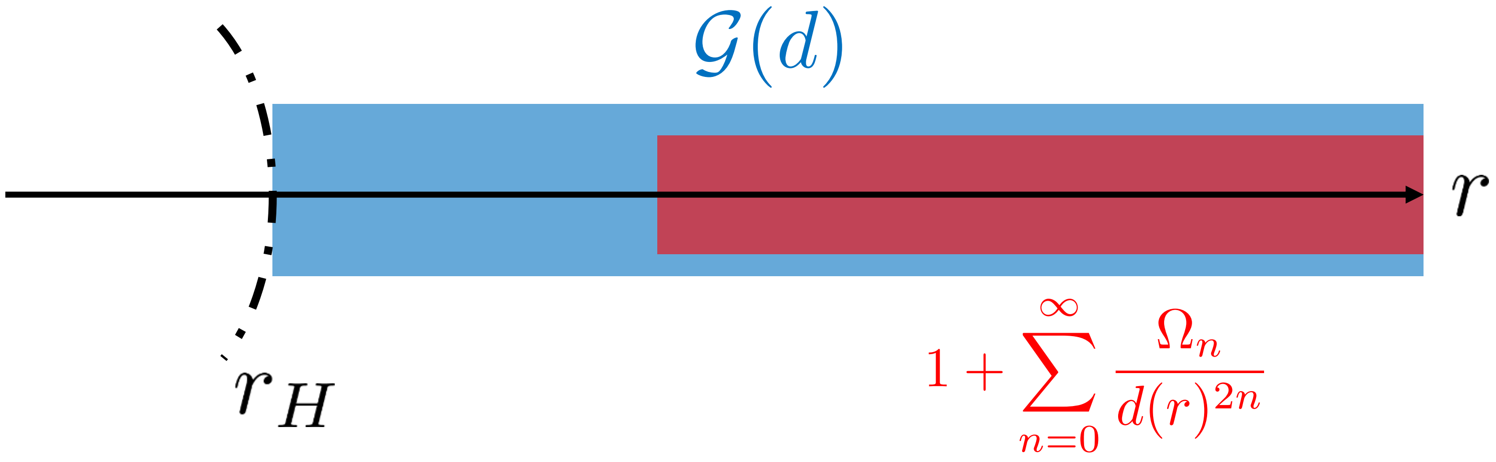
$$d(r) := \int_0^r \sqrt{|\mathrm{d}s^2|} = \int_0^r \frac{\mathrm{d}r'}{\sqrt{|f(r')|}}$$

4. Series expand $\mathcal{G}(d)$ at large distances

$$f(r) = 1 - \frac{2M}{r}\left(1 + \sum_{n=1}^{\infty} \frac{\Omega_n}{d(r)^{2n}}\right)$$

asymptotic flatness: ✓

the Ω_n's are dictated by a given theory of quantum gravity.

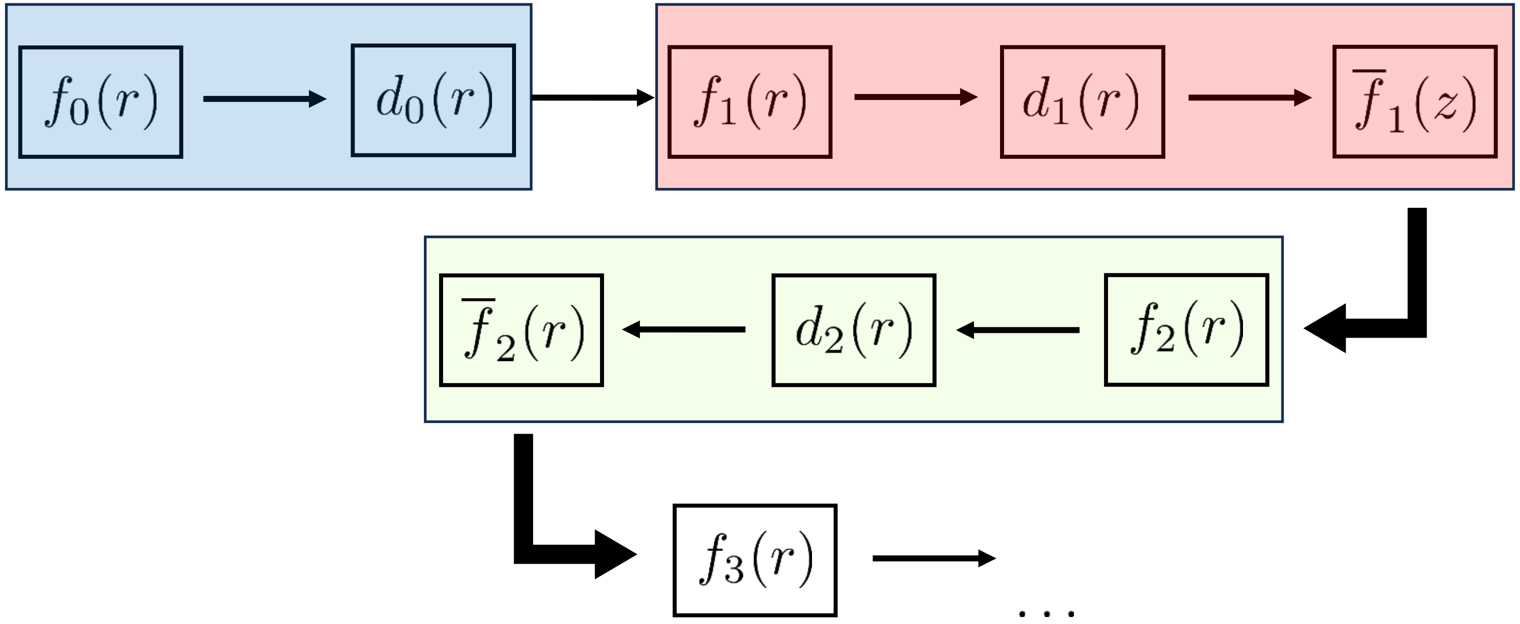


In the analysis we have no full metric.

Operatively, we must construct the metric order by order

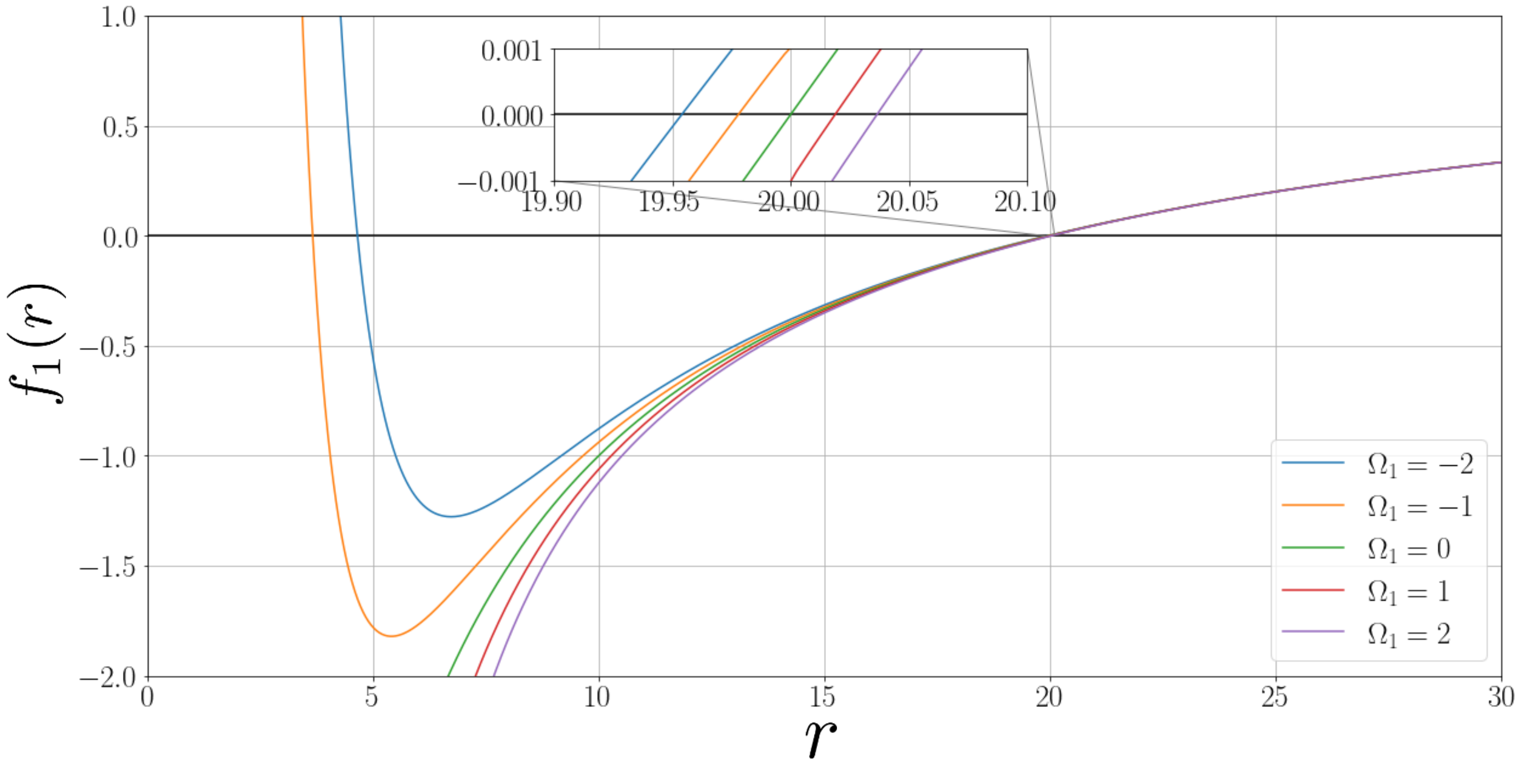
$$f_N(r) = 1 - \frac{2M}{r}\left(1 + \sum_{n=1}^{N} \frac{\Omega_n}{d_{N-1}(r)^{2n}}\right)$$

We study f iteratively up to order N through the following scheme



Let us start from the leading correction $N = 1$

$$d_0(z) = \int_0^r \frac{\mathrm{d}r'}{\sqrt{|1 - \frac{2M}{r'}|}} = \begin{cases} \pi M - 2M \tan^{-1}\left(\sqrt{\frac{2M}{r} - 1}\right) - \sqrt{r(2M - r)} & 0 < r \leq 2M \\ \pi M + 2M \tanh^{-1}\left(\sqrt{1 - \frac{2M}{r}}\right) + \sqrt{r(r - 2M)} & 2M \leq r < \infty \end{cases}$$



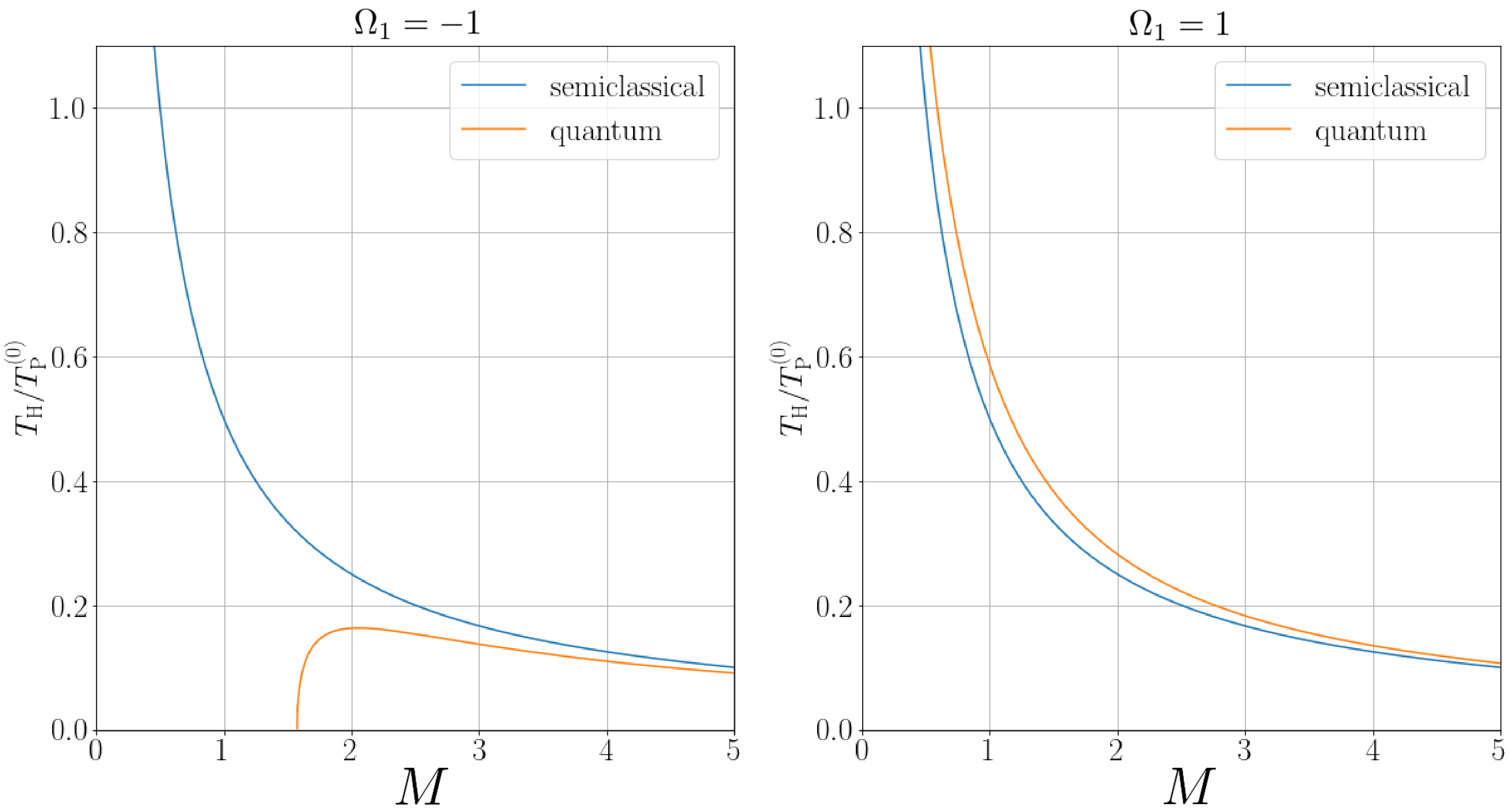
$$f_1(r) = 1 - \frac{2M}{r}\left(1 + \frac{\Omega_1}{d_0(r)^2}\right)$$

The position of the event horizon depends on the sign of Ω_1

$$r_H = 2M\left(1 + \frac{\Omega_1}{(\pi M)^2} + \mathcal{O}(\Omega_1^2)\right)$$

We studied the impact of these corrections on the Hawking temperature

$$T_{\mathrm{H}} = \frac{\kappa}{2\pi} = \frac{1}{4\pi} \left. \frac{\mathrm{d}f_1}{\mathrm{d}r} \right|_{r_H}$$



We computed the entropy

$$S = \int \frac{\mathrm{d}M}{T_{\mathrm{H}}(M)}$$

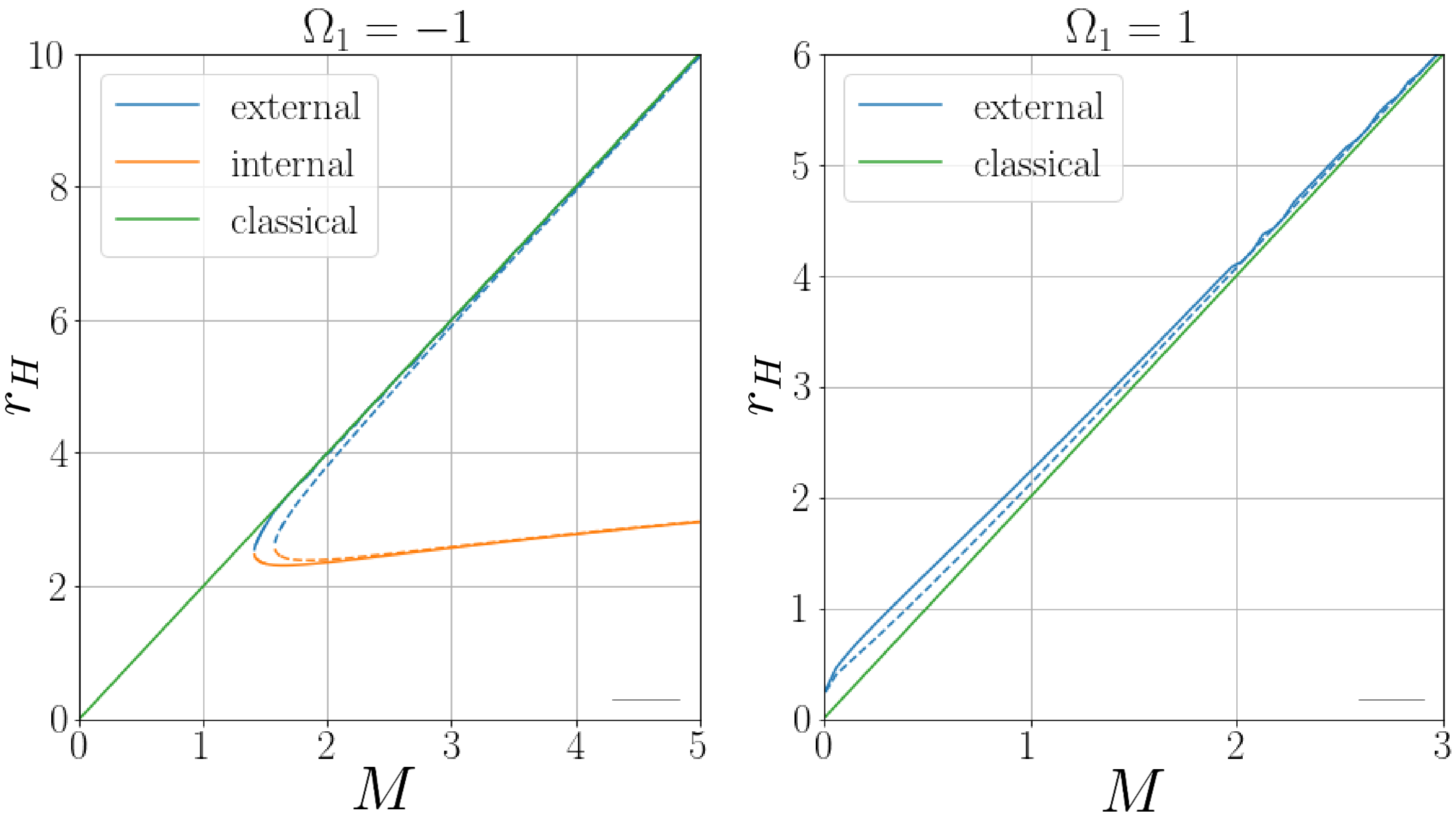
which results in

$$S \simeq \begin{cases} 4\pi M^2 \left[1 + \frac{8}{\pi}\sqrt{\frac{|\Omega_1|}{\pi^2 M^2}} - 4\left(1 - \frac{64}{\pi^2}\right)\frac{|\Omega_1|}{\pi^2 M^2}\log M\right] & \Omega_1 < 0 \\ 4\pi M^2 \left[1 - \frac{8}{\pi}\sqrt{\frac{|\Omega_1|}{\pi^2 M^2}} + 4\left(1 + \frac{64}{\pi^2}\right)\frac{|\Omega_1|}{\pi^2 M^2}\log M\right] & \Omega_1 > 0 \end{cases}$$

the leading correction is linear in the mass while the subleading is logarithmic

Self-consistency check with the **backreaction** on the proper distance

$$d_1(r) = \int_0^r \frac{\mathrm{d}r'}{\sqrt{\left|1 - \frac{2M}{r'}\left[1 + \frac{\Omega_1}{d_0(r')^2}\right]\right|}} \longrightarrow \overline{f}_1(r) = 1 - \frac{2M}{r}\left(1 + \frac{\Omega_1}{d_1(r)^2}\right)$$



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What we achieved

- Set up an effective self-consistent framework
- Determined the impact of the leading quantum corrections to the event horizon structure
- Investigated the corrections to the thermodynamic properties
- Tested the robustness of our results by further considering both the quantum proper distance backreaction as well as the effects of higher order corrections

Upcoming work:

M. Del Piano, S. Hohenegger, F. Sannino

Finitness condition from event horizons

- General spherically symmetric spacetimes

$$g_{tt} \neq g_{rr}^{-1}$$

- Generalize the framework to non perturbative deformations

$$\mathcal{G}_{tt}(d) = e^{\Psi(1/d)} \quad \mathcal{G}_{rr}(d) = e^{\Phi(1/d)}$$

- Conditions for regularity of temperature and curvature invariants at the horizon

Outlooks and remarks

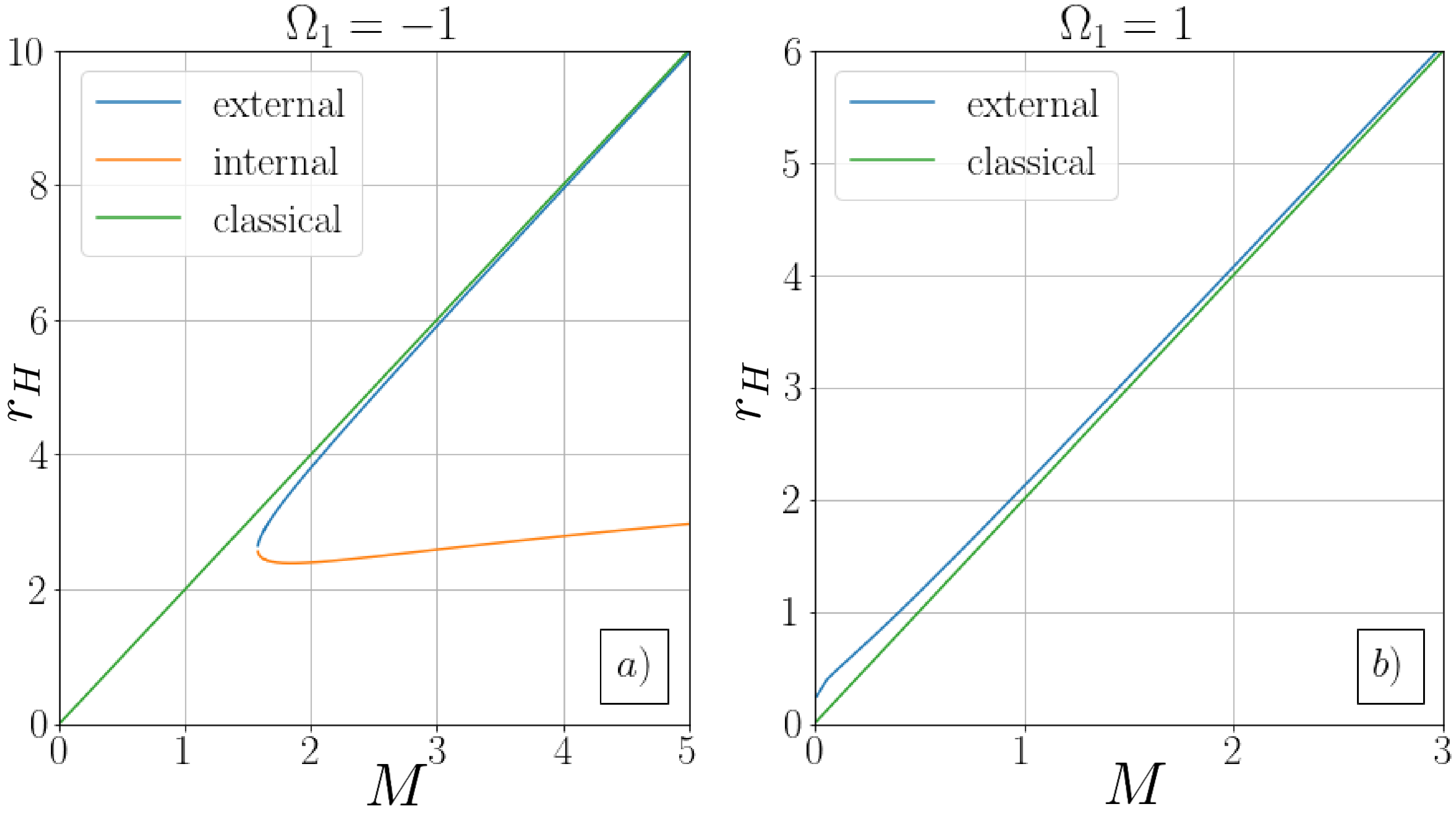
- Schwarzschild BH is the ideal laboratory for our initial studies, possible generalization to other spacetimes, such as Kerr
- Different models of quantum gravity provide specific values of Ω_n or could be experimentally determined, such as gravitational waves
- Corrections interpreted as effective EMT
- Test the stability and GWs
- Orbits and shadow

Positivity Conditions for Generalised Schwarzschild Space-Times
A. D'Alise *et al.* [2305.12965](2305.12965) [gr-qc]

Precession of bounded orbits and shadow in quantum black hole spacetime
Li, Kuang PhysRevD.107.064052

BACKUP Taylor expanding the positions of the horizons at the leading order

$$r_{H+} \approx 2M\left(1 \pm \frac{|\Omega_1|}{\pi^2 M^2} + \mathcal{O}\left(\left(\frac{|\Omega_1|}{\pi^2 M^2}\right)^{3/2}\right)\right) \quad r_{H-} \approx M\left(\frac{9\pi}{2}\left(\frac{|\Omega_1|}{\pi^2 M^2}\right)^{1/3} + \mathcal{O}\left(\left(\frac{|\Omega_1|}{\pi^2 M^2}\right)^{2/3}\right)\right)$$



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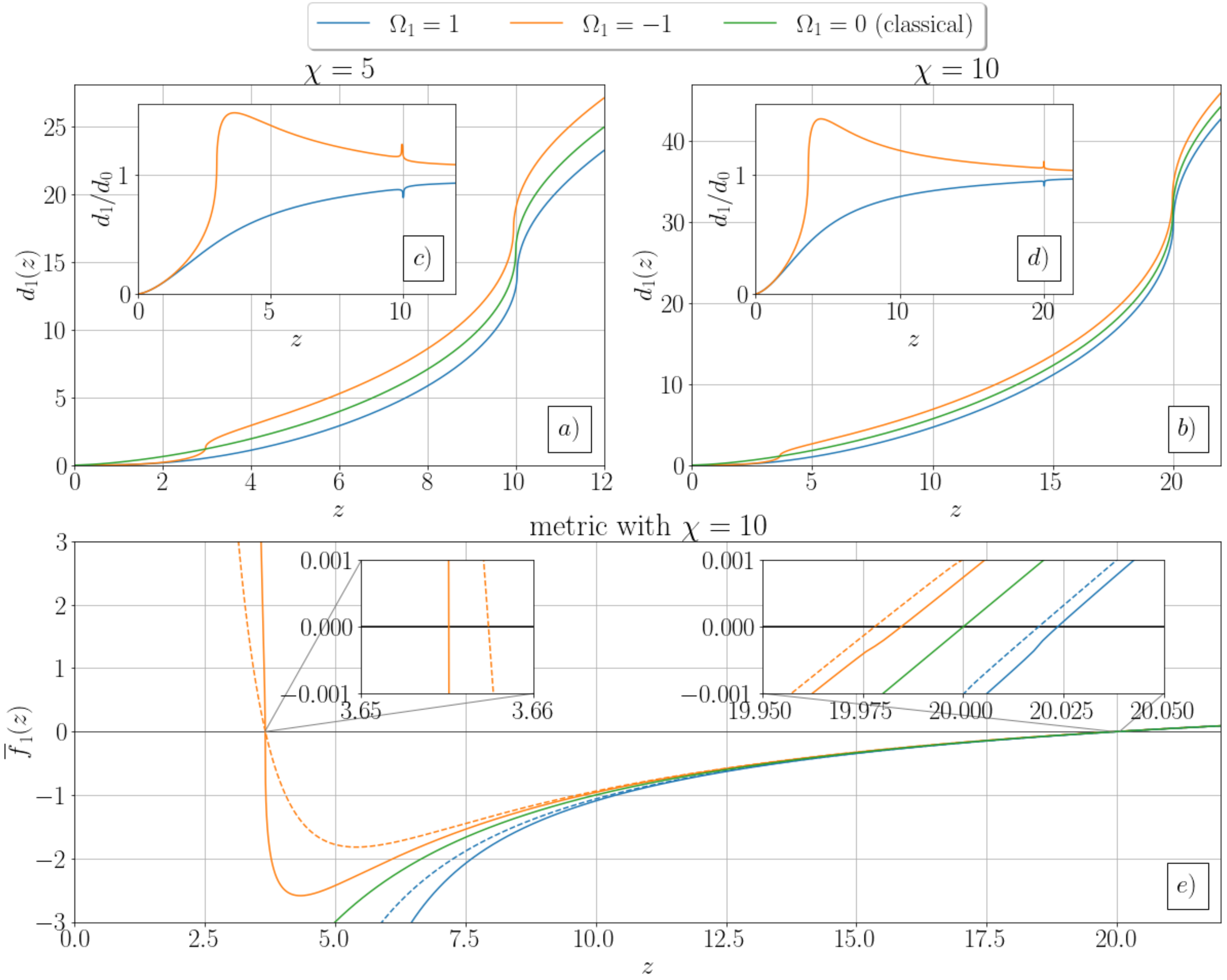
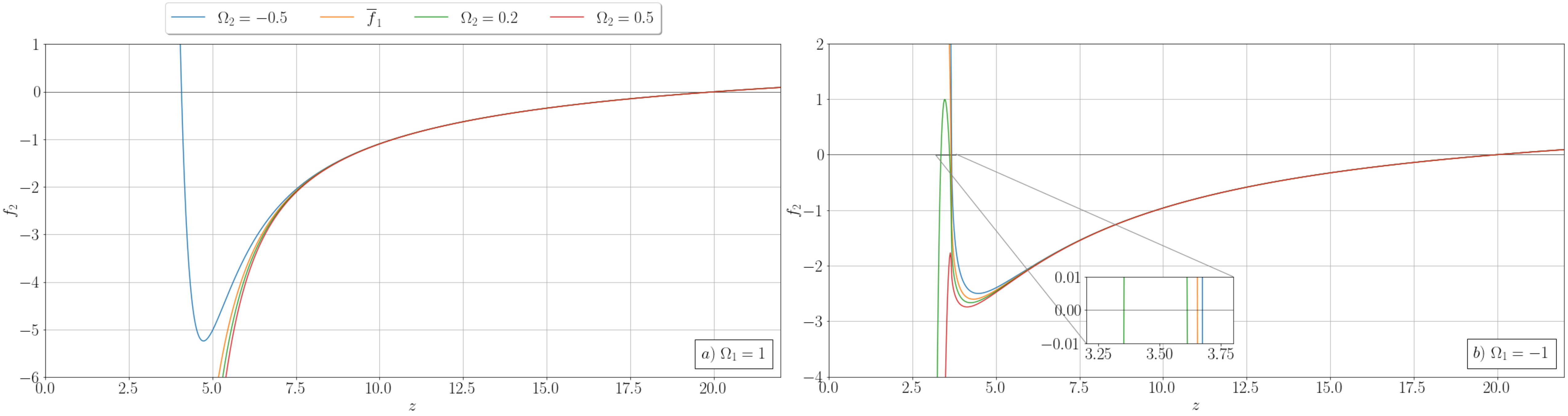
BACKUP



$\Omega_1 = 1$
$\Omega_1 = -1$
$\Omega_1 = 0$ (classical)
$\chi = 5$
$\chi = 10$
$d_1(z)$
d_1/d_0
z
a)
b)
c)
d)
e)
metric with $\chi = 10$
$\bar{f}_1(z)$

BACKUP The robustness of the results is confirmed by the second order expansion

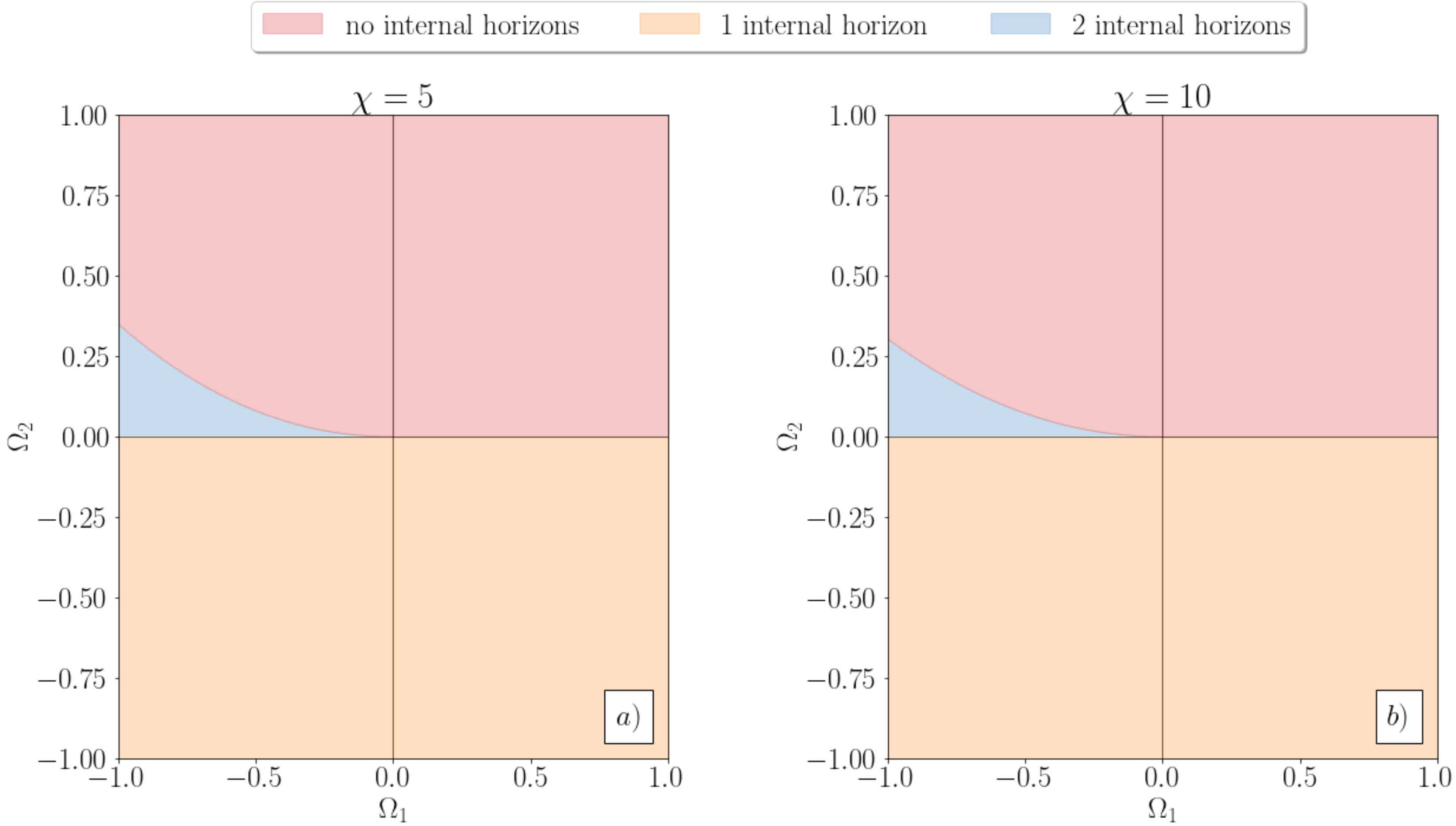
$$f_2(r) = 1 - \frac{2M}{r}\left(1 + \frac{\Omega_1}{d_1(r)^2} + \frac{\Omega_2}{d_1(r)^4}\right)$$



BACKUP

The Taylor expansion of the external event horizon when $\Omega_1 > 0$

$$r_{H+} = 2M\left[1 + \frac{\Omega_1}{\pi^2 M^2} + \bar{a}_3\left(\frac{|\Omega_1|}{\pi^2 M^2}\right)^{3/2} + \bar{a}_4\left(\frac{\Omega_1}{\pi^2 M^2}\right)^2 + \frac{\Omega_2}{\pi^4 M^4} + \mathcal{O}\left(\frac{|\Omega_1|^{5/2},\, |\Omega_1|^{1/2}\Omega_2}{\pi^5 M^5}\right)\right]$$



BACKUP

Proceed iteratively including higher order terms in the series expansion, as shown in the previous scheme

1. Compute the proper length at order N

$$d_N(r) = \int_0^r \frac{\mathrm{d}r'}{\sqrt{|f_N(r')|}} = \int_0^r \frac{\mathrm{d}r'}{\sqrt{\left|1 - \frac{2M}{r'}\left(1 + \frac{\Omega_1}{d_{N-1}(r')^2} + \frac{\Omega_2}{d_{N-1}(r')^4} + \cdots + \frac{\Omega_N}{d_{N-1}(r')^{2N}}\right)\right|}}$$

2. Define the back-reacted metric function

$$\bar{f}_N(r) = 1 - \frac{2M}{r}\left(1 + \frac{\Omega_1}{d_N(z)^2} + \frac{\Omega_2}{d_N(r)^4} + \cdots + \frac{\Omega_N}{d_N(r)^{2N}}\right)$$

BACKUP

The classical proper distance obeys the following behaviors

$$\lim_{r\to 0} d_0(r) \sim r^{3/2} \qquad \lim_{r\to\infty} d_0(z) \sim r$$

1. Universal behavior at asymptotic distances from BH

$$\lim_{r\to\infty} d_N(r) = \lim_{r\to\infty} d_0(r) \qquad \forall N \geq 1$$

ensures that the coefficients $\Omega_{i\leq N}$ can be defined in a consistent fashion, independent of the order N

2. Close to the origin the dominant term is the highest term in the sum

$$\lim_{r\to 0} \frac{d_N(z)}{d_0(z)} \sim \lim_{r\to 0} \frac{d_{N-1}(r)^N}{\sqrt{|\Omega_N|}} \xrightarrow{\text{iteratively}} \lim_{r\to 0} d_N(z) \sim \lim_{r\to 0} \frac{(d_0(r))^{e\Gamma(N+1,1)}}{\sqrt{\prod_{i=0}^N |\Omega_i|^{N!/i!}}}$$

if $\Omega_{1,2,\cdots,N-1,N} \neq 0$

BACKUP

for the case $N = 2$

$$f_2(r) = 1 - \frac{2M}{r}\left(1 + \frac{\Omega_1}{d_1(r)^2} + \frac{\Omega_2}{d_1(z)^4}\right)$$

$$\lim_{r\to 0} d_1(r) = \lim_{r\to 0} \frac{d_0(r)^2}{\sqrt{|\Omega_1|}} \qquad \lim_{r\to 0} d_2(r) = \lim_{r\to 0} \frac{d_0(r)^5}{|\Omega_1|\sqrt{|\Omega_2|}}$$

We can further investigate the relation

$$\frac{\mathrm{d}d_1}{\mathrm{d}z} = \frac{\mathrm{d}d_1}{\mathrm{d}d_0}\frac{\mathrm{d}d_0}{\mathrm{d}r} = \frac{1}{\sqrt{\left|1 - \frac{2M}{r}\left(1 + \frac{\Omega_1}{d_0^2}\right)\right|}} \longrightarrow \frac{\mathrm{d}d_1}{\mathrm{d}d_0} = \sqrt{\frac{\left|1 - \frac{2M}{r(d_0)}\right|}{\left|1 - \frac{2M}{r(d_0)}\left(1 + \frac{\Omega_1}{d_0^2}\right)\right|}}$$

$$d_0 \gg M \implies \frac{\mathrm{d}d_1}{\mathrm{d}d_0} \approx 1 \implies d_1 \approx d_0 \sim r$$

BACKUP

$$d_0 \ll M \implies \frac{\mathrm{d}d_1}{\mathrm{d}d_0} = \sqrt{\frac{1}{\left|1 + \frac{\Omega_1}{d_0^2}\right|}}$$

$$d_1 = \begin{cases} d_0\sqrt{1+\frac{\Omega_1}{d_0^2}} - \sqrt{\Omega_1} & \text{if} \quad \Omega_1 > 0\,, \\ \sqrt{-\Omega_1} - d_0\sqrt{-\frac{\Omega_1}{d_0^2} - 1} & \text{if} \quad \Omega_1 < 0 \text{ and } d_0 < \sqrt{-\Omega_1}\,, \\ \sqrt{-\Omega_1} + d_0\sqrt{1+\frac{\Omega_1}{d_0^2}} & \text{if} \quad \Omega_1 < 0 \text{ and } d_0 > \sqrt{-\Omega_1}\,. \end{cases} \xrightarrow{\text{small } d_0} d_1 \sim \frac{d_0^2}{2\sqrt{|\Omega_1|}} + \mathcal{O}(d_0^4)$$

BACKUP

for the case $N = 2$

$$\frac{\mathrm{d}d_2}{\mathrm{d}d_1} = \sqrt{\frac{\left|1 - \frac{2M}{r(d_1)}\left(1 + \frac{\Omega_1}{d_0(d_1)^2}\right)\right|}{\left|1 - \frac{2M}{r(d_1)}\left(1 + \frac{\Omega_1}{d_1^2} + \frac{\Omega_2}{d_1^4}\right)\right|}}$$

$$r \gg M \implies \frac{\mathrm{d}d_2}{\mathrm{d}d_1} \approx 1 \implies d_2 \approx d_1 \sim r$$

expand around $d_1 = 0$

$$\frac{\mathrm{d}d_2}{\mathrm{d}d_1} = \sqrt{\frac{1 + \frac{\Omega_1}{d_1(d_1 + 2\sqrt{\Omega_1})}}{1 + \frac{\Omega_1}{d_1^2} + \frac{\Omega_2}{d_1^4}}}$$

which yields

$$d_2 \sim \frac{\sqrt{2}}{5}\frac{\Omega_1^{1/4}}{\sqrt{\Omega_2}}\, d_1^{5/2} + \mathcal{O}(d_1^{7/2}) \sim \frac{d_0^5}{20\,\Omega_1\sqrt{\Omega_2}} + \mathcal{O}(d_0^7)$$