Formation of trapped vacuum bubbles during inflation, and consequences for PBH scenarios

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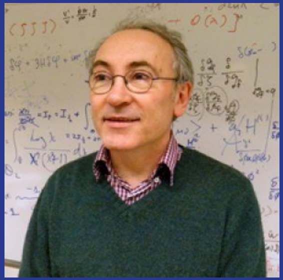


Based on: A. Escrivà, V. Atal and J. Garriga. ArXiv:2306.09990 (**today in Arxiv!**)

About PBHs…

- The most standard mechanism for PBH production is from the **collapse of sufficiently large (very rare event) adiabatic fluctuations generated during inflation**, which reenter the cosmological horizon during the radiation epoch.

- But indeed there are several mechanism for the production of PBHs: collapse of adiabatic fluctuations, isocurvature fluctuations, strings, domains walls, Q-balls , quark confinement, etc

What about baby Universes?

baby Universes: can be formed from false vacuum bubbles generated during inflation

Inside the bubble: an observer see an inflating Universe

Outside the bubble: an observer will see a black hole (PBH)

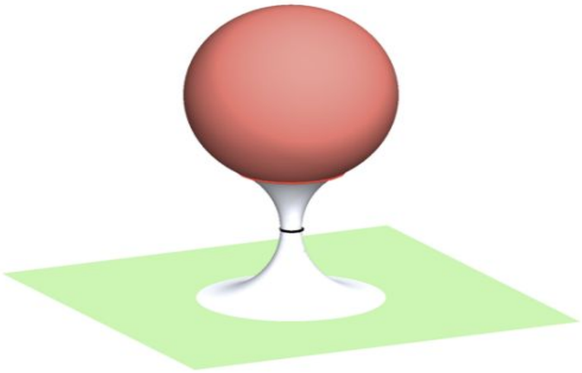
J. Garriga, A. Vilenkin. Arxiv: 1210.7540
J. Garriga, A. Vilenkin, J. Zhang. Arxiv:1512.01819
H. Deng, A. Vilenkin. Arxiv:1710.02865
A. Kusenko, M. Sasaki, S. Sugiyama, M. Takada, V. Takhistov, E. Vitagliano. Arxiv:2001.09160

vacuum bubbles may be produced by quantum tunneling during inflation.

In this scenarios, tunneling is assumed to be a Poissonian process which can happen with nearly constant probability per unit time and volume

↓

PBH mass function is rather broad.

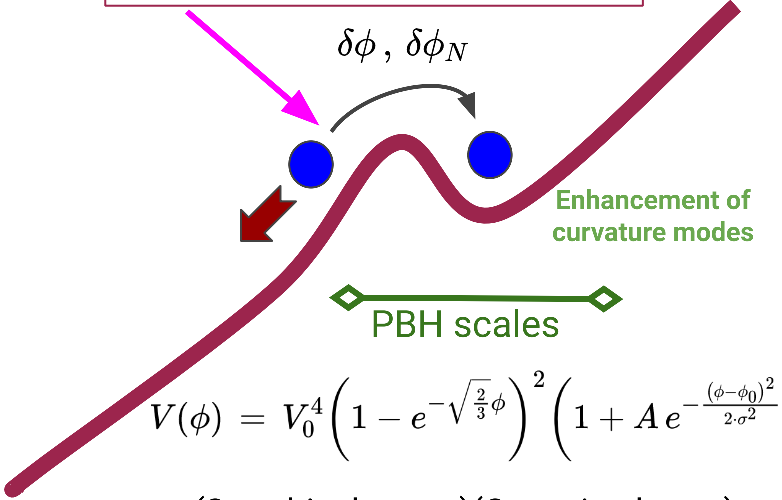


Another scenario

False vacuum bubbles can be formed from large backward fluctuations of the inflaton in single-field inflationary models containing a bump.

V. Atal, J. Garriga and A. M. Caballero. Arxiv:1905.13202
V. Atal, J. Cid, A. Escriva, J. Garriga. Arxiv:1908.11357

Include a bump in the inflationary potential



$$V(\phi) = V_0^4\left(1-e^{-\sqrt{\frac{2}{3}}\phi}\right)^2\left(1+A\,e^{-\frac{(\phi-\phi_0)^2}{2\cdot\sigma^2}}\right)$$

(Starobinsky type)(Gaussian bump)

The inflaton exit inflation, but sufficiently large backward fluctuations of the inflaton can prevent it from overshooting the barrier in horizon sized region

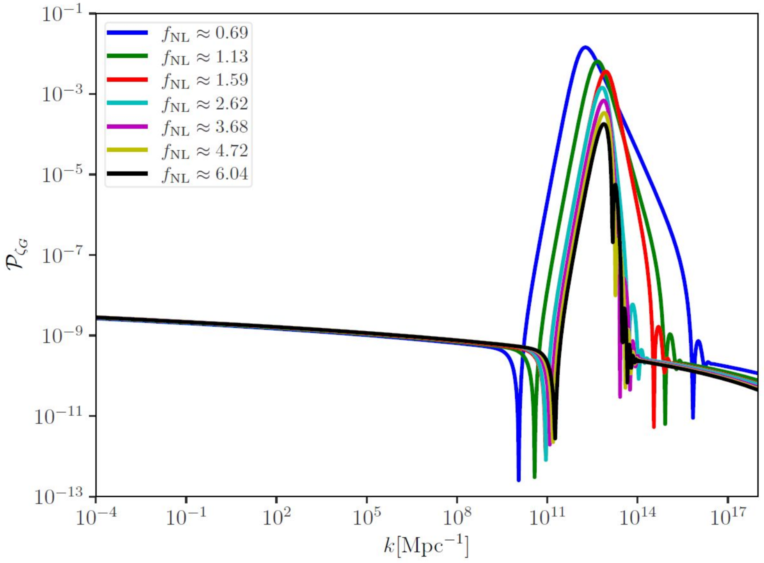
Formation of localized false vacuum bubbles!

Two coexistent channels for the production of PBHs

- From large adiabatic fluctuations
- From false vacuum bubbles → Never explored numerically!

Adiabatic curvature fluctuation

Solving the MS equation numerically, we obtain the power spectrum of the Gaussian curvature fluctuation.



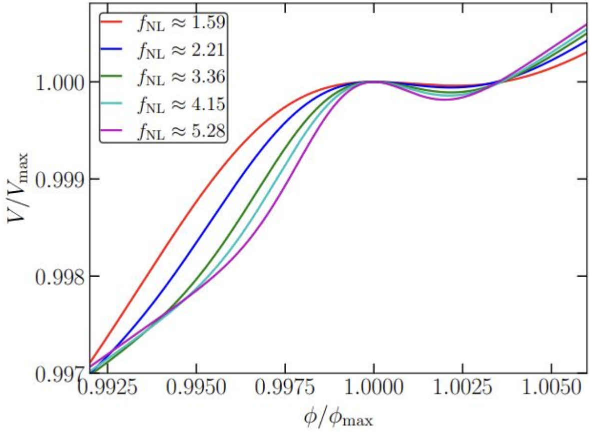
$$\zeta_G(r) = \mu_a \, \Psi_{\zeta_G}(r) = \mu_a \frac{1}{\sigma^2_{\zeta_G}} \int_{k_i}^{k_f} \mathcal{P}_{\zeta_G} \frac{\sin(kr)}{kr} d\ln k$$

(considering very large peaks)

The small bump induces a non-gaussian contribution->invalidates Gaussian assumption

$$\zeta \neq \zeta_G$$

$$f_{NL} = \frac{5}{12}\left[-3 + \sqrt{9 - 12\frac{V''(\phi_{\max})}{V(\phi_{max})}}\right]$$



Full-non gaussian curvature fluctuation

V. Atal, J. Garriga and A. Marcos-Caballero. Arxiv:1905.13202

$$\zeta \neq \zeta_G$$

From delta N formalism

$$\zeta = -\mu_\star \ln\left(1 - \frac{\zeta_G}{\mu_\star}\right)$$

$$\zeta_G = \mu(1 - e^{-\zeta/\mu_\star})$$

$$\mu_\star = \frac{5}{6\, f_{NL}}$$

$$\zeta \to \infty \Rightarrow \zeta_G \to \mu_\star$$

$P_G[\zeta_G]$ normalized

$P[\zeta]$ No normalized!

$$\int P[\zeta]\, D\zeta = \int_{\zeta_G < \mu_\star} P_G[\zeta_G]\, D\zeta_G < 1$$

→ The singularity indicates the presence of alternative channels for PBH production, which restores unitarity.

PDF for the NG curvature fluctuation

$$P[\zeta] = P_G[\zeta_G(\zeta)]\, \frac{d\zeta_G}{d\zeta}$$

"Exponential tail"

$$\frac{d\,\zeta_G}{d\,\zeta} = e^{-\zeta/\mu_\star} \quad P[\zeta] \propto e^{-\zeta/\mu_\star}, \quad (\zeta \to \infty)$$

$\zeta_G = \mu_\star$ Singular point

We need the threshold and mass to make estimation of PBH abundances…

We can follow two procedures to obtain the threshold:

- Make the full numerical simulation (very difficult for some cases of large NGs…) A.Escrivà. ArXiv:1907.13065
- Use the analytical estimate based on the compaction function

The compaction function has been shown to be useful in the context of PBH formation

M. Sasaki, M. Shibata. Arxiv:gr-qc/9905064
T. Harada, C.M. Yoo, T. Nakama, Y. Koga. Arxiv:1503.03934

$$\mathcal{C}(r) = 2\frac{M - M_b}{R}$$

twice the local excess-mass over the co-moving areal radius

(at super-horizon scales)

$$\mathcal{C}(r) = \frac{2}{3}\left[1 - \left(1 + r\,\zeta'(r)\right)^2\right]$$

We need the threshold and mass to make estimation of PBH abundances…

To find the threshold: Find iteratively the value of the amplitude such that satisfy the relation:

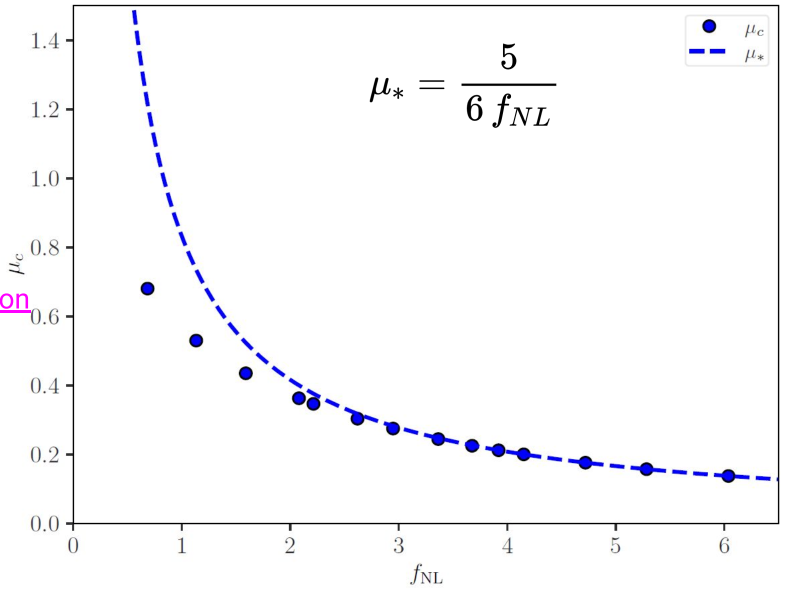
A.Escrivà, C.Germani, R. K.Sheth. Arxiv:1907.13311

$$\mathcal{C} = \frac{2}{3}\left[1 - \left(1 + r\,\zeta'\right)^2\right]$$

$$\bar{\mathcal{C}}_c = 2/5$$

Universal value! Independent on the shape of the fluctuation within 2%

$$\bar{\mathcal{C}}_c = \frac{3}{r_m^3(\mu_{a,c})e^{3\zeta(r_m(\mu_{a,c}))}} \int_0^{r_m(\mu_{a,c})} \mathcal{C}_c(r)(1 + r\zeta')e^{3\zeta(r)} r^2 dr.$$



What about the PBH mass near the critical regime?

N. Kitajima, Y. Tada, S.Yokoyama, C.M- Yoo. Arxiv:2109.00791

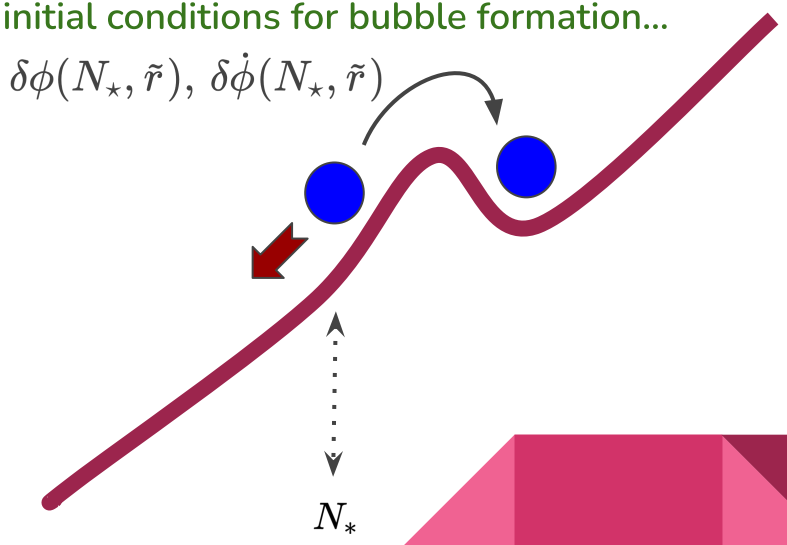
$$M_{\rm PBH}(\mu_a) = \mathcal{K}_a(\mu_{a,c}) M_k(k) x_m^2(\mu_a) e^{2\zeta(r_m(\mu_a))}(\mu_a - \mu_{a,c})^{\gamma_a}$$

Let's move to the bubble channel->Numerical formation of bubbles

We need to solve the KG field equation taking into account a radial dependence

$$\ddot{\phi} + \dot{\phi}\left(3 - \frac{1}{2}\dot{\phi}_b^2\right) - \left(\frac{a_I H_I}{a(N)H(N)}\right)^2 \Delta\phi + \frac{1}{H^2}\frac{V_\phi(\phi)}{V(\phi)} = 0$$

But we have a problem!: we need to find the correct initial conditions for bubble formation...



In general, we can consider:

$$\phi(N_\star, \tilde{r}) = \phi_b(N_\star) + \delta\phi(N_\star, \tilde{r})$$
$$\dot{\phi}(N_\star, \tilde{r}) = \dot{\phi}_b(N_\star) + \delta\dot{\phi}(N_\star, \tilde{r})$$

Initial conditions for bubble formation

Let's consider first the perturbation for the field space

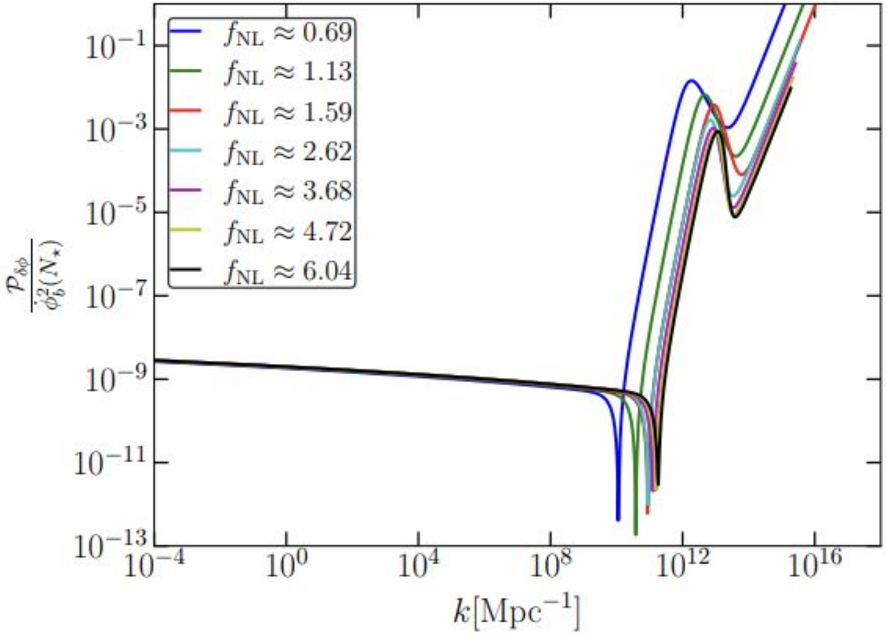
$$\mathcal{P}_{\delta\phi}(N_\star, k) = \frac{k^3}{2\pi^2}\dot{\phi}_b^2(N_\star) \mid \zeta_G(N_\star, k) \mid^2$$

we evaluate the " k " −modes at N_*

$$\Psi_b(N_\star, \tilde{r}) = \frac{1}{\sigma_b^2}\int_{k_i}^{k_f} \mathcal{P}_{\delta\phi}(N_\star, k)\,\mathrm{sinc}(k\tilde{r})\,d\ln k$$

$$\delta\phi(N_\star, \tilde{r}) = \mu_b \Psi_b(N_\star, \tilde{r})$$

(like in the adiabatic channel)



Initial conditions for bubble formation

What about the perturbation for the velocity?

During the attractor regime:
clear relation between position and momentum (which also holds for the perturbations)

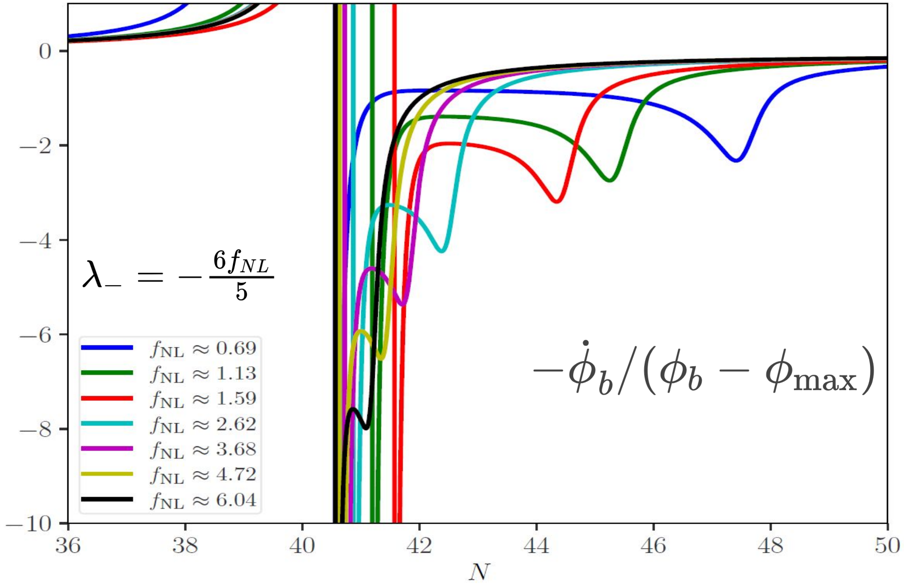
$$\dot{\phi}_b \approx -\lambda_-(\phi_b - \phi_{\max}) \quad \Longrightarrow \quad \delta\dot{\phi} \approx -\lambda_- \delta\phi$$

↓

Atractor regime

How do we choose?

N_* → When the numerics is closer to the attractor regime

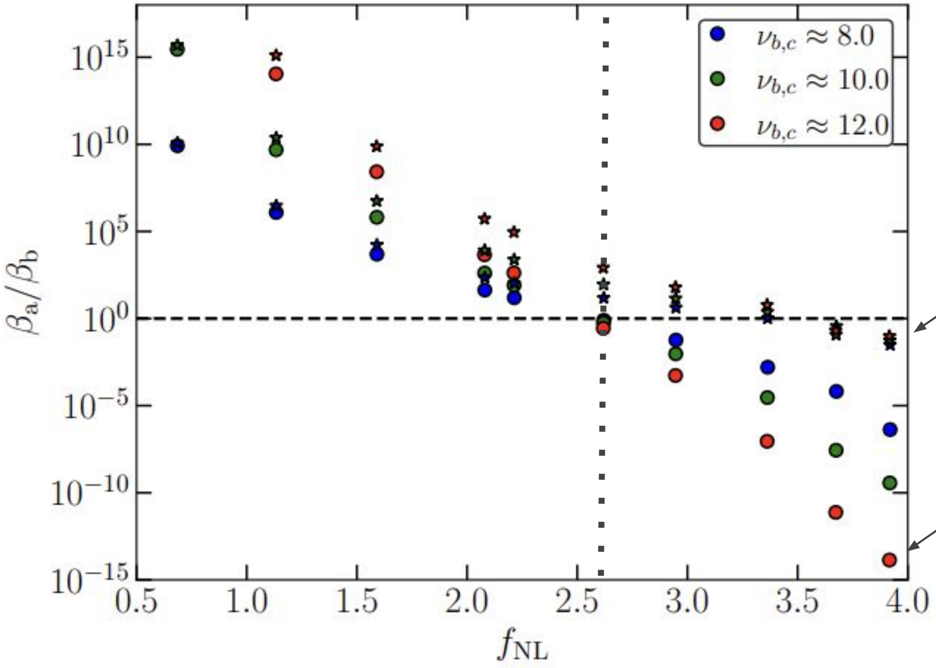


Ratio of PBH production between the two channels

The prov. distribution of bubbles is Gaussian

Peak theory

C.M. Yoo, T. Harada, J. Garriga, K. Kohri. ArXiv:1805.03946



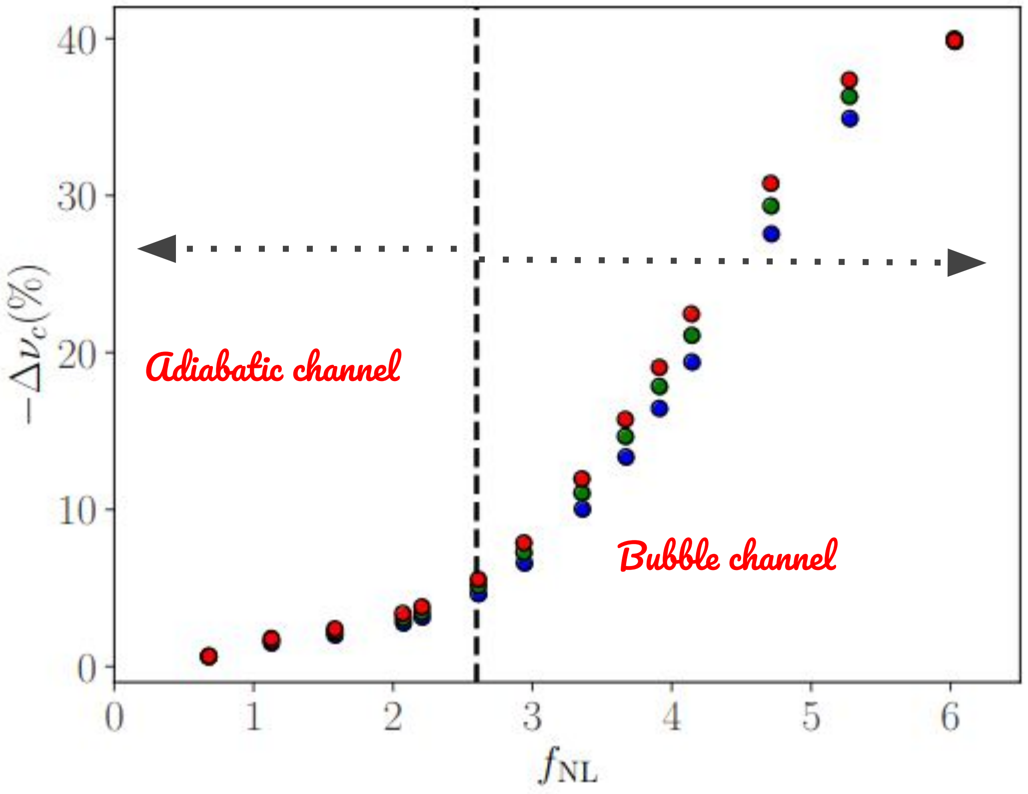
$f_{NL} \approx 2.6$ → Both channels give same abundance of peaks

Using the analytical estimate $\nu_\star$

Using the numerical results

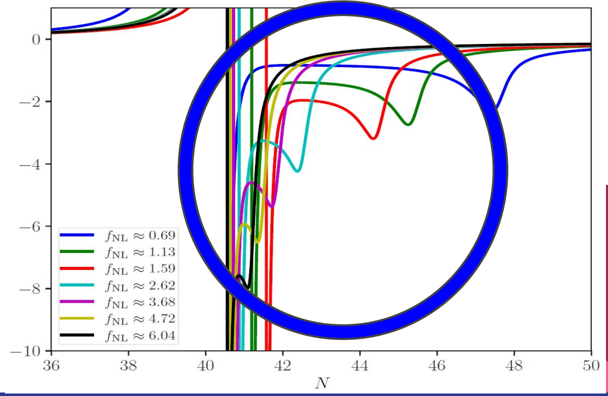
Vacuum bubbles are more easily formed than what we previously found!

Comparison between analytical and numerical

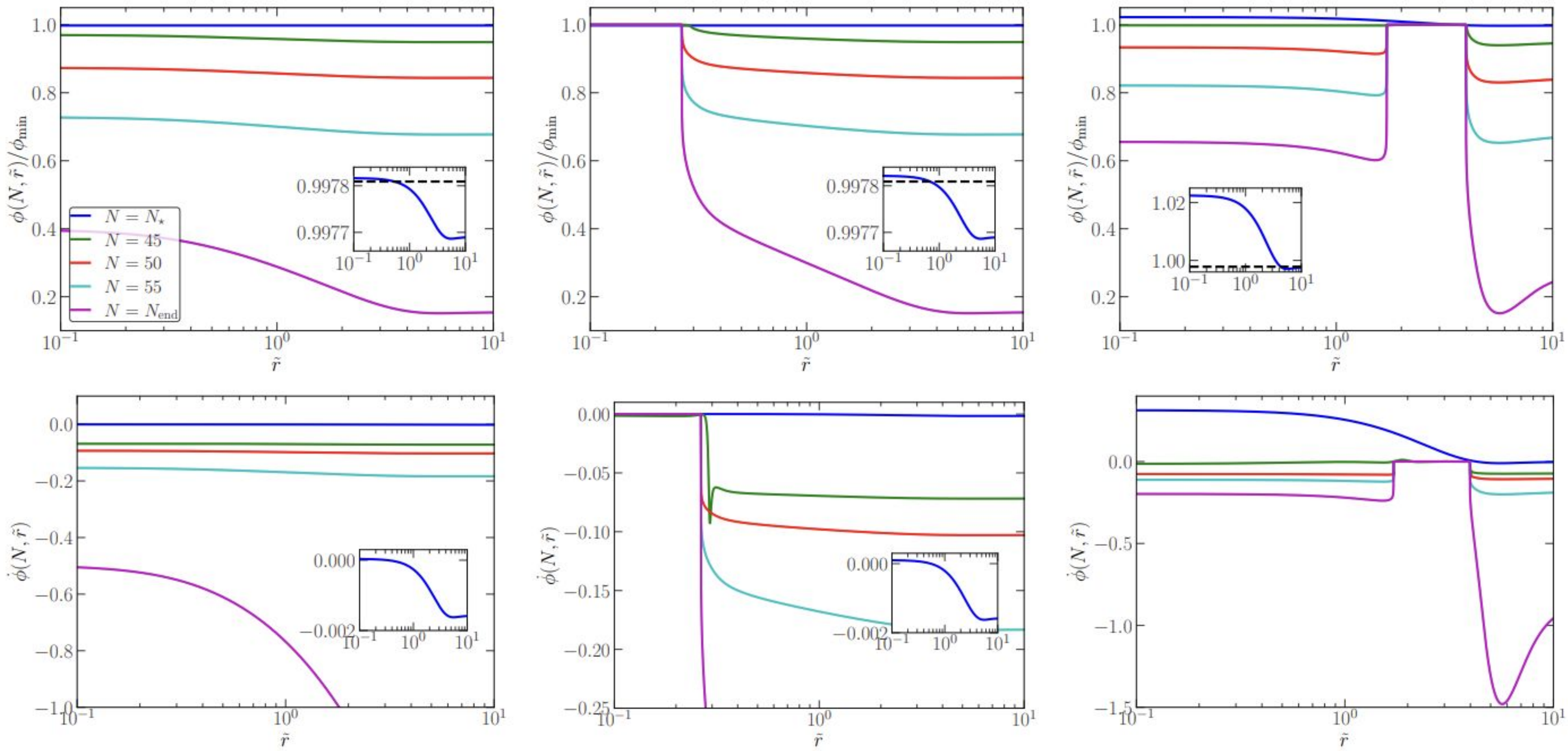


$$\Delta\nu_c(\%) = 100\frac{\nu_{b,c} - \nu_\star}{\nu_{b,c}}$$

The analytical estimate deviates for large non-gaussianity (expected, since we don't have attractor regime)



Dynamics of bubble formation

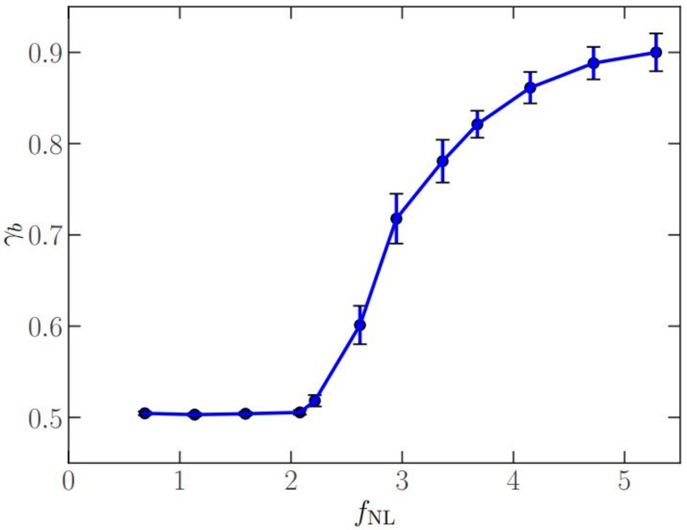


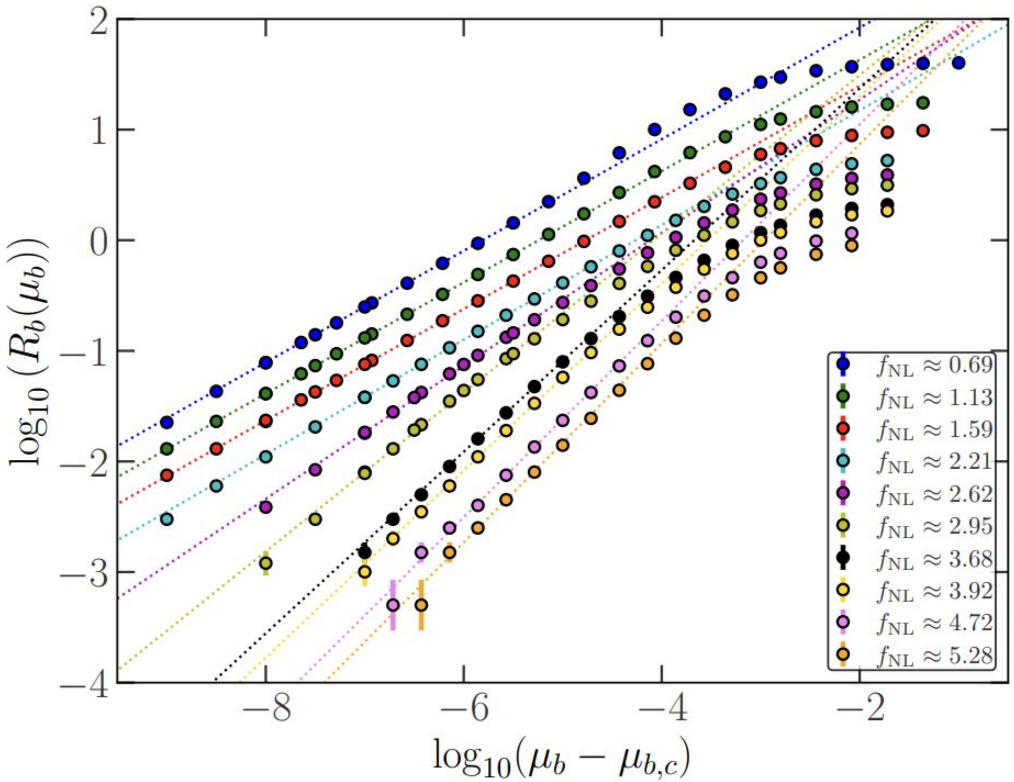
Let’s study now the bubble size

We find a critical regime for the bubble size!

Comoving size of the bubbles

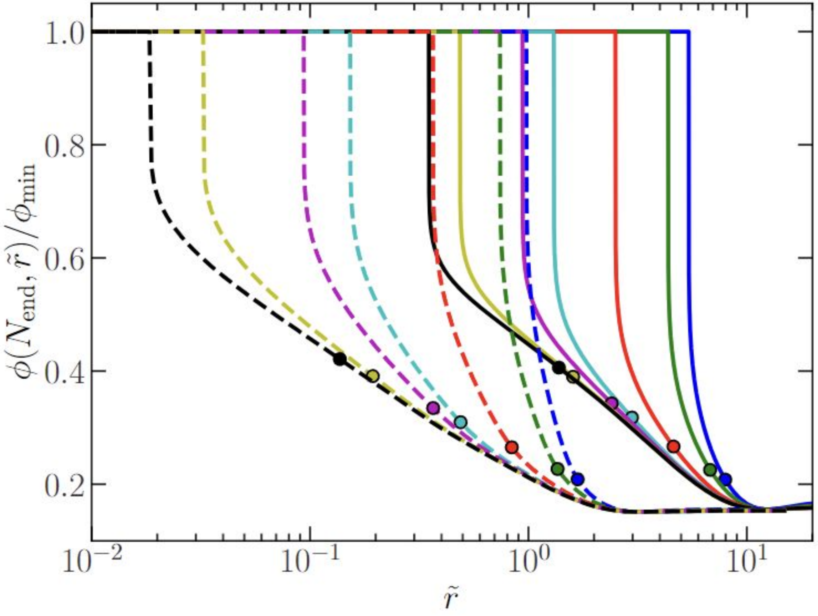
$$R_b(\mu_b) = \mathcal{K}_b(\mu_{b,c})(\mu_b - \mu_{b,c})^{\gamma_b(f_{\rm NL})}$$

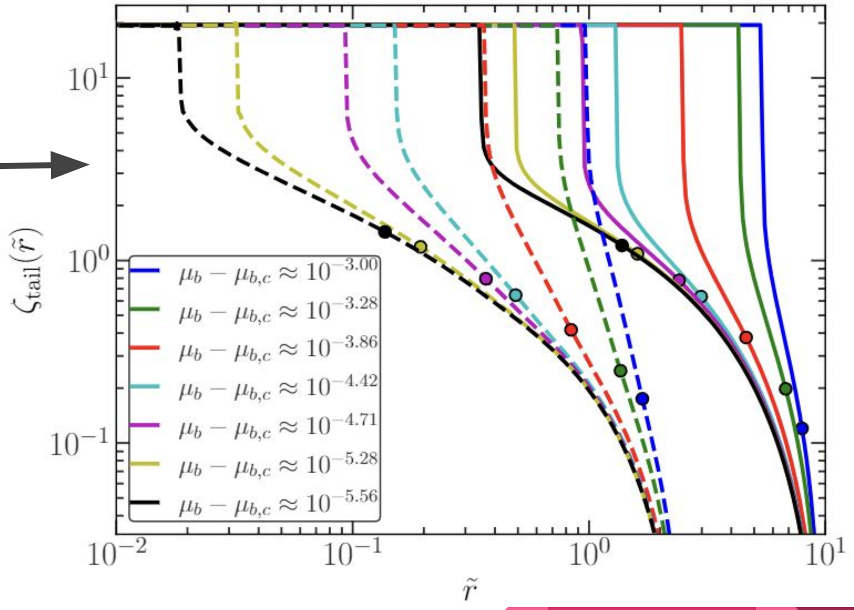




Inflaton at the end of inflation

Corresponding curvature fluctuation with delta N formalism





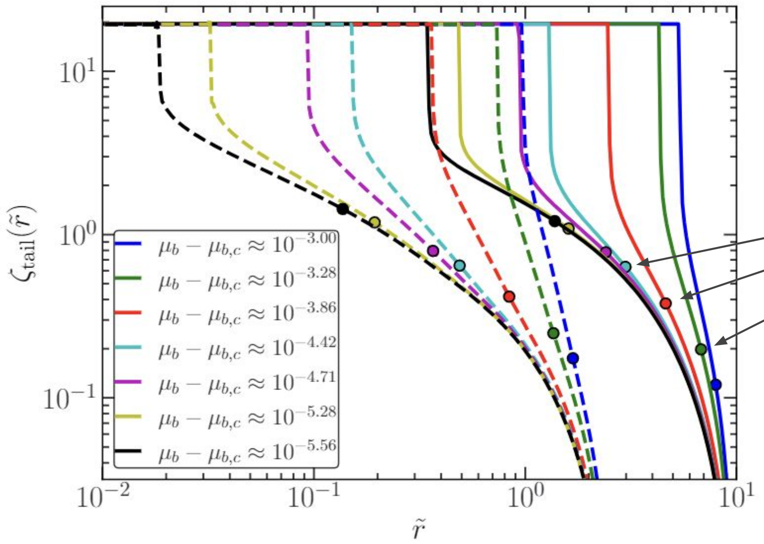
Surprisingly: surrounding fluctuation of type II!

Fluctuation of type II->Areal radius non-monotonic increasing function

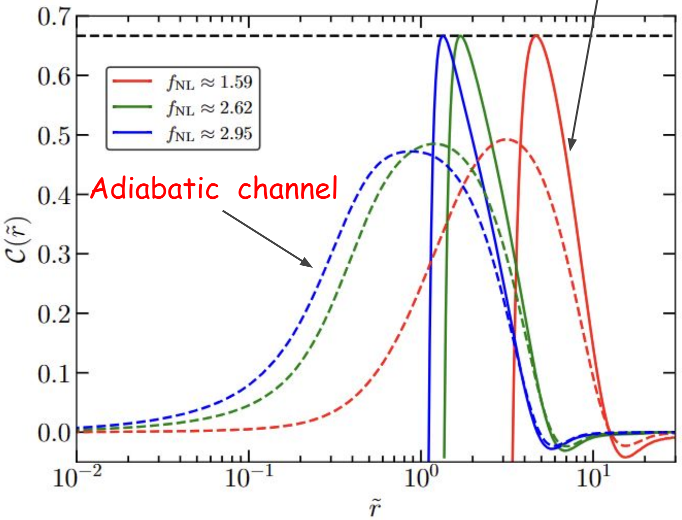
$$R = a\,r\,e^{\zeta_{tail}} \;\Rightarrow R' = a e^{\zeta_{tail}}\left(1 + r\,\zeta'_{tail}\right)$$

$$R' < 0 \;\Rightarrow 1 + r_{tail}\,\zeta'_{tail}(r_{tail}) = 0$$

$$\mathcal{C}(r_{tail}) = 2/3$$



Bubble channel



Adiabatic channel

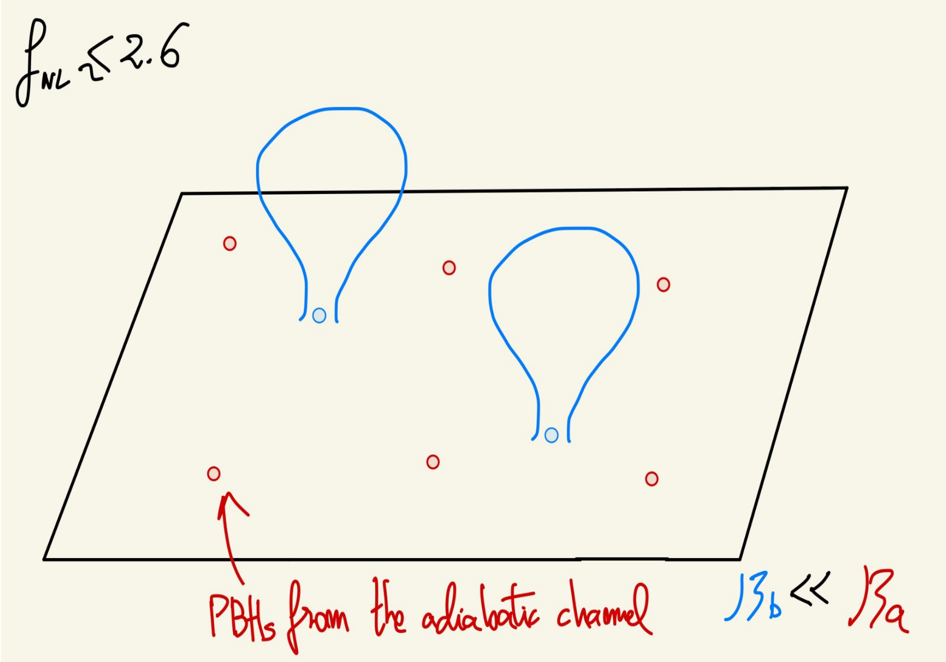
Fluctuations of type II corresponds to very large amplitudes far away from the threshold value->largely statistically suppressed in the adiabatic channel

But in the bubble channel, actually gives the dominant contribution!

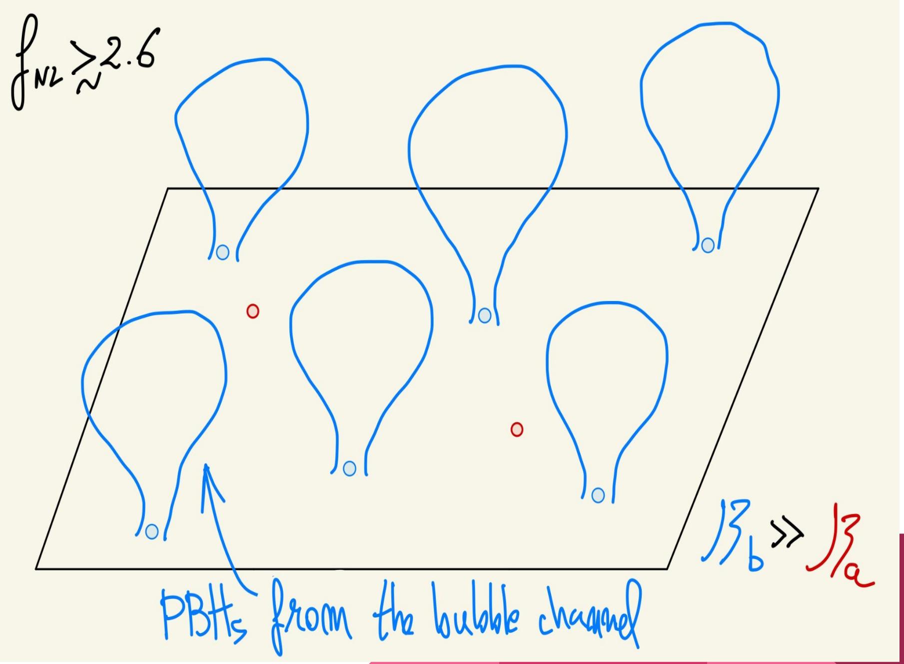
$$M_{bubble\ channel} \sim M_k(k_{tail})$$

Qualitative picture

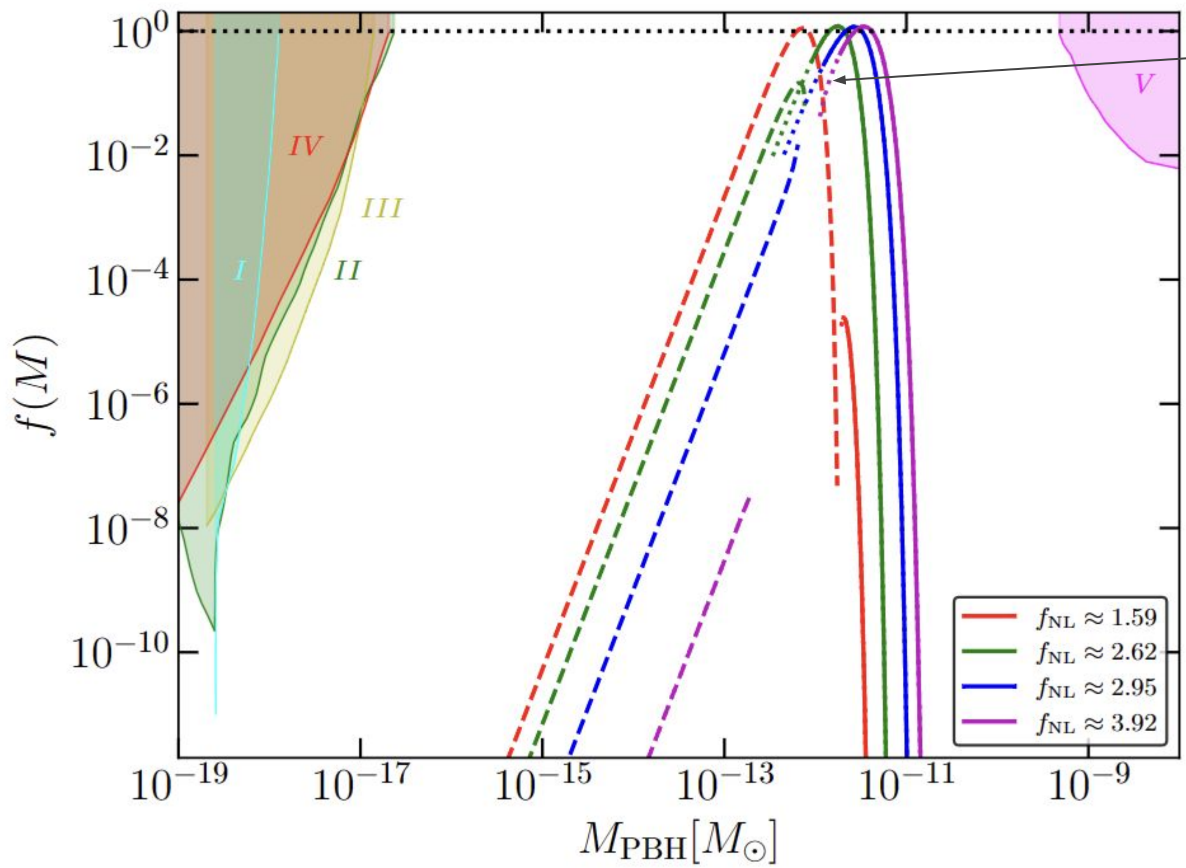
Fluctuations type I-> dominant contribution



Fluctuations type II-> dominant contribution



Mass function from both channels



$$\mu_{b,(\text{cut-off})} - \mu_{b,c} \approx \sqrt{-\ln(R_b(\mu_b))}\,\frac{H(N_\star)}{2\pi}$$

Cut-off for very small bubbles->quantum drift

Thanks for your attention!

Conclusions and messages to take home:

- The dynamics of vacuum bubble formation has been studied and clarified. We find a critical regime for the size of the bubbles.
- The log-relation for the full NG curvature fluctuation is successfully accurate to predict the bubble channel of PBH production for small non-gaussianity (atractor regime condition).
- Bubbles are more easily formed than previously expected. The bubble channel is dominant for fnl>2.6
- The mass of PBHs from the bubble channel is dominated by a surrounding fluctuation of type-II.
- The presence of alternative channels for PBH production in models with local type non-Gaussianity can be inferred from unitarity considerations.